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# Experimentally-efficient data-driven rational feedforward control for multivariable systems

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**ABSTRACT**
Feedforward control with task flexibility for MIMO systems is essential to meet the growing demands on throughput and accuracy of high-tech systems. The aim of this paper is to develop an experimentally efficient framework for data-driven tuning of rational feedforward controllers for general non-commutative MIMO systems. In the developed approach, the nonlinear terms in the non-convex optimisation problem of iterative learning control (ILC) are iteratively circumvented via approximation. This leads to a series of convex optimisation problems that can be solved offline to obtain the parameters of the rational feedforward controller. In addition, by limiting the number of offline iterations an experimentally intensive algorithm is derived, which could be beneficial in the case of severe model mismatch. A simulation study shows that the experimentally efficient approach converges fast through offline iterations and has improved convergence properties through the use of regularisation.

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Multivariable; iterative learning control; rational basis functions; non-convex optimisation

## 1. Introduction

Feedforward control is essential to meet the growing demands on throughput and accuracy in high-tech systems (Fleming & Leang, 2014). Throughput, e.g. the amount of products created, and accuracy, e.g. the amount of detail in a product, are key drivers in market placement of high-tech systems, such as semiconductor back-end machines (Boeren et al., 2016), lithography machines (Butler, 2011), and printer systems (Bolder et al., 2017). In addition to demands for performance, the industry sets the following requirements for motion control of high-tech systems. First, data-driven tuning for individual machines is required to deal with machine-to-machine variations and changing environments while achieving high accuracy. Second, flexibility in the task is crucial to ensure high throughput and accommodate industrial usage. Third, MIMO control is essential to deal with interaction between motion axes to achieve performance in multivariable systems. Feedforward control is envisaged to satisfy these requirements and achieve the performance objectives set for high-tech systems.

High-accuracy control for systems with trial-invariant motion tasks is enabled by iterative learning control (ILC) (Bristow et al., 2006). In ILC, a feedforward signal is learned iteratively using past trial data and an approximate model of the system. In a specific class of ILC, norm-optimal iterative learning control (NOILC) (Amann et al., 1996), a convex optimisation problem is solved to determine the feedforward signal of the next trial. This approach is considered a time-domain approach and is inherently applicable to multivariable systems, see, e.g. van Zundert, Bolder, Koekebakker, et al. (2016). Multivariable aspects are also taken into account in frequency domain ILC (Blanken & Oomen, 2020). In both domains, a key assumption is made on a trial-invariant task and extrapolation of the feedforward signal to other tasks generally leads to significant performance deterioration (van der Meulen et al., 2008), hampering widespread industrial adoption.

The requirements on task flexibility have spurred the development of new task-flexible ILC approaches. In Hoelzle et al. (2011), the task is divided into sub-tasks that are learned individually, but it restricts the task to only consist of sub-tasks that are in the library. Additionally, feedforward parametrisations using basis functions are introduced in van der Meulen et al. (2008), van de Wijdeven and Bosgra (2010) and Bolder and Oomen (2015), where the feedforward signal is parametrised in terms of the task, enabling the extrapolation capabilities of ILC. In van de Wijdeven and Bosgra (2010), polynomial basis functions are employed, which can be interpreted as using a finite impulse response (FIR) filter as a feedforward parametrisation that represents the inverse system, see van der Meulen et al. (2008). The parameters of the polynomial basis functions can easily be learned via the NOILC framework, as it retains its convex optimisation problem for the feedforward parameters. However, parametrisation via polynomial basis functions implies that the system has a unit numerator, which is not the case in physical systems modelled by rational models, which contain both poles and zeros. This leads to under-modelling and poor performance with respect to accuracy and extrapolation properties.

To improve the accuracy and extrapolation properties of ILC, rational feedforward is developed for ILC. In Boeren et al. (2014), an input shaping approach is developed in which

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the denominator and the numerator of the system are modelled, but this methodology focuses only on achieving settling performance. In Bolder and Oomen (2015), rational basis functions (RBFs) are introduced for ILC, enabling high tracking accuracy and task flexibility for SISO systems. Here, the analytic solution is lost as the optimisation problem becomes non-convex. In Bolder and Oomen (2015), an iterative solution based on Sanathanan-Koerner (SK) iterations, as introduced in Sanathanan and Koerner (1963), is presented to solve the optimisation problem through a series of weighted least-squares problems. The non-optimality of this SK-based approach is explained in van Zundert, Bolder, Oomen (2016) and has also been observed in related system identification algorithms (Steiglitz & McBride, 1965; Stoica & Söderström, 1981; Whitfield, 1987). In van Zundert, Bolder, Oomen (2016), it is shown that a locally optimal solution can be achieved by iteratively solving the non-convex optimisation problem using a gradient-based weighting, which is closely related to instrumental variable-based system identification, see, e.g. Söderström and Stoica (2002).

Since rational basis functions enhance the extrapolation properties of ILC algorithms in SISO systems, it is desired to use them to achieve high tracking accuracy and task flexibility for MIMO systems as well. However, an extension towards MIMO is challenging, since the derivation of the SK-based algorithm for optimising the parameters makes use of the fact that convolution matrices for SISO systems always commute, i.e. the commutative property of SISO systems. Since MIMO systems in general do not share this property, the same derivation for MIMO systems does not give a viable algorithm for MIMO systems in general. The same derivation *without* commuting the matrices also does not lead to a viable algorithm. The aim of this paper is to develop a general framework for data-driven tuning of rational feedforward controllers in ILC for non-commutative MIMO systems that is experimentally efficient and can deal with model mismatch.

The main contribution in this paper is a general framework for RBF in ILC for multivariable systems. More specifically, rational basis functions are optimised in ILC for non-commutative MIMO systems through an iterative offline optimisation scheme for experimentally efficient learning. Additionally, this framework introduces regularisation for MIMO RBFs that improves convergence properties and helps to deal with model mismatch. The sub-contributions are as follows.

(I) A general framework for data-driven tuning of MIMO RBFs in ILC for non-commutative MIMO systems.
(II) An approach that can deal with significant model mismatch through more experiments.
(III) An approach for fast and experimentally efficient learning using offline iterations.
(IV) A simulation study showing the efficacy of the developed approaches and the influence of regularisation and model mismatch on the convergence properties.
(V) An explanation of the differences with the SISO algorithm introduced by Bolder and Oomen (2015).

Preliminary results related to sub-contribution II appeared in Poot et al. (2023). This paper provides a general framework with contributions I, III–V, that includes an experimentally efficient approach with regularisation, enabling fast convergence despite model mismatch that is essential for rapid commissioning and industry adoption.

The paper is structured as follows. In the following section, the notation used in this paper is presented. In Section 2, the problem formulation is stated. The developed approaches are presented in Section 3. Then, in Section 4, a simulation study is presented. Section 5 contains conclusions and future work.

**Notation:** It is assumed that the systems are linear and time-invariant (LTI), discrete-time, and have $n_i$ inputs and $n_o$ outputs. Transfer functions that approximate these systems are usually rational in the complex variable $z$. Input signals are often assumed tacitly of length $n_iN \in \mathbb{Z}_+$ and output signals of lengths $n_oN \in \mathbb{Z}_+$. The $i$th element of a signal or vector $x$ is represented by $x[i]$. Let $h_k \in \mathbb{R}^{n_o \times n_i}$ with $k \in \mathbb{Z}_+$ be the Markov parameter at time step $k$ of the MIMO system $\underline{H}(z)$ with state-space matrices $A_d, B_d, C_d, D_d$. Given an input $u \in \mathbb{R}^{n_iN}$ and zero initial conditions, the output $y \in \mathbb{R}^{n_oN}$ is

$$y[k] = h_k \otimes u[k] = \sum_{l=0}^{\infty} h_l u[k-l],$$
$$h_k = \begin{cases} D_d, & k = 0, \\ C_d A_d^{k-1} B_d, & k \geq 1. \end{cases} \tag{1}$$

Assuming $u[k] = 0$ for $k < 0$ and $k > N-1$,

$$\underbrace{\begin{bmatrix} y^1[0] \\ \vdots \\ y^{n_o}[0] \\ y^1[1] \\ \vdots \\ y^{n_o}[N-1] \end{bmatrix}}_{y} = \underbrace{\begin{bmatrix} h_0 & 0 & \cdots & 0 \\ h_1 & h_0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ h_{N-1} & h_{N-2} & \cdots & h_0 \end{bmatrix}}_{H} \underbrace{\begin{bmatrix} u^1[0] \\ \vdots \\ u^{n_i}[0] \\ u^1[1] \\ \vdots \\ u^{n_i}[N-1] \end{bmatrix}}_{u}, \tag{2}$$

where $u^q[k], y^p[k]$ denote the input and output for input and output direction number $q$, $p$, respectively, where $q = 1, \dots, n_i$ and $p = 1, \dots, n_o$, and $H \in \mathbb{R}^{n_oN \times n_iN}$ is a MIMO block-Toeplitz matrix, which represents the convolution matrix of the system. The weighted 2-norm of a vector $x$ is expressed as $\| x \|_W := \sqrt{x^\top W x}$, where $W$ is a weighting matrix. A positive-definite matrix $W$, i.e. $x^\top W x > 0$ for all $x \neq 0$, is denoted by $W \succ 0$. On the other hand, a positive semi-definite matrix $W$, i.e. $x^\top W x \geq 0$ for all $x \neq 0$, is denoted by $W \succeq 0$.

## 2. Problem formulation

### 2.1 Problem setup

Consider the closed-loop control scheme in Figure 1 consisting of a feedback controller $C \in \mathbb{R}^{n_iN \times n_oN}$ and system $P \in \mathbb{R}^{n_oN \times n_iN}$ with $n_i$ inputs and $n_o$ outputs. Both are considered linear time-invariant (LTI) and causal. An experiment is denoted by index $j$ and has length $N \in \mathbb{N}$. The output $y_j \in \mathbb{R}^{n_oN}$ is measured for a given reference $r_j \in \mathbb{R}^{n_oN}$ and a feedforward $f_j \in \mathbb{R}^{n_iN}$. From Figure 1, the tracking error $e_j \in \mathbb{R}^{n_oN}$ in trial $j$

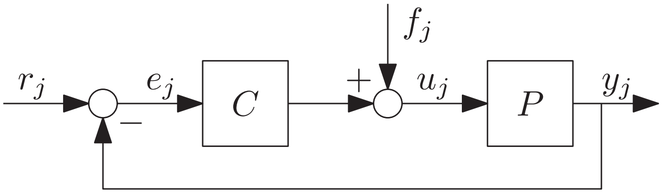

**Figure 1.** Closed-loop control scheme with plant $P$ with $n_i$-inputs and $n_o$-outputs, feedback controller $C$, feedforward $f_j$ and reference $r_j$.

is derived as

$$e_j = Sr_j - SPf_j, \tag{3}$$

where $S = (I + PC)^{-1}$ is the output sensitivity. Next, the feedforward signal $f_j$ is made a function of the reference $r_j$ to achieve task-flexible feedforward.

### 2.2 Rational basis functions

To achieve task-flexible feedforward, the feedforward $f_j$ is parametrised using rational basis functions as follows.

**Definition 2.1 (Rational basis functions for MIMO systems):** The MIMO rational feedforward is defined as

$$f_j := F(\theta_j)r_j \tag{4}$$

where $F : \mathbb{R}^{n_A+n_B} \rightarrow \mathbb{R}^{n_iN\times n_oN}$ is given by

$$F(\theta_j) := A(\theta_j)B(\theta_j)^{-1} \tag{5}$$

with

$$A(\theta_j) := \sum_{i=1}^{n_A} \xi_i^A \theta_j[i], \tag{6a}$$

$$B(\theta_j) := \xi_0^B + \sum_{i=1}^{n_B} \xi_i^B \theta_j[i + n_A]. \tag{6b}$$

Here, $\xi_i^A \in \mathbb{R}^{n_iN\times n_oN}$, $\xi_0^B \in \mathbb{R}^{n_oN\times n_oN}$, and $\xi_i^B \in \mathbb{R}^{n_oN\times n_oN}$ are convolution matrices of the user-defined polynomial matrices $\underline{\xi}_i^A(z) \in \mathbb{R}^{n_i\times n_o}[z]$ for $i = 1, \dots, n_A$, $\underline{\xi}_0^B(z) \in \mathbb{R}^{n_o\times n_o}[z]$, and $\underline{\xi}_i^B(z) \in \mathbb{R}^{n_o\times n_o}[z]$ for $i = 1, \dots, n_B$, respectively.

The following remarks are made on the defined rational feedforward parameterisation of Definition 2.1.

**Remark 2.1:** The transfer function of $F(\theta_j)$ is a polynomial matrix fraction description, see, e.g. Zhu (2001, Section 2.3), and is given by

$$\underline{F}(z, \theta_j) := \underline{A}(z, \theta_j)\underline{B}(z, \theta_j)^{-1}, \tag{7}$$

where $\underline{A}(z, \theta_j) \in \mathbb{R}^{n_i\times n_o}[z, \theta_j[1], \dots, \theta_j[n_A + n_B]]$ and $\underline{B}(z, \theta_j) \in \mathbb{R}^{n_o\times n_o}[z, \theta_j[1], \dots, \theta_j[n_A + n_B]]$ are polynomial matrices with real coefficients given by

$$\underline{A}(z, \theta_j) := \sum_{i=1}^{n_A} \underline{\xi}_i^A(z)\theta_j[i], \tag{8a}$$

$$\underline{B}(z, \theta_j) := \underline{\xi}_0^B(z) + \sum_{i=1}^{n_B} \underline{\xi}_i^B(z)\theta_j[i + n_A]. \tag{8b}$$

**Remark 2.2:** The expressions in (6a), given a signal $r_j$, can be written as

$$A(\theta_j)r_j = \Psi_{r_j}^A \theta_j^A, \tag{9a}$$

$$B(\theta_j)r_j = \Psi_{0,r_j}^B + \Psi_{r_j}^B \theta_j^B, \tag{9b}$$

where $\Psi_{r_j}^A := [\xi_1^A r_j \;\; \xi_2^A r_j \;\; \dots \;\; \xi_{n_A}^A r_j] \in \mathbb{R}^{n_iN\times n_A}$, $\Psi_{0,r_j}^B := \xi_0^B r_j \in \mathbb{R}^{n_oN\times 1}$, $\Psi_{r_j}^B := [\xi_1^B r_j \;\; \xi_2^B r_j \;\; \dots \;\; \xi_{n_B}^B r_j] \in \mathbb{R}^{n_iN\times n_B}$, and $\theta_j = \begin{matrix}\theta_j^A \\ \theta_j^B\end{matrix}.$

**Remark 2.3:** The specific choice of the user-defined polynomial matrices of in Definition 2.1 with $\xi_0^B$ independent of $\theta_j$ corresponds to a full-polynomial form, see, e.g. Söderström and Stoica (1989, Section 6.2), and guarantees the rational structure is well defined for all $\theta_j$. The convergence properties of the learning algorithm may depend on this choice and alternative parametrisations may be explored, see Söderström and Stoica (1989, Section 6).

**Remark 2.4:** The parametrisation in (5) is non-unique in $\theta_j$, see, e.g. Zhu (2001, Section 2.3) and Söderström and Stoica (1989, Section 6.3). Therefore, different choices of $\theta_j$ could lead to the same feedforward filter $\underline{F}(z, \theta_j)$, possibly hampering a learning algorithm to find a solution for $\theta_j$, as the solution might switch between multiple optima or diverge to infinity. To avoid divergence and switching behaviour, regularisation on the feedforward parameters is introduced in Section 3.4.

**Remark 2.5:** The rational parametrisation of (7) recovers a polynomial basis functions parametrisation in case $\underline{B}(z, \theta_j) = I$, then $\underline{F}(z, \theta_j) = \underline{A}(z, \theta_j)$, which is linear in the parameters and represents a FIR parametrisation, see van de Wijdeven and Bosgra (2010), Bolder et al. (2014), van der Meulen et al. (2008), depending on the choice of $\underline{\xi}_i^A$.

Given the rational feedforward of (4), the closed-loop tracking error of (3) can be expressed as

$$e_j = S\left(I - PF(\theta_j)\right) r_j. \tag{10}$$

Hence, the minimisation of the tracking error $e_j$ is achieved for all $r_j$ for $F(\theta_j) = P^{-1}$. In this paper, $F(\theta_j)$ consists of rational basis functions, see Definition 2.1, since typical physical systems are modelled using rational models, which contain both poles and zeros. Hence, a rational feedforward parametrisation can accurately describe the plant inverse $P^{-1}$ to obtain high tracking accuracy and task flexibility.

### 2.3 Iterative learning control with RBFs

The goal of RBF ILC is to minimise the norm of the tracking error of the next trial, $e_{j+1}$, by learning $f_{j+1}$, in an iterative manner similar to ILC (Bristow et al., 2006). The tracking error of the next trial is derived by substitution of (4) and (5) in (3), and is given by

$$e_{j+1} = Sr_{j+1} - SPA(\theta_{j+1})B(\theta_{j+1})^{-1}r_{j+1}, \tag{11}$$

hence, the goal of RBF ILC is to find a suitable choice of $\theta_{j+1}$ that would minimise this error. However, it is impossible to minimise $e_{j+1}$ directly, since the system is unknown and only approximate models $\hat{S} = (I + \hat{P}C)^{-1}$ and $\widehat{SP} = (I + \hat{P}C)^{-1}\hat{P}$ are known, where $\hat{P}$ is an approximate model of $P$.

Given $\hat{S}$ and $\widehat{SP}$, for example derived from parametric or first-principles modelling, the model-predicted error at trial $j$ is defined as

$$\hat{e}_j := \hat{S}r_j - \widehat{SP}A(\theta_j)B(\theta_j)^{-1}r_j, \tag{12}$$

which also holds for trial $j+1$ by increasing the index. Although it is possible to minimise $\hat{e}_{j+1}$, results can be improved by compensating for inherent model mismatch by using the available information on the difference between the model-predicted error and the true error at trial $j$, in a process called error propagation. Indeed, RBF ILC minimises $\tilde{e}_{j+1} := \hat{e}_{j+1} - (\hat{e}_j - e_j)$, which equals

$$\tilde{e}_{j+1} := e_j - \hat{S}r_j + \widehat{SP}f_j + \hat{S}r_{j+1} - \widehat{SP}A(\theta_{j+1})B(\theta_{j+1})^{-1}r_{j+1}, \tag{13}$$

which results in an iterative scheme based on data from past trials that compensates for the model mismatch.

**Remark 2.6:** The terms $\hat{S}r_j$ and $\hat{S}r_{j+1}$ in (13) cancel in the case $r_{j+1} = r_j = r$, but this is not necessarily the case in RBF ILC since $f_j$ is parametrised as a function of $r_j$ through the basis functions.

The objective in RBF ILC is to minimise the norm of the error propagation in (13) through the following cost function.

**Definition 2.2 (Cost function for RBF ILC for MIMO systems):** The cost function for RBF ILC for MIMO systems given the feedforward parametrisation of Definition 2.1 is defined by

$$\mathcal{J}_j(\theta_{j+1}) := \left\|\tilde{e}_{j+1}\right\|^2 \tag{14}$$

where $\tilde{e}_{j+1}$ is defined in (13), which is the predicted error for trial $j+1$ based on measurement $e_j$ and $f_j$ and models $\hat{S}$ and $\widehat{SP}$ of the system.

The cost function (14) is nonconvex in $\theta_{j+1}$ for the feedforward parametrisation as in Definition 2.1, as shown in Remark 2.7, and leads to the problem considered in this paper.

**Remark 2.7:** For $r = r_j = r_{j+1}$, $\mathcal{J}_j(\theta_{j+1})$ of (14) with (13) is non convex in $\theta_{j+1}$. Indeed, substitution of $\tilde{e}_{j+1}$ of (13) into (14) yields

$$\begin{aligned}
&\mathcal{J}_j(\theta_{j+1}) \\
&= e_j^\top e_j + f_j^\top \widehat{SP}^\top \widehat{SP} f_j \\
&\quad + r^\top B(\theta_{j+1})^{-\top} A(\theta_{j+1})^\top \widehat{SP}^\top \widehat{SP} A(\theta_{j+1}) B(\theta_{j+1})^{-1} r \\
&\quad + 2e_j^\top \widehat{SP} f_j \\
&\quad - 2e_j^\top \widehat{SP} A(\theta_{j+1}) B(\theta_{j+1})^{-1} r \\
&\quad - 2f_j^\top \widehat{SP}^\top \widehat{SP} A(\theta_{j+1}) B(\theta_{j+1})^{-1} r,
\end{aligned} \tag{15}$$

which is nonconvex in $\theta_{j+1}$.

### 2.4 Problem definition

The problem considered in this paper is to determine the optimal feedforward parameters $\theta_{j+1}^{\text{opt}}$ minimising the cost function of (14), i.e. minimise the norm of the error for the next experiment, as

$$\theta_{j+1}^{\text{opt}} := \operatorname{argmin}_{\theta_{j+1}} \mathcal{J}_j(\theta_{j+1}). \tag{16}$$

Note that (14) is in general non-convex in $\theta_{j+1}$, hence the solution of the optimisation problem is non-trivial. Moreover, the non-unique parametrisation, see Remark 2.4, might hamper the learning algorithm to find a solution, as the solution might diverge to infinity or alternate. The proposed approach for RBF in ILC for MIMO systems aims to solve the aforementioned problems, as presented in the next section.

## 3. Approach

In this section, the main contribution of this paper is presented, that is, the developed approaches that approximately solve the posed non-convex optimisation problem. Moreover, differences with the SISO algorithm, notes on optimality, and possible extensions are given.

### 3.1 Recasting the error propagation

To derive a solution strategy, the nonlinear error propagation of $\tilde{e}_{j+1}$ of (13) is approximated. Note that the nonlinearity in (13) originates from the nonlinear term $B(\theta_{j+1})^{-1}$ in the model-predicted error at trial $j+1$ of (12). Hence, this nonlinear term is circumvented in three steps.

First, recall $\hat{e}_{j+1}$ of (12). For brevity of notation, $A_k = A(\theta_k)$, $B_k = B(\theta_k)$, and the dependence on $\theta_k$ is often omitted throughout this paper. By introducing $B_{j+1}B_{j+1}^{-1} = I$ in the first term, $\hat{e}_{j+1}$ can be rewritten as

$$\hat{e}_{j+1} = \hat{S}B_{j+1}B_{j+1}^{-1}r_{j+1} - \widehat{SP}A_{j+1}B_{j+1}^{-1}r_{j+1}. \tag{17}$$

Second, the key idea is to replace the nonlinear term $B_{j+1}^{-1}$ in (17) with a known term $\underline{B}^{-1}$ that will be defined later, resulting in the approximated and linear $\hat{e}_{j+1,\text{lin}}$, that is given by

$$\hat{e}_{j+1,\text{lin}} := \hat{S}B_{j+1}\underline{B}^{-1}r_{j+1} - \widehat{SP}A_{j+1}\underline{B}^{-1}r_{j+1}. \tag{18}$$

Third, as in Section 2.3, the error propagation is derived by subtracting the difference between the model-predicted error and the true error at trial $j$, i.e. subtracting $\hat{e}_j - e_j$ from (18), hence,

$$\tilde{e}_{j+1,\text{lin}} := e_j - \hat{S}r_j + \widehat{SP}f_j + \hat{S}B_{j+1}\underline{B}^{-1}r - \widehat{SP}A_{j+1}\underline{B}^{-1}r. \tag{19}$$

Indeed, in case $B_{j+1}\underline{B}^{-1} = I$, the error propagation of (13) is achieved. This approximate error propagation is linear in $\theta_{j+1}$ and, as will be shown later in Theorem 3.1, results in a convex optimisation problem given $\underline{B}^{-1}$. The specific choice of $\underline{B}^{-1}$ leads to either an experimentally intensive or efficient approach, as elaborated on next.

### 3.2 Experimentally intensive approach

In this section, contribution II is presented, involving an approach that can deal with model uncertainty through more experiments. Here, $\underline{B}^{-1} := B(\theta_j)^{-1}$, which corresponds to replacing the unknown nonlinear term $B_{j+1}^{-1}$ in (19) by a known term $B(\theta_j)^{-1}$, i.e. using the feedforward parameters at trial $j$. Then, the approximate error propagation of (19) yields

$$\tilde{e}_{j+1,\text{lin}}^{<j>} = e_j - \hat{S}r_j + \widehat{SP}f_j + \hat{S}B_{j+1}B_j^{-1}r - \widehat{SP}A_{j+1}B_j^{-1}r. \quad (20)$$

Note that this is linear in $\theta_{j+1}$ and that for $\theta_{j+1} = \theta_j$ the original error propagation of (13) is obtained.

After substituting the approximate error propagation of (20) in the cost function of (14) from Definition 2.2, a convex optimisation problem based on $\theta_j$, $e_j$, and $f_j$ is obtained, see Theorem 3.1 with Remark 3.1. Hence, a single weighted least-squares problem can be analytically solved for $\theta_{j+1}$ at each ILC trial $j$, yielding the experimentally intensive approach.

### 3.3 Experimentally efficient approach

In this section, contribution III is presented, involving an approach for fast and experimentally efficient learning using offline iterations. Here, $\underline{B}^{-1} := B(\theta_{j+1}^{<k-1>})^{-1}$, which corresponds to replacing the unknown nonlinear term $B_{j+1}^{-1}$ in (19) by $B(\theta_{j+1}^{<k-1>})^{-1}$ with an auxiliary index $^{<k-1>}$, which is known from the previous offline iteration $^{<k-1>}$. Then, an auxiliary index $^{<k>}$ is introduced for the unknown $\theta_{j+1}$ terms such that it can be solved offline given $\theta_{j+1}^{<k-1>}$ of the previous iteration $^{<k-1>}$.

Recall (19), define $A_{j+1} := A(\theta_{j+1}^{<k>})$, $B_{j+1} := B(\theta_{j+1}^{<k>})$, and $\underline{B}^{-1} := B(\theta_{j+1}^{<k-1>})^{-1}$, hence the approximate error propagation of (19) can be written as

$$\tilde{e}_{j+1,\text{lin}}^{<k>} = e_j - \hat{S}r_j + \widehat{SP}f_j + \hat{S}B(\theta_{j+1}^{<k>})B(\theta_{j+1}^{<k-1>})^{-1}r_{j+1} \\ - \widehat{SP}A(\theta_{j+1}^{<k>})B(\theta_{j+1}^{<k-1>})^{-1}r_{j+1}. \quad (21)$$

Note that this is linear in $\theta_{j+1}^{<k>}$ and that for $\theta_{j+1}^{<k>} = \theta_{j+1}^{<k-1>}$ the original error propagation of (13) is obtained.

By substituting (21) into Definition 2.2, a convex optimisation problem is achieved that enables computation of $\theta_{j+1}^{<k>}$ given a $\theta_{j+1}^{<k-1>}$, as shown in the following theorem.

**Theorem 3.1:** *Let $r = r_j = r_{j+1}$ and $g_j := e_j - \hat{e}_j = e_j - \hat{S}r + \widehat{SP}f_j$, and given a known $\theta_{j+1}^{<k-1>}$. Then, (14) with (21), now denoted by $\mathcal{J}_{j,<k-1>}(\theta_{j+1}^{<k>})$, is non-negative and quadratic and, in particular, convex in $(\theta_{j+1}^{<k>})$.*

***Proof:*** Substitution of $\tilde{e}_{j+1,\text{lin}}^{<k>}$ of (21) into (14) yields

$$\begin{aligned}
&\mathcal{J}_{j,<k-1>}(\theta_{j+1}^{<k>}) \\
&= g_j^\top g_j + r^\top B(\theta_{j+1}^{<k-1>})^{-\top} B(\theta_{j+1}^{<k>})^\top \hat{S}^\top \hat{S} B(\theta_{j+1}^{<k>}) \\
&\quad B(\theta_{j+1}^{<k-1>})^{-1} r \\
&\quad + r^\top B(\theta_{j+1}^{<k-1>})^{-\top} A(\theta_{j+1}^{<k>})^\top \widehat{SP}^\top \widehat{SP} A(\theta_{j+1}^{<k>}) \\
&\quad B(\theta_{j+1}^{<k-1>})^{-1} r \\
&\quad + 2g_j^\top \hat{S} B(\theta_{j+1}^{<k>}) B(\theta_{j+1}^{<k-1>})^{-1} r - 2g_j^\top \widehat{SP} A(\theta_{j+1}^{<k>}) \\
&\quad B(\theta_{j+1}^{<k-1>})^{-1} r \\
&\quad - 2r^\top B(\theta_{j+1}^{<k-1>})^{-\top} B(\theta_{j+1}^{<k>})^\top \hat{S}^\top \widehat{SP} A(\theta_{j+1}^{<k>}) \\
&\quad B(\theta_{j+1}^{<k-1>})^{-1} r, \qquad (22)
\end{aligned}$$

which is non-negative and quadratic in $(\theta_{j+1}^{<k>})$. ■

The resulting optimisation problem of (14) with (21) can be sequentially solved for increasing index $k$ at each ILC trial $j+1$, i.e. a series of weighted least-squares problems in $k$ is obtained, starting with $\theta_{j+1}^{<0>} = \theta_j$. Indeed, this approach is computationally more demanding than the experimentally intensive approach due to solving a series of convex optimisation problems instead of a single one.

Henceforth, only the equations of the experimentally efficient approach are considered, as the experimentally intensive approach is easily recovered as follows.

**Remark 3.1:** The experimentally intensive approach of (20) can be recovered from (21) when $k_{\max} = 1$ and $\theta_{j+1}^{<0>} = \theta_j$. To show this, substitute $\theta_{j+1}^{<k-1>} = \theta_j$ in (21).

The two developed approaches have different strengths and weaknesses depending on the available computational resources and the degree of model mismatch. The computationally efficient approach is expected to converge faster than the computationally intensive approach, provided it converges, due to the offline iterations that are performed. However, because of these offline computations, more computational resources are required, and the approach may perform worse when the approximate model is inaccurate. In contrast, the computationally intensive approach is expected to have slower convergence, as only one iteration occurs after each experiment, but it demands less computational power. These statements are supported empirically by the simulation results in Section 4, though they are not mathematically proven.

Next, regularisation is introduced to facilitate convergence of the approach.

### 3.4 Adding regularisation to the optimisation problem

This section introduces regularisation to the optimisation problem that is part of contribution I to enable convergence despite model mismatch and the non-uniqueness of the parametrisation. The optimisation problem of Definition 2.2 does not necessarily lead to a convergent scheme. In particular, model mismatch and the non-uniqueness of the parametrisation, see Remark 2.4, could influence the convergence properties of the iterative optimisation approach (Zhu, 2001, Section 2.3). To facilitate convergence, regularisation terms on $\theta$ are introduced to the cost function of Definition 2.2, resulting in the following optimisation problem.

**Definition 3.2 (Cost function with regularisation for RBF ILC for MIMO systems):** The cost function with regularisation for RBF ILC given the feedforward parametrisation of Definition 2.1 yields

$$\mathcal{J}_{j,<k-1>}(\theta_{j+1}^{<k>}) := \left\| \tilde{e}_{j+1,\text{lin}}^{<k>} \right\|^2 + \left\| \theta_{j+1}^{<k>} \right\|_{W_\theta}^2 + \left\| \theta_{j+1}^{<k>} - \theta_j \right\|_{W_{\Delta\theta}}^2 \tag{23}$$

where $W_\theta, W_{\Delta\theta} \succeq 0$ and $\tilde{e}_{j+1,\text{lin}}^{<k>}$ is defined in (21), which is the predicted error for trial $j+1$ based on $\theta_{j+1}^{<k-1>}$ and measurement $e_j$ and $f_j$ and models $\hat{S}$ and $\widehat{SP}$ of the system.

The weight $W_\theta$ forces $\theta_{j+1}$ of (5) in Definition 2.1 to be small. This adds bias to the solution and might result in a local optimum, but helps to get smoother convergence of the iterative optimisation scheme, since it avoids possible switching between multiple optima and divergence to infinity due to the non-unique parametrisation. Additionally, the weight $W_{\Delta\theta}$ aids in the convergence of the iterative optimisation scheme by reducing large jumps in $\theta_{j+1} - \theta_j$, again forcing local optimum, but of a different cost function. The methodology for choosing $W_\theta$ and $W_{\Delta\theta}$ depends on the relative size of the terms in (23) and can be determined empirically. Higher values relative to the error term are appropriate if the error does not converge or fluctuates, while small values are suitable if the parameters remain small or do not change as the index $j$ increases. Next, the solution of the experimentally efficient approach is derived, which inherently includes the solution of the experimentally intensive approach.

### 3.5 Solution of the optimisation problem

The solution of the optimisation problem Definition 3.2 with (21) starts by substitution of the basis functions of (9a) from Remark 2.2, and by defining $g_j := e_j - \hat{e}_j = e_j - \hat{S}r_j + \widehat{SP}f_j$, which is the difference between the true error and the model-predicted error, and $x^{<k-1>} := B(\theta_{j+1}^{<k-1>})^{-1}r_{j+1}$. Then, the predicted error propagation of (21) results in

$$\tilde{e}_{j+1,\text{lin}}^{<k>} = g_j + \hat{S}\Psi_{x^{<k-1>}}^{B_0} - \underbrace{\begin{bmatrix} \widehat{SP}\Psi_{x^{<k-1>}}^{A} & -\hat{S}\Psi_{x^{<k-1>}}^{B} \end{bmatrix}}_{\Phi_{x^{<k-1>}}} \underbrace{\begin{bmatrix} \theta_{j+1}^{<k>,A} \\ \theta_{j+1}^{<k>,B} \end{bmatrix}}_{\theta_{j+1}^{<k>}}. \tag{24}$$

Clearly, $\tilde{e}_{j+1,\text{lin}}^{<k>}$ is linear in $\theta_{j+1}^{<k>}$ and is a function of $g_j$ and $x^{<k-1>}$.

Next, the solution of the optimisation problem defined in Definition 3.2 is obtained by sequentially minimising the cost function (23) as

$$\theta_{j+1}^{<k>} := \text{argmin}_{\theta_{j+1}^{<k>}} \mathcal{J}_{j,<k-1>}(\theta_{j+1}^{<k>}). \tag{25}$$

The solution is found by taking the derivative of (23), which is

$$\begin{aligned} &\frac{\partial \mathcal{J}_{j,<k-1>}(\theta_{j+1}^{<k>})}{\theta_{j+1}^{<k>}} \\ &= -2\Phi_{x^{<k-1>}}^{\top}\left(g_j + \hat{S}\Psi_{x^{<k-1>}}^{B_0} - \Phi_{x^{<k-1>}}\theta_{j+1}^{<k>}\right) \\ &\quad + 2W_\theta\theta_{j+1}^{<k>} + 2W_{\Delta\theta}\left(\theta_{j+1}^{<k>} - \theta_j\right), \end{aligned} \tag{26}$$

and by equating to zero. Hence, $\theta_{j+1}^{<k>}$ is given by

$$\begin{aligned} \theta_{j+1}^{<k>} &= \left(\Phi_{x^{<k-1>}}^{\top}\Phi_{x^{<k-1>}} + W_\theta + W_{\Delta\theta}\right)^{-1} \\ &\quad \cdot \left(\Phi_{x^{<k-1>}}^{\top}\left(g_j + \hat{S}\Psi_{x^{<k-1>}}^{B_0}\right) - W_{\Delta\theta}\theta_j\right). \end{aligned} \tag{27}$$

The optimal feedforward parameters $\theta_{j+1}^{<k>,\text{opt}}$ are found by iterating over the index $k$, and are achieved for $\theta_{j+1}^{<k-1>} = \theta_{j+1}^{<k>}$. Indeed, in practice a fixed amount of $k$ iterations can be performed such that $\theta_{j+1}^{<k-1>} \approx \theta_{j+1}^{<k>}$. Note that $W_\theta, W_{\Delta\theta} \succ 0$ ensures that the matrix inversion is possible, avoiding the need for a pseudo-inverse, which forces an unique solution as a consequence of regularisation.

**Remark 3.2:** In the optimisation problem in (25) the parameters $\theta_j$ are unconstrained, which means that unstable feedforwards $A(\theta_j)B(\theta_j)^{-1}$ are admissible. In case $B(\theta_j)^{-1}$ is unstable, stable inversion approaches can be used, see, e.g. van Zundert and Oomen (2018).

The resulting experimentally efficient MIMO RBF ILC algorithm with RBFs as defined in Definition 2.1 with the cost function of Definition 3.2 is given in Algorithm 1.

**Remark 3.3:** Note that the basis functions $\Psi^A$, $\Psi_0^B$ and $\Psi^B$ of Definition 2.1 can be selected using physical insight of the system. Furthermore, the weights $W_\theta$ and $W_{\Delta\theta}$ of (23) can be determined empirically to achieve the desired convergence properties. At last, the initial parameters $\theta_0$ can be initialised randomly or set from physical insight.

### 3.6 Differences with the SISO algorithm

The differences with the developed approach and the SISO algorithm introduced by Bolder and Oomen (2015) are illustrated. The developed approach is distinct from to the RBF ILC approach for SISO systems using SK iterations of Bolder and Oomen (2015), even if the commutative property of SISO systems holds, i.e. $AB = BA$. To illustrate this distinction, recall the problem defined in Section 2. In Bolder and Oomen (2015), the non-convex optimisation problem is approximated by weighting (13) in the cost function with $B(\theta_{j+1}^{<k>})B(\theta_{j+1}^{<k-1>})^{-1}$,

**Algorithm 1** Experimentally efficient MIMO RBF ILC algorithm

```
**Define:** $\Psi^A$, $\Psi_0^B$, $\Psi^B$, and $r$.
**Choose:** $W_\theta$ and $W_{\Delta\theta}$.
**Set:** $j = 0$ and $\theta_0$.
**while** $\theta_j$ not converged **do**
    Determine $f_j := A(\theta_j)B(\theta_j)^{-1}r$.
    Perform experiment with $r$ and $f_j$ and measure $e_j$, as in Figure 1.
    Set $k = 1$ and $\theta_{j+1}^{<0>} = \theta_j$.
    **while** $k \leq k_{\max}$ **do**
        Determine $x^{<k-1>} := B(\theta_{j+1}^{<k-1>})^{-1}r$.
        Construct $\Phi_{x^{<k-1>}}$ as defined in (24).
        Compute $\theta_{j+1}^{<k>}$ using (27).
        $k \rightarrow k + 1$.
    **end while**
    Set $\theta_{j+1} = \theta_{j+1}^{<k_{\max}>}$.
    $j \rightarrow j + 1$.
**end while**
**Output:** $\theta = \theta_j$
```

i.e. each term of (13) is weighted, resulting in a convex optimisation problem if the commutative property of SISO systems holds. In sharp contrast, in the developed approach, only specific terms corresponding to the model-predicted error $\hat{e}_{j+1}$ of (12) are weighted with $B(\theta_{j+1}^{<k>})B(\theta_{j+1}^{<k-1>})^{-1}$ in (13), resulting in (21). This avoids modification of the complete cost function by only weighting specific terms that are nonlinear. As a result, a different convex optimisation problem is obtained that is suited for MIMO systems, since it does not require commutation of several convolution matrices. Since both approaches iteratively solve the non-convex optimisation problem through a series of weighted least-squares problems, similar convergence properties are expected, which are discussed next.

### 3.7 Notes on optimality

The MIMO RBF ILC algorithm performs offline iterations with index $^{<k>}$ to iteratively solve the non-convex optimisation problem through a series of weighted least-squares problems, which is closely related to the SK iterations (Sanathanan & Koerner, 1963). Typically, algorithms that employ SK iterations have good convergence properties and are insensitive to local optima (Bohn & Unbehauen, 1998), regardless of the initial estimate. However, stationary points are generally not optima of the cost function (Whitfield, 1987). Similarly, for the Steiglitz and McBride (SM) identification algorithm, see Steiglitz and McBride (1965) and Stoica and Söderström (1981), which is similar to SK-based algorithms, it cannot be guaranteed that a global optima can be found either. Moreover, for RBF ILC for SISO systems (Bolder & Oomen, 2015), the non-optimality of SK-based approach is proven in van Zundert, Bolder, Oomen (2016). Despite the non-optimality of the SK and SM-based approaches, in some cases, a priori error bounds can be given regarding under-modelling in the SM-identification algorithm, see, e.g. Regalia and Mboup (1996) and Mboup et al. (1997).

### 3.8 Possible extensions

To address potential non-optimality of the developed approach, extensions to the framework can be considered. First, the developed approach can be used to provide an initial estimate that is subsequently used in a gradient-based optimisation algorithm, e.g. Levenberg-Marquardt (Moré, 1978), to find a locally optimum solution from the initial estimate. This sequential approach is often used in system identification, see, e.g. Voorhoeve et al. (2021), since the SK-based approaches do not necessarily converge to a local optima (van Zundert, Bolder, Oomen, 2016), but are insensitive to local optima (Bohn & Unbehauen, 1998), and gradient-based approaches rely on a decent initial estimate, but can converge to local optima. Second, the developed framework could be extended with an IV-based approach along the lines of van Zundert, Bolder, Oomen (2016), which could yield local optimal results since the approximation to make the problem convex takes place in the gradient, see van Zundert, Bolder, Oomen (2016) for proof. Both of these extensions are considered in future work. Next, the convergence properties of the algorithm are tested in simulation.

## 4. Simulation study

In this section a simulation study is presented that highlights the efficacy of the developed approach and the influence of regularisation and model mismatch on the convergence properties.

### 4.1 System

Consider the discrete-time system $\underline{P}(z)$ with 2 inputs and 2 outputs that is defined by the RMFD as

$$\underline{P}(z) := \underbrace{\begin{bmatrix} 3z^2 - 2z & 0 \\ z^2 & 6z^2 - 5z \end{bmatrix}}_{\underline{B}(z)} \underbrace{\begin{bmatrix} z^2 - 2z + 1 & -3z^2 - z \\ z^2 - z & 5z^2 - 7z + 4 \end{bmatrix}^{-1}}_{\underline{A}(z)^{-1}}. \tag{28}$$

A Bode diagram of the system is presented in Figure 2. The feedback controller is designed so that the closed-loop MIMO system is stable. Moreover, noise is added to the output of the system.

### 4.2 Setup

The simulation system $\underline{P}(z)$ is shown in Figure 2 and has interaction between inputs and outputs and is not symmetric, so the system does not commute. The models $\hat{S}$, $\widehat{SP}$ for ILC are based on an approximate model $\hat{P}$, which is defined as

$$\hat{\underline{P}}(z) := \begin{bmatrix} (3-\gamma)z^2 - 2z & 0 \\ (1-\gamma)z^2 & (6+\gamma)z^2 - 5z \end{bmatrix} \cdot \frac{1}{1-\gamma} \cdot \begin{bmatrix} z^2 - 2z + 1 & -3z^2 - z \\ z^2 - z & 5z^2 - 7z + 4 \end{bmatrix}^{-1}. \tag{29}$$

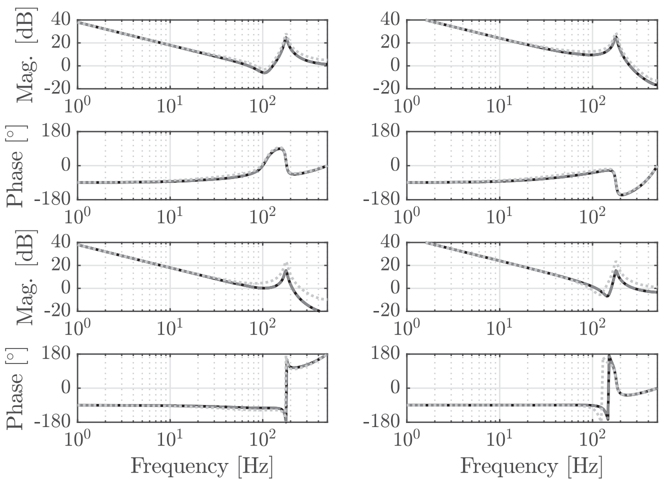

**Figure 2.** Bode diagram of the system $\underline{P}(z)$ used in simulation (—), and approximate model $\hat{\underline{P}}(z)$ with light (- -) and severe (⋯) model mismatch. Clearly, the MIMO systems has interaction between inputs and outputs and is not symmetric.

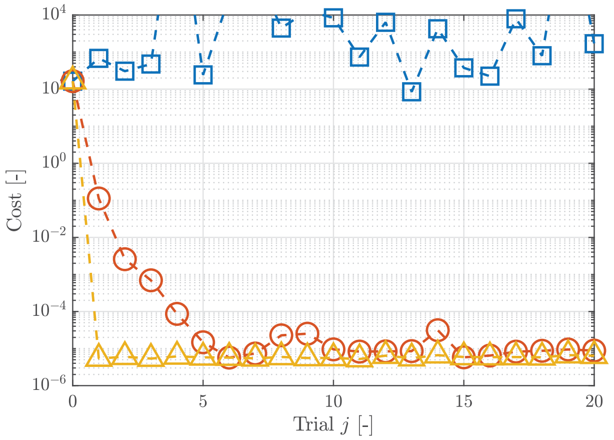

**Figure 3.** Error norm per trial $j$ for the commutative SK-based approach (-□-), the experimentally intensive MIMO RBF approach (-○-), and the experimentally efficient MIMO RBF approach (-△-) for the case of no model mismatch. Clearly, the later converges almost instantly and monotonically.

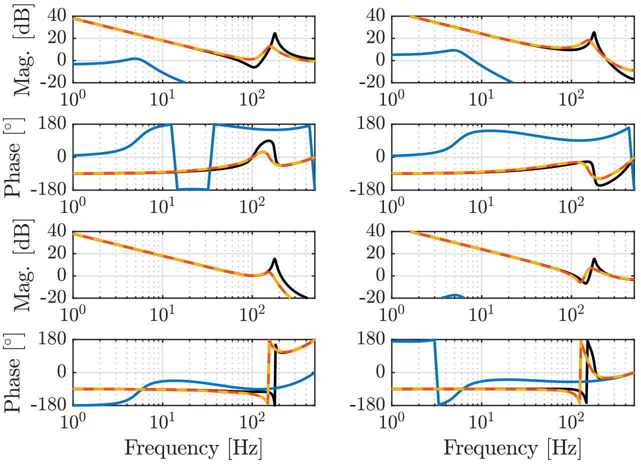

**Figure 4.** Bode diagram of the plant $\underline{P}(z)$ (—) and of the inverse feedforward filters $\underline{F}(z, \theta_{20})^{-1}$ achieved with commutative SK-based approach (—), experimentally intensive approach (- -), and the experimentally efficient approach (—) for the case of no model mismatch. The developed approaches approximate the system fairly well, suggesting task flexibility.

where first, $\gamma = 0.2$ is used for light model mismatch, and later, $\gamma = 0.4$ for severe model mismatch, see Figure 2. Note that when $\gamma = 0$, $\hat{\underline{P}}(z) = \underline{P}(z)$. The control objective is to track the reference depicted in Figure 4. The feedforward parametrisation is chosen according to Definition 2.1 and the same for both approaches. Moreover, $F(\theta) \approx P^{-1} = AB^{-1}$ is designed such that $P^{-1}$ is in the model set of $F(\theta)$ and $B$ is always invertible due to its structure. Furthermore, under-parametrisation is inherent due to the added output noise in the simulation. Regularisation is applied according to Definition 3.2 to aid in the convergence properties of the developed methods. The weights of the cost function in Definition 3.2 are empirically selected as $W_\theta = W_{\Delta\theta} = 10^{-7}$. These values ensure the regularisation terms in (23) are sufficiently large to achieve convergence but not too high to restrict the absolute value and changes in the parameters. For ILC, $j = 20$ trials are performed and $k_{\max} = 1$ to achieve the experimentally intensive approach, see Remark 3.1, and $k_{\max} = 19$ in the experimentally efficient approach.

In addition to applying the developed approaches, the commutative SK-based approach, i.e. RBF ILC algorithm developed for SISO systems using SK iterations, is applied to show the effect of wrongly exploiting the commutative property, see Section 3.6. The algorithm is derived along the lines of Bolder and Oomen (2015) with the cost function of Definition 3.2, and is implemented with the same cost function weights and feedforward parametrisation as the developed approaches. Moreover, $k_{\max} = 19$ SK iterations are performed. The results of the approaches are presented next.

### 4.3 Results

The simulation results for the experimentally intensive, experimentally efficient, and the commutative SK-based approach for the case of no model mismatch, i.e. $\gamma = 0$ in (29), are shown in Figures 3–6. The error norm per trial $j$ in Figure 3 shows that the experimentally efficient approach converges almost instantly and monotonically since there is no model mismatch. Furthermore, the experimentally intensive approach converges slower and achieves a low cost for certain trials, which coincides with the typical convergence behaviour of such iterative schemes.

In contrast, the commutative SK-based approach does not converge and does not attain a low cost, indicating that wrongly using the commutative property of SISO systems does not work. This is also reflected in the significant tracking error of Figure 4 for the feedforward signal of Figure 5.

Regarding task flexibility, recall that high tracking accuracy is achieved for any reference if $F^{-1} \approx P$, see (10). The inverse feedforward filters $\underline{F}(z, \theta_{20})^{-1}$ of the developed approaches approximate the system fairly well, as shown in Figure 6, suggesting extrapolation capabilities to other tasks. Regarding the effect of model mismatch on the convergence of the developed approaches, the model mismatch in $\hat{P}$ of Figure 2 is increased to the severe case, i.e. $\gamma = 0.4$. The error norm per trial is shown in Figure 7. Clearly, the experimentally intensive case converges slower in the first few trials if the model mismatch is present. For both the case of light and severe model mismatch the approach

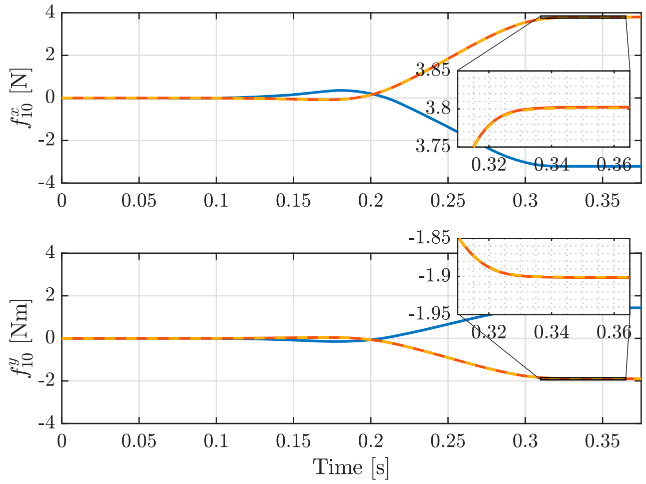

**Figure 5.** Feedforward signals in trial $j = 20$ of the commutative SK-based approach (—), experimentally intensive MIMO RBF approach (—), and the experimentally efficient MIMO RBF approach (- -) for the case of no model mismatch.

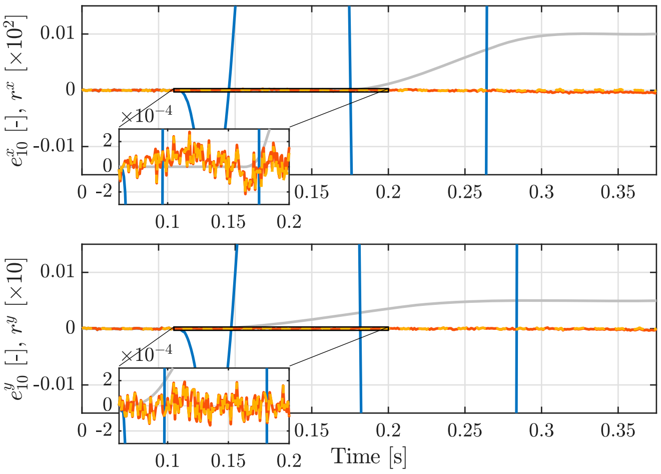

**Figure 6.** Error signals in trial $j = 20$ of the commutative SK-based approach (—), experimentally intensive approach (—), and the experimentally efficient approach (—) for the case of no model mismatch and for the scaled references trajectories $r^x, r^y$ (- -). The developed approaches achieve a significant performance gain in contrast to the commutative SK-based approach.

does not require increased regularisation and achieves a low cost for certain iterations, suggesting that the algorithm is less influenced by model mismatch. Furthermore, the experimentally efficient algorithm converges at a slower rate when there is light model mismatch present. For severe model mismatch there is clear divergence and no low cost is achieved. When regularisation is increased to $W_\theta = 10^{-6}$, the approaches converges at a slow rate and achieves a low cost for certain trials. At the last few trials, the cost seems to diverge which indicates that increased regularisation might be necessary. This highlights the effect of the number of $^{<k>}$-iterations and the importance of regularisation for convergence with respect to model mismatch.

## 5. Conclusions

The developed framework enables data-driven tuning of rational feedforward controllers for general non-commutative MIMO systems. In particular, by rewriting the nonlinear terms in the optimisation problem, a series of weighted least-squares problems is attained, enabling offline iterations for an experimentally efficient MIMO RBF scheme. Moreover, an experimentally intensive algorithm is obtained by limiting the number of offline iterations performed. The simulation results show that the experimentally efficient approach can converge almost monotonically despite mismatch through the use of regularisation and can achieve high tracking accuracy and task flexibility. Furthermore, the experimentally intensive approach was shown to be beneficial in the case of severe model mismatch without increasing regularisation, since the algorithm is less influenced by model mismatch. Moreover, it was shown that wrongly using the commutative property does not work, highlighting the importance of the developed framework.

Future developments involve achieving optimality through IV-based approaches along the lines of van Zundert, Bolder, Oomen (2016), extensions with gradient-based optimisation schemes, development of other approximation schemes, automating the tuning and weight selection procedure, quantifying the error in the cost function due to the approximation and model mismatch, and experimental validation.

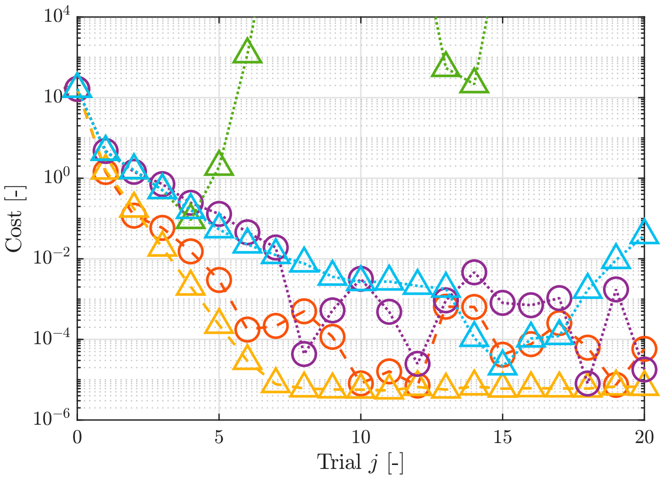

**Figure 7.** Error norm per trial $j$. The experimentally intensive approach for light (-⊖-) and severe (-⊖-) model mismatch shows that the latter converges more slowly, but still attains a low cost for certain trials without increasing regularisation. The experimentally efficient approach for severe model mismatch (-△-) and for increased regularisation (-△-) highlight the importance of regularisation, and the latter converges towards a slightly higher error norm, albeit at a slower rate, compared to the light model mismatch case (-△-).

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