Stimulated quantum phase slips from weak electromagnetic radiations in superconducting nanowires

Jafari-Salim, A; Eftekharian, A; Majedi, AH; Ansari, MH

DOI
10.1063/1.4944482

Publication date
2016

Document Version
Final published version

Published in
AIP Advances

AIP Advances

Stimulated quantum phase slips from weak electromagnetic radiations in superconducting nanowires

Cite as: AIP Advances **6**, 035209 (2016); https://doi.org/10.1063/1.4944482
Submitted: 28 October 2015 . Accepted: 29 February 2016 . Published Online: 14 March 2016

Amir Jafari-Salim, Amin Eftekharian, A. Hamed Majedi, and Mohammad H. Ansari

AIP ADVANCES **6**, 035209 (2016)

Stimulated quantum phase slips from weak electromagnetic radiations in superconducting nanowires

Amir Jafari-Salim,[1,2,a] Amin Eftekharian,[1] A. Hamed Majedi,[1,3,4,b] and Mohammad H. Ansari[5]
[1]*Department of Electrical and Computer Engineering, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1*
[2]*Institute for Quantum Computing, Waterloo, Ontario, Canada N2L 3G1*
[3]*Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada*
[4]*Waterloo Institute for Nanotechnology, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1*
[5]*Kavli Institute of Nanoscience, Delft University of Technology, P.O. Box 5046, 2600 GA Delft, The Netherlands*

(Received 28 October 2015; accepted 29 February 2016; published online 14 March 2016)

We study the rate of quantum phase slips in an ultranarrow superconducting nanowire exposed to weak electromagnetic radiations. The superconductor is in the dirty limit close to the superconducting-insulating transition, where fluxoids move in strong dissipation. We use a semiclassical approach and show that external radiation stimulates a significant enhancement in the probability of quantum phase slips. [http://dx.doi.org/10.1063/1.4944482]

I. INTRODUCTION

Quantum phase slip junctions are exact dual counterpart of the Josephson junctions. Recently these junctions have been successfully realized in ultranarrow superconducting nanowires, where quantum phase slip replaces tunneling Cooper pairs.[1,2] These nanowires are nonlinear elements performing similar physics as Josephson junctions with the roles of superconducting phase φ and charge q being interchanged.[3,4] This duality has been the motivation behind many of the recent applications of quantum phase slip (QPS) elements.[5–8] These elements have found interesting implications for fundamental metrology and information technology, for instance as photon pulse detectors, quantum current standard, and quantum bits.[7–10]

In a superconducting nanowire with small cross section, the supercurrent is determined by the phase difference φ between two ends of the nanowire from the sawtooth relation $I_s = \Phi_0\varphi/2\pi L$ with L being nanowire kinetic inductance and $\Phi_0 = h/2e$. In temperatures much lower than the superconducting critical temperature (i.e. $T \ll T_c$) quantum fluctuations may suppress the modulus of the order parameter in a region and turn it from superconductor to normal metal. This enables the superconducting phase to slip by $2n\pi$, with integer n, without any energy compensation. An individual phase slip takes place in a normal core, similar to the normal core of a magnetic flux vortex, therefore we can assume the core size is roughly the coherence length ξ.[11,12] QPS event takes place for a short period of time that is maximally of the order of inverse of superconducting gap $h/2\Delta$. Similar effect happens close to T_c due to thermal fluctuations of the order parameter.[13]

In superconducting nanowire made of clean materials with low normal resistance R, quantum phase slips rarely take place. To enhance the slip rate a nanowire should be made of highly disordered amorphous superconductor, which is in the dirty limit, with large R.[5,7] There is not a well-understood theory to describe the superconductivity in near superconductor-insulator transition (SIT). A candidate theory [14,15] proposes superconductivity at high disorder is maintained by a

[a]ajafaris@uwaterloo.ca
[b]ahmajedi@uwaterloo.ca

fragile coherence between electron pairs, which is characterized by an anomalous binding energy. If pairs are localized, they enter an insulating state, and if condense, a coherent zero-resistance state emerges. Based on this theory superconductor in SIT have regions of localized BCS-condensates nearly separated in different lakes.[16] The cores of QPS can coherently tunnel across superconducting regions and avoid dissipation. This is similar to the Cooper pairs that tunnel across a Josephson junction without much dissipation.[17,18]

The voltage across the nanowire is known to be periodic in charge of the crossing fluxoid; i.e. $V = V_0 \sin(2\pi q/2e)$. Individual phase slips in nanowires can be observed when a large bias voltage is applied on the wire. Under such bias voltage, effective potential becomes a tilted washboard with more slanted slop in larger bias. Depending on temperature, there are two general scenarios for the dynamics of a fluxoid. Close to the critical temperature T_c, fluxoid particle gains energy from thermal activation and overcomes potential barrier to slip across the wire.[13] Quite differently, in low temperature $T \ll T_c$ fluxoid particle becomes frozen in a minima of the washboard potential. The minima are called 'zero-current states' where Coulomb blockade occurs.[19–21] Vacuum fluctuations help the particle to tunnel into the barrier and slips away. Coherent tunneling is possible between two zero-current states where a quantum variable (phase or charge) has minimum fluctuations. Such coherent tunnelings have been previously observed in the superconducting-insulating transition limit.[19–21]

Exposing nanowire to strong electromagnetic radiation produces Shapiro steps[22,23] in the current-voltage dependence, which has been observed in Ref. 24. However, in many applications of superconducting nanowires, such as in qubits and photon detectors, weak radiation is applied where phase locking cannot occur.

In this paper, we qualitatively study the effect of a weak alternating electromagnetic field on the quantum phase slip rate in ultranarrow superconducting nanowire, where the width of the nanowire is smaller than the superconducting coherence length, i.e. $r < \xi$. We consider the nanowire is in the insulating phase. Our method is to map this problem into its well-studied analogue in Josephson junction in proper regime. We use the semiclassical quantum mechanical approach developed by Ivlev and Mel'nikov[25–27] in studying quantum tunneling in a high-frequency field to our problem. Similar to a Josephson junction under weak time-harmonic radiation,[27,28] we expect a significant enhancement in the stimulated phase slips at zero temperature. We show that a fluxoid gains energy from radiation and tunnel into the barrier more often than usual and slips away. This leads to the super-exponential enhancement in the rate of such 'stimulated quantum phase slips' (SQPs). In certain nanowires, this can result in larger DC resistivity with minimal fluctuations in a dynamical variable.

II. STIMULATED QUANTUM PHASE SLIPS

A superconducting nanowire with QPS is the dual to a Josephson junction with charge and phase (as well as current and voltage) interchanged. In Josephson junction, a Cooper pair tunneling across the junction picks up a phase $\exp(\pm i\varphi(t))$ corresponding to the superconducting phase $\varphi(t)$. This induces a coupling energy $E = E_J(1 - \cos\varphi)$. The current is defined $I = (2e/\hbar)\partial E/\partial\varphi$.

Analogously for a nanowire similar relations can be derived. A QPS fluxoid picks up a charge phase when tunneling $\exp(\pm iQ)$ with $Q \equiv 2\pi q/2e$ being a dimensionless charge parameter. Therefore the QPS energy becomes $E = E_S(1 - \cos Q(t))$. The voltage is defined $V = \partial E/\partial q$. Phase slips may take place everywhere in the wire whose induced current depends on the wire inductance. Therefore a narrow superconducting nanowire can be modelled as a voltage in series with an inductance as shown in the left part of Fig. 1. In the figure, the dissipation is modelled by a resistor and the AC and DC bias voltages are sources in series with the wire. This circuit is built based on.[3,4] The inductor L is the total of the kinetic inductance (L_k) and the geometric inductance (L_g) of the circuit. Since, in superconducting nanowire, the kinetic inductance is much larger than the geometric inductance, we have $L \approx L_k$. In the circuit of Fig. 1 voltage is $V = V_0 \sin(2\pi q/2e) + L\ddot{q} + R\dot{q}$ with the QPS and inductance energies

$$E_S = 2eV_0/2\pi, \qquad E_L = \Phi_0^2/2L_k. \tag{1}$$

where V_0 is the voltage scale of QPS energy E_S.

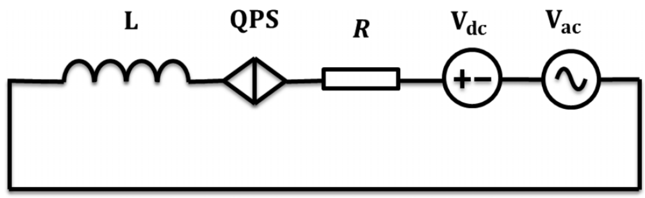

FIG. 1. The schematic circuit of a QPS junction including an ideal QPS element (a superconducting nanowire), the dissipative element R, the bias voltage V_{DC} and the driving source V_{AC}. L denotes the dominant kinetic inductance.

In the nanowire, a crossover from insulator to a superconducting inductor takes place when the inductance energy E_L is increased beyond QPS energy E_S. In the superconducting phase $E_L \gg E_S$, the fluxoid energy $E = E_S(1 - \cos Q) + E_L(\phi_f)$ is dominated by the parabolic inductance energy $E_L(\phi_f)$ associated with induced phase ϕ_f. The parabola associated with different winding integer n cross at certain energies where the small energy of QPS provide an avoided crossing gap. This makes nanowire energy to be multivalued in separated energy bands, similar to a capacitive Josephson junction.

In the opposite regime where $E_S \gg E_L$ the wire energy is dominated by QPS energy E_S $(1 - \cos Q)$ which oscillates in charge. Consider that the charge undergoes a fluctuation around its macroscopic value q so that the stochastic charge is $q = q + \delta q$. Following the analogue discussion for a Josephson junction phase, (see Refs. 29–31 and references therein) the charge fluctuation corresponds to the effective current fluctuations across the wire

$$\delta q(t) = \int_0^t \delta I(t')dt'. \tag{2}$$

A difference between QPS and thermally activated phase slips (TAPS) is that the dissipation in TAPS is due to stochastic energy activation in high temperature while QPS allows tunneling between distinct zero-current states. The latter is similar to zero-voltage states in Josephson junction.[19–21] For a nanowire in the insulating phase the quantum tunneling between zero-current states takes place without current fluctuations $\delta I(t)$. Therefore Eq. (2) shows that the charge in fluxoid behaves semiclassically, whereas superconducting phase can be subject to large fluctuations.[4]

The semiclassical charge associated with QPS fluxoid in the nanowire depicted in Fig. 1 evolves in the following way:

$$\frac{d^2Q}{dt^2} + \eta\frac{dQ}{dt} + \omega_p^2(\cos Q - k_0 - k_1 \cos \Omega t) = 0, \tag{3}$$

with Ω is the frequency of the driving voltage and

$$\eta = R/L, \quad \omega_p = \sqrt{2\pi V_0/2eL}, \quad k_i = V_i/V_0. \tag{4}$$

with the index $i = 0$ (1) corresponds to DC (AC) voltage. The definition of the plasma frequency ω_p is compatible with the definition based on the duality: $\hbar\omega_p = \sqrt{2E_SE_L}$, which is similar to the equation of RCSJ model of Josephson junction[32,33] with high-frequency driving field. For simplicity in writing Eq. (3) we had shifted $Q \to Q + \pi/2$ in order to have applied and QPS voltages in phase.

Our aim is to study the possibility of utilizing nanowire as a detector for time-harmonic radiations, therefore we restrict ourselves to weak alternating fields, i.e. $k_1 \ll 1$. This makes our problem to be different from the physics of the Shapiro steps[22] where the phase of nanowires (or its dual Josephson junction) is locked to the frequency of the driving field frequency and constant voltage steps are observed. For weak time-harmonic fields the driving force is very small and the wire is in zero-current state with $k_0 < 1$. The most important result is that the smallness of k_1 does not necessary mean that the its effect on the charge dynamics is small. In fact as we will show a weak time-harmonic field can significantly affect the wire by increasing the rate of QPS (see also Appendix A).

In the limit of weak dissipation, the tunneling rate of fluxoid can be studied using 1D quantum mechanics of the Lagrangian associated to Eq. (3) subject to $R = 0$. However we are interested to study the decay of the zero-current state in the limit of strong dissipation because resistance is large in wires with QPS effects. A dual effective theory has been developed for dissipative coherent tunneling in Josephson junction by Caldeira and Leggett,[17,18] and Larkin and Ovchinnikov.[34] In those theories, dissipation is modelled as the coupling to bosonic degrees of freedom. The low-energy effective action of the system is derived to properly take account of the dissipation.

The semiclassical theory of Josephson junction exposed to weak alternating current in Refs. 27 and 34 guides us to study the charge dynamics of radiation-stimulated quantum phase slips (SQPS) in nanowires with strong dissipation. Effectively the evolution of semi-classical charge that tunnel across the wire is:

$$\frac{d^2Q}{dt^2} + \omega_p^2(\cos Q - k_0 - k_1\cos\Omega t) - 2i\pi\eta\left(\frac{k_BT}{\hbar}\right)^2 \int_C dt_1 \frac{\sin[(Q(t) - Q(t_1))/2]}{\sinh^2(\pi k_B T(t_1 - t)/\hbar)} = 0, \tag{5}$$

where the contour C is shown in Fig. 3 and the principle value of the integral is implied (for general discussion of the method, see Appendix A).

In the limit of our interest, the nanowire is strongly dissipative $\eta \gg \omega_p$. For semi-classical description to be valid, it is required that $E_S \gg \hbar\Omega$.[27,34] Also the applied DC voltage is close to V_0, i.e. $V_0 - V_{DC} \ll V_0$, therefore the term with second derivative in Eq. (5) can be omitted. Also, we can assume that the exchange of energy between the wire and its environment takes place in the shortest time, thus the argument of sinh in the denominator of Eq. (5) can be replaced by its lowest order $\sinh x \approx x$. Technical analysis of this integration over the contour C shows that the integral tends to zero except that at the singularity $t = t_1$ where its proper residue must be counted (see Eq. 18 in Ref. 27). Therefore, in the lack of alternating radiation Eq.(5) in the regime of interest effectively reads: $-\eta dQ/dt + \omega_p^2(\cos Q - 1) = 0$, which has the following solution

$$Q(t) = i\ln\frac{t - i\tau_s}{t + i\tau_s}, \quad \tau_s = \frac{\eta}{\omega_p^2}, \tag{6}$$

with τ_s being the time of under-barrier motion.

In a system described by the classical action $S = -i\int_C \mathcal{L}dt$ with $\mathcal{L}$ being Lagrangian, the probability of quasiclassical tunneling is $\Gamma = \exp(-S)$. Regarding the alternating voltage being a small perturbation $k_1 \ll 1$, we can rewrite the action in the form of $S = S_0 + S_1$ with S_0 being the action in the lack of alternating field and S_1 is linear in V_{AC}.[35]

Let us assume the QPS probability in a nanowire with strong dissipation and DC voltage V_{DC} about V_0 is denoted as Γ_0. Above results easily show that in the presence of a weak high frequency radiation hitting the nanowire, the probability of QPS in the wire will change from Γ_0 to Γ in the following form:

$$\Gamma(V_{AC},\Omega) = \Gamma_0 \exp\left[\frac{4eV_{AC}}{\hbar\Omega}\sinh(\Omega\tau_s)\right], \tag{7}$$

Eq.(7) is the main result in this paper. It indicates that an alternating driving field with certain frequency and voltage can trigger occurance of a large number of QPS's in a proper nanowire at low temperature. For instance if a wire with dissipation factor $\eta/\omega_p = 10$ is driven by a weak time-harmonic radiation of the realtive amplitude $4eV_{AC}/\hbar\omega_p = 5\times10^{-4}$ and frequency $\Omega = \omega_p$, the QPS rate increases by a factor of about 250 times. A more careful analysis shows that this result is valid for nanowire temperature $T < T_0$ with T_0 being the crossover temperature between quantum and thermal activation regimes $T_0 = \sqrt{(1-k_0)/2}\,(\hbar\omega_p)^2/\pi k_B\eta$.[35]

From Eq.(7) one can see that the bigger the normal resistance R is, the larger the rate of SQPS becomes. Intuitively this can be understood from the definition of underbarrier time τ_s in Eq. (6). Upon increasing dissipation the under-barrier time grows larger. According to the semiclassical description of quantum tunneling, see Appendix A, during quantum tunneling time parameter becomes imaginary. This changes the bounded function of alternating potential in eq. (3) into the unbounded function $\cosh\Omega\tau$. The longer a fluxoid stays under the barrier, the more energy it absorbs from the alternating potential and this causes stimulation of quantum phase slips.

The QPS rate in the lack of an time-harmonic drive has been estimated by Mooij and Harmans in Ref. 6 to be nearly $\Gamma_0 \approx E_S/\hbar$. This in addition to substituting Eqs. (4) and (1) simplifies

Eq. (7) into:

$$\Gamma(V_{AC},\Omega) = (V_0/\Phi_0)\exp\left[\frac{4eV_{AC}}{\hbar\Omega}\sinh\left(\frac{eR\Omega}{\pi V_0}\right)\right], \tag{8}$$

Given that the QPS rate in the absence of time-harmonic radiation is proportional to V_0, one of the features of Eq. (8) is that the super-exponential enhancement of QPS rate is inversely proportional to the rate Γ_0, thus for small rate Γ_0 the exponential enhancement of QPS in the presence of high-frequency field is more significant. This enhancement is only due to the stimulated excitation in the zero-current states in the presence of external drive.

In the absence of the V_{DC}, there is no tilt in potential and the rate of fluxoid crossing to right or left are equal. As a result the average current becomes zero ($\bar{I} = 0$). However, in the presence of a positive value for DC voltage bias, the average current is given by:

$$\bar{I} = 2e\,(\Gamma_{\rightarrow} - \Gamma_{\leftarrow}) \tag{9}$$

with $\Gamma_{\rightarrow}$ ($\Gamma_{\leftarrow}$) the rate of crossing to the right (left) where the potential barrier decreases (increases). In the case the bias voltage is close to the critical voltage (V_0), $\Gamma_{\leftarrow}$ the crossing $\Gamma_{\rightarrow}$ dominantly exceeds that of the opposite direction. Hence $\Gamma = \Gamma_{\rightarrow}$ and

$$I = 2e\Gamma(V_{AC},\Omega), \tag{10}$$

where $\Gamma(V_{AC},\Omega)$ is given by Eq. (7). According to Eq. (10), the influence of high-frequency weak irradiation on superconducting nanowire biasd by the DC voltage $V_0 - V_{DC} \ll V_0$ is observable by measuring the crossing current.

The quality factor in nanowire Q_S is defined as:

$$Q_S = \frac{\omega_p}{\eta}. \tag{11}$$

In low quality factor QPS nanowire the dissipation is strong. The larger η leads to longer under-barrier time and consequently the enhancement of SQPS rate exponentially increases. The increase in the under-barrier motion due to higher dissipation can be seen in Fig. 4. Therefore, we expect that a low-Q_S nanowire to be a better candidate for observing tunneling enhancement.

A comment on the range of validity of the method we used in this section is in order. As it is seen from Eq. (7), the enhancement in the tunneling probability for $\Omega\eta \gg \omega_p^2$ is itself an exponentially large factor ($\sim eV_{AC}(\hbar\Omega)^{-1}\exp(\Omega\eta/\omega_p^2)$). This indicates that the range of the validity of the semi-classical approach in this case is limited to $eV_{AC} \sim (\hbar\Omega)\exp(-\Omega\eta/\omega_p^2)$. Beyond this, higher order correction in terms of V_{AC} to the calculations is required.[36]

An alternative approach in studying the QPS rate in superconducting nanowires under high-frequency radiation would be to use the the effective action method developed by Golubev and Zaikin[1,37–39] in a non-equilibrium setting. There are some challenges associated with this approach that are studied in Ref. 40.

III. CONCLUDING REMARKS

We studied the stimulation effect of a weak high-frequency field on the zero-current state tunneling of fluxoid particle. The approach chosen was to use the duality transformation between Josephson junction and a QPS junction to map the dynamics of QPS charge in a circuit model. Then we studied the effect of high-frequency alternating field on the coherent tunneling rate. The similar problem has been studied for the case of Josephson junction using semiclassical physics[25–27] which we adopted for the case of QPS junction. We observed that in a strongly dissipative superconducting ultranarrow nanowire, a high frequency field can enhance the probability of quantum tunneling super-exponentially. Interestingly we find that the enhancement of SQPS rate is more pronounced in wires with small non-stimulated QPS. This result will help to predict that quantum phase slip qubits should be better-working in the presence of weak driving field for specific regimes.

A potential application for the rate enhancement of the superconducting nanowire in high-frequency field is in designing a new class of high frequency detectors. Some of the advantages of a detectors made of superconducting nanowire would be small footprint, ease of fabrication and integration with other superconducting devices.

ACKNOWLEDGMENTS

This work is supported by the NSERC Discovery Grant. This research is also sponsored by CryptoWorks21, an NSERC funded collaborative research and training experience program. AJS and MHA thank Frank K. Wilhelm-Mauch for fruitful discussions.

APPENDIX: BRIEF REVIEW OF THE QUANTUM TUNNELING IN TIME-DEPENDENT POTENTIALS

In this appendix, we review the effect of a high-frequency field on quantum tunneling in the semi-classical description.[36] The approach will be based on the method developed in Refs. 25–27. First, a brief introduction is given to the semi-classical approach to quantum mechanics and quantum tunneling.

The semi-classical description is obtained from the stationary path approximation of the Feynman path integral approach to quantum mechanics. The stationary path of a Feynman path integral which is obtained from the variation of the action yields the Newtonian equation of motion (EOM):

$$\delta S = 0 \longrightarrow \text{EOM} \tag{A1}$$

This relation is familiar in classical mechanics for energetically allowed region; however, the natural question that arises is that: is this method applicable to energetically forbidden regions like in quantum tunneling? The answer is "yes"; however, it requires allowing the time to acquire an imaginary part.[41] To see this, let's consider the action of a point particle with mass m in a potential $V(x)$. The action can be written as:

$$S = \int dt \left\{ \frac{m}{2}\left(\frac{dx}{dt}\right)^2 - V(x) + E \right\}. \tag{A2}$$

The equation of motion is found to be

$$m\frac{d^2x}{dt^2} + \frac{dV(x)}{dx} = 0, \tag{A3}$$

and the total energy is given by

$$E = \frac{p^2}{2m} + V(x), \tag{A4}$$

where the momentum is defined as $p(t) = mdx/dt$. By integrating Eq. (A4), the required time t for the particle to reach infinity from point x is given by

$$t(x) = \int_x^\infty \frac{dx'\sqrt{m}}{\sqrt{2\left(E - V(x')\right)}}. \tag{A5}$$

From Eq. (A5), it is seen that as long as $E > V(x)$ the time remains real, but for $E < V(x)$ it acquires an imaginary part and becomes complex. Therefore, by allowing complex time, classically forbidden regions can be studied in the semi-classical approach. The tunneling of a point particle with energy E coming from left from the potential $V(x)$ is shown in Fig. 2. For $x > x_2$ time is real, because $E > V(x)$; however, for $x_1 < x < x_2$, the time goes in the imaginary direction. For $x < x_1$ time becomes complex, $t + i\tau_0$, where the constant imaginary part is

$$\tau_0 = \int_{x_1}^{x_2} \frac{dx\sqrt{m}}{\sqrt{2\left(V(x) - E\right)}}. \tag{A6}$$

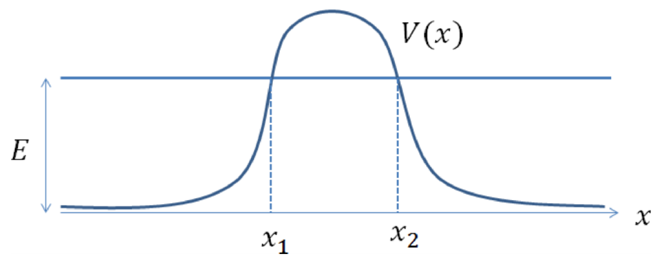

FIG. 2. Potential barrier for a particle moving from left to right with energy E. Classical turning points are indicated by x_1 and x_2. According to Eq. (A5), for $x_1 < x < x_2$, the time becomes complex.

Therefore, for a tunneling path of a particle moving from left to right, the time evolution is depicted in Fig. 3 by contour C_+.

According to the semi-classical description, the tunneling amplitude is found by calculating the action Eq. (A2) along contour C_+. In order to find the tunneling probability amplitude the contour C_- needs to be added, where the property $x(t^*) = x^*(t)$ has been used. Therefore, the tunneling probability with exponential accuracy is given by

$$\Gamma \approx \exp\left(-S_0\right), S_0 = -i \int_{C_-+C_+} dt \left[\frac{m}{2}\left(\frac{dx}{dt}\right)^2 - V(x) + E\right], \tag{A7}$$

where $x(t)$ is the solution to the classical equation of motion, i.e. Eq. (A3), along the contour.

The horizontal segments of C_+ and C_- cancel each other and only vertical segments corresponding to the under barrier motion survive. Using Eq. (A3) in the exponent of Eq. (A7) we get

$$\Gamma \approx \exp\left(i \int_{i\tau_0}^{-i\tau_0} dt\, m\dot{x}^2\right) = \exp\left(2im \int_{x_1}^{x_2} dx\dot{x}\right) = \exp\left(-2 \int_{x_1}^{x_2} dx\sqrt{2m(V(x)-E)}\right), \tag{A8}$$

which is the well-known WKB result in quantum mechanics.[41]

If the system is in thermodynamic equilibrium before the tunneling starts, then the tunneling probability needs to be statistically averaged over E

$$\langle\Gamma\rangle = \int dE \exp\left[-\frac{E}{k_B T} - S(E)\right], \tag{A9}$$

where E is given by Eq. (A7). The largest probability of tunneling occurs for energies that minimizes the exponent in Eq. (A9) and is given by

$$\frac{\partial S(E)}{\partial E} = -\frac{1}{k_B T}. \tag{A10}$$

The action for the underbarrier motion is given by

$$S(E) = 2 \int_{x_1}^{x_2} dx\sqrt{2m(V(x)-E)}, \tag{A11}$$

and the energy derivative of the action yields

$$\frac{\partial S(E)}{\partial E} = -2 \int_{x_1}^{x_2} dx \frac{\sqrt{m}}{\sqrt{2(V(x)-E)}} = -2\tau_0 \tag{A12}$$

where τ_0 is the time of the under barrier motion given by Eq. (A6). Comparing Eqs. (A10) and (A12) reveals that

$$\tau_0 = \frac{1}{2k_B T}. \tag{A13}$$

Eq. (A12) determines the energy of the tunneling particle. Therefore, in equilibrium, the probability of tunneling through the barrier is given by Eq. (A7) for real trajectories that satisfy Eq. (A13). The

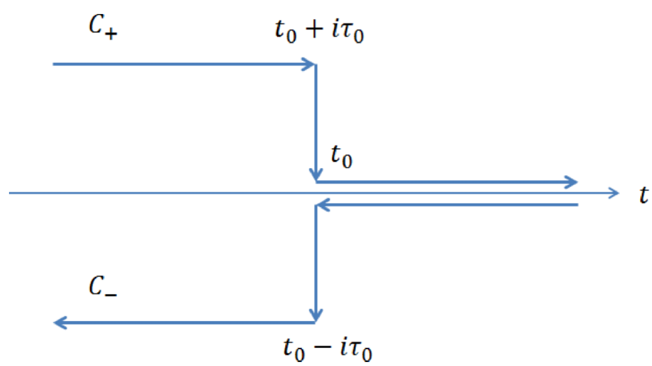

FIG. 3. The integration contour for the quantum tunneling probability. The vertical sections correspond to the underbarrier motion.

real trajectories condition comes from the analysis that shows that the time-averaged probability for semiclassical processes is entirely determined by real trajectories.[25]

The semi-classical method in which time can take on complex values is suitable for generalization to include tunneling from time dependent potentials. tunneling from periodically modulated potential barriers is the most common application of this method and since in this paper we are interested in sinusoidal alternating field we restrict this section of this review to potentials of the form[18,25–27]

$$U(x,t) = V(x) + \mathcal{E}x\cos\Omega t. \tag{A14}$$

According to the semiclassical description, the linear in the field-strength correction to the tunneling probability is then given by

$$S_1 = -i\mathcal{E}\int_{C_-+C_+} dt\, x(t)\cos\Omega t, \tag{A15}$$

where $x(t)$ is the solution of the unperturbed equation of motion Eq.(A3) along the contour. During the under-barrier motion in the vertical segment of Fig. 3, time is imaginary $t = i\tau$ and therefore the equation of motion becomes

$$m\frac{d^2x}{d\tau^2} - \frac{dV(x)}{dx} = 0, \tag{A16}$$

which in comparison to Eq. (A3) can be interpreted as the classical equation of motion in the inverted potential.

The contour in Eq. (A15) may be shifted to entails the singularities of the integrand. This enables calculating the integral based on the singularities of the $x(\tau)$. Therefore, the general trend of the A_1 in Eq. (A15) depends on the specific form of the potential. In some cases S_1 can be exponentially large, which is the case we encountered in this study. In order to have exponential enhancement, it is necessary for the function defined as

$$h(x) = \sqrt{E - V(x)}, \tag{A17}$$

to have singularities off the real axis in the x plane.[35] Assuming $V(x)$ has singularities of the form

$$V(x) \approx \begin{cases} \kappa(x - x_s)^\alpha, & \alpha < 0, \quad x \to x_s, \\ \kappa x^\alpha, & \alpha > 0, \quad x \to \infty, \end{cases} \tag{A18}$$

then, the solution to Eq. (A3) near x_s is of the form

$$x(t) = x_s + \left[-\frac{\kappa}{2m}(2-\alpha)^2(t - t_s)^2\right]^{\frac{1}{2-\alpha}}, \tag{A19}$$

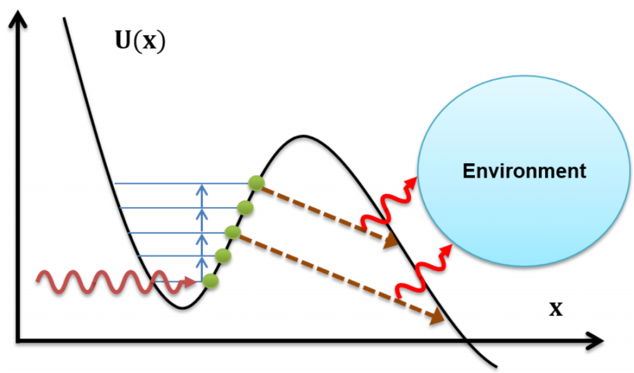

FIG. 4. Dissipative quantum tunneling of a particle from the metastable potential barrier under the influence of a high frequency field is shown. The incident radiation can excite the tunneling field to higher energy states with higher rate of tunneling; however, the probability for being excited to higher energy states would decreases. The dotted arrow indicates tunneling with dissipation where the field loses some energy to the environment. The emerging particle has less energy compared to the energy before tunneling and has to travel longer distance under the barrier.

where t_s is the complex time that takes going from x_2 in Fig. 2 to x_s given by

$$t_s = \int_{x_2}^{x_s} \frac{dx'\sqrt{m}}{\sqrt{2\,(E - V(x'))}}. \tag{A20}$$

By comparing to Eq. (A6), the time of the underbarrier motion τ_0 has the same order of the magnitude as

$$\tau_s = \operatorname{Im} t_s. \tag{A21}$$

but for analytical potentials always $\tau_S < \tau_0$. For $\Omega\tau_s \gg 1$, the main contribution to the integral in Eq. (A15) comes from branch-cut section in the vicinity of the singular points τ_s and τ_s^*. Therefore, the transition probability is given by

$$\Gamma(\mathcal{E},t) = \Gamma_0 \exp\left(a_1 \cos(\Omega t)\right), \tag{A22}$$

where

$$a_1 = \frac{2\pi\mathcal{E}}{\Omega}\left|\Gamma_E\left(\frac{2}{\alpha-2}\right)\right|^{-1}\left[\frac{|\kappa|(2-\alpha)^2}{2m\Omega^2}\right]^{\frac{1}{2-\alpha}} \exp(\Omega\tau_s), \tag{A23}$$

where Γ_E is the Euler Gamma function and Γ_0 is the tunneling rate in the absence of the alternating field. The time averaging of Eq. (A22) gives

$$\overline{\Gamma(\mathcal{E})} = \Gamma_0 \frac{1}{\sqrt{2\pi a_1}} \exp(a_1). \tag{A24}$$

Eq. (A24) shows that the semiclassical description is relevant only when

$$S_0 \gg a_1 \gg 1. \tag{A25}$$

If the condition $S_0 \gg a_1$ is not satisfied, in addition to linear expansion in $\mathcal{E}$, higher order corrections need to be considered.

In this semi-calssical method the probability of multi-photon processes in the enhancement of the quantum tunneling has been automatically included.[27,34] In a multi-photon process, the charge trapped in a local minima can absorb either one or more than one photon and tunnels away. Although, absorbing more than one photons is less probable; however, upon absorption of more photons the particle becomes exited to a higher level from which tunneling is facilitated, thus the probability of quantum tunneling increases (see Fig. 4).

[1] K. Arutyunov, D. Golubev, and A. Zaikin, Physics Reports **464**, 1 (2008).
[2] Journal of Physics: Condensed Matter **20**, 043202 (2008).

[3] A. J. Kerman, New Journal of Physics **15**, 105017 (2013).
[4] J. E. Mooij and Y. V. Nazarov, Nature Physics **2**, 169 (2006).
[5] J. E. Mooij, G. Schn, A. Shnirman, T. Fuse, C. J. P. M. Harmans, H. Rotzinger, and A. H. Verbruggen, New Journal of Physics **17**, 033006 (2015).
[6] J. E. Mooij and C. J. P. M. Harmans, New Journal of Physics **7**, 219 (2005).
[7] O. V. Astafiev, L. B. Ioffe, S. Kafanov, Y. Pashkin, Arutyunov, D. Shahar, O. Cohen, and J. S. Tsai, Nature **484**, 355 (2012).
[8] C. H. Webster, S. Giblin, D. Cox, T. J. B. M. Janssen, and A. Zorin, in *Precision Electromagnetic Measurements Digest, 2008. CPEM 2008. Conference on* (2008), pp. 628–629.
[9] A. M. Hriscu and Y. V. Nazarov, Phys. Rev. B **83**, 174511 (2011).
[10] T. T. Hongisto and A. B. Zorin, Phys. Rev. Lett. **108**, 097001 (2012).
[11] N. Giordano, Phys. Rev. Letters **61**, 2137 (1988).
[12] C. N. Lau, N. Markovic, M. Bockrath, A. Bezryadin, and M. Tinkham, Phys. Rev. Lett. **87**, 217003 (2001).
[13] M. Tinkham, *Introduction to Superconductivity* (New York: McGraw-Hill, 1996).
[14] M. Ma and P. A. Lee, Phys. Rev. B **32**, 5658 (1985).
[15] M. V. Sadovskii, Physics Reports **282**, 225 (1997).
[16] B. Sacépé, T. Dubouchet, C. Chapelier, M. Sanquer, M. Ovadia, D. Shahar, M. Feigel'Man, and L. Ioffe, Nature Physics **7**, 239 (2011).
[17] A. O. Caldeira and A. J. Leggett, Phys. Rev. Lett. **46**, 211 (1981).
[18] A. Caldeira and A. Leggett, Annals of Physics **149**, 374 (1983).
[19] M.-H. Bae, R. C. Dinsmore, M. Sahu, and A. Bezryadin, New Journal of Physics **14**, 043014 (2012).
[20] J. S. Lehtinen, K. Zakharov, and K. Y. Arutyunov, Phys. Rev. Lett. **109**, 187001 (2012).
[21] C. H. Webster, J. C. Fenton, T. T. Hongisto, S. P. Giblin, A. B. Zorin, and P. A. Warburton, Phys. Rev. B **87**, 144510 (2013).
[22] S. Shapiro, Phys. Rev. Lett. **11**, 80 (1963).
[23] A. H. Anderson and P. W. Dayem, Phys. Rev. Lett. 13 (1964).
[24] M.-H. Bae, R. C. Dinsmore, T. Aref, M. Brenner, and A. Bezryadin, Nano Letters **9**, 1889 (2009).
[25] B. I. Ivlev and V. I. Mel'nikov, Sov. Phys. JETP **63**, 1295 (1986).
[26] B. I. Ivlev and V. I. Mel'nikov, Phys. Rev. Lett. **55**, 1614 (1985).
[27] B. I. Ivlev and V. I. Mel'nikov, Sov. Phys. JETP **62**, 1298 (1985).
[28] M. H. Devoret, J. M. Martinis, D. Esteve, and J. Clarke, Phys. Rev. Lett. **53**, 1260 (1984).
[29] G.-L. Ingold and Y. V. Nazarov, e-print cond-mat/0508728 (2005).
[30] M. H. Ansari, F. K. Wilhelm, U. Sinha, and A. Sinha, Superconductor Science and Technology **26**, 125013 (2013).
[31] M. H. Ansari, Superconductor Science and Technology **28**, 045005 (2015).
[32] D. E. McCumber, Journal of Applied Physics **39**, 3113 (1968).
[33] W. C. Stewart, Applied Physics Letters **12**, 277 (1968).
[34] A. I. Larkin and Y. N. Ovchinnikov, Phys. Rev. B **28**, 6281 (1983).
[35] B. I. Ivlev and V. I. Mel'nikov, in *Quantum Tunneling in Condensed Media*, edited by Y. Kagan and A. Leggett (Elsevier Science, 1992).
[36] A. Kamenev, *Field Theory of Non-Equilibrium Systems* (Cambridge University Press, 2011).
[37] A. D. Zaikin, D. S. Golubev, A. van Otterlo, and G. T. Zimanyi, Physics-Uspekhi **41**, 226 (1998).
[38] D. S. Golubev and A. D. Zaikin, Phys. Rev. B **64**, 014504 (2001).
[39] A. D. Zaikin, D. S. Golubev, A. van Otterlo, and G. T. Zimányi, Phys. Rev. Lett. **78**, 1552 (1997).
[40] A. Jafari-Salim, "Superconducting Nanostructures for Quantum Detection of Electromagnetic Radiation," Ph. D. thesis, The University of Waterloo, Waterloo, Canada, 2014.
[41] L. Landau and E. Lifshits, *Quantum Mechanics: Non-relativistic Theory* (Butterworth-Heinemann, 1977).