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RESEARCH ARTICLE

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# Barrier Lypunov functions-based nonsingular fixed-time switching control for strict-feedback nonlinear dynamics with full state constraints

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**Abstract**

This work proposes a nonsingular adaptive fixed-time switching control method for a class of strict-feedback nonlinear dynamics subject to full state constraints. The peculiarity of this design lies in overcoming the singularity issue that typically appears in the existing backstepping-based fixed-time control methods caused by the iterative differentiation of fractional power terms as tracking errors approach to zero, while guaranteeing the nonviolation of full state constraints. Crucial in solving such singularity issue is to skillfully introduce a smooth switching between fractional power and integer power terms, which guarantees that fractional power term is confined within a positive interval all the time. An asymmetric time-varying barrier Lyapunov function is delicately incorporated into control design, rendering state variables to be within prescribed time-varying bounds. Besides, radial basis function neural network is employed to handle system unknown nonlinearities. It is rigorously proved that all the closed-loop signals eventually converge to small regions around origin within fixed-time. Comparative simulation results are finally given to validate the effectiveness and superiority of the proposed control strategy.

**KEYWORDS**

adaptive backstepping control, fixed-time stability, switching control, time-varying state constraints

## 1 | INTRODUCTION

Over the past few decades, the tracking problem of uncertain nonlinear systems has received considerable attention, due to its significance both in theory and practice. In addition, as an iterative design method, backstepping technique has provided a structured and systematic method for Lyapunov function design of complex nonlinear systems.[1-3] To go further, universal approximators such as neural network (NN) and fuzzy-logic system have been introduced to handle unknown dynamics.[4-6] Approximation-based adaptive backstepping controller design has been widely applied in industrial control systems including robot control,[7] flight control,[8-10] and so forth.[11-13]

In order to obtain a satisfactory tracking performance, there are many transient response indexes and steady-state indexes that are required to be taken into account. As one of the important factors that affects tracking performance, convergence rate has attracted extensive interest. It is worth mentioning that conventional works are solely able to make the system asymptotically or exponentially stable, that is, closed-loop signals converge to a residual set when time goes to infinity. Finite-time stability is therefore proposed to provide an upper bound of convergence time, while guaranteeing closed-loop stability. Based on finite-time stability, massive results have been acquired.[14,15] To list a few: in Reference 16, the finite-time adaptive fuzzy tracking control problem of pure-feedback nonlinear systems was addressed, and a criterion of semiglobal practical stability was first formulated. By fusing the command filter and backstepping control technique, the problem of finite-time control for a class of uncertain nonlinear systems with unknown actuator faults was studied in Reference 17. However, it has to be noted that the upper bound of convergence time in the above-mentioned works relies on their initial conditions, and the settling time might become larger when initial values stay far away from the equilibrium. To remove the dependence of convergence time on initial conditions, fixed-time stability is thus developed and has been attached tremendous attention since it was proposed.[18,19] For instance, a decentralized adaptive fuzzy fixed-time control design was given in Reference 20 for interconnected nonlinear systems. An adaptive practical fixed-time control strategy was investigated for strict feedback nonlinear systems in Reference 21. By employing tan-type barrier Lyapunov function (BLF) and NN techniques,[22] considered fixed-time control of nonstrict-feedback nonlinear system subject to dead zone and output constraint.

However, it is worth emphasizing that the main challenge of incorporating fixed-time control method under backstepping framework comes from the fact that the derivatives of intermediate control variables might become infinity as the tracking errors approach to zero, leading to a singularity issue. This is because the negative power terms appear in the derivatives of intermediate control variables.

On the other hand, state constraints are typically required to be satisfied in practical systems since their transgression might cause system performance degradation or even system instability.[23,24] Therefore, remaining the states within prescribed bounds has become a major research topic for the sake of safety consideration. BLF has been a powerful tool in guaranteeing full state constraints and some commonly seen forms include tan-type BLF,[25] log-type BLF,[26] and integral-type BLF.[27] Generally, most of the existing results are concentrated on strict/pure feedback nonlinear systems with output constraints or full state constraints.[28,29] In Reference 30, the asymptotic control laws were presented for single-input single-output nonlinear systems, while the symmetric and asymmetric BLFs were explored to prevent the output constraint violation. Later on, an asymmetric time-varying BLFs was employed to ensure output constraint satisfaction in Reference 31. Aiming at improving convergence speed, an adaptive finite-time tracking control strategy was investigated for strict-feedback nonlinear systems subject to time-invariant full state constraints and dead-zone in Reference 32. Independent of the initial conditions, an adaptive fixed-time control scheme was studied for nonlinear systems with the time-invariant full state constraints in Reference 33. However, these results fail to work in the case that fixed-time control and asymmetric time-varying full constrained states are considered simultaneously.

Motivated by the aforementioned discussion, this article focuses on the nonsingular fixed-time tracking control problem for a class of strict-feedback nonlinear systems with asymmetric time-varying full state constraints. The main contributions can be summarized as follows.

1. This article, to our best knowledge, presents a pioneering result about nonsingular fixed-time control of strict-feedback nonlinear systems subject to asymmetric time-varying full state constraints.
2. We devise a novel differentiable fixed-time adaptive control scheme by introducing a smooth switching between fractional power and integer power terms. Among this switching, the fractional power term used in controller design is confined within a positive interval, which can avoid the singularity issue that the negative power terms increase to infinity as the tracking errors approach to zero. Such singularity issue might appear in the derivatives of fractional power terms by incorporating fixed-time control method under backstepping framework.
3. In addition to guaranteeing fixed-time convergence, the proposed fixed-time design relies on a new corollary which first proves a smaller upper bound of convergence time than the existing literature.

The rest of this article is organized as follows. In Section 2, problem formulation and preliminaries are provided. In Section 3, the fixed-time tracking control scheme with full state constraints is developed. Subsequently, the stability analysis is given in Section 4. In Section 5, a numerical example and a practical example are, respectively, performed to demonstrate the effectiveness of the developed scheme. Finally, Section 6 concludes the work.

## 2 | PROBLEM DESCRIPTION AND PRELIMINARIES

Consider a class of nonlinear strict-feedback systems given by

$$\begin{cases} \dot{\chi}_i = f_i(\overline{\chi}_i) + g_i(\overline{\chi}_i)x_{i+1} + d_i(\overline{\chi}_i) \\ \dot{\chi}_n = f_n(\chi) + g_n(\chi)u + d_n(\chi) \\ y = \chi_1 \end{cases}, \tag{1}$$

where $\overline{\chi}_i = [\chi_1, \chi_2, \dots, \chi_i]^T \in R^i$ and $\chi = [\chi_1, \chi_2, \dots, \chi_n]^T \in R^n$ denote the state variables of the system, $u \in R$ is system control input and $y \in R$ is system output. $f_i(\overline{\chi}_i), i = 1, 2 \dots, n$ are unknown differentiable system functions, $g_i(\overline{\chi}_i)$ represent the unknown differentiable control-gain functions, $d_i(\overline{\chi}_i)$ denote the external disturbance and system uncertainties. Particularly, all the system states $\chi_i$ are constrained in the compact set $\Omega_\chi := \left\{ \chi \in R^n : \underline{k}_{c_i}(t) < \chi_i < \overline{k}_{c_i}(t) \right\}$ such that $\overline{k}_{c_i}(t) > \underline{k}_{c_i}(t)$, where $\overline{k}_{c_i}(t)$ and $\underline{k}_{c_i}(t)$ are prescribed time-varying constraint functions.

The control objective is to design an adaptive fixed-time tracking controller such that not only the system output $y$ can follow the desired trajectory $y_d$ in a fixed time, but also all the states do not violate the constrained set.

**Assumption 1.** (34) The desired trajectory $y_d$ is smooth, and its $n$ th order derivatives $y_d$, $\dot{y}_d$, and $\ddot{y}_d$ are bounded and satisfy $\Omega_0 := \left\{ \left[y_d, \dot{y}_d, \ddot{y}_d\right]^T \left| y_d^2 + \dot{y}_d^2 + \ddot{y}_d^2 \le B_0 \right. \right\}$, where $B_0$ is a positive constant. For any $t > 0$, there exist functions $\overline{Y}_0(t)$ and $\underline{Y}_0(t)$ such that the desired trajectory $y_d$ satisfy $\underline{Y}_0(t) \le y_d(t) \le \overline{Y}_0(t)$ and $\overline{Y}_0(t) < \overline{k}_{c_1}(t)$, $\underline{Y}_0(t) > \underline{k}_{c_1}(t)$. Moreover, the time derivatives of the desired trajectory satisfy $\underline{Y}_i(t) \le y_d^{(i)}(t) \le \overline{Y}_i(t)$, $i = 1, 2, \dots, n$.

**Assumption 2.** (35) There exist known positive constants $g_{im}$ and $g_{iM}$ such that $0 < g_{im} \le g_i(\overline{\chi}_i) \le g_{iM}$.

**Assumption 3.** (36) For the disturbance term $d_i(\overline{\chi}_i)$, there exist unknown positive constants $d_{Mi}$ such that $\left|d_i(\overline{\chi}_i)\right| \le d_{Mi}$, $i = 1, 2, \dots, n$.

**Lemma 1.** (37) *Suppose that there exists a continuous positive definite and radially unbounded function $V(\chi(t)) : R^\ell \to R^+ \cup \{0\}$ such that $\dot{V}(\chi(t)) \le -(\alpha V^p(\chi(t)) + \beta V^q(\chi(t)))^k + \eta$ for constants $\alpha, \beta, p, q, k, \eta > 0$ satisfying $pk < 1, qk > 1$, the residual set of the system solution with $0 < \theta_0 < 1$ is represented as*

$$\left\{ \lim_{t \to T_r} |V(\chi(t)) \le \min \left\{ \alpha^{-1/p} \left[\eta / \left(1 - \theta_0^k\right)\right]^{1/(pk)}, \beta^{-1/q} \left[\eta / \left(1 - \theta_0^k\right)\right]^{1/(qk)} \right\} \right\}. \tag{2}$$

*Furthermore, the origin is fixed-time stable with the convergence time being defined as*

$$T_r(\chi(0)) \le T_{\max} := \frac{1}{\theta_0^k \alpha^k (1 - pk)} + \frac{1}{\theta_0^k \beta^k (qk - 1)}. \tag{3}$$

**Corollary 1.** *Suppose that there exists a continuous positive definite and radially unbounded function $V(\chi(t))$ satisfying same conditions as Lemma 1, then the system is fixed-time stable. Furthermore, a smaller convergence time $T_r$ is given below*

$$T_r(\chi(0)) \le T_{\max} := \frac{1}{\theta_0^k \alpha^k (1 - pk)} \left(\frac{\alpha}{\beta}\right)^{\frac{1-pk}{q-p}} + \frac{1}{\theta_0^k \beta^k (qk - 1)} \left(\frac{\alpha}{\beta}\right)^{\frac{1-qk}{q-p}}. \tag{4}$$

*Proof.* In light of $\dot{V}(\chi(t)) \le -(\alpha V^p(\chi(t)) + \beta V^q(\chi(t)))^k + \eta$, we obtain for $V(\chi(t)) \le c$, it has $\dot{V}(\chi(t)) \le -\alpha^k V^{pk}(\chi(t)) + \eta$; For $V(\chi(t)) \ge c > 0$, it has $\dot{V}(\chi(t)) \le -\beta^k V^{qk}(\chi(t)) + \eta$.

Note that there exist $0 < \theta_0 < 1$, so the above inequality can be further expressed as the form that for $V(\chi(t)) \le c$, it obtains $\dot{V}(\chi(t)) \le -\theta_0^k \alpha^k V^{pk}(\chi(t)) - \left(1 - \theta_0^k\right) \alpha^k V^{pk}(\chi(t)) + \eta$; And for the case that $V(\chi(t)) \ge c > 0$, one can obtain that $\dot{V}(\chi(t)) \le -\theta_0^k \beta^k V^{qk}(\chi(t)) - \left(1 - \theta_0^k\right) \beta^k V^{qk}(\chi(t)) + \eta$.

To begin with, for any $\chi(t)$ such that $V(\chi(0)) \ge c$, when $V^{qk}(\chi(0)) > \frac{\eta}{(1-\theta_0^k)\beta^k}$, the inequality $\dot{V}(\chi(t)) \le -\theta_0^k \beta^k V^{qk}(\chi(t))$ exists. Divide both sides by $V^{qk}(\chi(t))$, and then integrating it over time $t$, one gets $\int_0^{T_1} \frac{\dot{V}(\chi(t))}{V^{qk}(\chi(t))} dt \le -\int_0^{T_1} \theta_0^k \beta^k dt$, that is to say $\frac{1}{1-qk}\left[V^{1-qk}(\chi(T_1)) - V^{1-qk}(\chi(0))\right] \leq -\theta_0^k\beta^k T_1$. To go further, we can rewrite the above inequality in the form of $T_1 \leq \frac{1}{\theta_0^k\beta^k(qk-1)}\left[V^{1-qk}(\chi(T_1)) - V^{1-qk}(\chi(0))\right] \leq \frac{1}{\theta_0^k\beta^k(qk-1)}V^{1-qk}(\chi(T_1))$.

When taking value $V(\chi(T_1)) = c$, we have

$$T_1 \leq \frac{1}{\theta_0^k\beta^k(qk-1)}c^{1-qk}. \tag{5}$$

Since $V(\chi(t))$ is a decreasing function, we can guarantee that for time $t \geq \frac{1}{\theta_0^k\beta^k(qk-1)}c^{1-qk}$, it holds that $V(\chi(t)) \leq c$.

Next, for any $\chi(t)$ such that $V(\chi(t)) \leq c$, the following two cases is considered. Denote $\Omega_\chi = \left\{\chi | V^{pk}(\chi(t)) \leq \frac{\eta}{(1-\theta_0^k)\alpha^k}\right\}$ and $\overline{\Omega}_\chi = \left\{\chi | V^{pk}(\chi(t)) > \frac{\eta}{(1-\theta_0^k)\alpha^k}\right\}$.

*Case 1* If $\chi(t) \in \overline{\Omega}_\chi$, the inequality $\dot{V}(\chi(t)) \leq -\theta_0^k\alpha^k V^{pk}(\chi(t))$ exists.

Similar to the above proof process, in order to get $\chi(t) \in \Omega_\chi$, then we can infer

$$\frac{1}{1-pk}\left[V^{1-pk}(\chi(T_r)) - V^{1-pk}(\chi(T_1))\right] \leq -\theta_0^k\alpha^k(T_r - T_1). \tag{6}$$

Then, the settling time is estimated by

$$T_r \leq T_1 + \frac{1}{\theta_0^k\alpha^k(1-pk)}\left[V^{1-pk}(\chi(T_1)) - V^{1-pk}(\chi(T_r))\right] \leq \frac{1}{\theta_0^k\alpha^k(1-pk)}c^{1-pk} + \frac{1}{\theta_0^k\beta^k(qk-1)}c^{1-qk}. \tag{7}$$

*Case 2* If $\chi(t) \in \Omega_\chi$, then the trajectory of $\chi(t)$ does not exceed the set $\Omega_\chi$.

To sum up, the settling time to reach the set $\Omega_\chi$ is bounded by a maximum value.

Define the function

$$T_r(c) = \frac{1}{\theta_0^k\alpha^k(1-pk)}c^{1-pk} + \frac{1}{\theta_0^k\beta^k(qk-1)}c^{1-qk}. \tag{8}$$

Let $T_r'(c) = 0$, and we can obtain $c^* = \left(\frac{\alpha}{\beta}\right)^{\frac{1}{q-p}}$.

Thus, the minimum settling time is given by $T_r(\chi(0)) \leq T_{\max} := T_r(c^*) = \frac{1}{\theta_0^k\alpha^k(1-pk)}\left(\frac{\alpha}{\beta}\right)^{\frac{1-pk}{q-p}} + \frac{1}{\theta_0^k\beta^k(qk-1)}\left(\frac{\alpha}{\beta}\right)^{\frac{1-qk}{q-p}}$ ∎

*Remark* 1. In the current work on fixed-time control, numerous research results are based on the classic fixed-time stability theory proposed by Polyakov,[18] in which the convergence time is described as $T_{\max} := \frac{1}{\alpha^k(1-pk)} + \frac{1}{\beta^k(qk-1)}$. However, only the special case that $c = 1$ in (8) is considered to get this settling time, leading to an incomplete analysis result. Actually, there exists a smaller convergence time, so an new upper bound of fixed-time $T_r(c^*)$ is derived in Corollary 1. It is noteworthy that according to the definition of the function $T_r(c)$, we can seen that its second-order derivative $T_r^{''}(c) > 0$, that is to say, the function $T_r(c)$ is a concave function. Moreover, there is just one point where its derivative is zero, so this stagnation point $c^* = (\alpha/\beta)^{\frac{1}{q-p}}$ must be the minimum point of the function and $T_r(c^*) \leq T_r(1)$. Therefore, it is proved for the first time that a smaller upper bound of convergence time can be obtained under conditions in Lemma 1, and this time is smaller than the time revealed by the existing fixed-time control schemes.[38]

*Remark* 2. It is worth noting that there is no loss of generality in Polyakov's derivation of this bound. The Lyapunov function can also be redefined as $W(\chi(t)) = \frac{1}{c}V(\chi(t))$, then the settling time can be estimated by $t \geq \frac{1}{\theta_0^k\alpha^k c^{pk-1}(1-pk)} + \frac{1}{\theta_0^k\beta^k c^{qk-1}(qk-1)}$ with $0 < \theta_0 < 1$. In addition to the classic fixed-time derivation result, the new function $T_r(c) = \frac{1}{\theta_0^k\alpha^k(1-pk)}c^{1-pk} + \frac{1}{\theta_0^k\beta^k(qk-1)}c^{1-qk}$ is defined and the concavity and convexity of this function is analyzed. As a result, the minimum settling time is given through rigorous and complete proof which is the main contribution that distinguish from the traditional fixed-time control result.

**Lemma 2.** (39) *For $x, y \in R$, and any real numbers $c, d, \gamma > 0$, it holds that*

$$|x|^c|y|^d \leq \frac{c}{c+d}\gamma|x|^{c+d} + \frac{d}{c+d}\gamma^{-\frac{c}{d}}|y|^{c+d}. \tag{9}$$

**Corollary 2.** *For $\Phi \in R^+$, and any real numbers $0 < h < 1$, the following holds*

$$\Phi^h \leq \Delta(h) + \Phi, \tag{10}$$

*where $\Delta(h) = (1-h)h^{\frac{h}{1-h}} > 0$.*

*Proof.* Inequality (10) is deduced from Lemma 2. In view of inequality $|x|^c|y|^d \leq \frac{c}{c+d}\gamma|x|^{c+d} + \frac{d}{c+d}\gamma^{-\frac{c}{d}}|y|^{c+d}$, choose $x = 1$, $y = \Phi$, $c = 1-h$, $d = h$, $\gamma = e^{(h/(1-h))\ln h}$, thus we can get $\Phi^h \leq \Delta(h) + \Phi$. ■

**Lemma 3.** (40) *Consider $\chi_i \in \mathbf{R}$, $i = 1, 2, \ldots, n$, and $0 < p < 1$, one gets*

$$\left(\sum_{i=1}^{n}|\chi_i|\right)^p \leq \sum_{i=1}^{n}|\chi_i|^p \leq n^{1-p}\left(\sum_{i=1}^{n}|\chi_i|\right)^p. \tag{11}$$

*In addition, the following inequality holds:*

$$\sum_{i=1}^{n}\chi_i^2 \leq \left(\sum_{i=1}^{n}\chi_i\right)^2 \leq n\sum_{i=1}^{n}\chi_i^2. \tag{12}$$

**Lemma 4.** (41) *The radial basis function neural network (RBFNN) is utilized to approximate the continuous function $S_i(Z_i)$ over a compact set $\Omega_{Z_i} \subset R^n$ as*

$$S_i(Z_i) = \Theta_i^{*T}\psi_i(Z_i) + \varepsilon_i(Z_i), \quad \forall Z_i \in \Omega_{Z_i} \subset R^n, \tag{13}$$

*where $\Theta_i^*$ is ideal weight vector, $\varepsilon_i(Z_i)$ is the approximation error such that $\|\varepsilon_i(Z_i)\| \leq \varepsilon_{Mi}$ with $\varepsilon_{Mi} > 0$ is an unknown constant, and $\psi_i(Z_i)$ is chosen as the commonly used Gaussian functions*

$$\psi_i(Z_i) = \exp\left[\frac{-(Z_i-\varphi_i)^T(Z_i-\varphi_i)}{\varpi_i^2}\right], \quad i = 1, 2, \ldots, w, \tag{14}$$

*where $\varphi_i$, $\varpi_i$, and $w$ represent the center, width, and number of the Gaussian function, respectively.*

## 3 | FIXED-TIME ADAPTIVE TRACKING CONTROL DESIGN

We start the design by giving the following coordinate transformation

$$\begin{cases} e_1 = \chi_1 - y_d \\ e_i = \chi_i - \alpha_{i-1}, i = 2, 3, \ldots, n \end{cases}, \tag{15}$$

where $e_1$ is the tracking error and $\alpha_{i-1}$ is the virtual control input that will be designed later.

The recursive design procedure contains $n$ steps. First, at each step of the backstepping design, the intermediate control $\alpha_{i-1}$ is designed to make the corresponding subsystem toward equilibrium. And at the final step, the stabilization of system can be achieved with the actual control input $u$ to be designed.

*Step 1*: In the design, the following definition will be needed.

$$r_i(e_i) = \begin{cases} 1, & e_i > 0 \\ 0, & e_i \leq 0 \end{cases}. \tag{16}$$

It can be easily seen that $r_i(e_i) = r_i^2(e_i)$ and $1 - r_i(e_i) = (1 - r_i(e_i))^2$.

Due to Assumption 1, the positive time-varying barrier functions are given by

$$k_{a_1}(t) := y_d(t) - \underline{k}_{c_1}(t), \quad k_{b_1}(t) := \overline{k}_{c_1}(t) - y_d(t), \tag{17}$$

where $\underline{k}_{b_1} \leq k_{b_1}(t) \leq \overline{k}_{b_1}, \underline{k}_{a_1} \leq k_{a_1}(t) \leq \overline{k}_{a_1}$ with positive constants $\underline{k}_{b_1}$, $\overline{k}_{b_1}$, $\underline{k}_{a_1}$, and $\overline{k}_{a_1}$.

By a change of error coordinates, we arrive at

$$\zeta_{a_1}(t)=\frac{e_1}{k_{a_1}(t)},\quad \zeta_{b_1}(t)=\frac{e_1}{k_{b_1}(t)},\quad \zeta_1(t)=r_1(e_1)\zeta_{b_1}(t)+(1-r_1(e_1))\zeta_{a_1}(t). \tag{18}$$

Consider the following BLF candidate:

$$L_1=\frac{r_1(e_1)}{2}\log\frac{k_{b_1}^2(t)}{k_{b_1}^2(t)-e_1^2}+\frac{1-r_1(e_1)}{2}\log\frac{k_{a_1}^2(t)}{k_{a_1}^2(t)-e_1^2}, \tag{19}$$

where log(•) represents the natural logarithm of •.

Then we can rewrite (19) into a form that does not depend explicitly on time

$$L_1=\frac{r_1(e_1)}{2}\log\frac{1}{1-\zeta_{b_1}^2(t)}+\frac{1-r_1(e_1)}{2}\log\frac{1}{1-\zeta_{a_1}^2(t)}=\frac{1}{2}\log\frac{1}{1-\zeta_1^2(t)}. \tag{20}$$

And it is clear that $L_1$ is positive definite and continuously differentiable in the set $|\zeta_1(t)|<1$.

Considering the following subsystem of (1) and noting $e_1=\chi_1-y_d$, one gets

$$\dot{e}_1=f_1(\chi_1)+g_1(\chi_1)\chi_2+d_1(\chi_1)-\dot{y}_d, \tag{21}$$

where $\chi_2$ is virtual control input.

To move on, the time derivative of $L_1$ along (19) is

$$\begin{aligned}\dot{L}_1=&\frac{r_1(e_1)\zeta_{b_1}(t)}{k_{b_1}(t)\left(1-\zeta_{b_1}^2(t)\right)}\left(S_1(Z_1)+g_1(x_1)x_2+d_1(x_1)-e_1\frac{\dot{k}_{b_1}}{k_{b_1}}\right)\\&+\frac{(1-r_1(e_1))\zeta_{a_1}(t)}{k_{a_1}(t)\left(1-\zeta_{a_1}^2(t)\right)}\left(S_1(Z_1)+g_1(x_1)x_2+d_1(x_1)-e_1\frac{\dot{k}_{a_1}}{k_{a_1}}\right),\end{aligned} \tag{22}$$

where $S_1(Z_1)=f_1(x_1)-\dot{y}_d$ with $Z_1=\left[\chi_1,\dot{y}_d\right]\in R^2$.

By employing the NN in the general form of (13) to approximate $S_1(Z_1)$, one obtains

$$\begin{aligned}\dot{L}_1=&\frac{r_1(e_1)e_1}{k_{b_1}^2(t)-e_1^2}\left(\Theta_1^{*T}\psi_1(Z_1)+\varepsilon_1(Z_1)+g_1(\chi_1)\chi_2+d_1(\chi_1)-e_1\frac{\dot{k}_{b_1}(t)}{k_{b_1}(t)}\right)\\&+\frac{(1-r_1(e_1))e_1}{k_{a_1}^2(t)-e_1^2}\left(\Theta_1^{*T}\psi_1(Z_1)+\varepsilon_1(Z_1)+g_1(\chi_1)\chi_2+d_1(\chi_1)-e_1\frac{\dot{k}_{a_1}(t)}{k_{a_1}(t)}\right)\\&\leq r_1(e_1)\left(\frac{e_1g_1(\chi_1)\chi_2}{k_{b_1}^2(t)-e_1^2}+\frac{e_1^2\theta_1\psi_1^T(Z_1)\psi_1(Z_1)}{2l_1\left(k_{b_1}^2(t)-e_1^2\right)^2}+\frac{l_1}{2}+\frac{e_1[\varepsilon_1(Z_1)+d_1(\chi_1)]}{k_{b_1}^2(t)-e_1^2}-\frac{e_1^2}{k_{b_1}^2(t)-e_1^2}\frac{\dot{k}_{b_1}(t)}{k_{b_1}(t)}\right)\\&+(1-r_1(e_1))\left(\frac{e_1g_1(\chi_1)\chi_2}{k_{a_1}^2(t)-e_1^2}+\frac{e_1^2\theta_1\psi_1^T(Z_1)\psi_1(Z_1)}{2l_1\left(k_{a_1}^2(t)-e_1^2\right)^2}+\frac{l_1}{2}+\frac{e_1[\varepsilon_1(Z_1)+d_1(\chi_1)]}{k_{a_1}^2(t)-e_1^2}-\frac{e_1^2}{k_{a_1}^2(t)-e_1^2}\frac{\dot{k}_{a_1}(t)}{k_{a_1}(t)}\right),\end{aligned} \tag{23}$$

where $l_1$ is a positive design parameter.

Define $\theta_i=\left\|\Theta_i^*\right\|^2$ and the generalized NN weight estimation error $\tilde{\theta}_i$ as $\tilde{\theta}_i=\theta_i-g_{im}\hat{\theta}_i$, where $\hat{\theta}_i$ is the estimate of $\theta_i$.

Devise the virtual fixed-time control $\alpha_1$ as follows:

$$\begin{aligned}\alpha_1=&(1-r_1(e_1))\left[-c_1\frac{e_1^3}{k_{a_1}^2(t)-e_1^2}-\lambda_1\frac{e_1}{k_{a_1}^2(t)-e_1^2}-\frac{e_1\hat{\theta}_1\psi_1^T(Z_1)\psi_1(Z_1)}{2l_1\left(k_{a_1}^2(t)-e_1^2\right)}-\hbar_1(t)e_1-\kappa_1\mathfrak{I}_1(e_1)\right]\\&+r_1(e_1)\left[-c_1\frac{e_1^3}{k_{b_1}^2(t)-e_1^2}-\lambda_1\frac{e_1}{k_{b_1}^2(t)-e_1^2}-\frac{e_1\hat{\theta}_1\psi_1^T(Z_1)\psi_1(Z_1)}{2l_1\left(k_{b_1}^2(t)-e_1^2\right)}-\hbar_1(t)e_1-\kappa_1\mathfrak{I}_1(e_1)\right],\end{aligned} \tag{24}$$

where $c_1$, $\lambda_1$, and $\kappa_1$ are the positive design parameters.

The time-varying gain $\hbar_1(t)$ is given by

$$\hbar_1(t) = \frac{1}{g_{1m}} \sqrt{\left(\frac{\dot{k}_{a_1}(t)}{k_{a_1}(t)}\right)^2 + \left(\frac{\dot{k}_{b_1}(t)}{k_{b_1}(t)}\right)^2 + o_1}, \tag{25}$$

where $o_1$ is a positive design parameter.

*Remark* 3. Currently the BLF-based adaptive control schemes mainly focus on tacking state constraints problem, which prescribed state constraints are usually defined to be time-invariance or symmetric. By contrast, we permit the barriers to vary with the desired trajectory in time and an asymmetric BLF is employed in fixed-time control. To realize the objective of time-varying state constraints, a change of tracking error coordinates is used to eliminate the dependence explicitly on time, and the time-varying gain is introduced to compensate the effect brought by dynamic barriers.

And the smooth switching law $\mathfrak{I}_1(e_1)$ is defined as

$$\mathfrak{I}_1(e_1) = \begin{cases} e_1^{2h-1}\left[r_1(e_1)\left(\left(k_{b_1}^2(t) - e_1^2\right)^{1-h}\right) + (1 - r_1(e_1))\left(\left(k_{a_1}^2(t) - e_1^2\right)^{1-h}\right)\right], & if\ |e_1| \geq \varsigma_1 \\ r_1(e_1)\left(\mu_{11}e_1 + \nu_{11}e_1^3\right) + (1 - r_1(e_1))\left(\mu_{12}e_1 + \nu_{12}e_1^3\right), & if\ |e_1| < \varsigma_1 \end{cases}, \tag{26}$$

with $\mu_{11} = \varsigma_1^{2h-2}\left(k_{b_1}^2(t) - \varsigma_1^2\right)^{1-h} - \nu_{11}\varsigma_1^2$, $\nu_{11} = (h-1)\varsigma_1^{2(h-2)}\left[\left(k_{b_1}^2(t) - \varsigma_1^2\right)^{1-h} + \varsigma_1^2\left(k_{b_1}^2(t) - \varsigma_1^2\right)^{-h}\right]$, $\mu_{12} = \varsigma_1^{2h-2}\left(k_{a_1}^2(t) - \varsigma_1^2\right)^{1-h} - \nu_{12}\varsigma_1^2$, $\nu_{12} = (h-1)\varsigma_1^{2(h-2)}\left[\left(k_{a_1}^2(t) - \varsigma_1^2\right)^{1-h} + \varsigma_1^2\left(k_{a_1}^2(t) - \varsigma_1^2\right)^{-h}\right]$, and $\varsigma_1$ is a small positive parameter satisfying $\varsigma_1 < k_{b_1}(t)$, $\varsigma_1 < k_{a_1}(t)$. From $0 < h < 1$, we have $\nu_{11}, \nu_{12} < 0$ and $\mu_{11}, \mu_{12} > 0$.

The estimation $\hat{\theta}_1$ is determined by the following adaptive control law:

$$\dot{\hat{\theta}}_1 = \rho_1\left(-\sigma_{11}\hat{\theta}_1 - \sigma_{12}\hat{\theta}_1^3 + r_1(e_1)\frac{e_1^2\psi_1^T(Z_1)\psi_1(Z_1)}{2l_1\left(k_{b_1}^2(t) - e_1^2\right)^2} + (1 - r_1(e_1))\frac{e_1^2\psi_1^T(Z_1)\psi_1(Z_1)}{2l_1\left(k_{a_1}^2(t) - e_1^2\right)^2}\right), \tag{27}$$

where $\rho_1$, $\sigma_{11}$, and $\sigma_{12}$ are the positive design parameters to be specified later. It can be inferred that $\hat{\theta}_1 \geq 0$ for $\forall t > 0$ after choosing $\hat{\theta}_1(0) \geq 0$.

Invoking $x_2 = e_2 + \alpha_1$, $r_1(e_1) \cdot (1 - r_1(e_1)) = 0$ and Assumption 2, substituting (24) into (23), we obtain

$$\begin{aligned}
\dot{L}_1 \leq{}& (1 - r_1(e_1))\left(\frac{g_1(\chi_1)e_1e_2}{k_{a_1}^2(t) - e_1^2} - \frac{c_1g_{1m}e_1^4}{\left(k_{a_1}^2(t) - e_1^2\right)^2} - \frac{\lambda_1g_{1m}e_1^2}{\left(k_{a_1}^2(t) - e_1^2\right)^2} - \frac{e_1^2g_{1m}\hat{\theta}_1\psi_1^T(Z_1)\psi_1(Z_1)}{2l_1\left(k_{a_1}^2(t) - e_1^2\right)^2} - \kappa_1g_1(\chi_1)\frac{e_1\mathfrak{I}_1(e_1)}{k_{a_1}^2(t) - e_1^2}\right) \\
&+ (1 - r_1(e_1))\left(\frac{e_1^2\theta_1\psi_1^T(Z_1)\psi_1(Z_1)}{2l_1\left(k_{a_1}^2(t) - e_1^2\right)^2} + \frac{l_1}{2} + \frac{e_1[\varepsilon_1(Z_1) + d_1(\chi_1)]}{k_{a_1}^2(t) - e_1^2} - \hbar_1(t)g_{1m}\frac{e_1^2}{k_{a_1}^2(t) - e_1^2} - \frac{e_1^2}{k_{a_1}^2(t) - e_1^2}\frac{\dot{k}_{a_1}}{k_{a_1}}\right) \\
&+ r_1(e_1)\left(\frac{g_1(\chi_1)e_1e_2}{k_{b_1}^2(t) - e_1^2} - \frac{c_1g_{1m}e_1^4}{\left(k_{b_1}^2(t) - e_1^2\right)^2} - \frac{\lambda_1g_{1m}e_1^2}{\left(k_{b_1}^2(t) - e_1^2\right)^2} - \frac{e_1^2g_{1m}\hat{\theta}_1\psi_1^T(Z_1)\psi_1(Z_1)}{2l_1\left(k_{b_1}^2(t) - e_1^2\right)^2} - \kappa_1g_1(\chi_1)\frac{e_1\mathfrak{I}_1(e_1)}{k_{b_1}^2(t) - e_1^2}\right) \\
&+ r_1(e_1)\left(\frac{e_1^2\theta_1\psi_1^T(Z_1)\psi_1(Z_1)}{2l_1\left(k_{b_1}^2(t) - e_1^2\right)} + \frac{l_1}{2} + \frac{e_1[\varepsilon_1(Z_1) + d_1(\chi_1)]}{k_{b_1}^2(t) - e_1^2} - \hbar_1(t)g_{1m}\frac{e_1^2}{k_{b_1}^2(t) - e_1^2} - \frac{e_1^2}{k_{b_1}^2(t) - e_1^2}\frac{\dot{k}_{b_1}}{k_{b_1}}\right)
\end{aligned} \tag{28}$$

To move on, it further holds that

$$
\begin{aligned}
\dot{L}_1 \le{} & (1 - r_1(e_1))\left(-\frac{c_1 g_{1m} e_1^4}{\left(k_{a_1}^2(t) - e_1^2\right)^2} + \frac{g_1(\chi_1) e_1 e_2}{k_{a_1}^2(t) - e_1^2} - \kappa_1 g_1(\chi_1)\frac{e_1 \mathfrak{I}_1(e_1)}{k_{a_1}^2(t) - e_1^2} + \frac{e_1^2 \tilde{\theta}_1 \psi_1^T(Z_1)\psi_1(Z_1)}{2 l_1\left(k_{a_1}^2(t) - e_1^2\right)^2}\right) \\
& + (1 - r_1(e_1))\left(\frac{l_1}{2} - \frac{\lambda_1 g_{1m} e_1^2}{\left(k_{a_1}^2(t) - e_1^2\right)^2} + \frac{e_1[\varepsilon_1(Z_1) + d_1(\chi_1)]}{k_{a_1}^2(t) - e_1^2}\right) - \frac{(1 - r_1(e_1))\, e_1^2}{k_{a_1}^2(t) - e_1^2}\left(\hbar_1(t) g_{1m} + \frac{\dot{k}_{a_1}(t)}{k_{a_1}(t)}\right) \\
& + r_1(e_1)\left(-\frac{c_1 g_{1m} e_1^4}{\left(k_{b_1}^2(t) - e_1^2\right)^2} + \frac{g_1(\chi_1) e_1 e_2}{k_{b_1}^2(t) - e_1^2} - \kappa_1 g_1(\chi_1)\frac{e_1 \mathfrak{I}_1(e_1)}{k_{b_1}^2(t) - e_1^2} + \frac{e_1^2 \tilde{\theta}_1 \psi_1^T(Z_1)\psi_1(Z_1)}{2 l_1\left(k_{a_1}^2(t) - e_1^2\right)^2}\right) \\
& + r_1(e_1)\left(\frac{l_1}{2} - \frac{\lambda_1 g_{1m} e_1^2}{\left(k_{b_1}^2(t) - e_1^2\right)^2} + \frac{e_1[\varepsilon_1(Z_1) + d_1(\chi_1)]}{k_{b_1}^2(t) - e_1^2}\right) - \frac{r_1(e_1)\, e_1^2}{k_{b_1}^2(t) - e_1^2}\left(\hbar_1(t) g_{1m} + \frac{\dot{k}_{b_1}(t)}{k_{b_1}(t)}\right)
\end{aligned} \tag{29}
$$

where $\hat{\theta}_1 \ge 0$ and $\tilde{\theta}_1 = \theta_1 - g_{1m}\hat{\theta}_1$.

By utilizing Cauchy's inequality and Young's inequality, one has

$$
\begin{aligned}
& -\frac{\lambda_1 g_{1m} e_1^2}{2\left(k_{a_1}^2(t) - e_1^2\right)^2} + \frac{e_1 \varepsilon_1(Z_1)}{k_{a_1}^2(t) - e_1^2} \le \frac{\varepsilon_1^2(Z_1)}{2\lambda_1 g_{1m}} \le \frac{\varepsilon_{M1}^2}{2\lambda_1 g_{1m}}, \quad -\frac{\lambda_1 g_{1m} e_1^2}{2\left(k_{a_1}^2(t) - e_1^2\right)^2} + \frac{e_1 d_1(\chi_1)}{k_{a_1}^2(t) - e_1^2} \le \frac{d_1^2(\chi_1)}{2\lambda_1 g_{1m}} \le \frac{d_{M1}^2}{2\lambda_1 g_{1m}} \\
& -\frac{\lambda_1 g_{1m} e_1^2}{2\left(k_{b_1}^2(t) - e_1^2\right)^2} + \frac{e_1 \varepsilon_1(Z_1)}{k_{b_1}^2(t) - e_1^2} \le \frac{\varepsilon_1^2(Z_1)}{2\lambda_1 g_{1m}} \le \frac{\varepsilon_{M1}^2}{2\lambda_1 g_{1m}}, \quad -\frac{\lambda_1 g_{1m} e_1^2}{2\left(k_{b_1}^2(t) - e_1^2\right)^2} + \frac{e_1 d_1(\chi_1)}{k_{b_1}^2(t) - e_1^2} \le \frac{d_1^2(\chi_1)}{2\lambda_1 g_{1m}} \le \frac{d_{M1}^2}{2\lambda_1 g_{1m}}
\end{aligned} \tag{30}
$$

With the aid of (25), we obtain

$$
\hbar_1(t) g_{1m} + r_1(e_1)\frac{\dot{k}_{b_1}(t)}{k_{b_1}(t)} + (1 - r_1(e_1))\frac{\dot{k}_{a_1}(t)}{k_{a_1}(t)} \ge 0. \tag{31}
$$

According to the inequalities (32) and (33), (31) can be rewritten as

$$
\begin{aligned}
\dot{L}_1 \le{} & r_1(e_1)\left(-c_1 g_{1m}\left(\frac{e_1^2}{k_{b_1}^2(t) - e_1^2}\right)^2 + \frac{g_1(\chi_1) e_1 e_2}{k_{b_1}^2(t) - e_1^2} - \kappa_1 g_1(\chi_1)\frac{e_1 \mathfrak{I}_1(e_1)}{k_{b_1}^2(t) - e_1^2} + \frac{e_1^2 \tilde{\theta}_1 \psi_1^T(Z_1)\psi_1(Z_1)}{2 l_1\left(k_{a_1}^2(t) - e_1^2\right)^2}\right) + \frac{l_1}{2} + \frac{\varepsilon_{M1}^2 + d_{M1}^2}{2\lambda_1 g_{1m}} \\
& + (1 - r_1(e_1))\left(-c_1 g_{1m}\left(\frac{e_1^2}{k_{a_1}^2(t) - e_1^2}\right)^2 + \frac{g_1(\chi_1) e_1 e_2}{k_{a_1}^2(t) - e_1^2} - \kappa_1 g_1(\chi_1)\frac{e_1 \mathfrak{I}_1(e_1)}{k_{a_1}^2(t) - e_1^2} + \frac{e_1^2 \tilde{\theta}_1 \psi_1^T(Z_1)\psi_1(Z_1)}{2 l_1\left(k_{a_1}^2(t) - e_1^2\right)^2}\right).
\end{aligned} \tag{32}
$$

*Step* $i\,(2 \le i \le n-1)$: A similar procedure is employed recursively for each step $i\,(2 \le i \le n-1)$. The positive time-varying barrier functions $k_{b_i}(t)$ and $k_{a_i}(t)$ are specified later on.

Consider the following quadratic Lyapunov function candidate:

$$
L_i = \frac{r_i(e_i)}{2}\log\frac{k_{b_i}^2(t)}{k_{b_i}^2(t) - e_i^2} + \frac{1 - r_i(e_i)}{2}\log\frac{k_{a_i}^2(t)}{k_{a_i}^2(t) - e_i^2}. \tag{33}
$$

Invoking $e_i = \chi_i - \alpha_{i-1}$, the dynamics of $e_i$-subsystem can be described as

$$
\begin{aligned}
\dot{e}_i ={} & -\left[\frac{r_{i-1}(e_{i-1})\, r_i(e_i)\left(k_{b_i}^2(t) - e_i^2\right)}{r_i^2(e_i)\left(k_{b_{i-1}}^2(t) - e_{i-1}^2\right)} + \frac{(1 - r_{i-1}(e_{i-1}))(1 - r_i(e_i))\left(k_{a_i}^2(t) - e_i^2\right)}{(1 - r_i(e_i))^2\left(k_{a_{i-1}}^2(t) - e_{i-1}^2\right)}\right] g_{i-1}(\overline{\chi}_{i-1}) e_{i-1} \\
& + S_i(Z_i) + g_i(\overline{\chi}_i)(e_{i+1} + \alpha_i) + d_i(\overline{\chi}_i),
\end{aligned} \tag{34}
$$

where $S_i(Z_i) = f_i(\overline{\chi}_i) + \left[ \frac{r_{i-1}(e_{i-1}) r_i(e_i)\left(k_{b_i}^2(t) - e_i^2\right)}{r_i^2(e_i)\left(k_{b_{i-1}}^2(t) - e_{i-1}^2\right)} + \frac{(1 - r_{i-1}(e_{i-1}))(1 - r_i(e_i))\left(k_{a_i}^2(t) - e_i^2\right)}{(1 - r_i(e_i))^2\left(k_{a_{i-1}}^2(t) - e_{i-1}^2\right)} \right] g_{i-1}(\overline{x}_{i-1}) e_{i-1} - \dot{\alpha}_{i-1}$ with $Z_i = \left[\overline{\chi}_i, \alpha_{i-2}, \alpha_{i-1}, \dot{\alpha}_{i-1}\right] \in R^{i+3}$ and $\alpha_0 = y_d$.

By employing the NN in the general form of (13) to approximate $S_i(Z_i)$, the time derivative of $L_i$ along (33) is

$$
\begin{aligned}
\dot{L}_i = & \frac{r_i(e_i) e_i}{k_{b_i}^2(t) - e_i^2} \left( \Theta_i^{*T} \psi_i(Z_i) + \varepsilon_i(Z_i) + g_i(\overline{\chi}_i)\chi_{i+1} + d_i(\overline{\chi}_i) - e_i \frac{\dot{k}_{b_i}(t)}{k_{b_i}(t)} \right) - \frac{r_{i-1}(e_{i-1}) g_{i-1}(\overline{\chi}_{i-1}) e_{i-1} e_i}{k_{b_{i-1}}^2(t) - e_{i-1}^2} \\
& + \frac{(1 - r_i(e_i)) e_i}{k_{a_i}^2(t) - e_i^2} \left( \Theta_i^{*T} \psi_i(Z_i) + \varepsilon_i(Z_i) + g_i(\overline{\chi}_i)\chi_{i+1} + d_i(\overline{\chi}_i) - e_i \frac{\dot{k}_{a_i}(t)}{k_{a_i}(t)} \right) - \frac{(1 - r_{i-1}(e_{i-1})) g_{i-1}(\overline{\chi}_{i-1}) e_{i-1} e_i}{k_{a_{i-1}}^2(t) - e_{i-1}^2} \\
\leq & \, r_i(e_i) \left( \frac{l_i}{2} + \frac{e_i[\varepsilon_i(Z_i) + d_i(\overline{\chi}_i)]}{k_{b_i}^2(t) - e_i^2} - \frac{e_i^2}{k_{b_i}^2(t) - e_i^2} \frac{\dot{k}_{b_i}(t)}{k_{b_i}(t)} \right) + (1 - r_i(e_i)) \left( \frac{l_i}{2} + \frac{e_i[\varepsilon_i(Z_i) + d_i(\overline{\chi}_i)]}{k_{a_i}^2(t) - e_i^2} - \frac{e_i^2}{k_{b_i}^2(t) - e_i^2} \frac{\dot{k}_{a_i}(t)}{k_{a_i}(t)} \right) \\
& - \frac{(1 - r_{i-1}(e_{i-1})) g_{i-1}(\overline{\chi}_{i-1}) e_{i-1} e_i}{k_{a_{i-1}}^2(t) - e_{i-1}^2} + (1 - r_i(e_i)) \left( \frac{g_i(\overline{\chi}_i) e_i e_{i+1}}{k_{a_i}^2(t) - e_i^2} + \frac{g_i(\overline{\chi}_i) e_i \alpha_i}{k_{a_i}^2(t) - e_i^2} + \frac{e_i^2 \theta_i \psi_i^T(Z_i) \psi_i(Z_i)}{2 l_i \left(k_{a_i}^2(t) - e_i^2\right)^2} \right) \\
& - \frac{r_{i-1}(e_{i-1}) g_{i-1}(\overline{\chi}_{i-1}) e_{i-1} e_i}{k_{b_{i-1}}^2(t) - e_{i-1}^2} + r_i(e_i) \left( \frac{g_i(\overline{\chi}_i) e_i e_{i+1}}{k_{b_i}^2(t) - e_i^2} + \frac{g_i(\overline{\chi}_i) e_i \alpha_i}{k_{b_i}^2(t) - e_i^2} + \frac{e_i^2 \theta_i \psi_i^T(Z_i) \psi_i(Z_i)}{2 l_i \left(k_{b_i}^2(t) - e_i^2\right)^2} \right),
\end{aligned}
\tag{35}
$$

where $l_i$ is a positive design parameter.

Devise a virtual fixed-time smooth control input $\alpha_i$ as follows:

$$
\begin{aligned}
\alpha_i = & (1 - r_i(e_i)) \left[ -c_i \frac{e_i^3}{k_{a_i}^2(t) - e_i^2} - \lambda_i \frac{e_i}{k_{a_i}^2(t) - e_i^2} - \frac{e_i \hat{\theta}_i \psi_i^T(Z_i) \psi_i(Z_i)}{2 l_i \left(k_{a_i}^2(t) - e_i^2\right)} - \hbar_i(t) e_i - \kappa_i \mathfrak{I}_i(e_i) \right] \\
& + r_i(e_i) \left[ -c_i \frac{e_i^3}{k_{b_i}^2(t) - e_i^2} - \lambda_i \frac{e_i}{k_{b_i}^2(t) - e_i^2} - \frac{e_i \hat{\theta}_i \psi_i^T(Z_i) \psi_i(Z_i)}{2 l_i \left(k_{b_i}^2(t) - e_i^2\right)} - \hbar_i(t) e_i - \kappa_i \mathfrak{I}_i(e_i) \right],
\end{aligned}
\tag{36}
$$

where $c_i$, $\lambda_i$, and $\kappa_i$ are the positive design parameters.

Along similar lines, the time-varying gain $\hbar_i(t)$ is given by

$$
\hbar_i(t) = \frac{1}{g_{im}} \sqrt{\left(\frac{\dot{k}_{a_i}(t)}{k_{a_i}(t)}\right)^2 + \left(\frac{\dot{k}_{b_i}(t)}{k_{b_i}(t)}\right)^2 + o_i}. \tag{37}
$$

And the smooth switching law $\mathfrak{I}_i(e_i)$ is defined as

$$
\mathfrak{I}_i(e_i) = \begin{cases} e_i^{2h-1} \left[ r_i(e_i) \left( \left(k_{b_i}^2(t) - e_i^2\right)^{1-h} \right) + (1 - r_i(e_i)) \left( \left(k_{a_i}^2(t) - e_i^2\right)^{1-h} \right) \right], & \text{if } |e_i| \geq \varsigma_i \\ r_i(e_i) \left(\mu_{i1} e_i + \nu_{i1} e_i^3\right) + (1 - r_i(e_i)) \left(\mu_{i2} e_i + \nu_{i2} e_i^3\right), & \text{if } |e_i| < \varsigma_i \end{cases}, \tag{38}
$$

with $\mu_{i1} = \varsigma_i^{2h-2}\left(k_{b_i}^2(t) - \varsigma_i^2\right)^{1-h} - \nu_{i1}\varsigma_i^2$, $\nu_{i1} = (h-1)\varsigma_i^{2(h-2)}\left[\left(k_{b_i}^2(t) - \varsigma_i^2\right)^{1-h} + \varsigma_i^2\left(k_{b_i}^2(t) - \varsigma_i^2\right)^{-h}\right]$, $\mu_{i2} = \varsigma_i^{2h-2} \left(k_{a_i}^2(t) - \varsigma_i^2\right)^{1-h} - \nu_{i2}\varsigma_i^2$, $\nu_{i2} = (h-1)\varsigma_i^{2(h-2)}\left[\left(k_{a_i}^2(t) - \varsigma_i^2\right)^{1-h} + \varsigma_i^2\left(k_{a_i}^2(t) - \varsigma_i^2\right)^{-h}\right]$, and $\varsigma_i$ is a small positive parameter satisfying $\varsigma_i < k_{b_i}(t)$, $\varsigma_i < k_{a_i}(t)$. From $0 < h < 1$, we have $\nu_{i1}, \nu_{i2} < 0$ and $\mu_{i1}, \mu_{i2} > 0$.

The estimation $\hat{\theta}_i$ is determined by the following adaptive control law:

$$
\dot{\hat{\theta}}_i = \rho_i \left( -\sigma_{i1}\hat{\theta}_i - \sigma_{i2}\hat{\theta}_i^3 + r_i(e_i) \frac{e_i^2 \psi_i^T(Z_i) \psi_i(Z_i)}{2 l_i \left(k_{b_i}^2(t) - e_i^2\right)^2} + (1 - r_i(e_i)) \frac{e_i^2 \psi_i^T(Z_i) \psi_i(Z_i)}{2 l_i \left(k_{a_i}^2(t) - e_i^2\right)^2} \right), \tag{39}
$$

where $\rho_i$, $\sigma_{i1}$, and $\sigma_{i2}$ are the positive design parameters to be specified later.

Then, substituting (36) into (35), it gives

$$\begin{aligned}\dot{L}_i \leq & -\frac{r_{i-1}(e_{i-1})g_{i-1}(\overline{\chi}_{i-1})e_{i-1}e_i}{k_{b_{i-1}}^2(t)-e_{i-1}^2} - \frac{(1-r_{i-1}(e_{i-1}))g_{i-1}(\overline{\chi}_{i-1})e_{i-1}e_i}{k_{a_{i-1}}^2(t)-e_{i-1}^2} \\ & + (1-r_i(e_i))\left(-\frac{c_i g_{im} e_i^4}{\left(k_{a_i}^2(t)-e_i^2\right)^2} + \frac{g_i(\overline{\chi}_i)e_ie_{i+1}}{k_{a_i}^2(t)-e_i^2} - \kappa_i g_i(\chi_i)\frac{e_i\mathfrak{J}_i(e_i)}{k_{a_i}^2(t)-e_i^2} + \frac{e_i^2\tilde{\theta}_i\psi_i^T(Z_i)\psi_i(Z_i)}{2l_i\left(k_{a_i}^2(t)-e_i^2\right)^2}\right) \\ & + (1-r_i(e_i))\left(\frac{l_i}{2} - \frac{\lambda_i g_{im} e_i^2}{\left(k_{a_i}^2(t)-e_i^2\right)^2} + \frac{e_i[\varepsilon_i(Z_i)+d_i(\overline{\chi}_i)]}{k_{a_i}^2(t)-e_i^2}\right) - \frac{(1-r_i(e_i))e_i^2}{k_{a_i}^2(t)-e_i^2}\left(\hbar_i(t)g_{im} + \frac{\dot{k}_{a_i}(t)}{k_{a_i}(t)}\right) \\ & + r_i(e_i)\left(-\frac{c_i g_{im} e_i^4}{\left(k_{b_i}^2(t)-e_i^2\right)^2} + \frac{g_i(\overline{\chi}_i)e_ie_{i+1}}{k_{b_i}^2(t)-e_i^2} - \kappa_i g_i(\chi_i)\frac{e_i\mathfrak{J}_i(e_i)}{k_{b_i}^2(t)-e_i^2} + \frac{e_i^2\tilde{\theta}_i\psi_i^T(Z_i)\psi_i(Z_i)}{2l_i\left(k_{b_i}^2(t)-e_i^2\right)^2}\right) \\ & + r_i(e_i)\left(\frac{l_i}{2} - \frac{\lambda_i g_{im} e_i^2}{\left(k_{b_i}^2(t)-e_i^2\right)^2} + \frac{e_i[\varepsilon_i(Z_i)+d_i(\overline{\chi}_i)]}{k_{b_i}^2(t)-e_i^2}\right) - \frac{r_i(e_i)e_i^2}{k_{b_i}^2(t)-e_i^2}\left(\hbar_i(t)g_{im} + \frac{\dot{k}_{b_i}(t)}{k_{b_i}(t)}\right),\end{aligned} \tag{40}$$

where $\hat{\theta}_i \geq 0$ and $\tilde{\theta}_i = \theta_i - g_{im}\hat{\theta}_i$.

By utilizing Cauchy's inequality and Young's inequality, one has

$$\begin{aligned} & -\frac{\lambda_i g_{im} e_i^2}{2\left(k_{a_i}^2(t)-e_i^2\right)^2} + \frac{e_i\varepsilon_i(Z_i)}{k_{a_i}^2(t)-e_i^2} \leq \frac{\varepsilon_i^2(Z_i)}{2\lambda_i g_{im}} \leq \frac{\varepsilon_{Mi}^2}{2\lambda_i g_{im}}, \quad -\frac{\lambda_i g_{im} e_i^2}{2\left(k_{a_i}^2(t)-e_i^2\right)^2} + \frac{e_i d_i(\chi_i)}{k_{a_i}^2(t)-e_i^2} \leq \frac{d_i^2(\chi_i)}{2\lambda_i g_{im}} \leq \frac{d_{Mi}^2}{2\lambda_i g_{im}} \\ & -\frac{\lambda_i g_{im} e_i^2}{2\left(k_{b_i}^2(t)-e_i^2\right)^2} + \frac{e_i\varepsilon_i(Z_i)}{k_{b_i}^2(t)-e_i^2} \leq \frac{\varepsilon_i^2(Z_i)}{2\lambda_i g_{im}} \leq \frac{\varepsilon_{Mi}^2}{2\lambda_i g_{im}}, \quad -\frac{\lambda_i g_{im} e_i^2}{2\left(k_{b_i}^2(t)-e_i^2\right)^2} + \frac{e_i d_i(\chi_i)}{k_{b_i}^2(t)-e_i^2} \leq \frac{d_i^2(\chi_i)}{2\lambda_i g_{im}} \leq \frac{d_{Mi}^2}{2\lambda_i g_{im}} \end{aligned} \tag{41}$$

With the aid of (37), we obtain

$$\hbar_i(t)g_{im} + r_i(e_i)\frac{\dot{k}_{b_i}(t)}{k_{b_i}(t)} + (1-r_i(e_i))\frac{\dot{k}_{a_i}(t)}{k_{a_i}(t)} \geq 0. \tag{42}$$

From (41) and (42), (40) can become

$$\begin{aligned}\dot{L}_i \leq & -\frac{r_{i-1}(e_{i-1})g_{i-1}(\overline{\chi}_{i-1})e_{i-1}e_i}{k_{b_{i-1}}^2(t)-e_{i-1}^2} - \frac{(1-r_{i-1}(e_{i-1}))g_{i-1}(\overline{\chi}_{i-1})e_{i-1}e_i}{k_{a_{i-1}}^2(t)-e_{i-1}^2} + \frac{l_i}{2} + \frac{\varepsilon_{Mi}^2 + d_{Mi}^2}{2\lambda_i g_{im}} \\ & + (1-r_i(e_i))\left(-c_i g_{im}\left(\frac{e_i^2}{k_{a_i}^2(t)-e_i^2}\right)^2 + \frac{g_i(\overline{\chi}_i)e_ie_{i+1}}{k_{a_i}^2(t)-e_i^2} - \kappa_i g_i(\overline{\chi}_i)\frac{e_i\mathfrak{J}_i(e_i)}{k_{a_i}^2(t)-e_i^2} + \frac{e_i^2\tilde{\theta}_i\psi_i^T(Z_i)\psi_i(Z_i)}{2l_i\left(k_{a_i}^2(t)-e_i^2\right)^2}\right) \\ & + r_i(e_i)\left(-c_i g_{im}\left(\frac{e_i^2}{k_{b_i}^2(t)-e_i^2}\right)^2 + \frac{g_i(\overline{\chi}_i)e_ie_{i+1}}{k_{b_i}^2(t)-e_i^2} - \kappa_i g_i(\overline{\chi}_i)\frac{e_i\mathfrak{J}_i(e_i)}{k_{b_i}^2(t)-e_i^2} + \frac{e_i^2\tilde{\theta}_i\psi_i^T(Z_i)\psi_i(Z_i)}{2l_i\left(k_{b_i}^2(t)-e_i^2\right)^2}\right).\end{aligned} \tag{43}$$

*Step n*: From $e_n = \chi_n - \alpha_{n-1}$, the dynamics of $e_n$-subsystem can be described by

$$\begin{aligned}\dot{e}_n = & g_n(\overline{\chi}_n)u - \left[\frac{r_{n-1}(e_{n-1})r_n(e_n)\left(k_{b_n}^2(t)-e_n^2\right)}{r_n^2(e_n)\left(k_{b_{n-1}}^2(t)-e_{n-1}^2\right)} + \frac{(1-r_{n-1}(e_{n-1}))(1-r_n(e_n))\left(k_{a_n}^2(t)-e_n^2\right)}{(1-r_n(e_n))^2\left(k_{a_{n-1}}^2(t)-e_{n-1}^2\right)}\right]g_{n-1}(\overline{\chi}_{n-1})e_{n-1} \\ & + S_n(Z_n) + d_n(\overline{x}_n),\end{aligned} \tag{44}$$

where $S_n(Z_n) = f_n(\overline{\chi}_n) + \left[ \frac{r_{n-1}(e_{n-1}) r_n(e_n) \left( k_{b_n}^2(t) - e_n^2 \right)}{r_n^2(e_n) \left( k_{b_{n-1}}^2(t) - e_{n-1}^2 \right)} + \frac{(1-r_{n-1}(e_{n-1}))(1-r_n(e_n)) \left( k_{a_n}^2(t) - e_n^2 \right)}{(1-r_n(e_n))^2 \left( k_{a_{n-1}}^2(t) - e_{n-1}^2 \right)} \right] g_{n-1}(\overline{\chi}_{n-1}) e_{n-1} - \dot{\alpha}_{n-1}$ with $Z_n = \left[ \overline{\chi}_n, \alpha_{n-2}, \alpha_{n-1}, \dot{\alpha}_{n-1} \right] \in R^{n+3}$.

Consider the following quadratic Lyapunov function candidate:

$$L_n = \frac{r_n(e_n)}{2} \log \frac{k_{b_n}^2(t)}{k_{b_n}^2(t) - e_n^2} + \frac{1 - r_n(e_n)}{2} \log \frac{k_{a_n}^2(t)}{k_{a_n}^2(t) - e_n^2}. \tag{45}$$

By employing the NN in the general form of (13) to approximate $S_n(Z_n)$, the time derivative of $L_n$ along (45) is

$$\begin{aligned} \dot{L}_n \leq & \frac{g_n(\overline{\chi}_n) e_n u}{k_{b_n}^2(t) - e_n^2} - \frac{g_{n-1}(\overline{\chi}_{n-1}) e_{n-1} e_n}{k_{b_{n-1}}^2(t) - e_{n-1}^2} + \frac{e_n^2 \theta_n \psi_n^T(Z_n) \psi_n(Z_n)}{2 l_n \left( k_{b_n}^2(t) - e_n^2 \right)^2} + \frac{l_n}{2} + \frac{e_n [\varepsilon_n(Z_n) + d_n(\overline{\chi}_n)]}{k_{b_n}^2(t) - e_n^2} \\ \leq & - \frac{(1 - r_{n-1}(e_{n-1})) g_{n-1}(\overline{\chi}_{n-1}) e_{n-1} e_n}{k_{a_{n-1}}^2(t) - e_{n-1}^2} + (1 - r_n(e_n)) \left( \frac{g_n(\overline{\chi}_n) e_n u}{k_{b_n}^2(t) - e_n^2} + \frac{e_n^2 \theta_n \psi_n^T(Z_n) \psi_n(Z_n)}{2 l_n \left( k_{b_n}^2(t) - e_n^2 \right)^2} - \frac{e_n^2}{k_{a_n}^2(t) - e_n^2} \frac{\dot{k}_{a_n}(t)}{k_{a_n}(t)} \right) \\ & - \frac{r_{n-1}(e_{n-1}) g_{n-1}(\overline{\chi}_{n-1}) e_{n-1} e_n}{k_{b_{n-1}}^2(t) - e_{n-1}^2} + r_n(e_n) \left( \frac{g_n(\overline{\chi}_n) e_n u}{k_{b_n}^2(t) - e_n^2} + \frac{e_n^2 \theta_n \psi_n^T(Z_n) \psi_n(Z_n)}{2 l_n \left( k_{b_n}^2(t) - e_n^2 \right)^2} - \frac{e_n^2}{k_{b_n}^2(t) - e_n^2} \frac{\dot{k}_{b_n}(t)}{k_{b_n}(t)} \right) \\ & + \frac{l_n}{2} + \frac{e_n [\varepsilon_n(Z_n) + d_n(\overline{\chi}_n)]}{k_{b_n}^2(t) - e_n^2}, \end{aligned} \tag{46}$$

where $l_n$ is a positive design parameter.

Construct the actual smooth control input $u$ as

$$\begin{aligned} u = & (1 - r_n(e_n)) \left( -c_n \frac{e_n^3}{k_{a_n}^2(t) - e_n^2} - \lambda_n \frac{e_n}{k_{a_n}^2(t) - e_n^2} - \frac{e_n^2 \hat{\theta}_n \psi_n^T(Z_n) \psi_n(Z_n)}{2 l_n \left( k_{a_n}^2(t) - e_n^2 \right)} - \hbar_n(t) e_n - \kappa_n \mathfrak{I}_n(e_n) \right) \\ & + r_n(e_n) \left( -c_n \frac{e_n^3}{k_{b_n}^2(t) - e_n^2} - \lambda_n \frac{e_n}{k_{b_n}^2(t) - e_n^2} - \frac{e_n^2 \hat{\theta}_n \psi_n^T(Z_n) \psi_n(Z_n)}{2 l_n \left( k_{b_n}^2(t) - e_n^2 \right)} - \hbar_n(t) e_n - \kappa_n \mathfrak{I}_n(e_n) \right), \end{aligned} \tag{47}$$

where $c_n$, $\lambda_n$, and $\kappa_n$ are the positive design parameters.

Along similar lines, the time-varying gain $\hbar_n(t)$ is given by

$$\hbar_n(t) = \frac{1}{g_{nm}} \sqrt{\left( \frac{\dot{k}_{a_n}(t)}{k_{a_n}(t)} \right)^2 + \left( \frac{\dot{k}_{b_n}(t)}{k_{b_n}(t)} \right)^2 + o_n}. \tag{48}$$

And the smooth switching law $\mathfrak{I}_n(e_n)$ is defined as

$$\mathfrak{I}_n(e_n) = \begin{cases} e_n^{2h-1} \left[ r_n(e_n) \left( \left( k_{b_n}^2(t) - e_n^2 \right)^{1-h} \right) + (1 - r_n(e_n)) \left( \left( k_{a_n}^2(t) - e_n^2 \right)^{1-h} \right) \right], & if\ |e_n| \geq \varsigma_n \\ r_n(e_n) \left( \mu_{n1} e_n + \nu_{n1} e_n^3 \right) + (1 - r_n(e_n)) \left( \mu_{n2} e_n + \nu_{n2} e_n^3 \right), & if\ |e_n| < \varsigma_n \end{cases}, \tag{49}$$

with $\mu_{n1} = \varsigma_n^{2h-2} \left( k_{b_n}^2(t) - \varsigma_n^2 \right)^{1-h} - \nu_{n1} \varsigma_n^2$, $\nu_{n1} = (h-1) \varsigma_n^{2(h-2)} \left[ \left( k_{b_n}^2(t) - \varsigma_n^2 \right)^{1-h} + \varsigma_n^2 \left( k_{b_n}^2(t) - \varsigma_n^2 \right)^{-h} \right]$, $\mu_{n2} = \varsigma_n^{2h-2} \left( k_{a_n}^2(t) - \varsigma_n^2 \right)^{1-h} - \nu_{n2} \varsigma_n^2$, $\nu_{n2} = (h-1) \varsigma_n^{2(h-2)} \left[ \left( k_{a_n}^2(t) - \varsigma_n^2 \right)^{1-h} + \varsigma_n^2 \left( k_{a_n}^2(t) - \varsigma_n^2 \right)^{-h} \right]$, and $\varsigma_n$ is a small positive parameter satisfying $\varsigma_n < k_{b_n}(t)$, $\varsigma_n < k_{a_n}(t)$. From $0 < h < 1$, we have $\nu_{n1}, \nu_{n2} < 0$ and $\mu_{n1}, \mu_{n2} > 0$.

The estimation $\hat{\theta}_n$ is determined by the following adaptive control law:

$$\dot{\hat{\theta}}_n = \rho_n \left( -\sigma_{n1}\hat{\theta}_n - \sigma_{n2}\hat{\theta}_n^3 + r_n(e_n)\frac{e_n^2\psi_n^T(Z_n)\psi_n(Z_n)}{2l_n\left(k_{b_n}^2(t)-e_n^2\right)^2} + (1-r_n(e_n))\frac{e_n^2\psi_n^T(Z_n)\psi_n(Z_n)}{2l_n\left(k_{a_n}^2(t)-e_n^2\right)^2} \right), \tag{50}$$

where $\rho_n$, $\sigma_{n1}$, and $\sigma_{n2}$ are the positive design parameters.

With the aid of (48), we obtain

$$\hbar_n(t)g_{nm} + r_n(e_n)\frac{\dot{k}_{b_n}(t)}{k_{b_n}(t)} + (1-r_n(e_n))\frac{\dot{k}_{a_n}(t)}{k_{a_n}(t)} \geq 0. \tag{51}$$

Similarly, substituting (47) into (46) and then utilizing Young's inequality and (51), we have

$$\begin{aligned}\dot{L}_n \leq & -\frac{r_{n-1}(e_{n-1})g_{n-1}(\overline{\chi}_{n-1})e_{n-1}e_n}{k_{b_{n-1}}^2(t)-e_{n-1}^2} - \frac{(1-r_{n-1}(e_{n-1}))g_{n-1}(\overline{\chi}_{n-1})e_{n-1}e_n}{k_{a_{n-1}}^2(t)-e_{n-1}^2} + \frac{l_n}{2} + \frac{\varepsilon_{Mn}^2+d_{Mn}^2}{2\lambda_n g_{nm}} \\ & + (1-r_n(e_n))\left( -c_n g_{nm}\left(\frac{e_n^2}{k_{a_n}^2(t)-e_n^2}\right)^2 - \kappa_n g_n(\overline{\chi}_n)\frac{e_n\mathfrak{I}_n(e_n)}{k_{a_n}^2(t)-e_n^2} + \frac{e_n^2\tilde{\theta}_n\psi_n^T(Z_n)\psi_n(Z_n)}{2l_n\left(k_{a_n}^2(t)-e_n^2\right)^2} \right) \\ & + r_n(e_n)\left( -c_n g_{nm}\left(\frac{e_n^2}{k_{b_n}^2(t)-e_n^2}\right)^2 - \kappa_n g_n(\overline{\chi}_n)\frac{e_n\mathfrak{I}_n(e_n)}{k_{b_n}^2(t)-e_n^2} + \frac{e_n^2\tilde{\theta}_n\psi_n^T(Z_n)\psi_n(Z_n)}{2l_n\left(k_{b_n}^2(t)-e_n^2\right)^2} \right),\end{aligned} \tag{52}$$

where $\hat{\theta}_n \geq 0$ and $\tilde{\theta}_n = \theta_n - g_{nm}\hat{\theta}_n$.

The design process of nonsingular fixed-time adaptive tracking controller has been completed.

## 4 | STABILITY ANALYSIS

We are at the position to present our main results in the following Theorem 1.

**Theorem 1.** *Consider the nonlinear system (1) under Assumptions 1–3. The virtual control laws are constructed as (24) and (36), with the adaptation laws (27) and (39). Based on the designed control laws, the actual control law is proposed as (47). If the initial conditions satisfy* $-k_{a_1}(0) < e_1(0) < k_{b_1}(0)$, *the proposed approach can ensure that: (1) the signals of the closed-loop system are bounded and converge into the arbitrarily small regions in a fixed time; (2) all the states constraints are never violated, that is, each state* $\chi_i$ *will remain in the set* $\Omega_\chi := \left\{ \chi \in \mathbb{R}^n : \underline{k}_{c_i}(t) < \chi_i < \overline{k}_{c_i}(t) \right\}$.

*Proof of Theorem* 1. To analyze the stability of the closed-loop system, we consider the following Lyapunov function candidate:

$$L = L_e + L_\theta, \tag{53}$$

where $L_e = \sum_{i=1}^n L_i = \sum_{i=1}^n \frac{r_i(e_i)}{2}\log\frac{k_{b_i}^2(t)}{k_{b_i}^2(t)-e_i^2} + \frac{1-r_i(e_i)}{2}\log\frac{k_{a_i}^2(t)}{k_{a_i}^2(t)-e_i^2}$ and $L_\theta = \sum_{i=1}^n \frac{1}{2\rho_i g_{im}}\tilde{\theta}_i^2$.

It follows from (32), (43), and (52) that the time derivative of $L_e$ is

$$\begin{aligned}\dot{L}_e \leq & (1-r_i(e_i))\left[ -\sum_{i=1}^n c_i g_{im}\left(\frac{e_i^2}{k_{a_i}^2(t)-e_i^2}\right)^2 - \sum_{i=1}^n \kappa_i g_i(\overline{x}_i)\frac{e_i\mathfrak{I}_i(e_i)}{k_{a_i}^2(t)-e_i^2} + \sum_{i=1}^n \frac{e_i^2\tilde{\theta}_i\psi_i^T(Z_i)\psi_i(Z_i)}{2l_i\left(k_{a_i}^2(t)-e_i^2\right)^2} \right] \\ & + r_i(e_i)\left[ -\sum_{i=1}^n c_i g_{im}\left(\frac{e_i^2}{k_{b_i}^2(t)-e_i^2}\right)^2 - \sum_{i=1}^n \kappa_i g_i(\overline{x}_i)\frac{e_i\mathfrak{I}_i(e_i)}{k_{b_i}^2(t)-e_i^2} + \sum_{i=1}^n \frac{e_i^2\tilde{\theta}_i\psi_i^T(Z_i)\psi_i(Z_i)}{2l_i\left(k_{b_i}^2(t)-e_i^2\right)^2} \right]\end{aligned}$$

$$+\sum_{i=1}^{n}\left(\frac{l_i}{2}+\frac{\varepsilon_{Mi}^2+d_{Mi}^2}{2\lambda_i g_{im}}\right). \tag{54}$$

In combination with $\dot{\tilde{\theta}}_i = \dot{\theta}_i - g_{im}\dot{\hat{\theta}}_i = -g_{im}\dot{\hat{\theta}}_i$, considering (27), (39), and (50) gives

$$\begin{aligned}\dot{L} &= \dot{L}_e + \dot{L}_\theta \\ &\le (1-r_i(e_i))\left[-\sum_{i=1}^{n}c_i g_{im}\left(\frac{e_i^2}{k_{a_i}^2(t)-e_i^2}\right)^2 - \sum_{i=1}^{n}\kappa_i g_i(\overline{\chi}_i)\frac{e_i\mathfrak{I}_i(e_i)}{k_{a_i}^2(t)-e_i^2}\right] + \sum_{i=1}^{n}\left(\sigma_{i1}\tilde{\theta}_i\hat{\theta}_i + \sigma_{i2}\tilde{\theta}_i\hat{\theta}_i^3\right) \\ &\quad + r_i(e_i)\left[-\sum_{i=1}^{n}c_i g_{im}\left(\frac{e_i^2}{k_{b_i}^2(t)-e_i^2}\right)^2 - \sum_{i=1}^{n}\kappa_i g_i(\overline{\chi}_i)\frac{e_i\mathfrak{I}_i(e_i)}{k_{b_i}^2(t)-e_i^2}\right] + \sum_{i=1}^{n}\left(\frac{l_i}{2}+\frac{\varepsilon_{Mi}^2+d_{Mi}^2}{2\lambda_i g_{im}}\right).\end{aligned} \tag{55}$$

Since $\tilde{\theta}_i\hat{\theta}_i \le -\frac{\tilde{\theta}_i^2}{2g_{im}} + \frac{\theta_i^2}{2g_{im}}$, one has

$$\begin{aligned}\dot{L} &\le r_i(e_i)\left[-\sum_{i=1}^{n}c_i g_{im}\left(\frac{e_i^2}{k_{b_i}^2(t)-e_i^2}\right)^2 - \sum_{i=1}^{n}\kappa_i g_i(\overline{\chi}_i)\frac{e_i\mathfrak{I}_i(e_i)}{k_{b_i}^2(t)-e_i^2}\right] + \sum_{i=1}^{n}\left(\frac{l_i}{2}+\frac{\varepsilon_{Mi}^2+d_{Mi}^2}{2\lambda_i g_{im}}\right) \\ &\quad + (1-r_i(e_i))\left[-\sum_{i=1}^{n}c_i g_{im}\left(\frac{e_i^2}{k_{a_i}^2(t)-e_i^2}\right)^2 - \sum_{i=1}^{n}\kappa_i g_i(\overline{\chi}_i)\frac{e_i\mathfrak{I}_i(e_i)}{k_{a_i}^2(t)-e_i^2}\right] \\ &\quad - \left(\sum_{i=1}^{n}\frac{\sigma_{i1}\tilde{\theta}_i^2}{2g_{im}}\right)^h + \left(\sum_{i=1}^{n}\frac{\sigma_{i1}\tilde{\theta}_i^2}{2g_{im}}\right)^h - \sum_{i=1}^{n}\frac{\sigma_{i1}\tilde{\theta}_i^2}{2g_{im}} + \sum_{i=1}^{n}\frac{\sigma_{i1}\theta_i^2}{2g_{im}} + \sum_{i=1}^{n}\left(\sigma_{i2}\tilde{\theta}_i\hat{\theta}_i^3\right).\end{aligned} \tag{56}$$

By utilizing Corollary 2, let $\Phi = \sum_{i=1}^{n}\frac{\sigma_{i1}\tilde{\theta}_i^2}{2g_{im}}$, it yields that

$$\left(\sum_{i=1}^{n}\frac{\sigma_{i1}\tilde{\theta}_i^2}{2g_{im}}\right)^h \le \Delta(h) + \sum_{i=1}^{n}\frac{\sigma_{i1}\tilde{\theta}_i^2}{2g_{im}}. \tag{57}$$

Substituting (57) into (56) arrives at

$$\begin{aligned}\dot{L} &\le r_i(e_i)\left[-\sum_{i=1}^{n}c_i g_{im}\left(\frac{e_i^2}{k_{b_i}^2(t)-e_i^2}\right)^2 - \sum_{i=1}^{n}\kappa_i g_i(\overline{\chi}_i)\frac{e_i\mathfrak{I}_i(e_i)}{k_{b_i}^2(t)-e_i^2}\right] + \sum_{i=1}^{n}\left(\frac{l_i}{2}+\frac{\varepsilon_{Mi}^2+d_{Mi}^2}{2\lambda_i g_{im}}\right) \\ &\quad + (1-r_i(e_i))\left[-\sum_{i=1}^{n}c_i g_{im}\left(\frac{e_i^2}{k_{a_i}^2(t)-e_i^2}\right)^2 - \sum_{i=1}^{n}\kappa_i g_i(\overline{\chi}_i)\frac{e_i\mathfrak{I}_i(e_i)}{k_{a_i}^2(t)-e_i^2}\right] \\ &\quad - \left(\sum_{i=1}^{n}\frac{\sigma_{i1}\tilde{\theta}_i^2}{2g_{im}}\right)^h + \Delta(h) + \sum_{i=1}^{n}\frac{\sigma_{i1}\theta_i^2}{2g_{im}} + \sum_{i=1}^{n}\left(\sigma_{i2}\tilde{\theta}_i\hat{\theta}_i^3\right).\end{aligned} \tag{58}$$

Since $\tilde{\theta}_i\hat{\theta}_i^3 = \frac{\tilde{\theta}_i}{g_{im}^3}\left(\theta_i^3 - 3\theta_i^2\tilde{\theta}_i + 3\theta_i\tilde{\theta}_i^2 - \tilde{\theta}_i^3\right)$, (53) can be expressed as

$$\begin{aligned}\dot{L} &\le r_i(e_i)\left[-\sum_{i=1}^{n}c_i g_{im}\left(\frac{e_i^2}{k_{b_i}^2(t)-e_i^2}\right)^2 - \sum_{i=1}^{n}\kappa_i g_i(\overline{\chi}_i)\frac{e_i\mathfrak{I}_i(e_i)}{k_{b_i}^2(t)-e_i^2}\right] + \sum_{i=1}^{n}\left(\frac{l_i}{2}+\frac{\varepsilon_{Mi}^2+d_{Mi}^2}{2\lambda_i g_{im}}\right) \\ &\quad + (1-r_i(e_i))\left[-\sum_{i=1}^{n}c_i g_{im}\left(\frac{e_i^2}{k_{a_i}^2(t)-e_i^2}\right)^2 - \sum_{i=1}^{n}\kappa_i g_i(\overline{\chi}_i)\frac{e_i\mathfrak{I}_i(e_i)}{k_{a_i}^2(t)-e_i^2}\right] - \left(\sum_{i=1}^{n}\frac{\sigma_{i1}\tilde{\theta}_i^2}{2g_{im}}\right)^h\end{aligned}$$

$$+\Delta(h)+\sum_{i=1}^{n}\frac{\sigma_{i1}\theta_i^2}{2g_{im}}+\sum_{i=1}^{n}\frac{3\sigma_{i2}\tilde{\theta}_i^3\theta_i}{g_{im}^3}+\sum_{i=1}^{n}\frac{\sigma_{i2}\tilde{\theta}_i\theta_i^3}{g_{im}^3}-\sum_{i=1}^{n}\frac{3\sigma_{i2}\theta_i^2\tilde{\theta}_i^2}{g_{im}^3}-\sum_{i=1}^{n}\frac{\sigma_{i2}\tilde{\theta}_i^4}{g_{im}^3}. \tag{59}$$

By using Young's inequality, we have

$$\sum_{i=1}^{n}\frac{3\sigma_{i2}\tilde{\theta}_i^3\theta_i}{g_{im}^3}\le\sum_{i=1}^{n}\frac{9\sigma_{i2}\epsilon^{4/3}\tilde{\theta}_i^4}{4g_{im}^3}+\sum_{i=1}^{n}\frac{3\sigma_{i2}\theta_i^4}{4\epsilon^4g_{im}^3},\quad \sum_{i=1}^{n}\frac{\sigma_{i2}\tilde{\theta}_i\theta_i^3}{g_{im}^3}\le\sum_{i=1}^{n}\frac{3\sigma_{i2}\tilde{\theta}_i^2\theta_i^2}{g_{im}^3}+\sum_{i=1}^{n}\frac{\sigma_{i2}\theta_i^4}{12g_{im}^3}. \tag{60}$$

Then using Lemma 3, (58) can be further expressed by

$$\begin{aligned}\dot{L}\le{}& r_i(e_i)\left[-\sum_{i=1}^{n}c_ig_{im}\left(\frac{e_i^2}{k_{b_i}^2(t)-e_i^2}\right)^2-\sum_{i=1}^{n}\kappa_ig_i(\overline{\chi}_i)\frac{e_i\mathfrak{I}_i(e_i)}{k_{b_i}^2(t)-e_i^2}\right]+\sum_{i=1}^{n}\left(\frac{l_i}{2}+\frac{\varepsilon_{Mi}^2+d_{Mi}^2}{2\lambda_ig_{im}}\right)\\&+(1-r_i(e_i))\left[-\sum_{i=1}^{n}c_ig_{im}\left(\frac{e_i^2}{k_{a_i}^2(t)-e_i^2}\right)^2-\sum_{i=1}^{n}\kappa_ig_i(\overline{\chi}_i)\frac{e_i\mathfrak{I}_i(e_i)}{k_{a_i}^2(t)-e_i^2}\right]+\Delta(h)\\&-\left(\sum_{i=1}^{n}\frac{\sigma_{i1}\tilde{\theta}_i^2}{2g_{im}}\right)^h-\sum_{i=1}^{n}\left(\frac{4\sigma_{i2}-9\sigma_{i2}\epsilon^{4/3}}{4g_{im}^3}\right)\left(\tilde{\theta}_i^2\right)^2+\sum_{i=1}^{n}\frac{\sigma_{i1}\theta_i^2}{2g_{im}}+\sum_{i=1}^{n}\frac{3\sigma_{i2}\theta_i^4}{4\epsilon^4g_{im}^3}+\sum_{i=1}^{n}\frac{\sigma_{i2}\theta_i^4}{12g_{im}^3}.\end{aligned} \tag{61}$$

For convenience, we rewrite (58) as

$$\begin{aligned}\dot{L}\le{}& r_i(e_i)\left[-\sum_{i=1}^{n}c_ig_{im}\left(\frac{e_i^2}{k_{b_i}^2(t)-e_i^2}\right)^2-\sum_{i=1}^{n}\kappa_ig_i(\overline{\chi}_i)\frac{e_i\mathfrak{I}_i(e_i)}{k_{b_i}^2(t)-e_i^2}\right]+\breve{\Xi}\\&+(1-r_i(e_i))\left[-\sum_{i=1}^{n}c_ig_{im}\left(\frac{e_i^2}{k_{a_i}^2(t)-e_i^2}\right)^2-\sum_{i=1}^{n}\kappa_ig_i(\overline{\chi}_i)\frac{e_i\mathfrak{I}_i(e_i)}{k_{a_i}^2(t)-e_i^2}\right]\\&-(\sigma_{i1}\rho_i)^h\left(\sum_{i=1}^{n}\frac{\tilde{\theta}_i^2}{2\rho_ig_{im}}\right)^h-\left(\frac{4\rho_i^2\sigma_{i2}-9\rho_i^2\sigma_{i2}\epsilon^{4/3}}{ng_{im}}\right)\left(\sum_{i=1}^{n}\frac{\tilde{\theta}_i^2}{2\rho_ig_{im}}\right)^2,\end{aligned} \tag{62}$$

where $\breve{\Xi}=\sum_{i=1}^{n}\left(\frac{l_i}{2}+\frac{\varepsilon_{Mi}^2+d_{Mi}^2}{2\lambda_ig_{im}}\right)+\Delta(h)+\sum_{i=1}^{n}\frac{\sigma_{i1}\theta_i^2}{2g_{im}}+\sum_{i=1}^{n}\frac{3\sigma_{i2}\theta_i^4}{4\epsilon^4g_{im}^3}+\sum_{i=1}^{n}\frac{\sigma_{i2}\theta_i^4}{12g_{im}^3}$.

From the definition of $\mathfrak{I}_i(e_i)$, $i=1,\ \dots\ ,n$ in (40), the following two cases should be considered.

*Case 1*: When $|e_i|<\varsigma_i$, $i=1,\ \dots\ ,n$.

Substituting $\mathfrak{I}_i(e_i)=r_i(e_i)\left(\mu_{i1}e_i+\nu_{i1}e_i^3\right)+(1-q_i(e_i))\left(\mu_{i2}e_i+\nu_{i2}e_i^3\right)$ into (61) gives

$$\begin{aligned}\dot{L}\le{}& r_i(e_i)\left[-\frac{c_ig_{im}}{n}\left(\sum_{i=1}^{n}\frac{e_i^2}{k_{b_i}^2(t)-e_i^2}\right)^2-\sum_{i=1}^{n}\kappa_ig_{im}\mu_{i1}\frac{e_i^2}{k_{b_i}^2(t)-e_i^2}-\sum_{i=1}^{n}\kappa_i\nu_{i1}g_i(\overline{\chi}_i)\frac{e_i^4}{k_{b_i}^2(t)-e_i^2}\right]\\&+(1-r_i(e_i))\left[-\frac{c_ig_{im}}{n}\left(\sum_{i=1}^{n}\frac{e_i^2}{k_{a_i}^2(t)-e_i^2}\right)^2-\sum_{i=1}^{n}\kappa_ig_{im}\mu_{i2}\frac{e_i^2}{k_{a_i}^2(t)-e_i^2}-\sum_{i=1}^{n}\kappa_i\nu_{i2}g_i(\overline{\chi}_i)\frac{e_i^4}{k_{a_i}^2(t)-e_i^2}\right]\\&-(\sigma_{i1}\rho_i)^h\left(\sum_{i=1}^{n}\frac{\tilde{\theta}_i^2}{2\rho_ig_{im}}\right)^h-\left(\frac{4\rho_i^2\sigma_{i2}-9\rho_i^2\sigma_{i2}\epsilon^{4/3}}{ng_{im}}\right)\left(\sum_{i=1}^{n}\frac{\tilde{\theta}_i^2}{2\rho_ig_{im}}\right)^2+\breve{\Xi}.\end{aligned} \tag{63}$$

By utilizing Corollary 2, let $\Phi=\sum_{i=1}^{n}\kappa_ig_{im}\mu_{i1}\frac{e_i^2}{k_{b_i}^2(t)-e_i^2}$, one reaches

$$\left(\sum_{i=1}^{n}\kappa_ig_{im}\mu_{i1}\frac{e_i^2}{k_{b_i}^2(t)-e_i^2}\right)^h\le\Delta(h)+\sum_{i=1}^{n}\kappa_ig_{im}\mu_{i1}\frac{e_i^2}{k_{b_i}^2(t)-e_i^2}. \tag{64}$$

*Remark* 4. Aiming to obtain the indispensable fractional power term $\left(\sum_{i=1}^{n}\kappa_i g_{im}\mu_{i1}\frac{e_i^2}{k_{b_i}^2(t)-e_i^2}\right)^h$ to implement the subsequent fixed-time stability analysis, thus it needs to convert the existing integer power terms into the form of fractional power term. Hence the inequality (10) builds the relationship between the fractional power term and integer power term, which can effectively make the fractional power term displayed in the stability analysis process.

In view of $\nu_{i1}, \nu_{i2} < 0$, $|e_i| < \varsigma_i$, $\varsigma_i < \min\left\{k_{b_i}(t), k_{a_i}(t)\right\}$ and (17), one has

$$-\sum_{i=1}^{n}\kappa_i\nu_{i1}g_i(\overline{\chi}_i)\frac{e_i^4}{k_{b_i}^2(t)-e_i^2} \leq -\sum_{i=1}^{n}\frac{\kappa_i g_{iM}\nu_{i1}\varsigma_i^4}{\underline{k}_{b_i}^2-\varsigma_i^2}, \quad -\sum_{i=1}^{n}\kappa_i\nu_{i2}g_i(\overline{\chi}_i)\frac{e_i^4}{k_{a_i}^2(t)-e_i^2} \leq -\sum_{i=1}^{n}\frac{\kappa_i g_{iM}\nu_{i2}\varsigma_i^4}{\underline{k}_{a_i}^2-\varsigma_i^2}. \tag{65}$$

Thus, we can rewrite (63) as

$$\begin{aligned}\dot{L} \leq{}& r_i(e_i)\left[-\frac{c_i g_{im}}{n}\left(\sum_{i=1}^{n}\frac{e_i^2}{k_{b_i}^2(t)-e_i^2}\right)^2-(\kappa_i g_{im}\mu_{i1})^h\left(\sum_{i=1}^{n}\frac{e_i^2}{k_{b_i}^2(t)-e_i^2}\right)^h\right]\\ &+(1-r_i(e_i))\left[-\frac{c_i g_{im}}{n}\left(\sum_{i=1}^{n}\frac{e_i^2}{k_{a_i}^2(t)-e_i^2}\right)^2-(\kappa_i g_{im}\mu_{i2})^h\left(\sum_{i=1}^{n}\frac{e_i^2}{k_{a_i}^2(t)-e_i^2}\right)^h\right]\\ &-\left(\frac{4\rho_i^2\sigma_{i2}-9\rho_i^2\sigma_{i2}\epsilon^{4/3}}{ng_{im}}\right)\left(\sum_{i=1}^{n}\frac{\tilde{\theta}_i^2}{2\rho_i g_{im}}\right)^2-(\sigma_{i1}\rho_i)^h\left(\sum_{i=1}^{n}\frac{\tilde{\theta}_i^2}{2\rho_i g_{im}}\right)^h+\Xi,\end{aligned} \tag{66}$$

where $\Xi = \breve{\Xi} + 2\Delta(h) - \sum_{i=1}^{n}\frac{\kappa_i g_{iM}\nu_i\varsigma_i^4}{\underline{k}_{b_i}^2-\varsigma_i^2} - \sum_{i=1}^{n}\frac{\kappa_i g_{iM}\nu_{i2}\varsigma_i^4}{\underline{k}_{a_i}^2-\varsigma_i^2}$.

*Case 2*: When $|e_i| \geq \varsigma_i$, $i = 1, \ldots, n$.

Substituting $\mathfrak{I}_i(e_i) = e_i^{2h-1}\left[r_i(e_i)\left(\left(k_{b_i}^2(t)-e_i^2\right)^{1-h}\right)+(1-r_i(e_i))\left(\left(k_{a_i}^2(t)-e_i^2\right)^{1-h}\right)\right]$ into (61) arrives at

$$\begin{aligned}\dot{L} \leq{}& r_i(e_i)\left[-\frac{c_i g_{im}}{n}\left(\sum_{i=1}^{n}\frac{e_i^2}{k_{b_i}^2(t)-e_i^2}\right)^2-\kappa_i g_{im}\left(\sum_{i=1}^{n}\frac{e_i^2}{k_{b_i}^2(t)-e_i^2}\right)^h\right]+\breve{\Xi}\\ &+(1-r_i(e_i))\left[-\frac{c_i g_{im}}{n}\left(\sum_{i=1}^{n}\frac{e_i^2}{k_{a_i}^2(t)-e_i^2}\right)^2-\kappa_i g_{im}\left(\sum_{i=1}^{n}\frac{e_i^2}{k_{a_i}^2(t)-e_i^2}\right)^h\right].\\ &-\left(\frac{4\rho_i^2\sigma_{i2}-9\rho_i^2\sigma_{i2}\epsilon^{4/3}}{ng_{im}}\right)\left(\sum_{i=1}^{n}\frac{\tilde{\theta}_i^2}{2\rho_i g_{im}}\right)^2-(\sigma_{i1}\rho_i)^h\left(\sum_{i=1}^{n}\frac{\tilde{\theta}_i^2}{2\rho_i g_{im}}\right)^h\end{aligned} \tag{67}$$

Summarizing above two cases leads to

$$\begin{aligned}\dot{L} \leq{}& -\overline{\omega}_1\left\{\left(\frac{r_i(e_i)}{2}\sum_{i=1}^{n}\frac{e_i^2}{k_{b_i}^2(t)-e_i^2}\right)^h+\left(\frac{1-r_i(e_i)}{2}\sum_{i=1}^{n}\frac{e_i^2}{k_{b_i}^2(t)-e_i^2}\right)^h\right\}-\hat{\omega}_1\left(\sum_{i=1}^{n}\frac{\tilde{\theta}_i^2}{2\rho_i g_{im}}\right)^h\\ &-\overline{\omega}_2\left\{\left(\frac{r_i(e_i)}{2}\sum_{i=1}^{n}\frac{e_i^2}{k_{b_i}^2(t)-e_i^2}\right)^2+\left(\frac{1-r_i(e_i)}{2}\sum_{i=1}^{n}\frac{e_i^2}{k_{b_i}^2(t)-e_i^2}\right)^2\right\}-\hat{\omega}_2\left(\sum_{i=1}^{n}\frac{\tilde{\theta}_i^2}{2\rho_i g_{im}}\right)^2+\Xi\\ \leq{}& -\omega_1\left\{\left(\frac{r_i(e_i)}{2}\sum_{i=1}^{n}\frac{e_i^2}{k_{b_i}^2(t)-e_i^2}\right)^h+\left(\frac{1-r_i(e_i)}{2}\sum_{i=1}^{n}\frac{e_i^2}{k_{b_i}^2(t)-e_i^2}\right)^h+\left(\sum_{i=1}^{n}\frac{\tilde{\theta}_i^2}{2\rho_i g_{im}}\right)^h\right\}\end{aligned}$$

$$
-\breve{\omega}_2\left\{\left(\frac{r_i(e_i)}{2}\sum_{i=1}^{n}\frac{e_i^2}{k_{b_i}^2(t)-e_i^2}\right)^2+\left(\frac{1-r_i(e_i)}{2}\sum_{i=1}^{n}\frac{e_i^2}{k_{b_i}^2(t)-e_i^2}\right)^2+\left(\sum_{i=1}^{n}\frac{\tilde{\theta}_i^2}{2\rho_i g_{im}}\right)^2\right\}+\Xi, \tag{68}
$$

where $\overline{\omega}_1=\min\left\{2^h\kappa_i g_{im},(2\kappa_i g_{im}\mu_{i1})^h,(2\kappa_i g_{im}\mu_{i2})^h\right\}$, $\hat{\omega}_1=(\sigma_{i1}\rho_i)^h$ and $\omega_1=\min\left\{\overline{\omega}_1,\hat{\omega}_1\right\}$. $\overline{\omega}_2=\frac{4c_i g_{im}}{n}$, $\hat{\omega}_2=\left(\frac{4\rho_i^2\sigma_{i2}-9\rho_i^2\sigma_{i2}\varepsilon^{4/3}}{ng_{im}}\right)$ and $\breve{\omega}_2=\min\left\{\overline{\omega}_2,\hat{\omega}_2\right\}$.

With the aid of $L=\sum_{i=1}^{n}\frac{r_i(e_i)}{2}\log\frac{k_{b_i}^2(t)}{k_{b_i}^2(t)-e_i^2}+\frac{1-r_i(e_i)}{2}\log\frac{k_{a_i}^2(t)}{k_{a_i}^2(t)-e_i^2}+\sum_{i=1}^{n}\frac{\tilde{\theta}_i^2}{2\rho_i g_{im}}$ and Lemma 3, it follows that

$$
\begin{aligned}
L^h &\le \left(\sum_{i=1}^{n}\frac{r_i(e_i)}{2}\log\frac{k_{b_i}^2(t)}{k_{b_i}^2(t)-e_i^2}+\frac{1-r_i(e_i)}{2}\log\frac{k_{a_i}^2(t)}{k_{a_i}^2(t)-e_i^2}\right)^h+\left(\sum_{i=1}^{n}\frac{\tilde{\theta}_i^2}{2\rho_i g_{im}}\right)^h \\
&\le \left(\frac{r_i(e_i)}{2}\sum_{i=1}^{n}\frac{e_i^2}{k_{b_i}^2(t)-e_i^2}\right)^h+\left(\frac{1-r_i(e_i)}{2}\sum_{i=1}^{n}\frac{e_i^2}{k_{b_i}^2(t)-e_i^2}\right)^h+\left(\sum_{i=1}^{n}\frac{\tilde{\theta}_i^2}{2\rho_i g_{im}}\right)^h,
\end{aligned} \tag{69}
$$

$$
\begin{aligned}
L^2 &\le 2n\left\{\left(\sum_{i=1}^{n}\frac{r_i(e_i)}{2}\log\frac{k_{b_i}^2(t)}{k_{b_i}^2(t)-e_i^2}+\frac{1-r_i(e_i)}{2}\log\frac{k_{a_i}^2(t)}{k_{a_i}^2(t)-e_i^2}\right)^2+\left(\sum_{i=1}^{n}\frac{\tilde{\theta}_i^2}{2\rho_i g_{im}}\right)^2\right\} \\
&\le 2n\left\{\left(\frac{r_i(e_i)}{2}\sum_{i=1}^{n}\frac{e_i^2}{k_{b_i}^2(t)-e_i^2}\right)^2+\left(\frac{1-r_i(e_i)}{2}\sum_{i=1}^{n}\frac{e_i^2}{k_{b_i}^2(t)-e_i^2}\right)^2+\left(\sum_{i=1}^{n}\frac{\tilde{\theta}_i^2}{2\rho_i g_{im}}\right)^2\right\}.
\end{aligned} \tag{70}
$$

Combining (68), (69), and (70), the following inequality holds

$$
\dot{L}\le-\omega_1 L^h-\omega_2 L^2+\Xi, \tag{71}
$$

where $\omega_2=\frac{\breve{\omega}_2}{2n}$.

According to Lemma 1 and Corollary 1, the parameters are selected as follows: $q=2, k=1$.

The fixed convergence time can be derived as

$$
T_{\max}:=\frac{1}{\theta_0\omega_1(1-h)}\left(\frac{\omega_1}{\omega_2}\right)^{\frac{1-h}{2-h}}+\frac{1}{\theta_0\omega_1}\left(\frac{\omega_1}{\omega_2}\right)^{\frac{-1}{2-h}}, \tag{72}
$$

for $\left\{\lim_{t\to T_r}L(x)\le\min\left\{\omega_1^{-\frac{1}{h}}\left(\frac{\Xi}{1-\theta_0}\right)^{\frac{1}{h}},\omega_2^{-\frac{1}{2}}\left(\frac{\Xi}{1-\theta_0}\right)^{\frac{1}{2}}\right\}\right\}$ with $0<\theta_0<1$.

Then, the internal error signals $e_i$ and $\tilde{\theta}_i^2$ will converge into the following compact sets:

$$
|e_i|\le\min\left\{k_{b_i}\sqrt{1-e^{-2\omega_1^{-\frac{1}{h}}\left(\frac{\Xi}{1-\theta_0}\right)^{\frac{1}{h}}}},k_{b_i}\sqrt{1-e^{-2\omega_2^{-\frac{1}{2}}\left(\frac{\Xi}{1-\theta_0}\right)^{\frac{1}{2}}}}\right\}, \tag{73}
$$

$$
|\tilde{\theta}_i|\le\min\left\{\sqrt{2\rho_i g_{im}}\sqrt{\omega_1^{-\frac{1}{h}}\left(\frac{\Xi}{1-\theta_0}\right)^{\frac{1}{h}}},\sqrt{2\rho_i g_{im}}\sqrt{\omega_2^{-\frac{1}{2}}\left(\frac{\Xi}{1-\theta_0}\right)^{\frac{1}{2}}}\right\}. \tag{74}
$$

As a consequence, it can be concluded that the error terms converge to an arbitrarily small neighborhood of the origin within fixed-time by appropriately online-tuning the design parameters.

Now, it is the time to prove that full state constraints are guaranteed.

If the initial conditions satisfy $-k_{a_1}(0) < e_1(0) < k_{b_1}(0)$, it is equivalent to $|\zeta_1(0)| < 1$, Then, we can show that $|\zeta_1(t)| < 1$. Since $|\zeta_1(t)| < 1$, we have $-k_{a_1}(t) < e_1(t) < k_{b_1}(t)$. Together with the fact that $\chi_1(t) = e_1(t) + y_d(t)$, we infer that $y_d(t) - k_{a_1}(t) < \chi_1(t) < k_{b_1}(t) + y_d(t)$ for all $t > 0$. From the definitions of $k_{a_1}(t)$ and $k_{b_1}(t)$ in (17), we conclude that $\underline{k}_{c_1}(t) < \chi_1 < \overline{k}_{c_1}(t)$. To verify that $\underline{k}_{c_2}(t) < \chi_2 < \overline{k}_{c_2}(t)$, it needs to show that there are positive functions $\overline{\alpha}_1$ and $\underline{\alpha}_1$ so that $-\underline{\alpha}_1 \leq \alpha_1 \leq \overline{\alpha}_1$. Invoking (74), one has that $\tilde{\theta}_1$ is bounded, then $\hat{\theta}_1$ is also bounded. Due to the fact that $\alpha_1$ is a function of $\hat{\theta}_1$, $x_1$, $e_1$ and $\dot{y}_d$, in which $\underline{k}_{c_1}(t) < \chi_1 < \overline{k}_{c_1}(t)$, $-k_{a_1}(t) < e_1(t) < k_{b_1}(t)$, $\underline{Y}_1(t) \leq y_d(t) \leq \overline{Y}_1(t)$, there must exists a bound of $\alpha_1$. And then define $k_{b_2}(t) = \overline{k}_{c_2}(t) - \overline{\alpha}_1$ and $k_{a_2}(t) = -\underline{k}_{c_2}(t) - \underline{\alpha}_1$, we have $-k_{a_2}(t) - \underline{\alpha}_1 \leq e_2 + \underline{\alpha}_1 \leq \chi_2 \leq e_2 + \overline{\alpha} < k_{b_2}(t) + \overline{\alpha}$, and thus, one has $\underline{k}_{c_2}(t) < \chi_2 < \overline{k}_{c_2}(t)$. Similarly and iteratively, we have that $\alpha_{i-1}$ for $i = 3, \ldots, n$ are bounded, together with $-k_{a_i}(t) < e_i(t) < k_{b_i}(t)$, we can in turn prove that $\underline{k}_{c_i}(t) < \chi_i < \overline{k}_{c_i}(t)$. Thus, the system states $\chi_i$, $i = 1, 2, \ldots, n$ will remain in the set $\Omega_\chi$ all the time.

This completes the proof. ∎

*Remark* 5. In (26), the appropriate parameters $\mu_{11}, \mu_{12} > 0$, and $\nu_{11}, \nu_{12} < 0$ are selected to guarantee the virtual control input $\alpha_1$ and its derivative $\dot{\alpha}_1$ are both continuous in the set $\Omega_\chi$. Dividing it into three kinds of situations and discuss: First, it can be shown that $\mathfrak{I}_1(e_1)$ and its derivative $\dot{\mathfrak{I}}_1(e_1)$ are indeed continuous in the case that $e_1 \in (-\varsigma_1, \varsigma_1)$ or $|e_1| \in (\varsigma_1, k_{b_1}(t))$. Second, by skillfully designing parameter $\mu_{11}, \mu_{12}$, and $\nu_{11}, \nu_{12}$, we can derive that both $\mathfrak{I}_1(\varsigma_1^+) = \mathfrak{I}_1(\varsigma_1^-)$ and $\dot{\mathfrak{I}}_1(\varsigma_1^+) = \dot{\mathfrak{I}}_1(\varsigma_1^-)$ hold in the case of $e_1 = \varsigma_1$. Third, the continuity of $\mathfrak{I}_1(e_1)$ and $\dot{\mathfrak{I}}_1(e_1)$ are ensured similarly when $e_1 = -\varsigma_1$. Hence, the virtual control input and its derivative are continuous, which makes them possible to implement the backstepping technique.

*Remark* 6. The main benefit of the proposed smooth switching is to eliminate singularity issue effectively, and achieve the fixed-time control subsequently. In view of (26), the equation is investigated by two cases: (1) when $|e_1| < \varsigma_1$, due to the possibility of existence that the tracking error approaches to zero, the switching law is designed in the integer power form, so there is no singularity problem with derivatives of integer power term. (2) when $|e_1| \geq \varsigma_1$, the fractional power term designed in the switching is limited in a positive interval, which can avoid the singularity issue that the derivatives of fractional power term might increase to infinity in the combination of fixed-time control and backstepping process. Moreover, this novel switching between the fractional and integer power form is proved to be smooth and continuous.

*Remark* 7. It is worth noting that the fixed-time control strategies have been widely studied, such as References 20-22. However, all of these works encounter singularity issue. Especially, the fractional power utilized for fixed-time controller design and stability proof is settled to a specific constant in References 20,21. In contrast to the above results, we present a smooth switching between the fractional and integer power forms to guarantee that fractional power term is confined within a positive interval, such that the singularity issue can be eliminated and the fractional power remains in an allowable range. Although the singularity-free fixed-time adaptive control has also been proposed in Reference 7, the robotic system can only convert to a second-order system. As improved, we develop a BLFs-based nonsingular fixed-time switching control approach for a high-order nonlinear dynamic system, this design controller plays a key role in ensuring the strong robustness and fast convergence rate of the closed-loop systems.

*Remark* 8. As a whole, there are only two scalar parameter adaptation laws (37) and (39) involved in our design besides the virtual and actual fixed-time smooth control law, which makes it simpler than time-varying BLFs-based adaptation laws without using NN technique in backstepping approach.[31] In addition, the proposed switching mechanism can be simply implemented as a static nonlinearity as in (38), which is comparable to the complexity of state-of-the-art approaches proposed for solving nonsingular issues or switched systems. On the whole, the computation complexity of the considered methods in this article is acceptable and the result is viable on the basis of its implementability.

## 5 | SIMULATION RESULTS

In this section, an application example of ship autopilot system is given to demonstrate the effectiveness of designed method. To begin with, the mathematical model of the ship dynamics is described as follows[42]

$$\begin{cases} \ddot{\phi} + (K/T)\,H(\dot{\phi}) = (K/T)\,\delta \\ \dot{\delta} + (1/T_E)\,\delta = (K_E/T_E)\,\delta_E \end{cases}, \tag{75}$$

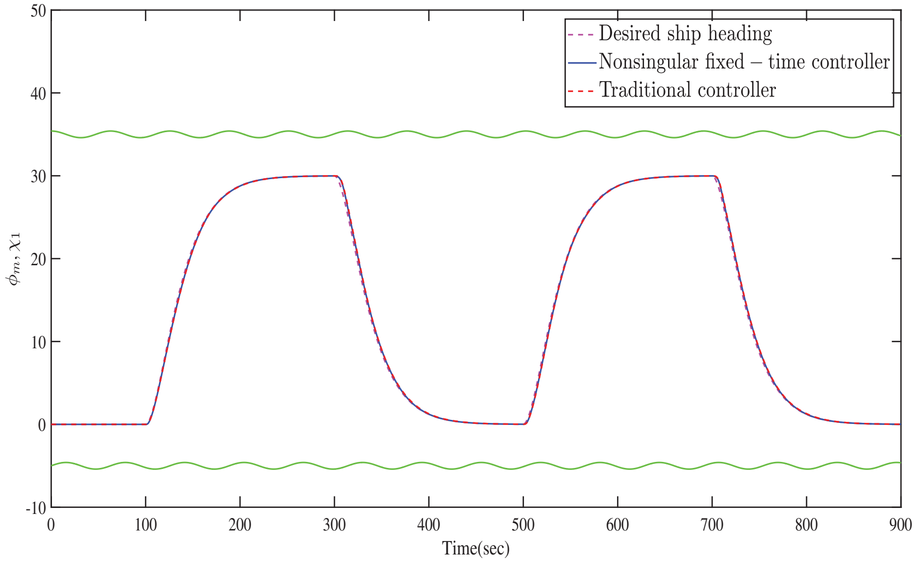


**FIGURE 1** Trajectories of $\chi_1$ and $\phi_m$

where $K = 0.2$ and $T = 64$ are parameters which are functions of ship's constants forward velocity and length, $\delta$ denotes the rudder angle and $\phi$ denotes the heading of the ship. $T_E = 2.5$ is the time delay constant, $K_E = 1$ is the control gain and $\delta_E$ denotes the order angle of the rudder actuator.

And the "spiral test" experiment can approximate the function $H(\dot{\phi})$ as

$$H(\dot{\phi}) = a_1\dot{\phi} + a_2\dot{\phi}^3 + a_3\dot{\phi}^5 + \ \dots \ , \tag{76}$$

where $a_1 = 1$, $a_2 = 30$, $a_3 = 0$ are real-valued constants.

The reference model satisfying realistic performance is selected as

$$\ddot{\phi}_m(t) + 0.1\dot{\phi}_m(t) + 0.0025\phi_m(t) = 0.0025\phi_r(t), \tag{77}$$

where $\phi_m$ is the desired system performance during the ship autopilot control, and $\phi_r$ is a command signal, which changes its value in the interval (0, 30 Deg) every 200 s.

Let the state variables be $\chi_1 = \phi$, $\chi_2 = \dot{\phi}$, $\chi_3 = \delta$, and control input be $u = \delta_E$, then (75) can be transferred into the following expressions

$$\begin{cases} \dot{\chi}_1 = \chi_2 \\ \dot{\chi}_2 = -(K/T)\,H(\chi_2) + (K/T)\,\chi_3 \\ \dot{\chi}_3 = -(1/T_E)\,\chi_3 + (K_E/T_E)\,u \\ y = \chi_1 \end{cases}. \tag{78}$$

In practice, the heading $\chi_1$, the heading velocity $\chi_2$, and the rudder angle $\chi_3$ are restrained by the compact sets $\Omega_\chi := \left\{ \chi \in \mathbb{R}^n : \underline{k}_{c_i}(t) < \chi_i < \overline{k}_{c_i}(t) \right\}$ with $\overline{k}_{c_1}(t) = 35 + 0.4\cos(0.1t)$ and $\underline{k}_{c_1}(t) = -5 + 0.4\sin(0.1t)$, $\overline{k}_{c_2}(t) = 1 + 0.2\cos(0.1t)$ and $\underline{k}_{c_2}(t) = -1 + 0.15\sin(0.1t)$, $\overline{k}_{c_3}(t) = 35 + 0.1\cos(0.05t)$, and $\underline{k}_{c_3}(t) = -35 + 0.1\sin(0.1t)$. The control objective is to ensure that the heading $\chi_1$ can follow the desired heading $\phi_m$ in a fixed time, and all the state variables do not violate the prescribed constraints.

In accordance with Theorem 1, the design parameters in Step 1 are set as: $c_1 = 1$, $\lambda_1 = 0.5$, $l_1 = 1$, $\kappa_1 = 0.06$, $h = 0.6$, $\rho_1 = 0.01$, $\sigma_{11} = \sigma_{12} = 0.5$ with barriers $k_{a_1}(t) = 1 + 0.4\sin(0.1t)$ and $k_{b_1}(t) = 2 + 0.4\cos(0.1t)$; In Step 2, the design parameters are set as: $c_2 = 1$, $\lambda_2 = 25$, $l_2 = 1$, $\kappa_2 = 5$, $\varsigma_2 = 0.01$, $\rho_2 = 0.01$, $\sigma_{21} = \sigma_{22} = 0.5$ with barriers $k_{a_2}(t) = 0.4 + 0.15\sin(0.1t)$ and $k_{b_2}(t) = 0.7 + 0.2\cos(0.1t)$; The design parameters in Step 3 are set as: $c_3 = 2$, $\lambda_3 = 1$, $l_3 = 1$,

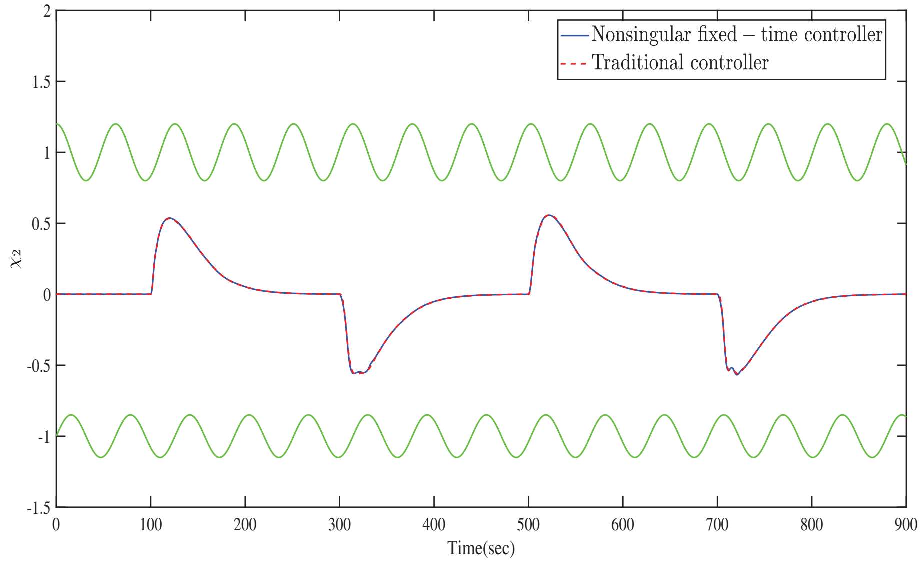


FIGURE 2 Trajectory of $\chi_2$

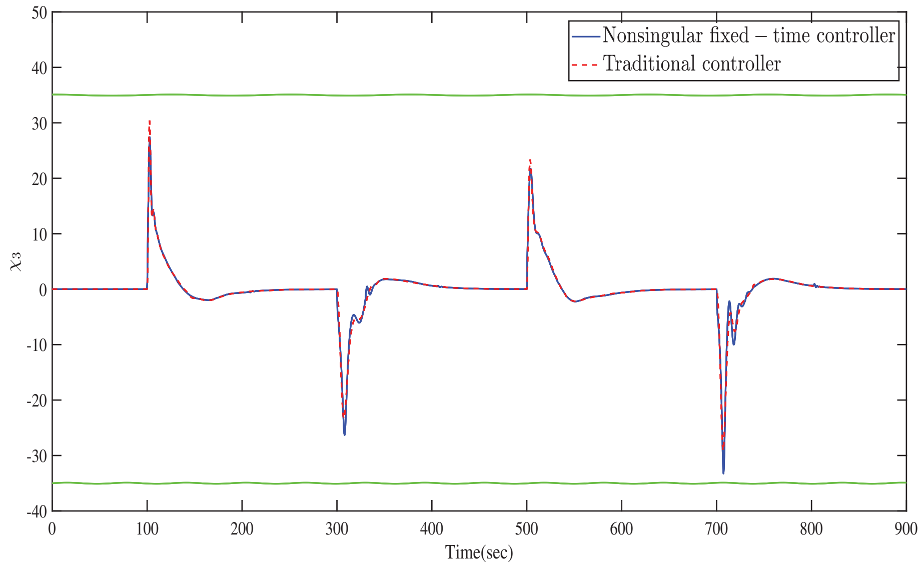


FIGURE 3 Trajectory of $\chi_3$

$\kappa_3 = 2$, $\varsigma_3 = 0.01$, $\rho_3 = 0.01$, $\sigma_{31} = \sigma_{32} = 0.5$ with barriers $k_{a_3}(t) = 0.3 + 0.1\sin(0.1t)$ and $k_{b_3}(t) = 0.4 + 0.1\cos(0.05t)$. Let the initial conditions for $\left[\chi_1(0), \chi_2(0), \chi_3(0)\right] = [0, 0, 0]$, $\left[\hat{\theta}_1(0), \hat{\theta}_2(0), \hat{\theta}_3(0)\right] = [0, 0, 0]$. According to the guideline for selecting parameters of Gaussian network in Reference 43, the RBFNN to approximate the function $S_1(Z_1)$ contains five nodes with centers evenly spaced in the interval $[-4, 4] \times [-4, 4]$ and the function $S_2(Z_2)$ contains 11 nodes in the interval $[-10, 10] \times [-10, 10] \times [-10, 10] \times [-10, 10] \times [-10, 10]$, the RBFNN for function $S_3(Z_3)$ contains 13 nodes spaced in $[-12, 12] \times [-12, 12] \times [-12, 12] \times [-12, 12] \times [-12, 12] \times [-12, 12]$ and the width of each one equals to two specifically.

It can be obviously observed from Figure 1 that the heading $\phi$ can follow the desired heading $\phi_m$ in a fixed time and fairly good tracking performance is obtained. The curves of the heading velocity $\chi_2$ and the rudder angle $\chi_3$ are shown in Figures 2 and 3 separately, and all the system state variables are restrained by the prescribed constraints. By employing the asymmetric time-varying BLFs, Figures 4–6 display the tracking errors $e_1$, $e_2$, and $e_3$, respectively, and prescribed barriers are not violated. The response curve of the bounded and continuous adaptive laws $\hat{\theta}_i$, the switching law $\mathfrak{I}_i$ and the virtual

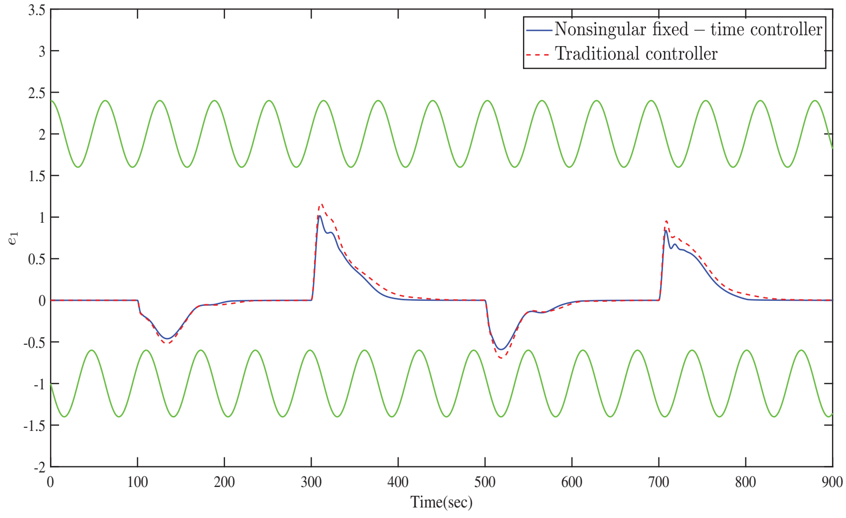


**FIGURE 4** Trajectory of $e_1$

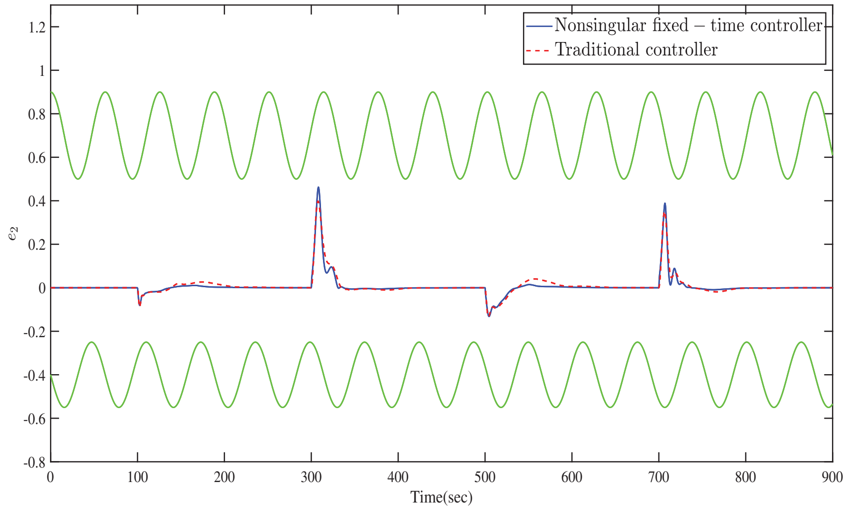


**FIGURE 5** Trajectory of $e_2$

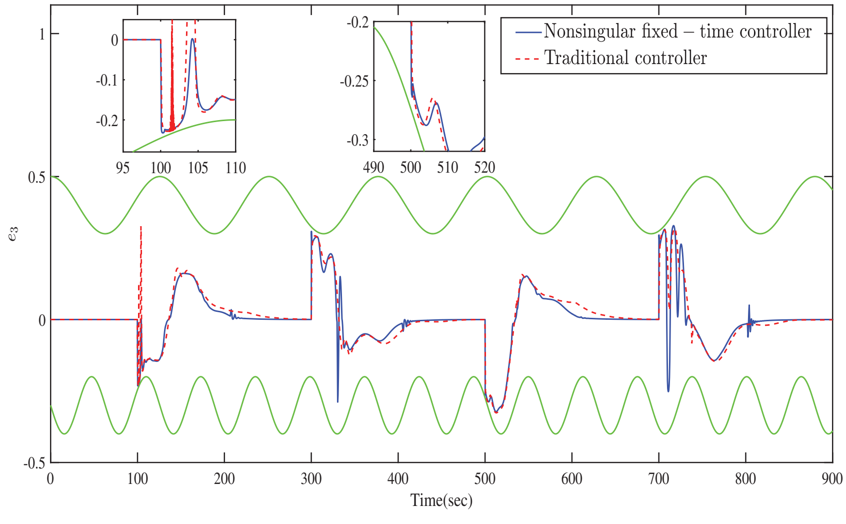


**FIGURE 6** Trajectory of $e_3$

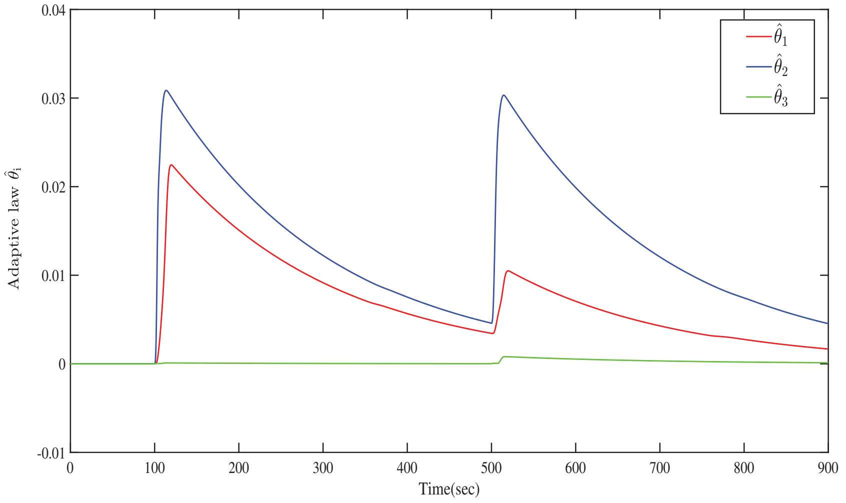


**FIGURE 7** Trajectory of $\hat{\theta}_i$

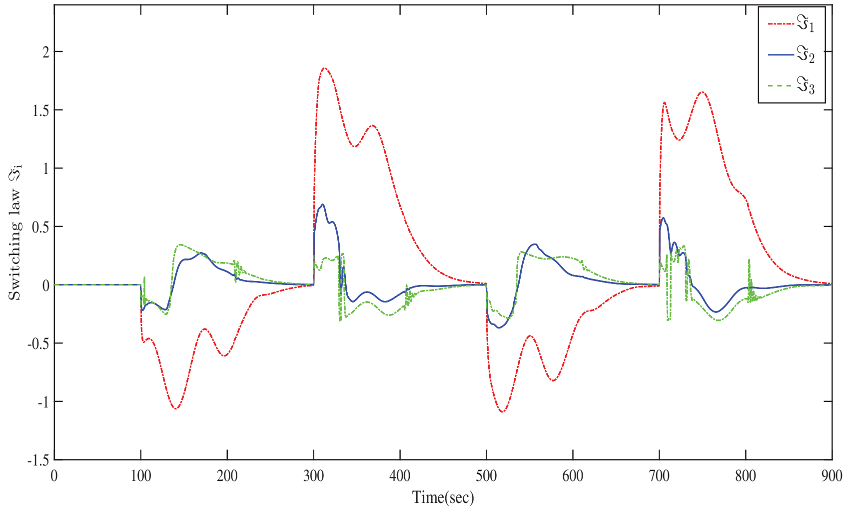


**FIGURE 8** Trajectory of $\mathfrak{I}_i$

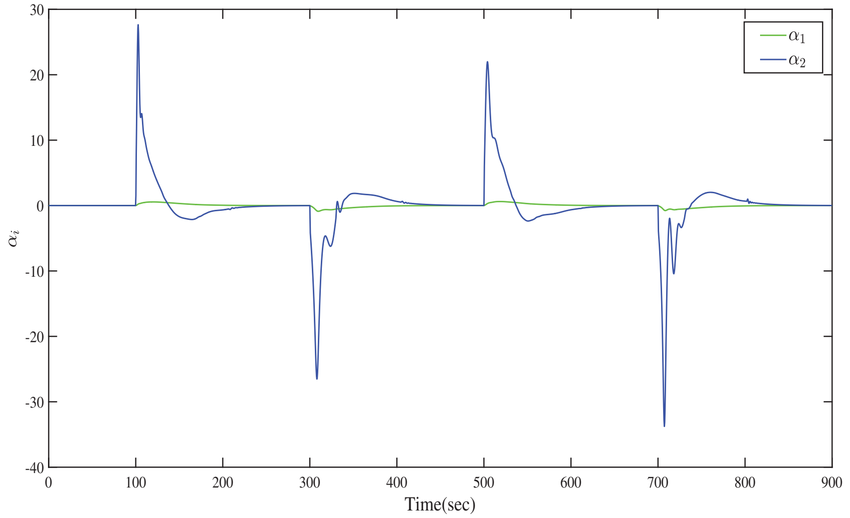


**FIGURE 9** Trajectory of $\alpha_i$

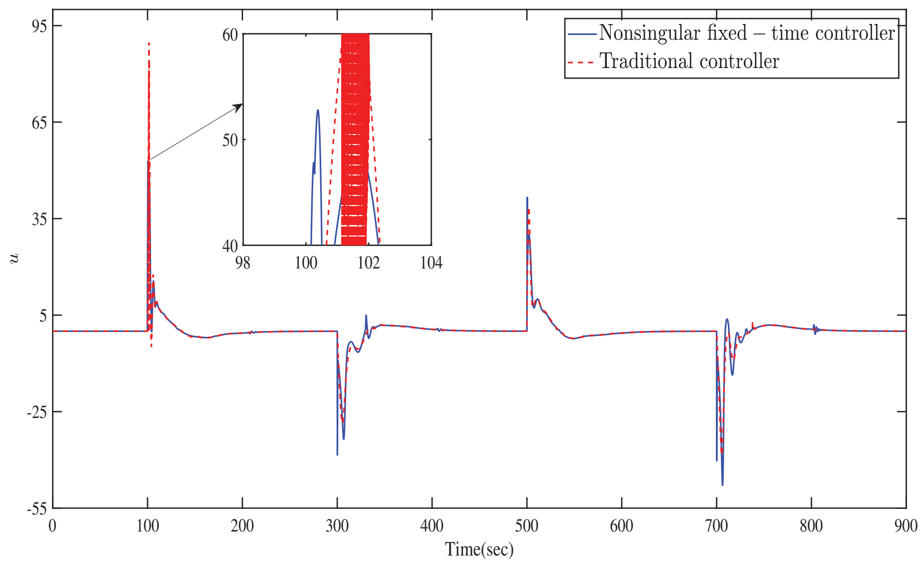


**FIGURE 10** Trajectory of $u$

control law $\alpha_i$ are depicted in Figures 7–9, respectively. In addition, it can be seen from Figure 10 that the control input is bounded and in an allowable range. In contrast to the traditional controller given by

$$
\begin{aligned}
u = r_n(e_n)&\left(-c_n e_n - \hbar_n(t)e_n - \kappa_n e_n - \lambda_n \frac{e_n}{k_{b_n}^2(t) - e_n^2} - \frac{e_n^2 \hat{\theta}_n \psi_n^T(Z_n)\psi_n(Z_n)}{2l_n\left(k_{b_n}^2(t) - e_n^2\right)}\right) \\
&+ (1 - r_n(e_n))\left(-c_n e_n - \hbar_n(t)e_n - \kappa_n e_n - \lambda_n \frac{e_n}{k_{a_n}^2(t) - e_n^2} - \frac{e_n^2 \hat{\theta}_n \psi_n^T(Z_n)\psi_n(Z_n)}{2l_n\left(k_{a_n}^2(t) - e_n^2\right)}\right),
\end{aligned}
\tag{79}
$$

our proposed method exhibits a faster convergence rate and higher tracking accuracy. Specially, Figure 9 produces a chattering behavior during the switching transient, especially when the tracking errors approach to the prescribed barriers, the control efforts would increase dramatically due to the fractional term and the barrier terms in the denominators. Even so, our nonsingular fixed-time controller maintains a less control effort than the traditional controller, which can reduce fuel consumption and enhance the practicability for the ship autopilot system.

## 6 | CONCLUSION

A nonsingular adaptive fixed-time switching control scheme is presented for a class of strict-feedback nonlinear disturbed systems under the full state constraints conditions. The main contribution is to address the singularity problem arising from that the negative power terms stem from the iterative differentiations of fractional power terms might increase to infinity as the tracking errors approach to zero. By skillfully employing a smooth switching between fractional power and integer power terms, the fractional power term is confined within a positive interval to avoid singularity. Compared with the common fixed-time control strategies, a less conservative convergence time is excavated for the first time. Moreover, by integrating NN techniques and the asymmetric time-varying BLF, the proposed control scheme can guarantee that the state variables constraints are not violated. Eventually, all the closed-loop signals can converge into the arbitrarily small regions with fast fixed-time convergence rate and high accuracy. Simulation results are given to confirm the effectiveness of the proposed control approach. In the future works, we will consider applying this nonsingular adaptive fixed-time control approach for high-order or nonstrict-feedback nonlinear systems[44-46] with dynamic surface control technique to reduce computing burden.

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## CONFLICT OF INTEREST
The authors declare no conflict of interest.

## DATA AVAILABILITY STATEMENT
All data generated or analyzed during this study are included in this article.

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