Fleet Level Multi-Unit Maintenance Optimization Subject to Degradation

Maintenance Scheduling For Aircraft Brakes Using Remaining-Useful-Life Prognostics

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by

S.A. Boekweit

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The research has been carried out in the midst of the COVID-19 pandemic, which complicated some aspects of the project and the world around us. I would sincerely like to thank my family and friends for their continued support during my studies and this research project.

S.A. Boekweit
Delft, January 29, 2021

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Nomenclature

Abbreviations

CBM	Condition-Based Maintenance
MCTF	Mean cycle to failure
MRO	Maintenance, Repair and Overhaul
OEM	Original Equipment Manufacturer
RUL	Remaining Useful Life
TBM	Time-Based Maintenance
WBS	Work Breakdown Structure
WFD	Work Flow Diagram

Symbols

$\bar{m}$	Average mean cycle to failure
Ψ	Optimization time
τ	Realization time
a	Scale parameter
b	Shape parameter
C	Cost value
c	Man hour rate of a single crew member
c_{brake}	New price of a single brake
$c_{k,i,t}$	Penalty cost function
c_{RUL}	Cost of the average waste of life at replacement
$c_{scheduled}$	Cost of a scheduled replacement
c_{setup}	Setup cost parameter
$c_{unscheduled}$	Cost of an unscheduled replacement
c_{visit}	Cost of a hangar visit
f	Flight cycles
$f^*_{k,t}$	Flight cycle at maintenance slot t
$f^{RUL}_{k,i}$	Estimated remaining useful life
f_r	Replacement flight cycle
H	Hangar availability
I_k	Set of components
K	Set of aircraft
N	Number of simulations
n	Assumed number of crew members required for a replacement
$r_{scheduled}$	Number of scheduled replacements
$r_{unscheduled}$	Number of unscheduled replacements
r_{visit}	Number of hangar visits

$S_{end,n}$	Time at the end of realization schedule n
$S_{start,n}$	Time at the start of optimization schedule n
T_k	Set of maintenance slots
$x_{k,i,t}$	Decision variable of optimization model
$y_{k,t}$	Auxiliary variable of optimization model
$Z(f)$	Degradation level

Introduction

This thesis puts forward a state-of-the-art research which combines three major subjects considered in maintenance literature, operations optimization, maintenance policies and stochastic simulation. The goal of this thesis is to contribute to the further development of the theories by providing insight, based on literature review and proof of concept, into the application of an optimization model for a fleet level multi-unit problem subject to degradation.

In short, the problem definition is: To determine the optimal maintenance schedule for a fleet of aircraft minimising cost while satisfying safety requirements considering multiple components per aircraft which states degrade over time according to a Gamma process and having limited hangar availability. The focus lies on the brake system of the aircraft which consists of eight brakes, four on each side of the undercarriage.

The condensed research questions read: How can the optimal maintenance schedule of the above stated problem be determined using degradation prognostics and an optimization model? When applying such a model to a case study, what would be the resulting maintenance schedule and key performance indicators? How do the results of the model compare when evaluating against a fixed replacement time-based maintenance strategy?

No prior research has been performed on multi-unit systems which combine prognostics and optimization. Thus, combining these two research disciplines makes this research unique. In time, the application of prognostics combined optimization in aircraft maintenance could increase operational efficiency.

The structure of this thesis report is as follows. First, the scientific paper is presented in Part I. Second, Part II contains the relevant literature review that supports the thesis research. At last, in Part III, some additional work is presented.

I

Scientific Paper

Fleet Level Multi-Unit Maintenance Optimization Subject To Degradation

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Abstract

This thesis paper puts forward a state-of-the-art research which combines three major subjects considered in maintenance literature, operations optimization, maintenance policies and stochastic simulation. The goal of this thesis paper is to contribute to the further development of the theories by providing insight and proof of concept, into the application of an optimization model for a fleet level multi-unit problem subject to degradation.

In short, the problem definition is: To determine the optimal maintenance schedule for a fleet of aircraft minimising cost while satisfying safety requirements considering multiple components per aircraft which states degrade over time according to a Gamma process and having limited hangar availability. The focus lies on the brake system of the aircraft which consists of eight brakes.

The condensed research questions read: How can the optimal maintenance schedule of the above stated problem be determined using degradation prognostics and an optimization model? When applying such a model to a case study, what would be the resulting maintenance schedule and key performance indicators? How do the results of the model compare when evaluating against a fixed replacement time-based maintenance strategy?

From literature it is known monotonic increasing processes, such as brake wear due to operations, is best estimated using a Gamma distribution. Thus the prognostic model used represents a Gamma distribution which estimates the future degradation. The parameters of the Gamma distribution are estimated using the method of moments based on available degradation data which is continuously monitored during operation. In literature maintenance of multi-unit systems has been considered especially in the context of opportunistic maintenance, which implies dependencies between components which can be exploited. In this research economic dependencies between components exists, which can be exploited by creating groups to reduce the overall schedule cost. The optimization model would be classified in literature as a multi-unit, continuous, stochastic, perfect and rolling maintenance model.

The prognostic model estimates the parameters of the Gamma distribution based on the available degradation data, after which it calculates the remaining useful life of the components. This remaining useful life becomes an input to the optimization model, which schedules the components to available maintenance slots while accounting for the constraints, optimizing with respect to cost. Then, the obtained schedule is fixed for a certain amount of time and realized assessing its performance. These steps are repeated using this newly realized degradation data until a schedule has been realized for a specified time. The approach of optimizing and realizing is called a rolling horizon, which allows for the continuation through time while accounting for previous decisions made. To achieve schedule results on which meaningful conclusions can be drawn Monte Carlo simulation is performed.

The model is applied to a case study where 15 aircraft are considered with eight brakes each for a total schedule time of 5 years. In total 5 maintenance strategies are considered: Condition-Based Maintenance (CBM) including grouping, CBM excluding grouping, Time-Based Maintenance (TBM) at Mean Cycle To Failure (MCTF), TBM at 97.5% MCTF and TBM at 95% MCTF. The CBM model is the created model, where grouping stands for the consideration of economic dependence between the components. TBM is a strategy used for evaluation, which is a fixed interval strategy which uses the mean cycle to failure and a percentage thereof.

From the Monte Carlo simulation results of the case study it can be concluded that the CBM including grouping outperforms the other strategies on all Key Performance Indicators (KPI) except for the waste of life KPI, which is expected as this strategy sacrifices remaining life at replacement for the grouping of components. When evaluating the total cost of the obtained maintenance schedules the maintenance schedule of the CBM including grouping performs best, followed by the CBM excluding grouping strategy, TBM at 97.5% of MCTF, TBM at MCTF and at last TBM at 95% of MCTF. From these results it can be observed that a CBM strategy is favoured over a TBM strategy with respect to overall cost. It can therefore be concluded that it is not only possible to combine degradation modelling prognostics with maintenance optimization, it also outperforms existing time-based maintenance strategies when evaluated for a case study.

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This research has contributed to the existing body of knowledge as it fits the gap which currently exists within literature. No prior research has been performed on multi-unit systems which combine prognostics and optimization. This research has shown that the application of a CBM strategy which incorporates prognostics can have a significant impact on the overall schedule cost compared to the traditional TBM strategies which are still commonly used among maintenance repair and overhaul companies. Especially the use of opportunistic maintenance, such as the economic dependence considered has shown promising results.

1 Introduction

The first papers regarding preventative maintenance got published around the 1960's, after which the popularity of the subject has steadily increased [J.J. McCall, 1963]. Then, around 1990, about 30 years later, the first maintenance decisions in industry were being made by optimisation models [R. Dekker, 1996]. It has been made clear that optimisation models have a significant impact on the operational efficiency of maintenance, repair and overhaul companies.

This thesis paper considers a fictitious airline for which a maintenance schedule is required to be created for the brake system of multiple aircraft and with multiple brakes per aircraft. The brake pads of the aircraft degrade over time due to operations. The maintenance is constraint by limited hangar availability and the aircraft's flight schedule. The research objective is to contribute to the further development of the theories regarding operations optimization, maintenance policies and stochastic simulation. This is achieved by providing insight and proof of concept, into the application of an optimization model and its evaluation.

The literature relevant to the defined problem is discussed in section 2. The literature regarding the degradation of components and their modelling is discussed. After, literature regarding maintenance strategies and their evaluation is looked at. Then, different approaches to modelling maintenance scheduling optimisation problems in literature are laid out. The classification scheme of such models is elaborated on and various papers are considered.

The model's development and implementation is discussed in section 3. First, the development of the prognostic model is discussed in detail after which its implementation is shown. Second, the optimization model is elaborated on. The concept, mathematical formulation and implementation is outlined. Final, a rolling horizon is used for the creation of the schedule over a long period of time. The development and implementation of this rolling horizon approach is elaborated on.

In order to evaluate the performance of the model a case study is created, which is elaborated on in section 4. Next to the case study, the key performance indicators of the resulting schedule are also elaborated on.

Then, section 5 shows the results achieved when performing the case study. These include the results of a single realization for different maintenance strategies. A Monte Carlo Simulation performed on the case study and evaluated for different maintenance strategies. At last, two local sensitivity analysis are performed on the number of aircraft and the total scheduling time in order to better understand the maintenance model's behaviour.

Final, the thesis research is concluded and discussed in section 6.3. First, the research scope is reevaluated. Second, the academic novelty of the performed research is discussed. Third, the conclusions regarding the model and its results are discussed. After which recommendations for future research are made.

2 Literature Review

Reviewing the available literature is of vital importance to the research as it gives insights into the considered theories and state-of-the-art research performed on related topics. First, the literature regarding degradation prognostics is reviewed. Second, the literature regarding maintenance strategy evaluation is elaborated on. At last, the theories and available literature on maintenance schedule optimization are discussed.

2.1 Degradation Prognostics

The first maintenance research papers which started to show signs of prognostics were the papers discussing preventative maintenance, as these papers discussed the replacement of working components which had not failed yet. Initially these strategies only considered fixed intervals with an assumed total life known as periodic policies [H. Wang, 2001]. Soon reliability was added in terms of a failure rate, resulting in failure limit policies. Usually the failure rate is expressed as a function of a state variable of the component, examples include: age, wear or accumulated damage. Some of the state variables can be physically measured however, they can also be modelled using various methods. Considerable research has been done in order to accurately predict these states. An example of such a state prediction model is the Gamma distribution, which can be used for irreversible processes where cumulative damage is the cause of degradation [H. Wang, 2001] [J.M. van Noortwijk, 2017]. A random variable X is said to be Gamma-distributed when X $\sim$ Gamma(α,β) resulting in the probability density function as seen in Equation 1 [F.M. Dekking, C. Kraaikamp, H.P. Lopuhaä, L.E. Meester, 2005].

$$f(x;\alpha,\beta) = \frac{\beta^{\alpha} x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)} \quad \text{for } x > 0, \;\; \alpha, \beta > 0 \tag{1}$$

Here α is called the shape parameter and β the rate parameter. The Remaining Useful Life (RUL) is one of the applications of such a prediction model where the state of the component has reached a given threshold were it no longer functions as intended. The parameters of the Gamma distribution can be estimated based on component degradation data. In previous research these parameters have been analyzed based on real observations of the state of degradation [J. Lee, M. Mitici, 2020]. For this research the real behaviour of the component degradation can thus be generated using the obtained parameters from the previous research. These parameters can be seen in Table 1.

Table 1: Real brake degradation Gamma parameters per location respectively [J. Lee, M. Mitici, 2020]

Brake ID	Parameter a_{real}	Parameter b_{real}	MCTF
1	3.350	0.0002063	1447.0
2	4.146	0.0001836	1313.7
3	3.546	0.0002217	1272.0
4	3.390	0.0002171	1358.8
5	4.667	0.0001715	1249.4
6	4.100	0.0001856	1314.1
7	3.068	0.0002329	1399.5
8	2.583	0.0002852	1357.5

Another very interesting paper is the conference paper of Q. Wei and D. Xu from 2014 from the Beihang University of Beijing, China [Q. Wei, D. Xu, 2014]. In their paper they consider a component which degrades according to a Gamma process and introduce a measurement error which can be treated as a Gaussian distribution. Where the independent increments $\Delta w_{\text{true}}(t) \sim Gamma(\alpha, 1/\lambda)$ and $\epsilon_{\text{error}}(t+\delta t) - \epsilon_{\text{error}}(t) \sim N(0, 2\sigma^2)$. Then to estimate the RUL of the component they want to estimate the parameters of both distributions. They propose the usage of the method of moments. After performing the derivations the resulting parameter estimators can be seen in Equation 2, Equation 3 and Equation 4.

$$\lambda = \left[\frac{E\left[\Delta w(t)^3\right]}{2E\left[\Delta w(t)\right]} - \frac{3}{2} E\left[\Delta w(t)^2\right] + E\left[\Delta w(t)\right]^2 \right]^{-\frac{1}{2}} \tag{2}$$

$$\alpha = \lambda E\left[\Delta w(t)\right] \tag{3}$$

$$\sigma^2 = \frac{1}{2}\left(E\left[\Delta w(t)^2\right] - E\left[\Delta w(t)\right]^2 - \frac{1}{\lambda} E\left[\Delta w(t)\right] \right) \tag{4}$$

2.2 Maintenance Strategy Evaluation

The problem definition considers a multi-unit system for which continuous monitoring is available. Other definitions used in literature for single-unit and multi-unit systems is simple and complex systems respectively [R. Dekker, 1996]. The reason a multi-unit system is considered to be more complex is due to dependencies between the components in a system. This has been extensively studied in multiple papers and led to the following classification scheme [S. Alaswad, Y. Xiang, 2016] [B. de Jonge, P.A. Scarf, 2019] [H. Ab-Samat, S. Kamaruddin, 2014]. The dependencies that are considered in literature are economic, structural and stochastic dependence. Economic dependence between components implies it is more economical to maintain multiple components in a single maintenance action than separate. An example would be a required set-up cost and an additional cost per component serviced. Structural dependence between components implies a more physical dependency, where a component can only be serviced if another component is also removed. The last dependency considered in literature is stochastic dependence which implies that the state of one component can influence the state of other components in the system.

Considering these dependencies a new maintenance approach has been proposed, namely opportunistic maintenance (OM) [H. Ab-Samat, S. Kamaruddin, 2014]. With the OM approach the researcher tries to exploit the dependencies of components in order to optimise the schedule, usually by minimising maintenance costs. One of the most promising applications are the papers considering grouping as an OM approach. Grouping is a term used to describe combining maintenance actions in order to optimise a maintenance schedule usually in systems which have a high economic dependency. Examples of such papers include [H.C. Vu, P. Do, A.

Barros, C. Bérenguer, 2014] [P. Do Van, A. Barros, C. Bérenguer, K. Bouvard, 2013] [K. Bouvard, S. Artus, C. Bérenguer, V. Cocquempot, 2010]], where OM is applied to complex systems.

The only dependence between components considered in the problem definition appears to be economic which makes the use of a grouping approach certainly viable.

2.3 Maintenance Schedule Optimization

As discussed in the previous subsections multiple distinctions with respect to the maintenance problem can be made. Examples include single-unit vs multiple-unit systems, failure rate vs deterioration and preventive vs corrective. Capturing the maintenance problem in a suitable optimisation model also comes with distinct properties which are captured in a classification scheme. Distinctions are made between continuous vs discrete time, deterministic vs stochastic processes, perfect vs imperfect repairs and finite vs rolling horizon. In Table 2 current research papers regarding maintenance optimisation are listed including their classification.

Table 2: Maintenance optimisation models considered in literature

Authors Year	**Continuous vs Discrete**	**Perfect vs Imperfect**	**Finite vs Rolling**	**Optimisation Objective**
[S. Wu, I.T. Castro, 2020]	Continuous	Imperfect	Finite	Minimize Maintenance Cost
[M.J. Kallen, J.M. van Noortwijk, 2004]	Continuous	Imperfect	Finite	Minimize Maintenance Cost
[X. Zhou, L. Xi, J. Lee, 2006]	Continuous	Imperfect	Rolling	Minimize Maintenance Cost
[H. Liao, E.A. Elsayed, L. Chan, 2005]	Continuous	Imperfect	Rolling	Maximize Availability
[H.C. Vu, P. Do, M. Fouladirad, A. Grall, 2020]	Continuous	Perfect	Rolling	Minimize Maintenance Cost
[C.R. Cassady, W.P. Murdock Jr, E.A. Pohl, 2000]	Discrete	Imperfect	Finite	Maximize Reliability
[S. Taghipour, D. Banjevic, A.K.S. Jardine, 2010]	Discrete	Imperfect	Finite	Minimize Maintenance Cost
[J.Y.J. Lam, D. Banjevic, 2015]	Discrete	Perfect	Finite	Minimize Maintenance Cost
[K. Schneider, C.R. Cassady, 2014]	Discrete	Perfect	Finite	Maximize Reliability, Minimize Cost, Maximize Minimum Reliability

Most of the choices regarding the optimisation model classification come from the problem they are being applied to. The data which is available, the system that is being maintained and the horizon of the problem. In relation to the problem definition as proposed in section 1, the most interesting models to consider are those which consider continuous time, stochastic processes, perfect repairs as only replacements are considered and a rolling horizon. The objective function which is required to optimise will be in relation to the costs associated to the maintenance. While the model is being constraint by hangar availability, the aircraft schedule and safety requirements.

3 Methodology

This section aims to elucidate the development and implementation of the condition-based maintenance model and the time-based maintenance model which shall be used to evaluate the achieved results. For elucidation purposes the condition-based model can be subdivided into a prognostic model, optimization model and rolling horizon approach.

3.1 Prognostic Model

This subsection shall discuss the prognostic model. First, the concept of the model is explained. After which, the mathematical formulation of the model is stated. Finally, the implementation of the model is shown.

3.1.1 Concept Description

The aim of the prognostic model is to determine the flight cycle at which the brake pads reach a degradation threshold. During operations the brake pads degrade, this degradation can be estimated per flight cycle as independent increments of a probability distribution. From literature it is known that monotonic increasing processes can best be estimated by the Gamma distribution. The wear process of brake pads on aircraft is an example of such monotonic increasing processes. Given initial degradation of the brake pads, the parameters of the Gamma distribution can be estimated by means of the method of moments. After which the remaining useful life in flight cycles can be determined by calculating the cumulative distribution function. The prognostic model assumes only nominal operation, meaning it does not consider hard landings or other such events which might expedite the degradation level of the brake pads.

3.1.2 Mathematical Formulation

Let $Z(f)$ be the degradation level of a brake pad at time f in flight cycles with $0 \leq Z(f) \leq 1$ and $f \leq 0$. Then, let $Z(f_0) = Z_0$ be the initial condition. Due to safety regulations the brake pad must be replaced at or before degradation level $Z_T = 0.75$. During a flight cycle the aircraft performs several tasks which require the brakes to be used such as taxiing, take-off and landing. The degradation level increment over a flight cycle can be estimated according to a Gamma distribution: $\Delta Z \sim Gamma(a, b)$, with a and b being the scale and shape parameters respectively. And so, given the initial conditions, the sum of Δf independent increments of ΔZ should be greater or equal to $Z_T = 0.75$ or $Z_0 + \sum_{\Delta f} \Delta Z \geq Z_T$. To find Δf the probability equation as stated in Equation 5 needs to be solved.

$$P(Z(f) \geq Z_T | Z(f_0) = Z_0) = P(Z_0 + \sum_{\Delta f} \Delta Z \geq Z_T) = P(\sum_{\Delta f} \Delta Z \geq Z_T - Z_0) \tag{5}$$

This equation can be solved by using the property of the Gamma distribution being: $\sum_{\Delta f} \Delta Z \sim Gamma(\Delta f a, b)$ and the definition of the cumulative distribution function of the Gamma distribution leading to Equation 6.

$$P(\textstyle\sum_{\Delta f} \Delta Z \geq Z_T - Z_0) = 1 - \frac{1}{\Gamma(\Delta f a)} \gamma(\Delta f a, b(Z_T - Z_0)) \tag{6}$$

Where Γ is the Gamma function and γ is the lower incomplete Gamma function. The aim of the prognostic model is to find $f_{k,i}^{RUL} = \Delta f$ such that Equation 6 equals 0.1.

The parameters a, b being the scale and shape parameters respectively are estimated based on the condition monitored degradation data available using the method of moments as initially proposed by Q. Wei and D. Xu in their paper from 2014 however slightly adapted to apply to this problem as seen in Equation 7.[Q. Wei, D. Xu, 2014]

$$b = -\frac{E[\Delta w(f)]}{E[\Delta w(f)]^2 - E[\Delta w(f)^2]}, \quad a = bE[\Delta w(f)] \tag{7}$$

Where Δw equals $Z(f+1) - Z(f)$ for all f in the available degradation data.

3.1.3 Implementation

The block diagram of the complete model can be seen in Figure 3, it provides an overview of the inputs, processing and outputs of the prognostic model. The input to the prognostic model is the monitored degradation data available for each component. First, the shape and scale parameters of the Gamma distribution fitting the degradation data are determined using the method of moments as discussed above. Second, the $f_{k,i}^{RUL}$ corresponding to each component i and aircraft k is calculated using the cumulative distribution function. Resulting in the output being the remaining-useful-life of each component, which will be used as input for the optimization model.

3.2 Optimization Model

This subsection shall elaborate on the optimization model. First, a concept description is given. Second, the mathematical formulation of the optimization is stated. Finally, the implementation of the model is shown.

3.2.1 Concept Description

The aim of the optimization model is to determine the optimal replacement maintenance slots of the aircraft brakes in terms of cost, while accounting for the component degradation, maintenance opportunities available and hangar availability. It should be able to perform this optimization for a given time frame, using the remaining useful life of the components which is determined by the prognostic model and using the aircraft flight schedule to determine the available maintenance slots. As discussed in the literature review significant cost savings can be achieved by grouping maintenance activities together, grouping maintenance activities shall increase the aircraft availability. Therefor, the model optimizes the cost in two ways, first by minimizing the waste of life, i.e. the remaining life of the component at replacement and second by minimizing the amount of maintenance groups, i.e. perform as much maintenance actions as possible in a single group. The definition of a group equals consecutive maintenance actions within the same ground time, a more formal mathematical definition will be given in the next subsection. The meaning of groups in the schedule represents the number of visits the aircraft is required to make to the hangar. Thus, one group represents one hangar visit. The optimization model considers maintenance slots as possible maintenance opportunities, it is assumed the replacement of a brake takes two hours to perform and therefor the time horizon of the optimization model is discretized in maintenance slots of 2 hours. Meaning there are multiple maintenance slots between flight cycles depending on the flight schedule per aircraft. The flight schedule used for the case study will be elaborated on in section 4.

3.2.2 Mathematical Formulation

To improve readability and clarity of the equations a table including all relevant nomenclature will be shown in Table 3.

Table 3: Nomenclature of optimization model

Sets	
K	Contains all aircraft considered
I_k	Contains all components i which are part of aircraft k
T_k	Contains all available maintenance slots which are part of aircraft K in the optimization window
Decision Variable	
$x_{k,i,t}$	Equals 1 if component i of aircraft k is assigned to maintenance slot t, 0 otherwise
Auxiliary Variable	
$y_{k,t}$	Equals 1 if aircraft k is assigned to maintenance slot t and not assigned to maintenance slot $t-1$, 0 otherwise
Parameter Definitions	
$c_{k,i,t}$	Penalty cost of assigning component i of aircraft k to maintenance slot t
H	Hangar availability
c_{setup}	Setup cost for performing maintenance on the aircraft

The objective function of the optimization model equals:

$$\min C = \sum_{k\in K}\sum_{i\in I_k}\sum_{t\in T_k} x_{k,i,t}\cdot c_{k,i,t} + c_{setup}\sum_{k\in K}\sum_{t\in T_k} y_{k,t} \tag{8}$$

Which will be subjected to the following constraints:

$$\sum_{t\in T_k} x_{k,i,t} = 1 \;\; \forall i \in I_k \;\; \forall k \in K \tag{9}$$

$$\sum_{k\in K}\sum_{i\in I_k} x_{k,i,t} \leq \text{H} \;\; \forall t \in T_k \tag{10}$$

$$y_{k,t} \geq \begin{cases} \sum_{i\in I_k} x_{k,i,t} - \sum_{i\in I_k} x_{k,i,t-1} & \forall t \in T_k, \forall k \in K \quad , t-1 \in T_k \\ \sum_{i\in I_k} x_{k,i,t} & \forall t \in T_k, \forall k \in K \quad \text{, otherwise} \end{cases} \tag{11}$$

Objective Function

The first term of the objective function represents the cost associated with the remaining useful life at the scheduled replacement, the larger the remaining useful life the higher the contribution to the objective. This ensures the optimization model drives the scheduled replacement to the $f_{k,i}^{RUL}$ as determined by the prognostic model. It does this by means of the cost function $c_{k,i,t}$ which depends on component i of aircraft k and maintenance slot t. It is not desired for the model to schedule replacements after $f^{RUL}_{k,i}$, as this would mean the degradation level threshold would be exceeded, this is also accounted for in $c_{k,i,t}$ which can be seen in Equation 12.

$$c_{k,i,t} = \begin{cases} f^{RUL}_{k,i} - f^{*}_{k,t} + \frac{t}{100} & , f^{*}_{k,t} < f^{RUL}_{k,i} \\ 1e^{6} & , \text{otherwise} \end{cases} \tag{12}$$

Where $f^{*}_{k,t}$ equals the flight cycle at maintenance slot t for aircraft k. A graphical representation of the above cost function can be seen in Figure 1.

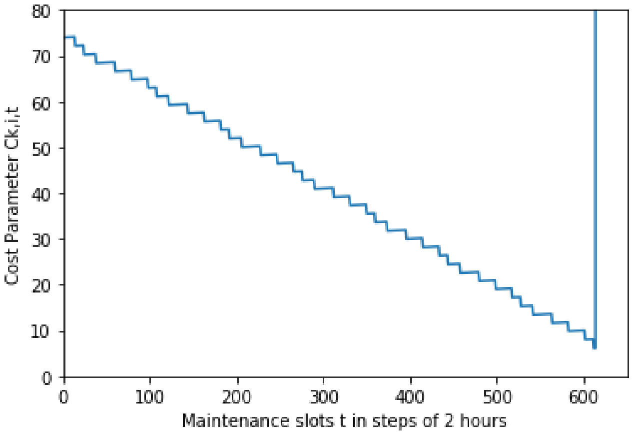


Figure 1: Example of the cost parameter $c_{k,i,t}$

The second term of the objective function represents the cost associated with the amount of groups in the maintenance schedule. All maintenance actions are part of a maintenance group, groups consist of 1 or more maintenance actions. If two or more maintenance actions on a single aircraft are performed on consecutive maintenance slots t they are considered to be a group. Thus, the sum of all groups over the schedule are multiplied with a fixed setup cost which is c_{setup}.

Constraints

The first constraint ensures all components which are within the optimization shall be scheduled exactly once. The constraint evaluates the sum of all decision variables over the maintenance slots t and equates them to one, for all components i and for all aircraft k in the optimization schedule.

The second constraint ensures the hangar availability is not exceeded, the hangar availability represents the amount of components i which can be serviced per maintenance slot t. Thus the constraint evaluates the sum of all decision variables of components i and aircraft k and evaluates them to the hangar availability H, this is done for all maintenance slots t.

The third constraint counts the number of groups within the schedule by evaluating the decision variables at t and $t-1$. There are two possible equations used, one for the case that $t-1$ is in T_k and another for all other cases. The other cases include when $t = 0$ is considered, as $t-1$ does not exist in this case, and when $t-1$ is not in T_k meaning $t-1$ is not a valid maintenance slot because it is during a flight. The constraint evaluates the sum of all decision variables over the components i for all aircraft k and all maintenance slots t.

3.2.3 Implementation

A block diagram of the complete model is created which provides an overview of the inputs, processing and outputs of the optimization model, this diagram can be seen in Figure 3. The inputs to the model include all data sets which include the aircraft, the components for each aircraft, the maintenance slots available for each aircraft, the setup cost value used in the optimization for the cost contribution of groups and the remaining useful life in flight cycles for each component as determined by the prognostic model. The input data is used to formulate the linear program after which the optimization is performed. The results of the optimization are the optimal times in maintenance slots at which maintenance should be performed for each component in the optimization. These maintenance slots are translated to $f_{scheduled}$ which represents the flight cycle before the scheduled maintenance slot.

3.3 Rolling Horizon Approach

In this subsection the rolling horizon approach for the realization of the maintenance schedule is elaborate upon. First, the concept description of the approach will be given. After which, the mathematical formulation is stated. Finally, the implementation of the rolling horizon approach is elucidated.

3.3.1 Concept Description

The rolling horizon approach is a method to step wise progress in time while accounting for decisions made. The maintenance schedule is optimized for a given period of time Ψ. Then, the optimized schedule is realized for a given period of time τ. By making use of this approach it is possible to evaluate the realized schedule and asses its performance. A figure indicating the different periods and progression of time can be seen in Figure 2.

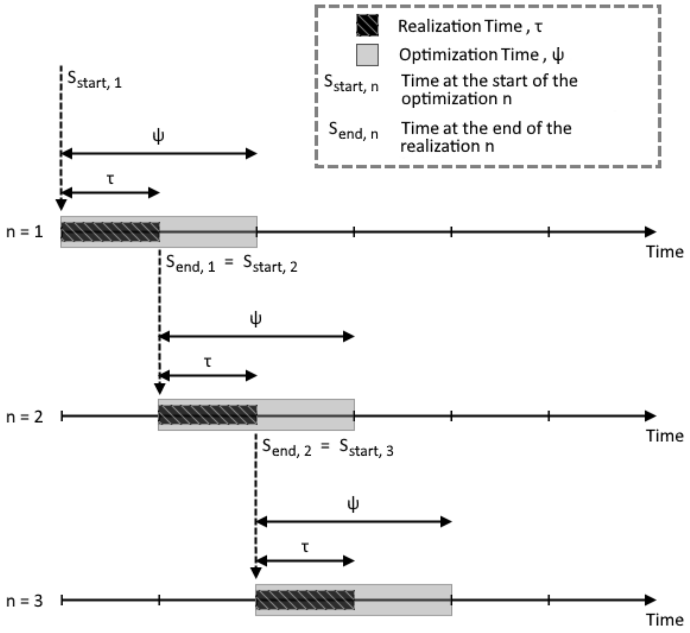


Figure 2: Rolling horizon approach visualization including relevant parameters

The degradation parameters used during the realization of the schedule are known based on observed data, this will be elaborated on in the next subsection. During realization the replacements are performed as scheduled. However, it is possible the degradation level reaches the degradation threshold before the replacement is scheduled. This results in an unscheduled replacement as the component needs to be replaced immediately when the degradation threshold is exceeded. A more detailed mathematical formulation is given in the next subsection.

When using the rolling horizon approach and when taking small time increments Ψ it is possible there are no components which require maintenance within the optimization schedule. If this is the case the degradation can simply be realized for the time τ. It is also possible some of the components are to be scheduled within the optimization and some are not. Therefor, before every optimization the model checks if the estimated $f_{k,i}^{RUL}$ by the prognostic model of every component falls within the next optimization time window n which equals $[S_{start,n},\, S_{start,n} + \Psi]$.

3.3.2 Mathematical Formulation

After the schedule optimization is performed the degradation is realized, for this realization degradation parameters, a_{real} and b_{real} which are the scale and shape parameters respectively from Table 1 are used.[J. Lee, M. Mitici, 2020] These parameters are estimated based on real component degradation observations for each brake in the brake system and depends on their location.

As mentioned in the previous subsection, during realization of the schedule it is evaluated whether the component is replaced or unscheduled replaced. Two checks are performed for each f during the realization, where f equals the current flight cycle of the realization. First, $f = f_{Scheduled}$ in which $f_{Scheduled}$ equals the flight cycle before which a replacement is scheduled by the optimization. Second, $Z(f) > 0.75$ where $Z(f)$ is the degradation level of the component after flight cycle f. If one of these two equations is true the component is replaced, depending on which equation is true it is replaced as scheduled or unscheduled respectively. In the case both are true for a given f the component is replaced as scheduled. In both cases the degradation level Z goes to zero, as the component is always replaced by a pristine component.

3.3.3 Implementation

The rolling horizon combines the prognostic and optimization model to realize the obtained schedules for each n to be able to evaluate them. For each schedule n the fixed schedule τ is realized. A block diagram visualizing the steps taken and links between the in- and outputs can be seen in Figure 3.

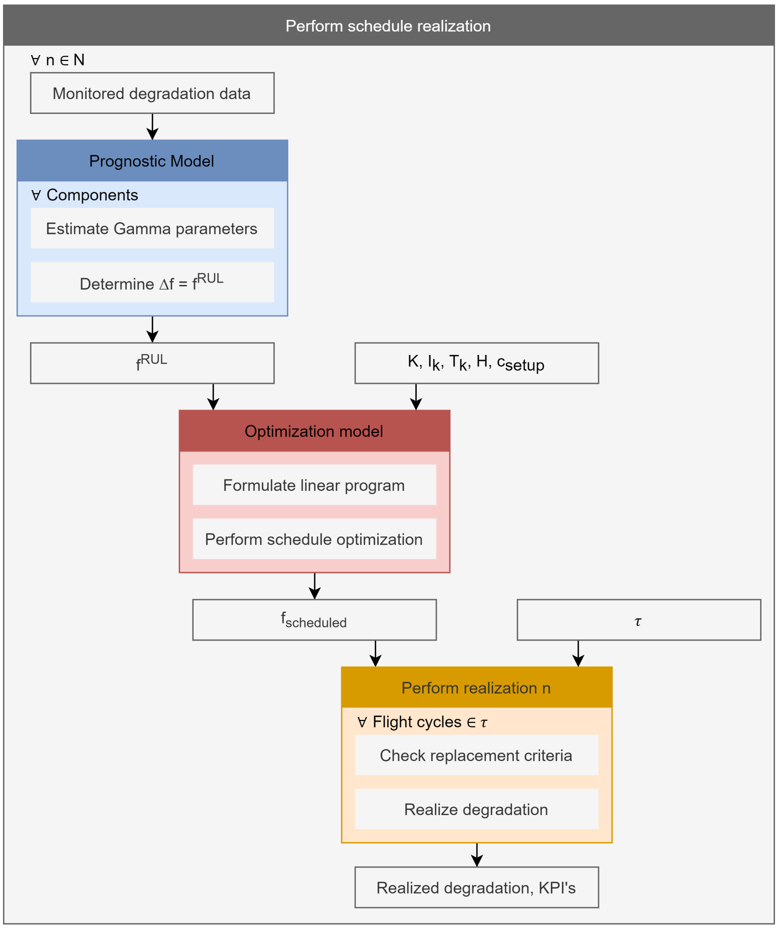


Figure 3: Block diagram of the implementation of the rolling horizon

3.3.4 Monte Carlo Simulation

The model is able to realize a maintenance schedule for a long period of time by realizing smaller periods of time N times, this full realization should be simulated in order to draw conclusions about the performance of the maintenance schedule strategy over a long period of time. This can be done using Monte Carlo simulation. Using Monte Carlo simulation the full schedule is realized repeatedly using the same initial conditions. By doing this it is possible to generate a more robust result which can be evaluated and compared to other schedules using a different strategy.

3.4 Time-Based Maintenance Model

In order to compare and verify the results achieved by using the condition-based maintenance method, a time-based maintenance method is developed. This second method is relatively simple, as all maintenance actions planned are only dependant on time, i.e. intervals between maintenance actions. First, the concept is elaborated on. Second, the mathematical formulation is shown.

3.4.1 Concept Description

For this maintenance strategy maintenance intervals need to be determined at which the replacements take place. There are multiple ways in which the time intervals can be chosen, for this strategy the mean cycle to failure of each component dependant to its position shall be used as interval this data is available from measured degradation data and can be found in Table 1. Similar to the realization of the condition-based maintenance model the degradation will be realized until either the replacement time is reached or the degradation level exceeds the threshold.

3.4.2 Mathematical Formulation

For the time-based maintenance approach the definitions remain the same as the condition-based maintenance approach, the derivations of the mean cycle to failure for this problem follows from Equation 13 up to Equation 16.

$$\sum_{\Delta f} \Delta Z \sim Gamma(\Delta f a, b) \tag{13}$$

$$E[\sum_{\Delta f} \Delta Z] = \Delta f ab \tag{14}$$

$$\Delta f ab = 0.75 - Z_0 \tag{15}$$

$$MCTF = \Delta f = \frac{0.75 - Z_0}{ab} \tag{16}$$

Again, for every realized flight cycle a check is performed to verify if a scheduled or unscheduled replacement is performed these checks are $f = MCTF + f_{LR}$ and $Z(f) > 0.75$ respectively. If the component is replaced multiple times for the given maintenance schedule the first check should be adapted accordingly. Therefor, the equation has the term f_{LR} which is the flight cycle at which the component is last replaced.

4 Description of the Case Study

First, the case study or experiment setup is discussed. Second, the key performance indicators which are used to evaluate the maintenance schedule are elaborated on.

4.1 Case Study

The maintenance schedule optimization shall be performed for the following case study using the parameters which can be seen in Table 4.

Table 4: Case study parameters

Parameter	Value	Unit	Elucidation
K	15	[Aircraft]	Number of aircraft
I_K	8	[Components]	Number of components per aircraft
Ψ	8	[Weeks]	Optimization time
τ	4	[Weeks]	Realization time
$\sum_n \tau$	5	[Years]	Total realization time
H	1	[Component]	Hangar availability

Two other important inputs to the maintenance schedule are the flight schedule for each aircraft and the initial degradation data which is available for each component. Because the model is making a long term planning where disturbances in the flight schedule are not considered, it can be assumed the flight schedule is repeating weekly for each aircraft. Based on historical flight data available a week schedule is generated and this schedule is repeated. The used weekly schedule for five aircraft can be seen in Figure 4, a more detailed overview of the flight schedule data can be found in Appendix A. In the figure below, red indicates the time away from the maintenance base.

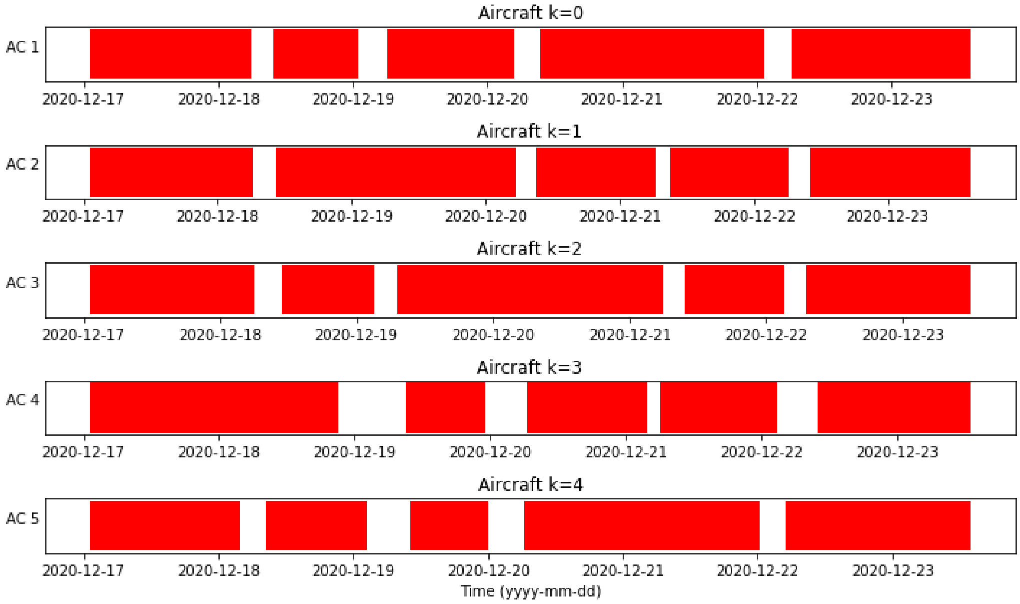


Figure 4: Single week of the flight schedule for five aircraft, red indicates time away from base

The initial degradation for each component is generated using the real component degradation parameters as discussed in section 3.3 for 20 flight cycles. The degradation level for each component does not start at zero, rather the starting degradation level is randomly distributed between 0 and 0.6. The inital degradation is generated once, and used for all maintenance strategies analyzed. The initial degradation of aircraft $k = 0$ for all its eight components can be seen in Figure 5.

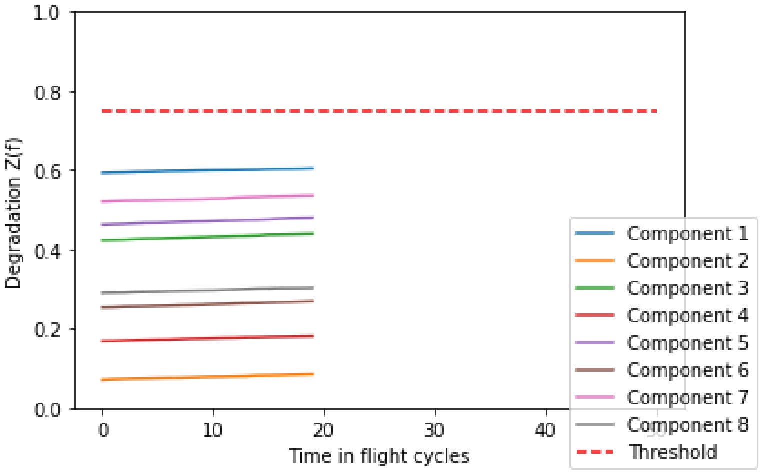


Figure 5: Initial degradation for aircraft k=0 with 8 components plotted over time in flight cycles

The above case study is evaluated by multiple maintenance strategies. First, the condition-based maintenance strategy where grouping, i.e. economic dependence between components, is applied. A c_{setup} equal to 40 is used. Second, the condition-based maintenance strategy is applied again, however this time not considering grouping. Meaning a c_{setup} equal to zero is used. Third, a classic strategy commonly used in industry is applied, being time-based maintenance using the mean cycle to failure to compare and evaluate the achieved results by the condition-based maintenance approach. As well as two more TBM strategies using percentages of the MCTF, being 97.5% and 95% respectively.

4.2 Key Performance Indicators

The key performance indicators are the metrics used to evaluate the obtained results. The most important key performance indicators include: the number of unscheduled replacements, the waste of life at replacement, the total number of hangar visits in the schedule and the amount of replacements per hangar visit. As discussed in section 3.3 during realization the component can either be replaced according to schedule or be an unscheduled replacement. This is a very important metric for the evaluation of the performance of a maintenance schedule as having to perform unscheduled replacements is undesirable for the MRO. The second metric is the waste of life at replacement, as the degradation threshold for replacements is equal to 0.75 it is desirable all components are replaced as close to 0.75 as possible such that the component is used to its full potential. By evaluating the difference between the components degradation level at replacement and the threshold an important performance metric becomes available. Finally, as discussed in section 3.2 all replacements are part of a group and consecutive replacements are considered to be part of the same group. A group represents an aircraft hangar visit, when one or more components are replaced. Counting the amount of hangar visits in the schedule is an interesting metric to consider. As the schedules will be evaluated using a Monte Carlo simulation approach for each key performance indicator the mean and standard deviation will be considered.

Using the key performance indicators as discussed above a quantitative cost analysis of the maintenance schedule can be performed. Using equations Equation 17 until Equation 21 the total cost of the maintenance schedules can be compared.

Equation 17 represents the cost formula for performing a single scheduled replacements. Where 2 stands for the time required to perform a replacement in hours. c Represents the man hour rate and n represents the number of crew required for a replacement.

$$c_{scheduled} = 2 \cdot c \cdot n \tag{17}$$

Equation 18 represents the cost formula for performing a single unscheduled replacement, which is assumed to cost equal to a scheduled replacement times a factor of 5.

$$c_{unscheduled} = 5 \cdot c_{scheduled} \tag{18}$$

Equation 19 represents the cost formula of hangar visits in the schedule by the aircraft, which includes the preparation of maintenance on the aircraft such as towing. The cost of a single hangar visit is assumed to be equal to two times $c_{scheduled}$.

$$c_{visit} = 2 \cdot c_{scheduled} \tag{19}$$

Equation 20 represents the cost formula of the remaining useful life. The cost associated with the remaining useful life equals the fraction of average remaining life at replacement $\bar{l}$ divided by the average mean cycle to failure $\bar{m}$ times the cost of a new brake.

$$c_{RUL} = \frac{\bar{l}}{\bar{m}} \cdot c_{brake} \tag{20}$$

Equation 21 represents the total cost of the maintenance schedule, which is the sum of all costs. Here $r_{scheduled}$ represents the number of scheduled replacements, $r_{unscheduled}$ represents the number of unscheduled replacements and r_{visit} represents the number of hangar visits in the maintenance schedule.

$$\begin{aligned} c_{total} =& c_{scheduled} \cdot r_{scheduled} + c_{unscheduled} \cdot r_{unscheduled} + \\ & c_{visit} \cdot r_{visit} + \frac{\bar{l}}{\bar{m}} \cdot c_{brake} \cdot r_{scheduled} \end{aligned} \tag{21}$$

To clarify the nomenclature of the parameters used in the above equations together with their respective value can be found in Table 5.

Table 5: Cost analysis parameter nomenclature including values

Parameter	Value	Unit	Elucidation
c	17	[€/h]	Man hour rate of a single crew member[ref, c]
n	3	[crew]	Assumed number of crew members required for a replacement
$\bar{m}$	1339	[FC]	Average mean cycle to failure calculated based on values in Table 1
c_{brake}	7 000	[€]	New price of a single brake assembly[ref, b]

5 Results

First, the results of a full schedule realization are displayed and discussed in section 5.1 for every maintenance strategy considered. Then, the results of the Monte Carlo simulation analysis will be discussed in section 5.2. In order to better understand the model's behaviour two sensitivity analysis have been performed, they are elucidated in section 5.3.

5.1 Single Realization Results

In this section the results of a single realization using the different maintenance strategies for the above mentioned case study are shown. First, the CBM including grouping is shown. Second, the CBM excluding grouping is displayed. Third, the TBM at MCTF is explained. Fourth, TBM at 97.5% of MCTF is elucidated. Finally, TBM at 95% of MCTF is shown. The plots in this section show the results for the first three aircraft, the full schedules can be found in Appendix B.

5.1.1 CBM, Incl Grouping

The first schedule is the result of applying the condition-based maintenance using opportunistic maintenance strategy or grouping. A plot of the schedule can be seen in Figure 6. As the legend of the figure shows scheduled and unscheduled replacements are indicated, as well as the $f_{k,i}^{RUL}$. Vertical dotted red lines are plotted to display the moments in time at which maintenance actions are performed and at last, when maintenance actions are considered groups they are indicated by a red box.

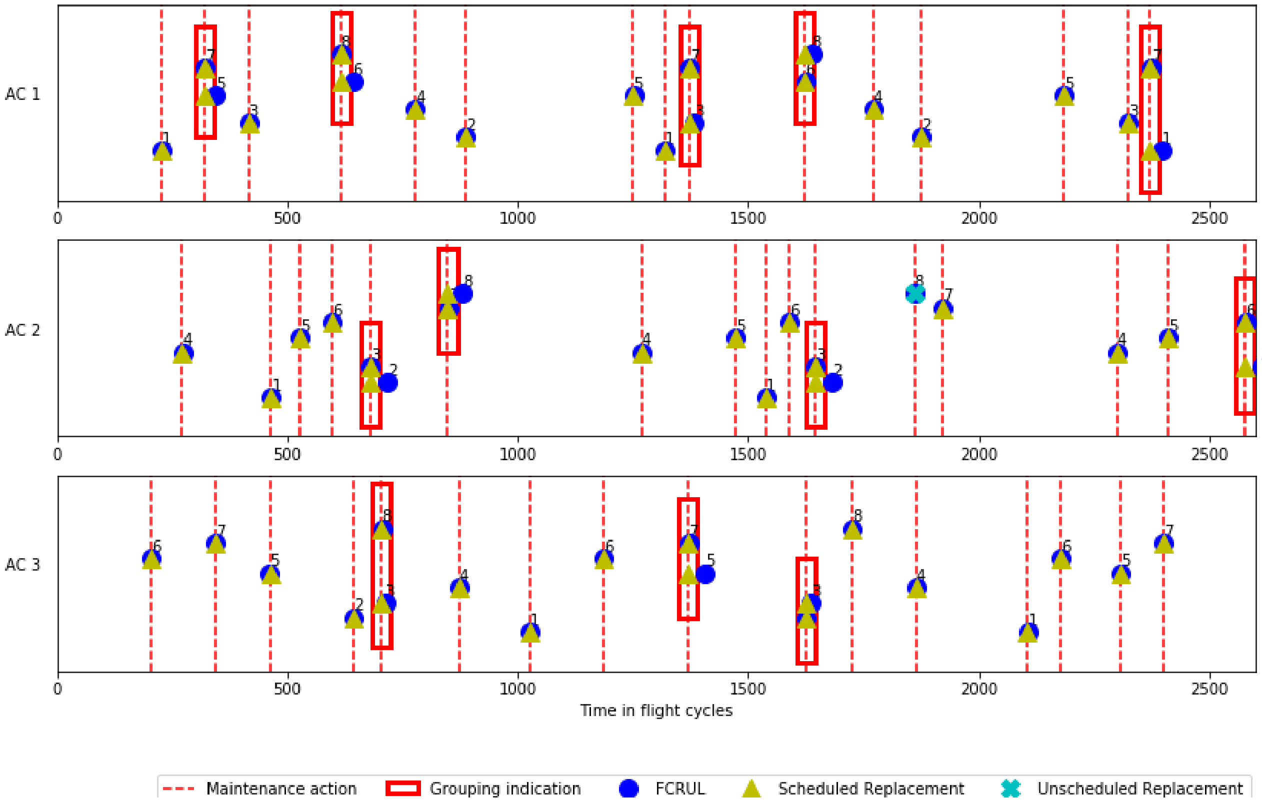


Figure 6: Plot of the maintenance schedule using CBM, including grouping for first three aircraft

In the figure multiple groups can be observed, the groups observable for the first three aircraft consist of two components, however larger groups can form. Cyclical behaviour with respect to the replacements can be observed due to their mean cycle to failure.

It can also be observed that the schedule has very little unscheduled replacements. Unscheduled replacements are highly undesirable for maintenance repair and overhaul companies. The fact that there are not many unscheduled replacements is considered positive.

Below in Table 6 the key performance indicators of the full schedule can be observed. Important to note is that here the number of hangar visits indicate all maintenance actions performed, including maintenance action on single components or more. For the average remaining life the mean of the remaining life at replacement for all components which are replaced is used, where Z_T is the degradation threshold and $Z(f_r)$ is the degradation level at replacement. To calculate the waste of life in flight cycles the remaining life in degradation is divided by the expected increment degradation of the Gamma distribution using a_{real} and b_{real} shown in Table 1.

Table 6: KPI's of the full schedule using CBM, including opportunistic maintenance

Replacements	Unscheduled Replacements	Average Remaining Life $E[Z_T - Z(f_r)]$	Number of Hangar Visits
302	6	7.29e-3 (10 FC)	203

To illustrate the degradation process the degradation level $Z(f)$ of the components can be plotted. This plot for the first aircraft and its eight components can be seen in Figure 7. As can be seen in the plot, when the brake is replaced the degradation level goes to zero. This is because at every replacement the component is pristine. In literature this type of maintenance is also considered perfect maintenance.

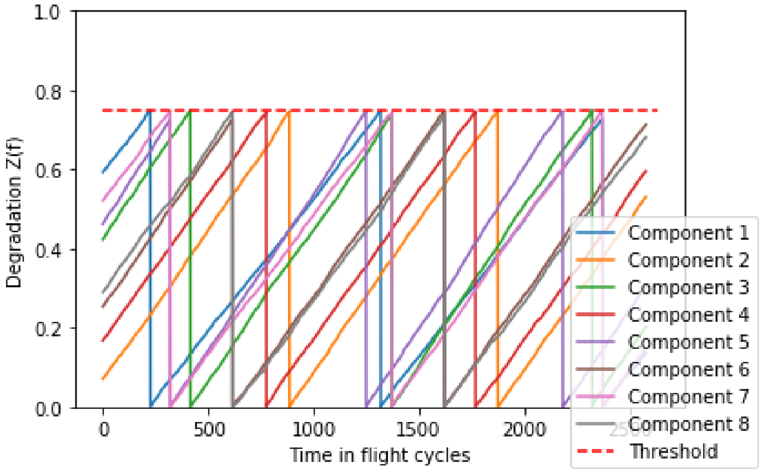


Figure 7: Degradation level of aircraft $k = 1$

5.1.2 CBM, Excl Grouping

The second maintenance strategy used to solve the maintenance schedule is a condition-based maintenance excluding grouping strategy. This strategy utilises the same prognostic and optimization model as the previous strategy, with the difference being the c_{setup} equal to zero. Meaning the model does not account for grouping of maintenance actions. This can clearly be observed in the schedule plotted in Figure 8.

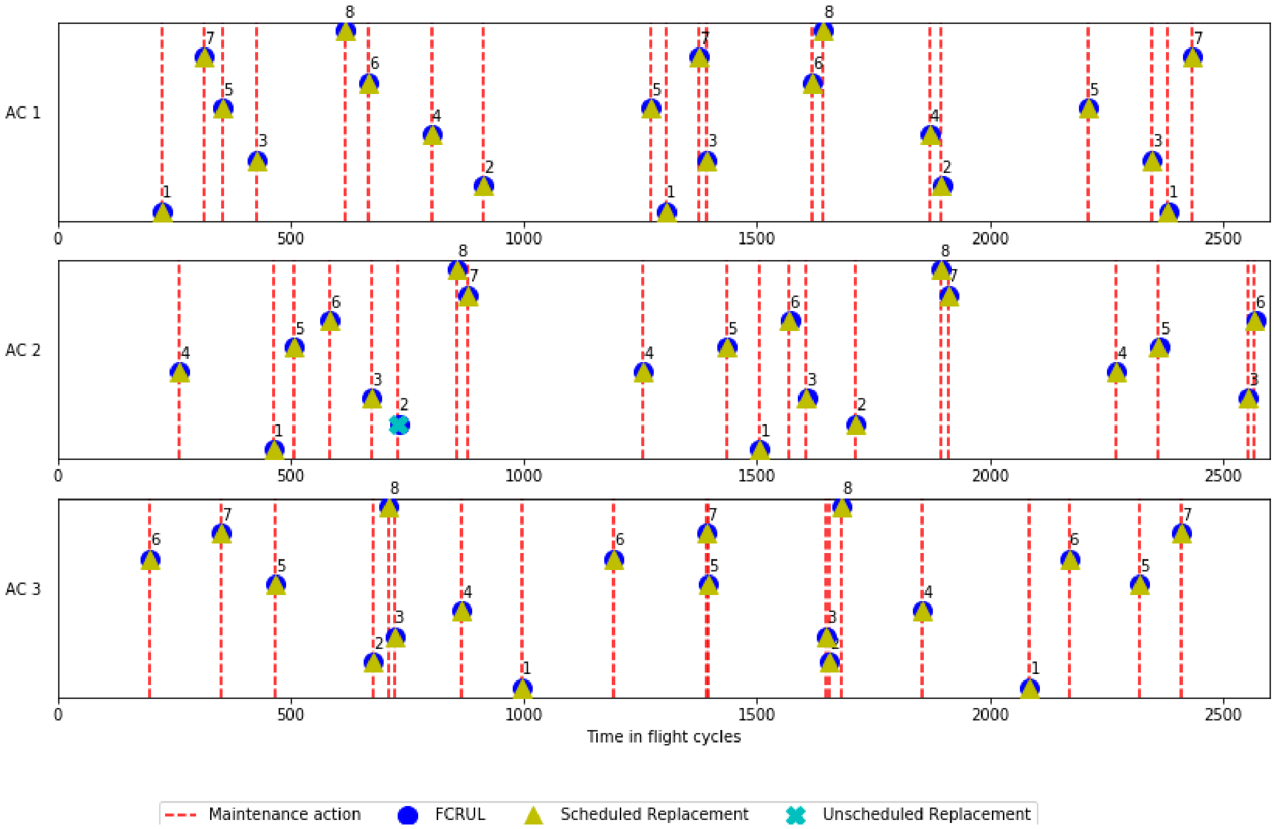


Figure 8: Plot of the maintenance schedule using CBM, excluding grouping for the first three aircraft

It is possible components of the same aircraft are replaced within the same ground time between flight cycles, meaning both components are serviced during the same ground time. Reducing the total amount of hangar visits. This is however pure coincidental and has not happened for the first three aircraft. What can clearly be seen is that the model always tries to schedule the component replacement as close to or at the flight cycle which is estimated by the prognostic model or $f_{k,i}^{RUL}$. To summarize the KPI's of the full schedule, Table 7 has been created. It can be seen the schedule has five unscheduled replacements, this implies the component degradation was reached before the maintenance action was scheduled by the optimization model five times.

Table 7: KPI's of the full schedule using CBM, excluding opportunistic maintenance

Replacements	Unscheduled Replacements	Average Remaining Life $E[Z_T - Z(f_r)]$	Number of Hangar Visits
299	5	3.18e-3 (4.24 FC)	298

5.1.3 TBM, MCTF

The third maintenance strategy considered is the time-based maintenance strategy at the mean cycle to failure. The maintenance schedule for the first three aircraft is plotted in Figure 9. A clear difference between the previous strategies results and these results shown can be observed in terms of unscheduled replacements.

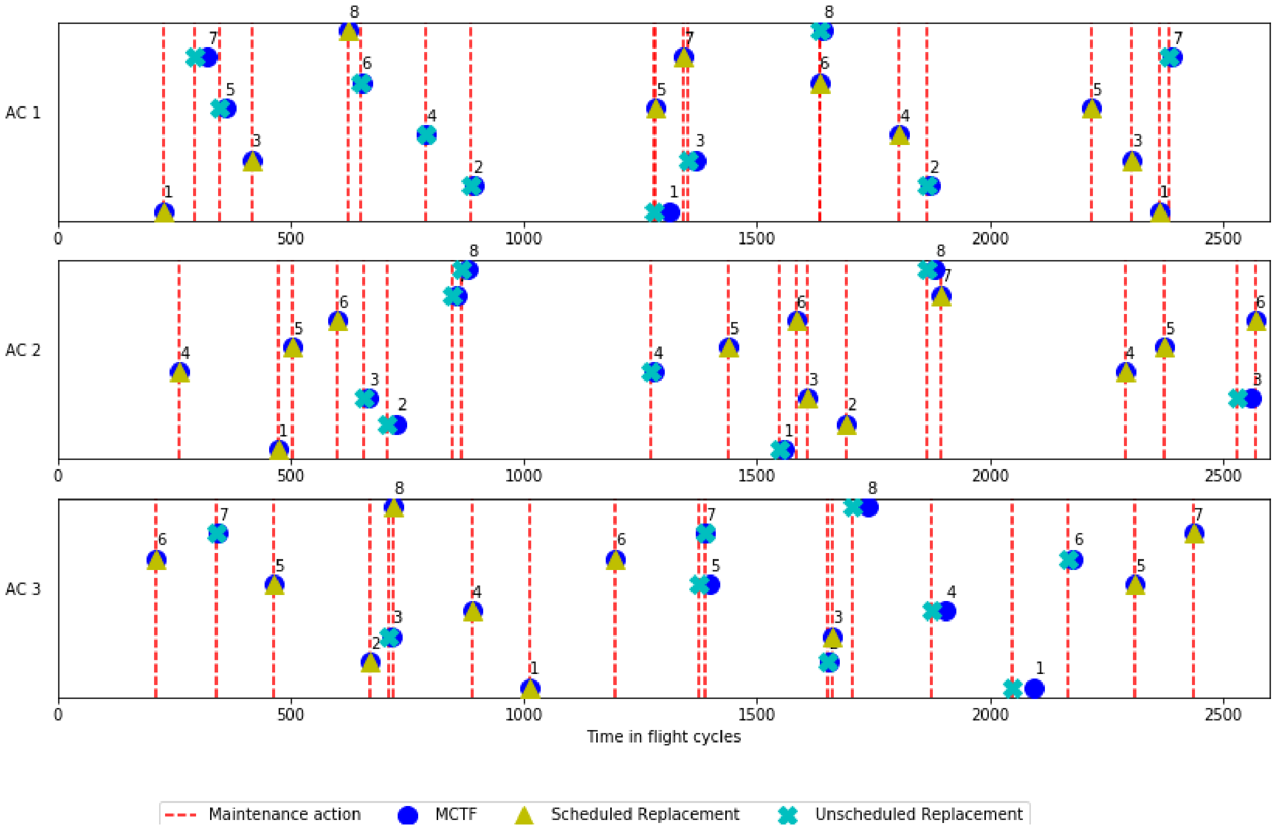


Figure 9: Plot of the maintenance schedule using TBM at MCTF, for the first three aircraft

A tabulated overview of the schedule's KPIs can be seen in Table 8. A reason for the high number of unscheduled replacements is the fixed time interval, MCTF, thus the result is expected.

Table 8: KPI's of the full schedule using CBM, excluding opportunistic maintenance

Replacements	Unscheduled Replacements	Average Remaining Life $\mathrm{E}[Z_T - Z(f_r)]$	Number of Hangar Visits
299	133	7.31e-3 (10 FC)	278

5.1.4 TBM, 97.5% MCTF

As the previous results have shown using the mean cycle to failure results in a significant amount of unscheduled replacements. Reducing the replacement interval shall result in less unscheduled replacement however at the cost of a higher remaining useful life at replacement. Therefor 97.5% of the MCTF is used resulting in the schedule which can be seen in Figure 10 for the first three aircraft.

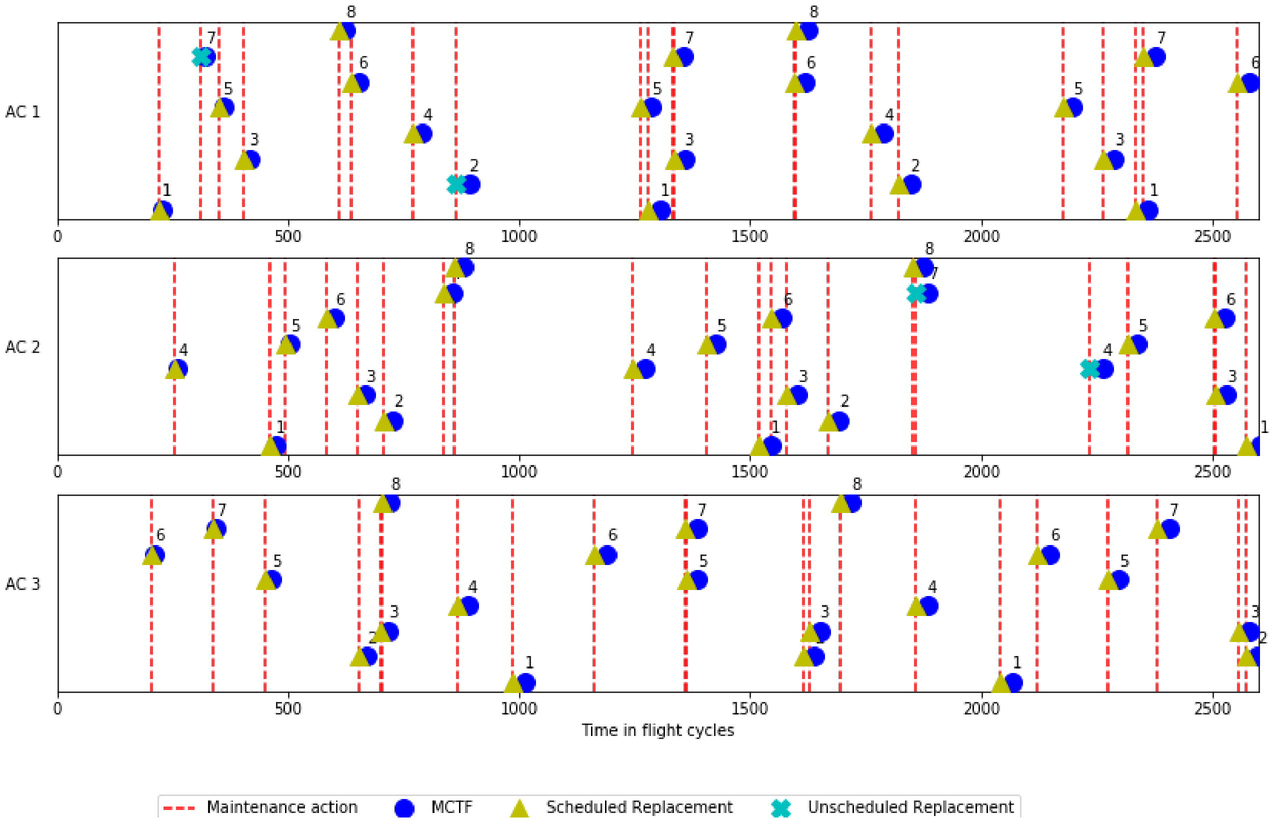


Figure 10: Plot of the maintenance schedule using TBM, using 97.5% of the MCTF for the first three aircraft

The figure shows that with respect to the amount of unscheduled replacement the schedule has improved compared to the previous TBM strategy. Table 9 shows the key performance indicators of the full schedule.

Table 9: KPI's of the full schedule using TBM, using 97.5% of the MCTF

Replacements	Unscheduled Replacements	Average Remaining Life $E[Z_T - Z(f_r)]$	Number of Hangar Visits
313	26	1.9e-2 (25 FC)	294

5.1.5 TBM, 95% MCTF

The results of the previous maintenance strategy were a significant improvement when comparing to the TBM strategy at MCTF. As mentioned the reduction of the fixed time interval resulted in less unscheduled replacement however at the cost of a higher remaining useful life at replacement. Now, reducing the fixed time interval even more, the number of unscheduled replacements should reduce further, while the waste of life shall increase. The resulting schedule when applying TBM at 95% of the MCTF for the first three aircraft can be seen in Figure 11.

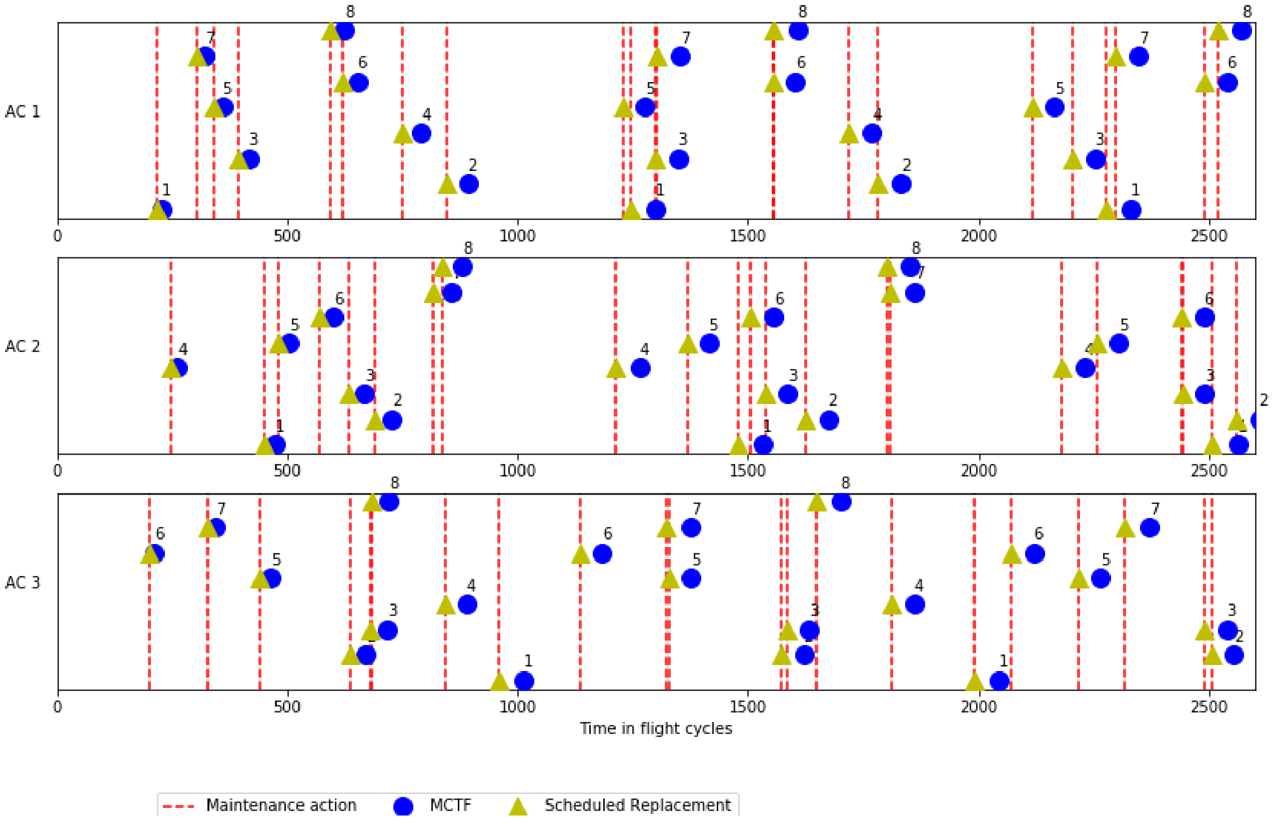


Figure 11: Plot of the maintenance schedule using TBM, using 95% of the MCTF for the first three aircraft

The figure confirms the predictions made about the number of unscheduled replacements. In Table 10 the key performance indicators of the schedule can be seen. Again, the predictions about the KPIs are confirmed, this time with respect to the waste of life. The remaining life at replacement is significantly higher when comparing the results of this schedule with the previous schedules.

Table 10: KPI's of the full schedule using TBM, using 95% of the MCTF

Replacements	Unscheduled Replacements	Average Remaining Life E$[Z_T - Z(f_r)]$	Number of Hangar Visits
327	2	3.3e-2 (44 FC)	303

5.2 Monte Carlo Simulation Results

In this section the results of the Monte Carlo simulation will be shown and evaluated. First, the results of the simulation are tabulated and analyzed. After which, a cost analysis will be performed.

For the Monte Carlo simulation the schedule is realized 200 times for each maintenance strategy. The results of the replacements key performance indicators can be seen in Table 11, these include the number of unscheduled replacements and the waste of life at replacement for each maintenance strategy. A second table, Table 12, tabulates the key performance indicators related to the number of hangar visits and the number of replacements per hangar visit. In both tables S.D. stands for the standard deviation from the mean.

Table 11: Replacement KPI's of the Monte Carlo Simulation results

	Unscheduled Replacements		Average Remaining Life E$[Z_T - Z(f_r)]$	
	Mean	S.D.	Mean	S.D.
CBM, grouping	6.6	2.8	7.3e-3 (9.7 FC)	3.3e-4 (0.44 FC)
CBM, no grouping	8.6	2.9	3.2e-3 (4.2 FC)	1.2e-4 (0.16 FC)
TBM, MCTF	133	7.5	8.9e-3 (12 FC)	5.1e-4 (0.69 FC)
TBM, 97.5% MCTF	21	4.7	1.8e-2 (24 FC)	5.8e-4 (0.78 FC)
TBM, 95% MCTF	1.4	1.5	3.3e-2 (44 FC)	7.8e-4 (1.1 FC)

Table 12: Grouping KPI's of the Monte Carlo Simulation results

	Average Number of Groups	Hangar Visits		Average Group Size	Replacements per Hangar Visit	
	Mean	Mean	S.D.	Mean	Mean	S.D.
CBM, grouping	204	204	5.3	1.48	1.48	0.45
CBM, no grouping	-	259	5.1	-	1.15	0.33
TBM, MCTF	-	278	3.8	-	1.08	0.31
TBM, 97.5% MCTF	-	290	3.8	-	1.08	0.21
TBM, 95% MCTF	-	303	3.3	-	1.08	0.31

From the above results it can be seen that the least unscheduled replacements occur when using the time-based maintenance at 95% of MCTF, followed by the condition-based maintenance including the opportunistic maintenance or grouping strategy. The most unscheduled replacements occur when using the time-based maintenance strategy considering the MCTF.

When looking at the waste of life at replacement, the best performance is achieved by the condition-based maintenance excluding opportunistic maintenance or grouping strategy. This is to be expected, as the CBM including opportunistic maintenance or grouping sacrifices remaining life at replacement to reduce the number of groups in the schedule. While the TBM strategy performs less at this KPI in general due to the fact it considers fixed intervals independent of the observed degradation.

It can be observed that only the CBM which includes active grouping has a value in the columns average number of groups and average group size. This is because the CBM which includes active grouping is the only maintenance strategy which, as the name implies, actively groups maintenance actions. This grouping metric does however represent a KPI in the schedule, which is then compared with the other maintenance strategies.

One of the KPIs tabulated is the number of hangar visits in the schedule, this result is certainly expected as the only strategy which actively tries to reduce the number of groups and thus hangar visits in the schedule is the condition-based maintenance strategy including grouping. The number of replacements performed per hangar visit is another KPI which is interesting to compare. As expected the CBM which incorporates grouping performs best at this KPI. In the table the mean and standard deviation are shown, however it would also be interesting to look at the underlying data of these replacements per hangar visit, thus this metric is plotted and can be seen in Figure 12 until Figure 14 for both CBM strategies and the TBM using MCTF.

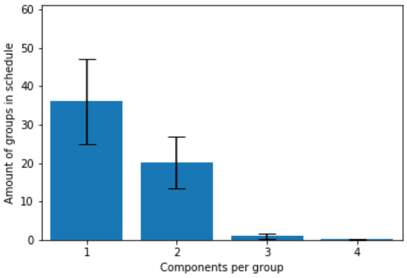


Figure 12: Replacements per hangar visit distribution plot for CBM, including grouping

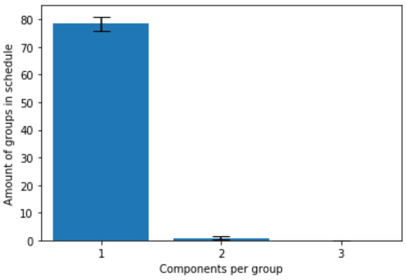


Figure 13: Replacements per hangar visit distribution plot for CBM, excluding grouping

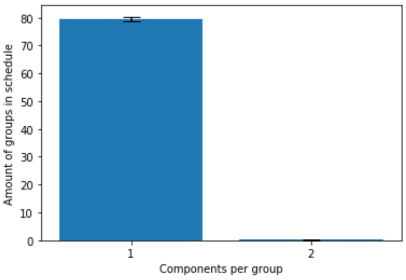


Figure 14: Replacements per hangar visit distribution plot for TBM for MCTF

It is clear to see from the above plots that the first plot, of the CBM including grouping, actively tries to group components. Still for most hangar visits only a single component is replaced however in some simulations even groups of four components are scheduled. The other plots have no such active grouping strategy thus during most hangar visits only contain a single components is maintained, with the exception of the coincidental groups of two components.

As discussed in section 4.2 it is possible to combine the resulting KPIs from the obtained schedule results and perform a costs analysis. By using the formulas which are elaborated on in section 4.2 the results as tabulated in Table 13 are obtained.

Table 13: Cost analysis of Monte Carlo schedule results, values in [€]

	Replacement Cost	Hangar Visit Cost	Remaining Life Cost	Unscheduled Cost	Total Cost
CBM, grouping	30906	41616	15365	3366	91253
CBM, no grouping	30396	52836	6543	4386	94161
TBM, MCTF	30498	56712	18757	67830	173797
TBM, 97.5% MCTF	31926	59160	39271	10710	141067
TBM, 95% MCTF	33354	61812	75217	714	171097

From the cost analysis results the following can be observed. First, the CBM including grouping performs best on overall total cost followed by CBM excluding grouping, TBM at 97.5% MCTF, then TBM at 95% and finally TBM at MCTF. The CBM including grouping performs best on all KPIs except the remaining life at replacement cost at which the CBM excluding grouping performs best. The worst performing schedule is the TBM at MCTF which has a significant higher cost with respect to the other strategies on unscheduled replacements. Unscheduled replacements, as mentioned before, have a significant impact on airline operations and thus should be prevented. The worst performing strategy when looking at the remaining life cost is the TBM at 95% of MCTF, which is surprising. CBM Which includes grouping actively sacrifices remaining life at replacement for the forming of groups, it would have been expected this strategy performs less at this cost factor. However, the fixed interval of 95% MCTF used by the TBM strategy results in a higher cost for remaining life while actually having the least unscheduled replacements of all other strategies.

From the cost analysis it can clearly be concluded that having a condition-based maintenance strategy which makes use of the available degradation data in order to predict the remaining useful life has a clear benefit over the time-based maintenance strategies. The CBM strategies outperform the TBM strategies at nearly every key performance indicator, which result in significant lower total schedule cost. When comparing both CBM strategies, the CBM strategy which includes grouping is the clear winner. The remaining useful life of the CBM which accounts for grouping is higher than that of the CBM excluding grouping, more than twice 9.7 flight cycles compared to 4.2 flight cycles respectively. However, it does not compare with respect to the benefit or gains on the other key performance indicators such as the number of unscheduled replacements and the number of hangar visits. It can therefor be concluded that it is not only possible to combine degradation modelling prognostics with maintenance optimization, it also outperforms existing time-based maintenance strategies when evaluated with a case study.

5.3 Sensitivity Analysis

In order to get a better understanding of the model's behaviour and the schedule results two local sensitivity analysis are performed. First, the number of aircraft which are part of the schedule is analyzed by ranging them from 5 to 25 using increments of 10 aircraft. Second, the total schedule time is analyzed by ranging it from 5 to 10 years using increments of 5 years.

5.3.1 Number of Aircraft Analysis

Increasing the number of aircraft considered shall increase the significance of the hangar availability. As increasing the number of aircraft increases the number of components and with the hangar availability the same, more components will require to be replaced earlier than desired due to hangar availability constraints. In turn this would have a negative effect on the remaining useful life at replacement, increasing the overall schedule cost. To support this claim the above case of CBM including grouping has been reevaluated however now ranging the number of aircraft from 5 to 25 aircraft using increments of 10, simulating 200 times has resulted in the schedule results seen in Table 14.

Table 14: Local sensitivity analysis simulation results on the number of aircraft

Number of Aircraft	Unscheduled Replacements		Average Remaining Life $\mathrm{E}[Z_T - Z(f_r)]$	Number of Hangar Visits		
	Mean	S.D.	Mean	S.D.	Mean	S.D.
5	1.67	1.25	8.01e-3 (10.7 FC)	8.6e-4 (1.16 FC)	55.4	2.8
15	6.6	2.8	7.3e-3 (9.7 FC)	3.3e-4 (0.44 FC)	204	5.3
25	9.9	3.04	7.4e-3 (10 FC)	3.7e-4 (0.5 FC)	319	5.98

It is important to note that the KPI's cannot be compared, because the number of aircraft vary thus resulting in higher values for unscheduled replacements and number of hangar visits. In order to compare these KPI's the results require to be normalized, this is done by normalizing with respect to one aircraft, the result can be seen in Table 15.

Table 15: Normalized mean schedule KPIs to KPI per single aircraft

Schedule	Unscheduled Replacements	Average Remaining Life $\mathrm{E}[Z_T - Z(f_r)]$	Number of Hangar Visits
5 aircraft	0.33	10.7 FC	12.09
15 aircraft	0.44	9.7 FC	13.6
25 aircraft	0.396	10 FC	12.76

The hypotheses made that the remaining life would increase as number of aircraft would increase does not seem to be proven based on the tabulated results. This is likely due to the fact that the hangar availability constraint is not stressed enough, meaning there are still enough maintenance opportunities available. For the described behaviour to show, the available maintenance slots for the aircraft should be reduced or the number of aircraft should be increased even more. In order to draw conclusions on the performance of the overall schedules, the KPIs of each normalized schedule can be combined in the cost analysis after which they can be compared. This cost analysis is tabulated in Table 16. From the table it can be seen that there is no significant difference in the cost per aircraft of the maintenance schedule when evaluating for different amount of aircraft.

Table 16: Cost analysis of the normalized results achieved in the local sensitivity analysis on number of aircraft, values in [€]

	Replacement Cost	Unscheduled Cost	Remaining Life Cost	Hangar Visits Cost	Total Cost
5 aircraft	2071	168	1136	2466	5841
15 aircraft	2060	224	1024	2774	6084
25 aircraft	2050	202	1051	2603	5906

5.3.2 Schedule Time Analysis

It is expected that when increasing the total time of the schedule more uncertainty is introduced. This will result in a higher variability in the Monte Carlo simulation results, resulting in a higher variance for the KPIs. To support this claim the CBM including grouping will be reevaluated however now the total schedule time is increased to 10 years from 5. The schedule will be simulated 100 times, the results of this simulation can be seen in Table 17.

Table 17: Local sensitivity analysis simulation results on the schedule time

Total Schedule Time	Unscheduled Replacements		Average Remaining Life $\mathrm{E}[Z_T - Z(f_r)]$		Number of Hangar Visits	
	Mean	Var	Mean	Var	Mean	Var
5 Years	6.6	7.84	7.3e-3 (9.7 FC)	1.1e-7 (0.19 FC)	204	28
10 Years	13.7	10.89	7.1e-3 (9 FC)	8.0e-7 (0.14 FC)	422	75

Looking at the variance of the resulting KPIs it can be seen that the difference is small but noticeable. Generating the schedule for a longer period of time does indeed increase the uncertainty of the results, resulting in a higher variance. However, important to note is that the simulations performed to achieve these results are on the low side, thus a critical stance should be adopted when evaluating standard deviation or variance.

6 Conclusion & Discussion

This section is dedicated to elaborate on the conclusions of the thesis paper. First, the research scope of the project is concluded. Second, the academic novelty of the project is elaborated on. Third, the conclusions regarding the results of the research are discussed. And final, recommendations for future research are made.

6.1 Research Scope

The goal of the proposed research is to contribute to the further development of the theories regarding operations optimization, maintenance policies and stochastic simulation. This goal is achieved by providing insight, based on literature study, prove of concept and the evaluation of the solution of a case study.

The problem can be divined as follows: Determining the optimal maintenance schedule for a fleet of aircraft minimizing cost while satisfying safety requirements considering multiple components per aircraft which states degrade over time according to a gamma process and having limited hangar availability.

The first research questions to be answered during this research concerns the formulation of the model, more specifically: The formulation of the prognostic model and optimization's model objective function and constraints. The second research question aims to evaluate a case study, by finding the optimal maintenance schedule and associated KPIs. Finally, the third research question requires the achieved results to the case study to be evaluated to a traditional time-based maintenance schedule.

6.2 Academic Novelty

In the available literature similar problems have been considered. Degradation is commonly modelled when considering the state itself using a Gamma distribution for monotonic increasing processes. Condition-based maintenance is a well-researched topic, therefor extensive classification schemes exist to structure the available research. This proposed research problem can be classified as multi-unit, continuous, stochastic, perfect and rolling. A very promising approach fitting this classification is the opportunistic maintenance which has been researched for multi-unit systems. It uses a grouping principle to cluster maintenance actions in order to improve the efficiency of the maintenance schedule.

However, no research on multi-unit systems combining prognostics and optimization has been considered in the available literature. The combination of these two research disciplines is currently unique and shows very promising results. Combining this with opportunistic maintenance which adds value to the research from a maintenance planning perspective positions this research at the forefront of development. The performed research significantly contributes to the existing body of knowledge as it fits the gap which currently exists and expands on existing theories.

6.3 Conclusions

The model created during this research consists of three main parts being: the prognostic model, the optimization model and the rolling horizon.

The prognostic model determines the remaining useful life in flight cycles of the components by making use of a Gamma distribution. First, the parameters of the Gamma distribution which describe the available condition monitored data are estimated. Second, the model uses the cumulative distribution function of the Gamma distribution to estimate the flight cycle at which the degradation threshold is reached. This then becomes the input to the optimization model.

The optimization model which generates the maintenance schedule is an integer linear programming model. The decision variables represent whether the component of an aircraft is assigned to a specific maintenance slot. The maintenance slots available per aircraft are determined by the flight schedule and the cost of assigning a component to a certain maintenance slot is driven by the remaining useful life estimation. When evaluating the maintenance schedule of a multi-unit system dependencies between components need to be considered. For this problem economic dependency is considered, where the cost of grouping multiple components together within the same ground time reduces the need for towing of the aircraft thus reducing the overall schedule cost.

In order to progress the schedule in time and evaluate the achieved result of the model a rolling horizon approach is used. This method accounts for decision made previously, it performs the optimization for the optimization time and then realizes this optimized schedule for the realization time. A component is replaced when its assigned maintenance slot is reached or when its degradation level reaches the threshold.

To evaluate the model and its results a case study is performed. The model including and excluding grouping is assessed as well as a variation of simple TBM strategies. The case study considers 15 aircraft with 8 components each, the schedule is generated for 5 years. Important key performance indicators of the schedule include the number of unscheduled replacements, the waste of life at replacement, the total amount of hangar visits and the number of replacements per hangar visit. With these KPIs it is possible to perform a cost analysis.

As expected the least unscheduled replacements occur when using the condition-based maintenance including the opportunistic maintenance or grouping strategy. The most unscheduled replacements occur when using the time-based maintenance strategy considering the MCTF. For the waste of life at replacement, the best performing strategy is the condition-based maintenance excluding opportunistic maintenance or grouping strategy. This is also to be expected, as the CBM including opportunistic maintenance or grouping sacrifices remaining life at replacement to reduce the number of groups in the schedule. While the TBM strategy performs less at this KPI in general due to the fact it considers fixed intervals independent of the observed degradation. For the KPI which depend on grouping, both the hangar visits and number of replacements per visit, the CBM including grouping performs best.

From the cost analysis it can clearly be concluded that having a condition-based maintenance strategy which makes use of the available degradation data in order to predict the remaining useful life has a clear benefit over the time-based maintenance strategies. The CBM strategies outperform the TBM strategies at nearly every key performance indicator, which result in significant lower total schedule cost. When comparing both CBM strategies, the CBM strategy which includes grouping is the clear winner. The remaining useful life of the CBM which accounts for grouping is higher than that of the CBM excluding grouping. However, it does not compare with respect to the benefit or gains on the other key performance indicators such as the number of unscheduled replacements and the number of hangar visits.

It can therefor be concluded that it is not only possible to combine degradation modelling prognostics with maintenance optimization, it also outperforms existing time-based maintenance strategies when evaluated with a case study.

6.4 Recommendations for Future Research

The current model could be improved by increasing its accuracy, considering scalability and expanding its application. The accuracy of the model's results can be improved in two ways. First, by reducing the number of assumptions made. An example of assumptions which can be improved are the assumptions made regarding the cost analysis. Second, the results accuracy can be improved by increasing the number of simulations of the Monte Carlo simulation. The model can easily be expanded to incorporate more aircraft, components and time. The downside to expanding however is the computational effort required and thus the speed of the model. Improving the computational speed shall improve the scalability. At last, incorporating more types of components which degradation potentially are governed by different probability distribution functions shall increase the model's applicability. Another interesting improvement for future research is incorporating more maintenance opportunities such as spare part stock management or other such opportunities mentioned in the literature regarding opportunistic maintenance.

References

[ref, a] Flightradar24, klm fleet. `https://www.flightradar24.com/data/airlines/kl-klm/fleet`. Online; accessed on (August 8, 2020).

[ref, b] Hydro, what you should know about aircraft brakes. `https://www.hydro.aero/en/newsletter-details/what-you-should-know-aboutaircraft-brakes.html#:~:text=For%20example%2C%20the%20list%20price,complete%2012%2Dpiece%20brake%20set`. Online; accessed on (November 2, 2020).

[ref, c] Iata, labor rate and productivity calculations for commercial aircraft maintenance. `https://www.iata.org/contentassets/bf8ca67c8bcd4358b3d004b0d6d0916f/2013-labor-rate.pdf`. Online; accessed on (November 2, 2020).

[B. de Jonge, P.A. Scarf, 2019] B. de Jonge, P.A. Scarf (September 2019). A review on maintenance optimization. *European Journal of Operational Research*, 285(1):805-824.

[C.R. Cassady, W.P. Murdock Jr, E.A. Pohl, 2000] C.R. Cassady, W.P. Murdock Jr, E.A. Pohl (May 2000). Selective maintenance for support equipment involving multiple maintenance actions. *European Journal of Operational Research*, 129(1):252-258.

[F.M. Dekking, C. Kraaikamp, H.P. Lopuhaä, L.E. Meester, 2005] F.M. Dekking, C. Kraaikamp, H.P. Lopuhaä, L.E. Meester (2005). *A Modern Introduction to Probability and Statistics, Understanding Why and How.* Springer, Delft, The Netherlands, 1th edition.

[H. Ab-Samat, S. Kamaruddin, 2014] H. Ab-Samat, S. Kamaruddin (March 2014). Opportunistic maintenance as a new advancement in maintenance approaches. *Journal of Quality in Maintenance Engineering*, 20(2):98-121.

[H. Liao, E.A. Elsayed, L. Chan, 2005] H. Liao, E.A. Elsayed, L. Chan (August 2005). Maintenance of continuously monitored degrading systems. *Computing, Artificial Intelligence and Information Management*, 175(1):821-835.

[H. Wang, 2001] H. Wang (April 2001). A survey of maintenance policies of deteriorating systems. *European Journal of Operational Research*, 139(1):469-489.

[H.C. Vu, P. Do, A. Barros, C. Bérenguer, 2014] H.C. Vu, P. Do, A. Barros, C. Bérenguer (August 2014). Maintenance grouping strategy for multi-component systems with dynamic contexts. *Reliability Engineering and System Safety*, 132(1):233-249.

[H.C. Vu, P. Do, M. Fouladirad, A. Grall, 2020] H.C. Vu, P. Do, M. Fouladirad, A. Grall (January 2020). Dynamic opportunistic maintenance planning for multi-component redundant systems with various types of opportunities. *Reliability Engineering and System Safety*, 198.

[J. Lee, M. Mitici, 2020] J. Lee, M. Mitici (October 2020). An integrated assessment of safety and efficiency of aircraft maintenance strategies using agent-based modelling and stochastic petri nets. *Reliability Engineering and System Safety*, 202.

[J.J. McCall, 1963] J.J. McCall (1963). Operating characteristics of opportunistic replacement and inspection policies. *Management Science*, 10(1):85-97.

[J.M. van Noortwijk, 2017] J.M. van Noortwijk (March 2017). A survey of the application of gamma processes in maintenance. *Reliability Engineering and System Safety*, 94(1):2-21.

[J.Y.J. Lam, D. Banjevic, 2015] J.Y.J. Lam, D. Banjevic (June 2015). A myopic policy for optimal inspection scheduling for condition based maintenance. *Reliability Engineering and System Safety*, 144(1):1-11.

[K. Bouvard, S. Artus, C. Bérenguer, V. Cocquempot, 2010] K. Bouvard, S. Artus, C. Bérenguer, V. Cocquempot (December 2010). Condition-based dynamic maintenance operations planning grouping. appliucation to commercial heavy vehicles. *Reliability Engineering and System Safety*, 96(1):601-610.

[K. Schneider, C.R. Cassady, 2014] K. Schneider, C.R. Cassady (October 2014). Evaluation and comparison of alternative fleet-level selective maintenance models. *Reliability Engineering and System Safety*, 134(1):178-187.

[M.J. Kallen, J.M. van Noortwijk, 2004] M.J. Kallen, J.M. van Noortwijk (December 2004). Optimal maintenance decision under imperfect inspection. *Reliability Engineering and System Safety*, 90(1):177-185.

[P. Do Van, A. Barros, C. Bérenguer, K. Bouvard, 2013] P. Do Van, A. Barros, C. Bérenguer, K. Bouvard (April 2013). Dynamic grouping maintenance with time limited opportunities. *Reliability Engineering and System Safety*, 120(1):51-59.

[Q. Wei, D. Xu, 2014] Q. Wei, D. Xu (2014). Remaining useful life estimation based on gamma process considered with measurement error. *International Conference on Reliability, Maintainability and Safety (ICRMS).*

[R. Dekker, 1996] R. Dekker (1996). Applications of maintenance optimization models: a review and analysis. *Reliability Engineering and System Safety*, 51(1):229-240.

[S. Alaswad, Y. Xiang, 2016] S. Alaswad, Y. Xiang (August 2016). A review on condition-based maintenance optimization models for stochastically deteriorating system. *Reliability Engineering and System Safety*, 157(1):54-63.

[S. Taghipour, D. Banjevic, A.K.S. Jardine, 2010] S. Taghipour, D. Banjevic, A.K.S. Jardine (April 2010). Periodic inspection optimization model for a complex repairable system. *Reliability Engineering and System Safety*, 95(1):944-952.

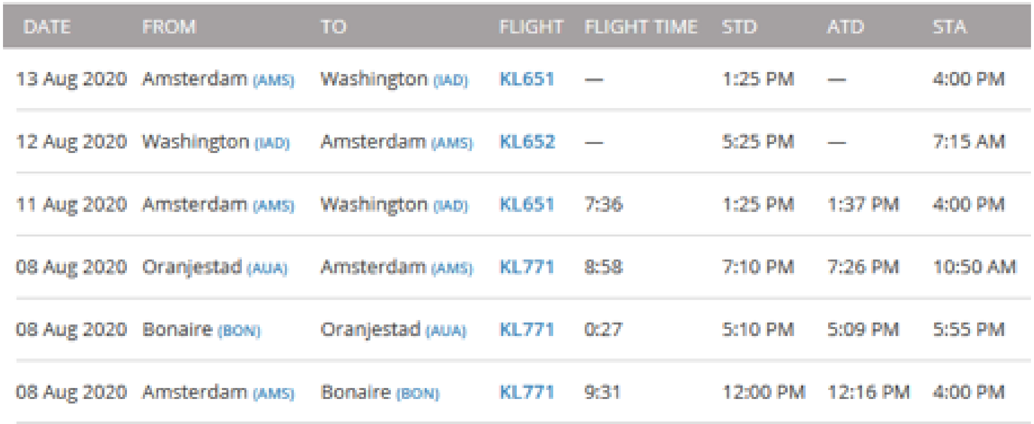
[S. Wu, I.T. Castro, 2020] S. Wu, I.T. Castro (June 2020). Maintenance policy for a system with a weighted linear combination of degradation processes. *European Journal of Operational Research*, 280(1):124-133.

[X. Zhou, L. Xi, J. Lee, 2006] X. Zhou, L. Xi, J. Lee (April 2006). Reliability-centered predictive maintenance scheduling for a continuously monitored system subject to degradation. *Reliability Engineering and System Safety*, 92(1):530-534.

Appendices

A Appendix 1

As mentioned in section 4.1, the flight schedule is created from historical flight data and extrapolated repeatedly. Since the model creates a long term planning, with a total schedule time of 5 years, this is a reasonable assumption. In total five unique flight schedules of one week were used, the source being the KLM A330 fleet using FlightRadar24.[ref, a]. An example of the historical flight data available for the PH-AKA can be seen in Figure 15.



DATE	FROM	TO	FLIGHT	FLIGHT TIME	STD	ATD	STA
13 Aug 2020	Amsterdam (AMS)	Washington (IAD)	KL651	—	1:25 PM	—	4:00 PM
12 Aug 2020	Washington (IAD)	Amsterdam (AMS)	KL652	—	5:25 PM	—	7:15 AM
11 Aug 2020	Amsterdam (AMS)	Washington (IAD)	KL651	7:36	1:25 PM	1:37 PM	4:00 PM
08 Aug 2020	Oranjestad (AUA)	Amsterdam (AMS)	KL771	8:58	7:10 PM	7:26 PM	10:50 AM
08 Aug 2020	Bonaire (BON)	Oranjestad (AUA)	KL771	0:27	5:10 PM	5:09 PM	5:55 PM
08 Aug 2020	Amsterdam (AMS)	Bonaire (BON)	KL771	9:31	12:00 PM	12:16 PM	4:00 PM

Figure 15: Flight history of the PH-AKA, KLM A330 aircraft from FlightRadar24

This historical flight data is then processes such that it can be used by the model. These flight schedules are presented in the text and can be seen in Figure 4. When increasing the number of aircraft to more than five the flight schedules are repeated and when increasing the time to more than one week the flight schedules are extrapolated. The flight schedules as used in the case study for 15 aircraft for 4 years is visualized in Figure 16.

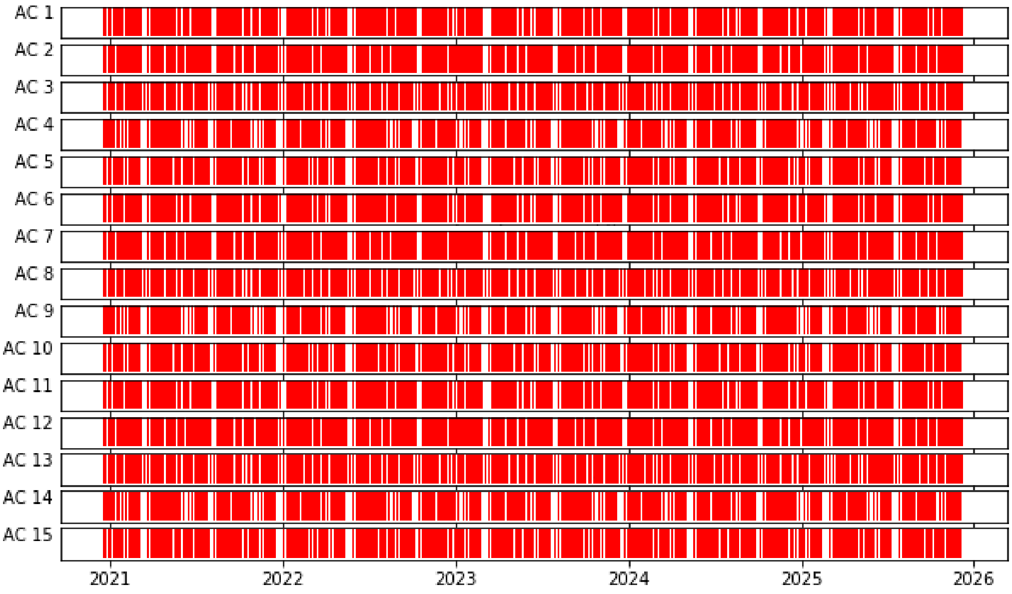


Figure 16: The flight schedule for 15 aircraft for a duration of 5 years

B Appendix 2

The purpose of this appendix is to visualize the full schedules as created by the model for all maintenance strategies considered and discussed in section 5. A full single realization of each maintenance strategy is plotted for all aircraft. First, the schedule using the CBM including grouping is displayed in Figure 17. Second, the schedule for CBM excluding grouping is shown in Figure 18. Third, the full schedule for TBM using MCTF is plotted in Figure 19. Fourth, the TBM using 97.5% of MCTF is plotted in Figure 20. Finally, TBM using 95% is shown in Figure 21.

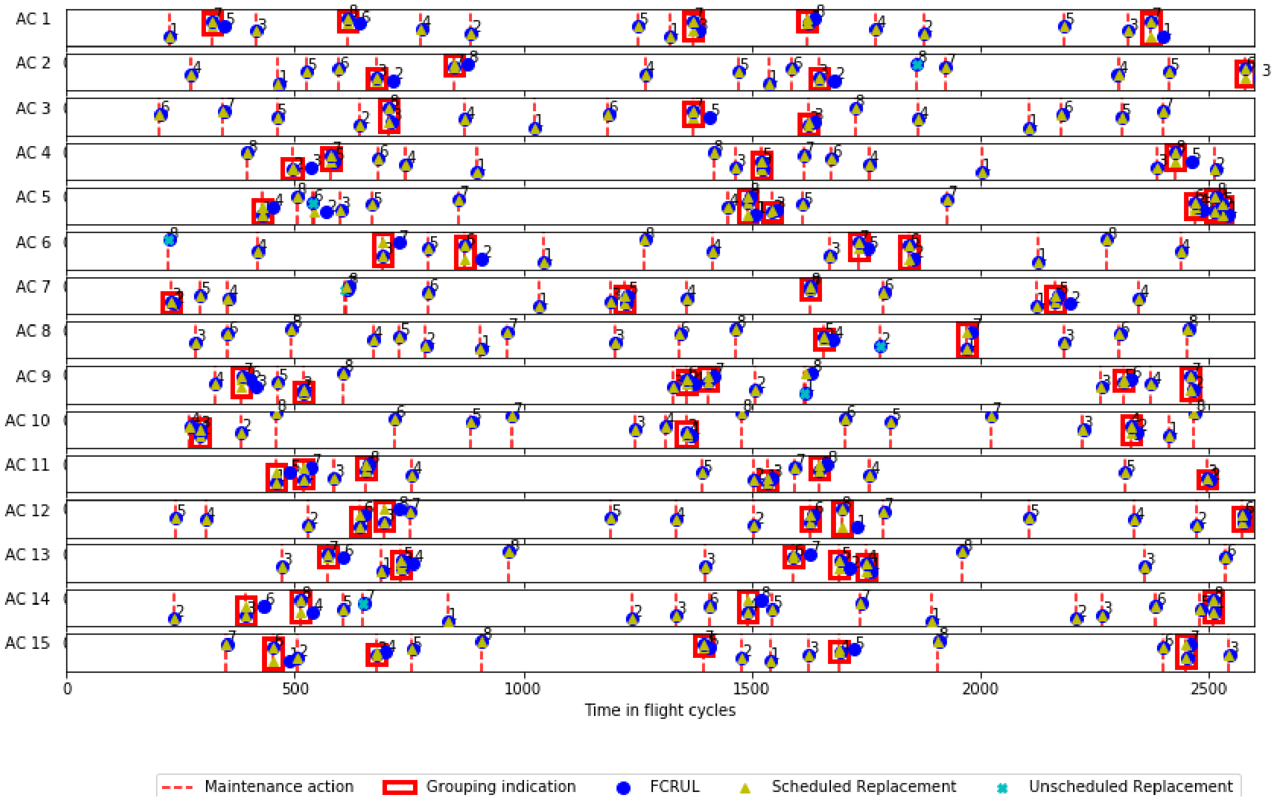


Figure 17: Full plot of the maintenance schedule using CBM, including grouping

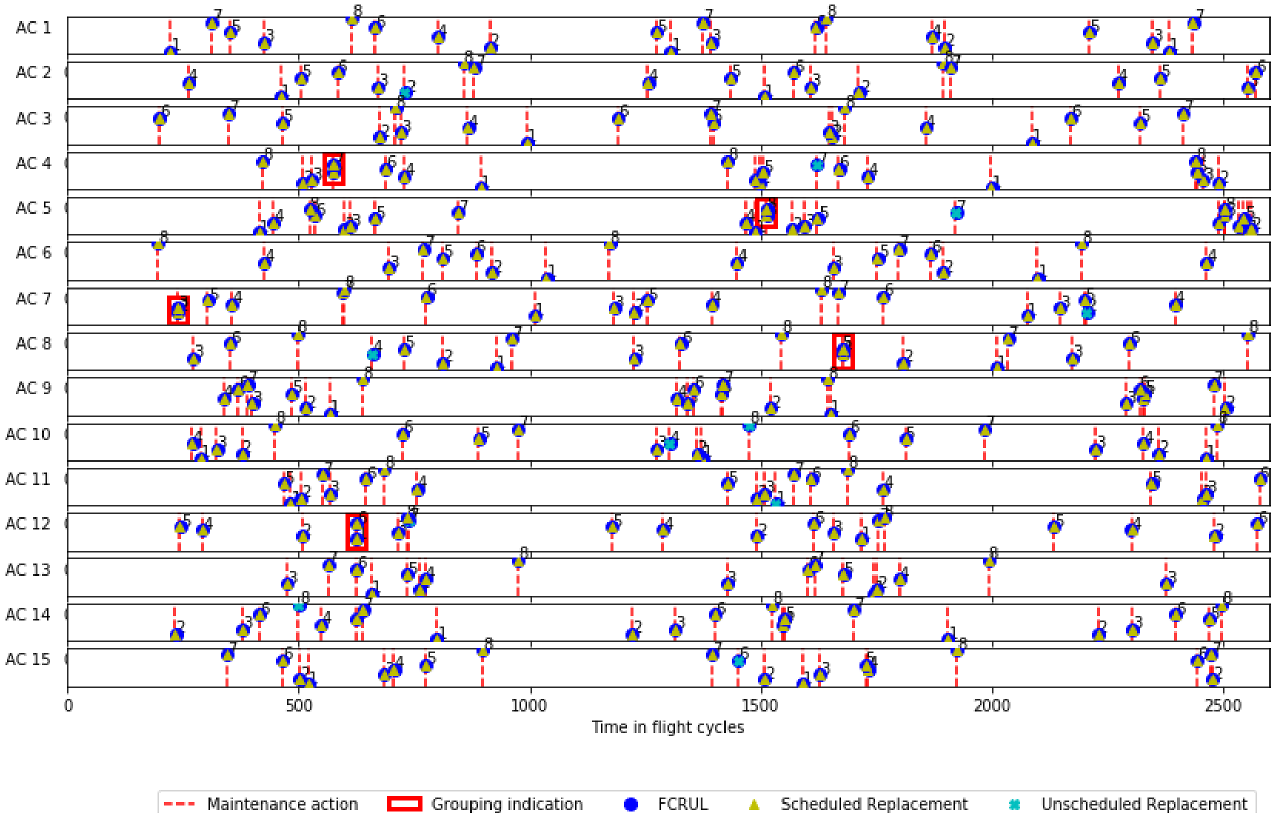


Figure 18: Full plot of the maintenance schedule using CBM, excluding grouping

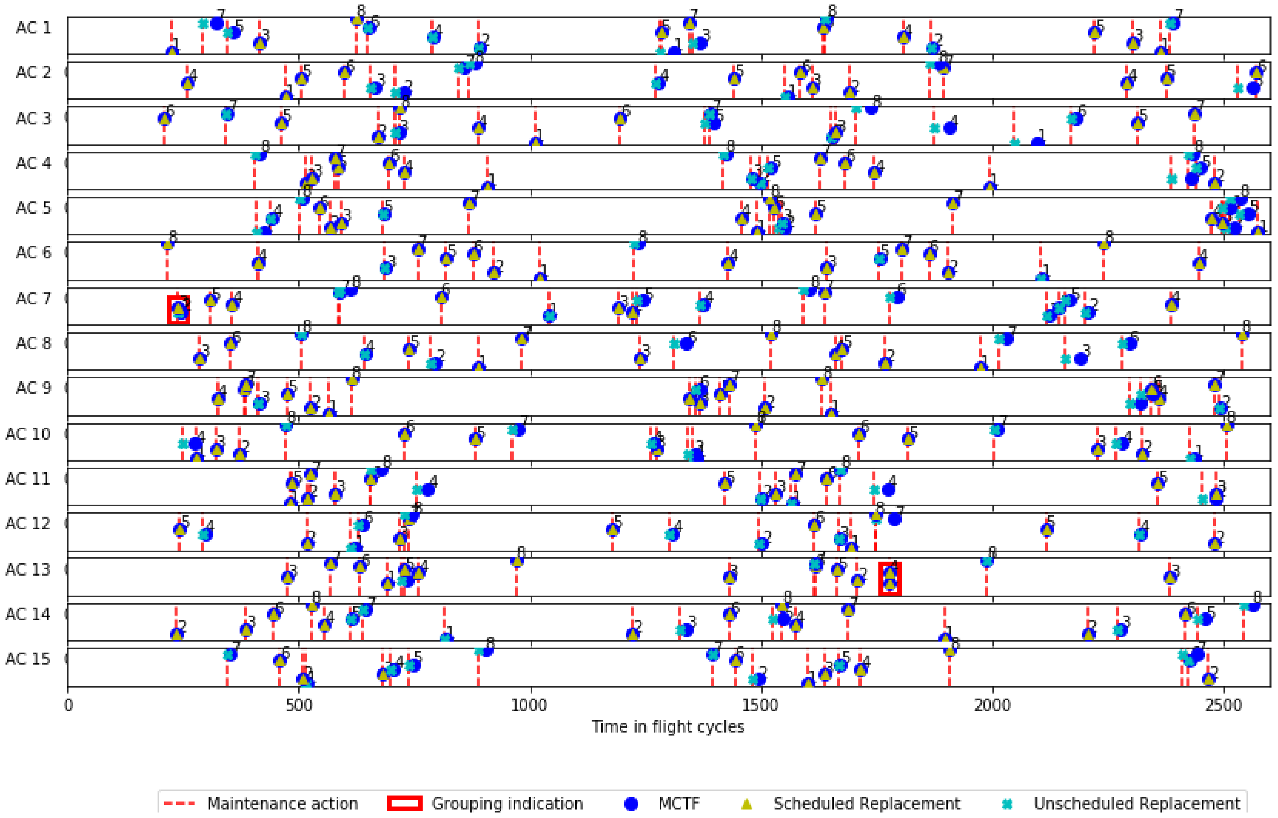


Figure 19: Full plot of the maintenance schedule using TBM at MCTF

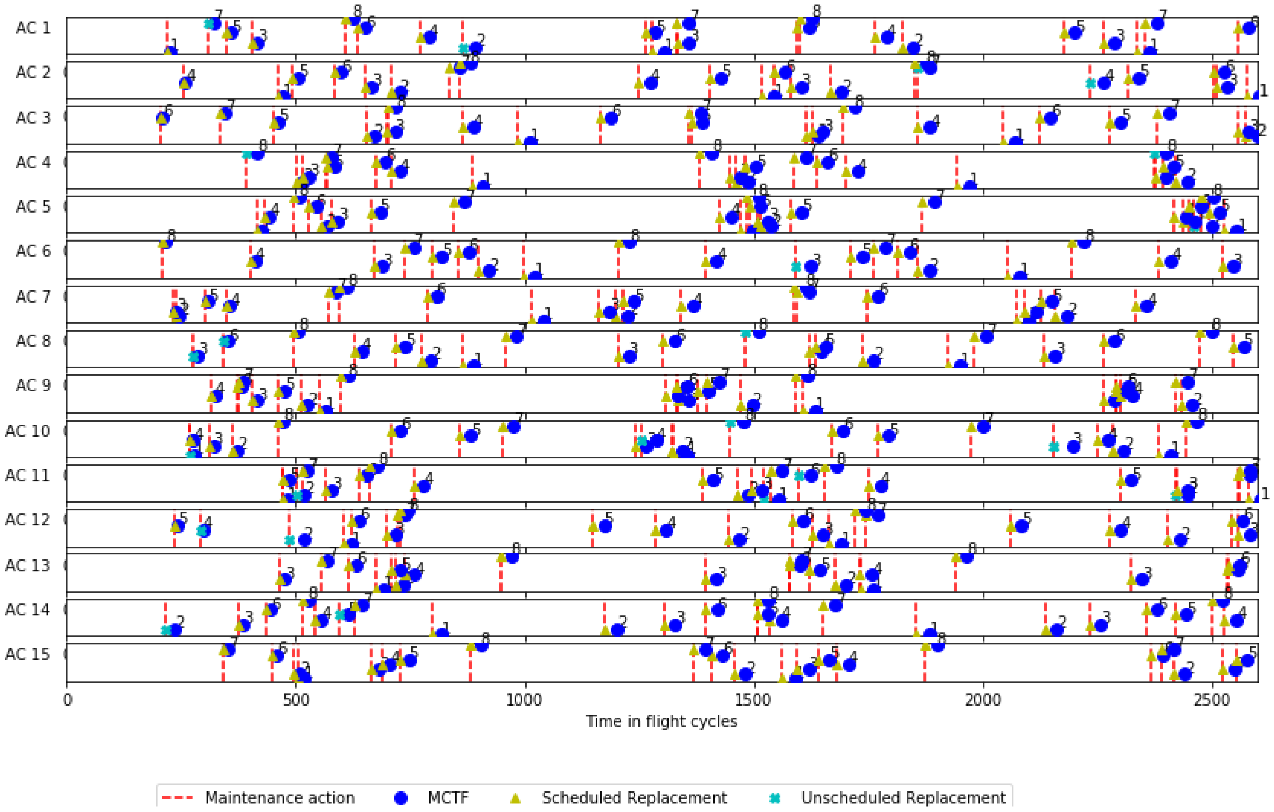


Figure 20: Full plot of the maintenance schedule using TBM, using 97.5% of the MCTF

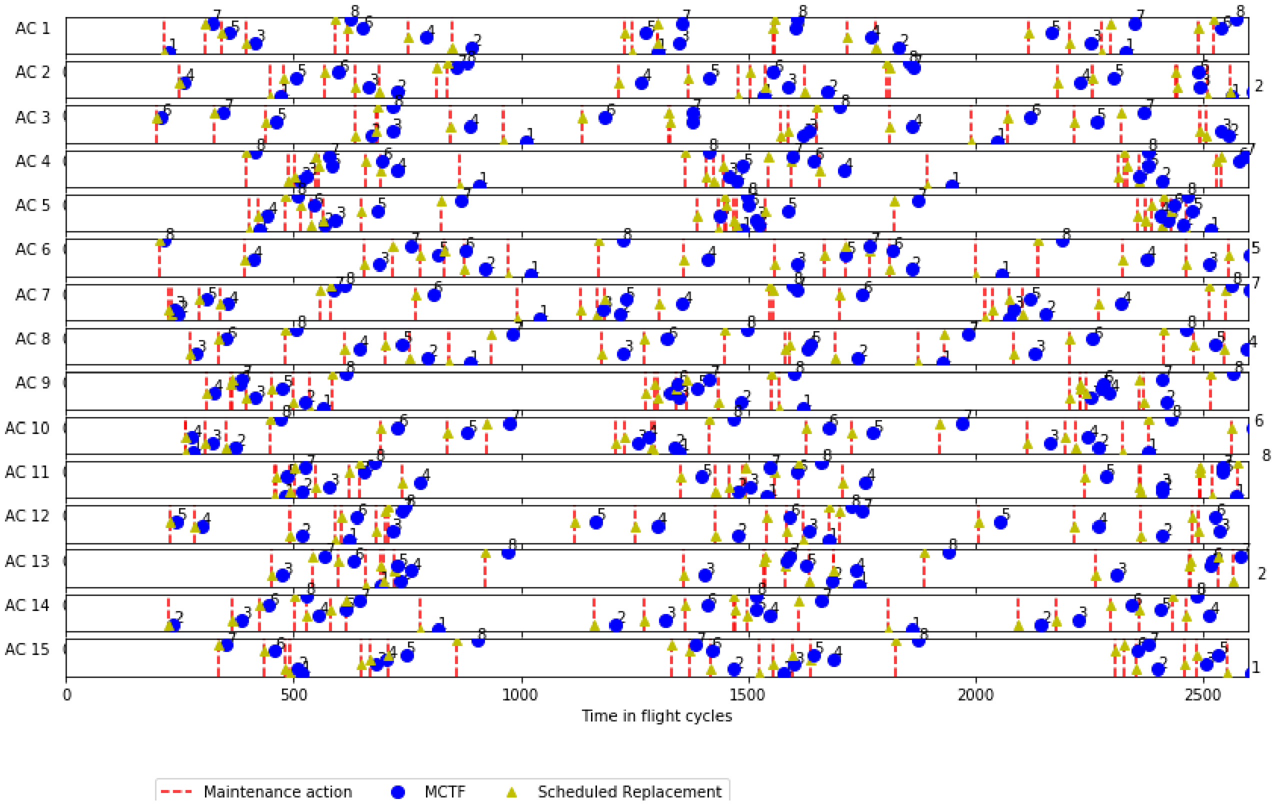


Figure 21: Full plot of the maintenance schedule using TBM, using 95% of the MCTF

II

Literature Study

1

Literature Review

The literature review is of vital importance to the research as it gives insights into the considered theories and state-of-the-art research performed on related topics. First, the literature regarding degradation prognostics is reviewed in section 1.1. Second, the literature regarding maintenance strategy evaluation is elaborated on in section 1.2. At last, in section 1.3, the theories and available literature on maintenance schedule optimization are discussed.

1.1. Degradation Prognostics

The goal of this section is to evaluate the available knowledge regarding degradation prognostics. Degradation is a term used to describe the process in which the quality of something is spoiled over a period of time [4]. Prognostics is a term, originally from the medical research field, which describes the advanced indication of a future event [5]. The combination of these two concepts result in the prediction of the future state of something which degrades over time. Historically this has been an interesting topic for maintenance engineers, especially with regards to remaining useful life (RUL). First, the historic approach and related historic research is discussed. Second, because of the more recent advancements in data collection during operation, new approaches and methods have been developed with respect to prognostic modelling. At last, the relation between the literature and the problem definition will be made.

1.1.1. History

Historically maintenance repair and overhaul (MRO) companies have had a more conservative approach with respect to their maintenance strategies [23], maintenance strategies are discussed in great detail in section 1.2. This conservative approach meant components were being serviced and replaced at fixed intervals, which is a time-based maintenance strategy, to ensure airworthiness of the system, negating the need for a prognostic approach. Because of current developments in condition monitoring and electronic computing it has become possible to apply condition-based strategies with which valuable conclusions can be drawn with respect to the state of components in a system [14].

The first maintenance research papers which started to show signs of prognostics were the papers discussing preventative maintenance, as these papers discussed the replacement of working components which had not failed yet. Initially these strategies only considered fixed intervals with an assumed total life known as periodic policies [18]. Soon reliability was added in terms of a failure rate, resulting in failure limit policies. Usually the failure rate is expressed as a function of a state variable of the component, examples include: age, wear or accumulated damage. An example of such a failure rate function is the Weibull distribution which is commonly used in literature, see Equation 1.1 [28][12][22][21][36][47].

$$\lambda(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \tag{1.1}$$

Where $\lambda(t)$ equals the failure rate at time t and β and η are the shape and scale parameter respectively. Some of the state variables can be physically measured however, they can also be modelled using various methods. Considerable research has been done in order to accurately predict these states. An example of such a state prediction model is the Gamma distribution, which can be used for irreversible processes where cumulative damage is the cause of degradation [18] [27]. A random variable X is said to be gamma-distributed when X ~ Gamma(α,β) resulting in the probability density function as seen in Equation 1.2 [13].

$$f(x;\alpha,\beta) = \frac{\beta^{\alpha} x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)} \quad \text{for} \quad x > 0, \quad \alpha, \beta > 0 \tag{1.2}$$

Here α is called the shape parameter and β the rate parameter. The RUL is one of the applications of such a prediction model where the state of the component has reached a given threshold were it no longer functions as intended. Over the years multiple similar classification schemes have been used by researchers to group the different types of RUL models, a recent proposed classification scheme can be seen in Figure 1.1 [29].

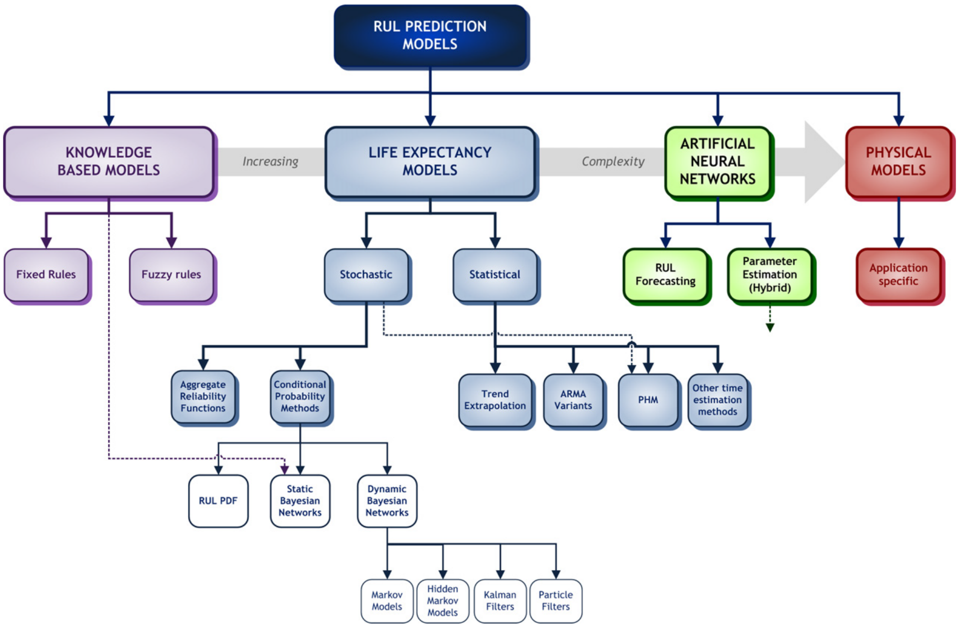


Figure 1.1: Proposed RUL prediction models classification [29]

1.1.2. Current Developments

Still a lot of research is being performed on the creation of accurate RUL models. First, multiple state-of-the-art models will be presented. After which they will be discussed in detail.

Types of Prognostic Models

As can be observed from the classification scheme RUL models can be very diverse depending on their application. Knowledge based models base their prediction on similarities between the observations and previous defined failures. Life expectancy models are a more mathematical approach where the RUL is calculated based on the expected deterioration. Artificial neural networks compute estimated outputs from a mathematical representation of observed failure data. At last physical models can be used to estimate RUL which require the underlying physical phenomenon to be modelled [29]. In Table 1.1 a collection of RUL models used in literature can be seen. The selection mostly considers life expectancy models as these are most relevant to the problem definition which will be elaborated on in subsection 1.1.3.

Case Studies of Prognostic Models

In Table 1.2 six examples of detailed case studies are presented. These case studies can be differentiated based on the distribution they consider and the technique they use to estimate the parameters of their models.

A very interesting paper is the conference paper of Q. Wei and D. Xu from 2014 from the Beihang University of Beijing, China [39]. In their paper they consider a component which degrades according to a Gamma process and introduce a measurement error which can be treated as a Gaussian distribution. Where the independent increments $\Delta w_{\text{true}}(t) \sim Gamma(\alpha, 1/\lambda)$ and $\epsilon_{\text{error}}(t+\delta t)-\epsilon_{\text{error}}(t) \sim N(0,2\sigma^2)$. Then to estimate

Table 1.1: RUL models from literature

Classification	Process	Sources
Conditional Probability Method	Weiner	[43]
	Gamma	[43], [17], [8], [31], [39], [30]
	Gaussian	[43], [46]
	Poisson	[44]
Aggregate Reliability Functions	Weibull	[28], [12], [22], [21], [36]
	Hazard Function	[48]
Trend Extrapolation	Linear	[25]
Physical Model	Physical	[35]

Table 1.2: Degradation prognostic articles

Source	Author	Year	Distribution	Technique	Classification
[8]	A. Grall et al.	2002	Gamma	Translation	Linear Regression
[39]	Q. Wei, D. Xu	2014	Gamma + Gaussian Noise	Method of Moments	Conditional Probability Method
[31]	K. Le Son et al.	2015	Gamma + Gaussian Noise	GIBS Filtering and SEM	Conditional Probability Method
[25]	J. Sun et al.	2020	Linear + Gaussian Noise	Bayesian Linear Regression	Linear Regression
[46]	S.J. Sheather	2004	Sample Data	Kernel Density Estimation	Conditional Probability Method
[35]	N.A. Stoica et al.	2018	None	Physical Model	Physical Model

the RUL of the component they want to estimate the parameters of both distributions. They argue using the method of maximum likelihood will work, however that the calculations will be heavy. Therefor they opt for the method of moments. After performing the derivations the resulting parameter estimators can be seen in Equation 1.3, Equation 1.4 and Equation 1.5.

$$\lambda = \left[\frac{E\left[\Delta w(t)^3\right]}{2E\left[\Delta w(t)\right]} - \frac{3}{2} E\left[\Delta w(t)^2\right] + E\left[\Delta w(t)\right]^2 \right]^{-\frac{1}{2}} \tag{1.3}$$

$$\alpha = \lambda E\left[\Delta w(t)\right] \tag{1.4}$$

$$\sigma^2 = \frac{1}{2}\left(E\left[\Delta w(t)^2\right] - E\left[\Delta w(t)\right]^2 - \frac{1}{\lambda} E\left[\Delta w(t)\right] \right) \tag{1.5}$$

Another very interesting paper is the journal paper of K. Le Son et al. from 2015 published in the Reliability Engineering and System Safety journal [31]. Again the researchers have chosen a combination of a Gamma distribution and a Gaussian distribution. Their solution approach is first to isolate the true degradation by filtering the noise from the data. Second they estimate the parameters of a underlying distribution using a SEM algorithm they have created based on the maximum likelihood method. As more measurement data becomes available over time their RUL estimation becomes more accurate as it keeps being iterated and generates new probability density functions. A clear depiction of how the degradation model works can be seen in Figure 1.2. The RUL at the threshold of L=100 is an estimated probability distribution.

1.1.3. Relation to the Problem Definition

Now that the history and current trends of prognostics in maintenance have been discussed it is important to position the problem as described in Part III within this literature. For the problem we are interested in the RUL of components, brakes to be more specific. Given the degradation follows an unknown Gamma distribution as can be assumed for monotonically increasing processes, estimating the probability density of the degradation seems most logical. This can be classified as a Conditional Probability Method RUL model. This approach would be very similar as applied in both studies discussed in the previous section [31] [39].

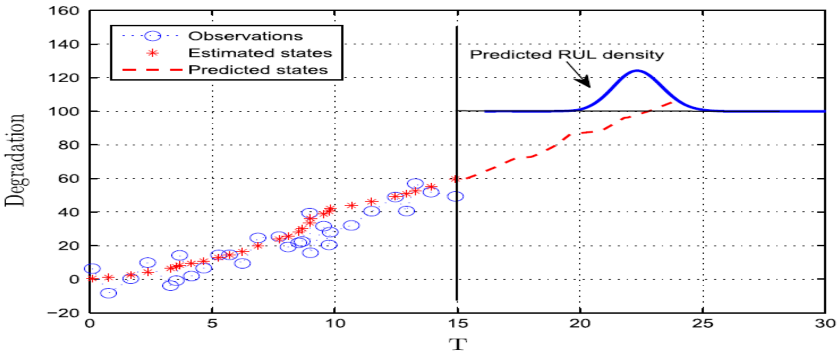


Figure 1.2: RUL probability density function constructed from data and extrapolated to threshold [31]

1.2. Maintenance Strategy Evaluation

This section is dedicated to expand on the literature regarding maintenance strategies and their evaluation. First, the historical perspective will be discussed in subsection 1.2.1. Second, the current developments regarding maintenance strategies will be laid out in subsection 1.2.2. Here a distinction will be made regarding time-based, condition-based and opportunistic maintenance strategies. At last, the relation between the problem definition and the discussed literature will be made.

1.2.1. History

There is extensive literature on maintenance strategies. The first papers started appearing around 1963 and the research has continued ever since [26]. More recent review papers have laid out this literature in which a clear classification scheme can be observed [18]. The first one being: corrective vs preventative maintenance. The difference being whether maintenance actions are performed when a failure has occurred or before a failure has occurred. When maintaining aircraft ensuring airworthiness is the number one priority, meaning corrective maintenance is not a suitable strategy [23]. Therefor only preventative maintenance strategies will be discussed. Another clear distinction in the available research is single-unit vs multi-unit systems. Which is a property of the considered system rather than of the applied strategy. When considering the literature regarding multi-component systems the main focus lies on the utilisation of dependencies between components in a system, this type of maintenance is categorised as opportunistic maintenance which will be elaborated upon in the next section [16].

In maintenance theory a distinction is made between repairable components and non-repairable components. This limits the available maintenance actions, which are repairing and/or replacing. Some papers consider the fact that not all repairs are perfect and thus not all maintenance actions make the system as good as new [19]. Another very clear distinction which will be elaborated upon in the next section is the time-based vs condition-based maintenance. Here the main distinction lies in the usage of data, in time-based maintenance the only important parameter is the time or interval between actions. While in condition-based maintenance the state of the component is the important parameter which is either continuously monitored or inspected [43].

Historically MRO companies have used time-based maintenance strategies as the data required for a condition-based maintenance strategy was not available or the strategy was not reliable enough [23].

1.2.2. Current Developments

In this section the more recent research developments regarding maintenance strategies will be discussed. As already mentioned in the previous section the main progress is currently being made in condition-based maintenance, however before discussing it is important to gain a good understanding of the research regarding time-based maintenance. At last, when considering multi-component systems the relation between the components in a system can be of importance. The research on this topic is categorised as opportunistic maintenance.

Time-Based Maintenance

Time-based maintenance is a relatively simple maintenance strategy. It uses specified maintenance or replacement intervals to maintain a system. It has been a popular choice as it is easy to implement and does not require complex electronics or inspections for data acquisition. Rather it uses predefined intervals, usually by the manufacturer of the component, such as operational time, time since last maintained or in case of aircraft flight cycles to determine the maintenance schedule [23]. This strategy has the advantage for the MRO that as long as they follow the manufacturers maintenance guidelines the airworthiness of the aircraft shall always be guaranteed. However, the use of this strategy also has its downside. The estimated life of components or service interval is often a very conservative estimate, because of the large safety margins involved especially in the case of critical operations in which failure could become catastrophic. This often results in replacements or scheduled repairs when they are not required, leading to waste costs. This realisation has led to the development of a more sophisticated strategy which makes use of data to determine the state of a component or system and base maintenance decision on them, named condition-based maintenance [11].

Condition-Based Maintenance

Condition-based maintenance is quickly becoming the most popular maintenance strategy at this moment [11]. As the name suggests it uses the condition of components or systems to determine the appropriate maintenance actions. This approach can significantly increases the useful life of components as they are only replaced or repaired when required, reducing the waste cost associated with traditional time-based maintenance strategies. How to monitor the state of components or systems can vary, a distinction is made between periodic, non-periodic and continuous [43]. Periodic means that the state of the component or system is observed according to a predefined schedule for example by means of inspection. Non-periodic means that the state is observed according to a dynamic, non predefined, schedule again an example could be by inspections. Continuous monitoring allows for the state to be observed at all times, not requiring the use of inspections rather sensors are used to measure the state.

Opportunistic Maintenance

Other definitions used in literature for single-unit and multi-unit systems is simple and complex systems respectively [40]. The reason a multi-unit system is considered to be more complex is due to dependencies between the components in a system. This has been extensively studied in multiple papers and let to the following classification scheme [43] [10] [16]. The dependencies that are considered in literature are economic, structural and stochastic dependence. Economic dependence between components implies it is more economical to maintain multiple components in a single maintenance action than separate. An example would be a required set-up cost and an additional cost per component serviced. Structural dependence between components implies a more physical dependency, where a component can only be serviced if another component is also removed. The last dependency considered in literature is stochastic dependence which implies that the state of one component can influence the state of other components in the system.

Considering these dependencies a new maintenance approach has been proposed, namely opportunistic maintenance (OM) [16]. With the OM approach the researcher tries to exploit the dependencies of components in order to optimise the schedule, usually by minimising maintenance costs. One of the most promising applications are the papers considering grouping as an OM approach. Grouping is a term used to describe combining maintenance actions in order to optimise a maintenance schedule usually in systems which have a high economic dependency. Examples of such papers include [21] [36] [30]], where OM is applied to complex systems. Figure 1.3 shows a snippet of the initial schedule and the schedule once it is grouped from the paper by H. Vu et al. [21].

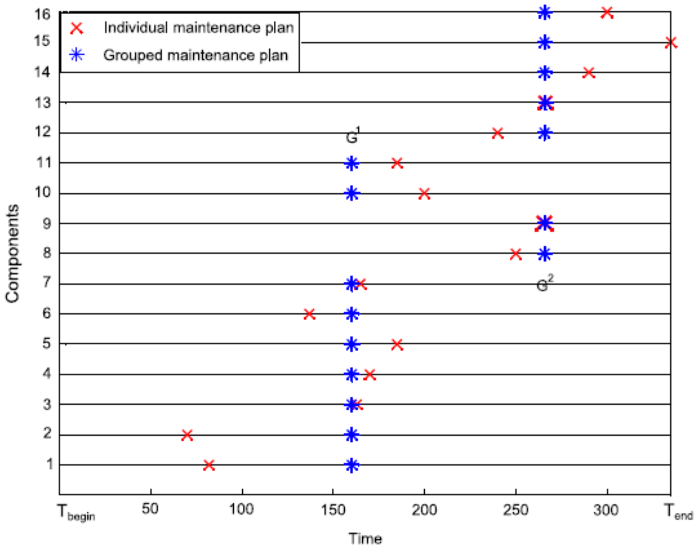


Figure 1.3: Grouping applied in an OM case study [21]

Next to trying to exploit the dependencies of a system another aspect of the OM approach is the consideration of maintenance opportunities, or periods of benefit [16]. Some maintenance actions are more preferable to perform when certain conditions are met, examples used in literature include a periodic decrease in spare part prices or in case of aviation the downtime during operations. Making effective use of such opportunities can make a significant impact on the efficiency of a maintenance schedule.

1.2.3. Relation to the Problem Definition

The problem definition considers a complex system for which continuous monitoring is available. A condition-based strategy is therefor the logical approach which can be verified by a time-based strategy. The only dependence between components appears to be economic which makes the use of a grouping approach certainly viable. As explained in the previous section a schedule can exhibit multiple opportunities which should be considered.

1.3. Maintenance Schedule Optimization

This section is dedicated to review the maintenance optimisation models used in literature. First, the historic approaches will be considered in subsection 1.3.1. After which, the more recent models and methods are reviewed in subsection 1.3.2. At last, the relation between the literature and the problem definition is made in subsection 1.3.3.

1.3.1. History

The first trends in scientific approaches to maintenance management are observed in 1950 to 1960 [40]. With the proposed approach preventative maintenance. Only around 1980 the first computers were brought to maintenance, at that time mostly for administrative processes. Only around 2000 the first models started to direct maintenance efforts for those components for which reliability is critical.

However, there is a historic gap between theory and practise, as is the case in most disciplines. In a review paper by R. Dekker he lays out six aspects which are considered to be factors for the gap [40]. The first aspect is that maintenance optimisation models are difficult to interpret. For the second aspect it is argued many papers on this subject only serve a mathematical purpose. A third aspect is the fact that companies are not interested in publishing. A fourth aspect is the fact that maintenance is a very generic term used to describe a multitude of aspects. As a fifth aspect the fact that optimisation is not always necessary is reasoned. The final aspect is that optimisation models are focused on the wrong type of maintenance. Despite these negative aspects there is scope for maintenance optimisation mainly due to two reasons. The first one being the technological push where computers are becoming cheaper, more powerful and information systems becoming integrated with intelligence embedded in business processes. The second reason is the economical necessity, a trend can be observed where the quality of decision making needs to be higher. Maintenance optimisation models which are integrated into decision support systems provide an objective, or unbiased, method for making such decisions.

1.3.2. Current Developments

With the rise of condition-based maintenance most current research papers propose optimisation models using this strategy. Within these models a clear classification scheme can be observed which is outlined below, followed by the objective functions and constraints which are considered in literature.

Classification Criteria of Models

As previously discussed in section 1.1 and section 1.2 multiple distinctions with respect to the maintenance problem can be made. Examples include single-unit vs multiple-unit systems, failure rate vs deterioration and preventive vs corrective. Capturing the maintenance problem in a suitable optimisation model also comes with distinct properties which are captured in a classification scheme. Distinctions are made between continuous vs discrete time, deterministic vs stochastic processes, perfect vs imperfect repairs and finite vs rolling horizon. In Table 1.3 current research papers regarding maintenance optimisation are listed including their classification.

Table 1.3: Maintenance optimisation models considered in literature

Ref	Authors	Year	Continuous vs Discrete	Perfect vs Imperfect	Finite vs Rolling	Optimisation Objective
[45]	S. Wu, I.T. Castro	2019	Continuous	Imperfect	Finite	Minimize Maintenance Cost
[34]	M.J. Kallen, J.M. van Noortwijk	2004	Continuous	Imperfect	Finite	Minimize Maintenance Cost
[48]	X. Zhou et al.	2006	Continuous	Imperfect	Rolling	Minimize Maintenance Cost
[17]	H. Liao et al.	2006	Continuous	Imperfect	Rolling	Maximize Availability
[22]	H.C. Vu et al.	2020	Continuous	Perfect	Rolling	Minimize Maintenance Cost
[12]	C.R. Cassady et al.	2001	Discrete	Imperfect	Finite	Maximize Reliability
[44]	S. Taghipour et al.	2010	Discrete	Imperfect	Finite	Minimize Maintenance Cost
[28]	J.Y.J. Lam, D. Banjevic	2015	Discrete	Perfect	Finite	Minimize Maintenance Cost
[32]	K. Schneider, C.R. Cassady	2015	Discrete	Perfect	Finite	Maximize Reliability, Minimize Cost, Maximize Minimum Reliability

Objective Functions

In order to optimise the maintenance schedule the optimisation criteria should be clear, this is the objective of the model. In literature multiple criteria are considered, these include; cost minimisation, reliability or available maximisation and multi-objective [43]. In cost minimisation the objective function describes the sum of all associated maintenance costs of the schedule. Examples of these costs can include preventative replacement, corrective replacement and inspection costs. Papers who considered such objective functions include [9] [33]. The second objective discussed in literature is the reliability or availability maximisation, here the goal is to maximise the systems availability to operate as intended (uptime) and minimise the time spend on maintenance or being inoperable (downtime). Availability is a function of uptime, downtime and the number of maintenance actions which can be seen in Equation 1.6 [43]. Papers who considered availability as an objective include [7] [20] [49].

$$\text{Availability} = \frac{E[\text{uptime}/(\text{N maintenance in a cycle})]}{E[\text{downtime}/(\text{N maintenance in a cycle})]} \tag{1.6}$$

Finally, sometimes a model is required which can optimise multiple objectives simultaneously which can prove to be a challenge as some of them may be in conflict. Several conflicting objective functions must be evaluated with respect to their decision variable. With the goal being the identification of the best compromise between all various objectives. An example paper considering such a model is R.J. Ferreira et al. from 2009 who developed a multi-criteria decision model optimising for expected cost and expected downtime [42]. Another paper applying such a multi-criteria model is a paper by P.D. Van and C. Bérenguer from 2012 who consider maintenance cost and productivity [38].

Constraint Functions

How the maintenance optimisation model is constraint largely depends on the problem it is solving. In most papers in literature there is a constraint categorised as resource availability. This could be in terms of spare parts [15], maintenance slots (e.g. labor, equipment, etc) [32]. Another very prevalent constraint in literature regarding aircraft maintenance is the safety. Some components are not allowed to fail and other, which have redundancy in their design, are to some extend. Other constraints include budget or workforce limitations [10].

1.3.3. Relation to the Problem Definition

Most of the choices regarding the optimisation model classification come from the problem they are being applied to. The data which is available, the system that is being maintained and the horizon of the problem. In relation to the problem definition as proposed in Part III, the most interesting models to consider are those which consider continuous time, stochastic processes, perfect repairs as only replacements are considered and a rolling horizon. The objective function which is required to optimise will be in relation to the costs associated to the maintenance. While the model is being constraint by hangar availability, the aircraft schedule and safety requirements.

III

Supporting Work

1

Research Plan

In this supplemental chapter the thesis research plan will be presented. First, the problem definition will be given in section 1.1, together with background information. Second, the conceptual design will be shown in section 1.2, this conceptual design has largely been based on the definitions and structure as proposed in [37]. Here the scope of the research and the research objective will be elaborated upon. third, the research questions are presented in section 1.3. At last, the technical research design will be discussed which includes the work breakdown structure (WBS), the work flow diagram (WFD) and the proposed planning in section 1.4.

1.1. Problem Definition

This section is dedicated to elucidate and define the maintenance problem which is to be researched. First the background to the problem is given. After which, the component degradation is explained.

1.1.1. Background Information

Let us consider a fictitious airline which operates multiple wide-body aircraft of the same type. In order to ensure that the aircraft remain operational over time they must be subjected to a maintenance program. For this research the main focus lies on the schedule optimisation of the maintenance tasks of the brake system. The aircraft considered makes use of eight brakes which are all mounted on the main undercarriage. A representative image can be seen in Figure 1.1. It can be noted that there are four brakes on each side of the aircraft. Due to wear of operation the thickness of the brake pads reduces, after some time the minimum thickness threshold is reached and a replacement must be performed. Brakes are considered to be safety critical, thus regulations state their minimum thickness before replacement equals 50% of the thickness of a new brake. The brakes are fitted with sensors allowing for continuous monitoring of their state. In Figure 1.2 an example of a single aircraft brake assembly can be seen. The aim is to develop an optimisation model which finds the optimal times to replace the brakes of each aircraft given limited maintenance hangar availability.

1.1.2. Component Degradation

Given that the deterioration state of the brakes is continuously monitored estimation of their remaining useful life (RUL) can be made. A prognostic model must therefor be created which can predict the future state of the components. The components degrade independently according to a gamma distribution. In previous research these parameters have been analyzed based on real observations of the state of degradation.[24] For this research the real behaviour of the component degradation can thus be generated using the obtained parameters from the previous research. These parameters can be seen in Table 1.1.

For the estimation of the remaining useful life the parameters are considered to be unknown. In order to fully utilise the brake it must be replaced exactly at the threshold, however other factors may require the brake to be replaced earlier. The threshold of degradation is a set value which is determined based on safety requirements by the airline, MRO or OEM. For this research this degradation threshold is equal to 0.75. A mathematical definition of the degradation modelling can be found in Part I. Each brake is fitted with sensors, their state can be continuously monitored.

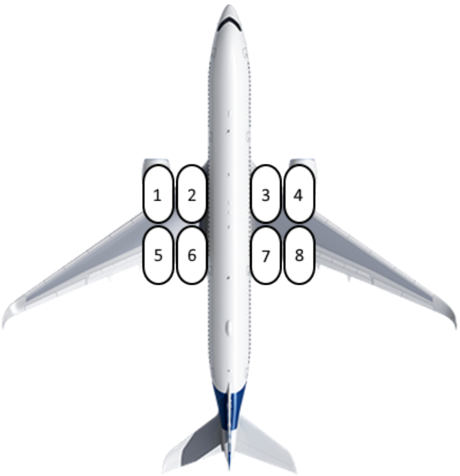


Figure 1.1: Topview of an A350-900 overlayed with the brake assembly lay-out [1]

Figure 1.2: Example brake assembly of an 737NG main landing gear [3]

Table 1.1: Real brake degradation Gamma parameters per location respectively [24]

Brake ID	Parameter a_{real}	Parameter b_{real}	MCTF
1	3.350	0.0002063	1447.0
2	4.146	0.0001836	1313.7
3	3.546	0.0002217	1272.0
4	3.390	0.0002171	1358.8
5	4.667	0.0001715	1249.4
6	4.100	0.0001856	1314.1
7	3.068	0.0002329	1399.5
8	2.583	0.0002852	1357.5

1.2. Conceptual Research Design

The conceptual research design is the foundation of the project and serves as the basis of the thesis. The thesis is placed within the context of knowledge available by defining the scope and the objective of the research is elucidated.

1.2.1. Scope Definition

The scope of this thesis is optimising and analysing the maintenance schedule of a single type fleet of aircraft which have multiple degrading components of the same type with respect to cost while satisfying hangar availability and safety requirements applying a condition-based maintenance strategy.

In Part II the results of an in-depth literature review are shown containing all relevant research disciplines which fall within the scope, includes stochastic simulation, maintenance theory and operations optimization. More specifically, degradation prognostics, maintenenace strategy evaluation and maintenance schedule optimization. However, between these subjects a clear gap in literature is observed. These subjects have been touched upon separately but have not been performed in combination. Currently, no research is available on the optimisation of a maintenance schedule on fleet-level considering multi-component systems using prognostics.

1.2.2. Research Objective

The projects research objective is to contribute to the further development of the theories regarding operations optimisation, maintenance policies and stochastic simulation. In particular, the focus lies on the following issues: Determining the optimal maintenance schedule for a fleet of aircraft minimising cost while satisfying safety requirements considering multiple components per aircraft which states degrade over time according to a gamma process and having limited hangar availability. This objective is achieved by providing insight, based on literature study and proof of concept, into the application of an optimisation model for the particular problem stated and evaluation of a condition-based maintenance strategy.

1.3. Research Questions

From the conceptual research design it is possible to construct multiple research questions which are to be answered at the end of the research. The questions can be subdivided into: setting up the model, solving for a case study using the condition-based maintenance strategy and evaluating the schedule. Resulting in the following research questions:

1. Considering an aircraft fleet with multiple degrading components and limited hangar availability, how can the optimal maintenance schedule be determined using degradation prognostics and an optimisation model?
    - 1.1 What prognostic can be used to estimate the component degradation?
    - 1.2 What are the possible objectives of the problem and how can the objectives be formulated in terms of a function?
    - 1.3 What are the constraints of the problem and how can the constraints be mathematically formulated?
2. When considering the application of a condition-based maintenance strategy, what would be the optimal maintenance schedule and the resulting key performance indicators?
    - 2.1 What is the resulting schedule and costs associated with the solution?
    - 2.2 What are the key performance indicators of the solution?
3. When evaluating the proposed condition-based maintenance schedule to a fixed replacement time-based maintenance schedule, how do they compare in terms of costs and key performance indicators?
    - 3.1 How do the costs of the solution compare between the two strategies?
    - 3.2 How do the key performance indicators of the solution compare between the two strategies?

1.4. Technical Research Design

Now that the aim of the research is defined the approach towards the solutions should be laid out. This is done in terms of a technical research design which can be seen as the road map of the research. First, a work breakdown structure is presented which shows the tasks to be performed independent of time. Second, a work flow diagram is presented which displays the chronological order of the work packages of the work breakdown structure. Final, a detailed planning will be presented in terms of a Gantt planning.

1.4.1. Work Breakdown Structure

The to be performed work has been divided into six discrete work packages, each of which have their own tasks. The structure can be represented as a work breakdown structure as can be seen in Figure 1.3. Each task has been given an unique work package code which is used to identify the task. The work packages are subdivided into the model definition, model implementation, model verification, experiments, results analysis and defence preparation.

1.4.2. Work Flow Diagram

The progress of the project over time can be indicated by a work flow diagram. In this diagram the continuation of the work packages is shown together with important deliverables, deadlines and the time that is allocated to the completion of each work package. This diagram can be seen in Figure 1.4.

1.4.3. Planning

Now that the technical research design is almost complete a detailed planning can be created which represents the full execution of the project. A Gantt planning is chosen, which represent all required work, milestones and interdependence of tasks over time. The detailed Gantt planning can be seen in Figure 1.5.

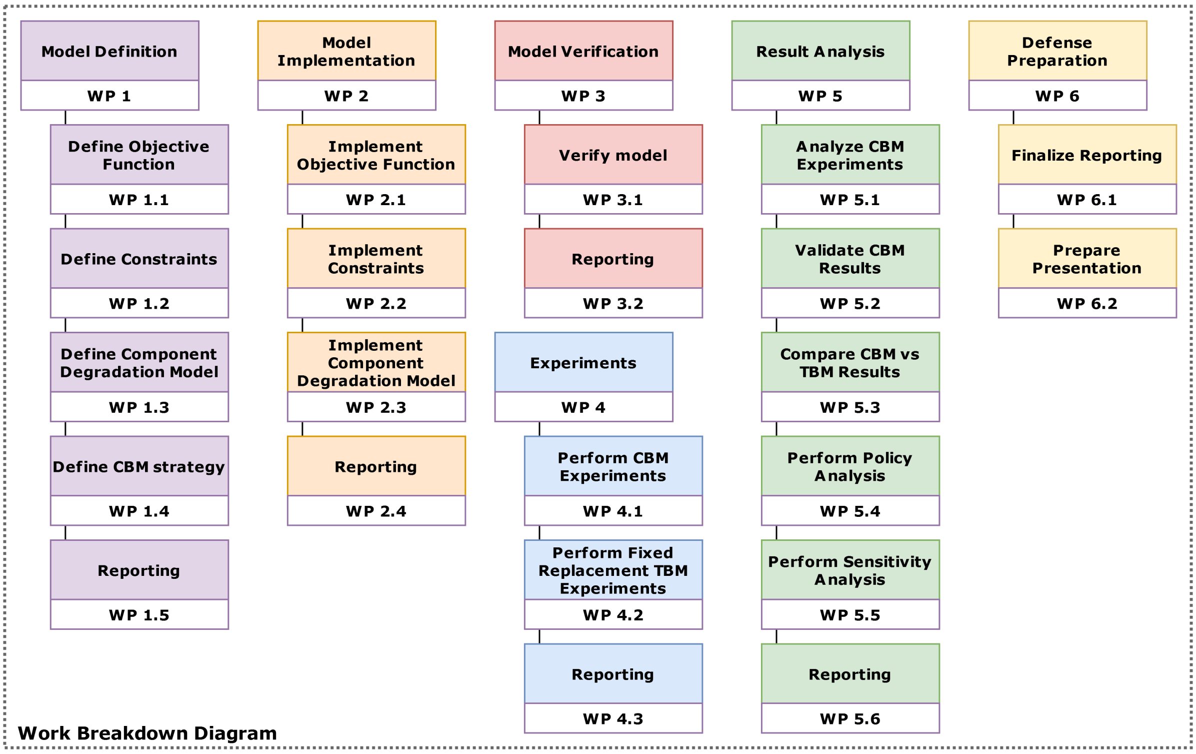


Figure 1.3: Work breakdown structure of the thesis research plan

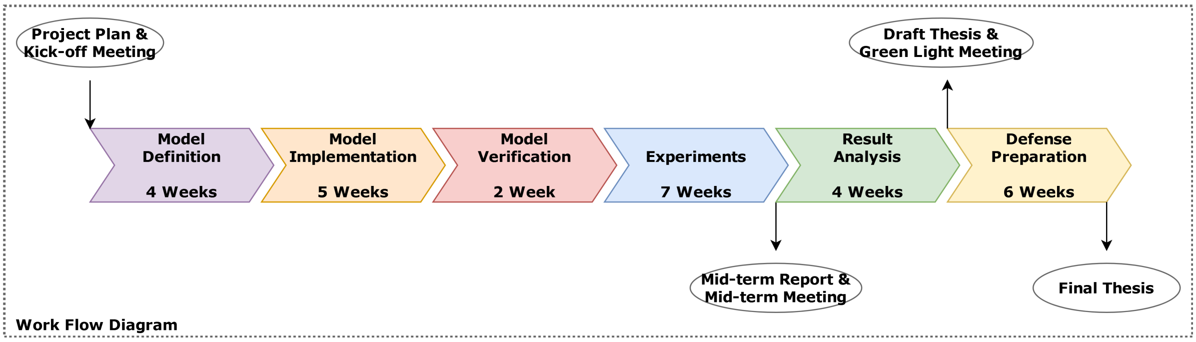


Figure 1.4: Work flow diagram of the thesis research plan

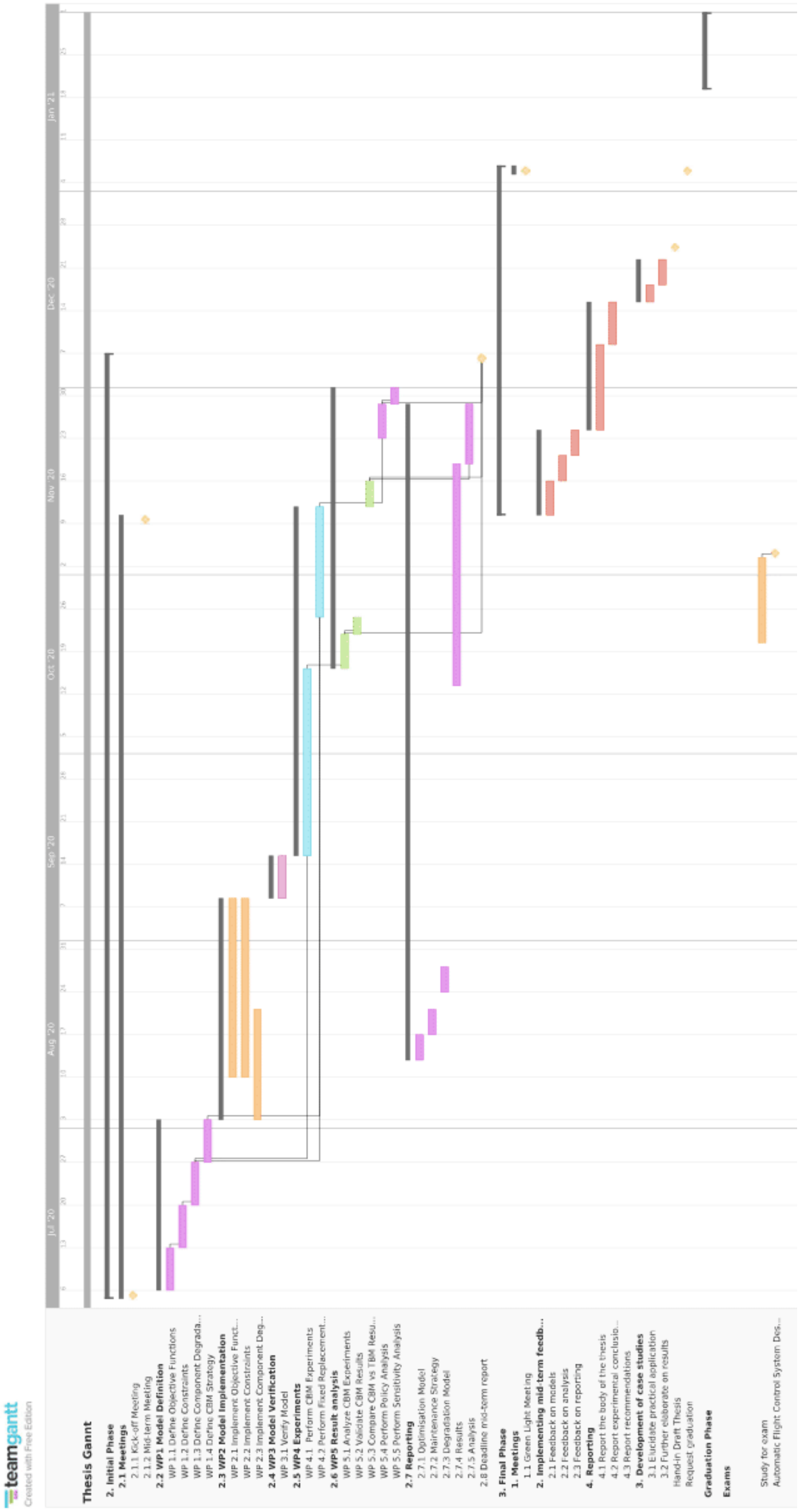


Figure 1.5: Full Gantt chart including all work packages

2

Verification & Validation

This supplemental chapter is dedicated to elaborate on the verification and validation methods used and their application. Verification and validation is of importance to see if the developed model is providing feasible and valuable results, thus indirectly determine the applicability of the model.

The verification and validation approach used in this research is based on the work proposed by R. Sargent [41]. He has outlined a process in which simulation models such as the maintenance model created during this research can be verified and validated. First, the verification and validation strategy is discussed in section 2.1. Second, the model verification is elaborated on in section 2.2. At last, the model validation is shown in section 2.3.

2.1. Verification and Validation Strategy

First, it would be wise to explain or restate the definitions of the terms verification and validation in the context as discussed here. Namely, because in literature multiple definitions of these two terms are used. Because the verification and validation approach is based on the work proposed by R. Sargent, the same definition of the terms shall be used which are the following:[41]

- **Model verification** Ensuring that the computer program of the computerized model and its implementation are correct.

- **Model validation** Substantiation that a computerized model within its domain of applicability possesses a satisfactory range of accuracy consistent with the intended application of the model.

In short, verification focuses on the implementation of the model and the correctness of the programming. While the validation focuses on the value of the models outcome and its applicability to its intended application. To evaluate these criteria the following subjects will be elaborated on.

- **Model input verification** Verification of the model input and the processing performed

- **Model function verification** Verification of the programmed code, evaluation of the correctness of computations performed

- **Conceptual validation** Validating that the assumptions and theories are correct and reasonable for the intended purpose

- **Operational validation** Validating the model's outcome with its intended purpose and applicability

2.2. Model Verification

This section shall elaborate on the verification process. The verification is performed in two parts. The first part being the model input verification and the second part being the model function verification.

2.2.1. Model Input Verification

The flight schedule is an important input to the model, as it determines when maintenance slots are available and it translates time to flight cycles. The flight schedule used by the model is derived from actual flight data of the KLM A330 fleet, however only for five aircraft for one week was available. This means the schedule is repeated when analyzing more than five aircraft and repeated over time, weekly. As the model is evaluating a maintenance schedule over a long period of time the repeating nature of the available flight schedule does not have a negative influence on the results. An elaboration on the flight schedule used can be found in Part I.

The realization of the component degradation makes use of Gamma distribution parameters which are determined from real aircraft brake data in prior research.[24] These parameters have been verified by evaluating the mean cycle to failure of the brakes with respect to the available literature.

The initial degradation of the components is an input to the model. It has been generated using the available degradation parameter data of Table 1.1.[24] For the case study performed the initial degradation of 20 flight cycles is kept constant for all maintenance strategies evaluated.

2.2.2. Model Function Verification

Now that the input of the model has been evaluated, the functions that make use of this input can be verified. This has been a continuous process during the project. The model has been subdivided into three distinct blocks namely, the prognostic model, the optimization model and the rolling horizon model.

Prognostic Model

The prognostic model consists of two main functions, calculating the degradation Gamma distribution parameters by means of the method of moments and determining the remaining useful life by means of the cumulative distribution function. The more degradation data available the better the parameter estimation becomes. Since the model applies a rolling horizon approach, when nearing the remaining useful life there is a lot of degradation data available resulting in reliable estimations of the Gamma parameters. Multiple test cases have been performed in order to rule out errors from the processing.

Optimization Model

The optimization model assigns components to maintenance slots by optimizing for the available data. This data needs to be converted into the parameters used by the model. Such as the available maintenance slots and penalty function used in the constraints and objective function. The functions which compute these parameters have been individually verified by testing using simple examples as well as taking random samples and evaluating the result by plotting. The optimization itself is difficult to verify as the optimization is performed by using a solver, Gurobi Optimization for Python.[6] It is very difficult to perform verification on an industry solver such as Gurobi and goes beyond the scope of this research. However, Gurobi is academically acclaimed and an industry leader in optimization thus it is assumed the solver is verified and validated.

Rolling Horizon Model

The rolling horizon model or approach combines the above mentioned models to create the desired maintenance schedule over a long period of time. It performs some necessary steps in order to progress the schedule, which require to be verified. First, the model determines which components are within the current optimization window. This is verified by performing several sample tests and evaluating the result, looking at the remaining useful life as calculated by the prognostic model and the start and ending of the optimization window. Second, the realization of the schedule is performed. In this computation the replacement criteria are evaluated for every realized flight cycle. By evaluating the key performance indicators which are the results of these computations the realization process is verified.

2.3. Model Validation

Now that the verification process has been elaborate on, this section shall elaborate on the validation process. The validation can be subdivided into conceptual and operational validation. Conceptual validation focuses on the assumptions and theories behind the model and validates their correctness with respect to the intended purpose. While operational validation focuses on the outcome of the model itself and its correctness with respect to the intended purpose.

2.4. Conceptual Validation

The theories behind the model and their respective assumptions have been discussed in detail in the literature review in Part II. Rather than evaluate these theories and their assumptions, it is more important to validate the application of these theories to this specific research and its intended purpose.

The intended purpose of the research is to contribute to the development of the theories regarding operation optimization, maintenance policies and stochastic simulation by providing insight into the application of an optimization model and evaluation of a condition-based maintenance strategy. The focus lies on the case study where a fleet of aircraft is considered with multiple degrading components for which the maintenance schedule is required to be optimized with respect to cost while satisfying hangar availability.

The first theory applied is degradation modelling from the field of stochastic simulation. More specifically, brake degradation modelling which is a monotonic increasing process. For monotonic increasing processes such as brake degradation Gamma distributions best suit their behaviour. This is a valid assumption recognised in literature.[43]

With respect to operation optimization the theories regarding integer linear programming have been used. Since the problem considers the scheduling of maintenance an optimization model has been created which considers all available maintenenace slots within the optimization window and determines whether there should be a replacement scheduled on them or not. This is a binary question, thus an integer linear program is a good fit for this type of problem.

The choice of maintenance strategy has largely been determined by the case study which is to be considered. However, in order to validate and evaluate the results achieved by the CBM strategy multiple strategies are considered. The main focus lies on the condition-based maintenance strategy, which is evaluated using multiple time-based maintenance strategies. Given the fact there is condition monitored data available, a CBM strategy is the most logical choice. Followed by a TBM strategy as this is still an industry standard in aviation maintenance.

2.5. Operational Validation

In operational validation the outcome of the model is validated with respect to its intended purpose and applicability. The model could be viewed as a basis or first step towards a tool to help maintenance engineers determine the optimal maintenance schedule for their aircraft fleet. However it should be noted that this is not the intended purpose as the intended purpose has a more academic foundation and is stated in the section above. First, the compliance of the constraints is evaluated. After which, the usefulness of the results is analyzed.

Constraint Compliance

When evaluating the constraints itself, there are two hard constraints and one soft constraint. Hard constraints give the decision variables set conditions which need be satisfied, while the soft constraint penalizes the objective function when unfavourable conditions are chosen. The compliance with the hard constraints is easily checked as the optimization solution would be infeasible if these constraints cannot be met during optimization. The soft constraints are harder to evaluate as the solution would always be feasible however it should be checked whether the solution makes sense.

Usefulness of Results

In order to validate the usefulness of the results it is important to keep the intended purpose in mind. The obtained CBM schedule is evaluated to the TBM schedule, which is in itself a validation of the CBM strategy. Applying a CBM strategy to the problem at hand has proven to be valuable and useful, thus serving the intended purpose.

The results achieved are logical, however they are hard to compare to real maintenance schedules. The model only considers the brake system of the aircraft which makes it hard to compare to maintenance schedules which consider more types of components. The research project does not have active ties with an industry partner in order to receive an actual maintenance schedule. This makes the validation of the obtained schedule result difficult.

From literature it is possible to find suggested aircraft brake lifetime estimations which are in line with the maintenance schedule obtained.[2]

Bibliography

[1] Airbus a350-900 details. `https://www.airbus.com/aircraft/passenger-aircraft/a350xwb-family/a350-900.html#details`, . Online; accessed on (June 25, 2020).

[2] Aviation maintenance, wheel brake repair and overhaul. `https://www.avm-mag.com/wheel-brake-repair-and-overhaul/`, . Online; accessed on (November 10, 2020).

[3] Skybrary, brake assembly.jpg. `https://www.skybrary.aero/index.php/File:Brake_Assembly.jpg`, . Online; accessed on (June 25, 2020).

[4] Cambridge dictionary, meaning of degradation in english. `https://dictionary.cambridge.org/dictionary/english/degradation`, . Online; accessed on (June 10, 2020).

[5] Lexico dictionary, meaning of prognostic in english. `https://www.lexico.com/definition/prognostic`, . Online; accessed on (June 10, 2020).

[6] Gurobi optimization. `https://www.gurobi.com/`, . Online; accessed on (November 10, 2020).

[7] A. Biswas, J. Sarkar, S. Sarkar. Availability of a periodically inspected system, maintained under an imperfect-repair policy. *IEEE Transactions on Reliability*, 52(3):311-318, 2003.

[8] A. Grall, C. Bérenguer, L. Dieulle. A condition-based maintenance policy for stochastically deteriorating systems. *Reliability Engineering and System Safety*, 76(1):167-180, December 2001.

[9] A. Grall, L. Dieulle, C. Berenguer, M. Roussigol. Continuous-time predictive-maintenance scheduling for a deteriorating system. *IEEE Transactions on Reliability*, 51(2):141-150, 2002.

[10] B. de Jonge, P.A. Scarf. A review on maintenance optimization. *European Journal of Operational Research*, 285(1):805-824, September 2019.

[11] B. de Jonge, R. Teunter, T. Tinga. The influence of practical factors on the benefits of condition-based maintenance over time-based maintenance. *Reliability Engineering and System Safety*, 158(1):21-30, 2017.

[12] C.R. Cassady, W.P. Murdock Jr, E.A. Pohl. Selective maintenance for support equipment involving multiple maintenance actions. *European Journal of Operational Research*, 129(1):252-258, May 2000.

[13] F.M. Dekking, C. Kraaikamp, H.P. Lopuhaä, L.E. Meester. *A Modern Introduction to Probability and Statistics, Understanding Why and How*. Springer, Delft, The Netherlands, 1th edition, 2005.

[14] G. Vachtsevanos, F.L. Lewis, M. Roemer, A. Hess, B. Wu. *Intelligent Fault Diagnosis and Prognosis for Engineering Systems*. John Wiley Sons Inc, USA, 1th edition, October 2006.

[15] G.Q. Cheng, B.H. Zhou, L. Li. Integrated production, quality control and condition-based maintenance for imperfect production systems. *Reliability Engineering System Safety*, 175(1):251-264, 2018.

[16] H. Ab-Samat, S. Kamaruddin. Opportunistic maintenance as a new advancement in maintenance approaches. *Journal of Quality in Maintenance Engineering*, 20(2):98-121, March 2014.

[17] H. Liao, E.A. Elsayed, L. Chan. Maintenance of continuously monitored degrading systems. *Computing, Artificial Intelligence and Information Management*, 175(1):821-835, August 2005.

[18] H. Wang. A survey of maintenance policies of deteriorating systems. *European Journal of Operational Research*, 139(1):469-489, April 2001.

[19] H. Wang, H. Pham. Optimal maintenance policies for several imperfect maintenance models. *International Journal of Systems Science*, 27(6):543-549, 1996.

[20] H. Xu, W. Hu. Availability optimisation of repairable system with preventive maintenance policy. *International Journal of Systems Science*, 39(6):655-664, 2008.

[21] H.C. Vu, P. Do, A. Barros, C. Bérenguer. Maintenance grouping strategy for multi-component systems with dynamic contexts. *Reliability Engineering and System Safety*, 132(1):233-249, August 2014.

[22] H.C. Vu, P. Do, M. Fouladirad, A. Grall. Dynamic opportunistic maintenance planning for multi-component redundant systems with various types of opportunities. *Reliability Engineering and System Safety*, 198, January 2020.

[23] J. Hessburg. *Air Carrier MRO Handbook*. McGraw Hill Education, USA, 1th edition, 2000.

[24] J. Lee, M. Mitici. An integrated assessment of safety and efficiency of aircraft maintenance strategies using agent-based modelling and stochastic petri nets. *Reliability Engineering and System Safety*, 202, October 2020.

[25] J. Sun, F. Wang, S. Ning. Aircraft air conditioning system health state estimation and prediction for predictive maintenance. *Chinese Journal of Aeronautics*, 33(3):947-955, January 2019.

[26] J.J. McCall. Operating characteristics of opportunistic replacement and inspection policies. *Management Science*, 10(1):85-97, 1963.

[27] J.M. van Noortwijk. A survey of the application of gamma processes in maintenance. *Reliability Engineering and System Safety*, 94(1):2-21, March 2017.

[28] J.Y.J. Lam, D. Banjevic. A myopic policy for optimal inspection scheduling for condition based maintenance. *Reliability Engineering and System Safety*, 144(1):1-11, June 2015.

[29] J.Z. Sikorska, M. Hodkiewicz, L. Ma. Prognostic modelling options for remaining useful life estimation by industry. *Mechanical Systems and Signal Processing*, 25(1):1803-1836, December 2010.

[30] K. Bouvard, S. Artus, C. Bérenguer, V. Cocquempot. Condition-based dynamic maintenance operations planning grouping. appliucation to commercial heavy vehicles. *Reliability Engineering and System Safety*, 96(1):601-610, December 2010.

[31] K. Le Son, M. Fouladirad, A. Barros. Remaining useful life time estimation and noisy gamma deterioration process. *Reliability Engineering and System Safety*, 149(1):76-87, January 2016.

[32] K. Schneider, C.R. Cassady. Evaluation and comparison of alternative fleet-level selective maintenance models. *Reliability Engineering and System Safety*, 134(1):178-187, October 2014.

[33] M. Fouladirad, A. Grall, L. Dieulle. On the use of on-line detection for maintenance of gradually deteriorating systems. *Reliability Engineering System Safety*, 93(12):1814-1820, 2008.

[34] M.J. Kallen, J.M. van Noortwijk. Optimal maintenance decision under imperfect inspection. *Reliability Engineering and System Safety*, 90(1):177-185, December 2004.

[35] N.A. Stoica, A. Petrescu, A. Tudor. Modelling the wear processes of the automotive brake pad and disc. *INCAS*, 10(4):169-179, 2018.

[36] P. Do Van, A. Barros, C. Bérenguer, K. Bouvard. Dynamic grouping maintenance with time limited opportunities. *Reliability Engineering and System Safety*, 120(1):51-59, April 2013.

[37] P. Verschuren. *Designing a Research Project*. Eleven International Publishing, The Hague, The Netherlands, 2th edition, 2010.

[38] P.D. Van, C. Bérenguer. Condition-based maintenance with imperfect preventive repairs for a deteriorating production system. *Quality and Reliability Engineering International*, 28(6):624-633, 2012.

[39] Q. Wei, D. Xu. Remaining useful life estimation based on gamma process considered with measurement error. *International Conference on Reliability, Maintainability and Safety (ICRMS)*, 2014.

[40] R. Dekker. Applications of maintenance optimization models: a review and analysis. *Reliability Engineering and System Safety*, 51(1):229-240, 1996.

[41] R.G. Sargent. Verification and validation of simulation models. *Institute of Electrical and Electronics Engineers*, 37(2): 166 - 183, January 2011.

[42] R.J. Ferreira, A.T. de Almeida, C.A. Cavalcante. A multi-criteria decision model to determine inspection intervals of condition monitoring based on delay time analysis. *Reliability Engineering System Safety*, 94(5):905-912, 2009.

[43] S. Alaswad, Y. Xiang. A review on condition-based maintenance optimization models for stochastically deteriorating system. *Reliability Engineering and System Safety*, 157(1):54-63, August 2016.

[44] S. Taghipour, D. Banjevic, A.K.S. Jardine. Periodic inspection optimization model for a complex repairable system. *Reliability Engineering and System Safety*, 95(1):944-952, April 2010.

[45] S. Wu, I.T. Castro. Maintenance policy for a system with a weighted linear combination of degradation processes. *European Journal of Operational Research*, 280(1):124-133, June 2020.

[46] S.J. Sheather. Density estimation. *Institute of Mathematical Statistics*, 19(4):588-597, 2004.

[47] T. Nakagawa. *Maintenance Theory of Reliability*. Springer, London, UK, 1th edition, 2005.

[48] X. Zhou, L. Xi, J. Lee. Reliability-centered predictive maintenance scheduling for a continuously monitored system subject to degradation. *Reliability Engineering and System Safety*, 92(1):530-534, April 2006.

[49] Y. Zhu, E.A. Elsayed, H. Liao, L.Y. Chan. Availability optimization of systems subject to competing risk. *European Journal of Operational Research*, 202(3):781-788, 2010.