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Price dynamics in the oil market: a bond-graph modeling approach

Coen Hutters, * **Nicolaas G. Orie,** ** **Max B. Mendel** *

* *Delft Center for Systems and Control, Delft University of Technology, The Netherlands. (e-mail: C.Hutters-1@tudelft.nl; coenhutters@gmail.com)*
** *Shell Global Solutions International B.V., The Netherlands*

Abstract: Current oil modeling techniques lack a comprehensive approach, as long-term oil prices are qualitatively modeled based on first principles, while short-term price transients are modeled using econometric methods. In this paper we propose a comprehensive bond-graph modeling approach in which price dynamics follow from first principles. The first principles that we use are derived from the recently developed economic-engineering theory in which price dynamics are modeled using Newtonian mechanics and price drivers are identified as forces. We reformulate a qualitative first-principles model developed by the Energy Information Administration (EIA) as a bond graph by modeling six identified price-driving factors as port-elements. The constitutive laws of these port-elements generate the price drivers, which through the interconnection structure of the bond graph yield the price dynamics. We demonstrate the bond-graph model by identifying its parameters and letting it estimate the oil price given historic oil supply data. Compared to a benchmark black-box model, we find that the bond graph has two advantages: (i) it achieves a better performance, and (ii) we know what its parameters and variables represent. The latter advantage allows us to validate the bond-graph model by reconstructing the oil inventory stocks and to manually adjust parameters by expert input.

Keywords: Economic Dynamics, Bond graphs, Grey-box modeling, Oil markets, Energy markets

1. INTRODUCTION

The global energy system is undergoing a mayor shift. Carbon-based energy carriers, such as crude oil and natural gas, are being partly replaced by electrification alternatives such as electricity storage and hydrogen (Keramidas et al. (2020)). The non-equilibrium effects of the shift to a hybrid and more complex energy system call for a new type of economic-energy models (Hafner et al. (2020); Haldane and Turrell (2017)).

Current oil-economic modeling techniques can be subdivided in three categories (Huntington et al. (2013)): (i) structural models, (ii) computational models, and (iii) reduced-form econometric models. Structural models (i) are first-principle models, consisting of networks of decision-making agents. Although they have promising capability in terms of scalability and efficiency (see e.g. (Chen et al. (2022))), structural models lack the ability to yield reliable quantitative results. Computational models (ii) are first-principle equilibrium models of economic systems. These models yield reliable quantitative results, but lack the ability to model non-equilibrium scenarios. Reduced form econometric models (iii) are regression models based on time-series data. Such models can accurately model short-term variability, but are not able to make predictions outside the variable space on which these models are trained. Furthermore, regression models have limited interpretability with regards to the model parameters and states, which is a crucial shortcomings given the shifting energy system.

In this paper, we use economic-engineering theory (Mendel (2023)) to derive a model of the global oil market in which price is a dynamical state variable. Specifically, we use a bond-graph modeling approach to derive an interpretable bond-graph model of the price (and inventory dynamics) in the oil market. We base this model on a structural model developed by the Energy Information Administration (EIA)(EIA (2022); Lang and Auer (2020)).

The EIA model identifies six price-driving factors for oil prices, being supply from (i) non-OPEC and (ii) OPEC countries, demand from the (iii)Organisation for Economic Co-operation and Development (OECD) countries and (iv) non-OECD countries, (v) oil inventories, and (vi) the financial markets. The EIA model only considers qualitative relations between these price-driving factors and the oil price but does not provide a quantitative model for oil price dynamics.

To derive a quantitative model for oil price dynamics, we model the price-driving factors as port elements in a bond graph. Bond-graph modeling is a generalized technique to model dynamical system as graphs (see e.g. Karnopp et al. (2012)). A bond-graph model consists of port elements that are linked by power bonds and exchange power through pairs of *flow* and *effort* variables, e.g. pairs of velocities and forces, respectively. Each port element has a constitutive law that relates its associated flow and effort variables.

In the literature, bond-graph modeling has been applied to economic systems in which economic port elements exchange cash flows in the form of flows of commodities (flow) and prices (effort) (Brewer and Craig (1982); Wong (2001); Machado and Mata (2015)). However, modeling prices as effort signals does not result in price dynamics.

In the economic-engineering theory (Mendel (2023)), we model the oil-economic system as a bond-graph model in which the flow and effort variables are pairs of a *flow of oil* and a

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price driver, respectively. Price dynamics then follow from the balancing of price drivers that result from the price-driving factors. This is analogous to how dynamics follow from the balancing of forces in a mechanical system.

The main contribution of this paper is to derive a bond-graph model of the oil economy that on the one hand has the interpretability of a structural model and on the other hand has the computational performance of a regression model. In the remainder of this paper, we devlop this model as follows. In Section 2 we lay out the bond-graph fundamentals for oil-economic systems with price dynamics based on the economic-engineering approach. These fundamentals include the model variables (inventory stocks, flows of oil, oil prices, and price drivers), the price-driving factors (supply, demand, inventories, and financial markets), and the interconnection structures. In Section 3 we use these fundamentals to derive a bond-graph model that corresponds to the structural EIA-model. The bond-graph model includes the dynamics of crude-oil prices and of crude-oil inventory stocks. In Section 4 we identify the parameters of the bond-graph model and reconstruct the oil price in a simulation study on the Venezuelan oil strike. Furthermore, we show that the interpretability of the parameters and state variables of the bond-graph model allows for the estimation of the inventory stocks and for manual parameter adjustments by expert input.

2. BOND-GRAPH THEORY FOR OIL MARKETS

2.1 Flows and stocks of oil

Similar to e.g. (Brewer and Craig (1982); Macchelli (2013); Machado and Mata (2015); Wong (2001)) we define the flow variable as follows

$$f := \text{flow of oil (in bbl/dy)} \tag{1}$$

We let a positive-valued flow of oil run from producer to consumer. Integrating a flow of oil over time yields a stock of oil in barrels (bbl):

$$q(t_1) = q(t_0) + \int_{t_0}^{t_1} f\,\mathrm{d}t, \tag{2}$$

The stock is a state variable, analogous to generalized position. A positive-valued stock represents a physical amount of oil, whereas a acnegative stock represents a shortage.

2.2 Economic effort as price driver

In contrast to (Brewer and Craig (1982); Macchelli (2013); Machado and Mata (2015); Wong (2001)) we define the effort variable as (Mendel (2023))

$$e := \text{price driver (in \$/(bbl·dy))}. \tag{3}$$

We let a positive-valued price driver causes a price increase and vice versa. Integrating a price-driver over time yields the oil price in dollars per barrel ($\$/bbl$

$$p(t_1) = p(t_0) + \int_{t_0}^{t_1} e\,\mathrm{d}t \tag{4}$$

Price is analogous to generalized momentum, a price driver is analogous to a generalized force, and (4) is analogous to Newton's second law (Mendel (2023)).

2.3 Inventories as C-elements

Oil stocks build up in inventories in accordance with (2). Inventories act as price-driving factors; an abundance of inventory

$$\mathbf{C} \longrightarrow\!|$$

stocks has a decreasing effect on the price, whereas an inventory shortage has an increasing price effect (see e.g. Ye et al. (2005)). Thus, we express the price driver resulting from inventory as

$$e_C = \phi_C(q) \tag{5}$$

This expression represents the constitutive relation of a C-element (Karnopp et al. (2012)). In the linear case, the price driver is given by $e_C = C^{-1}q$, where C is the inventory elasticity.

2.4 Consumers and Producers as I-elements

$$\mathbf{I} \;|\!\longrightarrow$$

A price driver changes the reservation price that a consumer is willing to pay and a producer is willing to receive for a barrel of oil. The theory of supply and demand (see e.g. Mas-Colell et al. (1995)), states that the flow of oil bought by consumers and sold by producers depends on the unit price of oil. We express such a relation between price and flow of oil as

$$f_I = \phi_I(p) \tag{6}$$

This expression motivates us to model consumers and producers as I-elements. In the linear case, the constitutive relation the I-element defines a linear demand or supply curve $f_I = \varepsilon p$, where ε is the price elasticity.

2.5 Brokers as R-elements

$$\mathbf{R} \longrightarrow\!|$$

Brokers on the financial markets do not store stocks in inventory and do not have a reservation price. Instead, brokers are a price-driving factor that have an increasing effect on price if there is an excess demand for oil flow and a decreasing effect on price if there is an excess supply of oil flow. This is captured by

$$e_R = \phi(f) \tag{7}$$

Therefore, we model brokers as R-elements that relate a flow of (excess) oil to a price driver. In the linear case $e = \delta f$, the broker elasticity δ corresponds to speed-of-adjustment parameter used in Walrasian economics.

2.6 Time-dependent demand and supply as source elements

$$\mathbf{S}_f \;|\!\longrightarrow$$

Not all demand and supply depends on the price. Demand and supply may depend on external conditions, be inelastic, or be scheduled according to long-term strategies rather than short-term prices. We model demand and supply that does not depend on price as flow sources, where the oil flow explicitly depends on time

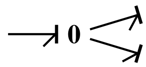
$$f_S = f(t) \tag{8}$$

2.7 Bonds and Causality

Port elements are interconnected by bonds. A bond consists of a pair of a flow of oil and a price driver (effort) and is depicted by a half arrow (Karnopp et al. (2012)), as shown in Fig. 1. The causal stroke at one end of the bond indicates whether an element determines the price driver or the flow of oil. The element that is away from the causal stroke determines the price driver.

Fig. 1. Causal strokes on a bond. Left: A determines the price driver, B determines the flow. Right: vice versa.

2.8 Oil balancing at 0-junctions



The 0-junction models the balancing of oil between connected price-driving factors. The net flow through a 0-junction must always clear:

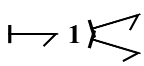
$$f_1 + f_2 + \cdots + f_n = 0 \tag{9}$$

For example, any quantity of unsold oil must flow into inventory. The 0-junction structure prescribes that all price-driving elements at the market are affected by a common price driver:

$$e_1 = e_2 = \cdots = e_n \tag{10}$$

Each connected element thus experiences the same price driver.

2.9 Price formation at 1-junctions



The 1-junction represents the accumulation and distribution of price drivers. For a 1-junction, the net price driver must clear:

$$e_1 + e_2 + \cdots + e_n = 0 \tag{11}$$

For example, any unbalanced price driver must result in a change in the market price. The structure of the 1-junction prescribes that the connected price-driving elements have a common flow

$$f_1 = f_2 = \cdots = f_n \tag{12}$$

This represents that the connected price-driving factors react to the same supplied, demanded, or excess flow of oil.

In a bond-graph model, the 0- and 1-junctions simultaneously account for the balancing of oil flows (kinematics) and price drivers (dynamics).

3. BOND GRAPH OF THE EIA OIL MODEL WITH PRICE AND STOCK DYNAMICS

In this section we summarize the structural EIA model and introduce a corresponding bond-graph model that explicitly models price drivers as effort signals.

3.1 EIA oil model

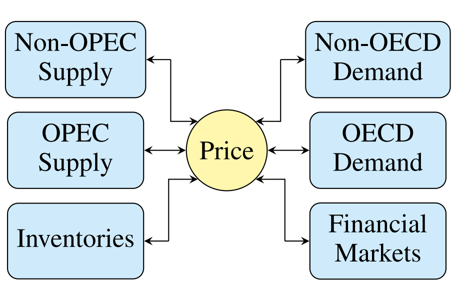


Fig. 2. EIA model of what drives oil price (EIA (2022)).

The EIA model, illustrated in Fig. 2, identifies six essential price-driving factors of crude oil prices, being (i) supply from OPEC countries, (ii) supply from non-OPEC countries, (iii) demand from OECD countries, (iv) demand from non-OECD countries, (v) inventories, and (vi) financial markets (EIA (2022); Lang and Auer (2020)). We briefly summarize the behavior of each price driver.

Supply (i-ii) is split into competitive non-OPEC supply and supply from the OPEC cartel. Non-OPEC supply reacts to the market price and accounts for roughly 60% of global supply. OPEC on the other hand actively manages its supply by setting production targets. These targets have an influence on the price through the mechanism of supply and demand.

Similarly, demand (iii-iv) is split into developed OECD countries and emerging non-OECD countries. The demand from (developed) OECD countries reacts to the price; OECD demand decreases when the oil price increases. Demand from emerging non-OECD countries is less dependent on the price as these countries strongly rely on the use of oil for manufacturing, transportation, feedstock, and power generation. Furthermore, many non-OECD countries control or subsidize domestic oil prices, making them less susceptible to global price changes

Inventories (v) act as a buffer between supply and demand. Apart from physically storing oil, inventories also affect oil prices. Low inventory levels typically result in rising prices, whereas high inventory levels result in decreasing prices.

Financial Markets (vi) influence the oil price by trading oil and related financial derivatives to efficiently match supply and demand. In reality, the relation between oil prices and activities on financial markets is complex and changing over time, see (Lang and Auer (2020)) and references therein.

3.2 Bond-graph model

Using the prerequisites presented in Section 2 and price-drivig factors identified in the EIA model, we derive the bond graph in Fig. 3.

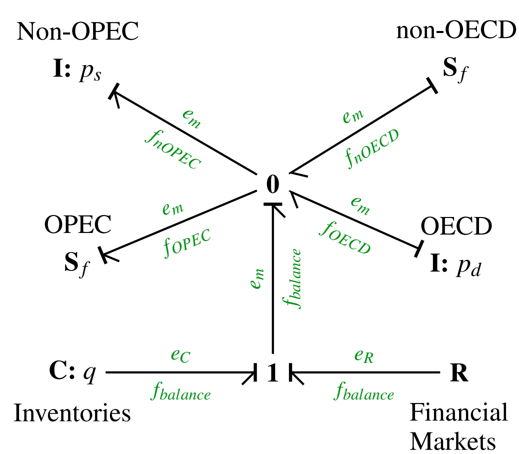


Fig. 3. Bond-graph model based on the EIA model in Fig. 2. Each price-driving factor is modeled as a port element. The flow variables represent the flow of oil. The effort variables represent price drivers. The corresponding state-space model (for $p_s = p_d = \lambda$) is given in (24).

3.3 Non-OPEC supply and OECD demand as I-elements

We model non-OPEC producers and OECD consumers as linear I-elements connected to a 0-junction that represents the oil market. The reservation prices of the non-OPEC producers, $p_s \in \mathbb{R}$ (\$/bbl), and the OECD consumers, $p_d \in \mathbb{R}$, vary over time due to the net market price driver $e_m \in \mathbb{R}$ (\$/bbl·dy):

$$\frac{\mathrm{d}}{\mathrm{d}t} p_s = \frac{\mathrm{d}}{\mathrm{d}t} p_d = e_m \tag{13}$$

If we assume the integration constants to be zero, integrating the above expression yields that both non-OPEC producers and OECD consumers are price takers at the *market price*, i.e. $p_s = p_d = \lambda$, where

$$\lambda = \int e_m \mathrm{d}t \tag{14}$$

This is an important result for two reasons. First, because competitive suppliers and demanders are assumed to be price takers. Second, because for model identification and simulation we only have availability to the market price data and not to the reservation prices of suppliers and demanders.

The constitutive relations of the I-elements relate the market price to the non-OPEC supply $f_{nOPEC} \in \mathbb{R}$ (bbl/dy) and the OECD demand $f_{OECD} \in \mathbb{R}$

$$f_{nOPEC}(\lambda) = \varepsilon_s \lambda, \tag{15}$$

$$f_{OECD}(\lambda) = \varepsilon_d \lambda, \tag{16}$$

where, $\varepsilon_s \in \mathbb{R}$ is the elasticity of supply and $\varepsilon_d \in \mathbb{R}$ the elasticity of demand.

3.4 OPEC supply and non-OECD demand as flow sources

As the supply from OPEC countries and the consumption from non-OECD demand mainly depends on policies rather than on price, we model these price-driving factors as flow sources that explicitly depend on time

$$f_{OPEC} = f_{OPEC}(t) \tag{17}$$

$$f_{nOECD} = f_{nOECD}(t) \tag{18}$$

All four supplying and demanding price-driving factors have the price-driver e_m as input and a flow of oil as output. They thus do not directly drive the oil price. This is consistent with the economic literature in which supply and demand are considered as long-term price-driving factors (Lang and Auer (2020)).

3.5 Oil flow balancing at 0-junction

Supply and demand do however drive the price indirectly through their interconnection to the rest of the market. The balance between supply and demand is computed at the 0-junction as

$$f_{balance} = f_{nOPEC} + f_{OPEC} - f_{OECD} - f_{nOECD} \tag{19}$$

The balance results in a market price driver e_m, which is formed at the 1-junction.

3.6 Inventories as C-element

The balance accumulates in the inventory as

$$q = \int f_{balance} \mathrm{d}t \tag{20}$$

The price driver resulting from the inventory stock is given by the constitutive relation of the C-element

$$e_C = -C^{-1} q, \tag{21}$$

where C is the elasticity of inventory.

3.7 Financial markets as R-element

Simultaneously, the financial markets broker between supply and demand by offering oil at a premium when oil supply is scarce ($f_{balance} < 0$) and at a discount when oil supply is abundant ($f_{balance} > 0$). The premiums and discounts follow from the price-driver

$$e_R = -\delta f_{balance}, \tag{22}$$

where δ is the broker elasticity.

3.8 Price formation at 1-junction

The market price driver e_m is the sum of the price drivers from the inventories and financial markets formed at the 1-junction as

$$e_m = e_C + e_R \tag{23}$$

Considering (14), the 1-junction represents the market-price formation process.

Evidently from (23), the inventories and financial markets are short-term price-driving factors. We have seen that e_C and e_R are both functions of the balance between supply and demand. Hence supply and demand are long-term price-driving factors.

3.9 Input-state-output dynamics of the bond graph

Finally, we capture the dynamics from the input $u = (f_{OPEC} - f_{nOECD})$ to the output $y = \lambda$ of the bond-graph model in Fig. 3 in the linear state-space

$$\begin{aligned} \begin{pmatrix} \dot{\lambda} \\ \dot{q} \end{pmatrix} &= \begin{pmatrix} -\delta(\varepsilon_s - \varepsilon_d) & -1/C \\ (\varepsilon_s - \varepsilon_d) & 0 \end{pmatrix} \begin{pmatrix} \lambda \\ q \end{pmatrix} + \begin{pmatrix} -\delta \\ 1 \end{pmatrix} u \\ y &= \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} \lambda \\ q \end{pmatrix}, \end{aligned} \tag{24}$$

4. SIMULATION STUDY: VENEZUELAN OIL STRIKE

In thist section we use the state-space representation (24) to identify the model parameters of the bond-graph model in Fig. 3, using historical data of u and λ. Subsequently we generate a reconstruction of the price, $\hat{\lambda}$ using input data u from an identification and validation set. We compare the performance of the bond-graph model to a benchmark black-box model identified on the same historical data. In addition, we validate the bond-graph model by reconstructing the inventory, $\hat{q}$, given the identification input data, and compare it to historical inventory data. The inventory data reconstruction cannot be compared to the black-box model, as for that model we do not know what each state represents.

4.1 Data description

The data we use for identification, simulation, and validation is publicly available data on (i) monthly OPEC oil supply and (ii) quarterly non-OECD demand for the input $u = f_{OPEC} - f_{nOECD}$ (OPEC (2022)), (iii) monthly data West Texas Intermediate (WTI) crude oil spot prices for the output $y = \lambda$ (EIA (2022)), and (iv) weekly US oil inventories for validation of the state q .

For identification, we select the period from July 2002 until May 2003, during which a supply shock occurred due to the Venezuelan oil strike, causing global oil supply to drop by 3.4% (Mendoza and Vera (2010)). This shock is more isolated from factors outside the model compared to for example shock resulting from the COVID-19 crisis in which many macroeconomic factors that are outside the model scope were affected.

4.2 Data pre-processing

Before we can use the data for the identification of the bond-graph model, we need to pre-process them in two steps. First, we detrend all data as the bond-graph model does not account for long-term effects such as the increase of oil rigs, technological advancements due to rising oil prices, and economic growth. The trend is identified using a third-order polynomial fit. Second, we smooth and interpolate the data with local regression using weighted linear least squares and a first degree polynomial model. This step serves two purposes; it ensures that each data set is of equal size, and second it increases the amount of data points for identification. The data pre-processing is illustrated for the price data in Fig. 4.

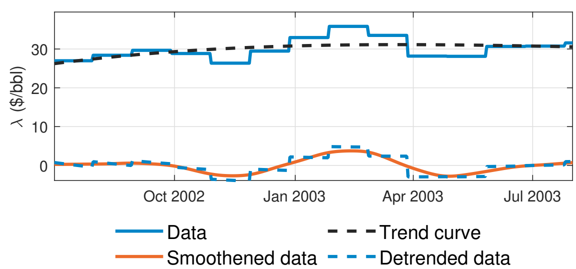


Fig. 4. Illustration of data pre-processing method as described in Section 4.2 for WTI price data. The data is first detrended and subsequently smoothed. This pre-processing method is applied to all four data sets.

4.3 Model identification

We identify the bond-graph model parameters using the linear grey-box model identification `greyest` in MATLAB, utilizing a Gauss-Newton line search method. The input data (u in 24)) consists of the difference between the pre-processed OPEC oil supply and non-OECD demand data. The pre-processed WTI spot price is used as output data. The pre-processed input and output data are shown in Fig. 6.

The values of the identified parameters are listed in Table 1. The variance accounted for (VAF) is 83%. The elasticity of demand and supply are of the same order of magnitude as the price elasticities of crude oil identified in the literature: -0.1 and 0.1, respectively (Caldara et al. (2016)). The value of the inventory elasticity implies that the price-driving effect of inventories is small with respect to that of financial markets.

Table 1. Identified parameter values

Parameter	Value
ε_d elasticity of demand	-0.19
ε_s elasticity of supply	0.19
C inventory elasticity	$8.2{\cdot}10^8$
δ speed-of-adjustment	0.27

4.4 Simulation results and validation

We compare the computational performance of the identified bond-graph model against a third-order black-box model as

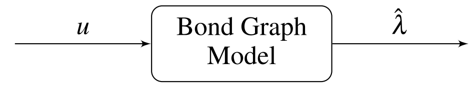


Fig. 5. Simulation setup: u is historical data and $\hat{\lambda}$ is the market price estimated by the identified bond-graph model in Fig. 3 and (24)

a benchmark. The black-box model is identified on the same input and output data using the `ssest` function in MATLAB, utilizing the MOESP identification method. The model order is motivated by a singular-value decomposition. Both models are simulated using the `lsim` function, using the same input sequence u.

The simulation results for both models are shown in Fig. 6. The results show that the bond-graph model (VAF=83%) has a better performance than the black-box model (VAF=55%). This implies that the imposed structure of the bond-graph model results in a more efficient identification process using the scarce identification data.

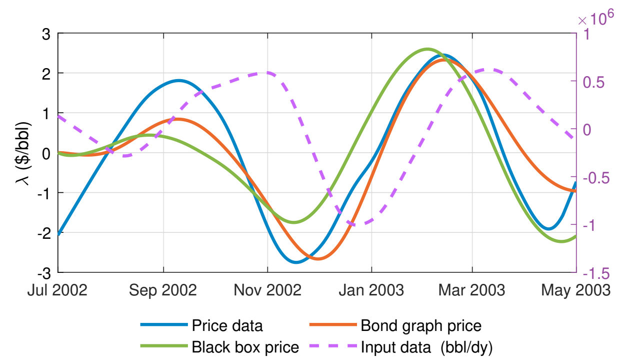


Fig. 6. Estimated price $\hat{\lambda}$ by the bond-graph model and black-box model given the input data u vs. price data λ (identification data set). The bond graph model (VAF=83%) performs better than the black-box model (VAF=55%).

As we know what the parameters and state-variables of the bond-graph model represent, we can validate the identified model by comparing the simulated inventory stocks, state q in (24), to pre-processed historic inventory data, which has not been used for identification. Due to the lack of interpretability, this is not possible for the black-box model. The results of the validation are shown in Fig. 7. The VAF for the validation is now 39%. However, the bond-graph model is able to predict when inventories increases and decreases.

Finally, we validate the bond-graph model using validation data between July 2015 and May 2016, during another OPEC supply shock (Lang and Auer (2020)). The results are shown in Fig. 8. Although the results of the bond-graph model are considerably worse than for the identification set with a VAF of only 26%, it still outperforms the black-box model, which scores a VAF of 18%. Again, the interpretability allows us to update the price elasticities, e.g. by expert input. Adjusting the price-elasticities to $\varepsilon_s = 0.3$ and $\varepsilon_d = -0.3$, i.e. we assume that oil has become more price elastic due to alternative energy sources, improves the performance to VAF=51%.

5. CONCLUSIONS

By utilizing the economic-engineering approach we have derived a comprehensive bond-graph model that quantitatively ex-

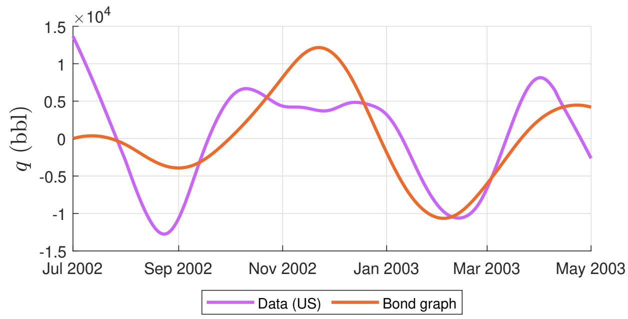


Fig. 7. Estimated inventory $\hat{q}$ given input data u (from identification set) vs. inventory data q (validation set). The simulated data has been scaled by a factor $5 \cdot 10^{-4}$ to match the historical data's amplitude. Although the scaling factor is large, scaling is appropriate as the bond-graph model represents the global oil market and the data only involves US inventories. The obtained VAF is 39%.

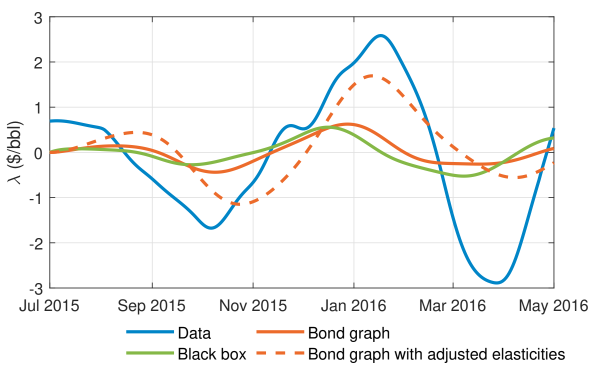


Fig. 8. Estimated price $\hat{\lambda}$ by the bond-graph model and black-box model given the input data u vs. price data λ (validation data set). In the validation set, the bond-graph model reconstructs the price with a VAF of only 26%, still outperforming the benchmark black-box model (VAF=18%). However, the economic interpretability of the bond-graph model allows us to selectively update the price elasticities, resulting in a better performance (VAF=51%)

plains oil price dynamics. The bond-graph model is consistent with the structural model identified by EIA. However, whereas the EIA model only provides qualitative relations between price drivers and the oil price, our bond-graph model provides quantitative price dynamics. Furthermore, the bond-graph model provides interpretability of the involved states and parameters, allowing for manual adjustments of the model states in accordance with economic insights. This is an advantage over commonly used econometric models. Although the bond-graph model is a simple linear second-order system we found that it was able to reconstruct the oil price and inventory dynamics given the used data set. Of course, to derive a model useful for industry purposes more research is needed. In a broader scope, the contributions made in this paper motivates us to do further research on, among others, the application to other (complex) economic systems, non-linear constitutive laws, and the application of (optimal) control theory.

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