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Simulating Interaction Between Level Ice and Conical Structures with a 2D Lattice Model

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ABSTRACT

Structures with a downward sloping waterline shape in Arctic or sub-Arctic regions experience reduced loading from ice on the structure and its foundation compared to vertical structures as the slope causes the ice to fail in bending. For the design of these structures, a numerical model is desired that can predict the loading from the ice on the structure by simulating the physical processes that occur when the ice fails.

A numerical 2D lattice model has been developed to simulate level ice behavior. The model is composed of masses and interconnecting springs, taking into account deformation in tension, compression, bending, shear and torsion in the ice sheet. Deformation and failure criteria in the model are based on first principles, enabling a physically sound simulation of breaking processes. In addition, the discrete lattice model avoids stress singularities in fracture modeling since the model only describes displacements and forces in the connections and no stresses are present.

In this paper interaction between ice and a downward sloping conical structure is simulated with the lattice model and compared with data from scale model tests. From the model test results it was observed that level ice and larger ice floes fail in sequential bending during interaction with a structure, giving a repetitive load pattern. Smaller ice floes split and break in bending into smaller parts and a sequential breaking pattern does not develop. The lattice model is capable of simulating failure in bending, splitting and combinations of these. In contrast, predictions of the model presented herein suffer from limitations of the contact model between the ice and the structure, the lack of a clearing mechanism and the use of a regular lattice mesh.

KEY WORDS: Ice; Failure; Lattice model.

INTRODUCTION

To build structures in Arctic and sub-Arctic regions, assessment and minimization of loading from ice on structures is required. Structures with a downward sloping waterline shape experience reduced loading from ice on the structure and its foundation compared to vertical structures as the slope causes the ice to fail in bending. The manner in which the ice floes fail when encountering the structure depends on the size and material parameters of the floes; most observed failure modes are in bending, splitting or a combination of these. By means of numerical assessment, a multitude of structures and loading conditions can be analyzed and structural shapes can be optimized for ice load reduction. Ice management processes can be optimized as well in case the effects of managed floe sizes on the loading of a structure are known. A numerical model is used in this paper that can predict the loading from the ice on the structure by simulating the physical failure processes and effects of floe parameters.

Several research institutes and companies have developed numerical assessment tools to study ice-sloping-structure interaction, in the form of 1D, 2D or 3D models. Aksnes (2010) and Wille, Kuiper, & Metrikine (2011) simulated bending failure of ice in interaction with a downward sloping conical structure using a beam model. Beam models are limited in the prediction of loading frequencies and breaking length of the ice in case of circular or ship-shaped structures as 2D effects have significant influence on the breaking pattern for these structures. To simulate real-time ice-structure interaction Lubbad and Løaset (2011) adopted a model of wedge-shaped beams failing adjacently based on theory of a semi-infinite plate resting on an elastic foundation, for which a solution was presented by Nevel (1992). Ice failure was modeled in a similar manner to simulate a floating drillship in level ice with SIBIS software by Metrikin et al. (2015). The discrete element tool described by Lau et al. (2008) has a module that allows for pre-defined crack patterns in simulations for ships in level ice. Bonnemaire et al. (2014) used a semi-empirical model to simulate the response of a moored floating structure in ice. The discrete element method was used to analyze the breaking process of sea ice by Ji et al. (2015) by connecting particles with parallel-bonds with associated moment and force and breaking the bonds when internal stresses exceeded a certain threshold.

In this paper a numerical lattice model developed by the authors is employed to simulate interaction between ice and a conically shaped structure. The lattice model is composed of masses and springs of various types, taking into account deformation in tension, compression, bending, shear and torsion in the ice sheet. Lattice models have been used in the past to simulate ice-structure interaction. Sayed (1997) used a lattice model for fracture of sea-ice, using elastic and visco-elastic bonds in normal direction between the masses in the model. Dorival, Metrikine, & Simone (2008) developed a lattice model to simulate ice crushing against a structure. Van den Berg (2016) presented a 3-dimensional lattice model to simulate ice-structure interaction. The 2-dimensional model of an ice plate that is used in this paper applies deformation and failure criteria that are based on first principles, enabling physically sound simulation of failure processes that occur in ice-structure interaction. Another advantage of the lattice model is that stress singularities in fracture modeling are avoided as the model only describes displacements and forces in the connections instead of strains and stresses inherently present in any continuum model.

The lattice model is validated in this paper against model test results for a rigid downward breaking conical structure interacting with ice floes of various sizes. These tests were performed as part of a Joint Industry Project and described by Bruun et al. (2011). Focus in this validation study is on combined bending and splitting behavior of the ice, which depends

on the floe size. It is shown that the physical failure processes of combined bending and splitting can be simulated with the lattice model.

Although fracture in bending and splitting can be predicted with the lattice model, there are limitations to the fracture patterns that can be simulated as the square mesh that is applied in the current study causes the fractures in the ice plate to follow mesh-related patterns. This is partly resolved by application of local variations to spring stiffness and nodal locations in the model. Further improvement could be achieved in the future by further study of local variations, improvement of the contact model, adding ice clearance and local mesh refinement.

The paper is structured as follows. In the next section the numerical lattice model is described. The third section presents results of experimental measurements that are used for validation of the model predictions. This is followed by a comparison is made between experimental results and simulations in section 4. In the last section conclusions are listed.

DESCRIPTION OF THE LATTICE MODEL

A lattice model has been developed to simulate ice-structure interaction. This 2-dimensional model is composed of masses and springs, taking into account deformation in tension, compression, bending, shear and torsion in the ice sheet. The lattice, which has a square mesh, and its connections are depicted in Figure 1. The model can be applied in both time- and frequency-domain simulations. Due to strongly non-linear fracture modeling, in this paper simulations are carried out in the time domain.

In axial directions, parallel with the x - and y - axes, the lattice model has more types of connectors than in diagonal directions. The connectors in axial and diagonal directions have different stiffness and their stiffness is assigned such that the global bending deformations of the plate match the Mindlin-Reissner equations in out-of-plane and those of the classical elastic continuum in in-plane direction in the long-wave approximation. The method used to derive and verify stiffness of the springs in the lattice model for in-plane-deformation are described in Suiker et al. (2001). Linear viscous elements are added to all connectors, to represent material damping.

The Mindlin-Reissner equations, to which the out-of-plane deformations of the lattice model are matched in the long-wave limit, describe the dynamic out-of-plane behavior of shear-deformable plates. The lattice model deformations are based on first principles and not limited in width-to-thickness ratio of the ice plate deformations that are simulated with it.

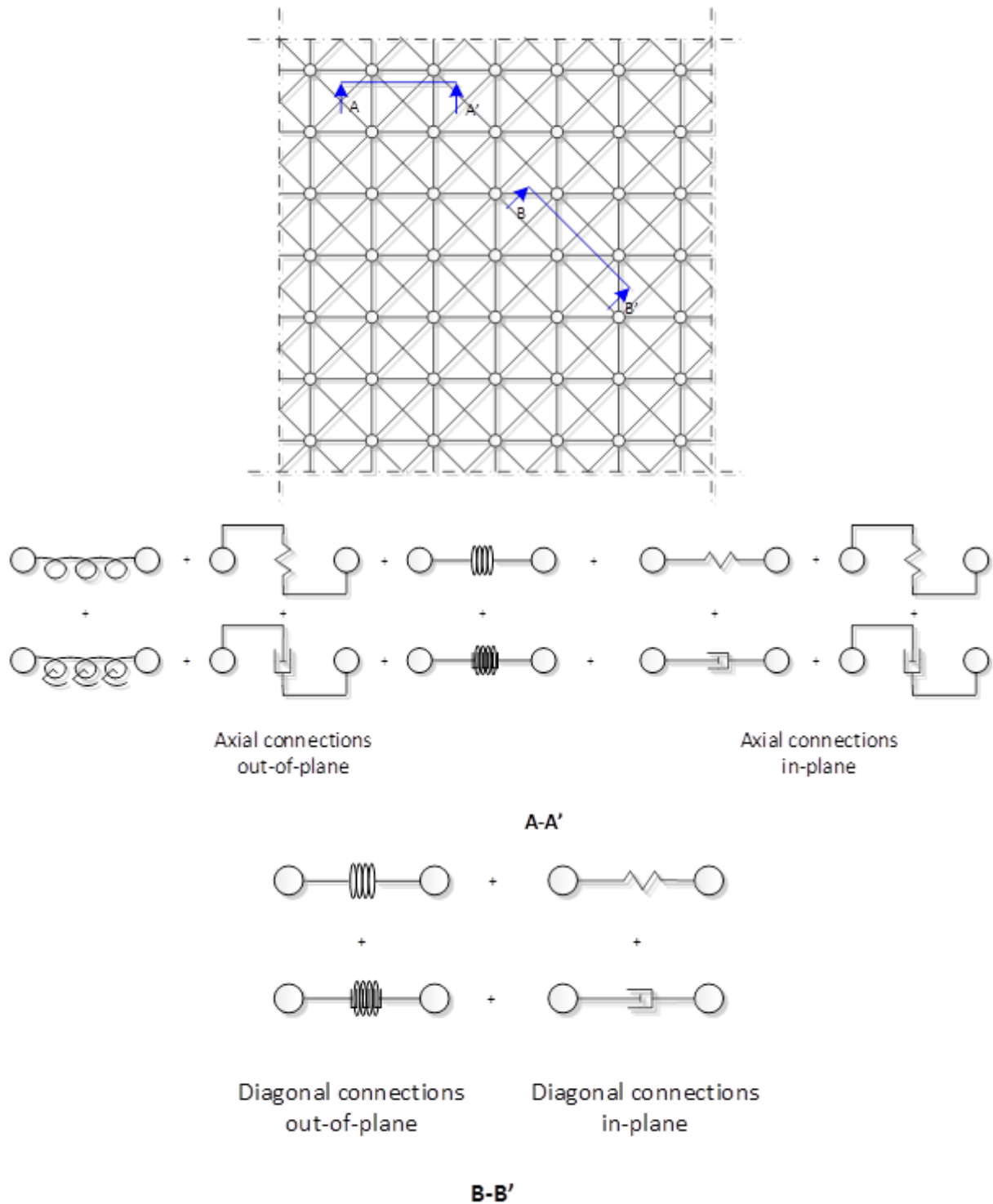


Figure 1. The lattice model and its connections working in-and out-of-plane direction. For section A-A' from left to right out-of-plane-bending, out-of-plane-shear, torsion, axial deformation and in-plane shear. For section B-B' from left to right torsion and axial deformation.

A Kelvin foundation is applied to the model as a simplistic manner to account for stiffness and damping associated with the fluid the ice plate is assumed to float on. Friction working

parallel to the ice sheet is included by applying form friction and skin friction drag loads that are related to the squared velocity of the ice plate.

An algorithm for connection failure is included in the lattice model to study fracture. The criterion for failure of the connections is based on relative displacement and rotation of the springs. When the combined bending and tensile deformation in a connection reaches a critical value, the stiffness of all springs forming that connection are set to zero. Critical deformation is linked to the flexural strength of the ice via linear stress-strain relation accounting for the varying distance between the masses for different lattice cell sizes.

The advantage of the lattice model is that stress singularities in fracture modeling are avoided as the model only describes displacements and forces. Other benefits are that the shape of the model can easily be varied by cutting-out the required shape and size along the mesh. Natural variations of a material, which are of significant relevance in ice-mechanics, can be easily implemented by varying the spring stiffnesses in the different connections or displacing the nodes in the mesh of the lattice model.

To simulate contact load between the lattice and a conical structure, a simplified contact algorithm is applied. At each time step of the simulation the amount of overlap between the ice and the structure is determined, based on the structural shape, its indentation and the deflection and tip rotation of the ice. From this overlap the contact area, a_{crush} in Figure 2, is determined. Please note that Figure 2 is a side view of the overlap and that contact forces are calculated in three dimensions. The force from the structure on the ice, which is a linear function of the contact area and the crushing strength of the ice, and works perpendicular to the structure, is calculated and applied to the ice. A friction force is applied in opposite direction to the motion of the ice along the structure and is related to the contact force via a friction coefficient.

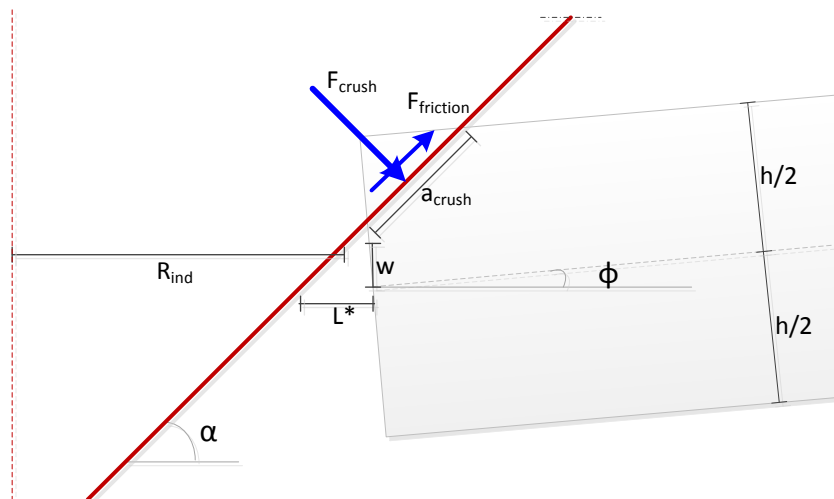


Figure 2. Side view indicating important parameters in determining the contact load: R_{ind} is the radius of the cone, α is the slope of the cone, L^* is the distance between the center of the ice edge and the cone, w is the deflection of the ice, ϕ is the tip rotation of the ice, h is ice thickness, a_{crush} is the contact length between the ice and the structure, F_{crush} is the crushing load, perpendicular to the structure and F_{friction} is the friction load, directed opposite to the motion of the ice along the structure.

OBSERVATIONS FROM MODEL TESTING

Three tests that were executed as part of the JIP model testing campaign, of which an overview is presented by Bruun et al. (2011), are studied in this paper. All of the tests were executed using the same fixed structure, depicted in Figure 3, which was moved through ice with a thickness of 3m at a velocity of 0.1 m/s. All reported values in this paper are in full scale.

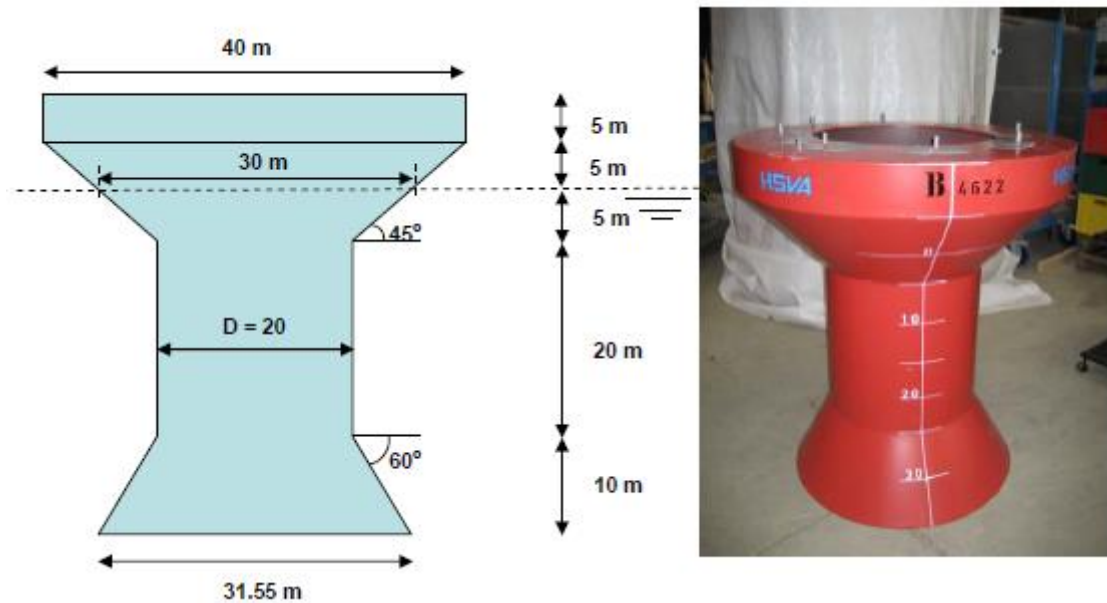


Figure 3. Dimensions of the structure that was used during the ice basin tests. The waterline diameter was 30m. (HSVA 2010a)



Figure 4. Overview of the floe distribution in the ice basin a) large floes and b) smaller floes (HSVA 2010a)

In the three tests the structure encountered ice plates of various sizes: one test was executed in level ice, one test in large ice floes and one test in smaller ice floes. Figure 4 presents an overview of the ice floe distribution for the two tests in broken ice.

Figure 5 shows the structure during the test in level ice. Only bending failure can be observed when looking at images from the tests, but the failure pattern is not very clear. For tests in broken ice, the failure patterns are clearer as the ice floes drift apart and reveal fractures. Figure 6, Figure 7 and Figure 8 show images from test video's for interaction between the structure and finite size ice floes. Events were selected where the structure encounters the ice floe edge in near the centre. Failure occurs as a combination of bending and splitting.

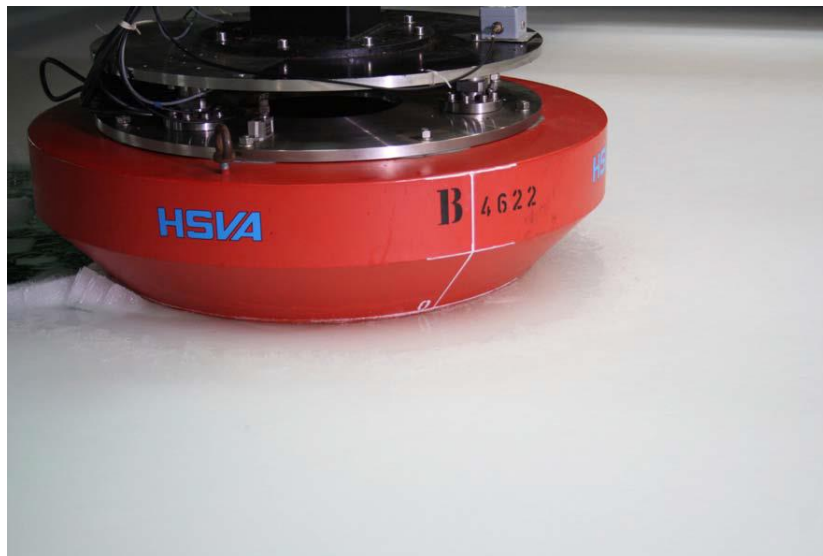


Figure 5. Structure in level ice during model test (HSVA 2010b)

During interaction with a large ice floe of 165x165m, first a piece of ice fails in bending, showing a circumferential breaking pattern. Shortly after the structure encounters the new broken edge of the ice, the ice plate splits in two. Then the ice fails close to the structure in combined radial and circumferential fracture as can be seen on the right hand side of Figure 6. During the remaining encounters between the structure and the ice floe, splitting and circumferential and radial fracture alternate.

During interaction with smaller ice floes of 69x34m and 55x55m in both cases first a piece of ice breaks off circumferentially. The rectangular floe immediately splits apart during this first interaction. The square floe splits apart at the second encounter. The remaining ice pieces experience confinement from the surrounding pieces of ice, loading the structure during the clearing process. Since all ice floes in the basin are of different size and the structure approaches each ice floe at a different angle, breaking patterns of each ice floe are different. In case an ice floe is encountered at the edge, typically the edge breaks off before the ice floe splits during further encounter with the structure.



Figure 6. Image from model test video. Encounter between the structure and a large ice floe of 165x165m (HSVA 2010b)

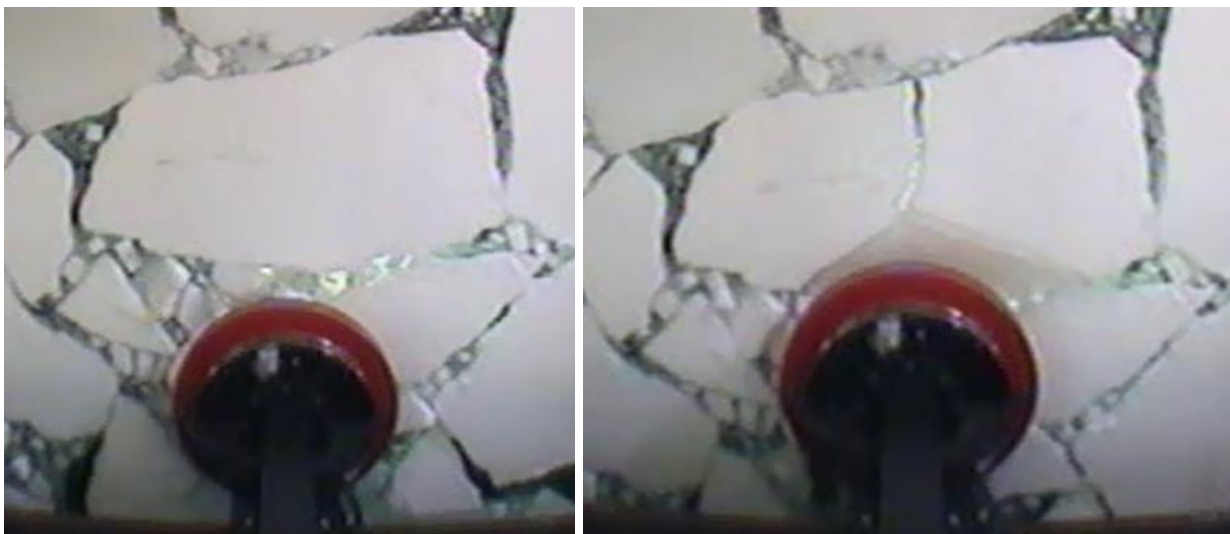


Figure 7. Image from model test video. Encounter between the structure and a small rectangular ice floe of 69x34m (HSVA 2010b)

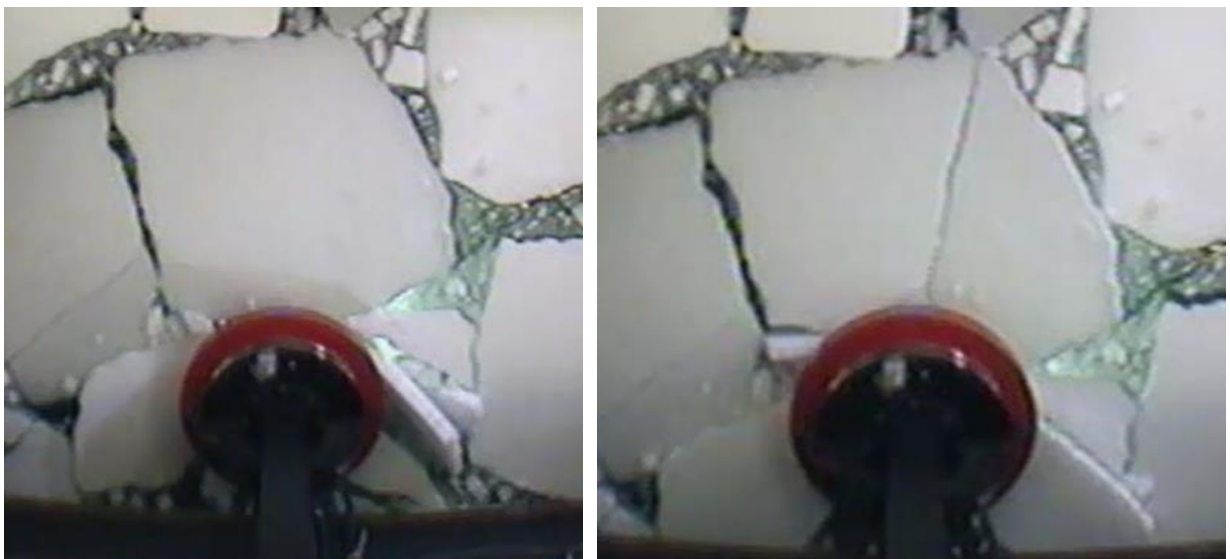


Figure 8. Image from model test video. Encounter between the structure and a small square ice floe of 55x55m (HSVA 2010b)

The measured force on the structure as function of time is presented in Figure 9 for vertical Z-direction and in Figure 10 for horizontal X-direction, in-line with the motion of the structure. For the tests in level ice (blue curve) there is a clear sequential pattern of ice loading building up until the ice fails and reducing again as the ice clears around the structure, both for loading in Z-direction and for loading in X-direction. The difference in height between the peaks indicates natural variations in the failure process. For the large ice floes, these sequential peaks are also observed, but not continuously during the tests. The peaks coincide with events of bending failure and associated clearance during encounters with larger ice floes. Circumferential breaking lengths were approximately 6-10m. Despite the splitting fracture of the ice floe, similar load levels and failure periods as in level ice are achieved, both in X-and Z-direction. It should be noted here that the three tests were executed at different timing, such that the ice properties in the basin were not exactly the same. For the smaller ice floes the sequential failure patterns are not observed. The smaller ice floes break apart almost immediately after encounter with the structure, such that sequential bending failure will not occur. The higher horizontal loading peaks in the plots for the structure in broken ice, are associated with pushing broken pieces of ice against other ice pieces in the basin. The horizontal loading peaks can reach similar levels for all floe sizes. Periodic failure is less likely to occur as the floe sizes get smaller.

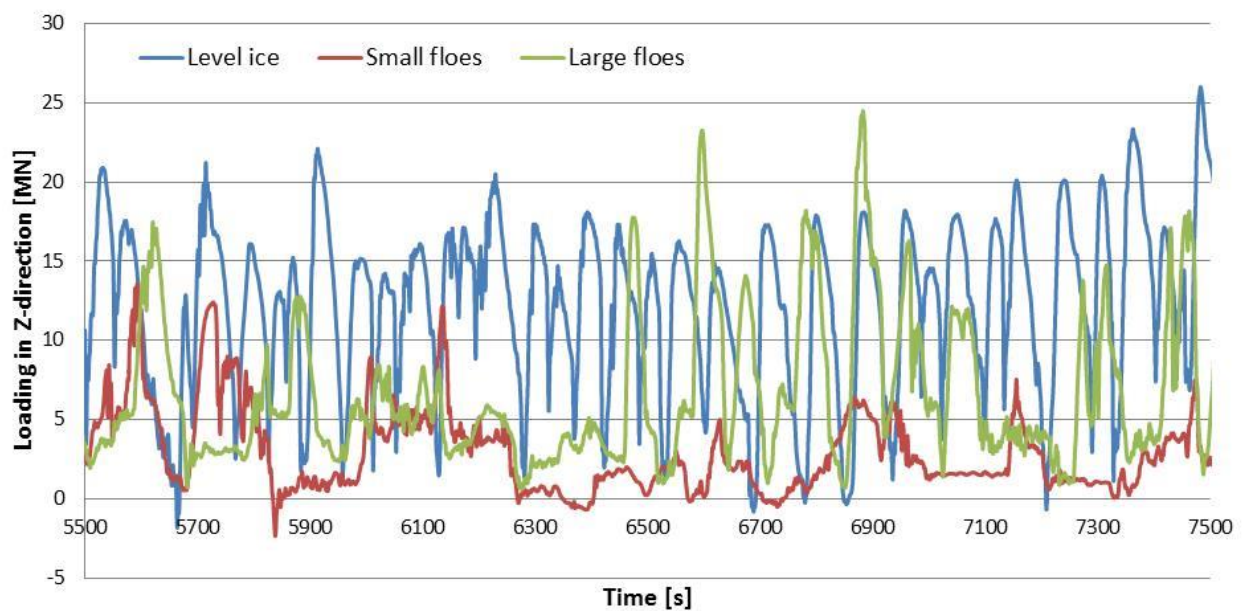


Figure 9. Loading in Z-direction, measured during the model testing campaign for a sloping structure interacting with level ice and ice floes of different sizes

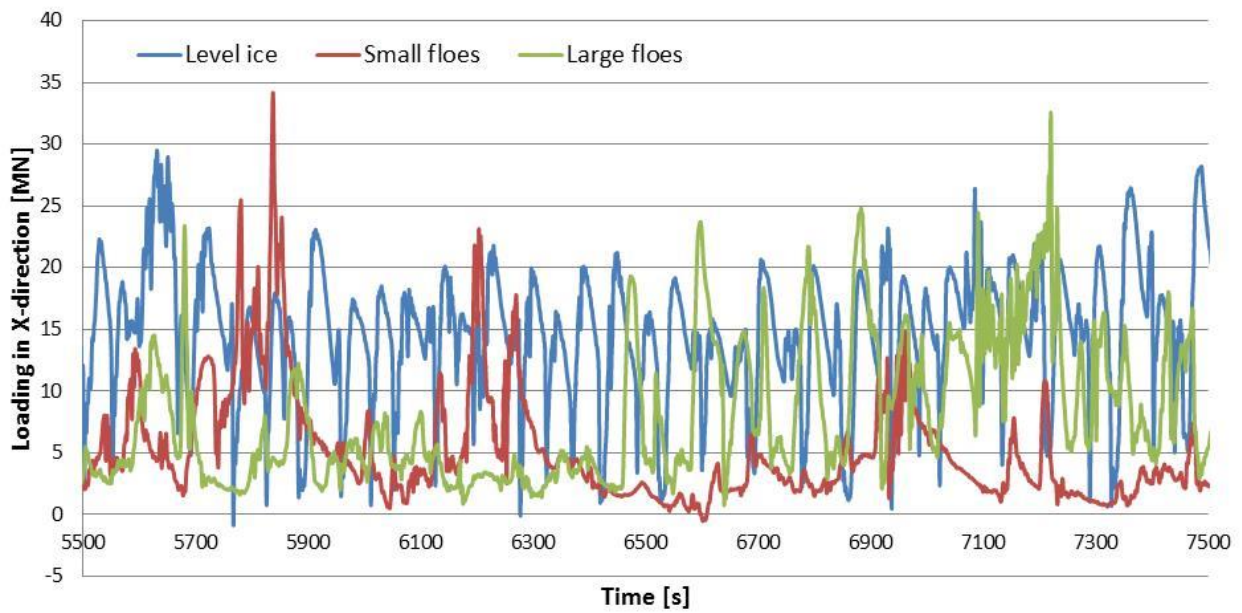


Figure 10. Loading in X-direction, measured during the model testing campaign for a sloping structure interacting with level ice and ice floes of different sizes

The load levels in Figure 9 and Figure 10 are achieved after running through the ice for a longer period of time. At initial encounter with the ice, the load levels are lower, as shown in Figure 11 for level ice. This is due to: 1) at first encounter the structure hits the free ice edge whereas at steady-state, the structure is surrounded by ice, except for its wake and 2) the ice rubble that accumulates underneath the ice near the structure increasing the load that is required to push the ice downward and break in bending, as shown in Figure 12.

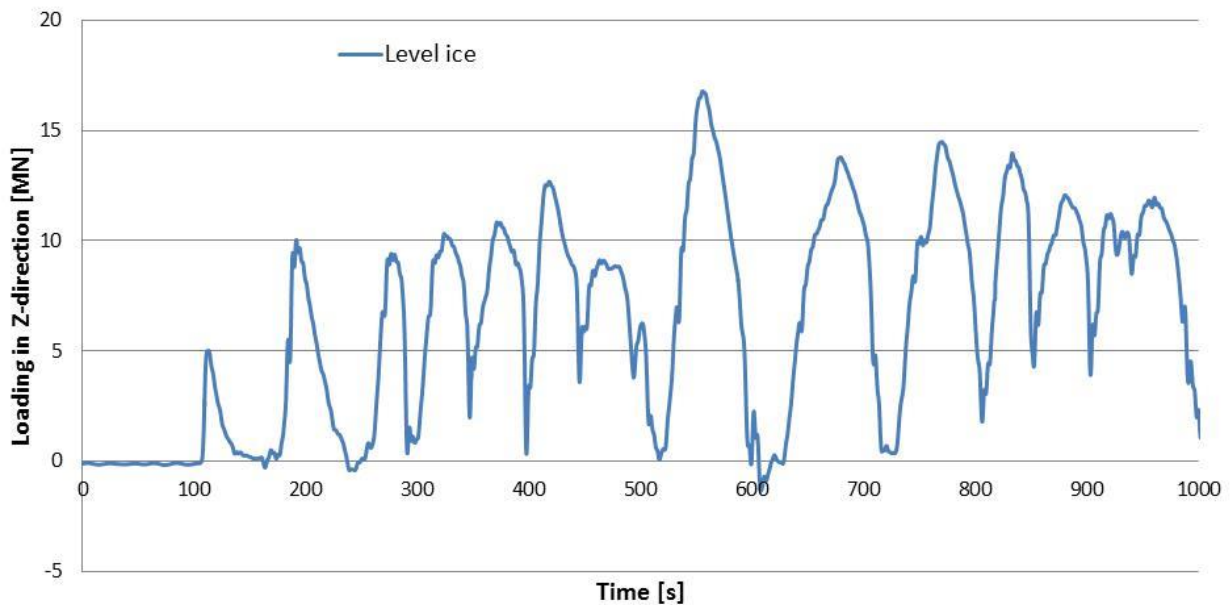


Figure 11. Loading in Z-direction, measured during the model testing campaign for a sloping structure interacting with level ice during the first stage of the test



Figure 12. Ice rubble accumulates near the structure, beneath the ice, still from model test video (HSVA 2010a)

COMPARISON OF LATTICE MODEL AND MODEL TEST RESULTS

Interaction between the conical structure and level ice and ice floes of 165x165m, 55x55m and 34x69m is simulated with the lattice model, focusing on the moment of first encountering the ice. The input parameters that are used are the same as in the model tests and presented in Table 1. It is assumed that the structure hits the ice floes at the mid position of the edge. Confinement is applied at the back of the ice floes, representing confinement from the other ice floes in the basin. A square mesh was selected for the lattice model in this study to keep the model as simple as possible.

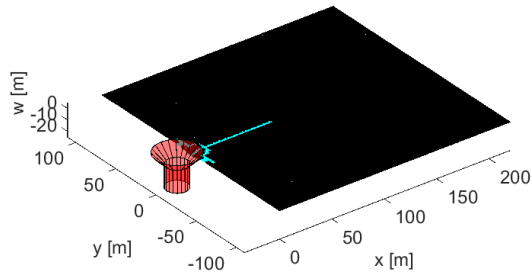
Table 1. Input parameters for ice-structure interaction simulations with the lattice model, based on model test input parameters.

Parameter	Quantity
Ice thickness	3 m
Flexural strength	750 kPa
Young's modulus	8.7 GPa
Ice density	846 kg/m ³
Waterline diameter of the cone	30 m
Velocity of the cone	0.1 m/s

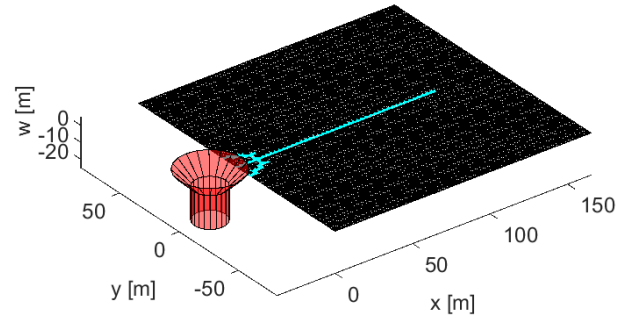
The resulting fracture patterns of the simulations with the lattice model are presented in Figure 13 with light blue lines. In interaction with the level ice (Figure 13a) bending fracture only occurs near the structure. This is in line with observations from the model tests. A split runs into the ice sheet in the numerical simulations, which was not seen in the model tests. It could be that the split in the numerical simulations is caused by effects from the boundary or simplifications in the contact model. For the finite size ice floes (Figure 13 b-d) the fracture patterns consist of splitting of the ice floe and bending fracture near the structure, this is in line with observations from the model tests. For the 165x165m ice floe, first a small piece breaks off during the model tests, before the floe splits during the second encounter. In the numerical simulations a split occurs, which does not reach the end of the ice-sheet at the first encounter, but is likely to reach the end of the sheet at the second encounter between the ice edge and the structure. In interaction with the 34x69m ice floe, the ice floe splits apart and two triangular-shaped pieces of ice break off. In the numerical simulation a similar failure pattern is observed. The numerical pattern is slightly mesh-dependent as can be seen by the straight fracture lines along the mesh near the structure. This could be avoided by applying randomness in nodal locations or local strength or stiffness properties in the mesh. For the 55x55m ice floe the ice fails near the structure and splits apart. In the model tests the structure hits the ice floe to the side and under an angle with the x-axis, which causes a piece of ice to break off first, before the ice splits.

The maximum load the structure imposes on the ice for the case with level ice and the 165x165m ice floe is 3.6MN. This is of the same order of magnitude, but lower than the loading of 5MN that was observed during the model testing for the first encounter with the ice, where no rubble was present ahead of the structure and the structure encountered the ice at the edge. Uncertainties and local variations in ice strength and contact simulation may be a source of these differences. A lower load of 1.6MN for the 34x69m ice floe and 1.3MN for the 55x55m ice floe are simulated. This is in line with the model tests, where lower loads were found for encounter with smaller ice floes. The build-up time of the load in the simulations is similar to the build-up time in the model tests; a few seconds. The ice clearance phase around the structure is not simulated.

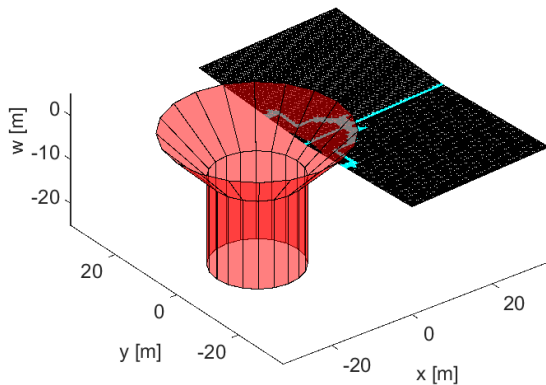
The breaking radius of circumferential fracture in the level ice and 165x165m ice floe is approximately 11m in the simulations with the lattice model. This is a bit higher than the 6-10m that was observed in model testing. Including local imperfections (lower strength of connections or weaker connections) could make the breaking length estimate more accurate. Mesh refinement near the structure may also lead to more accurate failure length calculation. The triangular piece that breaks off in the 34x69m ice floe is a bit wider in the model tests than in the numerical simulation. The ice rubble that was present under the ice in the model test, but was not simulated numerically, could play a role in discrepancies between numerical results and model tests.



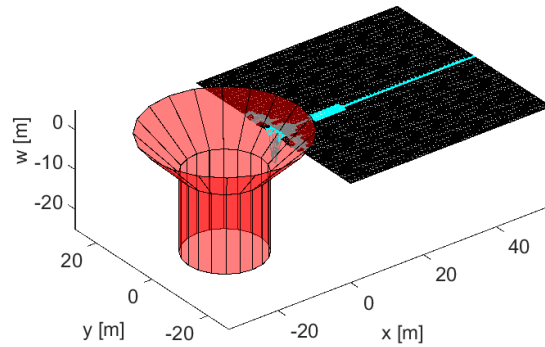
a.



b.



c.



d.

Figure 13. Fracture patterns of interaction between the lattice model and the structure. a) level ice b) 165x165m ice floe c) 34x69m ice floe d) 55x55m ice floe

To simulate natural variations in the ice, the stiffness of each spring in the mesh was changed by adding random fluctuation using a normal distribution with zero mean and unit variance, multiplied with 1/10th of the original stiffness. The nodes of the lattice were displaced by adding random fluctuation using a normal distribution with zero mean and unit variance, multiplied with 1/16th of the distance between the nodes. Results are presented in Figure 14 for the 69x34m ice floe. The fracture pattern is now influenced by the irregular mesh and initially a piece of ice breaks off towards one side. The failure load in the numerical simulation is 1.7MN. The randomness in the model could be further optimized to improve accuracy of failure predictions with the lattice model.

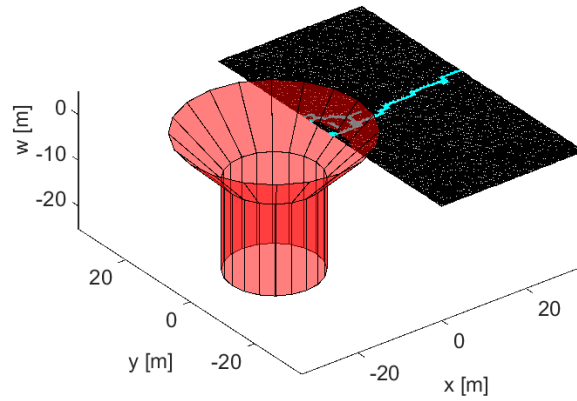


Figure 14. Fracture patterns of interaction between the lattice model for a 34x69m ice floe and the structure with randomized stiffness of the connections.

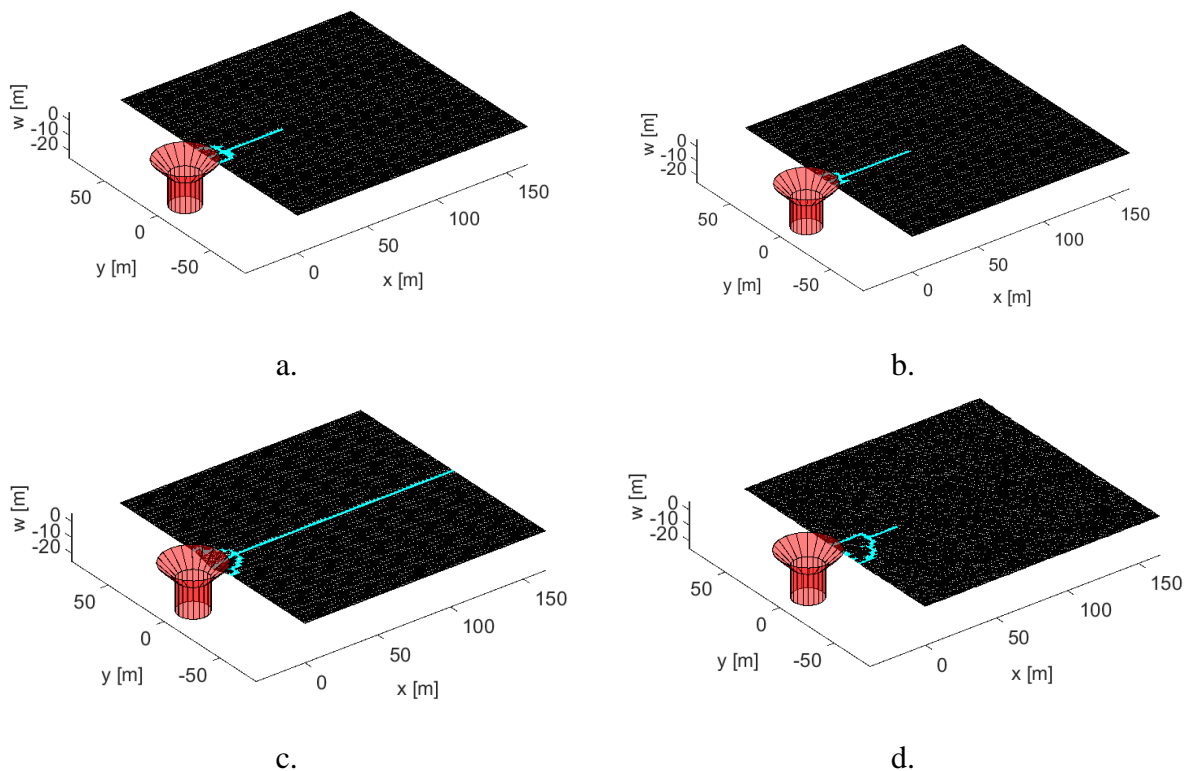


Figure 15. Fracture patterns of interaction between the lattice model and the structure for 165x165m ice floe. Sensitivity checks for a) Young's Modulus 2GPa b) Ice thickness 2m c) Crushing strength 0.1MPa d) Randomized mesh

Four sensitivity checks are done for the 165x165m ice floe. The fracture patterns are presented in Figure 15. First the Young's Modulus is reduced to 2GPa, which reduces the length of the split. The maximum load on the structure remains 3.6 MN. Next, the ice thickness is reduced to 2m and again reduction the length of the split is observed, and the maximum load reduces to 1.5MN. Then the crushing strength of the ice is reduced from 2MPa to 0.2MPa causing the ice plate to split over its full length. The failure load slightly increases to 3.9 MN and the failure process takes longer. Finally, the effect of randomization of the ice is tested for the large ice floe, resulting in a different fracture pattern; the split does not follow the mesh in a straight line. The failure load for this realization is 2.1MN.

The lattice model is capable of simulating failure in bending, splitting and combinations of these. The deformation and fracture algorithm are based on first principles, enabling correct simulation of physical processes without any calibration to the model. Currently sequential failure cannot be simulated with the model as a clearing mechanism of the ice blocks after failure has not been developed. Remaining differences in failure pattern and ice load between model test results and lattice model simulations could be reduced by making further improvements such as local mesh refinement, optimized mesh randomization and an improved contact model. The numerical lattice model could be linked to existing numerical models, for example discrete element method, to make optimum use of existing codes and developments.

CONCLUSIONS

A numerical lattice model has been used to simulate level ice behavior and its capabilities of predicting fracture patterns and loads have been compared to results from model testing. Deformation and failure criteria in the 2-dimensional lattice model are based on first principles, enabling sound simulation of physical processes. In addition, the discrete lattice model avoids stress singularities in fracture modeling since the model only describes displacements and forces in the connections and no stresses are present. By means of numerical assessment, a multitude of structures and loading conditions can be analyzed and structural shapes can be optimized for interaction with ice. Ice management processes can be optimized in case the effects of managed floe sizes on the loading of a structure are known.

In model tests with a conical structure in 3m thick level ice and ice floes of different sizes it was found that for level ice and larger ice floes the floes fail in bending near the structure, whereas the smaller ice floes split apart in interaction with the structure. Sequential loading peaks are observed in interaction with level ice and larger floe sizes, but not for smaller floe sizes. Load on the structure in vertical direction is smaller in interaction with smaller floes. Horizontal loading, however, can reach similar levels for all floe sizes, in case of high confinement, as pieces of ice are pushed against other ice pieces forming load bridges.

Interaction between ice and a downward sloping conical structure is simulated with the lattice model and compared with observations from the model tests, focusing on the first encounter with the ice floe. The lattice model is able to predict whether an ice floe splits or whether failure occurs near the structure in bending. Slightly lower load levels are found in simulations with the lattice model. A sensitivity study is done to analyse the influence of heterogeneities by varying the stiffness of the connections in the lattice model and nodal locations, which could make the lattice model less mesh sensitive. The fracture pattern becomes less sensitive to the mesh, but further improvements to the randomization process

are required. Other additions such as ice clearance mechanisms, local mesh refinement, optimized randomization and an improved contact model would enable more accurate fracture pattern predictions and maximize the efficiency of the lattice model. The numerical lattice model could be linked to existing numerical models, for example a discrete element method, to make optimum use of existing codes and developments.

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