Model predictive trajectory tracking control and thrust allocation for autonomous vessels

Haseltalab, Ali; Garofano, Vittorio; van Pampus, Maurits; Negenborn, Rudy R.

DOI
10.1016/j.ifacol.2020.12.1457

Publication date
2020

Document Version
Final published version

Published in
IFAC-PapersOnline

Citation (APA)
Haseltalab, A., Garofano, V., van Pampus, M., & Negenborn, R. R. (2020). Model predictive trajectory tracking control and thrust allocation for autonomous vessels. *IFAC-PapersOnline*, *53*(2), 14532-14538. https://doi.org/10.1016/j.ifacol.2020.12.1457

Available online at www.sciencedirect.com

ScienceDirect

IFAC PapersOnLine 53-2 (2020) 14532–14538

Model Predictive Trajectory Tracking Control and Thrust Allocation for Autonomous Vessels ⋆

Ali Haseltalab ∗,∗∗ Vittorio Garofano ∗
Maurits van Pampus ∗,∗∗ Rudy R. Negenborn ∗

∗ *Department of Maritime and Transport Technology, Delft University of Technology, Delft, the Netherlands (e-mail: {a.haseltalab, v.garofano, r.r.negenborn}@ tudelft.nl).*
∗∗ *Amsterdam Institute for Advanced Metropolitan Solutions (e-mail: {ali.haseltalab, maurits.vanpampus}@ams-institute.org)*

Abstract: The maneuvering control of autonomous vessels has been under extensive investigations by academic and industrial communities since it is one of the primary steps towards enabling unmanned shipping. In this paper, a model predictive control (MPC) approach is presented for trajectory tracking control of vessels which takes into account the thrust allocation (TA) problem in the presence of rotatable thrusters. In this approach, the TA problem is formulated over a finite horizon and solved with regard to the power consumption, changes in the angle and speed of actuators, and the operating constraints. In the proposed control approach, several linearization techniques have been employed to enable the adoption of quadratic programming approaches for solving the MPC's and TA's optimization problems. The performance of the proposed approach is evaluated through several simulation experiments using a replica vessel model.

Keywords: Autonomous vessels, model predictive control, maneuvering control, thrust allocation, feedback linearization, quadratic programming.

1. INTRODUCTION

Maneuvering control is one of the most critical challenges on the way of enabling autonomous vessels. The problem has been considered in several research works in the last few years where different control approaches have been adopted at address different aspects of this challenge, e.g., see Haseltalab and Negenborn (2019a,b); Sørensen and Breivik (2015); Ashrafiuon et al. (2008) .

One of the main adopted approaches for trajectory tracking control is Model Predictive Control (MPC) where the maneuvering model of the vessel is used for building the prediction model and solving the optimization problem of MPC (Haseltalab and Negenborn (2019b); Zheng et al. (2016)). To use MPC, in most of the research works, the maneuvering model of the vessel is linearized using Taylor's approximation scheme and then, discretized (e.g., see Zheng et al. (2016); Chen et al. (2019a,b)) so that quadratic programming approaches can be adopted to solve the optimization problem of the MPC. In Zheng et al. (2016), a Model Predictive Control (MPC) algorithm is proposed to address the problem of trajectory tracking control with knowledge over arrival time where the nonlinear model of the vessel is linearized to decrease computational complexity. Nonlinear MPC algorithms are adopted in Abdelaal et al. (2016, 2018); Zheng et al (2013) to address the problem of trajectory tracking.

The outputs of the ship maneuvering controller are the forces that should be applied to the ship's Center of Gravity (CoG). These forces should be generated by the actuating propellers. As a result, a Thrust Allocation (TA) problem should be solved. The complexity of this problem depends on the type and configuration of the propelling thrusters (Fossen (2011)). In most of the research works on the maneuvering control of autonomous vessels, the thrust allocation problem is either not considered or trivially considered.

In this paper, the objective is to integrate the thrust allocation problem into the ship maneuvering controller in the presence of rotatable thrusters. After presenting the ship maneuvering model and formulating it in a state space format, an MPC control approach is proposed for trajectory tracking in which Input-Output Feedback Linearization (IOFL) as well as a linearization technique are utilized to enable the use of quadratic programming for solving the MPC's optimization problem. Then, the TA problem is considered where it makes use of the predictions of the MPC over a finite horizon with the objective of minimizing the power consumption as well as the rotation of rotatable thrusters. In order to solve the TA problem using quadratic programming approaches, the TA problem is linearized over the prediction horizon. For evaluating the performance of the proposed approach, a replica model of

⋆ This research is supported by the project "Impulse Autonomous Shipping for Amsterdam" of the Amsterdam Institute for Advanced Metropolitan Solutions (AMS Institute).

10.1016/j.ifacol.2020.12.1457

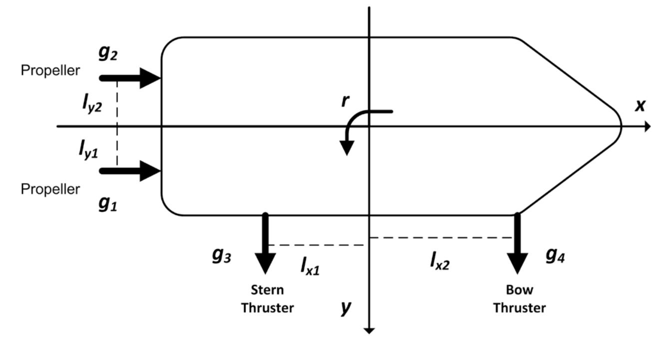


Fig. 1. A vessel with two propellers, a bow thruster, and a stern thruster.

a tug vessel is considered and its voyage is simulated in the city of Amsterdam waterways. The contributions of the paper can be summarized as:

1. Proposing an MPC control approach incorporating maneuvering control and thrust allocation.
2. Enabling the use quadratic programming methods for solving the optimization problems through adopting a set of linearization approaches.

The remainder of the paper is as follows. In Section 2, the maneuvering model of the vessel is presented in a state space format. In Section 3, the proposed MPC and TA approaches are formulated and presented. The simulation experiment results are shown and discussed in Section 4. In Section 5, concluding remarks are given.

2. SYSTEM DESCRIPTION AND THE PROBLEM FORMULATION

In this section, maneuvering model of vessels and the thrust configuration problem in 3 Degrees of Freedom (3DoF) are presented.

2.1 3DoF maneuvering model

In this paper, the 3DoF maneuvering model is considered (Fossen (2011); Skjetne et al. (2004)) which is suitable for maneuvering control applications of surface vessels. The model includes information about the mass of the vessel and displacement, centrifugal and Coriolis forces, drag forces, and configuration of actuators. In Figure 1, the layout of a vessel with two propellers and two thrusters is illustrated.

The maneuvering model of the ship can then be described as:

$$\begin{aligned} &\dot{\eta}_s(t) = R\big(\eta_s(t)\big)v_{\rm s}(t) \\ &M_{\rm s}\dot{v}_{\rm s}(t) + C_{\rm s}\big(v_{\rm s}(t)\big)v_{\rm s}(t) = \tau_{\rm s}(t) + \tau_{\rm drag}\big(v_{\rm s}(t), \eta_{\rm s}(t)\big), \end{aligned} \quad (1)$$

where $\eta_{\rm s}(t) = [x(t), y(t), r(t)]^T$ is the ship position and orientation at time t, $v_{\rm s}(t) = [v_{\rm x}(t), v_{\rm y}(t), v_{\rm r}(t)]^T$ is the 3DoF ship speed and τ_s is the vector of forces applied to the ship center of gravity. M_s is the Inertial Mass matrix which consists of rigid body and added mass matrices:

$$M_s = M_{RB} + M_A \quad (2)$$

where

$$M_s = \begin{bmatrix} m_{\rm b} & 0 & 0 \\ 0 & m_{\rm b} & 0 \\ 0 & 0 & I_z \end{bmatrix}, M_A = \begin{bmatrix} m_{\rm ax} & 0 & 0 \\ 0 & m_{\rm ay} & 0 \\ 0 & 0 & I_{\rm a} \end{bmatrix}. \quad (3)$$

Parameter $m_{\rm b}$ is the mass of the vessel, $I_{\rm z}$ is the moment of inertia, $m_{\rm ax}$ and $m_{\rm ay}$ are the added mass in x and y direction, respectively, and $I_{\rm a}$ represents the added moment of inertia.

Matrix $C_s(\cdot)$ is the Coriolis and Centrifugal matrix defined as:

$$C_s(v_{\rm s}) = \begin{bmatrix} 0 & 0 & -m_{\rm b}v_{\rm y} \\ 0 & 0 & m_{\rm b}v_{\rm x} \\ m_{\rm b}v_{\rm y} & -m_{\rm b}v_{\rm x} & 0 \end{bmatrix}. \quad (4)$$

Function $\tau_{\rm drag}(.)$, which is a function of ship speed and course angle, represents drag forces in 3DoF applied to the craft. The details of this function are provided in Haseltalab and Negenborn (2019b).

A method to present drag forces is by establishing added Coriolis and damping matrices. In this regard,

$$\tau_{\rm drag}\big(v_{\rm s}(t), \eta_{\rm s}(t)\big) = -C_{\rm A}\big(v_{\rm s}(t)\big) - D_{\rm s}\big(v_{\rm s}(t)\big) \quad (5)$$

where

$$C_{\rm A}(v_{\rm s}) = \begin{bmatrix} 0 & 0 & c_{13}(v_{\rm s}) \\ 0 & 0 & c_{23}(v_{\rm s}) \\ -c_{13}(v_{\rm s}) & -c_{23}(v_{\rm s}) & 0 \end{bmatrix}, \quad (6)$$

with $c_{13}(v_{\rm s}) = Y_{\dot{\rm v}}v_{\rm s} + \frac{1}{2}(N_{\dot{\rm v}} + Y_{\dot{\rm r}})$ and $c_{23}(v_{\rm s}) = -X_{\dot{\rm u}}v_{\rm x}$.

The damping matrix D_s is constructed by addition of a linear and a nonlinear matrices, i.e.,

$$D_{\rm s}(V) = D_{\rm L} + D_{\rm NL}\big(v_{\rm s}\big), \quad (7)$$

where

$$\begin{aligned} D_{\rm L} &= \begin{bmatrix} -X_{\rm u} & 0 & 0 \\ 0 & -Y_{\rm v} & -Y_{\rm r} \\ 0 & -N_{\rm v} & -N_{\rm r} \end{bmatrix} \\ D_{\rm NL}\big(v_{\rm s}\big) &= \begin{bmatrix} -d_{11}(v_{\rm s}) & 0 & 0 \\ 0 & -d_{22}(v_{\rm s}) & -d_{23}(v_{\rm s}) \\ 0 & -d_{32}(v_{\rm s}) & -d_{33}(v_{\rm s}) \end{bmatrix}, \end{aligned} \quad (8)$$

with $d_{11}(v_{\rm s}) = X_{|u|u}|v_{\rm x}| + X_{uu}v_{\rm x}^2$, $d_{22}(v_{\rm s}) = Y_{|v|v}|v_{\rm y}| + Y_{|r|v}|v_{\rm r}|$, $d_{23}(v_{\rm s}) = Y_{|v|r}|v_{\rm y}| + Y_{|r|r}|v_{\rm r}|$, $d_{32}(V) = N_{|v|v}|v_{\rm r}| + N_{|r|v}|v_{\rm r}|$ and $d_{33}(v_{\rm s}) = N_{|v|r}|v_{\rm x}| + N_{|r|r}|v_{\rm r}|$. For more information on the model and the parameters, see Fossen (2011); Skjetne et al. (2004).

Matrix $R(\eta_{\rm s})$ transforms ship velocity from body-fixed into inertial velocities and is defined as:

$$R(\eta_{\rm s}) = \begin{bmatrix} \cos(r) & -\sin(r) & 0 \\ \sin(r) & \cos(r) & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (9)$$

Vector τ_s is the vector of forces generated by propellers applied to the ship center of gravity, defined as:

$$\tau_s(t) = \begin{bmatrix} \tau_x(t) \\ \tau_y(t) \\ \tau_r(t) \end{bmatrix}, \quad (10)$$

where τ_x and τ_y are surge and sway forces and τ_r is the yaw moment.

2.2 Thrust allocation

Considering rotatable and non-rotatable thrusters, the relationship between the thrust produced by actuators and the vector of forces is Fossen (2011):

$$\tau_s = \Xi_{3\times m}F = \Xi_{3\times m}\begin{bmatrix} f_1 \\ \vdots \\ f_m \end{bmatrix}, \quad (11)$$

where $f_1, \dots, f_m$ are propelling thrust generated by actuators, m is the number of actuators, and Ξ is the thrust configuration matrix defined as:

$$\Xi = [\gamma_1 \; ... \; \gamma_m], \tag{12}$$

with $\gamma_1, \gamma_2, ..., \gamma_m$ column vectors for standard actuators. If the actuator is a non-rotatable thruster, then:

$$\gamma_i = \begin{bmatrix} 1 \\ 0 \\ -l_{\mathrm{y}_i} \end{bmatrix}, \tag{13}$$

if the actuator is a rotatable thruster, then:

$$\gamma_i = \begin{bmatrix} \cos(\alpha_i) \\ \sin(\alpha_i) \\ l_{\mathrm{x}_i}\sin(\alpha_i) - l_{\mathrm{y}_i}\cos(\alpha_i) \end{bmatrix}, \tag{14}$$

and if the actuator is a stern or bow thruster, then:

$$\gamma_i = \begin{bmatrix} 0 \\ 1 \\ l_{\mathrm{x}_i} \end{bmatrix}, \tag{15}$$

where l_{y_i} and l_{x_i} represent the position of the actuator i in the vessel's reference frame.

The TA problem can be formulated as (Fossen (2011)):

$$\mathbb{P}_{\mathrm{n}} : \min_{f,\alpha} J_{\mathrm{n}}(f,\alpha) \tag{16}$$

subject to:

$$\begin{aligned} &\Xi(\alpha)F = \tau \\ &F_{\min} \le F \le F_{\max} \\ &\alpha_{\min} \le \alpha \le \alpha_{\max} \\ &\Delta\alpha_{\min} \le \Delta\alpha \le \Delta\alpha_{\max} \\ &\Delta F_{\min} \le \Delta F \le \Delta F_{\max} \end{aligned} \tag{17}$$

where objective function J_{n} is:

$$\begin{aligned} J_{\mathrm{n}}(f,\alpha) =& F^T P F + (\alpha-\alpha_0)^T Q (\alpha-\alpha_0) \\ &+ \frac{\mu}{\epsilon + det\left(\Xi(\alpha)\Xi^T(\alpha)\right)}. \end{aligned} \tag{18}$$

In the above optimization problem, parameter α is the vector of rotatable actuators' angle, α_0 is the vector of measured angles, $\Delta\alpha = \alpha - \alpha_0$ is the difference between the current angles and the angles in the next sampling time, $\Delta F = F - F_0$ is the difference in thrust generation between two consecutive sampling times, and P is a positive definite diagonal matrix.

3. PREDICTIVE TRAJECTORY TRACKING CONTROL AND THRUST ALLOCATION

In this section, a control approach is proposed for maneuvering control and TA.

3.1 Predictive Trajectory Tracking Control

Let us rewrite the speed dynamics of the ship as:

$$\dot{v}_{\mathrm{s}}(t) = M_{\mathrm{s}}^{-1}\Big(\tau_{\mathrm{s}} + \tau_{\mathrm{drag}}\big(v_{\mathrm{s}}(t),\eta_{\mathrm{s}}(t)\big) - C_{\mathrm{s}}\big(v_{\mathrm{s}}(t)\big)v_{\mathrm{s}}(t)\Big). \tag{19}$$

With the following IOFL law the above system can be linearized:

$$\tau_{\mathrm{s}} = M_{\mathrm{s}}\Big(-\tau_{\mathrm{drag}}\big(v_{\mathrm{s}}(t),\eta_{\mathrm{s}}(t)\big) + C_{\mathrm{s}}\big(v_{\mathrm{s}}(t)\big)v_{\mathrm{s}}(t) + A_{\mathrm{s}}v_{\mathrm{s}} + B_{\mathrm{s}}\nu_{\mathrm{s}}\Big) \tag{20}$$

where ν_{s} is the input vector of linearized system, v_{s} represents its states and A_{s} and B_{s} are states and input matrices of the linear system, respectively. As a result, the transformed linear system can be written as:

$$\dot{v}_{\mathrm{s}} = A_{\mathrm{s}}v_{\mathrm{s}} + B_{\mathrm{s}}\nu_{\mathrm{s}}. \tag{21}$$

After discretization, MPC is applied where the objective is to keep the ship as close as possible to the reference trajectory. In this regard, the following MPC problem is defined with sample time T_k:

$$\mathbb{P}(v_{\mathrm{s}}) : \min_{\nu_{\mathrm{s}}} \Big(V_{\mathrm{N}}(v_{\mathrm{s}},\nu_{\mathrm{s}}) = \sum_{i=0}^{N-1} l\big(v_{\mathrm{s}}(k+i),\nu_{\mathrm{s}}(k+i)\big)\Big) \tag{22}$$

subject to:

$$\begin{aligned} &v_{\mathrm{s}}(k+i+1) = A_{\mathrm{s}}(T_{\mathrm{k}})v_{\mathrm{s}}(k+i) + B_{\mathrm{s}}(T_{\mathrm{k}})\nu_{\mathrm{s}}(k+i) \\ &v_{\min}(k+i) \le v_{\mathrm{s}}(k+i)(k) \le v_{\max}(k+i) \\ &\nu_{\min}(k+i-1) \le \nu_{\mathrm{s}}(k+i-1) \le \nu_{\max}(k+i-1), \\ &\forall i \in [0,N], \end{aligned} \tag{23}$$

where

$$\begin{aligned} l\big(v_{\mathrm{s}}(k),\nu_{\mathrm{s}}(k)\big) =& \big(v_{\mathrm{s}}(k) - v_{\mathrm{s_{ref}}}(k)\big)^T W_{\mathrm{s}}\big(v_{\mathrm{s}}(k) - v_{\mathrm{s_{ref}}}(k)\big) \\ &+ \nu_{\mathrm{s}}^T(k)\nu_{\mathrm{s}}(k). \end{aligned} \tag{24}$$

In the above MPC problem, parameter N is the prediction horizon and W_{s} is the positive definite weight matrix of the cost function.

The reference ship speed $v_{\mathrm{s_{ref}}}(k)$ is approximated using (1) as:

$$v_{\mathrm{s_{ref}}}(k+1) = R^{-1}\big(\eta_{\mathrm{s}}(k)\big)\Big(\frac{\eta_{\mathrm{ref}}(k+1) - \eta_{\mathrm{s}}(k)}{T_{\mathrm{k}}}\Big). \tag{25}$$

The adoption of IOFL for MPC results in clear advantages since the optimization problem is simplified, however, due to non-linearity of input constraints, quadratic programming cannot be adopted for solving the optimization problem. In the following, using the results in te Braake et al. (1999), we adopt a methodology for linearizing the input constraints in (23) to further simplify the optimization problem which leads to major reduction of computational costs.

The main idea behind this methodology is linear estimation of non-linear constraints. Let us present the constraints acting on the thrust vector τ_{s}:

$$\tau_{\min} \le \tau_{\mathrm{s}}(k) \le \tau_{\max}. \tag{26}$$

If the IOFL rule is rewritten as:

$$\begin{aligned} \nu_{\mathrm{s}}(t) =& \Psi_{\mathrm{s}}\big(v_{\mathrm{s}}(t),\tau_{\mathrm{s}}(t)\big) = \\ & B_{\mathrm{s}}^{-1}\Big(M_{\mathrm{s}}^{-1}\tau_{\mathrm{s}}(t) + \tau_{\mathrm{drag}}\big(v_{\mathrm{s}}(t),\eta_{\mathrm{s}}(t)\big) - C_{\mathrm{s}}\big(v_{\mathrm{s}}(t)\big)v_{\mathrm{s}}(t) \\ & - A_{\mathrm{s}}v_{\mathrm{s}}(t)\Big), \end{aligned} \tag{27}$$

then, ν_{s} can be approximated around $\big(v_{\mathrm{s}}(t_0),\tau_{\mathrm{s}}(t_0)\big)$ as:

$$\begin{aligned} \nu_{\mathrm{s}}(t) \approx \hat{\Psi}_{\mathrm{s_{t_0}}}\big(v_{\mathrm{s}}(t),\tau_{\mathrm{s}}(t)\big) =& \Psi_{\mathrm{s}}\big(v_{\mathrm{s}}(t_0),\tau_{\mathrm{s}}(t_0)\big) \\ &+ \frac{\partial\Psi_{\mathrm{s}}}{\partial v_{\mathrm{s}}}\bigg|_{\left(v_{\mathrm{s}}(t_0),\tau_{\mathrm{s}}(t_0)\right)}\big(v_{\mathrm{s}}(t) - v_{\mathrm{s}}(t_0)\big) \\ &+ \frac{\partial\Psi_{\mathrm{s}}}{\partial \tau_{\mathrm{s}}}\bigg|_{\left(v_{\mathrm{s}}(t_0),\tau_{\mathrm{s}}(t_0)\right)}\big(\tau_{\mathrm{s}}(t) - \tau_{\mathrm{s}}(t_0)\big). \end{aligned} \tag{28}$$

Let $v_{\rm s}(k+i|k)$ denote the value of $v_{\rm s}$ at time $(k+i)t_{\rm k}$ predicted at time $kt_{\rm k}$, then using (28), the linear constraints can be found as:

$$\begin{aligned}&\nu_{\min}(k+i-1)=\\&\min_{\tau_s(k+i-1)}\hat{\Psi}_{s_{k+i|k-1}}\big(v_{\rm s}(k+i|k-1),\tau_{\rm s}(k+i-1)\big)\\&\nu_{\max}(k+i-1)=\\&\max_{v_2(k+i-1)}\hat{\Psi}_{s_{k+i|k-1}}\big(v_{\rm s}(k+i|k-1),\tau_{\rm s}(k+i-1)\big)\end{aligned}\quad(29)$$

subject to,

$$\tau_{\min}\le\tau_{\rm s}(k+i-1)\le\tau_{\max},\forall i\in[0,N-1].\quad(30)$$

Note that for time instant $(k+N-1)t_{\rm k}$, we have:

$$\begin{aligned}\nu_{\min}(k+N-1)=\nu_{\min}(k+N-2)\\\nu_{\max}(k+N-1)=\nu_{\max}(k+N-2).\end{aligned}\quad(31)$$

Note also that, due to the linearity of $\hat{\Psi}_{s_{k+i|k-1}}(.)$, the optimization problems in (29) are trivial to solve.

The adoption of this methodology leads to simplification of the optimization problem within MPC and to the possibility of using a quadratic programming scheme.

At every sample time k, the proposed control algorithm generates a set of control inputs $\nu_{\rm s}(k|k),...,\nu_{\rm s}(k+N-1|k)$ and $v_{\rm s}(k|k),...,v_{\rm s}(k+N-1|k)$. Using these sets and (20), the set of future control inputs $\tau_{\rm s}(k|k),...,\tau_{\rm s}(k+N-1|k)$ can be estimated.

3.2 Predictive Thrust Allocation

The optimization problem in (16) and (17) is a non-convex nonlinear problem which needs a significant amount of computation. Moreover, it does not use the future control inputs $\tau_{\rm s}(k|k),...,\tau_{\rm s}(k+N-1|k)$. In this section, the optimization problem in (16) is regulated and approximated with a convex quadratic programming problem which utilizes the prediction of future required propelling forces.

The first term in the optimization problem $\mathbb{P}_{\rm n}$ can be represented as $(F(k-1)+\Delta F(k))^T P\,(F(k-1)+\Delta F(k))$ where $F(k-1)$ is the vector of generated thrusts by actuators in the previous sampling time. The second term can be shown as $\Delta\alpha(k)^T P\Delta\alpha(k)$ and the third term, i.e., the singularity avoidance penalty can be approximated by a linear term around $\alpha(k-1)$ that is $\frac{\partial}{\partial\alpha}\left(\frac{\mu}{\epsilon+det(\Xi(\alpha)\Xi^T(\alpha))}\right)\Big|_{\alpha(k-1)}\Delta\alpha(k)$. As a result, the linearized TA problem $\mathbb{P}_{\rm l}$ can be written as:

$$\mathbb{P}_{\rm l}:\min_{\Delta F,\Delta\alpha}J_{\rm l}\big(\Delta F(k),\Delta\alpha(k)\big)\quad(32)$$

subject to:

$$\begin{aligned}\Xi\big(\alpha(k-1)\big)\Delta F(k)+\left(\frac{\partial}{\partial\alpha}\Xi(\alpha)F(k-1)\Big|_{\alpha(k-1)}\right)\Delta\alpha(k)\\=\tau(k)-\Xi\big(\alpha(k-1)\big)F(k-1)\\F_{\min}-F(k-1)\le\Delta F(k)\le F_{\max}-F(k-1)\\\alpha_{\min}-\alpha(k-1)\le\Delta\alpha(k)\le\alpha_{\max}(k-1)\\\Delta\alpha_{\min}\le\Delta\alpha(k)\le\Delta\alpha_{\max}\\\Delta F_{\min}\le\Delta F(k)\le\Delta F_{\max}\end{aligned}\quad(33)$$

with objective function $J_{\rm l}$ defined as:

$$\begin{aligned}&J_{\rm l}\left(\Delta F(k),\Delta\alpha(k)\right)=\\&\quad(F(k-1)+\Delta F(k))^T P\,(F(k-1)+\Delta F(k))\\&\quad+\Delta\alpha(k)^T P\Delta\alpha(k)\\&\quad+\frac{\partial}{\partial\alpha}\left(\frac{\mu}{\epsilon+det\left(\Xi(\alpha)\Xi^T(\alpha)\right)}\right)\Big|_{\alpha(k-1)}\Delta\alpha(k).\end{aligned}\quad(34)$$

The above TA problem can be extended to a predictive TA problem over finite horizon N. In this regard, the generated thrust at time step $k+i$ can be formulated as:

$$\begin{aligned}F(k+i)=&F(k+i-1)+\Delta F(k+i)\\=&F(k-1)+\sum_{j=0}^{i}\Delta F(k+j).\end{aligned}\quad(35)$$

If $\chi_{\rm F}(k)=\{\Delta F(k),...,\Delta F(k+N-1)\}$ and $\chi_\alpha(k)=\{\Delta\alpha(k),...,\Delta\alpha(k+N-1)\}$ are the sets of generated thrust and thrusters' angle, respectively, over horizon N, then, using (34) and (35), the cost function of the predictive TA problem can be formulated as:

$$\begin{aligned}&J_{\rm p}\big(\chi_{\rm F}(k),\chi_\alpha(k)\big)=\\&\sum_{i=0}^{N-1}\Big(F(k-1)+\sum_{j=0}^{i}\Delta F(k+j)\Big)^T P\Big(F(k-1)\\&+\sum_{j=0}^{i}\Delta F(k+j)\Big)+\Delta\alpha(k+i)^T Q\Delta\alpha(k+i)\\&+\frac{\partial}{\partial\alpha}\left(\frac{\mu}{\epsilon+det\left(\Xi(\alpha)\Xi^T(\alpha)\right)}\right)\Big|_{\alpha(k-1)}\sum_{j=0}^{i}\Delta\alpha(k+j)\\&+(N-i)\Delta F(k+i)^T P\Delta F(k+i).\end{aligned}\quad(36)$$

In the above objective function, the fourth term $\Delta F(k+i)^T P\Delta F(k+i)$ is added to guarantee the convexity of the problem and to explicitly take into account the changes of the thrust generated by actuators during the operation.

Function $J_{\rm p}$ can be represented in a quadratic programming format as:

$$J_{\rm p}\big(\chi_{\rm F}(k),\chi_\alpha(k)\big)=u^T Hu+L^T u\quad(37)$$

where $u=[\Delta F^T(k),...,\Delta F^T(k+N-1),\Delta\alpha^T(k),...,\Delta\alpha^T(k+N-1)]^T$,

$$H=\begin{bmatrix}2NP&2(N-1)P&\cdots&2P&0_{\rm m\times r}&0_{\rm m\times r}&\cdots&0_{\rm m\times r}\\2(N-1)P&2(N-1)P&\cdots&2P&0_{\rm m\times r}&0_{\rm m\times r}&\cdots&0_{\rm m\times r}\\\vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\ddots&\vdots\\2P&2P&\cdots&2P&0_{\rm m\times r}&0_{\rm m\times r}&\cdots&0_{\rm m\times r}\\0_{\rm r\times m}&0_{\rm r\times m}&\cdots&0_{\rm r\times m}&Q&0_{\rm r\times r}&\cdots&0_{\rm r\times r}\\0_{\rm r\times m}&0_{\rm r\times m}&\cdots&0_{\rm r\times m}&0_{\rm r\times r}&Q&\cdots&0_{\rm r\times r}\\\vdots&\vdots&\cdots&\vdots&\vdots&\vdots&\ddots&\vdots\\0_{\rm r\times m}&0_{\rm r\times m}&\cdots&0_{\rm r\times m}&0_{\rm r\times r}&0_{\rm r\times r}&\cdots&Q\end{bmatrix},$$

and

$$L = \begin{bmatrix} 2NF^T(k-1)P \\ 2(N-1)F^T(k-1)P \\ \vdots \\ 2F^T(k-1)P \\ N\dfrac{\partial}{\partial\alpha}\left(\dfrac{\mu}{\epsilon + det\left(\Xi(\alpha)\Xi^T(\alpha)\right)}\right)\Big|_{\alpha(k-1)} \\ (N-1)\dfrac{\partial}{\partial\alpha}\left(\dfrac{\mu}{\epsilon + det\left(\Xi(\alpha)\Xi^T(\alpha)\right)}\right)\Big|_{\alpha(k-1)} \\ \vdots \\ \dfrac{\partial}{\partial\alpha}\left(\dfrac{\mu}{\epsilon + det\left(\Xi(\alpha)\Xi^T(\alpha)\right)}\right)\Big|_{\alpha(k-1)} \end{bmatrix}.$$

The constraints of the predictive TA optimization problem are:

$$\begin{aligned} Au &\leq b \\ A_{\text{eq}}u &= b_{\text{eq}} \\ l_{\text{b}} \leq u &\leq u_{\text{b}} \end{aligned} \tag{38}$$

where

$$A_{\left(2N(m+r)\times N(m+r)\right)} =$$
$$\begin{bmatrix} I_{\text{m}} & 0_{m\times m} & \cdots & 0_{m\times m} & 0_{m\times r} & 0_{m\times r} & \cdots & 0_{m\times r} \\ I_{\text{m}} & I_{\text{m}} & \cdots & 0_{m\times m} & 0_{m\times r} & 0_{m\times r} & \cdots & 0_{m\times r} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ I_{\text{m}} & I_{\text{m}} & \cdots & I_{\text{m}} & 0_{m\times r} & 0_{m\times r} & \cdots & 0_{m\times r} \\ -I_{\text{m}} & 0_{m\times m} & \cdots & 0_{m\times m} & 0_{m\times r} & 0_{m\times r} & \cdots & 0_{m\times r} \\ -I_{\text{m}} & -I_{\text{m}} & \cdots & 0_{m\times m} & 0_{m\times r} & 0_{m\times r} & \cdots & 0_{m\times r} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -I_{\text{m}} & -I_{\text{m}} & \cdots & -I_{\text{m}} & 0_{m\times r} & 0_{m\times r} & \cdots & 0_{m\times r} \\ 0_{r\times m} & 0_{r\times m} & \cdots & 0_{r\times m} & I_{\text{r}} & 0_{r\times r} & \cdots & 0_{r\times r} \\ 0_{r\times m} & 0_{r\times m} & \cdots & 0_{r\times m} & 0_{r\times r} & I_{\text{r}} & \cdots & 0_{r\times r} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0_{r\times m} & 0_{r\times m} & \cdots & 0_{r\times m} & 0_{r\times r} & 0_{r\times r} & \cdots & I_{\text{r}} \\ 0_{r\times m} & 0_{r\times m} & \cdots & 0_{r\times m} & -I_{\text{r}} & 0_{r\times r} & \cdots & 0_{r\times r} \\ 0_{r\times m} & 0_{r\times m} & \cdots & 0_{r\times m} & 0_{r\times r} & -I_{\text{r}} & \cdots & 0_{r\times r} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0_{r\times m} & 0_{r\times m} & \cdots & 0_{r\times m} & 0_{r\times r} & 0_{r\times r} & \cdots & -I_{\text{r}} \end{bmatrix}$$

and

$$b_{\left(2N(m+r)\times 1\right)} = \begin{bmatrix} F_{\max} - F(k-1) \\ \vdots \\ F_{\max} - F(k-1) \\ -F_{\max} + F(k-1) \\ \vdots \\ -F_{\max} + F(k-1) \\ \alpha_{\max} - \alpha(k-1) \\ \vdots \\ \alpha_{\max} - \alpha(k-1) \\ -\alpha_{\max} + \alpha(k-1) \\ \vdots \\ -\alpha_{\max} + \alpha(k-1) \end{bmatrix}.$$

Matrix I_{m} is an $m \times m$ identity matrix and I_{r} is an $r \times r$ identity matrix.

The equality constraints matrix A_{eq} is:

Fig. 2. Tito-Neri vessel.

$$A_{\text{eq}\left(3N\times N(m+r)\right)} =$$
$$\begin{bmatrix} \Xi_{\alpha(\text{k}-1)} & 0_{3\times\text{m}} & \cdots & 0_{3\times\text{m}} & D_{\Xi\text{F}} & 0_{3\times\text{r}} & \cdots & 0_{3\times\text{r}} \\ \Xi_{\alpha(\text{k}-1)} & \Xi_{\alpha(\text{k}-1)} & \cdots & 0_{3\times\text{m}} & D_{\Xi\text{F}} & D_{\Xi\text{F}} & \cdots & 0_{3\times\text{r}} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \Xi_{\alpha(\text{k}-1)} & \Xi_{\alpha(\text{k}-1)} & \cdots & \Xi_{\alpha(\text{k}-1)} & D_{\Xi\text{F}} & D_{\Xi\text{F}} & \cdots & D_{\Xi\text{F}} \end{bmatrix}$$

where $\Xi_{\alpha(\text{k}-1)} = \Xi\big(\alpha(k-1)\big)$ and $D_{\Xi\text{F}} = \frac{\partial}{\partial\alpha}\Xi(\alpha)F(k-1)\big|_{\alpha(k-1)}$. The equality constraints vector is:

$$b_{\text{eq}\left(3N\times 1\right)} = \begin{bmatrix} \tau(k) - \Xi_{\alpha(\text{k}-1)}F(k-1) \\ \tau(k+1) - \Xi_{\alpha(\text{k}-1)}F(k-1) \\ \vdots \\ \tau(k+N-1) - \Xi_{\alpha(\text{k}-1)}F(k-1) \end{bmatrix}.$$

The vector bounds on u can be derived from (33) as,

$$l_{\text{b}\left(N(m+r)\times 1\right)} = \begin{bmatrix} \Delta F_{\min} \\ \vdots \\ \Delta F_{\min} \\ \Delta\alpha_{\min} \\ \vdots \\ \Delta\alpha_{\min} \end{bmatrix}, u_{\text{b}\left(N(m+r)\times 1\right)} = \begin{bmatrix} \Delta F_{\max} \\ \vdots \\ \Delta F_{\max} \\ \Delta\alpha_{\max} \\ \vdots \\ \Delta\alpha_{\max} \end{bmatrix}.$$

Then, using (37), the predictive TA problem can be formulated as:

$$\mathbb{P}_{\text{p}} : \min_{\chi_{\text{F}},\chi_\alpha} J_{\text{p}}\big(\chi_{\text{F}}(k), \chi_\alpha(k)\big) \tag{39}$$

subject to constraints in (38).

4. SIMULATION EXPERIMENTS

In this section, the performance of the proposed control approach is evaluated using a high fidelity 1:30 replica vessel model known as Tito-Neri (Figure 2). The maneuvering model parameters of the vessel is presented and discussed in Haseltalab and Negenborn (2019b).

For the simulation experiment, a trajectory of real vessels in IJ river, in Amsterdam metropolitan areas, is chosen that is shown in Figure 3. The trajectory is scaled down using Froude scaling so that it is applicable to Tito Neri vessel. Matlab Simulink 2018a is used for simulations.

The prediction horizon is chosen as $N = 20$ with controller sampling time of $T_{\text{s}} = 3\text{s}$. The MPC weight matrix is $W_{\text{s}} = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 10 \end{bmatrix}$, and the predictive TA weighting matrices are chosen as $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $Q = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$.

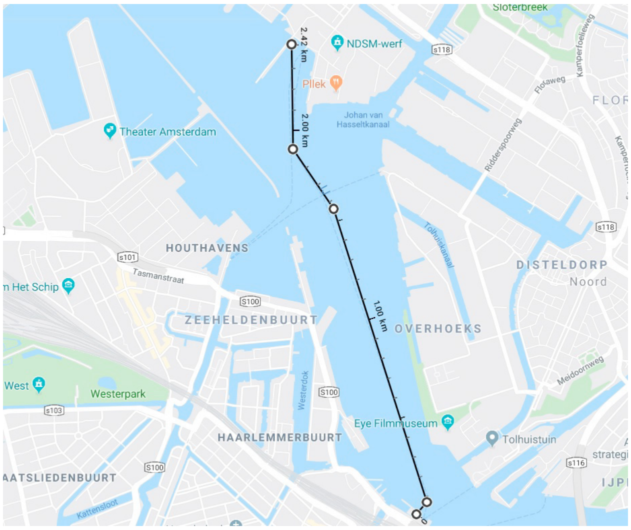


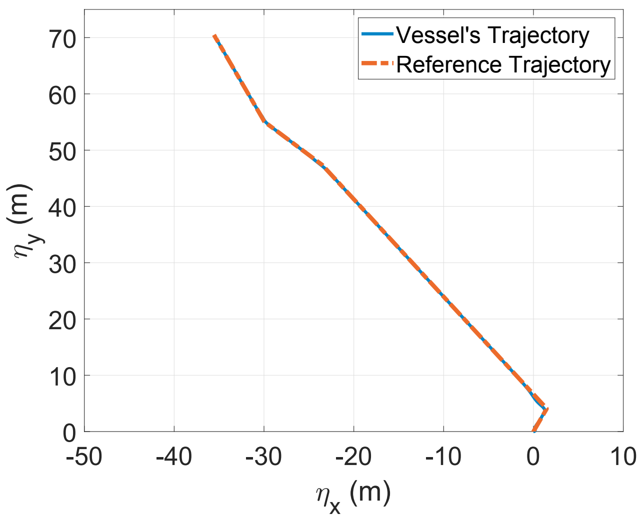
Fig. 3. Tito-Neri vessel.

The constraints on τ_s are chosen as $\tau_{\min} = -\tau_{\max} = [-2, -2, -2]^T$ and the constraints on thrusters are $F_{\max} = -F_{\min} = [3, 3, 3]^T$ and $\alpha_{\max} = -\alpha_{\max} = [\pi/2, \pi/2]^T$.

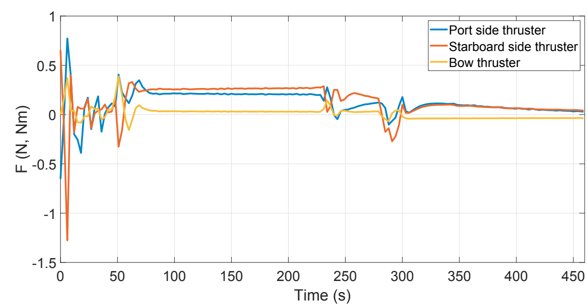
The simulation results are provided in Figure 4 where the vessels starts its trajectory tracking from $[0, 0]$. The trajectory tracking performance is shown in Figure 4a. The propelling forces generated by the thrusters are shown in Figure 4b and the angle of azimuth thrusters during the voyage is presented in Figure 4c.

The speed of the vessel in its own reference frame and its power consumption are shown in Figure 5. By integrating over the power consumption, the overall energy consumption during the voyage is calculated using the thrusters model which is 19.86 W.

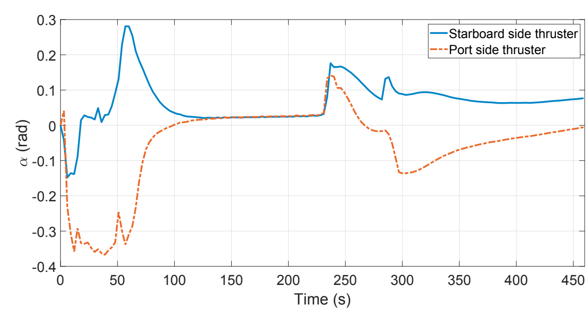
This experiment is also carried out using a conventional discrete PI-based approach where the PI controller parameters are chosen as $K_p = 1.15$ and $K_I = 0.025$. This approach is used in combination with non-predictive TA approach in (32). The Root-Square Error (RSE) of trajectory tracking is shown in Figure 6. By integrating over the RSE results in Figure 6a, the overall RSE of PI-based approach is derived as 955 while this for the MPC-based approach is 719.3. Moreover, the overall energy consumption of the PI-based approach is 24.66 W.



(a) The reference and vessel's trajectory.



(b) Applied thruster forces.



(c) The angle of thrusters.

Fig. 4. Trajectory tracking performance of the proposed control approach.

5. CONCLUSION

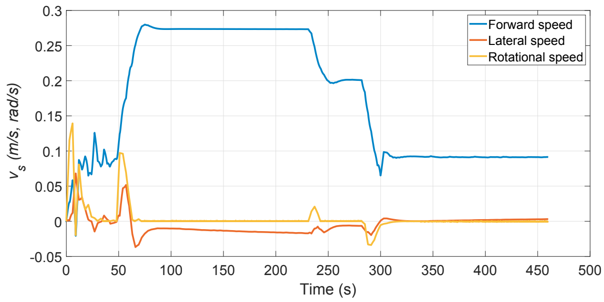
In this paper, Model Predictive Control (MPC)-based approaches have been proposed for trajectory tracking control and Thrust Allocation (TA) of autonomous vessels. Several linearization techniques have been adopted including Input-Output Feedback Linearization (IOFL) to enable the use of quadratic programming approaches for solving the optimization problems of MPC and TA problem. For the simulation experiment, a replica vessel model known as Tito-Neri has been adopted and the trajectory of real vessels in Amsterdam metropolitan waterways are chosen. The results show that the proposed approaches are capable of improving the system performance and decreasing the energy consumption.

REFERENCES

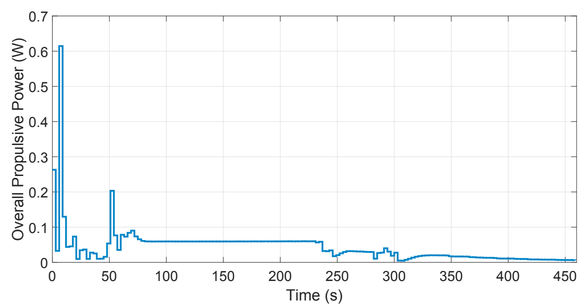
Abdelaal, M., Franzle, M., and Hahn, A. (2018). Nonlinear model predictive control for trajectory tracking and collision avoidance of underactuated vessels with disturbances. *Ocean Engineering*, 160, 168 – 180.

Abdelaal, M., Frnzle, M., and Hahn, A. (2016). Nmpc-based trajectory tracking and collison avoidance of underactuated vessels with elliptical ship domain. *IFAC-PapersOnLine*, 49(23), 22 – 27. 10th IFAC Conference on Control Applications in Marine SystemsCAMS 2016.

Ashrafiuon, H., Muske, K.R., McNinch, L.C., and Soltan, R.A. (2008). Sliding-mode tracking control of surface vessels. *IEEE Transactions on Industrial Electronics*, 55(11), 4004–4012.



(a) The speed of the vessel.



(b) Propelling power during the voyage.

Fig. 5. Speed and propelling power of the vessel

Chen, L., Haseltalab, A., Garofano, V., and Negenborn, R.R. (2019a). Eco-vtf: Fuel-efficient vessel train formations for all-electric autonomous ships. In *2019 18th European Control Conference (ECC)*, 2543–2550.

Chen, L., Huang, Y., Zheng, H., Hopman, H., and Negenborn, R. (2019b). Cooperative multi-vessel systems in urban waterway networks. *IEEE Transactions on Intelligent Transportation Systems*, 1–14.

Fossen, T.I. (2011). *Handbook of Marine Craft Hydrodynamics and Motion Control*. Wiley, West Sussex, UK.

Haseltalab, A. and Negenborn, R.R. (2019a). Adaptive control for autonomous ships with uncertain model and unknown propeller dynamics. *Control Engineering Practice*, Vol. 91, 2019.

Haseltalab, A. and Negenborn, R.R. (2019b). Model predictive maneuvering control and energy management for all-electric autonomous ships. *Applied Energy*, 251, 113308.

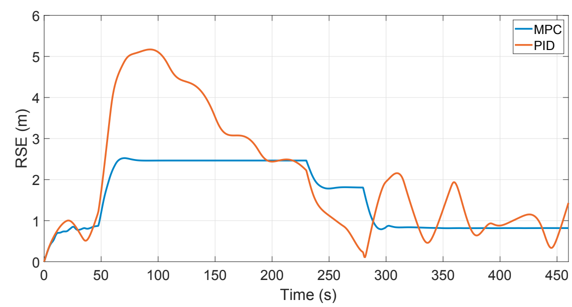
Zheng, H., Negenborn, R.R., and Lodewijks, G. (2013). Trajectory tracking of autonomous vessels using model predictive control. In *Proceedings of IFAC Congress, Cape Town, South Africa*, vol. 19.

Skjetne, R., Smogeli, Y.N., and Fossen, T.I. (2004). A Nonlinear Ship Manoeuvering Model: Identification and adaptive control with experiments for a model ship. *Modeling, Identification and Control*, 25(1), 3–27.

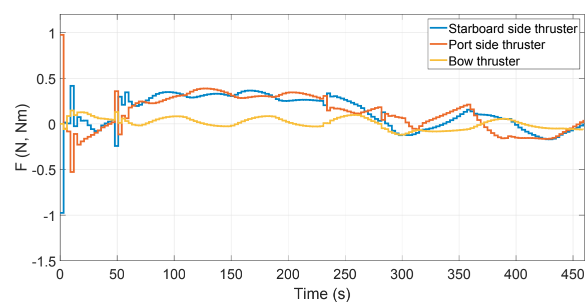
Sørensen, M.E.N. and Breivik, M. (2015). Comparing nonlinear adaptive motion controllers for marine surface vessels. *IFAC-PapersOnLine*, 48(16), 291 – 298.

te Braake, H.A.B., Botto, M.A., van Can, H.J.L., da Costa, J.S., and Verbruggen, H.B. (1999). Linear predictive control based on approximate input-output feedback linearisation. *IEE Proceedings - Control Theory and Applications*, 146(4), 295–300.

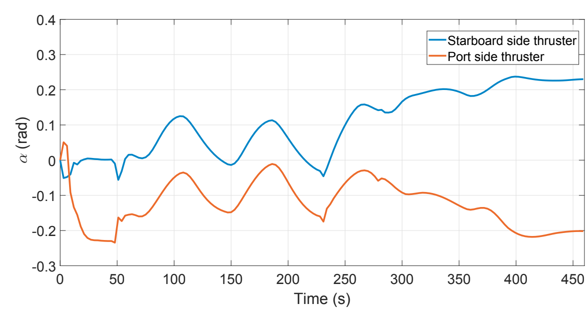
Zheng, H., Negenborn, R.R., and Lodewijks, G. (2016). Predictive path following with arrival time awareness for waterborne agvs. *Transportation Research Part C: Emerging Technologies*, 70, 214 – 237.



(a) Trajectory tracking RSE: MPC vs. PI.



(b) Applied thruster forces.



(c) The angle of thrusters.

Fig. 6. Trajectory tracking performance of the conventional PI-based approach.