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# Adaptive and High-Resolution Estimation of Specific Differential Phase for Polarimetric X-Band Weather Radars

RICARDO REINOSO-RONDINEL, CHRISTINE UNAL, AND HERMAN RUSSCHENBERG

*Faculty of Civil Engineering and Geosciences, Delft University of Technology, Delft, Netherlands*

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## ABSTRACT

One of the most beneficial polarimetric variables may be the specific differential phase  $K_{DP}$  because of its independence from power attenuation and radar miscalibration. However, conventional  $K_{DP}$  estimation requires a substantial amount of range smoothing as a result of the noisy characteristic of the measured differential phase  $\Psi_{DP}$ . In addition, the backscatter differential phase  $\delta_{hv}$  component of  $\Psi_{DP}$ , significant at C- and X-band frequency, may lead to inaccurate  $K_{DP}$  estimates. In this work, an adaptive approach is proposed to obtain accurate  $K_{DP}$  estimates in rain from noisy  $\Psi_{DP}$ , whose  $\delta_{hv}$  is of significance, at range resolution scales. This approach uses existing relations between polarimetric variables in rain to filter  $\delta_{hv}$  from  $\Psi_{DP}$  while maintaining its spatial variability. In addition, the standard deviation of the proposed  $K_{DP}$  estimator is mathematically formulated for quality control. The adaptive approach is assessed using four storm events, associated with light and heavy rain, observed by a polarimetric X-band weather radar in the Netherlands. It is shown that this approach is able to retain the spatial variability of the storms at scales of the range resolution. Moreover, the performance of the proposed approach is compared with two different methods. The results of this comparison show that the proposed approach outperforms the other two methods in terms of the correlation between  $K_{DP}$  and reflectivity, and  $K_{DP}$  standard deviation reduction.

## 1. Introduction

Weather observations are conventionally performed by single-polarization S- or C-band weather radars. Although these radars have substantially improved weather monitoring, researchers have found several limitations. For example, the spatial and temporal resolutions obtained from these conventional radars seem to be undesirable for the early detection of small but threatening features of convective weather as well as the detection of localized and heavy rainfall storms (Heinselman and Torres 2011; Schellart et al. 2012; Berne and Krajewski 2013). In contrast, single-polarization X-band weather radars are suited to obtain localized weather observations at resolutions higher than those of conventional radars. For example, a network of X-band weather radars in Hamburg, Germany, is used to observe precipitation at high resolutions (Lengfeld et al. 2016). Nonetheless, power attenuation and radar miscalibration may reduce the accuracy of single-polarization radar observables (Gourley et al. 2009).

One technique to potentially mitigate these issues is polarimetric radar technology (Doviak et al. 2000; Bringi and Chandrasekar 2001). For instance, in western Europe, polarimetric X-band weather radars are used to obtain high-resolution rainfall rates in order to cope with urban flooding (Ochoa-Rodriguez et al. 2015). In the United States, a network of X-band radars with polarimetric capabilities is used to improve the coverage of convective weather at low levels (McLaughlin et al. 2009), while in France a similar radar network is used to fill the gaps from operational radars in mountainous regions (Beck and Bousquet 2013). Moreover, in Germany, polarimetric Doppler X-band radars together with Doppler lidars are installed in international airports to provide local measurements of precipitation type and wind shear conditions, which are difficult to obtain from a national weather radar network (Ernsdorf et al. 2015).

In this work the polarimetric radar variable of interest is the specific differential phase  $K_{DP}$  because it is independent of attenuation and miscalibration; therefore, it can improve rainfall-rate estimation compared to power-based measurements, usually in heavy rain (Wang and Chandrasekar 2010). However, the accuracy

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*Corresponding author:* Ricardo Reinoso-Rondinel, r.r.reinosorondinel@tudelft.nl

of  $K_{DP}$  is not yet sufficient because the measured differential phase  $\Psi_{DP}$  could be significantly noisy in light and moderate rain. In addition, in moderate and heavy rain,  $\Psi_{DP}$  can include a nonnegligible scattering component resulting from the Mie scattering region, which is known as the backscatter differential phase  $\delta_{hv}$  (Matrosov et al. 2002; Trömel et al. 2013). Thus, accurate estimation of  $K_{DP}$  is necessary in order to unleash the potential of polarimetric weather radars.

Literature review reveals a large and continuous study to estimate  $K_{DP}$  and for simplicity it is divided into two groups. For the first group and in situations where  $\delta_{hv}$  can be neglected (e.g., at S-band frequency or light rain), straightforward approaches based on autoregressive average models are applied to smooth  $\Psi_{DP}$  (Bringi and Chandrasekar 2001; Matrosov et al. 2006; Vulpiani et al. 2012). For the same group but in cases where  $\delta_{hv}$  is of significance, Hubbert and Bringi (1995) introduced an iterative filtering approach to smooth  $\Psi_{DP}$  and filter  $\delta_{hv}$ . A common problem in this group is that  $K_{DP}$  is estimated with inadequate spatial resolutions that could result in an underestimation of  $K_{DP}$  peaks and therefore lead to an inaccurate phase-based rainfall estimation (Ryzhkov and Zrnić 1996). This limitation was reduced by Wang and Chandrasekar (2009), who developed an algorithm to filter  $\delta_{hv}$  and to control the smoothing degree on  $\Psi_{DP}$  while maintaining its spatial resolution. For the second group, the  $K_{DP}$  approaches included polarimetric relations in rain, such as the self-consistency (SC) relation, which formulates a dependency between  $K_{DP}$ , reflectivity  $Z$ , and differential reflectivity  $Z_{DR}$  (Scarchilli et al. 1996; Goddard et al. 1994; Gorgucci et al. 1992), and the relation between  $\delta_{hv}$  and  $Z_{DR}$ ,  $\delta_{hv}-Z_{DR}$  (Scarchilli et al. 1993; Testud et al. 2000). Otto and Russchenberg (2011), were able to estimate  $K_{DP}$  at range resolution scales using both relations, while Schneebeli and Berne (2012) included the  $\delta_{hv}-Z_{DR}$  relation in their Kalman filter approach. Recently, Giangrande et al. (2013) introduced a linear programming method that includes  $Z$  measurements, whereas Huang (2015) used the SC relation to estimate  $K_{DP}$  at S-, C-, and X-band frequencies. A disadvantage of using polarimetric relations is that uncertainties on  $Z$  and  $Z_{DR}$  measurements might reduce the performance of these approaches. Last, approaches in both groups could be associated with significant errors when  $K_{DP}$  is estimated at range resolution scales (Grazioli et al. 2014; Hu et al. 2015).

In this work an adaptive approach that includes polarimetric relations is presented to estimate accurate  $K_{DP}$  from  $\Psi_{DP}$  in rain, whose  $\delta_{hv}$  is of significance, at high spatial resolution. This paper is a follow-up of Otto and Russchenberg (2011) and is organized as follows. Two  $K_{DP}$  methods—given by Hubbert and Bringi

(1995), first group, and Otto and Russchenberg (2011), second group—are shortly described in section 2. They will be used for comparison with the proposed technique because (i) the method of Hubbert and Bringi (1995) is widely accepted for  $K_{DP}$  estimation at C- and X-band frequency and (ii) the focus of this work is to improve the technique introduced by Otto and Russchenberg (2011). In section 3, the adaptive high-resolution approach is introduced to estimate  $K_{DP}$  and model its standard deviation. To demonstrate the capability of this approach in terms of  $\delta_{hv}$  contamination and spatial resolution, one storm event is analyzed in section 4. In section 5, the performance of the adaptive approach is compared with those from section 2, using four storm events. In section 6, conclusions are drawn. Finally, the standard deviation of the proposed  $K_{DP}$  estimator is derived in appendix A, while the filter required by Hubbert and Bringi (1995) is designed in appendix B.

## 2. Specific differential phase: Background and estimation

In polarimetric weather radars, the difference between the horizontal and vertical polarization phases is defined as  $\Psi_{DP}$  ( $^{\circ}$ ). A conceptual model for a  $\Psi_{DP}$  profile is expressed as

$$\Psi_{DP}(r) = \Phi_{DP}(r) + \delta_{hv}(r) + \varepsilon, \quad (1)$$

where  $\Phi_{DP}(r)$  ( $^{\circ}$ ) represents the cumulative propagation phase shift along its course, while  $\delta_{hv}(r)$  ( $^{\circ}$ ) indicates local backscattering phase shifts manifested as “blips” or “bumps.” Random noise is represented by  $\varepsilon$  ( $^{\circ}$ ) and the range by  $r$  (km). Depending on the weather environment, the standard deviation error of  $\Psi_{DP}$ ,  $\sigma_P$  ( $^{\circ}$ ), varies between  $2^{\circ}$  and  $5^{\circ}$  (Bringi and Chandrasekar 2001).

The one-way  $K_{DP}$  ( $^{\circ} \text{ km}^{-1}$ ) is half the derivative of  $\Phi_{DP}$ :

$$K_{DP}(r) = \frac{1}{2} \frac{d\Phi_{DP}(r)}{dr}; \quad (2)$$

however, the estimation of accurate  $K_{DP}$  is not straightforward. For rainfall-rate applications,  $K_{DP}$  should be estimated such that the normalized standard error (NSE) of rainfall rate is less than 20% (Gorgucci et al. 1999; Bringi and Chandrasekar 2001). For instance, assuming a standard deviation of  $\Phi_{DP}$  is equal to  $3^{\circ}$  and estimating  $K_{DP}$  as in Eq. (2) over a pathlength of 2 km, the standard deviation of  $K_{DP}$ ,  $\sigma_K$ , would be  $1^{\circ} \text{ km}^{-1}$ . If the rainfall rate and  $K_{DP}$  are given by a power-law relation with a power coefficient of 0.8 (Ryzhkov et al. 2005; Wang and Chandrasekar 2010), then the value of  $\sigma_K$  is theoretically sufficient for  $K_{DP}$  larger than  $4^{\circ} \text{ km}^{-1}$ .

However, for  $K_{DP}$  values smaller than  $4^\circ \text{ km}^{-1}$ ,  $\sigma_K$  values are required to decrease accordingly, which can be achieved by increasing the pathlength with the sacrifice of spatial resolution.

*a. Conventional method*

Hubbert and Bringi (1995) introduced an iterative range filtering technique based on two steps. In the first step a low-pass filter is designed such that fluctuations resulting from  $\delta_{hv}$  and  $\varepsilon$  at scales of the radar resolution [ $\Delta r$  (km)] are eliminated from the  $\Psi_{DP}$  profile. The magnitude response  $H$  of this filter is given in the range domain by defining its Nyquist frequency as  $1/(2\Delta r)$ . In this domain,  $H$  is specified by the filter order and  $\Delta r/r_c$ , where  $r_c$  (km) is the required cutoff scale such that  $r_c > \Delta r$ . This means that the filter will maintain (or “pass”) spatial variations of  $\Psi_{DP}(r)$  at scales larger than  $r_c$ . However, spatial variations smaller than  $r_c$  will not be effectively suppressed because  $H$  is characterized by a transition bandwidth from the “pass” band to the “stop” band. This transition can be faster if the order of the filter is heavily increased. However, a high-order filter will strongly smooth the spatial variability of  $\Psi_{DP}$ , leading to a coarse spatial resolution of  $K_{DP}$ . Thus, the order of the filter is selected such that  $H$  will suppress spatial variations at scales slightly larger than  $\Delta r$  without compromising the spatial resolution of  $K_{DP}$ . This filter is referred to as a “light” filter that will lead to reduced  $\sigma_P$  and thereby  $\sigma_K$ .

In the second step the light filter is applied several times to eliminate extended  $\delta_{hv}$  fluctuations, up to  $r_c$  scales, as it would result from a “heavy” filter but without excessive smoothing. This process begins by filtering  $\Psi_{DP}$ , resulting in a first estimation of  $\Phi_{DP}$  ( $\hat{\Phi}_{DP}$ ). The absolute difference between  $\Psi_{DP}$  and  $\hat{\Phi}_{DP}$  profiles at each range gate are employed to generate a modified  $\Psi_{DP}$  profile ( $\check{\Psi}_{DP}$ ): if the absolute difference is larger than a threshold  $\tau$ , then  $\check{\Psi}_{DP}(r) = \hat{\Phi}_{DP}(r)$  holds; otherwise,  $\check{\Psi}_{DP}(r) = \Psi_{DP}(r)$  holds. This threshold is predefined from the interval  $[1.25; 2]\sigma_P$ . The process is iterated until  $\check{\Psi}_{DP}$  from two consecutive iterations show insignificant changes. Next,  $\check{\Psi}_{DP}$  from the last iteration is filtered one more time to obtain  $\Phi_{DP}$  and thereby  $K_{DP}$  according to Eq. (2).

Even though this is an elegant approach to estimate  $K_{DP}$  in real time, the following issues limit its purpose. First, spatial fluctuations larger than  $r_c$  will not be completely eliminated by the iterative step. Second, its performance is sensitive to the value of  $\tau$ . For example,  $\tau = 2\sigma_P$  will lead to a  $\check{\Psi}_{DP}$  more correlated to  $\Psi_{DP}$  than to  $\hat{\Phi}_{DP}$ , which might not be sufficient to eliminate unwanted fluctuations. Third, if  $r_c$  is increased or  $\Delta r$  is decreased, then the order of the filter should be

increased (Wang and Chandrasekar 2009), which may exacerbate these issues. In summary, in the design of the filter and the selection of the threshold, there is a trade-off between the smoothing extent and the spatial resolution required to estimate  $K_{DP}$  with small standard deviation and reduced bias.

*b. High-resolution method*

In contrast to the conventional approach, Otto and Russchenberg (2011) included  $\delta_{hv}$ - $Z_{DR}$  and SC relations in rain to estimate  $K_{DP}$  at X-band frequency. The first relation is represented by  $\delta_{hv} = e_1 Z_{DR}^{e_2}$  and the second by

$$K_{DP}^{SC} = c_1 10^{(c_2 Z/10)} 10^{(c_3 Z_{DR})}, \quad (3)$$

where  $Z$  and  $Z_{DR}$  are given in dBZ and decibels (dB), respectively, while coefficients  $e_1, e_2, c_1, c_2,$  and  $c_3$  establish the average fit from a set of drop size distributions (DSDs), drop shape models, and temperatures. The specific differential phase in Eq. (3) is indicated by  $K_{DP}^{SC}$  to distinguish between  $K_{DP}$  from the SC relation and  $K_{DP}$  from the high-resolution approach. Otto and Russchenberg (2011) used a normalized gamma distribution to model DSDs. To represent rain variability, 1500 DSDs were modeled by varying the parameters of the distribution (i.e., median volume diameter, concentration, and shape parameter). In addition, a hybrid drop shape model that consists of three models and temperatures in the range of  $1^\circ$ - $25^\circ\text{C}$  was considered to express a wide range of possibilities. The coefficients for the  $\delta_{hv}$ - $Z_{DR}$  relation are  $e_1 = 1$  and  $e_2 = 1.8$ , while for the SC relation they are  $c_1 = 1.37 \times 10^{-3}$ ,  $c_2 = 0.68$ , and  $c_3 = -0.042$ .

The measurements for  $Z$  and  $Z_{DR}$  are corrected for attenuation and differential attenuation, respectively, according to Bringi et al. (1990). The path-integrated attenuation (PIA) in reflectivity and in differential reflectivity ( $PIA_{DP}$ ) is given by  $0.34\Delta\Psi_{DP}$  and  $0.05\Delta\Psi_{DP}$ , respectively, where both coefficients are obtained from scattering simulation. For a  $\Psi_{DP}$  profile represented by Eq. (1), the difference of  $\Psi_{DP}$  in a path  $\overline{ab}$  can be approximated as  $\Delta\Psi_{DP} = \Delta\Phi_{DP} + \Delta\delta_{hv}$ , where  $\Delta\Psi_{DP} = \Psi_{DP}(b) - \Psi_{DP}(a)$  and  $b > a$ . The length of  $\overline{ab}$  could be as small as  $\Delta r$  or as large as the maximum unambiguous range. To identify whether  $\Delta\delta_{hv}$  is negligible, the  $\delta_{hv}$ - $Z_{DR}$  relation is used in the following assumption: if  $|Z_{DR}(b) - Z_{DR}(a)|$  is smaller than 0.3 dB, then  $\delta_{hv}(b) - \delta_{hv}(a) \approx 0^\circ$ . If that case is satisfied, then  $\Delta\Psi_{DP}$  is retained for further processes; otherwise,  $\Delta\Psi_{DP}$  is discarded. Multiple  $\Delta\Psi_{DP}$  samples associated with negligible  $\Delta\delta_{hv}$  are obtained by considering more paths. These samples are weighted by  $Z$  and  $Z_{DR}$  using the SC relation to obtain  $\Delta\Psi_{DP}$  samples at  $\overline{\Delta r}$  scale. For instance, the weight  $w$  at gate  $i$  within path  $\overline{ab}$  is

$$w(i) = \frac{K_{DP}^{SC}(i)}{\sum_a^b K_{DP}^{SC}(g)}; \quad \text{with } g = a, \dots, b. \quad (4)$$

Weighted  $\Delta\Psi_{DP}$  samples are used according to Eq. (2) to obtain multiple  $K_{DP}$  samples. Finally,  $K_{DP}(i)$  is estimated by the arithmetic mean of these samples.

Otto and Russchenberg (2011) demonstrated their method using one rainfall event, showing a visual consistency between  $K_{DP}$  and  $Z$  as well as between  $\delta_{hv}$  and  $Z_{DR}$ . Nevertheless, fewer consistent results were observed for  $K_{DP}$  values smaller than  $4^\circ \text{ km}^{-1}$ . In addition, estimates of  $K_{DP}$  were associated with values of  $\sigma_K$  as high as  $3^\circ \text{ km}^{-1}$ . Moreover, the sensitivity of the SC relation to DSD, drop shape, and temperature variability and its impact on estimated weights were not discussed. In addition, the effect of uncertain measurements of  $Z$  and  $Z_{DR}$  caused by, for example, attenuation, partial beam blockage (PBB), and miscalibration on the performance of this approach remains an open question. Besides its limited validation, another matter is the computational time required to weigh a significant amount of  $\Delta\Psi_{DP}$  samples at each range gate, which might be inefficient for operational purposes.

### 3. Adaptive high-resolution approach

The presented approach is an improved version of the high-resolution method in order to address limitations associated with the conventional and high-resolution methods, mainly the low accuracy of  $K_{DP}$  in light rain, contamination resulting from unfiltered  $\delta_{hv}$ , sensitivity to  $Z$  and  $Z_{DR}$  measurements, and computational time. The inputs are radial measurements of  $\Psi_{DP}$ ,  $Z$ , and  $Z_{DR}$  in rain. In addition, a predetermined length interval  $[L_{min}; L_{max}]$  is required to control the selection of pathlengths. This interval is assumed to be defined by a user; however, possible values are discussed in section 3e. The adaptive approach consists of three processes: preprocessing, pathlength selection, and  $K_{DP}$  estimation. A flowchart to estimate  $K_{DP}$ , among other variables, for a given radial profile, is presented in Fig. 1.

#### a. Preprocessing

To correct  $Z$  and  $Z_{DR}$  profiles for attenuation and differential attenuation, respectively, Otto and Russchenberg (2011) used noisy  $\Delta\Psi_{DP}$  instead of  $\Delta\Phi_{DP}$ , which may decrease the accuracy of the method given by Bringi et al. (1990). In contrast, in the adaptive high-resolution approach, a linear regression fit over a 3-km window is applied to a  $\Psi_{DP}$  profile, resulting in a  $\Phi_{DP}$  profile ( $\Phi_{DP}^i$ ). Thus, attenuation-corrected reflectivity ( $Z^i$ ) and differential reflectivity ( $Z_{DR}^i$ ) are given as  $Z^i = z + 0.34\Delta\Phi_{DP}^i$  and  $Z_{DR}^i = z_{dr} + 0.05\Delta\Phi_{DP}^i$ ,

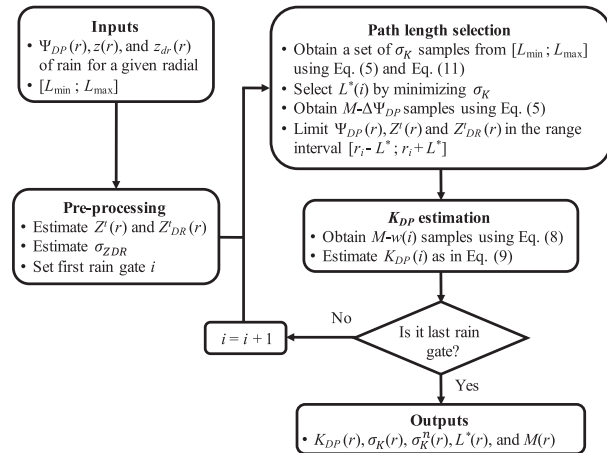


FIG. 1. Flowchart for the adaptive high-resolution  $K_{DP}$  approach.

respectively, where  $z$  (dBZ) and  $z_{dr}$  (dB) represent attenuated and differential attenuated measurements, respectively. This attenuation correction method might be sensitive to measurements errors, constant coefficients, and  $\delta_{hv}$  contamination. However, Gorgucci and Chandrasekar (2005) studied the method of Bringi et al. (1990) at X-band frequency and showed that this method performs fairly well with only a slight degradation of the average error for attenuation correction.

In this approach an estimate of the standard deviation of  $Z_{DR}^i$  profile,  $\sigma_{ZDR}$  (dB), is required by the pathlength selection process. A moving window of five gates is applied to the  $Z_{DR}^i$  profile so that local  $\sigma_{ZDR}$  samples are obtained. Then,  $\sigma_{ZDR}$  is estimated by averaging these  $\sigma_{ZDR}$  samples. The estimation of  $K_{DP}$  is achieved gate by gate, starting from ranges near the radar and continuing downrange. Assuming that the first gate with measurements of rain is located at  $r_i$ , the estimation of  $K_{DP}$  begins at gate  $i$ .

#### b. Pathlength selection

In the high-resolution technique,  $\Delta\Psi_{DP}$  samples were obtained from  $\Psi_{DP}$  using paths of any possible length. However,  $K_{DP}$  results were associated with high values of  $\sigma_K$  and significant computational time. In this work a pathlength  $L$  (km) for gate  $i$  is selected from  $[L_{min}; L_{max}]$  such that a theoretical  $\sigma_K$  is minimized. The formulation of a theoretical  $\sigma_K$  is shown in section 3e but now let the theoretical  $\sigma_K$  be a function of parameters  $L$  and  $M$ , where  $M$  represents the number of  $\Delta\Psi_{DP}$  samples with negligible  $\Delta\delta_{hv}$ . To identify negligible  $\Delta\delta_{hv}$ , the high-resolution technique used a fixed threshold to constrain  $|\Delta Z_{DR}^i|$ . However, a fixed threshold might not capture the possible variability of  $Z_{DR}$  within the storm. In this work the condition to identify negligible  $\Delta\delta_{hv}$  is given by

$$|\Delta Z'_{\text{DR}}| < \sigma_{\text{ZDR}}. \tag{5}$$

Equation (5) can be considered to be independent of any parameterized  $\delta_{\text{hv}}\text{-}Z_{\text{DR}}$  relation because this relation is not used quantitatively. Instead, Eq. (5) relies on the existing correlation between  $\delta_{\text{hv}}$  and  $Z_{\text{DR}}$ . Furthermore, the sudden variability in microphysics is taken into consideration by using  $\sigma_{\text{ZDR}}$  rather than an arbitrary threshold. Issues such as  $Z_{\text{DR}}$  miscalibration are mitigated by the estimation of  $\Delta Z'_{\text{DR}}$ . Equation (5) is referred to as the  $\Delta\Psi_{\text{DP}}$  filter condition.

The pathlength selection starts with  $L = L_{\text{min}}$ . For simplicity, a pathlength is of the form  $L = n\Delta r$ , where  $n$  is an integer larger than 1. Then, a range interval centered at gate  $i$  is defined as  $[r_i - L; r_i + L]$ . This range interval is used to limit the extent of  $Z'_{\text{DR}}$  and to obtain multiple  $\Delta Z'_{\text{DR}}$  samples. These samples are achieved by shifting a path of length  $L$ , within the interval starting at  $r_i - L$ ,  $n$  times with steps of  $\Delta r$ . In this manner,  $(n + 1)\text{-}\Delta Z'_{\text{DR}}$  samples are obtained. Next,  $M$  is calculated by counting the number of samples that satisfy Eq. (5). Note that  $M \leq (n + 1)$ . From  $L$  and  $M$ , a  $\sigma_K$  value is determined. To have a set of  $\sigma_K$  values, the same procedure is repeated for the next value of  $L$  until  $L = L_{\text{max}}$ . Finally, the pathlength that leads to the minimum  $\sigma_K$  is selected and is represented by  $L^*$ .

Repeating a similar procedure but with  $L^*$  [i.e., shifting a path of length  $L^*$  and using Eq. (5)],  $M\text{-}\Delta\Psi_{\text{DP}}$  samples with negligible  $\Delta\delta_{\text{hv}}$  are retrieved from  $\Psi_{\text{DP}}$  to estimate  $K_{\text{DP}}(i)$ . The remaining  $\Delta\Psi_{\text{DP}}$  samples are discarded to avoid bias on  $K_{\text{DP}}(i)$ . For the next steps, only  $\Psi_{\text{DP}}$ ,  $Z'$ , and  $Z'_{\text{DR}}$  profiles in the interval  $[r_i - L^*; r_i + L^*]$  are used.

*c.  $K_{\text{DP}}$  estimation*

To estimate  $K_{\text{DP}}(i)$ ,  $M\text{-}\Delta\Psi_{\text{DP}}$  samples should be downscaled from  $L^*$  to  $\Delta r$  scales. A downscaling weight  $w(i)$  was suggested by the high-resolution method according to Eq. (4). In contrast, in the adaptive high-resolution approach, a different formulation of  $w(i)$  is proposed in order to reduce its sensitivity to possible sources of uncertainty that were discussed in section 2b, mainly the sensitivity of the SC relation to rain variability, radar miscalibration, and PBB effects. Moreover, this formulation allows us to study statistics of  $w(i)$  and  $K_{\text{DP}}(i)$  for quality control purposes.

Consider a theoretical  $\Delta\Psi_{\text{DP}} \geq 0^\circ$  from a path  $\overline{ab}$  of length  $L$ . For gate  $i$  in the interval  $[a + 1; b]$ , the downscaling weight  $w(i)$  is expressed as a factor that weighs  $\Delta\Psi_{\text{DP}}$  such that  $\Delta\Psi_{\text{DP}}(i) = w(i)\Delta\Psi_{\text{DP}}$ , where  $\Delta\Psi_{\text{DP}}(i)$  represents the difference  $\Psi_{\text{DP}}(i) - \Psi_{\text{DP}}(i - 1)$  (i.e., at  $\Delta r$  scale). For derivation purposes, let  $w(i)$  be bounded by the interval  $[0; 1]$  and  $\sum_{a+1}^b w(i) = 1$ . Using

Eqs. (1) and (2),  $\Delta\Psi_{\text{DP}}(i)$  and  $\Delta\Psi_{\text{DP}}$  are expressed in terms of  $K_{\text{DP}}$  and  $w(i)$  as

$$w(i) = \frac{2\Delta r K_{\text{DP}}(i) + \delta_{\text{hv}}(i) - \delta_{\text{hv}}(i - 1)}{2LK_{\text{DP}} + \delta_{\text{hv}}(b) - \delta_{\text{hv}}(a)}. \tag{6}$$

Both  $K_{\text{DP}}(i)$  and  $K_{\text{DP}}$  are estimated using the SC relation according to Eq. (3) at scales of  $\Delta r$  and  $L$ , respectively. In the numerator of Eq. (6), adjacent  $\delta_{\text{hv}}$  values are assumed to be similar, so  $\delta_{\text{hv}}(i) - \delta_{\text{hv}}(i - 1)$  is approximately  $0^\circ$ . In the denominator, assuming that  $\Delta\Psi_{\text{DP}}$  satisfies Eq. (5), the difference  $\delta_{\text{hv}}(b) - \delta_{\text{hv}}(a)$  is negligible. In this manner,  $w(i)$  is formulated as

$$w(i) = \frac{\Delta r}{L} \frac{K_{\text{DP}}^{\text{SC}}(i)}{K_{\text{DP}}^{\text{SC}}}, \tag{7}$$

where the SC relation is used two times in contrast to the high-resolution method, which is used  $(n + 2)$  times [see Eq. (4) and replace  $b$  with  $a + n$ ]. In this way, added errors associated with the SC relation are reduced.

To downscale  $M\text{-}\Delta\Psi_{\text{DP}}$  samples (i.e.,  $\Delta\Psi_{\text{DP}}^{(j)}$  with  $j = 1, 2, \dots, M$ ) associated with  $M\text{-}\overline{ab}$  paths of length  $L^*$ , Eq. (7) is used and the  $j$ th weight is given as

$$\hat{w}^{(j)}(i) = \frac{\Delta r}{L^*} 10^{c_2[Z'(i) - \bar{Z}^{(j)}]/10} 10^{c_3[Z'_{\text{DR}}(i) - \bar{Z}'_{\text{DR}}]}, \tag{8}$$

where  $\bar{Z}^{(j)}$  and  $\bar{Z}'_{\text{DR}}^{(j)}$  represent the arithmetic mean of  $Z'(i)$  and  $Z'_{\text{DR}}(i)$  values in path  $\overline{ab}^{(j)}$ , respectively. Repeating Eq. (8) over the remaining paths,  $M\text{-}\hat{w}^{(j)}$  samples are obtained and  $K_{\text{DP}}(i)$  is estimated as

$$\hat{K}_{\text{DP}}(i) = \frac{1}{M} \sum_{j=1}^M \frac{\Delta\Psi_{\text{DP}}^{(j)} \hat{w}^{(j)}(i)}{2\Delta r}, \quad \text{with } j = 1, 2, \dots, M. \tag{9}$$

Once  $K_{\text{DP}}(i)$  is estimated, the pathlength selection and  $K_{\text{DP}}$  estimation processes are applied to gate  $i = i + 1$  until the last gate measured in rain. Hence, a  $K_{\text{DP}}$  profile is obtained as well as associated  $L^*$  and  $M$  profiles.

*d.  $K_{\text{DP}}$  uncertainty*

The  $K_{\text{DP}}$  estimator is a function of variables  $\Delta\Psi_{\text{DP}}^{(j)}$  and  $\hat{w}^{(j)}$ , which result from the  $\Delta\Psi_{\text{DP}}$  filter condition and  $\Delta\Psi_{\text{DP}}$  downscaling, respectively. Therefore, it is important to discuss errors associated with both variables. For this purpose,  $\Delta\Psi_{\text{DP}}^{(j)}$  is expressed as  $\Delta\Psi_{\text{DP}} = \Delta\Phi_{\text{DP}} + \varepsilon_\delta$ , where  $\varepsilon_\delta$  indicates possible errors from neglecting  $\Delta\delta_{\text{hv}}$ . Using the scattering simulation introduced in section 2b and setting  $\sigma_{\text{ZDR}}$  equal to 0.2 dB, the estimated mean and standard deviation of  $\varepsilon_\delta$  are  $0.04^\circ$  and  $0.6^\circ$ , respectively. The uncertainty of  $\hat{w}^{(j)}$  depends on the SC relation in rain. Trabal et al. (2014) demonstrated that the coefficients of the SC relation shown in Eq. (3) are sensitive to

temperature variability, while DSD and drop shape variabilities are well represented by a normalized gamma distribution and a hybrid drop shape model. Similar findings were reported by [Gourley et al. \(2009\)](#) and [Adachi et al. \(2015\)](#). Although  $\hat{w}^{(j)}$  is independent of  $c_1$ , any possible sensitivity to  $c_2$  and  $c_3$  is modulated by the difference  $Z^t(i) - \bar{Z}^{(j)}$  and  $Z_{\text{DR}}^t(i) - \bar{Z}_{\text{DR}}^{(j)}$ , respectively (i.e., it depends on the spatial variability of  $Z^t$  and  $Z_{\text{DR}}^t$  within path  $\overline{ab}^{(j)}$  instead of their absolute values). For example, in a uniform path,  $\hat{w}^{(j)}$  might be constant and equal to  $\Delta r/L^*$ . Moreover,  $\hat{w}^{(j)}$  is independent of constant biases in  $Z^t$  and  $Z_{\text{DR}}^t$  within  $\overline{ab}^{(j)}$  as well as radar miscalibration. This independence could reduce the impact of biases on  $Z^t$  and  $Z_{\text{DR}}^t$  areas caused by PBB. For simplicity the estimated weight is rewritten as  $\hat{w}^{(j)} = \hat{\alpha}^{(j)} \Delta r/L$ , where  $\hat{\alpha}^{(j)}$  is referred to as the SC ratio [see Eqs. (7) and (8)].

The uncertainty of  $\hat{\alpha}^{(j)}$  can be quantified by its NSE. [Scarchilli et al. \(1996\)](#) derived an expression for NSE of  $K_{\text{DP}}^{\text{SC}}$ , hereinafter  $\text{NSE}(K)$ , that is a function of  $c_2$ ,  $c_3$ , and variances of  $Z^t$  and  $Z_{\text{DR}}^t$ . Using this expression and basic properties of the variance, the NSE of  $\hat{\alpha}^{(j)}$  is given by  $\text{NSE}[\hat{\alpha}^{(j)}] = \sqrt{1 + 1/n} \text{NSE}(K)$ . For example, setting  $n$  equal to 10 (i.e.,  $L$  is 10 times  $\Delta r$ ) and using values of  $c_2$  and  $c_3$  given in [section 2b](#) and conventional accuracies of 1 and 0.2 dB for  $Z^t$  and  $Z_{\text{DR}}^t$ , respectively,  $\text{NSE}[\hat{\alpha}^{(j)}]$  results in 15.7%. This analysis can be used as guidance to identify which elements associated with the  $\Delta\Psi_{\text{DP}}$  filter condition and  $\Delta\Psi_{\text{DP}}$  downscaling can lead to an incorrect estimation of  $K_{\text{DP}}$ .

The uncertainty of  $\hat{K}_{\text{DP}}(i)$  is measured by its standard deviation as

$$\sigma_K(i) = \sqrt{\frac{1}{M} \sum_{j=1}^M \left[ \frac{\Delta\Psi_{\text{DP}}^{(j)} \hat{w}^{(j)}(i)}{2\Delta r} - \hat{K}_{\text{DP}}(i) \right]^2}; \quad \text{with} \quad (10)$$

$$j = 1, 2, \dots, M.$$

Equation (10) is referred to as the actual  $\sigma_K$ . In addition, the NSE of  $\hat{K}_{\text{DP}}(i)$ , hereinafter  $\sigma_K^n(i)$ , is given by  $[\sigma_K(i)/\hat{K}_{\text{DP}}(i)]100\%$ . Both actual  $\sigma_K$  and  $\sigma_K^n$  profiles are added to the output of [Fig. 1](#), which can be used for quality control purposes. In a similar manner but for the  $M-\hat{\alpha}(i)$  samples, profiles of their actual mean ( $\mu_\alpha$ ), standard deviation ( $\sigma_\alpha$ ), and NSE ( $\sigma_\alpha^n$ ) are also obtained.

#### e. $\sigma_K$ modeling

The uncertainty of  $\hat{K}_{\text{DP}}$  can be controlled by modeling its actual  $\sigma_K$ . Therefore, a theoretical  $\sigma_K$  is derived from Eq. (9), where  $\hat{w}^{(j)}$  is replaced by  $\hat{\alpha}^{(j)} \Delta r/L$ . Values of  $\Delta r$ ,  $L$ , and  $M$  are assumed to be given, while  $\Delta\Psi_{\text{DP}}^{(j)}$  and  $\hat{\alpha}^{(j)}$  are defined as random variables. Using properties of the variance that involve the sum and product of random

variables, a theoretical  $\sigma_K$  is approximated as (for a detailed derivation, refer to [appendix A](#))

$$\sigma_K = \frac{\mu_\alpha}{L} \sqrt{\frac{2\sigma_P^2 + \sigma_\varepsilon^2}{4M}}, \quad (11)$$

where  $\sigma_P^2$  and  $\sigma_\varepsilon^2$  are the variance of  $\Psi_{\text{DP}}$  and  $\varepsilon_\delta$ , respectively. The value of  $\mu_\alpha$  depends on the spatial variability of  $Z^t$  and  $Z_{\text{DR}}^t$ . For example,  $\mu_\alpha$  may be near 1 in stratiform rain, but it can be between 2 and 5 in convective rain. The value of  $\sigma_P$  also depends on rain type, while  $\sigma_\varepsilon$  is assumed equal to  $0.6^\circ$  as given in [section 3d](#). To illustrate theoretical  $\sigma_K$ , let us assume that  $\mu_\alpha$  and  $\sigma_P$  are equal to 3 and  $3^\circ$ , respectively. A similar value for  $\sigma_P$  was given by [Lim et al. \(2013\)](#).

Theoretical  $\sigma_K$  curves as a function of  $M$  and for different combinations of  $L$  are presented in [Fig. 2](#). Although  $\sigma_K$  in Eq. (11) is independent of  $\Delta r$ , the maximum value of  $M$  for a fixed pathlength  $L$  is given by  $(n + 1)$ , which is equivalent to  $L/\Delta r + 1$ . For  $\Delta r$  equal to 0.03 km, [Fig. 2a](#) shows that if  $L$  is near 1 km, then it is expected to obtain  $\sigma_K$  values larger than  $1^\circ \text{ km}^{-1}$ . However,  $\sigma_K$  values smaller than  $0.5^\circ \text{ km}^{-1}$  are expected if  $L$  is equal to 2 km and  $M$  is larger than 40. In terms of probability,  $M$  larger than 40 indicates that at least 60% of the total number of paths  $(n + 1)$  satisfies Eq. (5). For  $L$  larger than 3 km,  $\sigma_K$  curves continue for values of  $M$  beyond 100 but are not shown here. Note that the gap between consecutive  $\sigma_K$  curves decreases as  $L$  increases (e.g.,  $\sigma_K$  curves corresponding to 5 and 6 km are almost identical). Therefore, it is recommended to avoid large values of  $L$  (i.e., lengthy paths) associated with a small reduction of  $\sigma_K$ .

Recall that for a given value of  $L$ ,  $M$  is determined only after evaluating Eq. (5). To search for the combination of  $L$  and  $M$  that leads to the smallest theoretical  $\sigma_K$ , a pathlength interval  $[L_{\text{min}}; L_{\text{max}}]$  is considered instead of a single pathlength. For example, a length interval equal to [3;5] km could be defined from [Fig. 2a](#) as an input to the adaptive high-resolution approach. In that case,  $\sigma_K$  is expected to be small with a sufficient number of paths  $M$ . Similar  $\sigma_K$  curves can be produced for a coarser spatial resolution. For instance, if  $\Delta r = 0.25$  km, then a set of  $L$  values ranging from 4 to 14 km would be used to generate  $\sigma_K$  curves as shown in [Fig. 2b](#) and an interval equal to [6;10] km would be defined. Note that for larger  $\Delta r$ , a smaller number of paths  $M$  might lead to reasonable  $\sigma_K$  values (cf. [Figs. 2a and 2b](#)).

In this  $\sigma_K$  modeling, it is assumed that a user predefines a theoretical value for  $\mu_\alpha$  and  $\sigma_P$  according to the storm type, and sets  $[L_{\text{min}}; L_{\text{max}}]$ . For example, in a uniform  $Z$  field (i.e., low  $\mu_\alpha$  and  $\sigma_P$ ) or a more variable  $Z$  field (i.e., high  $\mu_\alpha$  and  $\sigma_P$ ),  $[L_{\text{min}}; L_{\text{max}}]$  can be decreased

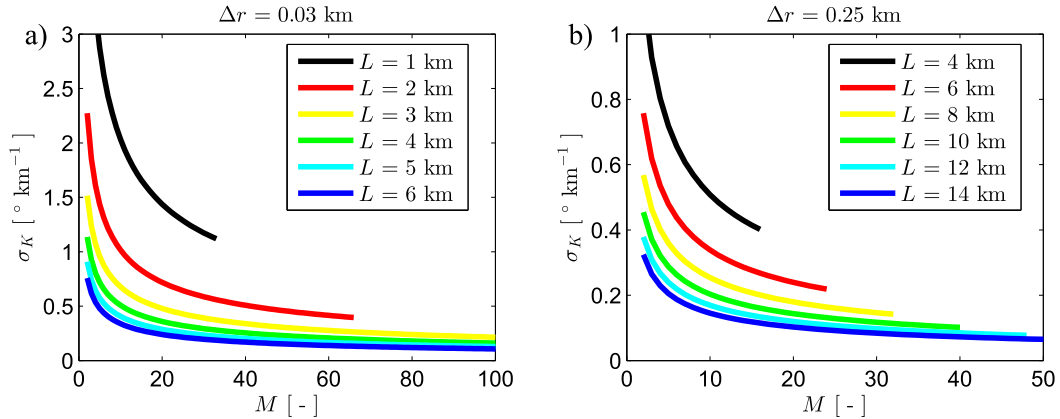


FIG. 2. Theoretical standard deviation of the adaptive high-resolution  $K_{\text{DP}}$  estimator as a function of the number of paths  $M$ . (a) Theoretical  $\sigma_K$  curves for  $\mu_{\alpha} = 3$  and  $\Delta r = 0.03 \text{ km}$ . (b) As in (a), but for  $\Delta r = 0.25 \text{ km}$ .

or increased according to theoretical  $\sigma_K$  curves. In summary, setting  $[L_{\min}; L_{\max}]$  allows us to avoid high values of actual  $\sigma_K$  as well as unnecessary lengthy paths associated with an increased dependence on the SC ratio and a large computational time.

#### 4. Analysis of the adaptive high-resolution approach

##### a. Data settings

The polarimetric X-band weather radar the International Research Centre for Telecommunications and Radar (IRCTR) drizzle radar (IDRA) is a frequency-modulated continuous wave system whose operational range is 15.3 km with a resolution of 0.03 km (Figueras i Ventura 2009). IDRA is located at the Cabauw Experimental Site for Atmospheric Research (CESAR) observatory in the Netherlands at a height of 213 m from ground level (Leijnse et al. 2010). It scans at a fixed elevation of  $0.50^{\circ}$  and rotates the antenna over  $360^{\circ}$  in 1 min with a beamwidth of  $1.8^{\circ}$ . Clutter echoes are removed by a filter based on spectral polarimetric processing (Unal 2009). Moreover, the measured linear depolarization ratio  $L_{\text{DR}}$  is used to filter areas that include particles other than rain and have low SNR, such that range gates with  $L_{\text{DR}}$  larger than  $-18 \text{ dB}$  are removed (Bringi and Chandrasekar 2001). This simple filtering procedure should be extended in the case of an automatic algorithm. The unwrapping of differential phase profiles is performed by detecting a differential phase jump between two adjacent gates, equal to 80% of the maximum differential phase of  $180^{\circ}$ .

A convective storm event was observed by IDRA on 10 September 2011. The plan position indicators (PPIs) of  $z$ ,  $z_{\text{dr}}$ , and  $\Psi_{\text{DP}}$  are shown in Fig. 3.

Attenuation-affected areas can be identified behind strong reflectivity echoes. The radial pattern observed in the fields of  $z_{\text{dr}}$  and  $\Psi_{\text{DP}}$  is probably due to a metallic fence that surrounds the IDRA platform. This introduces an azimuthal-dependence bias in the  $z_{\text{dr}}$  field in a similar way as PBB effects (Giangrande and Ryzhkov 2005). In addition,  $z$  might suffer from radar miscalibration as reported by Otto and Russchenberg (2011). Here, we use the opportunity to study the impact of rain attenuation and biases associated with PBB and miscalibration on the adaptive high-resolution approach.

##### b. Methodology

The proposed  $K_{\text{DP}}$  approach is analyzed using the storm event observed by IDRA. Besides the  $z$ ,  $z_{\text{dr}}$ , and  $\Psi_{\text{DP}}$  fields, a pathlength interval is required. For this requirement, Fig. 2a can be used for guidance because the theoretical  $\sigma_K$  curves were built for the same range resolution as for IDRA and with theoretical  $\mu_{\alpha}$  and  $\sigma_P$  equal to 3 and  $3^{\circ}$ , respectively, which express the spatial variability of the observed  $z$  and  $\Psi_{\text{DP}}$  fields. Thus, from Fig. 2a, a length interval of  $[3;5] \text{ km}$ , which is associated with  $\sigma_K < 0.5^{\circ} \text{ km}^{-1}$  for  $M > 20$ , is selected. This interval is used by the pathlength selection process for all radials. For simplicity,  $L^*(i)$  is indicated by  $L(i)$  and is determined by minimizing  $1/(L\sqrt{4M})$  instead of  $\sigma_K$ , as shown by Eq. (11), because theoretical  $\mu_{\alpha}$ ,  $\sigma_P$ , and  $\sigma_{\epsilon}$  remain constant in all radials. The coefficients  $c_2$  and  $c_3$  given in section 2b are used by the  $K_{\text{DP}}$  estimation process.

To study the performance of the  $K_{\text{DP}}$  approach in terms of spatial resolution and  $\delta_{\text{hv}}$  filtering, three variations on applying the  $\Delta\Psi_{\text{DP}}$  filter condition and  $\Delta\Psi_{\text{DP}}$  downscaling, given by Eqs. (5) and (9) respectively, are defined and indicated in Table 1. In case I,  $K_{\text{DP}}$  is estimated using the  $\Delta\Psi_{\text{DP}}$  filter condition without



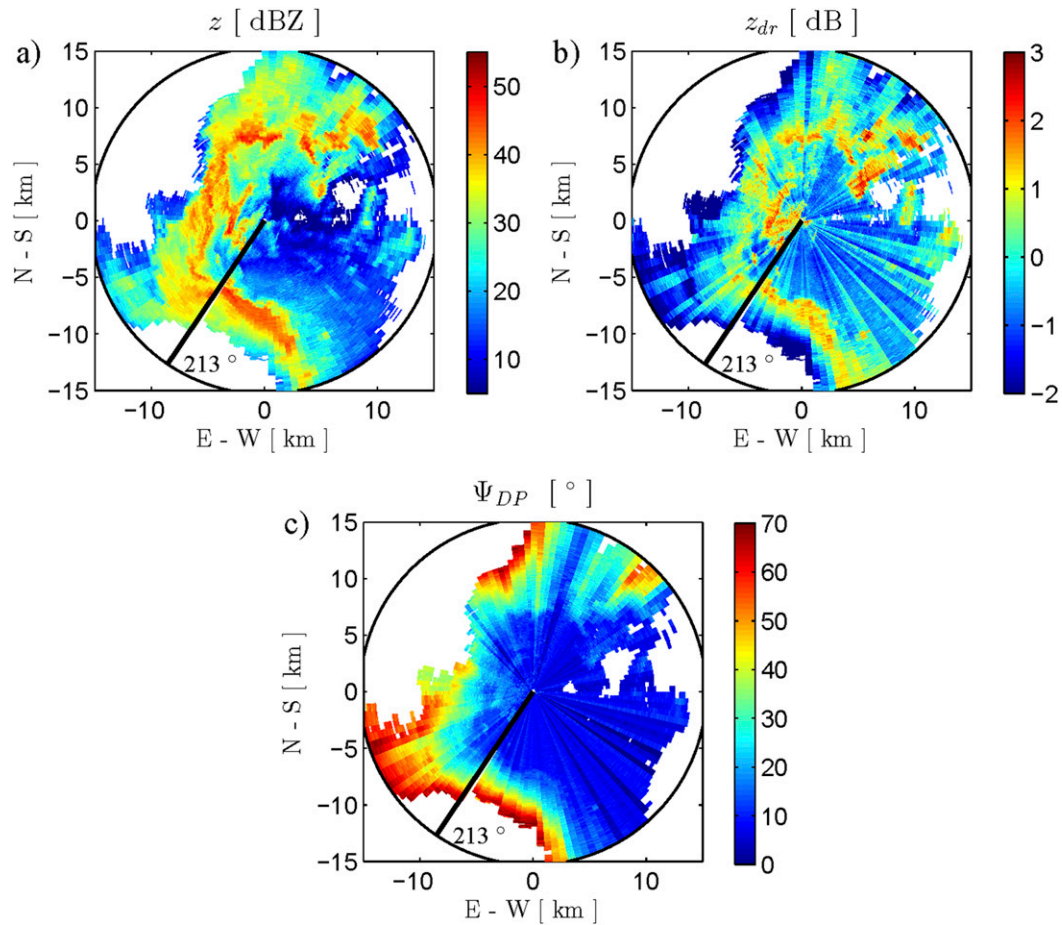


FIG. 3. Observations by the IDRA radar at an elevation angle of  $0.5^\circ$  in the Netherlands at 1950 UTC 10 Sep 2011. Fields of (a) attenuated reflectivity, (b) attenuated differential reflectivity, and (c) differential phase. Also shown is the 15.3-km range coverage of IDRA (black circles). Range resolution is 0.03 km.

downscaling  $\Delta\Psi_{DP}^{(j)}$  (i.e.,  $\hat{w}^{(j)} = \Delta r/L$  and the SC ratio is equal to 1). This case is denoted by  $\Delta\delta_{hv} \approx 0^\circ$  and  $\alpha \approx 1$ . In case II, the  $\Delta\Psi_{DP}$  filter condition is not used but  $\Delta\Psi_{DP}^{(j)}$  is downscaled. This case is expressed by  $\Delta\delta_{hv} \neq 0^\circ$  and  $\alpha \neq 1$ . In case III, the  $\Delta\Psi_{DP}$  filter condition is applied and  $\Delta\Psi_{DP}^{(j)}$  is downscaled, and this case is represented by  $\Delta\delta_{hv} \approx 0^\circ$  and  $\alpha \neq 1$ . Note that case III follows the proposed  $K_{DP}$  approach and that cases I and II are defined only for analysis purposes.

After a  $K_{DP}$  profile is obtained, a  $\Phi_{DP}$  profile is reconstructed by integrating Eq. (2) and  $\delta_{hv}$  is determined as  $\delta_{hv} = \Psi_{DP} - \Phi_{DP}$ . Moreover,  $Z$  and  $Z_{DR}$  are obtained by correcting  $z$  and  $z_{dr}$  in a similar manner as in section 3a but replacing  $\Delta\Phi_{DP}^i$  with  $\Delta\Phi_{DP}$ . This correction method could be improved by using more sophisticated techniques, such as those given by Park et al. (2005a) and Snyder et al. (2010). However, attenuation correction for  $z$  and  $z_{dr}$  is beyond the scope of this work, as our goal is to assess the performance of the  $K_{DP}$  approach.

### c. $K_{DP}$ and $\sigma_K$ results

#### 1) FOR A PPI RADIAL

The  $K_{DP}$  approach specified by cases I–III is applied to the azimuthal radial of  $213^\circ$  and its results are shown in Fig. 4. The downscaling aspect of  $K_{DP}$  is examined by comparing the  $\Psi_{DP}$ ,  $\Phi_{DP}$  (case I), and  $\Phi_{DP}$  (case III) profiles as shown in Fig. 4a. Observe that the total  $\Delta\Phi_{DP}$  for cases I and III is equal to  $45^\circ$ . However,  $\Phi_{DP}$  from case III captures the spatial variability and rapid increments of  $\Psi_{DP}$  better than  $\Phi_{DP}$  from case I. This can be seen by their corresponding  $K_{DP}$  profiles, which are also shown in Fig. 4a but they are shifted by  $-10^\circ \text{ km}^{-1}$ , where two  $K_{DP}$

TABLE 1. Three cases for  $K_{DP}$  estimation using the adaptive HR approach.

Cases	I	II	III
$\Delta\delta_{hv}$	$\Delta\delta_{hv} \approx 0^\circ$	$\Delta\delta_{hv} \neq 0^\circ$	$\Delta\delta_{hv} \approx 0^\circ$
$\alpha$	$\alpha \approx 1$	$\alpha \neq 1$	$\alpha \neq 1$

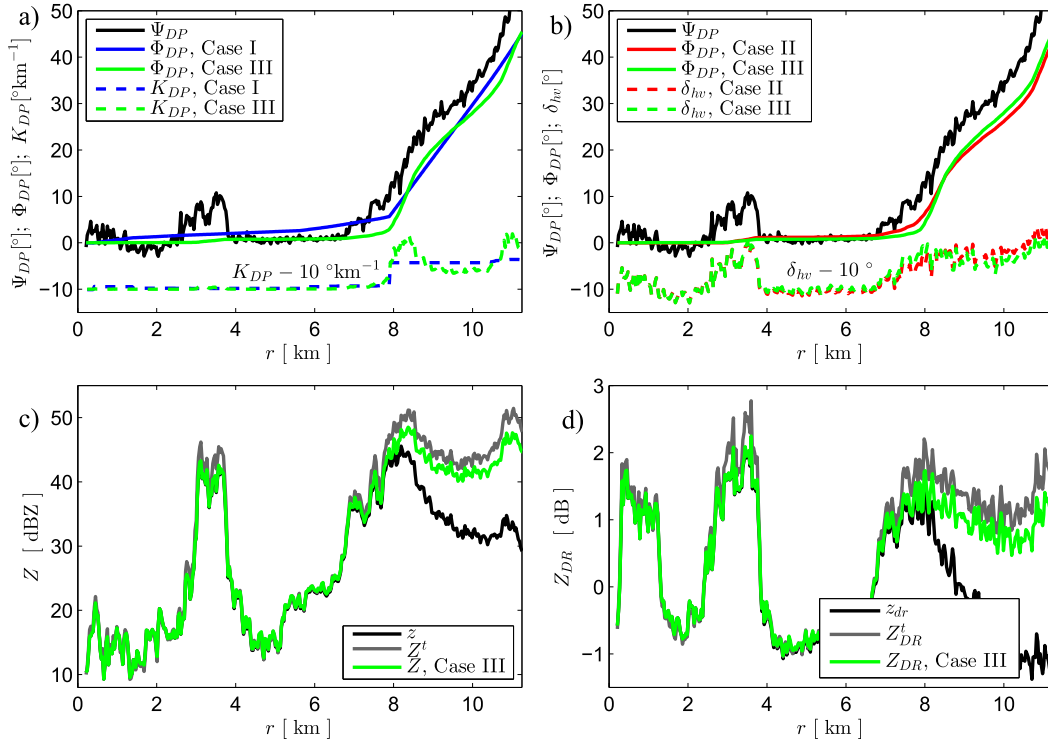


FIG. 4. Profiles from estimating  $K_{DP}$  using cases I–III at azimuthal radial of  $213^\circ$ . Shown are (a) the  $\Psi_{DP}$  profile (black) and the reconstructed  $\Phi_{DP}$  using case I (blue) and case III (green). Corresponding  $K_{DP}$  profiles, with an offset of  $-10^\circ \text{ km}^{-1}$ , are shown in a similar manner. (b) As in (a), but using cases II (red) and III (green) and showing  $\delta_{hv}$  instead of  $K_{DP}$ . (c) Profiles of  $z$  (black),  $Z^t$  (gray), and  $Z$  (green). (d) As in (c), but for  $Z_{DR}$ .

peaks (both of approximately  $10^\circ \text{ km}^{-1}$ ) from case III correspond to fast increments of  $\Psi_{DP}$  located downrange in convective areas. Observe that the  $\Psi_{DP}$  profile includes a  $\delta_{hv}$  “bump” in the range 2–4 km. To analyze the  $\delta_{hv}$  contamination aspect of  $K_{DP}$ ,  $\Phi_{DP}$  from cases II and III are shown in Fig. 4b as well as their corresponding  $\delta_{hv}$  profiles shifted by  $-10^\circ$ . This  $\delta_{hv}$  bump of 2-km length does not show an impact on  $\Phi_{DP}$  (case II) because its length is smaller than  $L_{\min} = 3$  km. Also,  $\Psi_{DP}$  values outside the bump are similar and therefore most  $\Delta\Psi_{DP}$  samples have values near  $0^\circ$ . In summary,  $K_{DP}$  estimation is not affected by this  $\delta_{hv}$  bump in both cases, II and III. Note that in the range 7–11 km,  $\Psi_{DP}$  increases rapidly, which probably means that raindrops of moderate to large size are present and thus significant  $\delta_{hv}$  values are expected. However, both  $\Phi_{DP}$  profiles are similar. This similarity may be due to  $\delta_{hv}$  values hardly varying in this range and thus  $\Delta\delta_{hv}$  samples do not impact  $\Delta\Psi_{DP}$  samples. Such a feature can be seen in  $\delta_{hv}$  (case III), where it shows a slight spatial variability. Further, the estimation of  $\delta_{hv}$  depends on  $\Phi_{DP}$  estimation, which can include accumulated  $K_{DP}$  errors (e.g., toward the end of the range). A rigorous estimation and analysis of  $\delta_{hv}$  are beyond the scope of this work; the focus here is on  $K_{DP}$  estimation.

Figure 4c shows attenuated  $z$ , corrected  $Z^t$ , and corrected  $Z$  profiles—the last profile being associated with case III. The correction of  $z$  is evident toward the convective range 7–11 km, where PIA reaches 15 dB at 11 km. Note that the  $Z^t$  profile shows values slightly larger than those of the  $Z$  profile because of unfiltered  $\delta_{hv}$ . In a similar manner, the correction of the  $z_{dr}$  profile is shown in Fig. 4d, where  $\text{PIA}_{DP}$  equals 2.3 dB at 11 km. Note that the  $\delta_{hv}$  and  $Z_{DR}$  profiles show a correlated behavior as expected from the  $\delta_{hv}$ – $Z_{DR}$  relation.

## 2) FOR A FULL PPI

The results from applying the  $K_{DP}$  approach, specified by case III, to all radials of Fig. 3 are shown in Fig. 5. The field of  $Z$  is plotted in Fig. 5a and the field of  $L$ , selected from the interval [3; 5] km, in Fig. 5b. The spatial variability of  $L$  exhibits an adaptive performance with the purpose of minimizing  $\sigma_K$ . The  $K_{DP}$  and  $\Phi_{DP}$  fields are shown in Figs. 5c and 5d, respectively. It can be seen that the  $K_{DP}$  field maintains the structure and resolution of the storm illustrated by the  $Z$  field, whereas the  $\Phi_{DP}$  field displays the propagation phase component of the  $\Psi_{DP}$  field depicted in Fig. 3c. Note that the  $K_{DP}$  field presents some gaps in

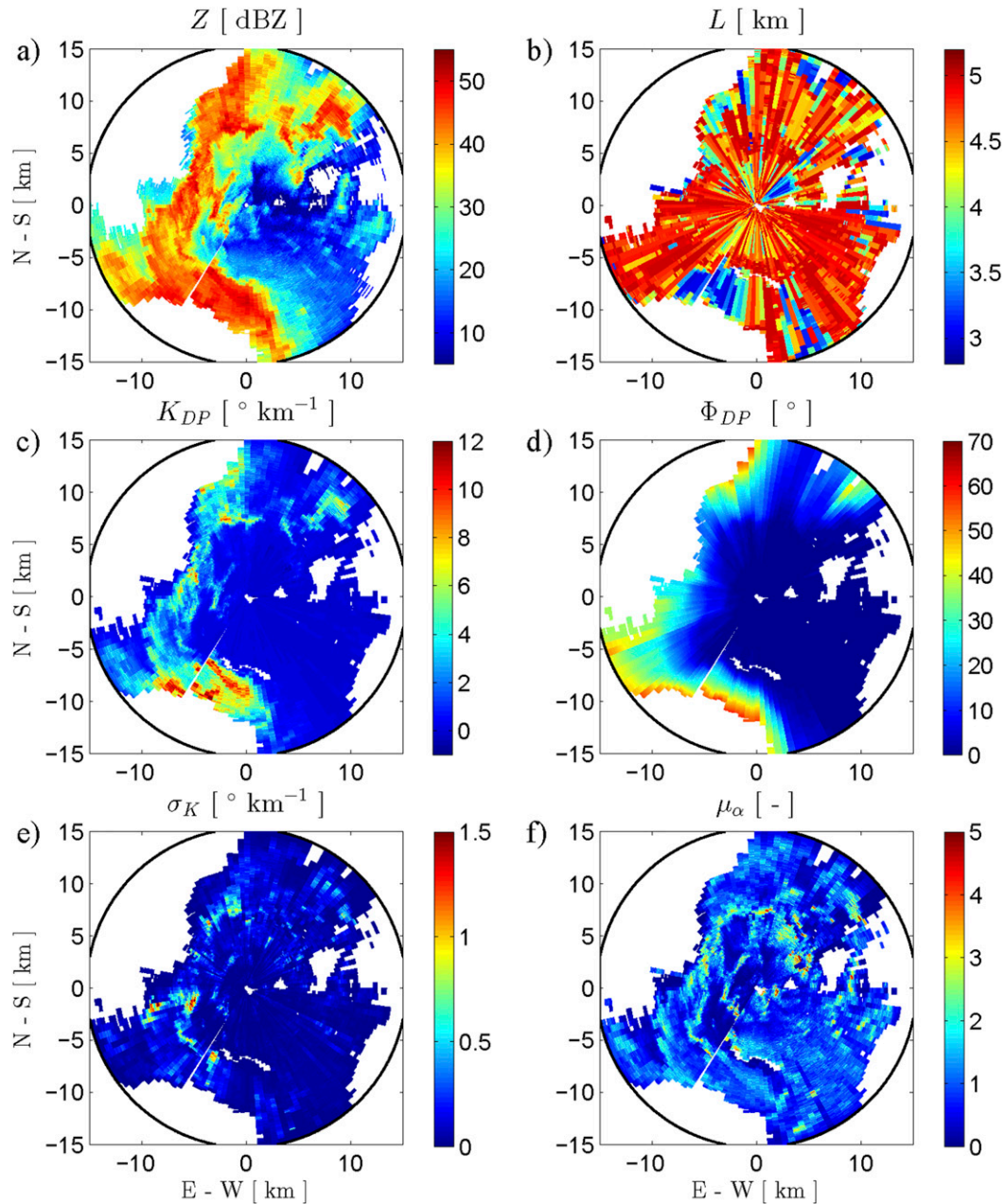


FIG. 5. PPI fields from estimating  $K_{DP}$  using case III: (a) attenuation-corrected  $Z$ , (b) selected  $L$ , (c) estimated  $K_{DP}$ , (d) reconstructed  $\Phi_{DP}$ , (e) actual  $\sigma_K$ , and (f) actual  $\mu_\alpha$ .

areas of measured  $Z$  (e.g., at approximately 8 km south from IDRA). In these areas,  $M$  is equal to 0, which means that for any bounded value of  $L$ , none of the  $(n + 1) - \Delta\Psi_{DP}$  samples satisfies Eq. (5). Such an issue could be avoided if, for instance,  $K_{DP}$  is estimated using case II instead of case III or interpolation algorithms are used. Figure 5e shows the actual  $\sigma_K$ , whose arithmetic mean is equal to  $0.1^\circ \text{ km}^{-1}$ . However, values as high as  $1^\circ \text{ km}^{-1}$  are visible near convective edges. This

increase in  $\sigma_K$  is partly due to a reduced number of  $\Delta\Psi_{DP}$  samples that satisfy Eq. (5). The actual  $\mu_\alpha$  are represented in Fig. 5f, whose values are mostly between 0 and 5. Thus, setting theoretical  $\mu_\alpha$  equal to 3 in this convective storm is a reasonable predefined. The field of  $\mu_\alpha$  also shows an adaptive characteristic of the  $K_{DP}$  approach as it handles the spatial variability of  $\Psi_{DP}$ . In a similar manner,  $K_{DP}$  is estimated using cases I and II.

To study the impact of unfiltered  $\delta_{\text{hv}}$  on the standard deviation of  $K_{\text{DP}}$ , actual  $\sigma_K$  resulting from cases II and III over all radials is displayed in Fig. 6 as a function of its corresponding number of paths  $M$ , with  $M > 1$ . Because of the large number of  $\sigma_K$  and  $M$  samples, they are plotted as 2D histograms for better visualization. Note that the  $\sigma_K$ – $M$  histogram from case II shows a very small dependence on  $M$  as opposed to Eq. (11). This holds for values of  $M$  up to 167 (i.e., the nearest integer less than  $L_{\text{max}}/\Delta r + 1$ ). Such behavior occurs because the  $\Delta\Psi_{\text{DP}}$  filter condition is not applied in case II and thereafter adjacent paths are employed to obtain  $\Delta\Psi_{\text{DP}}$  samples, which leads to an increased correlation coefficient between these samples. In a hypothetical situation with a correlation coefficient equal to 1, theoretical  $\sigma_K$  is no longer a function of  $M$  [see Eq. (A2) in appendix A]. In addition to these adjacent samples, unfiltered  $\Delta\delta_{\text{hv}}$ , and thereby  $\delta_{\text{hv}}$ , compromises the estimation of  $K_{\text{DP}}$  and increases the variability of actual  $\sigma_K$ . In contrast, the 2D histogram from case III shows a dependence on  $M$  because  $\sigma_K$  decreases when  $M$  increases as expressed by the theoretical  $\sigma_K$  in Eq. (11). Two theoretical  $\sigma_K$  curves, for the same range resolution of IDRA, are also plotted in Fig. 6 to compare theoretical  $\sigma_K$  with actual  $\sigma_K$  from case III. The upper curve is set with  $L = 3$  km (i.e.,  $L_{\text{min}}$ ) and  $\mu_\alpha = 5$  and the lower curve with  $L = 5$  km (i.e.,  $L_{\text{max}}$ ) and  $\mu_\alpha = 0.5$ , assuming the same theoretical values for  $\sigma_P$  and  $\sigma_\varepsilon$  as given in section 3e. For a fair comparison between actual  $\sigma_K$  near  $0^\circ \text{ km}^{-1}$  and the lower  $\sigma_K$  curve, only actual  $\sigma_K$  values slightly larger than  $0^\circ \text{ km}^{-1}$ —for example,  $\geq 0.05^\circ \text{ km}^{-1}$ —are considered. In this comparison, 91% of the  $\sigma_K$ – $M$  scatters are in between both curves, while only 2% are located above the upper curve.

### 3) $Z$ – $K_{\text{DP}}$ RELATION

Another manner to study the downscaling and  $\delta_{\text{hv}}$  contamination aspects of the  $K_{\text{DP}}$  approach is by examining the consistency between  $Z$  and  $K_{\text{DP}}$  (Park et al. 2005b). For this purpose, Fig. 7a compares the  $Z$ – $K_{\text{DP}}$  histograms from cases I–III. In case I  $K_{\text{DP}}$  is estimated at coarse resolution and its values are smaller than  $8^\circ \text{ km}^{-1}$ , while in cases II and III  $K_{\text{DP}}$  is estimated at  $\Delta r$  scales and  $K_{\text{DP}}$  values can be as high as  $12^\circ \text{ km}^{-1}$ . However, in case II the 2D histogram shows multiple negative outliers because of unfiltered  $\delta_{\text{hv}}$ , resulting in underestimated and overestimated  $K_{\text{DP}}$  values. Among these three cases, case III provides the best consistency because of the application of the  $\Delta\Psi_{\text{DP}}$  filter condition and  $\Delta\Psi_{\text{DP}}$  downscaling specified by the proposed  $K_{\text{DP}}$  approach.

For evaluation purposes, Fig. 7b shows the  $Z$ – $K_{\text{DP}}$  histogram from case III, the  $Z$ – $K_{\text{DP}}$  scatterplot using

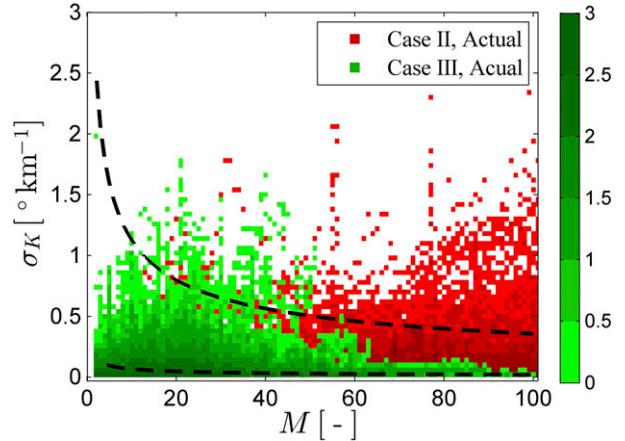


FIG. 6. 2D histograms of the actual  $\sigma_K$  and number of paths  $M$  resulting from case II (red) and case III (green). Theoretical  $\sigma_K$  curves, at  $\Delta r = 0.03$  km, are indicated (dashed black), where the upper curve corresponds to  $L = 3$  km and  $\mu_\alpha = 5$  and the lower curve to  $L = 5$  km and  $\mu_\alpha = 0.5$ . Frequency of occurrence is given in logarithm scale and applies to all subsequent histograms.

scattering simulation from section 2b, and its theoretical fit  $Z$ – $K_{\text{DP}}$  relation given by  $K_{\text{DP}} = 8.7 \times 10^{(0.69Z/10)-4}$ . Note that simulated  $Z$  values were shifted by  $-8$  dB in order to match those from case III, which could be due to incorrect attenuation correction and/or bias associated with PBB and miscalibration. In contrast, the  $K_{\text{DP}}$  axis shows a noticeable agreement between simulation and estimation. As a first step to analyze the discrepancy in the  $Z$  axis, a similar histogram is shown but with attenuated  $z$  instead of corrected  $Z$ , keeping estimated  $K_{\text{DP}}$ . From both plots it is clear that attenuation is not the major reason for this inconsistency but rather PBB and miscalibration.

### d. $K_{\text{DP}}$ in solid or mixed precipitation

As part of the presented analysis,  $K_{\text{DP}}$  estimation at X-band frequency over areas of solid and melting hydrometeors, such as graupel, hail, and snow, are shortly discussed. Because the shape and orientation response of particles are strongly related to their dielectric response, polarimetric signatures of solid hydrometeors is reduced because their dielectric constant factor is 20% or less that of raindrops. For example, Dolan and Rutledge (2009) and Snyder et al. (2010) simulated  $K_{\text{DP}}$  values at X band for solid and melting particles, showing a limited range of  $-1$  to  $1$  ( $^\circ \text{ km}^{-1}$ ) except for melting graupel, which can be between  $-2$  to  $7$  ( $^\circ \text{ km}^{-1}$ ). In addition, values of  $\delta_{\text{hv}}$  from solid hydrometeors are small except for large nonspherical hail or melting hail, in which  $\delta_{\text{hv}}$  can be in the order of  $4^\circ$ – $7^\circ$  (Trömel et al. 2013). Moreover, Schneebeli et al. (2014) calculated  $K_{\text{DP}}$  in snow using a Kalman filter–based approach and

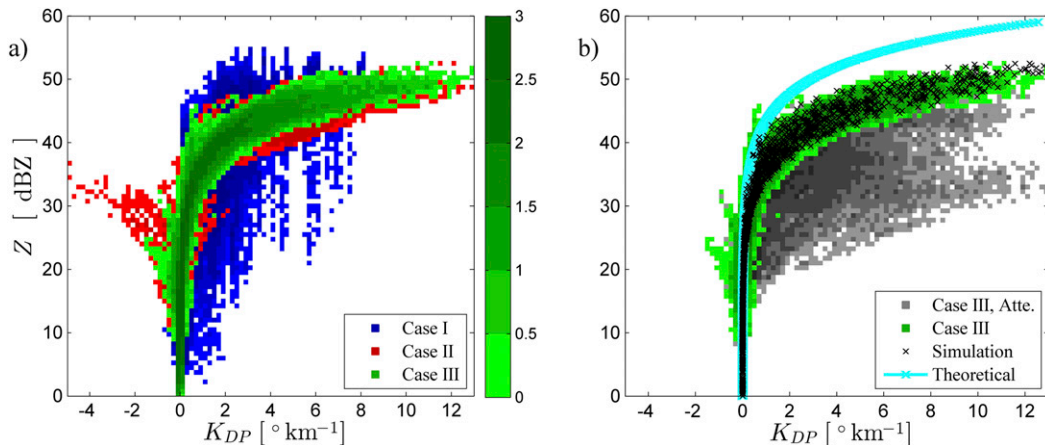


FIG. 7. (a) Histograms of  $Z$ - $K_{DP}$  resulting from cases I (blue), II (red), and III (green) are compared. (b) As in (a), but only for case III, where the  $Z$  axis before (gray) and after (green) attenuation correction is shown. In addition, a simulated  $Z$ - $K_{DP}$  scatterplot (1500 cross symbols) and its theoretical fit (cyan). Simulated  $Z$  values are shifted by  $-8$  dB.

found similar results when  $K_{DP}$  is estimated by the conventional technique. In this context the spatial variability of  $K_{DP}$  in nonrain regions may be less significant than in rain regions. Thus, the conventional approach or an autoregression-based model can be considered to complement  $K_{DP}$  estimates in nonrain regions as demonstrated by Lim et al. (2013). Alternately, the adaptive high-resolution approach can be also used by setting  $\hat{w}^{(i)}$  equal to  $\Delta r L^{-1}$  and  $M$  equal to  $n + 1$ , which is similar to case I but without the  $\Delta\Psi_{DP}$  filter condition, at expenses of low-resolution and possibly  $\delta_{hv}$  infiltration. In this scenario the theoretical  $\sigma_K$  is simplified to  $1.5\sqrt{2}\Delta r L^{-1.5}$  assuming  $\sigma_P = 3^{\circ}$ ,  $\mu_{\alpha} = 1$ , and  $M = n$ . For instance, for  $\Delta r$  values of 0.03, 0.25, and 1 km,  $L$  can be set to 3, 6, and 9 km, respectively, in order to obtain a theoretical  $\sigma_K$  equal to  $0.07^{\circ}$  km $^{-1}$ . However, further research is required to test the suggested alternative.

## 5. Assessment of the adaptive high-resolution approach

In this section the performance of the proposed  $K_{DP}$  approach specified by case III is compared with the conventional and high-resolution (HR) techniques. For this purpose, four storm events (E1–E4) observed by IDRA are described in Table 2. Although only observations of E3 at 1950 UTC were shown in section 4, the other events also display patterns related to attenuation, PBB, and miscalibration. In the conventional technique, the filter is designed using a 36th-order filter with  $r_c = 1$  km and  $\tau = 1.5\sigma_P$ . More details on the filter design are included in appendix B.

### a. During 1 min

The corresponding times for E1–E4 are 2151, 2225, 1950, and 0550 UTC, respectively. The  $Z$ - $K_{DP}$  histograms resulting from the three  $K_{DP}$  approaches applied to each 1-min event (i.e., one PPI) are shown in Fig. 8. The  $Z$ - $K_{DP}$  scatters from the conventional technique are significantly spread because  $K_{DP}$  is estimated at coarse resolution and  $\delta_{hv}$  is not properly filtered, which leads to negative and positive  $K_{DP}$  bias. In contrast, results from the HR method show more condensed relations. However, for  $Z$  values smaller than approximately 40 dBZ, multiple outliers are noticeable. Those outliers are substantially eliminated by the adaptive HR approach, which exhibits an enhanced consistency for weak and strong  $Z$ . To quantify the consistency of the results, the correlation coefficient between  $Z$  and  $K_{DP}$ ,  $\rho_{Z,K}$ , obtained from each approach is given by the second, third, and fourth columns of Table 3. From this quantification, the adaptive HR approach outperforms the other two techniques. For reference purposes, the  $\rho_{Z,K}$  from the simulated  $Z$ - $K_{DP}$  shown in Fig. 7b was also estimated and is equal to 0.75, which is similar to those resulted from the adaptive HR technique. Note that  $\rho_{Z,K}$  values resulting from simulation or observations can change according to the range of  $Z$  and  $K_{DP}$  values because the theoretical  $Z$ - $K_{DP}$  relation is nonlinear and thereby  $\rho_{Z,K}$  may be used as a relative quantity.

The results of  $Z$  and  $K_{DP}$  from the conventional and adaptive HR methods, presented in Fig. 8, show a similar discrepancy in the  $Z$  axis as indicated in Fig. 7b. Although the degree of discrepancy is not the same in all events, the  $Z$  values reached by the conventional

TABLE 2. Description of four storm events observed by IDRA with coverage of 15.3 km and  $\Delta r = 0.03$  km.

Events	Date	Period (UTC)	Storm type
E1	10 May 2011	2100–2300	Single cells, <sup>a</sup> light and moderate rain
E2	28 Jun 2011	2200–0000	A cluster of multicells (moderate rain) followed by a widespread area of light rain
E3	10 Sep 2011	1940–2140	A cell larger than 100 km <sup>2</sup> followed by single cells, moderate and heavy rain
E4	7 Oct 2011	0420–0620	Cells of irregular shapes, light and moderate rain

<sup>a</sup> Area of a cell is found in the range 10–100 km<sup>2</sup> with Z larger than 30 dBZ. Similar cell characteristics are defined by Johnson et al. (1998).

technique are in the same order as those from the adaptive HR approach. This indicates that Z is most likely biased and that the estimation of  $K_{DP}$  by the adaptive HR approach may not be affected by attenuation and biases associated with PBB and/or miscalibration.

A second manner to quantify the performance of the adaptive HR approach is by comparing its actual  $\sigma_K$  and  $\sigma_K^n$ , which were introduced as quality measures in section 3d, with the HR method for each event. The mean values of the  $\sigma_K$  field ( $\overline{\sigma_K}$ ) and  $\sigma_K^n$  field ( $\overline{\sigma_K^n}$ ) resulting from these techniques are given in Table 3. For the calculation of  $\overline{\sigma_K^n}$ , only gates with  $|K_{DP}| \geq 1^\circ \text{ km}^{-1}$  are considered. Note that  $\overline{\sigma_K}$  from the adaptive HR

approach is, on average, one-tenth of the HR method. In addition,  $\overline{\sigma_K^n}$  results from the adaptive HR approach are smaller than a reasonable error of 20%, while those from the HR method are much larger than 20%. Another quality measure, also given in section 3d, is  $\sigma_\alpha^n$ , which measures the percentage error of the actual  $\mu_\alpha$  estimated by the adaptive HR approach. The mean of the  $\sigma_\alpha^n$  field ( $\overline{\sigma_\alpha^n}$ ) for each event is found reasonably small as indicated in Table 3. Finally, in terms of computational time required by both techniques, the adaptive HR approach needs, on average, one-third of the time required by the HR method, which is on the order of a few minutes for 1 min of data, while for the conventional technique it is on the order of seconds.

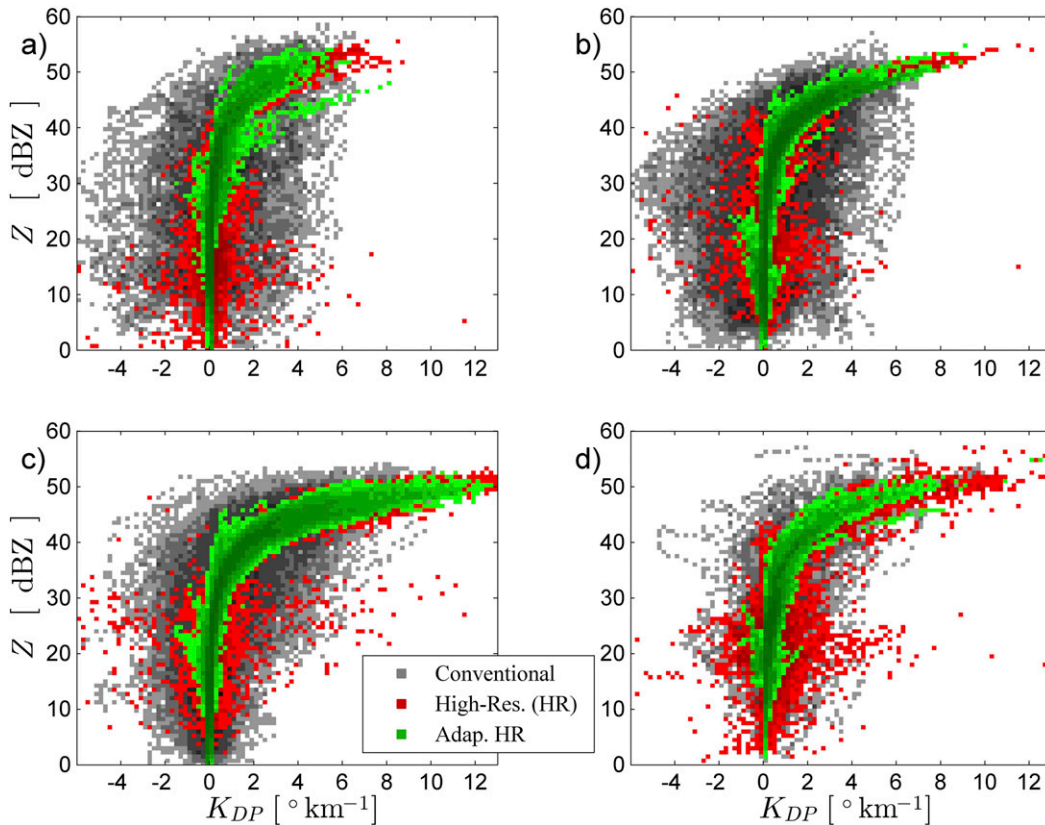


FIG. 8. (a) Histograms of Z– $K_{DP}$  for event E1. Conventional (gray), HR (red), and adaptive HR (green) approaches. (b)–(d) As in (a), but for E2–E4, respectively.

TABLE 3. Comparison of the three  $K_{DP}$  approaches—conventional (conv), HR, and adaptive HR (adap HR)—for the four storm events quantified by  $\rho_{Z,K}$ ,  $\overline{\sigma_K}$ ,  $\overline{\sigma_K^2}$ , and  $\overline{\sigma_\alpha^2}$ .

Event	$\rho_{Z,K}$ (-)			$\overline{\sigma_K}$ ( $^{\circ} \text{ km}^{-1}$ )		$\overline{\sigma_K^2}$ (%)		$\overline{\sigma_\alpha^2}$ (%)
	Conv	HR	Adap HR	HR	Adap HR	HR	Adap HR	Adap HR
E1	0.38	0.61	0.72	1.12	0.09	85	16	11
E2	0.48	0.70	0.74	0.93	0.09	50	9	13
E3	0.66	0.73	0.75	0.83	0.10	42	7	16
E4	0.53	0.67	0.75	1.22	0.06	83	6	10

### b. During 2 h

Next, the three  $K_{DP}$  techniques are compared and evaluated using the same quality measures  $\rho_{Z,K}$ ,  $\overline{\sigma_K}$ ,  $\overline{\sigma_K^2}$ , and  $\overline{\sigma_\alpha^2}$  and storm events E1–E4 but during 2-h periods as illustrated in Figs. 9–12, respectively. In general, it can be observed that the adaptive HR approach outperforms the other two methods, although the performance of each technique varies according to the storm scenario. For example, the conventional technique can lead to reasonable results when a storm consists of a large area of heavy rain because  $\Psi_{DP}$  profiles carry sufficient data samples with high SNR levels, reducing the impact from  $\Psi_{DP}$  outliers. In the HR method, acceptable results can be obtained in a scenario given by a large cell or a cluster of multiple cells with moderate to heavy rain because it allows for consideration of multiple  $\Delta\Psi_{DP}$  samples over extended paths, which reduces the impact of small and sometimes negative  $\Delta\Psi_{DP}$  values. In addition, this scenario may reduce the sensitivity of the downscaling weight  $w(i)$  to noisy measurements of  $Z$  and  $Z_{DR}$ . In contrast, the adaptive HR approach yields reliable results even when storm cells cover relatively small areas with light rain because  $\Delta\Psi_{DP}$  samples are adaptively selected over paths with lengths determined from a predefined interval so that a theoretical value of  $\sigma_K$  is minimized. Moreover, uncertainties associated with the SC relation are reduced as a result of the improved formulation of  $w(i)$  in this method. In case a storm cell becomes significantly small, such that the extents of  $\Psi_{DP}$  profiles are on the order of  $L_{min}$ , estimates of  $K_{DP}$  by the adaptive HR approach are not possible; this feature could be beneficial because accurate estimation of  $K_{DP}$  from limited data samples is rarely achieved.

The resulting time series of  $\rho_{Z,K}$  for event E3 indicate that during the first hour, the three  $K_{DP}$  approaches performed in a similar manner because  $K_{DP}$  estimates were obtained from a large cell with heavy rain in which the conventional and HR methods perform at their best. During the second hour, small cells with moderate rain were observed, leading to decreased performance of the conventional and HR methods. In event E2, during

2220–2300 UTC, the HR and adaptive HR approaches provided similar results and performed better than the conventional technique because this period was associated with a cluster of cells with moderate rain. After this period the adaptive HR approach maintained a satisfactory performance, while the performance of the conventional and HR methods decreased because values of  $K_{DP}$  were estimated from light rain. For events E1 and E4,  $\rho_{Z,K}$  time series obtained from the conventional and HR techniques are similar but smaller than those from the adaptive HR approach because in these events single cells with irregular shapes and light rain

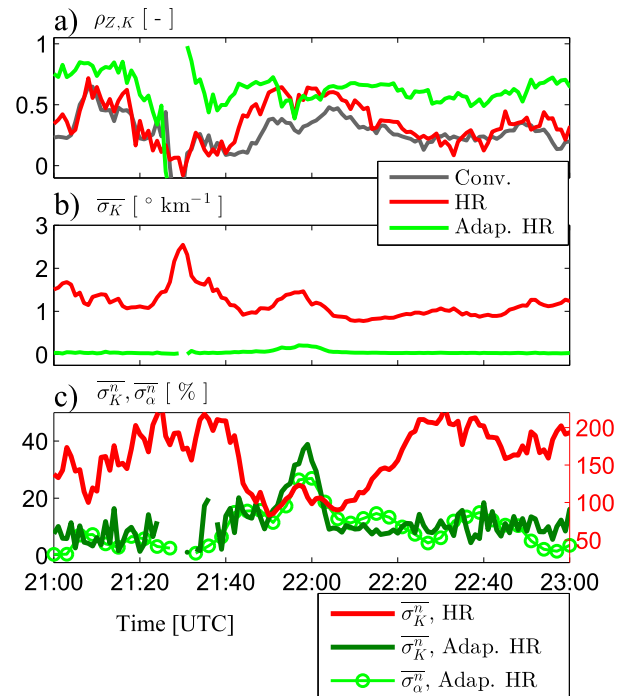


FIG. 9. Time series of quality measures from the three  $K_{DP}$  approaches for event E1. (a) Correlation coefficients from the conventional (gray), HR (red), and adaptive HR (green) techniques. (b) Mean values  $\overline{\sigma_K}$  from the HR (red) and adaptive HR (green) approaches. (c) The  $\overline{\sigma_K^2}$  from the HR (red, right y axis) and adaptive HR (green, left y axis) approaches. Also  $\overline{\sigma_\alpha^2}$  from the adaptive HR approach (green line with circles, left y axis).

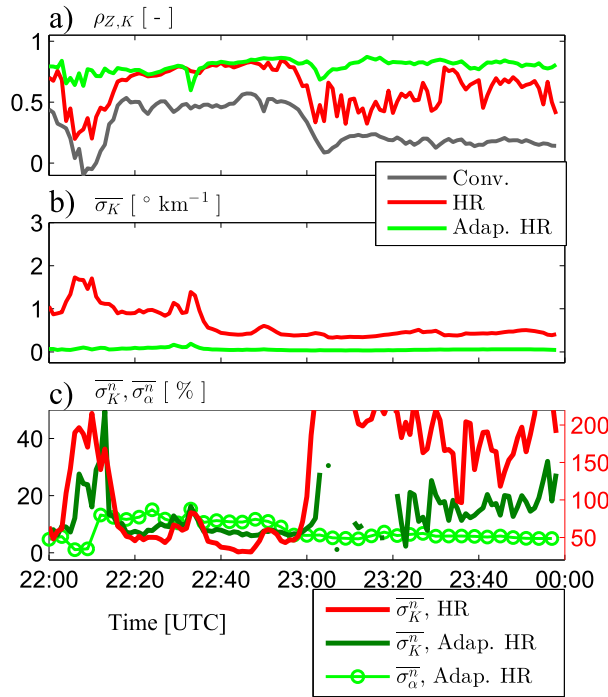


FIG. 10. As in Fig. 9, but for event E2.

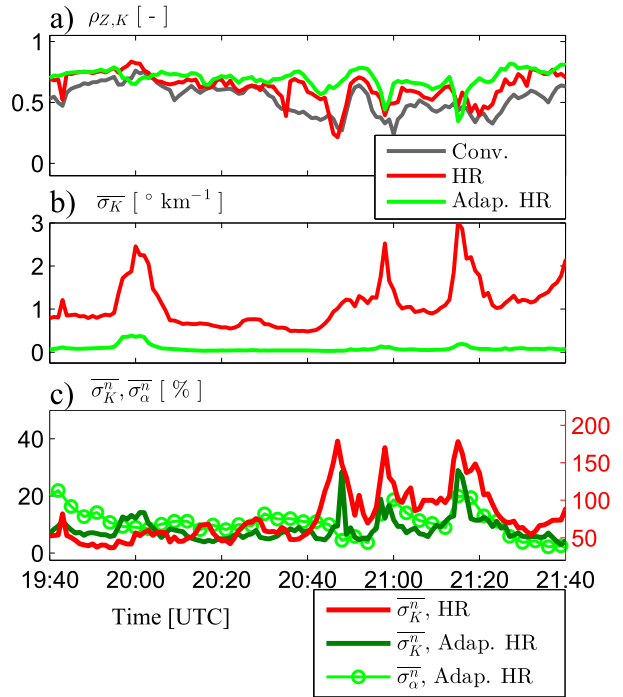


FIG. 11. As in Fig. 9, but for event E3.

were observed. Although  $\rho_{Z,K}$  time series from the three  $K_{DP}$  techniques can show similar results for a given storm scenario,  $Z$ - $K_{DP}$  relations might include multiple scatters as illustrated in Fig. 8, especially for weak  $Z$  values. As a result, the adaptive HR approach provides the best  $Z$ - $K_{DP}$  consistency for storm scenarios with different cell sizes and rain amounts.

Time series of  $\overline{\sigma_K}$  from the HR and adaptive HR techniques for the four events exhibited values between 1 and 3 and near 0 ( $^{\circ} \text{ km}^{-1}$ ). However, the results from the HR technique for event E2 showed values smaller than  $1^{\circ} \text{ km}^{-1}$  because of a widespread area of rain with low variability on  $Z$ ,  $Z_{DR}$ , and  $\Psi_{DP}$  fields. Nonetheless, its performance measured by  $\rho_{Z,K}$  remains below the adaptive HR approach. This shows a persistent accuracy in estimating  $K_{DP}$  by the adaptive HR approach. Furthermore, time series of  $\overline{\sigma_K^n}$  and  $\overline{\sigma_{\alpha}^n}$  resulting from the adaptive HR approach depicted, in all events, consistent percentage errors smaller than 20%. However, for events E1 (around 2200 UTC) and E3 (during some periods after 2040 UTC), the percentage errors increased because of inaccurate measurements of  $Z$  and  $Z_{DR}$  resulting from storm cells with heavy rain located adjacent to or on top of the radar, leading to power saturation in the receiver. This is an example of how  $\overline{\sigma_K^n}(i)$  or  $\overline{\sigma_K}$  and  $\overline{\sigma_{\alpha}^n}(i)$  or  $\overline{\sigma_{\alpha}}$  can be used to identify areas where  $K_{DP}$  estimates could be compromised. Another example of large  $\overline{\sigma_K}$  can be seen in E2 around 2210

UTC, when  $\overline{\sigma_K}$  values are as high as 40% because of small areas of light rain with a reduced number of  $\Delta\Psi_{DP}$  samples, affecting the accuracy of  $K_{DP}$  estimates. The discontinuity seen between 2300 and 2320 UTC is due to the constraint  $|K_{DP}| \geq 1^{\circ} \text{ km}^{-1}$ . A similar discontinuity is observed in event E1 around 2130 UTC. During the same event, a decreasing and discontinued behavior of  $\rho_{Z,K}$  is observed in the period 2120–2140 UTC. Such behavior is associated with a progressive reduction of storm cells in intensity and size, which led to light rain echoes with areas smaller than  $5 \text{ km}^2$  where the extents of  $\Psi_{DP}$  profiles are not sufficient for the estimation of  $K_{DP}$ . This means that  $K_{DP}$  cannot be estimated over a  $\Psi_{DP}$  segment whose length is on the order of or smaller than  $L_{\min}$ . For the case of the HR method, the time series of  $\overline{\sigma_K^n}$  indicated a limited performance, as  $\overline{\sigma_K^n}$  values are mostly larger than 50%.

## 6. Conclusions

Polarimetric studies have continuously focused on the estimation of  $K_{DP}$  because of its capability to overcome power attenuation and radar miscalibration. However, accurate estimation of  $K_{DP}$  at scales of the range resolution is challenging because  $K_{DP}$  requires significant spatial smoothing because of noisy  $\Psi_{DP}$  profiles, for example, in light rain. This problem is intensified at short wavelengths when  $\Psi_{DP}$  profiles include  $\delta_{hv}$  components,



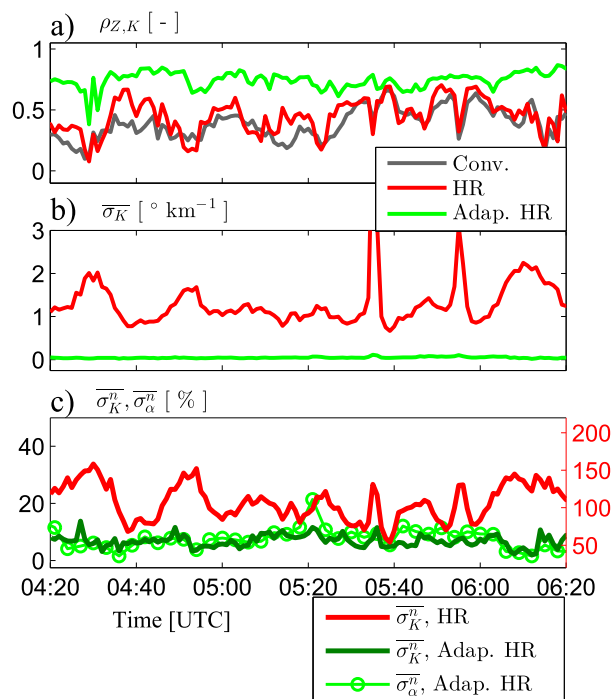


FIG. 12. As in Fig. 9, but for event E4.

for example, in moderate and heavy rain. In this work an adaptive HR approach has been presented to address these problems. The standard deviation of the proposed  $K_{DP}$  estimator has been derived and formulated in order to provide a pathlength interval that could lead to  $K_{DP}$  estimates with reduced error. This formulation takes into account the spatial variability of the storm and the radar range resolution.

A storm event observed by a polarimetric X-band weather radar during 1 min was used to analyze and test the performance of the  $K_{DP}$  estimator. Results showed that the estimated  $K_{DP}$  field kept the structure of the attenuation-corrected  $Z$  field without significant spatial distortion and that its estimation was associated with reduced errors indicated by the actual standard deviation (i.e., the  $\sigma_K$  field). The consistency between  $Z$  and  $K_{DP}$  showed that negative values of  $K_{DP}$ , associated with weak  $Z$ , can be reduced and that high  $K_{DP}$  values, associated with strong  $Z$ , can be obtained. To assess the performance of the adaptive HR approach to obtain accurate  $K_{DP}$  at range resolution scales, four storm events observed by the same radar but during 2-h periods were considered and the  $K_{DP}$  results were compared with the conventional and HR techniques. In general, the proposed approach was able to provide a correlation coefficient between  $Z$  and  $K_{DP}$  higher than the other two methods. In terms of standard deviations, the adaptive HR approach showed significant

improvements compared to the HR technique. The actual  $\mu_\alpha$  field, introduced by the adaptive HR approach, was associated with reduced uncertainty as indicated by the  $\overline{\sigma_\alpha^n}$  results. However, it was observed that the adaptive HR approach is able only to estimate  $K_{DP}$  over  $\Psi_{DP}$  segments larger than  $L_{min}$  and where the number of  $\Delta\Psi_{DP}$  samples is larger than 0.

Although the adaptive HR approach considers measurements of  $Z$  and  $Z_{DR}$  and constant coefficients related to the SC relation and attenuation correction in rain, the results of this method did not highlight issues related to radar miscalibration, radial patterns in  $Z_{DR}$  as a result of PBB, power attenuation, and variability on DSD and drop shape. Consequently, the adaptive HR approach, which uses the correlation between  $\delta_{hv}$  and  $Z_{DR}$  and the SC relation, is able to filter  $\delta_{hv}$  and maintain the spatial variability of  $\Psi_{DP}$ . Therefore, accurate  $K_{DP}$  profiles at high spatial resolution in light and heavy rain are achieved. However, if measurements of  $Z$  and  $Z_{DR}$  are associated with low SNR levels or are affected by offsets that fluctuate along a given radial, then it is expected that the accuracy of the downscaling weights and thereby  $K_{DP}$  estimates will be reduced. In general, quality control variables  $\sigma_K^n$  or  $\sigma_\alpha^n$ , which are associated with percentage errors larger than, for example, 20%, might lead to inaccurate  $K_{DP}$  estimates.

To achieve the ambitions of implementing the proposed  $K_{DP}$  algorithm for real-time operation, further studies are required. This effort includes estimating and updating the coefficients, which are used in the attenuation correction method and in the SC principle, to operational C- and S-band frequencies and, if possible, taking into account temperature variability. Note that at S-band frequency, the  $\Delta\Psi_{DP}$  filter condition can be excluded from the  $K_{DP}$  algorithm because  $\delta_{hv}$  is usually negligible. Moreover, an automatic algorithm might be needed to classify areas of rain, nonrain, and nonhydrometeors. For solid or mixed rain areas, either the SC ratio should be set to 1, so that  $K_{DP}$  is given at expenses of coarse spatial resolution, or another  $K_{DP}$  algorithm should be employed. Furthermore,  $[L_{min}; L_{max}]$  should be selected according to values of  $\Delta r$  and to predefined values of  $\mu_\alpha$  and  $\sigma_P$ ; the latter two can be set as a function of the rain scenario (e.g., 4 and 3° for convective and 1 and 2° for stratiform rain). Last, it is recommended to evaluate the proposed approach at longer ranges, where attenuation and nonuniform beamfilling affect the  $Z$  and  $Z_{DR}$  measurements, and in scenarios where PBB is associated with complex terrain features.

A reliable  $K_{DP}$  is one of the most powerful observables from polarimetric weather radars. The adaptive HR approach may prove to be key in addressing the dilemma between the spatial resolution and the accuracy of  $K_{DP}$

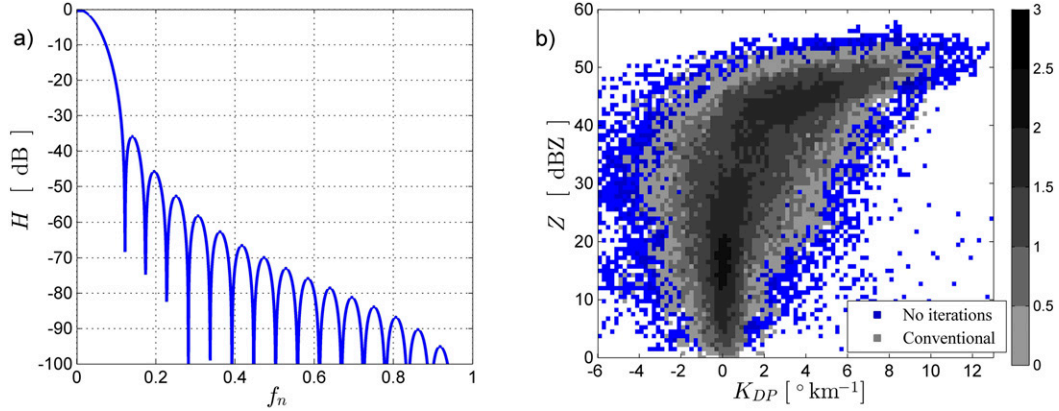


FIG. B1. (a) Magnitude response of a FIR low-pass filter specified by  $r_c = 1$  km,  $\Delta r = 0.03$  km, filter order of 36, and Hann window. (b) Histograms of  $Z$ - $K_{DP}$  from the first step (blue) and both steps (gray) of the conventional approach.

in rain. Moreover, the formulation of the theoretical  $\sigma_K$  and the capability of calculating the uncertainty of  $K_{DP}$  estimates gate by gate allow for reducing and controlling the errors in the estimation of  $K_{DP}$ . Even though this approach still needs to be tested in operational environments, urban hydrology and weather forecast communities may benefit from the proposed approach in terms of spatial resolution, accuracy, and quality control of  $K_{DP}$  estimates, which can lead to significant improvements in  $K_{DP}$ -based products.

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## APPENDIX A

### Standard Deviation of the $K_{DP}$ Estimator

For the purpose of deriving  $\sigma_K$ , the weight in Eq. (8) is expressed as  $\hat{w}^{(j)} = \hat{\alpha}^{(j)} \Delta r / L$  and therefore the  $K_{DP}$  estimator given by Eq. (9) is rewritten as

$$K_{DP} = \frac{1}{2L} \left[ \frac{1}{M} \sum_{j=1}^M \Delta \Psi_{DP}^{(j)} \alpha^{(j)} \right]; \quad \text{with } j = 1, 2, \dots, M. \quad (\text{A1})$$

Note that this expression includes the pathlength  $L$  and omits the index  $i$  for simplicity. Both  $\Delta \Psi_{DP}^{(j)}$  and  $\alpha^{(j)}$  are considered to be independent random variables (rvs) and hence their product is also an rv and denoted by  $\kappa^{(j)}$  with  $j = 1, 2, \dots, M$ . Assume that  $\kappa^{(1)}, \kappa^{(2)}, \dots, \kappa^{(M)}$  have

the same variance  $\sigma_\kappa^2$  and that the pair  $[\kappa^{(j)}, \kappa^{(m)}]$ , with  $j \neq m$ , has a constant covariance  $\gamma_\kappa$ . Using the variance property of the sum of correlated rvs and the relation  $\gamma_\kappa = \sigma_\kappa^2 \rho_\kappa$ , where  $\rho_\kappa$  is the correlation coefficient, the variance of the  $K_{DP}$  estimator  $\sigma_K^2$  is expressed as

$$\sigma_K^2 = \frac{1}{(2ML)^2} M \sigma_\kappa^2 [1 + (M-1) \rho_\kappa]. \quad (\text{A2})$$

If the term  $\sigma_\kappa^2$  is rewritten using  $\Delta \Psi_{DP}^{(j)}$  and  $\alpha^{(j)}$ , we obtain

$$\sigma_K^2 = \frac{1}{(2L\sqrt{M})^2} [\mu_\alpha^2 \sigma_\Delta^2 + \sigma_\alpha^2 (\mu_\Delta^2 + \sigma_\Delta^2)] [1 + (M-1) \rho_\kappa], \quad (\text{A3})$$

where  $\mu_\alpha$  and  $\mu_\Delta$  represent the mean of  $\alpha^{(j)}$  and  $\Delta \Psi_{DP}^{(j)}$ , respectively. Similarly,  $\sigma_\alpha^2$  and  $\sigma_\Delta^2$  indicate their variance. Equation (A3) can be reduced if  $\rho_\kappa$  and  $\sigma_\alpha^2$  are assumed to be significantly small. In consequence, Eq. (A3) is simplified to

$$\sigma_K^2 = \frac{\mu_\alpha^2 \sigma_\Delta^2}{(2L\sqrt{M})^2}. \quad (\text{A4})$$

To include residuals from the  $\Delta \Psi_{DP}$  filter condition (i.e., neglecting  $\Delta \delta_{hv}$ ), the difference in differential phase is represented by the sum of two uncorrelated rvs:  $\Delta \Psi_{DP}^{(j)} + \varepsilon_\delta^{(j)}$ , where  $\varepsilon_\delta^{(j)}$  has a mean and variance equal to  $0^\circ$  and  $\sigma_\varepsilon^2$ , respectively. Thus, Eq. (A4) is rewritten as

$$\sigma_K^2 = \frac{\mu_\alpha^2 (\sigma_\Delta^2 + \sigma_\varepsilon^2)}{(2L\sqrt{M})^2}. \quad (\text{A5})$$

For a given path  $\overline{ab}$ ,  $\Delta \Psi_{DP}^{(j)}$  is expressed as  $\Psi_{DP}(b) - \Psi_{DP}(a)$ , which is the difference between two rvs. If both rvs have the same variance  $\sigma_p^2$ , then the variance of  $\Delta \Psi_{DP}^{(j)}$  is

given by  $\sigma_{\Delta}^2 = 2\sigma_p^2$ . As a result, a theoretical  $\sigma_K$  for the  $K_{DP}$  estimator is defined as

$$\sigma_K = \frac{\mu_{\alpha}}{L} \sqrt{\frac{2\sigma_p^2 + \sigma_{\varepsilon}^2}{4M}}. \quad (\text{A6})$$

## APPENDIX B

### Filter Design to Estimate $K_{DP}$ by the Conventional Technique

A finite impulse response (FIR) filter type is selected for designing the light filter (first step). The filter order is set to 36, while the required  $r_c$  is set to 1 km. In addition, a Hann window function is included to obtain a magnitude response  $H$  (dB) with small sidelobes. The response  $H$  is shown in Fig. B1(a) as a function of the normalized range scale  $f_n = \Delta r/r_s$ , where  $r_s \geq \Delta r$  and  $r_s$  in km. Note that  $H$  reaches approximately  $-40$  dB at  $f_n = 0.1$ . This means that the light filter is designed such that spatial variations at range scales smaller than 0.3 km are suppressed. Next, this filter is iterated several times (second step) by setting  $\tau$  equal to  $1.5\sigma_p$ . The storm event E3 at 1950 UTC is used for demonstration purposes. The  $Z$ - $K_{DP}$  histograms from the first and second steps are shown in Fig. B1(b). Observe that the iterative step eliminates several outliers without excessively smoothing  $K_{DP}$ .

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