A stochastic approach on predicting the economic life of assets

A case study on HVAC systems of petrol stations assets in The Netherlands

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Master of Science Thesis

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Abstract

Many firms are occupied with determining the optimal replacement time of machinery. Machine replacement is a complex investment decision that requires the estimation of future cash flows and other parameters. The non-deterministic character of future cash flows has given rise to stochastic models, that take into account this uncertainty. This study has applied a theoretical stochastic asset replacement model in practice. It was found that the stochastic replacement model can be used on real data by performing a weighted least squares (WLS) regression. Decision-makers should however be aware of the model assumptions and limitations of the model. The replacement decision-making process can be automated using a Python script that is provided in this study. However, the CMMS that was used in the case study needs to be upgraded to have additional features.

When one wants to perform an analysis of assets on a system level, the expected replacement year value can be used. Until now, the probability distribution of the expected replacement year had to be computed by means of Monte Carlo simulation. In this report, a closed form solution is used for the expected replacement year distribution when operating cost follows a geometric Brownian motion (GBM). With this contribution, decision-makers in engineering asset management now have the opportunity to rapidly analyse and perform probabilistic computations on the expected economic life of deteriorating machinery on system levels, such as geographic systems or weather systems. As only few studies on stochastic asset replacement are empirical, a second important contribution of this study is the application of the model in a case study. The case study concerns HVAC systems of petrol stations in the Netherlands. The paper describes how to perform a weighted least squares (WLS) regression so that model parameters can be easily estimated for real cases. Finally, several new insights and barriers on implementing theoretical models in practice are introduced.

Note: part of this project has been transformed into a research article that is to be sent out for publication in a journal. Appendix B contains the draft.

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Delft, University of Technology — R. Hilwerda
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Chapter 1

Introduction

This chapter describes the context and problem formulation of the research project. The research objectives and the research question are described and finally the research methodology is discussed. A distinction is made between the **scientific** part and the **consultancy** part, which will both be elaborated in this chapter. The scientific part aims to contribute to the domain of stochastic asset replacement in the research field of engineering asset management. The consultancy part relates to the empirical part of the research project. The case study is elaborated in more detail in Chapter **??**.

1-1 Context

Scientific

A study that was conducted in the US found that manufacturers had spent $50 billion on maintenance costs in 2016 [5]. It is safe to say that maintenance costs take up a significant part of life cycle costs of assets and machinery. [2] found that on average 20.8% of the total budgets of firms in manufacturing industries is assigned to maintenance, which explains why firms aim for maintenance excellence. Among maintenance activities is the replacement of assets and machinery.

Costs might increase inordinately if asset replacement is badly timed. Determining the optimal replacement year of assets is an important and interesting study, but also a complex one. The optimum replacement age in terms of cost is often referred to as economic life. The aim is to maximise the net present value (NPV) of all related costs and revenues of an asset. A decision-maker thus needs to estimate these future cash flows. In reality, maintenance and operating costs behave in a non-deterministic way and require a stochastic approach. This characteristic has led rise to stochastic models, which take into account uncertainty of life cycle costs. Uncertainty and the option to wait are believed to have value that should be accounted for in investment decisions. Conventional methods that assume deterministic cash flows fail to incorporate this uncertainty [6].

There are several methods and tools available that support firms in making their equipment replacement decision. A common method that is often used for investment decisions is the discounted cash flow method (DCF). This method assumes that a decision-maker makes his investment decision based on the information that is available at present [3]. Information that becomes available later cannot change the value of the investment. This characteristic is often regarded as a limitation in investment decision-making [3, 7, 6, 8].

Consultancy

The research project was performed in collaboration with an asset manager in The Netherlands. This firm provides facility and asset management for a network of petrol stations in the Netherlands. The goal is to optimise as much as possible the availability, reliability and safety of the gas stations and to minimise costs. The asset manager is always looking for ways to optimise the asset management process and to increase the quality of service level of the petrol stations. Part of the asset management process involves the planning of replacing equipment. The replacement policy resembles the age replacement model. An asset is either replaced correctively upon failure or when the asset is considered deteriorated. In 2019, a transition was made to a new Computerised Maintenance Management System. In the CMMS, the asset manager stores all work orders, assets and cost related data.

1-2 Problem formulation

Scientific

[9] state in their review that only 10 percent of the contributions discussing uncertainty, investment and stochastic models were empirical studies. The authors suggest that more empirical work is needed to bring theoretical concepts closer to practice. There are several views and studies that investigate other reasons behind stochastic modelling lagging behind in practice. [10] found that infrastructure projects have characteristics that create barriers, making it more difficult to implement models that incorporate uncertainty in practice. In their paper, they describe barriers that prevent the adoption of stochastic models. The authors argue that these barriers can only be overcome by including more project data in research and by testing the theoretical tools that are available in practice. This thesis project aims to bring existing theoretical models closer to practice by testing an application in practice.

Several authors did studies on the integration of stochastic models with asset replacement, both theoretical and empirical. In their paper, [11] performed an empirical study and used a stochastic approach on the replacement of public infrastructure assets when political decisions, structural integrity and prices are uncertain. They observed that in the application of theoretical models in practice, there exists confusion about the choice of using either decision tree analysis (DTA) or a random walk process. Among the reasons for the lag of the use of stochastic models in practice, the authors point to the complexity and the difficulties in estimating market variables. A study on ship fleet replacement concluded that extending their model to different scenarios requires computational capacity and new algorithm designs [12]. [8] study replacement decisions of heavy machinery when operational expenditure costs and lead-time of orders are uncertain. Reviewing the literature, there seems to be debate on how to model costs, as some papers use a geometric Brownian motion (GBM), whereas other papers model costs via an arithmetic Brownian motion (ABM). They also found that it is particularly difficult in practice to estimate volatility parameters. [13] developed a theoretical model that optimises the replacement decision of equipment when the salvage value of assets is uncertain. In conclusion, none of the found studies describe explicitly how to perform a statistical regression and how to transform existing real data to model parameters such as drift and volatility. Furthermore, it is observed that none of the existing case studies reflect on the validity of using a Brownian motion to model operating costs.

Although several authors did empirical studies on the application in practice of stochastic models and equipment replacement, none of them have yet tested the application in an automated real-time process. As new information becomes available every day, the replacement strategy could constantly evolve. The studies that were performed all require a decision-maker that gathers the data and performs the calculations. However, today's software allows for computing data in such a way that this process can be automated. It is not clear if existing software has characteristics that lead to additional barriers in implementing stochastic asset replacement in practice. Research on this topic has not yet been done.

Consultancy

Once every year, the asset manager related to the case study draws up a consultancy report, advising which assets and components to replace. Until now, this advice has been merely based on a gut-feeling, rather than a quantitative calculation. Consequently, the asset manager faces the risk of wasting value by keeping equipment alive while replacement would create more net value, and vice versa. Also, replacement requests are often rejected (and considered too expensive) because of the lack of quantitative support for the request. The maintenance management system that is used offers a feature that allows for generating reports based on real-time data. Consequently, it should be possible to compute real-time data in such a way that a replacement strategy is automatically provided by the system. It is suggested that the use of a stochastic asset replacement model allows for a more cost-effective and accurate method to decide on asset replacement decisions. Quantitative methods for equipment replacement are not yet utilised within the asset management firm as research needs to be done on how to successfully implement this. This proposed research project aims to fill this gap.

1-3 Research objectives

Scientific

To cover the research gap, this research project studies an application of a theoretical stochastic asset replacement model. A case study is chosen because it allows for an analysis on the validation of the assumptions of this model. In addition, a weighted least squares (WLS) regression is carried out that transforms real data into the required parameters of the stochastic model. Barriers associated with implementing the theoretical model in the case study are presented. Besides filling a scientific research gap, the project has an additional goal, in that it also introduces a technical innovation: it aims to develop a script that is able to automate the equipment replacement decision-making process.

A major novelty of the study is the use of a closed form solution of the probability density function of the expected replacement year, which has not been found before in the engineering asset management literature. The expected replacement year allows for a collective analysis and comparison of different asset groups or systems. Furthermore, using a closed form solution is an important contribution because Monte Carlo simulations can only obtain point estimates. Closed form solutions allow analysts to better understand the impact of variations in price and cost [14]. The case study allows for an analysis on the validation of the assumptions of this model. A weighted least squares (WLS) regression is carried out that transforms real data into the required parameters of the stochastic model. Barriers associated with implementing the theoretical model in the case study are presented. The contributions of this study are summarised as follows; (a) the use of a closed formulation of the probability distribution of the expected economic life and (b) a new application of stochastic asset replacement in engineering asset management practice.

Consultancy

The consultancy assignment relates to the requirement to deliver a quantitative model that is able to automatically select the most cost-effective course of action in making equipment replacement strategies, based on real-time asset data. Successful implementation will allow the delivery of an accurate replacement strategy. An automated process will also save time, which will increase labour productivity.

1-4 Research question

The research objectives can be transformed into the following research question:

How can theoretical stochastic replacement models be used to automate the equipment replacement decision-making process in practice?

1-5 Research methodology

The proposed research topic deals with an operation-related problem. The goal of the project is to improve and optimise the working methods by designing a mathematical model. A common approach that is often used for improving and optimising systems and processes is the operation research approach [15]. Figure 1-1 depicts the seven steps that are to be taken in order to carry out an operations research process [1]. This approach will be used in the case study for carrying out the proposed research project. The orientation stage and the problem formulation stage have already been explained above. The remainder of the stages are described below.

Data collection

Nowadays, stakeholders within the asset management industry have the opportunity to make use of modern information systems such as Computerised Management Maintenance Systems (CMMS). Consequently, larger amounts of data can be generated. This provides firms with the opportunity to gain value and to obtain a more competitive advantage [16]. In the CMMS database, data such as downtime, cost, age, asset data and service level agreements are registered for every workorder that is carried out. In another database, the asset manager has information on replacement costs of assets. These data are available in the form of large databases. In the data collection phase, the aim is to bring together the data from the relevant sources and to centralise the data. Another task in this phase is the rearrangement of these data into an orderly form so that the data can be used in a proper way.

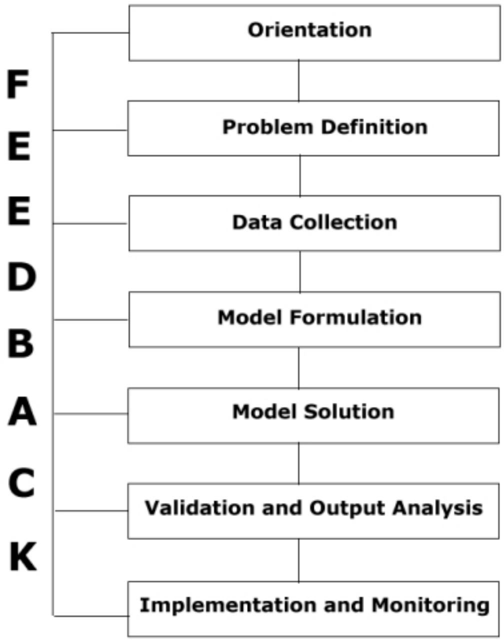


Figure 1-1: The operations research approach [1]

Model formulation

During this phase, a stochastic asset replacement model is searched that is able to select an equipment replacement strategy. When an appropriate model is found, the required data and the format of the data will be known. A feedback loop to the previous stage is required, and the data will be adjusted to the format that is required.

Model solution

The model solution stage involves the integration of the model to the CMMS. The model needs to be compatible with the CMMS that is used in the asset management firm. Computations on data in this software requires knowledge of Python programming skills. In order for the model to work properly, the Python script has to be written in a way that makes the software automatically select the correct data, perform the correct computations and generate a report.

Validation and Analysis

In the validation stage, the correctness of the model results will be discussed (internal validation) and the applicability of the model in a wider system will be tested (external validation). First, it is assessed if the model solution itself makes sense. The model should be correct and serve as an accurate representation of the reality. The results of the model will be compared to the expected results according to the theoretical models, but also to empirical results that are found in other case studies. If it turns out that the model solution is not consistent, the model needs to be adjusted. This is a cyclical process. A sensitivity analysis will be performed to investigate the impact of the results when model variables are altered. Secondly, it is assessed whether the model is applicable for wider systems. What are the limitations of the model and to what extent is the model applicable in a wider context on the subsystem level? The relevance of the model will be discussed with the agents of the asset management firm that are involved in the equipment replacement decision-making process. Assumptions, risks and limitations of the model will be discussed. The last stage of the validation stage involves the assessment of external validity. The model that is to be developed will be a tailor-made model that applies to petrol stations in The Netherlands. It will be investigated to what extent the model can be generalised. Assumptions, variables and boundary conditions that apply in the model and in the case study will be tested for other situations that are found in practice.

Implementation

The implementation phase entails the handover of the model to the asset management firm. Based on the results of the validation and analysis phase, additional recommendations will be made, and risks will be discussed.

Chapter 2

Asset replacement: deterministic versus stochastic

2-1 Introduction

Conventional Discounted Cash Flow (DCF) methods for investment decisions require a decision-maker to estimate all future cash flows in a deterministic manner. In the case of engineering asset management, this method yields a replacement strategy that takes into account the information available only at present, that is, information that becomes available in a later stage cannot change the outcome of the investment value. This is often considered an important drawback [3][17]. Stochastic models can overcome this drawback by taking into account uncertainty of future information and cash flows. The uncertainty approach is also known as Real Option Analysis (ROA), which finds its roots in financial option valuation. This chapter first discusses the deterministic approach. Regarding the stochastic approach, the analogy with financial options is discussed and finally, three stochastic asset replacement models are described.

2-2 Deterministic asset replacement

The basic model for asset replacement is illustrated in Figure 2-1 [2]. The model assumes that operating and maintenance (O&M) costs increase as assets age. The ownership cost reflects the capital outlay for a new asset minus the salvage value at the time of replacement, divided by the replacement age. The decreasing trend in the ownership cost shows that the salvage value decreases in time. Finally, fixed cost reflects costs that are independent of the age of an asset, such as utilisation cost of an asset. The common convention in deterministic asset replacement is to compute the total life cycle cost of an asset in terms of its Equivalent Annual Cost (EAC). A present value can be easily transformed into its EAC by multiplying the present value by the capital recovery factor (CRF) which is given by:

$$\text{CRF} = \frac{r(1+r)^t}{(1+r)^t - 1} \tag{2-1}$$

where r denotes the discount rate and t the number of periods. If it is assumed that replacement occurs over an infinite time interval, and that O&M costs are identical in each cycle, the total cost $C(t)$ of a replacement policy (with replacement in year t) can be computed as follows:

$$C(t) = \frac{\sum_{i=1}^{t} C_i r^i + r^t(K - S_t)}{1 - r^n} \tag{2-2}$$

with C_i equals the O&M cost in year i in terms of EAC, K the capital value of a new asset and S_t the salvage value of the asset in year t. The optimal replacement year can then be selected by minimising the total cost C_n in terms of EAC. In formula form:

$$\text{argmin}_{t,t>0} \frac{\sum_{i=1}^{t} C_i r^i + r^t(K - S_t)}{1 - r^n} \tag{2-3}$$

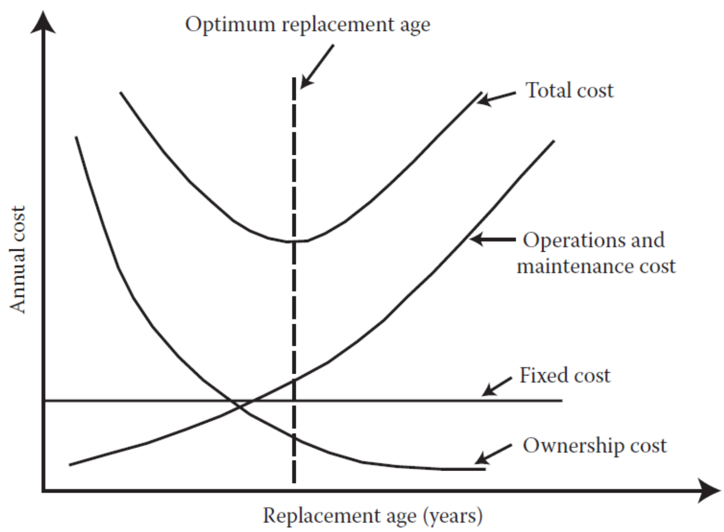
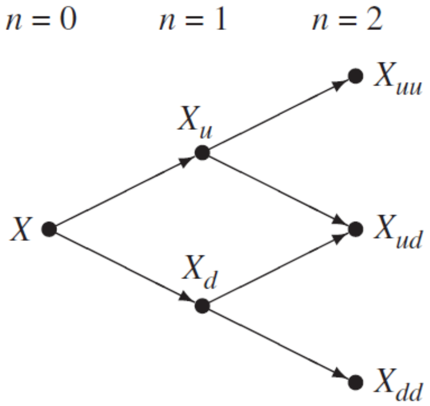


Figure 2-1: The deterministic approach of asset replacement [2]

2-3 Three approaches of stochastic asset replacement

[11] distinguish three valuation methods that incorporate uncertainty in investment decisions. They are the decision tree analysis (DTA), the risk neutral approach and the single discount rate approach. These three approaches are discussed in this paragraph.



⇔

$X(i, n)$	0	1	2
0	X	X_u	X_{uu}
1		X_d	X_{ud}
2			X_{dd}

Figure 2-2: A decision tree with up and down moves [3]

2-3-1 The decision tree approach

The decision tree approach (DTA) captures flexibility but requires a decision-maker to know all possible future outcomes. A single discount rate is used to discount future cash flows back to a present value. Future cash flows are expressed in the form of a binomial tree that comprises of one up move and one down move (Figure 2-2). This approach does not apply a stochastic process and thus requires the estimation of future cash flows. When the entire decision tree has been mapped out, backwards recursion of the decision tree leads to the determination of the best course of action at each node.

2-3-2 The risk neutral approach

The risk neutral approach makes use of so called risk-neutral probabilities, whereas the decision-tree approach uses actual probabilities. The risk neutral approach can be applied when a decision-maker does not know exactly all future outcomes in advance as it makes use of a stochastic process [11].

The risk neutral approach assumes that a state variable can go up to X_u or go down to X_d. The approach is to build a replicating portfolio with two components: risk-free bonds and a so-called spanning asset that together resemble identical cash flows. The price of the replicating portfolio V can then be computed by

$$V = \frac{\pi_u Y_u + \pi_d Y_d}{R_f} \tag{2-4}$$

where π_u and π_d equal the risk-free probabilities of an up and down move respectively, Y_u and Y_d equal the market values of cash flows in an up move and in a down move respectively, and R_f is the risk free interest rate. The risk-free probabilities can be computed by:

$$\pi_u = \frac{ZR_f - X_d}{X_u - X_d} \tag{2-5}$$

and

$$\pi_d = \frac{X_u - ZR_f}{X_u - X_d} \tag{2-6}$$

where Z equals the current price of the spanning asset.

There are several ways to model the values of the up and down moves. The risk-neutral approach makes use of a stochastic process for modelling the price. One of the most common processes is the Brownian motion with drift [3]. The Brownian motion is also called a Wiener process. A Wiener process is a continuous-time stochastic process with three important properties [17]. First, it is a Markov process, which means that the distribution of future values depends only on the current value. Second, the process has independent increments. Third, price changes in any time interval are normally distributed and their variance increases linearly in time.

The Brownian motion with drift can be modelled as an arithmetic Brownian motion (ABM) or a geometric Brownian motion (GBM). In the case of GBM, the logarithm of the price follows a Brownian motion. In formula form, a price that follows a GBM can be expressed as follows:

$$dx/x = \theta dt + \sigma d\omega_t \tag{2-7}$$

where x is the price, ω_t denotes the Wiener process, θ equals the drift or growth rate and σ equals the volatility. An important feature of Equation (2-7) is that the change in a logarithm of x is normally distributed with mean $(\theta - \frac{1}{2}\sigma^2)$ and variance $\sigma^2 t$. If the growth rate θ and volatility σ are known, Monte Carlo computations can be carried out in order to simulate paths that x can follow.

When translating the theory as described above to asset replacement, one can think of x as the operating cost over a life cycle of a physical asset that deteriorates in time. Suppose that the operating cost increases in time geometrically with a growth rate θ and volatility σ. Suppose further that at some cost limit it will be economically more efficient to replace the asset for a new one. Different simulations can then be performed by means of Monte Carlo simulations. Figure 2-3 illustrates ten samples that were computed for a geometric Brownian motion.

It can be observed that the times at which the samples hit the cost limit are different and are in fact stochastic variables. The distribution of the hitting times can be computed as well. The use of a closed form solution for the hitting time distribution is a scientific contribution in the area of engineering asset management. The red dashed line in the figure highlights the deterministic operating cost process, in which there is no uncertainty (i.e. volatility is zero). A deterministic approach assumes a single growth rate and does not take into account the possibility that operating cost might decrease (e.g. because of maintenance interventions on the asset).

2-3-3 The single discount rate approach

The single discount rate approach is only slightly different from the previous approach, from the fact that a single discount rate is used for discounting future cash flows, rather than

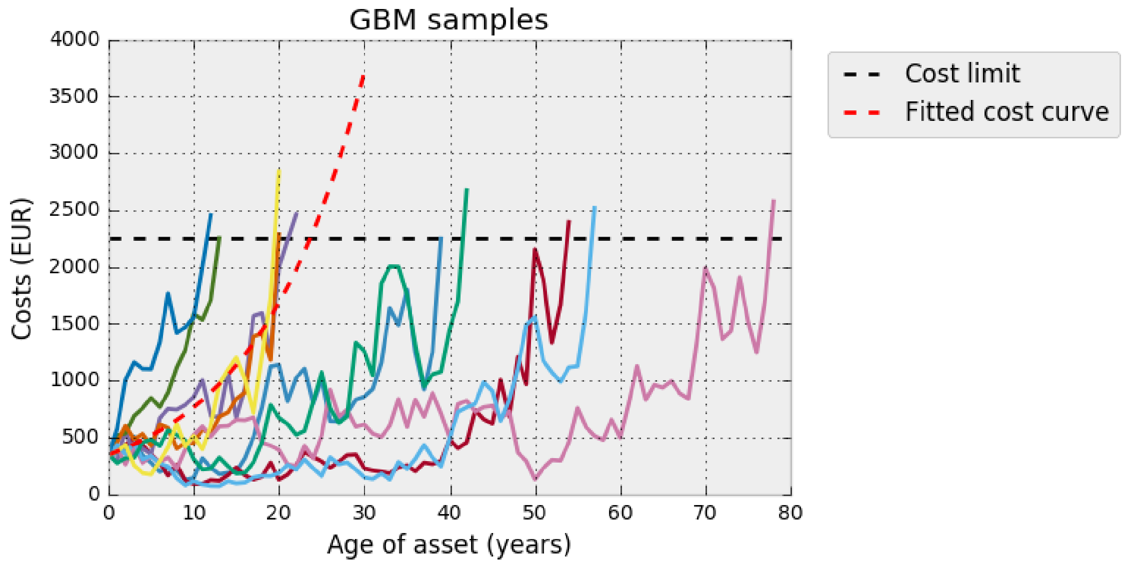


Figure 2-3: Samples of a geometric Brownian motion with drift

risk-neutral probabilities. Actual probabilities are used for the operating cost process. [11] reason that the single discount rate approach is not entirely mathematically correct, because it allows for arbitrage in financial markets. However, the authors reason that in situations when one cannot reasonably predict future variables, it is unlikely that the single discount rate approach will yield an outcome that significantly differs from the outcome of the risk neutral approach. In the case study it was found that the operating cost process is a very uncertain process with high volatility. Therefore, the single discount rate approach can be applied. During the literature review, a model was found that is specifically developed for modelling operating cost as a geometric Brownian motion with drift [18]. This model is explained in the next chapter.

2-4 From financial to real options

Stochastic uncertainty modelling finds its roots in financial option valuation. To illustrate the analogy with a financial option, the valuation process of a call option in finance is explained.

A call option gives the holder the right to buy stock at a given exercise price (or strike price) on or before a particular date [7]. European call options can only be exercised on the maturity date, whereas American call options can be exercised on or before the maturity date. The flexibility that options give has value which is reflected in the option price that a holder needs to pay. The profit that an option holder makes depends on the future stock price. The payoff diagram in Figure 2-4 illustrates this. It can be observed that from a certain stock price, the payoff increases with the stock price.

Determining the value of a call option has been problematic in the past, until Fischer Black and Myron Scholes presented their famous paper on option pricing [19]. The Black and Scholes formula uses a lognormal distribution to model the probability of stock prices. The formula is as follows:

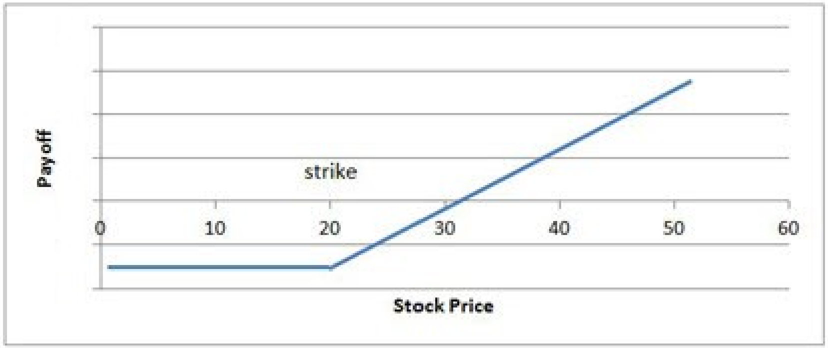


Figure 2-4: Payoff diagram of a call option [4]

$$\text{Value of call option} = [N(d_1) * P] - [N(d_2) * PV(EX)] \tag{2-8}$$

where

$$d_1 = \frac{\log[P/PV(EX)]}{\sigma\sqrt{t}} + \frac{\sigma\sqrt{t}}{2} \tag{2-9}$$

$$d_2 = d_1 - \sigma\sqrt{t} \tag{2-10}$$

with

$N(d)$ = the normal distribution of d;

EX = the strike price of the option;

$PV(EX)$ = the risk-free discounted strike price;

t = number of periods until strike;

P = the current stock price at t_0;

σ = the standard deviation of the stock return

A holder of an American call option has the opportunity to wait for more information - that is, stock price - before making the decision to exercise. This option to wait for more information could easily be translated to the physical asset world. In this case, the option is known as a real option. When a decision-maker wants to make an investment, he or she has the option to wait with investing. For example, a decision-maker who wants to invest in a steel plant might want to wait for higher steel prices, which will affect the future cash flows. This option also has value and this value can be computed. The real option philosophy can also be extended to asset replacement: a decision-maker who wants to replace an asset is in fact making an investment decision. Every year, the decision-maker will gain new insight and information on the asset, in this case in the form of incurred maintenance and operating cost. If it turns out that maintenance and operating cost in a certain year was low, the decision-maker has the option to wait with replacement until maintenance and operating cost rises to a higher level. Or, when maintenance and operating costs turn out to be higher than predicted, the decision-maker can decide to replace earlier. The option to wait with replacement is important because information that becomes available later has value.

Chapter 3

Model and case study

This part has been removed from the report. It has been captured in the form of a journal article. For reference, please see Appendix B.

Chapter 4

Automating the replacement policy

4-1 Introduction

If one wants to apply the stochastic asset replacement model, input data such as operating cost and asset price information is required. Nowadays, firms have the opportunity to centralise this data into Computerised Maintenance Management Systems (CMMS). Another feature of CMMS is that decision-makers can assign information and data to assets, perform computations and generate business intelligence reports. Consequently, an opportunity arises that allows for automating this process. This chapter introduces a Python script that can transform real time data into model parameters, such as drift and volatility of the geometric Brownian motion, and output values, such as the trigger replacement level and the average replacement year. First, the concept of CMMS is explained. The requirements for automated replacement are laid out and finally a recommendation is made that applies to the case study within the asset management firm.

4-2 Computerised Maintenance Management Systems (CMMS)

A CMMS can be regarded as a data storage platform for maintenance works. It also functions as a central communication platform that allows involved stakeholders to adjust workorders and update work related information. Consequently, the progress of a workorder, such as an inspection or repair, can be tracked by all stakeholders. Regarding the case study, the asset manager manages the petrol stations by means of this central information system. The system comprises of an online environment, in which the maintenance leads can track all workorder related information. The core of the database requires an asset matrix and a repair matrix, so that the stakeholders, such as contractors and staff that works on-site, can select the correct asset and the correct repair task, or upload invoices. Stakeholders can do so by using the corresponding CMMS application on their smartphones. An additional module is the Business Intelligence (BI) module, in which BI reports can be drawn up.

4-3 Recommendation

If a decision-maker wants to automate the stochastic replacement model, several requirements need to be met. They can be distinguished into three categories:

1) Real time input parameters of the model

The input parameters of the model are listed in Table 4-1.

Input parameter	**Symbol**	**Description**
O&M cost	$C(t)$	Operating and maintenance cost linked to the asset in CMMS
Age of the asset	t_i	The age of the asset or the current year minus the building year
Interest rate	r	The rate at which life cycle costs of the asset are discounted
Price of new asset	K	The price of a new asset and linked to the asset in CMMS
Salvage value	S	The salvage value of the asset and linked to the asset in CMMS

Table 4-1: Input parameters of the stochastic model

It is important that all financial data are linked to an asset in the asset matrix. Missing data might lead to an underestimation of O&M cost. The model will then yield an incorrect replacement policy.

2) A CMMS module that allows for computation of real time data

It is required that the CMMS contains a module that is able to gather the input data and can transform the data to the output values by means of a programming language. In the case of the CMMS used in the case study, Python can be used in combination with the software.

3) A programming script that transforms input parameters to the output values of the stochastic model

The script acts as the black box that transforms input parameters into the output values. In the case study, Python was used to write a script that is able to do this. The Python script that was written to automate the mathematical stochastic model that was used in the project can be found in Appendix A. If the requirements as described above are met, an automated replacement policy can be made possible.

Recommendation for the asset manager The CMMS that is used in the case study does not yet meet all the requirements. It was found that out of the five input parameters needed, only real time O&M cost is stored. A recommendation for the other parameters is given below.

Age of the asset - In the CMMS, only the age of the asset components are stored (such as a 3-in-1 system or a boiler). In order for the model to work, the installation date of the entire HVAC system has to be stored as well. In the software, this is not yet possible. It is recommended to have an additional parameter input per location added in the software, so that this information can be added.

Interest rate - The interest rate can be either added within the CMMS or the value can also be captured within the Python script. A disadvantage is that the latter makes it more difficult for a decision-maker to adjust the value. It is recommended to store the value within the script, only if the interest rate is assumed to remain relatively stable over time.

Price of new asset and salvage value - The price of a new asset should also be available in the CMMS. In the current software version, it is not possible yet to add this information. It is recommended to have an additional parameter input per asset added in the software. This also goes for the salvage value of the asset.

The second and third requirement are available in the case study. The BI module can be used to make the replacement policy. The script that is needed is provided in the Appendix.

Chapter 5

Conclusion and recommendations

This study has introduced the use of a closed form solution of the expected economic life distribution of asset systems when operating cost follows a geometric Brownian motion (GBM). Secondly, this study has tested a practical application of a theoretical stochastic asset replacement model. The case study that was used involves HVAC systems of a network of petrol stations in the Netherlands. New insights and barriers are introduced when implementing stochastic models in practice, an explicit parameter estimation method is provided and the validity of using a geometric Brownian motion for modelling operating cost is discussed. Finally, the modelling process can be automated and a Python script is provided that can perform this.

With the closed form solution, it is now possible to analyse asset groups on a system level. For instance, assets can be analysed or compared on different geographical systems (e.g. on provincial level or national level), different weather systems or different asset types. The closed form solution yields the exact distribution of the expected replacement year, making discrete Monte Carlo simulations obsolete. An advantage of the expected economic life distribution is that data of multiple assets can be used. This is beneficial for situations in which operating cost data is absent, as is often the case.

It was found that using geometric Brownian motion for operating cost modelling is possible, but regression diagnostics of the data showed that the data points are not perfectly scattered. The linear increase of the variance in the model did not correspond with the actual variance development of the data. Although a GBM proved successful in modelling uncertainty, it was also found that it cannot model the reality perfectly. Minimum inspection costs and fixed maintenance intervals could not be captured because of the random walk process. The model has additional limitations, such as fixed capital outlay values, salvage values and discount rates. It would be interesting to test other theoretical models that have different assumptions. Secondly, when implementing the model in practice, firms are required to have all operating cost data available per asset. In reality, this might not always be available. It should also be noted that the expected replacement year is highly elastic to changes in the drift parameter (θ). Therefore, caution should be taken when estimating the model parameters. Finally, it should be noted that the model assumes a continuous GBM, whereas in the case study, data was provided on a monthly basis. It is not clear whether this has a significant impact on the outcome of the model.

The main research question involved the question on how theoretical stochastic asset replacement models can be used in practice and how they can be automated. It was found that using the theoretical model as described in this paper in practice is possible, but one should be aware of model assumptions and limitations. When one wants to apply the theoretical model in a real case, a weighted least squares (WLS) regression can be carried out to transform data into the required model parameters. Automating the process through a Computerised Maintenance Management System (CMMS) is also possible, but the CMMS used in the case study needs additional features to support this. Within the CMMS, several new input value fields are required, such as the interest rate, capital value and salvage value. More research is needed to test if other CMMS systems are capable of running the asset replacement model in an automated process.

Appendix A

Python script

Step 1 - Plotting the data points and computing the model parameters

Input

```
# import the modules needed for the script
import pandas as pd
import matplotlib.pyplot as plt
%matplotlib inline
import numpy as np
#plt.style.use('grayscale')
import random
from scipy.stats import norm

# import the excel file and select the relevant columns
df = pd.read_excel(r'C:\Users\robert.hilwerda\Downloads\PYTHON\datafinal.xlsx',
usecols= ['Accumulatiewaarde', 'Leeftijd'])
df = df[df.Leeftijd != 0]

#plot data points
t_data = df['Leeftijd']
C_data = df['Accumulatiewaarde']
plt.plot(t_data , C_data, 'x', c = 'black', label = 'Asset')

#### Weighted Least Squares ####
#Assign weights
w = np.zeros(len(df))
for i in range(0, len(df)):
    if df['Leeftijd'].iloc[i] == 0.0:
        w[i] = 1.0
    else:
```

```
        w[i] = 1.0 / df['Leeftijd'].iloc[i]

#Calculate weighted means:
t_w = np.sum((w * t_data)) / np.sum(w)
C_w = np.sum((w * np.log(C_data))) / np.sum(w)

#Calculate estimates
beta_1 = (sum(w *(t_data - t_w)*(np.log(C_data) - C_w)))
/   sum((w *((t_data - t_w)**2)))
beta_0 = C_w - beta_1 * t_w
C_0 = np.exp(beta_0)

print 'Estimate for C_0 =', C_0
print 'Estimate for (theta - 0.5 sigma^2) =', beta_1

#Calculate variance and standard deviation:
var = (np.sum( w *((np.log(C_data) - beta_0 - beta_1*t_data)**2))) / (len(df)-2)
sigma = var**0.5
print 'Estimate for sigma = ', sigma
theta = beta_1 + sigma**2/2
print 'Estimate for theta = ', theta

# Plot fitted curve
print 'Weighted Least Squares fitted curve:'
print 'Fitted curve: C_0 =', C_0.round(2), '* exp(', beta_1.round(4), '* t)'

def fitted_curve():
    x = np.linspace(0, df['Leeftijd'].max(), df['Leeftijd'].max() + 1)
    y = C_0 * np.exp(beta_1 * x)
    return y

plt.plot(fitted_curve(), '--', label = 'Fitted cost curve')
#plt.legend(bbox_to_anchor=(1.5, 1.0))
#plt.title('Costs of HVAC systems of in 2018')
plt.xlabel('Age of asset in years')
plt.ylabel('Operating cost in EUR');

# Determine variance of estimators
var_beta_0 = ( 1 / sum(w)) +( ( t_w**2) / sum (w*(t_data - t_w)**2))* sigma**2
var_beta_1 = sigma**2 / sum ( w * (t_data - t_w)**2)
print 'Variance of estimator C_0 =', var_beta_0
print 'Variance of estimator (theta-0.5*sigma^2) =', var_beta_1

print 'SD of C_0 =', 100*np.sqrt(var_beta_0), '%'
print 'SD of (theta-0.5*sigma^2) =', 100*np.sqrt(var_beta_1), '%'

# RESIDUAL PLOT
```

```
t_data = df['Leeftijd']
C_data = df['Accumulatiewaarde']
#plt.plot(t_data , np.log(C_data), 'x', c = 'black', label = 'Asset')

def fitted_curve_log():
    x = np.linspace(0, df['Leeftijd'].max(), df['Leeftijd'].max() + 1)
    y = C_0 * np.exp(beta_1 * x)
    return np.log(y)
#plt.plot(fitted_curve_log(), '--', c = 'black', label = 'Fitted cost curve')

### RESIDUAL PLOT ###
#print C_data

residual = np.zeros(len(C_data))

for i in range(0, len(C_data)):
    #print np.log(C_data[i]) - fitted_curve_log()[ df['Leeftijd'].iloc[i] ]
    residual[i] = (( np.log(C_data[i]) ) - fitted_curve_log()
    [ df['Leeftijd'].iloc[i] ]) / df['Leeftijd'].iloc[i]
#print residual
plt.plot(t_data, residual, 'x', c = 'black')
plt.axhline(c = 'black')
plt.xlabel('Age of asset in years')
plt.ylabel('Residuals')
```

Output

```
Estimate for C_0 = 1848.59310194
Estimate for (theta - 0.5 sigma^2) = 0.0264237709624
Estimate for sigma =  0.10323826293
Estimate for theta =  0.0317528404288
Weighted Least Squares fitted curve:
Fitted curve: C_0 = 1848.59 * exp( 0.0264 * t)
Variance of estimator C_0 = 0.0293459690591
Variance of estimator (theta-0.5*sigma^2) = 2.15743770738e-05
SD of C_0 = 17.1306652116 %
SD of (theta-0.5*sigma^2) = 0.46448226095 %
```

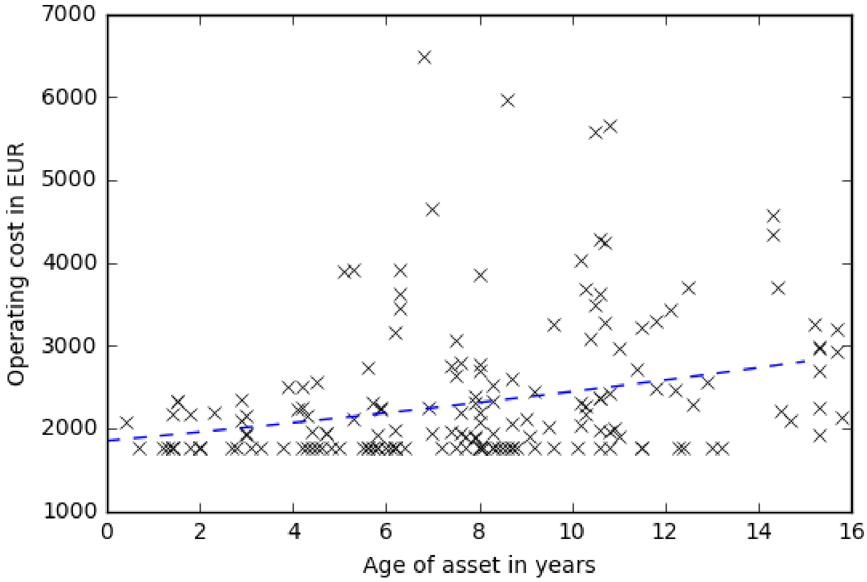


Figure A-1: Data plot

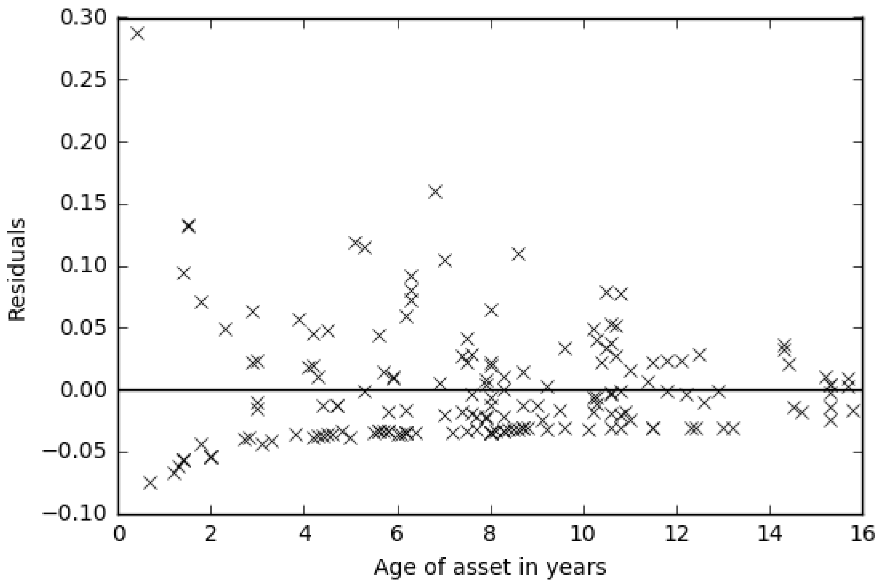


Figure A-2: Residual plot

Step 2 - Determinstic replacement policy calculation

Input

```
# Calculate minimum present value
# SPECIFY VARIABLES
r = 0.04 # DISCOUNT RATE
K = 15000 # PRICE OF NEW ASSET
S = 3000 # SALVAGE VALUE

def PV(x,y):
    return y
x = np.linspace(1,30,31)
y = ((1-np.exp(-r*x))**-1)*((((C_0)*(np.exp((theta-r)*x)-1)/(theta - r))) + K -
(S * np.exp(-r*x)))
plt.plot(x,y, c = 'black', label = 'Present value of life cycle costs')
#plt.axhline(y = np.min(PV(x,y)), c = 'r', ls = '--')
plt.axvline(x = np.argmin(PV(x,y)), c = 'black', ls = '--', label =
'Optimum replacement year')

print 'Operational cost limit: EUR', C_0 * np.exp((theta) * np.argmin(PV(x,y)))
print 'Optimum replacement in year', np.argmin(PV(x,y))
print 'Present value: EUR', np.min(PV(x,y))
plt.legend(bbox_to_anchor=(1.8, 1.0))
plt.title('Present Value of life cycle costs')
plt.xlabel('t, Age of asset [years]')
plt.ylabel('y(t), Present Value [EUR]');
```

Output

```
Operational cost limit: EUR 3273.88054409
Optimum replacement in year 18
```

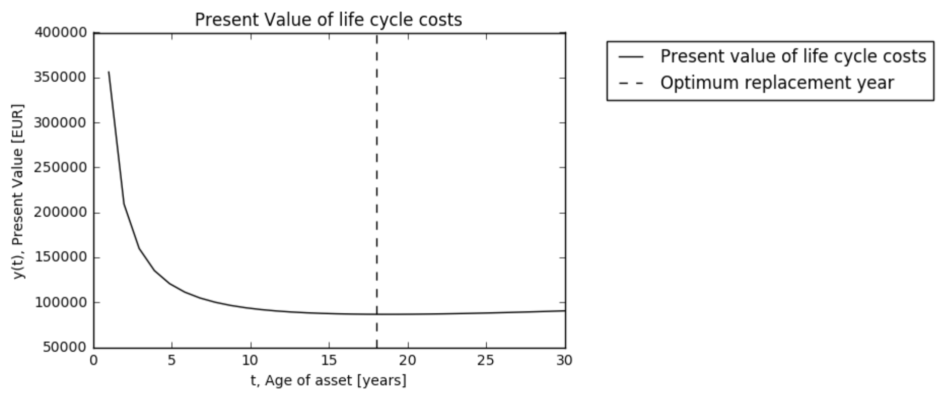


Figure A-3: Deterministic replacement year

Step 3 - Stochastic asset replacement

Input

```
#### PROBABILISTIC REPLACEMENT ####

# importing the other modules needed for the script
import random
from scipy.stats import norm

########### CALCULATE COST LIMIT #############

# Define roots of the differential equation

R1 = beta_1
R2 = (R1**2 + 2*sigma**2*r)**0.5
labda_1 = (-R1 + R2) / sigma**2

# Numerically solving for C
Initial_Guess = C_0
ans = (Initial_Guess*((1-labda_1)-(C_0/Initial_Guess)**(labda_1))) + ((C_0+(K-S)*
(r-theta)) * labda_1)
for n in range(0,500):
    while ans < -5 or ans > 5:
        Initial_Guess += 10
        ans = (Initial_Guess*((1-labda_1)-(C_0/Initial_Guess)**(labda_1)))
        + ((C_0+(K-S)*(r-theta)) * labda_1)
        #print('Current Guess: C =', Initial_Guess, 'Ans =', ans)
    else:
        C = Initial_Guess
        break
print 'Operational cost limit: EUR', C
```

Output

```
    Operational cost limit: EUR 3478.59310194
```

Step 4 - PDF of expected replacement year

Input

```
V_0 =  np.log(C) - np.log(C_0)

def pdf():
    x = np.linspace(0,40,41)
    y_pt1 = V_0 / ( (sigma) * np.sqrt(2 * np.pi * x**3))
    y_pt2 = np.exp(-(( V_0 - beta_1*x) **2)/( 2*sigma**2*x))
```

```
    y = y_pt1 * y_pt2
    plt.plot(y, c = 'black')
pdf()

beta_1 = theta - sigma**2/2
print 'Expected hitting time =', np.log(C / C_0)/beta_1

plt.axvline(np.log(C / C_0)/beta_1, ymax = 0.46, c = 'black', ls = '--')
plt.xlabel('Age of asset in years')
plt.ylabel('Probability')
#plt.title('Probability density function of hitting times');
```

Output

```
    Expected hitting time = 23.9255429345
```

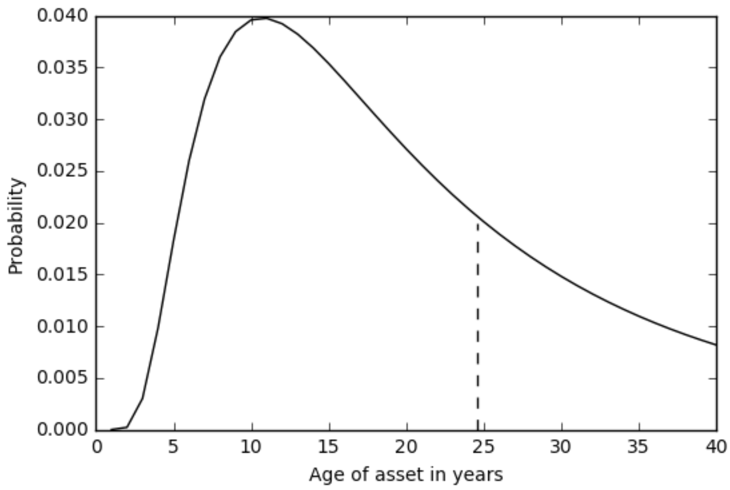


Figure A-4: PDF of expected economic life

Appendix B

Journal article

ARTICLE TEMPLATE

A closed-form solution of the expected economic life distribution when operating and maintenance cost is uncertain: a case study

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ABSTRACT
Many firms are occupied with determining the optimal replacement time of assets. Asset replacement is a complex investment decision that requires the estimation of future cash flows and other parameters. The non-deterministic character of future cash flows has given rise to stochastic models, that take into account this uncertainty. When one wants to perform an analysis of assets at a system level, the expected replacement year value can be used. Until now, the probability distribution of the expected replacement year had to be computed by means of Monte Carlo simulation. In this paper, the authors use a closed-form solution of the expected replacement year distribution when operating cost follows a geometric Brownian motion (GBM). With this contribution, decision-makers in engineering asset management now have the opportunity to rapidly analyse and perform probabilistic computations on the expected economic life of deteriorating assets at a system level. For example, wind turbine fields can be compared on different levels of maritime conditions . The approach has been implemented based on a real case study, which concerns HVAC systems of petrol stations in the Netherlands. The paper describes how to perform a weighted least squares (WLS) regression that considers the time-dependent variance of the operating cost, so that model parameters can be easily estimated for real cases. Finally, several new insights and barriers on implementing theoretical models in practice are discussed.

KEYWORDS
Stochastic asset replacement; geometric Brownian motion; Expected economic life; Parameter estimation; Case study

1. Introduction

A recent study conducted in the US found that manufacturers had spent $50 billion on maintenance costs in 2016 (Thomas 2018). It is safe to say that maintenance costs take up a significant part of life cycle costs of assets and machinery. Jardine and Tsang (2005) found that on average 20.8% of the total budgets of firms in manufacturing industries is assigned to maintenance, which explains why firms aim for maintenance excellence. The replacement of assets and machinery, which is among maintenance activities, might increase inordinately if it is badly timed. Determining the optimal

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replacement year of assets is an important and interesting study, but also a complex one. The aim is to maximise the net present value (NPV) of all related cash flows that are associated with the asset; thus, a decision-maker needs to estimate these future cash flows. In reality, maintenance and operating costs behave in a non-deterministic way and require a stochastic approach. This characteristic has led rise to stochastic methods, which take into account the uncertainty of life cycle cost development. Although uncertainty and the option to wait are believed to have value that should be accounted for in investment decisions, conventional methods that assume deterministic cash flows fail to incorporate this uncertainty (Martins *et al.* 2013).

Several authors did studies on stochastic asset replacement, both theoretical and empirical. It is observed that two types of research gaps exist; namely, lack of knowledge on how to use stochastic asset replacement models and issues around implementation of these models in practice. Regarding the lack of knowledge, Van Den Boomen *et al.* (2019) used a stochastic approach on the replacement of public infrastructure assets when political decisions, structural integrity and prices are uncertain. They observed that in the application of stochastic methods in practice, there is no consensus on the choice of using either decision tree analysis (DTA) or using a stochastic process. A study on ship fleet replacement concluded that extending the theoretical framework to include new assumptions requires computational skills and new algorithm designs (Zheng and Chen 2018). Richardson *et al.* (2013) study replacement decisions of heavy machinery when operational expenditure costs and lead-time of orders are uncertain. Regarding the cost modelling, some authors suggest the use of the geometric Brownian motion (GBM), whereas others propose the arithmetic Brownian motion (ABM). Such a choice can result in large differences in the estimated cost. Among the issues of implementation of stochastic asset replacement models in practice, Van Den Boomen *et al.* (2019) point to the complexity and the difficulties in estimating model variables, in particular Zambujal-Oliveira and Duque (2011) highlight the difficulty of estimating the volatility parameters.

It is observed that none of the found studies describe explicitly how to perform a statistical regression that accounts for time-dependent variance and how to transform existing real data to model parameters such as drift and volatility. Furthermore, it is observed that none of the existing case studies reflect on the validity of using a Brownian motion to model operating costs.

To cover both research gaps mentioned above, this paper presents a novel framework to determine the optimal asset replacement accounting for the stochasticity of the problem. A major novelty of the study is the use of a closed-form solution of the probability density function of the expected replacement year, which has not been found before in the engineering asset management literature. The expected replacement year allows for a collective analysis and comparison of different asset groups or systems. In addition, the conventional applied methods, e.g., Monte Carlo simulation, can only obtain point estimates, thus closed-form solutions allow analysts to better understand the impact of variations in price and cost (Lima and Suslick 2005). Furthermore, the proposed approach has been applied to a real case, allowing for an analysis on the validation of the assumptions of this model. A weighted least squares (WLS) regression accounting for the increasing variability with time is carried out that transforms real data into the required parameters of the stochastic model.

The remaining document is organised as follows; Section 2 gives a brief introduction of stochastic asset replacement, Section 3 describes the theoretical model that is used throughout this paper and presents the closed form solution of expected economic life. In Section 4 the case study is introduced and discussed in Section 5. Finally, in

Section 6 some conclusions and future research lines are drawn.

2. Asset replacement: deterministic versus stochastic

Conventional Discounted Cash Flow (DCF) methods for investment decisions require a decision-maker to estimate all future cash flows in a deterministic manner. In the case of engineering asset management, this method yields a replacement strategy that takes into account the information available only at present, that is, information that becomes available in a later stage cannot change the outcome of the investment value. This is often considered an important drawback (Dixit *et al.* 1994, Guthrie 2009).

The stochastic character of cash flows can be incorporated by taking into account different possibilities with regard to future values of uncertain cash flows (Van Den Boomen *et al.* 2019). In the literature, two main approaches to uncertainty modelling can be distinguished. They are the decision-tree analysis (DTA) and the stochastic process, a modelling technique that models cash flows by means of random walks. The approaches differ in the way that future values and their probability of occurring are determined. In the former, future values of cash flows in combination with their expected probabilities need to be determined by the decision-maker. These future values are then captured in a decision-tree, for which the expected present value can finally be computed, based on the given probabilities and values. This approach thus requires the decision-maker to predict both future costs and their corresponding probabilities. The latter approach uses a stochastic process for modelling cash flow development. The cash flows are modelled by the stochastic process which are partially determined by a random walk, also called a Wiener process. Future cash flows and probabilities do not require a decision-maker to predict these values. Instead, input parameters are required to generate a random path of the cost process. These input parameters are often estimated by analysing historical data or by means of expert judgment. The stochastic process approach can be applied when a decision-maker does not know future outcomes in advance, as it makes use of a stochastic model that generates random cost paths. Within the stochastic process approach, future costs can be discounted by using a single discount rate and actual probabilities. However, it is also possible to make use of so-called risk-neutral probabilities, a method which is also known as the replicating portfolio method (Guthrie 2009). Van Den Boomen *et al.* (2019) argue that using a single discount rate is not entirely mathematically correct, because it allows for arbitrage. However, the authors reason that in situations when one cannot reasonably predict future variables, it is unlikely that using a single discount rate will yield a different outcome than the outcome of the replicating portfolio method.

There seems to be a wide consensus among authors about the need for more empirical studies on the topic of investment and uncertainty. Trigeorgis and Tsekrekos (2018) state in their review that only 10 percent of the contributions discussing uncertainty, investment and stochastic processes were empirical studies. The authors suggest that more empirical work is needed to bring theoretical concepts closer to practice. Echoing this issue, the present document makes use of a real case to show the challenges of implementing the proposed approach.

The next section discusses the mathematical model that is used throughout the paper. As stated before, operating cost development is uncertain and cost might increase or decrease. This probabilistic characteristic of the cost process can be accounted for by using a stochastic process in which operating cost can either go up or down. In the model used in this paper, a random walk (Wiener process) is used to capture this uncertainty. It is assumed that in the long run, operating cost increases as a result of asset deterioration. This increase is captured by a drift parameter. Drift and uncertainty are then combined in the form of a Brownian motion with drift.

3. Mathematical model

3.1. *Introduction of the model*

The model that is used throughout this paper is the stochastic machine replacement model as proposed by Dobbs (2004), which provides the basic mathematical overview of the model. This model is the basis of a number of more complex approaches that consider several extensions, such as assuming stochastic capital values or salvage values. For the aim of this paper, the most simple version of this model is used. It should be noted that the model uses a single discount rate to discount future costs. The operating cost process is modelled by a stochastic process. Secondly, it is important to note that the operating cost process is modelled by a continuous process. Third, the model assumes an infinite replacement cycle of the asset. Finally, an important assumption in this model is that the initial outlay, the operating cost profile and salvage value are constant in each replacement cycle. The model does not allow for technological progress or price changes. The implications of these assumptions in practice are discussed in Section 5. Dobbs assumes that operating costs increase by a constant rate. This is then extended by incorporating uncertainty through a geometric Brownian motion with drift, in which the increment of operating costs c_t is described by:

$$dc_t/c_t = \theta dt + \sigma dW_t \tag{1}$$

where θ describes the growth rate of operating costs, σ represents the volatility and W_t is a Wiener process.

The assumption of operating costs behaving as a Brownian Motion with drift is not new. Richardson *et al.* (2013) found that using a Brownian motion to model operating costs allows for a compact presentation. Beichelt (2006) claims that in general, all linear processes that are permanently disturbed by random influences can be successfully modelled by a Brownian motion with drift. Another advantage of using Brownian motion is the attractive mathematical properties it carries (Lindqvist and Skogsrud 2008). However, it has also been found that validation of these models when used in case studies is often absent (Nicolai 2008). Dinwoodie *et al.* (2015) explain that it can be difficult for researchers to obtain suitable data to perform model validation, as historical data is often not available. Shi and Min (2014) also experienced this. They state that, even though some applications might not be fully supported by a GBM, operating costs share important features with a GBM: they increase in time, which is captured by the drift parameter; the uncertainty of operating costs can then be captured by the volatility parameter. As models can only represent reality in part, they are subject to several limitations, as occurs with GBM. The most important limitation is the assumption of a constant volatility rate. The model does not allow for time-varying volatility.

At some point, it will be more cost efficient to replace an asset, rather than keep the asset alive. This point is captured by a cost limit $\bar{c}$, also called the trigger level.

The replacement level can then be regarded as an optimisation problem which can be solved by finding the value $\bar{c}$ that minimises the expected present value costs for the entire replacement chain. The trigger value is a deterministic function of K (the initial outlay), S (the salvage value), c_0 (the level of operating costs in year zero), θ (the growth rate of operating costs), r (the discount rate) and σ (the volatility of operating costs). The trigger level is the solution to a differential equation and can be described by the following equation (Dobbs 2004):

$$\bar{c}\left[(1-\lambda)-(c_0/\bar{c})^{\lambda}\right]+\left[c_0+(K-S)(r-\theta)\right]\lambda=0 \tag{2}$$

with

$$\lambda=(-R_1+R_2)/\sigma^2 \tag{3}$$

where

$$R_1 \equiv \theta-\frac{1}{2}\sigma^2 \tag{4}$$

$$R_2 \equiv \left(R_1^2+2\sigma^2 r\right)^{1/2} \tag{5}$$

Equation (2) is a non-linear equation that can be solved numerically.

According to the model, the economic life of an asset is a random variable. One way of obtaining this value is by performing a number of Monte Carlo simulations and take the average of the trigger level hitting times. The hitting time T_i is here defined as the time it takes for a sample path i to hit the replacement trigger level (see Figure 1). The estimator for the expected economic life $\overline{T}$, simulating n runs, is thus:

$$\overline{T}=\sum_{i=1}^{n} T_i/n \tag{6}$$

The outcome will be an approximation, as the simulations need to be conducted within a discrete time framework.

3.2. *Closed-form solution*

According to Dobbs (2004), an analytical method or closed-form solution for obtaining the expected replacement year is not available. However, given that a GBM is a stochastic process in which the logarithm of the variable follows an ABM with drift, the probability density function of the hitting times of a GBM can be obtained by means of an extension of the hitting time distribution of an ABM with drift. The

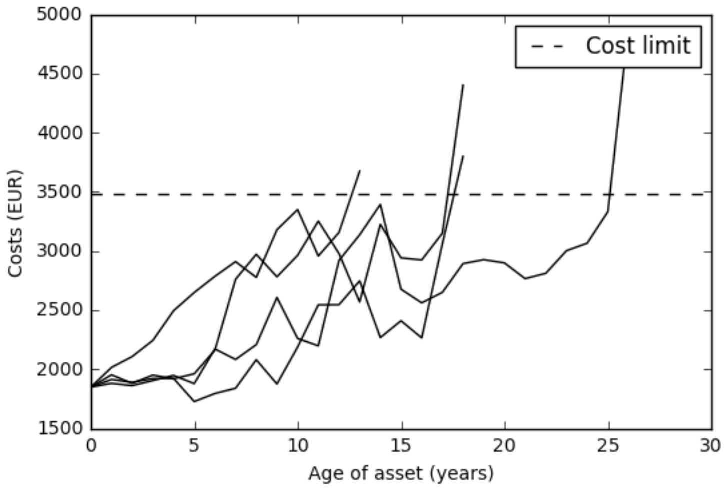


Figure 1. GBM sample paths can be generated by means of Monte Carlo simulation

density function of the hitting time distribution of arithmetic Brownian motion equals (Karlin and Taylor 2014):

$$f(t; c_0, \overline{c}) = \frac{\overline{c} - c_0}{\sigma\sqrt{2\pi t^3}} \exp\left[-\frac{(\overline{c} - c_0 - \theta t)^2}{2\sigma^2 t}\right] \tag{7}$$

For the case of geometric Brownian motion, Equation (7) can be adjusted by substituting θ with $(\theta - \frac{\sigma^2}{2})$ and the barrier $(\overline{c} - c_0)$ with $(\ln \overline{c}/c_0)$. The new probability density function becomes:

$$f(t; c_0, \overline{c}) = \frac{\ln(\overline{c}/c_0)}{\sigma\sqrt{2\pi t^3}} \exp\left[-\frac{\left[\ln\,(\overline{c}/c_0) - \left(\theta - \frac{\sigma^2}{2}\right)t\right]^2}{2\sigma^2 t}\right] \tag{8}$$

This closed-form solution has not yet been applied in the area of engineering asset management. The closed-form solution defined in Eq.(8) allows decision makers to obtain the probability of average life of a given asset, based on the historical data of such asset. However, as mentioned, these data are usually scarce, which reduces the applicability of the approach.

The authors note that for a group of assets belonging to the same system (e.g., wind turbines in a WT field), the historical data can be aggregated and used to obtain the *average economic life at a system level.* The average economic life is the expected value of the above density function:

$$E\left[f(t; c_0, \overline{c})\right] = \frac{\ln(\overline{c}/c_0)}{(\theta - \frac{\sigma^2}{2})} \tag{9}$$

It should be noted that the average economic life might be different than the economic life of an individual asset. In the next section, the model as described above is applied in practice.

4. Application of the model

4.1. *The petrol station network case*

The case study concerns the life cycle costs of Heating, Ventilation and Air Conditioning (HVAC) systems of a petrol station network in The Netherlands. Part of the asset management process involves the planning of replacing HVAC systems. The replacement policy resembles the age replacement model. HVAC systems are either replaced correctively upon failure or when the system is considered deteriorated.

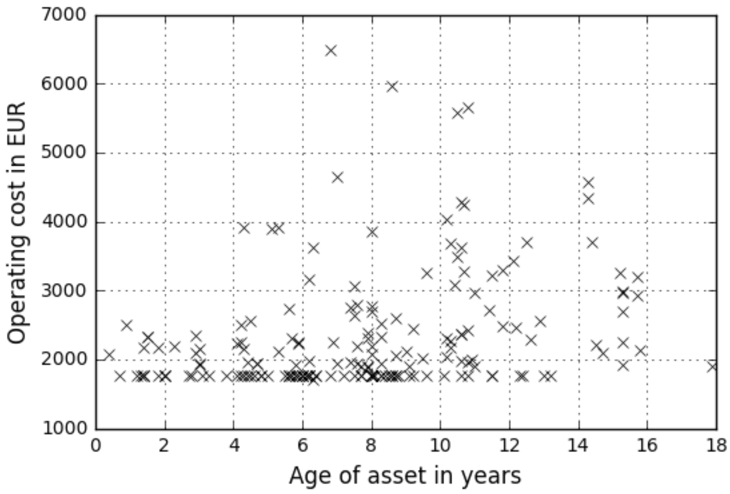


Figure 2. Operating costs of petrol station HVAC systems in 2018

Figure 2 shows the operating cost of all individual HVAC systems of the petrol stations. A first observation of the figure reveals that operating cost is increasing as assets age, but with high variance. A second observation is a horizontal asymptote, which equals the minimum operating cost per year. This minimum of EUR 1755 equals the yearly inspection cost of an HVAC system. In order to obtain the optimal replacement time by applying the proposed mathematical framework, the model parameters should be obtained based on the observed data. With this aim, a regression model is proposed. It is noted that the regression model should capture the increasing variance of the data with time. The methodology is presented in the following section.

4.2. *Parameter estimation*

The cost data as shown in Figure 2 can be used to estimate drift (θ) and volatility (σ) parameters for the GBM in the model of Equation (1). To obtain the parameters, a statistical regression model can be fitted to the data. Standard linear regression techniques assume a constant variance (Rawlings *et al.* 2001). However, the variance of a Wiener process grows linearly with the time horizon. This characteristic should thus be accounted for in the regression. A weighted least squares (WLS) regression can overcome this by assigning different weights to the data points. The variance of the errors can be described by

$$\mathrm{Var}(\epsilon_i) = \frac{\sigma^2}{w_i} \tag{10}$$

in which w_i is a positive constant that is inversely proportional to its corresponding variance, describing the weight in the regression. If we suppose that the model can be written in the form $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$ with $i = 1, 2, ..., n$ and $\epsilon_i \sim N(0, \sigma^2/w_i)$, then the estimates of β_0 and β_1 minimise the weighted sum of squares $S_w(\hat{\beta}_0, \hat{\beta}_1)$. The estimates are unbiased and are computed as follows:

$$\hat{\beta}_0 = \overline{y}_w - \hat{\beta}_1 \overline{x}_w \tag{11}$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} w_i (x_i - \overline{x}_w)(y_i - \overline{y}_w)}{\sum_{i=1}^{n} w_i (x_i - \overline{x}_w)^2} \tag{12}$$

with weighted means

$$\overline{x}_w = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i} \tag{13}$$

$$\overline{y}_w = \frac{\sum_{i=1}^{n} w_i y_i}{\sum_{i=1}^{n} w_i} \tag{14}$$

The variances of the estimators can be computed as follows:

$$\mathrm{Var}(\hat{\beta}_0) = \left[\frac{1}{\sum_{i=1}^{n} w_i} + \frac{\overline{x}_w^2}{\sum_{i=1}^{n} w_i (x_i - \overline{x}_w)^2} \right] \sigma^2 \tag{15}$$

$$\mathrm{Var}(\hat{\beta}_1) = \frac{\sigma^2}{\sum_{i=1}^{n} w_i (x_i - \overline{x}_w)^2} \tag{16}$$

An unbiased estimator for σ^2 can also be obtained. It equals the weighted error mean square:

$$\hat{\sigma^2} = \frac{S_w(\hat{\beta}_0, \hat{\beta}_1)}{n-2} = \frac{\sum_{i=1}^{n} w_i (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2}{n-2} \tag{17}$$

in which $S_w(\hat{\beta}_0, \hat{\beta}_1)$ equals the weighted sum of squares. The continuous form solution of the differential equation as described in Equation (1) equals

$$c_t = c_0 \exp\left[\left(\theta - \frac{\sigma^2}{2}\right) t + \sigma W_t\right] \tag{18}$$

with W_t being a Wiener process. By taking the logarithms of both sides, a linear form that is compatible with WLS can be obtained:

$$\ln c_t = \ln c_0 + \left(\theta - \frac{\sigma^2}{2}\right) t + \sigma W_t \tag{19}$$

Here, σW_t represents the error term of the linear regression. Also note that the variance of a Wiener process increases linearly in time. The variance of the error is then:

$$\text{Var}(\sigma W_t) = \sigma^2 \text{Var}(W_t) = \sigma^2 t \tag{20}$$

The weights in the regression are therefore:

$$w_i = \frac{1}{t_i} \tag{21}$$

The linear regression model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$, can then be used by substituting y_i into $\ln c_{t_i}$, β_0 into $\ln c_0$ and β_1 into $(\theta - \frac{\sigma^2}{2})$ and estimates of the model parameters can be obtained.

4.3. *Results*

The input parameters that were used in the case study are summarised in Table 1. Weighted least squares regression resulted in a drift (θ) of 0.0318 and a volatility (σ) of 0.103. The estimate for the operating cost in year zero, c_0, is EUR 1848. The standard deviations of the estimates were also obtained. For $\hat{c}_0$ and the estimator for $(\theta - \frac{\sigma^2}{2})$ they are 17,1% and 0.46% respectively. The trigger level for replacement, $\bar{c}$, as given by Equation (2), equals EUR 3479. Asset replacement is thus triggered when the operating cost in a particular year reaches this level. The probability function of the hitting times as described by Equation (8) can then be computed and is shown in Figure 3, along with the expected economic life. In the case study, the expected economic life is 23,9 years, as highlighted by the vertical dashed line in Figure 3. A conventional discounted cash flow method without uncertainty was also performed, yielding a replacement in year 18. This verifies the real option philosophy, stating that uncertainty creates value. In the case of asset replacement, increasing uncertainty means postponing the optimal replacement year.

Table 1. Input parameters that were used in the case study

Description	Symbol	Value
Discount rate	r	4 %
Initial outlay of new asset	K	EUR 15.000
Salvage value	S	EUR 3.000

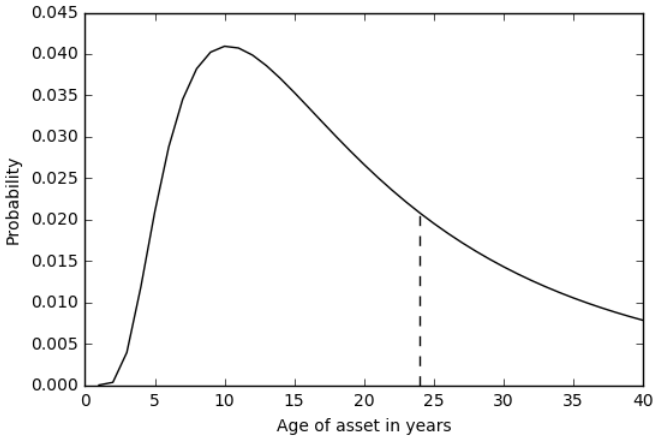


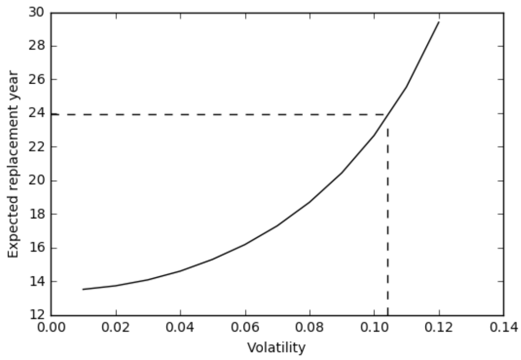
Figure 3. Probability density function of trigger level hitting times

4.4. *Sensitivity analysis*

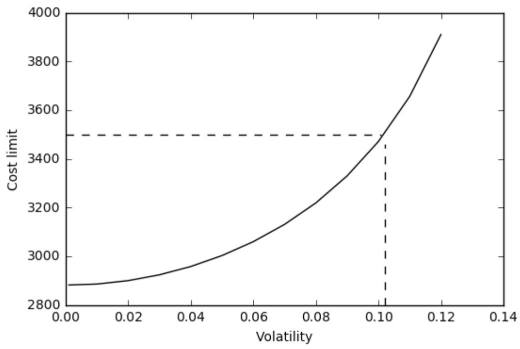
It is of interest to determine the influence of the input parameters on the final output, for that reason, a sensitivity analysis has been conducted to gain further insights on the model. For this study, a sensitivity analysis is performed specifically on the case study in order to observe the impact of a 10 percent parameter increase on both the replacement trigger $\overline{c}$ and the replacement year $\overline{T}$. The results are listed in Table 2. It can be observed that expected economic life decreases significantly when the drift (θ) increases. It is therefore crucial for decision-makers to determine the drift parameter accurately, because a small change can have a significant impact on the expected economic life. A second observation is that the expected economic life is rather inelastic with respect to interest rate (r). this is a very interesting result, given the uncertainty that this value involves. This corresponds with the findings of other authors, e.g. [here]. Also, as the initial capital outlay K increases, or when the salvage value S decreases, the expected economic life increases, as it will be economically more efficient to wait with the investment. The impact of uncertainty on decision-making is central in this paper. Therefore, an additional analysis is carried out with regard to the impact of volatility (σ) in the model. The impact of changes on the replacement trigger and the replacement year are shown in Figure 4. It can be observed that both the trigger level for replacement and the expected replacement year increase exponentially with increasing volatility. Higher uncertainty leads to a higher expected economic life. This characteristic has been found in many other studies [author]. It has also been adopted as the option effect, which states that the option to wait has value and should thus be accounted for in investment decision-making.

Table 2. Sensitivity analysis on the case study data, showing the impact of a 10% parameter increase on $\overline{c}$ and $\overline{T}$

Parameter	Value	$\overline{c}$	Percentage change	$\overline{T}$	Percentage change
Benchmark value →		*3479*		*24.6*	
θ	0.0349	3498.59	+0.56%	21.57	-12.3%
σ	0.113	3504.97	+0.75%	25.22	+2.52%
r	4.4 %	3524.39	+1.30%	24.42	-0.73%
K	16.500	3.602.80	+3.56%	25.25	+2.64%
S	3.300	3.455.12	-0.69%	23.68	-3.74%



(a) Volatility impact on the trigger level



(b) Volatility impact on the expected replacement year

Figure 4. Volatility impact on the trigger level hitting times and average replacement year

5. Discussion

It is of interest to find out the barriers and limitations that arise in practice when applying a theoretical stochastic asset replacement model, that is, to which extent the assumptions of the model are supported by the data.

As mentioned before, the operating cost development is assumed to be similar in each cycle. This implies that the model does not allow for technological progress or varying growth rates. In the case study, existing HVAC systems that were powered by gas were replaced by all-electric systems, so the model fails to capture this property. These new systems might have significant different characteristics in terms of prices and operating cost.

Moreover, the validation of using geometric Brownian motion (GBM) for modelling the cost data should be discussed, which is done by performing a residual plot of the WLS in which the residuals are corrected for the weights (Figure 5). A GBM

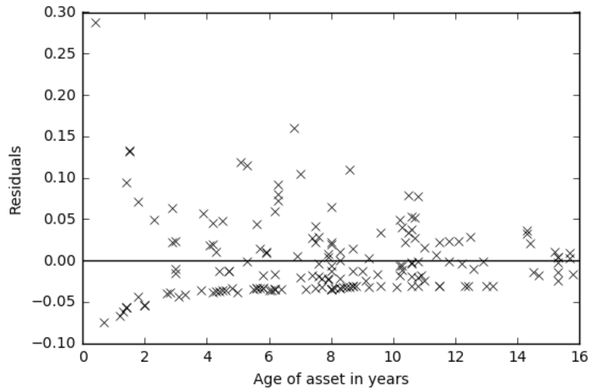


Figure 5. Weighted residual plot in which the residuals are corrected for their weights in the regression

for modelling operating cost development would be a good fit if the points in the residual plot are dispersed around the horizontal axis in a random manner (Larsen and McCleary 1972). Observation of the residual plot shows that the data points are not perfectly random dispersed. The residuals seem to converge, which is caused by the increasing variance of the GBM in time. This characteristic might indicate that a linear increase of the variance in time does not correspond with the data. One might therefore argue that a GBM is not a good representation of the operating cost process. However, because there is no significant systematic pattern disturbing the residual plot, the assumption of using a GBM is supported.

A mathematical limitation arises in the fact that the model assumes a continuous geometric Brownian motion, whereas the data points in the case study were only available on a monthly basis. The implication is that both the model estimates and the weight variances that were obtained are different from the outcomes that would be obtained in a discrete framework. The impact of neglecting this is hard to investigate analytically.

Some practical issues were found with regard to the case. It was observed that data of asset groups other than HVAC often could not be used, because costs could not be assigned to single assets, which was caused by the fact that for some asset groups, lump-sum contracts were held with subcontractors. Cash flows that follow from lump-sum contracts were not assigned to separate assets.

Finally, it should be noted that the expected economic life value does not imply that all assets should be replaced on that time. Therefore, to determine the economic life of a single asset, historical cost data of the one asset is required.

6. Conclusion

This study has introduced the use of a closed-form solution of the expected economic life distribution of asset systems when operating cost follows a geometric Brownian motion (GBM). Secondly, this study has tested a practical application of a theoretical stochastic asset replacement model. The case study involves HVAC systems of a network of petrol stations in the Netherlands. New insights and barriers are introduced when implementing stochastic models in practice, an explicit parameter estimation method is provided and the validity of using a geometric Brownian motion for modelling operating cost is discussed.

With the closed-form solution, it is now possible to analyse asset groups at a system level. Assets can be analysed and compared on different geographical systems or systems that have different environmental conditions. For example, volatility values can be obtained for the decommissioning of wind turbine parks, or compared on different levels of maritime conditions or construction materials. HVAC systems could be analysed and compared for levels of different air quality.

The closed-form solution yields the exact distribution of the expected replacement year, making discrete Monte Carlo simulations obsolete. An advantage of the expected economic life distribution is that data of multiple assets can be used. This is beneficial for situations in which operating cost data is absent, as is often the case.

It was found that using geometric Brownian motion for operating cost modelling is possible, but regression diagnostics of the data showed that a validation of the fitness of the selected approach for the analysed dataset is required. The linear increase of the variance in the model did not correspond with the actual variance development of the data. Although a GBM proved successful in modelling uncertainty, it is also important to take into account the limitations in modeling reality. Minimum inspection costs and fixed maintenance intervals could not be captured because of the random walk process. The model has additional limitations, such as fixed capital outlay values, salvage values and discount rates.

Future research will analyse how to extend the closed-form solution to more flexible initial assumptions. Moreover, the effect of other weight functions for the WLS will be analysed. Secondly, when implementing the model in practice, firms are required to have all operating cost data available per asset. In reality, this might not always be available. It should also be noted that the expected replacement year is highly elastic to changes in the drift parameter (θ). Therefore, caution should be taken when estimating the model parameters. Finally, it should be noted that the model assumes a continuous GBM, whereas in the case study, data was provided on a monthly basis. It is not clear whether this has a significant impact on the outcome of the model.

The limitations of the proposed approach are mainly linked to the stochastic model given that it assumes fixed values for the initial outlay and salvage value and it fails to capture fixed maintenance interventions. Moreover, the modelled operating costs can reach values below a minimum maintenance and operating cost level. More complex models and extensions should be explored to address these limitations.

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Glossary

Acronyms

BI Business Intelligence

CMMS Computerised Maintenance Management System

CRF Capital Recovery Factor

DCF Discounted Cash Flow

DT Decision Tree

DTA Decision Tree Approach

EAC Equivalent Annual Cost

MRDP Machine Replacement Dynamic Programming

NPV Net Present Value

O&M Operations and Maintenance

ROA Real Options Analysis

WLS Weighted Least Squares