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DOI
10.1016/j.strusafe.2020.102019

Publication date
2021

Document Version
Final published version

Published in
Structural Safety

Structural Safety 89 (2021) 102019

Contents lists available at ScienceDirect

Structural Safety

journal homepage: www.elsevier.com/locate/strusafe

Adaptive approaches in metamodel-based reliability analysis: A review

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ARTICLE INFO

Keywords:
Adaptive metamodeling
Reliability analysis
Response surfaces
Polynomial chaos expansion
Support vector machines
Kriging

ABSTRACT

The present work reviews the implementation of adaptive metamodeling for reliability analysis with emphasis in four main types of metamodels: response surfaces, polynomial chaos expansions, support vector machines, and Kriging models. The discussion presented is motivated by the identified spread and little interaction between metamodeling techniques in reliability, which makes it challenging for practitioners to decide which one to consider in a context of implementation. The conceptual problem of reliability analysis and the theoretical description of the four models is presented, and complemented by a comparative discussion of applications with identification of new areas of interest. The different considerations that influence the efficiency of adaptive metamodeling are reviewed, with extension to applicability discussions for the four models researched. Despite all adaptive techniques contributing to achieve significant gains in the amount of effort required for reliability analysis, and with minimal trade-off in accuracy, they should not be expected to perform equally in regard to the dependence on the reliability problem being addressed.

Cross application of methodologies, bridging the gap between methodology and application, and ensembles are some of new areas of research interest identified. One of the major critical considerations for adaptive metamodeling, and that has been target of limited research, is the need for comprehensive techniques that allow a blind selection of the most adequate model with relation to the problem in–hand.

To conclude, the extensive and comprehensive discussion presented aims to be a first step for the unification of the field of adaptive metamodeling in reliability; so that future implementations do not exclusively follow individual lines of research that progressively become more narrow in scope, but also seek transversal developments in the field of adaptive metamodeling for reliability analysis.

1. Introduction

One of the key challenges for engineers since the emergence of computational methods has been the development of modelling techniques that enable fast, cheap, and accurate evaluation of engineering systems. Modeling engineering systems has become progressively more accurate with the growth of computationl availability, but also complex. *In tandem* with the development of high-fidelity computational algorithms that model engineering systems, greater data availability has been continuously stressing the demand for approaches that rapidly solve problems that are critical to engineers, such as the problem of reliability analysis.

One of the approaches that showed a large potential in tackling engineering analyses that involve complex time-consuming problems is the application of metamodeling techniques. Metamodeling relies in constructing models that act as surrogates of complex problems.

In their most fundamental form, metamodels are easily understood as black-box functions that relate an input variable x to an output $Y(x)$, allowing cheap evaluation of $Y(x)$ at any input value x, Fig. 1.

Hence, a metamodel is described as a function $G(x)$ that surrogates a function $g(x)$ and allows costless evaluation of the relationship between $x \subseteq \mathrm{IR}^d$ and $Y(x)$, the value of the output at a generic x given by $G(x)$ and that surrogates the true response given by $g(x)$. d is the dimension of the input space. The common approach to metamodeling is to define $G(x)$ using a set of $x_{ED} = [x_{ED_1}, \ldots, x_{ED_k}] \subseteq x$ and $Y_{ED}(x_{ED}) = [g(x_{ED_1}, \ldots, x_{ED_k})] \subseteq g(x)$ observations, also called the experimental design (ED).

It is known that reliability analysis pursues to find the few occurrences that will result in the failure of an engineering system. That is, if a designer wants to study an engineering structure or system (described by $g(x)$) that has a 1 in N probability of failure in operation, he/she will need to search in $[x_i, g(x_i); i = 1, \ldots, N]$ evaluations for the $g(x_i)$ that results in failure. As a result of aleatory uncertainty, he/she is bound and mandated to repeat this procedure multiple times. The aforementioned idea of applying metamodels in reliability analysis is that of creating a surrogate $G(x)$ of the performance function $g(x)$. Since $G(x)$ is virtually costless to evaluate it is possible to avoid the evaluation of $g(x)$. In this

https://doi.org/10.1016/j.strusafe.2020.102019
Received 15 November 2019; Received in revised form 14 April 2020; Accepted 1 September 2020
Available online 28 November 2020

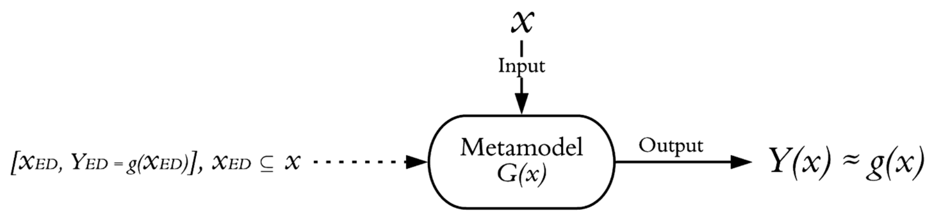

Fig. 1. Generic description of a metamodel as a black-box function defined on a support ED.

context, a crucial aspect of metamodels is that their interest is bounded to how accurate they can act as representations of $g(x)$. If an accurate surrogate of $g(x)$ is set, then it is expected to produce accurate reliability estimations. Otherwise, its interest is limited. At the same time, it is of interest to minimize the resources that are spent in building a metamodel for a certain level of accuracy. This requirement to exploit the characteristics of metamodeling in order to fully harness the benefits of their application originated a research topic that has captivated significant interest, the adaptive metamodeling.

Adaptive metamodeling refers to the methods that in some way use a measure of improvement to enhance the capability to surrogate $g(x)$. In these, the surrogate prediction is improved (in iteration $i+1$) with basis on the current (at iteration i) stage of the surrogate using a pre-established target (*e.g.*, accuracy). In computational experiments this process of improvement in sequence is also frequently denominated as the process of learning [1].

Because metamodels have showed that they can perform well to solve the problem of reliability, their application as surrogates in this field proliferated and distinct adaptive techniques for metamodeling emerged. It is difficult in the present for a new practitioner of reliability to grasp the existing adaptive metamodeling techniques to their full extent. At the same time, little interaction has been identified between fields of metamodeling [2]. It is expected that enabling practitioners to overview the field of adaptive metamodeling and fomenting transversal interaction in it, will have an important role in improving the current state-of-the-art in metamodeling for reliability. In this context, the present work pursuits to establish a comprehensive review of the adaptive metamodeling in time-invariant reliability analysis for scalar performance functions in order to provide practitioners with an overview, while not disregarding its contribution to the state-of-the-art. For such goal, Section 2 frames the problem of time-invariant reliability and introduces the theoretical basis of the different metamodels used. Section 3 discusses adaptive implementations in reliability with particular emphasis on aspects that influence the performance of adaptive metamodeling in reliability. Section 4 presents a comparative discussion with basis on results from the literature, and discusses applicability for the models studied. Section 5 discusses the contribution of the developed analysis beyond the state-of-the-art, i.e., areas of further improvement. Finally, the main conclusions of the work developed are drawn in Section 6.

2. Metamodeling for reliability analysis

In the context of metamodeling for reliability analysis, [3] distinguishes two sub-disciplines of metamodeling, regression and classification. The distinction is related to the definition of the variable $Y(x)$. In regression, the metamodel surrogates $Y(x)$ as a continuous variable within the x continuous space. In classification the x space is also covered but attributing discrete labels to Y. In reliability analysis, even considering that the ultimate goal is to perform a classification (*i.e.*, failure and non-failure), regression is more prevalent. In both cases metamodeling can be further classified in subtopics, such as, global and local approximation. In the local approximation the goal is to establish an accurate predictor of $g(x)$ for the region of interest, *i.e.,* the region of failure. The idea is mainly to characterize locally the boundary that will separate failures and non-failures, and this is of interest when confined regions of x dominate the estimation of P_f. For highly complex problems this approach is not sufficient, and global approximation should be pursued. In it, $G(x)$ pursues to establish a global description of $g(x)$ while capturing also aspects enclosed by the local approximation.

In the present work the general framework for time-invariant reliability analysis [4,5] is addressed, where the probability of failure (P_f) is expressed as the probability $P[]$ of the performance function having values smaller or equal than a threshold of 0. That is,

$$P_f = P[g(x) \leqslant 0] = \int_{g(x) \leqslant 0} f_x(x) dx \tag{1}$$

where $f_x(x)$ is the continuous[1] joint distribution of the d - dimensional vector of x input variables. $g(x)$, the performance or limit-state function, divides x in two domains: the safe-domain, $g(x) > 0$, and the failure domain, $g(x) \leqslant 0$. An efficient strategy to evaluate the complex integral in Eq. (1) is to classify the performance function $g(x)$ in x as failure or non-failure accordingly to,

$$I_f(x) = \begin{cases} 0, & if \ \ g(x) > 0 \\ 1, & if \ \ g(x) \leqslant 0 \end{cases} \tag{2}$$

with I_f being a binary performance evaluator of failure that is, $I_f(x) = 1$ for failure and $I_f(x) = 0$ for non-failure. Accuracy in metamodeling for reliability is related to how well the regressor or classifier represents the true $I_f(x)$ given by $g(x)$.

One of the fundamental alternatives to solve the integral of Eq. (1) is to use the Monte Carlo simulation (MCS). In MCS, the I_f classification supports the construction of a statistical estimator of the approximate probability of failure, that is,

$$P_f \approx \widehat{P}_f = \frac{1}{N_{MCS}} \sum_{i=1}^{N_{MCS}} I_{f_i}(x) \tag{3}$$

where N_{MCS} is the total number of assessed x for reliability calculations. The coefficient of variation (CoV) of this calculation is given by,

$$CoV_{\widehat{P}_f} = \sqrt{\frac{1-\widehat{P}_f}{N_{MCS}\widehat{P}_f}} \tag{4}$$

It is understandable that since it is common for P_f to be of $\mathcal{O}(10^{-3})$, $\mathcal{O}(10^{-4})$, or even smaller, evaluations of N_{MCS} can become a burden. This resulted in a need for developing alternative techniques to calculate P_f, such as, Importance Sampling (IS) [6], the First Order Reliability Method (FORM) [7,8], or Subset Simulation (SS) [9]. Metamodels [10] are just another tool that is used to solve this complex evaluation.

2.1. Types of metamodels

Common application metamodels for reliability analysis, Fig. 2, are: response surfaces [11,12], support vector machines [13,14] (SVM),

[1] It is noted that continuity is intrinsically related to metamodeling but it is not inherent in the definition of reliability.

polynomial chaos expansion [15,16] (PCE), Kriging models also known as Gaussian process predictors [17,18], and artificial neural networks (ANN) [19]. Applications of the first four in the context of reliability are extensively discussed in the present work. It is important to highlight that application in reliability of other metamodels not addressed here can be identified, *e.g.* logistic regression [20]. In the present discussion the interest is on adaptive implementations; a topic that has been most widely discussed for these four models.

Artificial neural networks (ANN) are an alternative metamodel that has been successfully implemented in reliability analysis [21,22]. These are considered in the discussion but not extensively covered in the present work. Implementations of adaptive ANN for reliability are limited. While the concept of network is broad, most of the implementations identified consider sequential enrichment of the training samples and/or definition of the best ANN configuration [23,19,24]. The complexity of the hidden layers of the network may be one of the reasons that has hindered further applications of adaptive approaches that fully exploit ANN in its most interesting form, with multiple layers. The challenge of finding an adequate architecture for the network commonly demands the usage of a training sample in addition to an ED. From the perspective of the present analysis, which focuses on adaptive approaches, most of the ANN works for reliability fall into a slightly different category of implementation. Nevertheless, the interested reader is directed to the recent comprehensive review of [19] that addresses the application of ANN in the context of reliability analysis.

The term Response Surfaces (RS) has been consistently used for metamodels that use linear regression of polynomial functions since the origins of the idea of metamodeling complex systems [10,25]. Later, the term RS would be applied also to refer to other applications, such as SVM [26], or ANN [19], but not consistently. In the present work, RS methods describe metamodeling that uses linear regression in its simplest forms with different basis functions. Other metamodels that can be also understood as RS are discussed separately, in particular due to the fact that some of these originated extensive separate research trends (*e.g.*, Polynomial Chaos Expansions). The present discussion follows then the diagram of Fig. 2 in order to distinguish the different metamodels. Additionally, Table 1 summarizes the definition and characterization of; (i) RS in three of its main forms: using polynomial basis functions, radial basis functions, and spline basis functions; (ii) PCE; (iii) SVM and (iv) Kriging; and its complemented by a brief discussion on each model in the following sections.

2.1.1. *Response surfaces*

The most widely established technique to metamodel $g(x)$ using $G(x)$ uses a linear combination of basis functions, which gives form to the RS method. RS have been applied to many different fields in reliability engineering [27,28,25,29–34]. Despite widely applied with polynomial basis functions, RS also appear constructed on radial basis functions (RBF), spline functions, or other less common forms, such as the exponential form proposed by [35]. Despite the appearance of more complex alternatives, according to [36] polynomial basis RS are still the most popular metamodeling technique for reliability.

In the application of polynomial regression RS, three major factors that have large influence on the performance of $G(x)$ as a surrogate of $g(x)$ can be highlighted; the order of the regression (number and degree of basis functions, including mixed terms); the technique used to estimate a; and the ED. Due to their wider establishment in different fields, extensive literature covers the distinct problems that emerge in application to reliability and that are frequently related to the polynomial RS simplicity, such as biased or inaccurate predictions of P_f due to saturated designs [37] (ED has strictly the size necessary to define the vector a), or ill-conditioned problems [38,39].

When RBF are applied at least one hyperparemeter needs to be adjusted to the ED. This demands additional cost in the RS definition. Cross-validation has been previously implemented to adjust RBF hyperparameters[2] in reliability problems [40]. Its intrinsic measure relating to the ED points through a distance metric and an adjustable hyperparemeter indicates a larger capability of RBF regressions to adapt locally (due to the nature of their kernel, RBF act as interpolants, and are expected to approximate other models that use the similar kernel).

When constructed with basis on spline functions, RS become piecewise functions defined using sub-functions in subset domains, and divided by the so-called knots (Ξ). Considering the range of definition $[a, b]$, this interval can be subdivided into Q subintervals denoted by $[a,\Xi_1]$, $[\Xi_1,\Xi_2],\dots,[\Xi_{Q-1},b]$. In each subinterval, different polynomials $P_i(x)$ (or other basis) are used to fit the objective function, making the spline function a set of Q pieces. Their interest emerged as a response to the limitations of the polynomial RS to perform well for large intervals of x and large ED. A comprehensive discussion on splines is presented in [41–43]. Common RS that use splines, such as B-splines functions, can be defined with the individual application of established techniques such as least squares regression [44].

2.1.2. *Polynomial chaos expansion*

Polynomial chaos expansions (PCE) are a metamodel that is able to expand finite variance $g(x)$ processes using a combination of multivariate basis functions that are orthogonal with respect to the joint probability density function f_x of input variable x. For example, if x are independent standard Gaussian variables, there is a multivariate polynomial basis that is orthogonal with respect to f_x.

In reliability they are implemented in their non-intrusive form. As f_x can take multiple forms, a common approach for reliability is to represent x in the standard normal space via a transformation of variable [49], which makes a type of orthogonal, Hermite, polynomials particularly interesting. In order to estimate the a_i coefficients popular methods are the projection with quadrature methods or least-square minimization. A discussion on these is presented in [50]. In PCE the value of d poses a significant threat to its efficiency. As d increases the size of the required ED explodes, making PCE highly susceptible to the *curse of dimensionality*[3] [2]. The orthogonal property of the PCE representation is one of its most interesting merits. It allows for these to perform efficiently in the capture of the global stochastic behaviour of $g(x)$ [2]. If a polynomial regression is applied using p order polynomial functions *i.e.*, $[x_i, x_i^2, \dots, x_i^p]$, as this polynomial basis is not orthogonal, for $x > 0$ the prediction value considering the basis functions may increase rapidly, while the same is not verified for $x < 0$. The approximation may highly depend on the estimated weights of the basis functions. The interested reader is directed to the works of [51,49,52,50,53] where the PCE theory and its merits are comprehensively discussed.

2.1.3. *Support vector machines*

Support vector machines (SVM) is a kernel based metamodeling technique initially formulated for classification problems, and later extended to regression problems. These are frequently, and respectively identified as SV Classifier (SVC) and SV Regressor (SVR). In reliability they can be applied in both forms [54,13,55,5]. In binarySVC, $g(x)$ is classified in a $c \pm 1$ category so that a boundary can be set in-between the two classes such that $G(x) > +1$ for $c = +1$ and $G(x) < -1$ for $c = -1$. This boundary is given by an hyperplane whose expression is $G(x) = 0$. In SVR, the problem is formulated such that the SVR that defines $G(x)$ is found to have at most a deviation of ε from observed $g(x)$

[2] Hyperparameters refer to the parameters that are not directly learnt from the data and demand tuning to improve the metamodel performance.

[3] Computational demand increases exponentially with the increase of the number of dimensions, or variables in the present context.

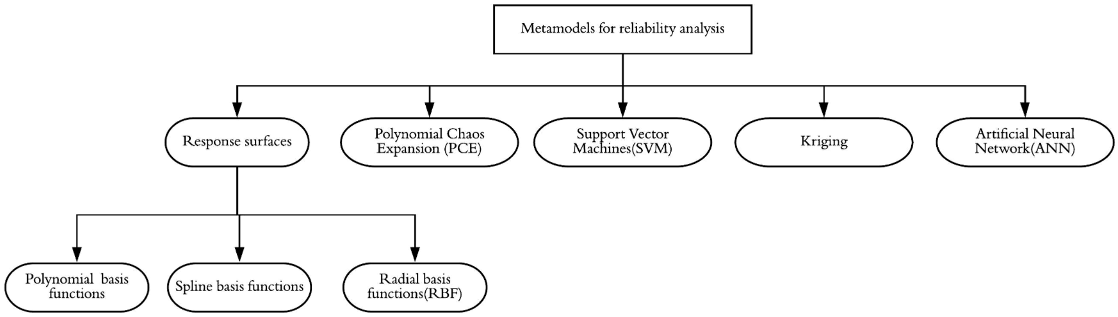

Fig. 2. Types of metamodels identified in reliability analysis for design.

Table 1
Summary of the main features of the different metamodels discussed in the present work in relation to their applicability to the reliability analysis.

Method		Model	Characterization
RS	Polynomial regression	Definition of $G(x)$, surrogate of $g(x)$, with polynomial regression uses, $$G(x)=\sum_{i=1}^{p} a_i f_i(x)$$ where $a=[a_1,\dots,a_p]^T$ is a set of weight factors dependent on the pair $[x_{ED},g(x_{ED})]$ used to define $G(x)$. In its most common form, p polynomial functions of type x_i^0,x_i^1,x_i^2,x_i^p for $i=1,\dots,d$ are applied.	In their polynomial regression form, RS are fast to construct (they depend only on the weight factors a). As d increases the minimum size of ED increases, and for large d only a limited set of basis functions may be feasible to consider. Prediction on polynomial regression, or splines based on these, has little demand. New predictions are evaluated on basis functions weighted with the polynomial coefficients (splines based metamodels are not restricted to polynomial basis functions). Radial basis functions (RBF) demand the training of the kernel shape that increases the demand to train a model. Depending on the kernel used, these interpolants are expected to approximate other metamodels as surrogates of $g(x)$, but not enclosing unique properties such as uncertainty in the Kriging, or sparsity in SVM. RBF demand the computation of distances, which is expected to increase the time and memory demand when the ED increases. Distance metrics such as the Euclidean may also under-perform in very high dimensional spaces [45]. *Main features*: Simple. Fast computation.
	Radial Basis Functions	In regression RBF takes the form $$G(x)=\sum_{i=1}^{k} a_i\varphi(\|x-x_i\|)$$ over a set of k size ED and where $\varphi(.)$ relates the Euclidean norm ($\|.\|$) to a given support point x_i. Different types of RBF kernel may be applied, such as linear $\varphi(x)=c\|x-x_i\|$, or Gaussian $\varphi(x)=\exp(-\alpha\|x-x_i\|)$, with c and α as positive shape parameters (hyperparameters) that need to be adjusted. While the most conventional polynomial regression depends on the set of p parameters (under the consideration that $p<k+1$ for the problem to be well-posed) that is an input to the model characterization, RBF is adjusted to the size k.	
	Spline Functions	A spline of degree p with $Q-1$ knots can be expressed in the form of, $$f(x)=\sum_{i=0}^{p} a_i x^i+\sum_{j=1}^{Q-1} l_j(x-\Xi_j)_+^p,\quad \text{with}\quad (x-\Xi_j)_+^p=\begin{cases}0, & x<\Xi_j\\ (x-\Xi_j)^p, & x\geq\Xi_j\end{cases}$$ and $a=[a_0,a_1,\dots,a_p]$, $l=[l_1,l_2,\dots,l_{Q-1}]$ as the model coefficients. This form of splines is denominated the truncated power basis. [41] generalizes these in the so-called B-splines.	
PCE		Considering that x is characterized by its f_x, the polynomial chaos expansion of $g(x)$ (on a truncated basis) can be simply written as $$G(x)=\sum_{i=1}^{P} a_i\Phi_i(x)$$ where a_i are a series of deterministic coefficients and $\Phi_i(x)$ is a basis of multivariate orthogonal polynomials. These multivariate basis polynomials are defined as a tensor of the univariate polynomials related to $x=[x_1,\dots,x_d]$. The order of the expansion is set to be function of the number of input variables and maximum order ot the $\phi(x)$ basis, both define the minimum ED required for the PCE definition to be well-posed [16].	It uses a regression that expands the metamodel representation on an orthogonal basis. Training or fitting of the PCE uses simple and fast methods such as the least squares minimization. It has no hyperparameters to train, however PCE are sensitive to d. And in their most efficient form they demand some training of the orthogonal basis, which requires an iterative search. After the basis functions are set, prediction involves only the evaluation of new points in the univariate polynomials and only few additional calculations. *Main features:* Efficient global prediction given by orthogonality characteristics. Direct evaluation of statistical moments. Relatively fast computation.
SVM	SVC	In SVC, $g(x)$ is classified in a $c\pm1$ category so that a boundary can be set in-between the two classes such that $G(x)>+1$ for $c=+1$ and $G(x)<-1$ for $c=-1$. This boundary is given by an hyperplane whose expression is $G(x)=0$, and that maximizes the margin between the points (minimizing the norm of vector normal to the separating hyperplane w) . The SVC problem and the $G(x)$ classifier are formulated as, $$SVC\ :\ c[\langle w,x\rangle+b]\geq 1 \text{ and } G(x)=\langle w,x\rangle+b$$ with b being a constant bias term.	It is virtually insensitive to the number of random variables [46, 47]. In the simplest form it has two hyperparameters to be tuned which may need to be adjusted for the dataset via grid-search and cross-validation. A third parameter may also require adjustment in the SVR form. Prediction demands the evaluation of a kernel matrix that for very large data-sets can require costly calculations, nonetheless it only depends on a subset of the ED, the support vectors (terms with coefficient non-zero). Are versatile interpolants with access to distinct kernel functions. *Main features:* They can be applied in the form of classification and regression, and are fast to compute for small ED [48]. They have synergy with high-dimensional problems, and their accuracy is expected to approach that of other metamodel that use similar kernel (function of the optimization).
	SVR	In SVR the problem is formulated such that a $G(x)$ is found to have at most a deviation of a ε loss function from observed $g(x)$ evaluations (or $g(x_{ED})$). $$SVR\ :\ Y-\langle w,x\rangle-b\leq\varepsilon\cap\langle w,x\rangle+b-Y\leq\varepsilon \text{ and } G(x)=\langle w,x\rangle+b$$ As it is not always possible to solve the problem of optimization demanded by the SVR under the "rigid" constraint of ε, slack variables (ξ,ξ^*) are introduced to by-pass this limitation [46]. In addition to ξ,ξ^*, a penalisation term C is also considered to further expand the flexibility of this problem.	
Kriging		The Kriging metamodel approximates the true response function $g(x)$ as $$G(x)=f(a;x)+Z(x)\ \text{ with }\begin{cases}f(a;x)=a_1f_1(x)+\dots+a_pf_p(x)\\ Z(x)=\mathcal{N}(0,C(x))\end{cases}$$ where $f(a;x)$ is a polynomial regression in its standard form with p ($p\in\mathbb{N}^+$) basis trend functions $f_p(x)$ and p regression coefficients a to be defined. $Z(x)$ is a Gaussian stochastic process with zero mean, defined with basis on a covariance matrix (C) that relates generic x points by using a constant process variance (σ^2) and a correlation function $R(x;\theta)$. A prediction for the true realisation $g(u)$ in a point u in the space given by the Kriging has expected value $G_\mu(u)$ and a variance $G_\sigma^2(u)$ component.	In the form commonly applied to reliability, with Gaussian correlation, have θ_d hyperparameters to be tuned. Other kernels can be applied and are of interest for involved problems (may add additional parameters to be adjusted). In high dimensional spaces and ED the number of hyperparameters is expected to increase the requirement to define the metamodel. [48] show that Kriging are inefficient when applied to relatively large ED (+2000 points). Furthermore, they may not be stable in large ED. It is noted that this is hardly a limitation as in reliability analysis relatively low samples sizes are commonly used. Reliability applications are frequently set to depend on a distance measure, which may pose a challenge for large d and ED. *Main features:* They enclose an intrinsic measure of uncertainty. They perform as interpolants without loss of generalization.

(See above-mentioned references for further information.)

evaluations, see Table 1.

One of the particularities of SVM is that the solution of the optimization that finds w uses Lagrange multipliers (α), which allows for w to be represented as a linear combination of x_{ED} and α. The solution to this linear combination shows that only a subset of x_{ED} is required to generate $G(x)$, the points that have on-zero value of the α multiplier. Therefore, by construction there is sparsity in the resulting model (origin of the name Support Vector). This is the most relevant property of the SVM in its both forms, the definition of SVM has limited dependence on the d dimension of the input space [47,46]. In SVC, α is non-zero in the points that define the margin, whereas in SVR only the samples outside the ε-region will enclose relevant information to characterize w.

In common reliability problems, $G(x)$ needs to metamodel highly complex $g(x)$. In SVM the approximation to complex classification and regression is achieved by using a *kernel trick* [56], which projects the support data in a feature space where the projection of x in a separable inner product can be solved.

In their most fundamental form, SVM may use a tuning over kernel parameter from the kernel function, the C parameter that controls the complexity of the regression and the loss function. Common implementations for reliability analysis adjust these using cross validation error with root-mean-squared-error [57,58]. A comprehensive discussion on the parameter selection for classification and regression is presented in [59,60].

2.1.4. Kriging or Gaussian process models

Kriging models, or Gaussian process models, are a particular case of metamodels that interpolate $g(x)$ (and that in their stochastic form, approximate $g(x)$) considering that the model response follows a Gaussian process indexed by input random variables, with the ED acting as conditioning points. Because Kriging models enclose a measure of uncertainty, they intrinsically perform as self-improving functions.

The application of Kriging is kernel based and demands the selection of a correlation function and a polynomial basis. The correlation is commonly assumed to be stationary and to take the *separable* form [61]. Nonetheless, other types of correlation can be applied [1]. In reliability applications a Gaussian correlation function, or kernel, and constant trend function are frequently used [62].

In the Kriging, $G(x)$ predictions depend on a, σ^2 and a correlation $R(x;\theta)$, which depend then on a series of θ hyperparameters to be estimated. In common kernel forms applied to reliability, one θ needs to be trained for each dimension, however, research on more advanced kernels is of interest for reliability problems [63]. For a given sample of support points the problem of prediction can then be solved through a generalised least squares formulation, where the estimators for β and σ^2 depend uniquely on θ. In order to adjust $G(x)$ to the ED, an optimization is performed using a maximum likelihood search for θ. The final form of $G(x)$ is that of an interpolation function that encloses infinite possibilities of curve predictions under the assumption that in the x points the prediction follows a $\mathscr{N}(G_\mu(x), G_{\sigma^2}(x))$, with $G_\mu(x_{ED}) = Y_{ED}$ and $G_{\sigma^2}(x_{ED}) = 0$.

To conclude the present section, it is noted that implementations of the presented metamodels are machine, algorithm and assumption dependent. This is particularly relevant when readily available models are used. The introduction of the different models shows that in metamodeling different decisions rely on the user. Selection of basis functions, fitting techniques, kernels, hyperparameter optimization algorithm, or parameter space constraints are some examples of variables that depend on the user, that may have large influence on the performance, and that many times are not researched to the extent they should. Depending on the codes, variations may be found depending on the algorithm construction (e.g., comparative cases of the Kriging for the ooDACE [64] and UQLab [50]). The aim of the present paper is that of reviewing adaptive approaches, therefore, despite of significant relevance, no further discussion is pursued in relation to these important assumptions. The interested reader is directed to the extensive literature referred to in the present Section that discusses the fundamentals of these models.

3. Adaptive approaches in metamodeling for reliability analysis

The progressive increase in applications of adaptive metamodeling in reliability analysis resulted in a multiplication of singular or unique research implementations that adaptively pursue to set accurate $G(x)$ surrogates of $g(x)$. Four main general aspects can be highlighted to play a major role in the metamodeling and adaptive implementations:

- Initial Experimental Design (ED);
- ED enrichment and stopping criterion;
- ED size and domain;
- Metamodel parameters (assumption and estimation);

The ED has large influence on the capability of $G(x)$ to approximate $g(x)$. This influence is prevalent by means of the initial ED or the ED enrichment, *i.e.*, the process of enlarging the ED with new evaluations of $[x, g(x)]$. It was seen that defining $G(x)$ demands a sample of support points, an initial ED that may be posteriorly enriched based on a specified criterion. ED enrichment uses criteria that select new candidates to be added to the ED and a halting condition that balances the gains of further ED enrichment. In addition to the ED, all metamodeling techniques depend by construction on a set of parameters and assumptions that are selected/fitted/adjusted to the ED (such as, correlation functions or hyperparameters), and that can be exploited in adaptive approaches. The procedure of estimating and tuning the model parameters in machine learning language is frequently referred to as training. Finally, the ED size and domain used to enrich, evaluate $G(x)$ or to set the ED also influences the efficiency of the metamodel approximation, and recent works have exploited this fact in the rationale of adaptive implementations. Fig. 3 presents four representative examples on how these different considerations influence $G(x)$ as a surrogate of $g(x)$. Case I presents how the choice of initial ED defines in a first instance $G(x)$ (RS in this case). II shows how an appropriate choice of the point to enrich the ED contributes to improve the $G(x)$ surrogate of $g(x)$. In III the same ED and enrichment approach are used considering two different candidate samples (a MCS and a Sobol Sequence), with direct influence on the improvement attained in the $G(x)$ capability to represent $g(x)$. Finally, in IV the same ED is fitted with two PCE, with different considerations on model parameters, showing that an adequate choice of model assumptions substantially improves the approximation to $g(x)$.

It is important to highlight that what distinguishes adaptive approaches in metamodeling is that they enclose some notion of improvement that pursues to enhance the performance of $G(x)$ as an accurate predictor of $g(x)$ or as an accurate classifier of $I_F(x)$. The origin of the term adaptive is related to the ED enrichment. Nonetheless, adaptivity may be possible with measures of adjustment such as, sparse rationales [16,65,66], learning functions [18,67], sampling enrichment [13], sub-framing of ED regions [68], and design space transformation [69]. With the increasing interest on metamodeling in reliability analysis, several methods have started to combine multiple approaches that cover more than one of the previous [70,36,71]. The following sections review and discuss how the four aspects highlighted have been addressed in the problem of adaptive metamodeling for reliability analysis. And in order to facilitate the screening of different methods, Table 2 summarizes adaptive implementations in reliability by type of metamodel, its features and adaptive measures enclosed.

3.1. Initial Experimental Design (ED)

In the early days of metamodeling with RS, [25,72] rapidly identified that the simple application of metamodels was not a guarantee of efficient effort reduction for reliability analysis; hence, highlighting the requirement for adequately approaching the ED. Such perception of improvement generated an initial spectra of ED alternatives to define $G(x)$ [73,74,11,75,76].

Random sampling techniques, such as MCS, are the most fundamental techniques to define the initial ED, however, as these do not obey any criterion other than the random description of x, they do not provide the most efficient approach to it. Star-shaped designs would emerge as an alternative technique for efficient RS metamodeling. Star shaped ED consist in using a center point and two on-axis complementary points, a pair for each dimension, with a distance of k standard deviations from the center. It has synergies with the RS, however, their application lacks generalization. With the requirement for progressively more complex metamodeling techniques, the Latin Hypercube Sampling (LHS) became the most widely implemented technique in adaptive metamodeling for reliability analysis. LHS consists in sampling points in equal intervals of probability guaranteeing a balanced coverage of the x space. In cases where the ED is not adaptive, LHS have been preferred due to their global description of the ED [77,78,16]. An initial LHS also allows to use an iterative refinement of the initial sample while preserving the LHS properties in what is frequently called a nested-LHS [65]. In order to meet the demand for more comprehensive approaches to the ED, [79,69] recently proposed the usage of uniform ED, and [80] of Sobol Sequences. The uniform ED allows for a global coverage of x, while relaxing the probability constraints of the LHS. The Sobol Sequence is a low discrepancy sequence that also pursues a uniform distribution of points. [81,82] show, based on the works of [83,84], that optimum ED considerations can be found for the initial ED and its improvement in PCE implementations. Nevertheless, the rationale and relevance of

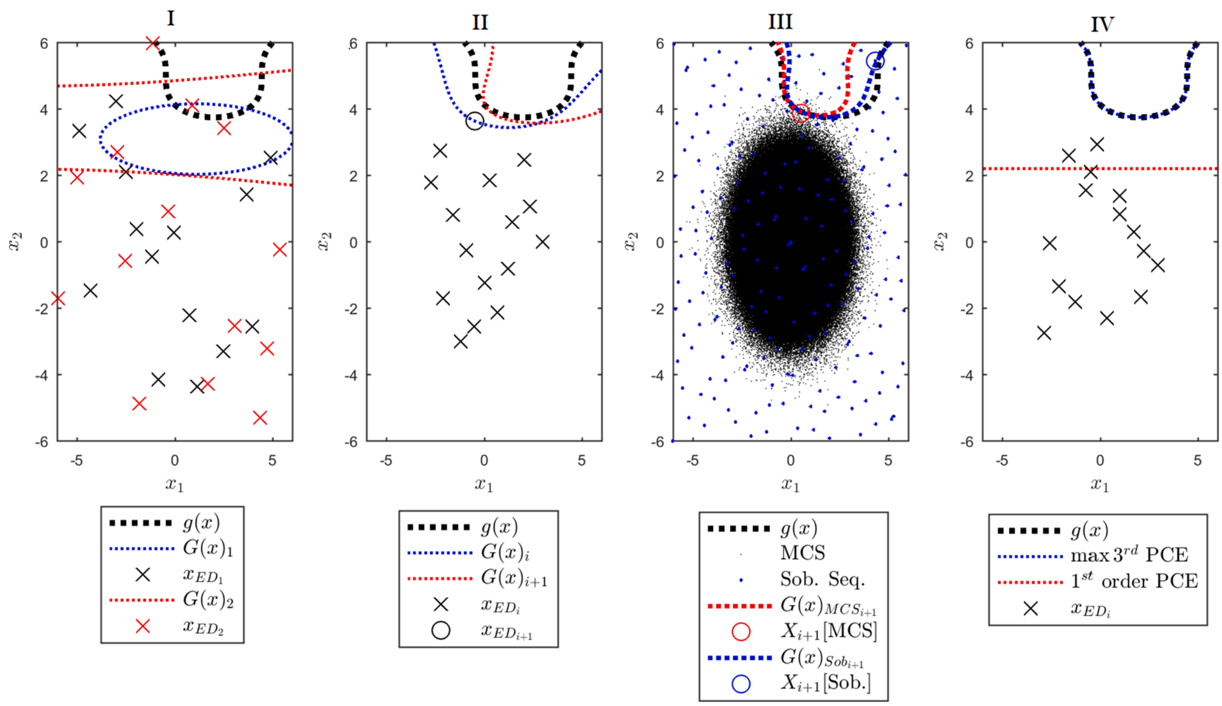

Fig. 3. Example of the influence of the four considerations discussed. **I** - Example of the influence of the initial ED in a quadratic RS approximation, showing that the initial sample of support points is of relevance for further enhancement of the surrogate. **II** - Example of the influence of ED enrichment with a learning function that uses Kriging, and its contribution to improve $G(x)$ as a surrogate of $g(x)$ in iteration $i+1$. **III** - Example of the influence of the candidate sample, using a MCS and a Sobol Sequence, and its influence in the sequential improvement of $G(x)$ as a surrogate of $g(x)$. **IV** - Example of the influence of the model parameter (considering PCE) in the capability to surrogate $g(x)$. $g(x)$ represents the separation between the domains 0 and 1 of $I_f(x)$.

optimal ED considerations are yet to be researched to a larger extent in adaptive metamodeling for reliability applications.

No practical guidance on the adequate size of the initial ED for reliability applications has been systematically investigated yet. [36] previously highlighted this fact, while discussing some alternatives for initial sample sizes. [85] recommended a sample size of $4/3d$ in their introductory works to LHS. In practice different literature works use distinct initial ED sizes, and frequently with limited information on the criteria for selection.

3.2. ED enrichment and stopping criterion

Despite the importance of the initial ED, the possibility of adaptively enriching the initial ED is one of the main characteristics of adaptive metamodeling. It consists in establishing a measure of improvement in the capability of $G(x)$ to surrogate $g(x)$ in order to select the additional ED points that are expected to improve this approximation. This occurs iteratively until a stopping criterion halts the enrichment. Despite an accessible concept, it was not until the work of [11] that its relevance would start to be fully exploited. In the context of the literature reviewed, four main approaches are discussed hereafter:

3.2.1. ED enrichment using interpolated ED

The idea of using interpolated ED is related to a redefinition or update of the ED in a region of interest where the new ED point or the redefined ED is selected or interpolated within the present metamodel. Interpolated ED have been mainly used, and are of interest, in local regression (approximate the limit-state function locally).

In the influential work of [11], the authors proposed an adaptive scheme that seeks to improve a RS with basis on its current iteration (e.g., updating a new star-shaped ED centre) in order to improve the characterization of the failure region. [12,34] later exploited the insights given by the former highlighting the need for an iterative update of $G(x)$ and a criterion to halt the search, *i.e.*, a stopping criterion. [86] further elaborated on this approach using a gradient-projection technique to rotate the interpolated ED. Ref. [87] adapted this gradient technique, combining it with the first order reliability method (FORM), higher-order polynomial functions, and highlighting the interest of using selective information about previous iterations. In order to deal with more complex $g(x)$, [88] applied interpolated ED on shifted axis for multiple failure region identification; and, [89] proposed an iterative ED complemented on projection points. This rationale of iteratively (re) interpolating ED influenced a large spectra works in adaptive metamodeling for reliability with distinct metamodels [90–92,57,26,93,94,69].

3.2.2. ED enrichment with multi-stage algorithms

Multi-stage algorithms emerged from the initial need to tackle the inherent limitations of the relative simplicity of RS, and to deal with increasing demand for methods capable of addressing complex reliability problems. Multi-stage algorithms use different stages of improvement obeying distinct enrichment conditions and halting criteria, which result in efficient surrogates. When improvement of the metamodel is fulfilled in a given stage, it passes to the following one until all the stages are progressively fulfilled.

[87] proposed one of the first multi-stage algorithms with increasing metamodel and ED complexity as an alternative to enhance the performance of RS for reliability estimation. [68] also pioneered this idea with a multi-stage algorithm with framed domains. Ref. [5] applies a multi-stage approach using SVC. [95] implemented a multi-stage adaptive Latin Hypercube Sampling (LHS) ED inspired by the implementation of [96], using three stages of convergence. [36] further elaborated on multi-stage refinement of an initial LHS.

The main drawback of multi-stage algorithms is related to their relative complexity. In each stage multiple parameters may need to be selected, and this may decrease their generalization capability.

Table 2
Relevant adaptive metamodeling approaches applied in structural reliability analysis. ED - uses adaptive enrichment. Stop – refers to the usage of a stop criterion, other than max i. Model – uses model parameter or assumptions. Par. – Uses parallel computation. Cand. – Adaptivity in candidate sample or candidate domain. + – Star-shaped design [11]. VC – Voronoi cells. LDS – Low-discrepancy sample. U – Uniform sample. Rand – Random sample. $\beta(x_D)$ – Method uses design point identified on metamodel/FORM to estimate P_f.

				Measures of improvement implemented					
Ref.	Approach	Initial ED	Iterative	ED	Stop	Model	Par.	Cand.	$\widehat{P}_f$
Adaptive RS metamodeling									
[11]	Re-interpolated centre.	+		✓					$\beta(x_D)$
[12]	Proposed an iterative approach to [11].	+	✓	✓	✓				$\beta(x_D)$
[34]	Proposed an iterative approach to [11].	+	✓	✓	✓				$\beta(x_D)$
[86]	Gradient-perturbed ED near $g(x) = 0$.	+	✓	✓	✓			✓	$\beta(x_D)$
[87]	Multi-stage approach based on [86].	+	✓	✓	✓			✓	$\beta(x_D)$
[68]	CQ2RS method.	U	✓	✓	✓			✓	$\beta(x_D)$
[88]	k-shifted axis re-interpolation.	k-+	✓	✓					MCS
[91]	ADAPRES method.	half-+	✓	✓					$\beta(x_D)$
[92]	Adaptive weighted interpolation.	half-+	✓	✓	✓			✓	$\beta(x_D)$
[89]	ED projected enrichment.	+	✓	✓	✓			✓	$\beta(x_D)$
[153]	Adaptive RS order.	Uniform	✓			✓			MCS
[144]	Double weighted adaptive interpolation.	half-+	✓	✓	✓				$\beta(x_D)$
[145]	FORM re-centred ED	half-+	✓	✓	✓			✓	$\beta(x_D)$
[154]	Re-centred rotated ED.	half-+	✓	✓	✓				MCS/IS
[95]	Adaptive LHS in region of interest.	LHS	✓	✓	✓				MCS/IS
[69]	Re-centred ED, RS fitted in spline space.	U	✓	✓	✓				$\beta(x_D)$
[36]	Adaptive LHS in region of interest.	LHS	✓	✓	✓	✓		✓	IS
[40]	Radial basis RS and optimization search.	LHS	✓	✓	✓				MCS
Adaptive SVM metamodeling									
[13]	Margin based ED enrichment.	(2)±1 ED	✓	✓					MCS
[14]	Method of [13] combined with IS	(8)±1 ED	✓	✓	✓				IS
[97]	Explicit-Design-Space-Decomposition (EDSD) SVC.	VC	✓	✓	✓			✓	MCS
[5]	Adaptive SVC combined with SS.	Rand	✓	✓	✓			✓	SS
[57]	Re-centred SVM comparision.	+	✓	✓	✓				$\beta(x_D)$
[26]	Re-centred SVM with FORM gradient.	+	✓	✓	✓				$\beta(x_D)$
[124,155]	Adaptive Metropolis search SVR.	Rand	✓	✓	✓			✓	MCS/IS
[99]	Adaptive SVC with virtual samples.	LHS/VC	✓	✓	✓				MCS
[93]	Re-centred SVM with FORM gradient.	+	✓	✓	✓				$\beta(x_D)$
[24,156]	Adaptive LHS multi-wavelet kernel SVR.	LHS	✓	✓	✓				MCS
[157]	SVC with directional sampling.	LS	✓	✓				✓	MCS
[101]	SVR with SS in a 3-stage algorithm.	MCS	✓	✓	✓			✓	SS
[100]	SVC distance-based ED enrichment.	LHS	✓	✓	✓				MCS
[158]	SVC with division of the search space.	LHS/U	✓	✓	✓				MCS
[80]	SVC of [97] applied to interval variables.	Sobol	✓	✓	✓				MCS
Adaptive PCE metamodeling									
[15]	RBDO with PCE fixed-variables.	+	✓						MCS
[16]	Sparse-PCE.	LHS	✓		✓	✓			MCS
[65]	Sparse-PCE and nested-LHS.	LHS	✓	✓	✓	✓		✓	IS
[151]	Sparse PCE with adaptive ED.	U	✓	✓	✓	✓			MCS
[66]	Least-Angle-Regression (LAR) PCE.	LHS	✓	✓	✓	✓			IS
[150]	Bootstrapped PCE order selection.	LHS	✓	✓	✓				MCS
[81]	Optimal ED for PCE.	LHS/MCS	✓	✓				✓	MCS
[118]	Hybrid saprse PCE-SVR.	Rand	✓		✓	✓			–
[108]	Bootstrapped sparse-PCE (bPCE)	*e.g.* LHS	✓	✓	✓	✓			MCS
[120]	PCE with d reduction.	VC	✓	✓	✓	✓			MCS
[109]	PCE with BIP learning.	LDS	✓	✓	✓	✓			MCS
[159]	Bayesian sequential PCE.	Rand	✓	✓	✓			✓	IS
Adaptive Kriging metamodeling									
[18]	Expected Feasibility Function (EFF) in AKMCS.	LHS	✓	✓	✓				MCS
[102]	Margin of uncertainty+IMSE AK.	LHS	✓	✓	✓				MCS
[67]	U-function AKMCS.	LHS	✓	✓	✓				MCS
[103]	Margin k-centres AK.	Rand	✓	✓	✓				MCS/SS
[125]	Quasi-optimum IS density AK.	MCS/LHS	✓	✓	✓				IS
[123]	AKMCS with IS.	LHS	✓	✓	✓				IS
[127]	meta-AK-IS^2.	LHS	✓	✓	✓		✓	✓	IS
[160]	AK to system reliability.	LHS	✓	✓	✓				IS
[105]	LS and H-function AK.	MCS	✓	✓	✓				LS
[2]	PC-Kriging (hybrid PCE and Kriging).	LHS	✓		✓	✓			MCS
[70]	PC-Kriging and AK-MCS of [67].	LHS	✓	✓	✓	✓	✓		MCS
[130]	ISKRA method.	MCS	✓	✓	✓		✓	✓	MCS
[126]	AKMCS with SS.	LHS	✓	✓	✓			✓	SS
[161]	Complementary candidate update.	LHS	✓	✓	✓				MCS

(continued on next page)

Table 2 (*continued*)

Ref.	Approach	Initial ED	Iterative	Measures of improvement implemented					$\widehat{P}_f$
				ED	Stop	Model	Par.	Cand.	
[106]	Least-Improvement-Function (LIF) AK.	LHS	✓	✓	✓				MCS
[122]	AKMCS and IS with trust region.	LHS	✓	✓	✓			✓	IS
[129]	AK-ARBIS procedure.	Rand	✓	✓	✓			✓	MCS
[140]	AKMCSi.	LHS	✓	✓	✓		✓	✓	MCS
[107]	General learning function applied to AK.	LHS	✓	✓	✓				MCS
[114]	Failure-pursuing sampling (FPS) AK.	LHS	✓	✓	✓				MCS
[110]	REIF and REIF2 AK.	LHS	✓	✓	✓			✓	MCS
[132]	REAK.	LHS	✓	✓	✓			✓	MCS
[136]	AK with biased randomisation.	LHS	✓	✓	✓				MCS
[162]	AKEE-SS algorithm.	LHS	✓	✓	✓			✓	SS
[131]	AKMCS-IS with χ adaptation.	LHS	✓	✓	✓			✓	IS
[133]	AKOIS method.	LDS	✓	✓	✓			✓	IS
[115]	Density-based parallel enrichment.	LHS	✓	✓	✓		✓	✓	MCS
[163]	AK with Bayesian Updating (BUAK).	LHS	✓	✓	✓			✓	MCS/ SS
[135]	Adaptive candidate PAK-Bn method.	LHS	✓	✓	✓			✓	MCS
[164]	SALK for system reliability in RBDO.	LHS	✓	✓	✓			✓	MCS

Moreover, multi-stage algorithms may use one or more enrichment techniques, which may generate convoluted applications.

3.2.3. ED enrichment using the margin of classification

The pioneer work in SVM of [13] proposes an adaptive ED that sequentially enriches an initial SVC using the random samples within the SVC margin. ED points that fall within the margin in SVM are the points of interest that are expected to have larger uncertainty in classification, and reducing the margin is expected to improve the $G(x)$ capability to surrogate $g(x)$. Despite intrinsically related to SVC, the concept of margin was, and can be further, extended to other implementations. [97,5,98–100] use margin considerations in SVC as a measure to set a notion of improvement in $G(x)$. [101] further elaborated on ED enrichment of [5], but instead using SVR. According to the authors SVR (and consideration of absolute output values) is more informative about the problem in-hand. [57] had highlighted earlier the more informative character of SVR.

Inspired by the concept of margin in SVM, [102,103] would later extend the application of the margin in enrichment to other metamodel, with the usage of a margin of uncertainty in order to select new points for ED enrichment and evaluation of convergence. A margin of uncertainty can be built using estimators of uncertainty in the metamodel implementation, *e.g.* resampling or leave-one-out estimators. Usage of the margin to select new points in the ED is of interest because it guarantees that the selected points will have an explicit relation to the problem of reliability analysis (approximating the region of $G(x) = 0$ to $g(x) = 0$), however, it is bound to the accuracy with which $g(x)$ is represented by $G(x)$. It is noted that the margin rationale is also related to the idea of framing the ED, discussed in the following section.

3.2.4. ED Enrichment using learning functions

Learning functions are convenient mathematical functions that weight the metamodel properties to seek for the best candidate to improve the ED. They evaluate a set of candidates with criteria that are essentially built on considerations of uncertainty in the model approximation and proximity to the failure region, and select the new most promising to enrich the ED. Learning functions are the present state-of-art technique for ED enrichment.

Learning functions became popular due to their efficiency in the so-called Adaptive Kriging (AK) applications, and then progressively extended to other metamodeling techniques. [18,67] introduced two of the most relevant works in this context. [18] introduced the Efficient Global Reliability Analysis (EGRA), proposing the Expected Feasibility Function (EFF) to enrich the ED. And [67] the AKMCS that uses the so-called U-function, which uses the probability of misclassifying a candidate to enrich the ED. [104] also used the misclassification error. [105] introduces the H learning function, built on entropy considerations. [106] proposed the Least Improvement Function (LIF), that uses misclassification, but that also considers the influence of neighbour candidates. [107] proposes three new learning functions of universal application (*i.e.*, applicable to all metamodels), built on distance and uncertainty considerations. [108] proposed the bPCE for reliability that iteratively enriches the ED using a learning function built on bootstrapping. [40] proposes the SSRM that uses an optimization learning function to enrich the ED. [109] proposes a learning function in PCE that models uncertainty with a Bayesian approach. Recently, [110] proposed the Reliability Expected Improvement Function (REIF), which relates to the expected improvement (EI) of [111]; while [112] proposed yet another search function fo AK, the Most Probable Learning Function (MPLF). All the discussions on adaptive implementations have been accompanied and benefited from research on stopping conditions that can be adapted for different adaptive metamodeling techniques [62,113–115].

In general, adaptive implementations pursue one of two: an accurate surrogate of $g(x)$, or a confident prediction of P_f. In this context, learning functions perform well even with complex $g(x)$. To approximate these, learning functions to be robust need to enclose global and local considerations in the enrichment. This is commonly differentiated in the literature as exploration and exploitation. The first is related to global identification of trends and description of $g(x)$, while the second is related to the local characterization of sub-areas of x and $g(x)$.

3.3. ED size and domain

In the definition of the initial and posterior ED there is interest in considering the number of x variables that are strictly necessary to define an accurate metamodel. High dimensional spaces demand additional effort in the analysis. Sensitivity analyses are an effective method to reduce the ED to the variables of interest. Adaptive reduction of the ED random variables, such as applied in the PCE-RBDO of [15], is an efficient method to address dimensionality in complex problems. Recent research works of [116–120] are an indicative of the relevance that dimensional dependence still has in metamodeling implementations in reliability.

An important consideration in relation to metamodeling, and that largely influences the performance of adaptive methods, is related to the fact that metamodels can be constructed in different spaces. Examples of commonly used spaces are the initial space of x and the standard normal space (if a transformation is assumed [121]). It is usually convenient to work in the standard normal space. When applying PCE it simplifies the definition of the basis, or if a learning function that depends on distances is applied, *e.g.,* [107], it mitigates the influence of the relative description of the x variables. Nonetheless, other spaces may be used to construct metamodels, and such feature is expected to be of interest in implementations of adaptive metamodeling. [69], for reference, improves the RS implementation by fitting the metamodel in a transformed spline space.

3.3.1. Framing of the ED domain

Research on adaptive implementation has shown that significant gains could be attained with framing the initial and iterated domains in regions of interest. [92] highlights the importance of the ED to be realistic. [68] frame the ED in their search for the design point in reliability. [122] improve the methodology of [123] by using a trust region that efficiently searches for the design point. [36] were able to achieve efficient results using screening to identify a promising domains for implementation.

In ED enrichment, new points can be directly interpolated from the surrogate model or from a sample of $x \in \mathbb{R}$. In the present, most adaptive implementations use a random pool or batch of candidates (χ) that frame the learning space. The usage of appropriate samples is also an efficient technique to define realistic ED and improve adaptive metamodeling implementations, in particular when learning functions are applied.

3.3.2. Adaptive candidate sample

MCS is the most common technique to define the batch of candidates to be used in the enrichment of the ED. MCS does not discriminate on *a priori* knowledge about $g(x)$ other than using the adequate sample size for a reliable estimation of P_f. Despite being of general application, MCS is not always the most efficient methodology to generate samples of candidates. Importance sampling (IS) [14,124,123,125], Subset Simulation (SS) [5,126], or Line Sampling (LS) [105] are examples of implemented methods to improve the generation of an adequate batch of candidates in adaptive metamodeling. Other methods further elaborate on the strategies presented in these by combining one or more of these approaches or improving the sampling strategies [127–129]. Global sampling techniques, such as low discrepancy samples of candidates, have also been applied as an alternative to mitigate the large cost of handling MCS samples [110].

Adaptivity in metamodeling may also use an adaptive batch of candidates (χ) for the ED, since there is always a sample of candidates that produces the best implementation performance. This sample can also perform as efficient stopping criterion for the adaptive implementation. IS, SS or DS already use this rationale to some extent, however, even within these an improvement sample can be attained. [130] identified this fact and proposed an adaptive χ size for AK in reliability. [71] addresses the influence of χ by proposing adaptivity with dependence on the i P_f prediction. [131] uses a re-sampling χ technique (with updated centre for the sample). [132] proposes an adaptive χ that uses the candidate sample error-rate influence in P_f. [133] uses discrepancy samples and local subsets for enrichment. In [134], χ is sequentially partitioned depending on the estimated P_f and radial spheres that adapt it, and [135] use uniform samples in a radial domain. [136] weight the choice of χ with a randomised bias. This idea, called biased randomisation, is that of using a filter function in order to weight on the adaptive approach with *a priori* knowledge about the problem [137,138].

Despite pioneered for Kriging, the techniques discussed have transversal interest in future implementations of adaptive metamodeling. Recent research has shown that adaptive candidates have a relevant contribution to significantly improve the efficiency of adaptive approaches.

3.3.3. Parallel computation

While most of the works seek to iteratively improve the ED, research for this effect originated innovative complementary ideas of implementation, such as parallel $g(x)$ evaluation. Parallel $g(x)$ evaluation uses a division of the candidate space or sample to select more than one candidate to enrich the ED. With larger computational availability, it is expected for the reduction of the number of iterations in adaptive metamodeling to gain leverage in the search for efficient metamodel implementations. [5] introduced this idea of parallel processing in reliability, using k-means. [130,70] extended parallel computations to AK (with k-means and the Kriging believer of [139]). [70] uses the concept of margin for parallelization. [140,141] further elaborated on the application of k-means in AK. A drawback of parallel processing is the requirement of additional $g(x)$ evaluations. Recently, [115] tackled this issue in parallel $g(x)$ evaluations by using density-based partitions.

3.4. Metamodel parameters

It was seen in Section 2 that all metamodels have *a priori* assumptions and parameters to be estimated and that these are expected to have a large influence on the performance of a surrogate of $g(x)$. This fact originated a spectra of techniques to improve metamodel parameter estimation in reliability. Notwithstanding, [142] highlight that the relevance of model assumption and parameter estimation is still to a large extent underestimated and overlooked in the application of metamodels in engineering. Some alternatives for model estimation were presented, in the present work the analysis is explicitly extended to the context of reliability analysis.

3.4.1. Weighted parameter estimation

The idea of weighted parameter estimation is related to using some measure of randomized bias in order to improve the fitting of the model parameters in a region of interest. The resulting metamodel fitted with this technique is expected to approximate better this region of interest, *i.e.*, in reliability, the failure region.

Weighted regression has been extensively used to improve the RS approximation in the failure region [143,91,144–146,94], and recently it was extended to other models in reliability. [147] weighted the PCE in a region of interest, however, not exploiting an adaptive scheme to its full extent. It is noted that the techniques identified in weighted parameter estimation do not explicitly pursue a notion of improvement (frequently only minimize a quantity), nonetheless, their widespread application and efficiency is of relevance to be potentially researched in adaptive schemes that also enclose a notion of improvement in weighted parameter estimation.

3.4.2. Sparse implementation

The rationale behind sparse implementations consists in iteratively searching for the metamodel parameters (*e.g.* basis functions) that are of interest for its efficiency; discarding the ones that are identified as non-relevant. This approach is different from optimizing metamodel parameters because there is a sequential notion of improvement by reconstructing the metamodel.

[16] pioneered sparse implementations in reliability by proposing an iterative approach that selects the adequate number of PCE coefficients. A similar rationale had been previously implemented in [148,149]. [65] would improve sparse implementation by using nested-LHS ED. And, [66] would extend this rationale using Least-angle-regression (LAR).

Due to the aforementioned limitations of PCE in high dimensions, significant research on sparsity has involved these models [150–152,117]. However, other metamodels have benefited from enclosing sparse rationales in their implementation, such as the expansion of sparsity in SVR [24], or the sparse RS in [36].

3.4.3. Hierarchical implementation

[142] recently discussed the importance of metamodel parameters and assumptions, and proposed a procedure that iteratively searches for the most appropriate model from a batch of fitted models with different model parameters and assumptions. This procedure was motivated by the identification in the literature of a lack of comprehensiveness in

metamodel construction.

The idea of a hierarchical implementation is therefore a level of complexity above the sparse implementation. In the hierarchical implementation, improvement is not only related to selected model parameters, but instead, it is extended to model assumptions, such as, correlation functions or set of basis functions.

In metamodeling, it is frequent to find works that consider a limited number of basis functions or parameters primarily justified by the need to avoid the risk posed by more involved surrogates (e.g. higher order functions); than by the lack of gains that may be achieved by higher or different order functions. [153,98] showed that a more involved analysis of these in RS and SVC can be an efficient measure of improvement.

Table 3
Comparative results for distinct AK implementations in literature. g_{eval} refers to the number of $g(x)$ evaluations. x_1 and x_2 are standard normal variables. For representation purposes, black horizontal bars are illustrative of a g_{eval} that is much larger than the other results.

$$g(x) = \min \begin{cases} g_1(x) = q_a + 0.1(x_1 - x_2)^2 - \frac{x_1 + x_2}{\sqrt{2}} \\ g_2(x) = q_a + 0.1(x_1 - x_2)^2 + \frac{x_1 + x_2}{\sqrt{2}} \\ g_3(x) = (x_1 - x_2) + \frac{q_b}{\sqrt{2}} \\ g_4(x) = (x_2 - x_1) + \frac{q_b}{\sqrt{2}} \end{cases}$$

Method/Reference	$P_f\,(10^{-3})$ (mean)	$e_r(\%)$ (reported)	g_{eval}
$q_a = 3$; $q_b = 6$			
MCS [67, 2]	[4.41,4.46]	-	10^6
RS			
IS + RS [67]	4.90	1.53	1469
SVM			
ASVM-MCS of [100]	4.46	1.36	99
PCE			
PC-Kriging (stop uses $U > 2$) [70]	4.471	0.24	127.8
PC-Kriging (Stop uses P_f margin) [70]	4.458	0.07	73.2
Parallel PC-Kriging (Stop uses P_f margin) [70]	4.458	0.04	98.4
PC-bootrap of [165]	4.460	-	284
A-bPC [108]	4.62	3.59	167
Kriging			
AK-MCS+U [67]	4.416	**	126
AK-MCS+EFF [67]	4.416	0.004	124
AK-MCS+U with criterion of [2]	4.440	0.45	78.3
Fast candidate AKMCS+LIF of [106]	[4.27, 4.54]*	[0.8, 3.3]	[26, 51]
FPS of [114]	[4.411, 4.497]	[0.05, 0.97]	[56.3, 64.7]
$q_a = 3$; $q_b = 7$			
MCS [67, 36]	[2.23, 2.24]	-	10^6
RS			
iRS of [36]	2.24	0.12	33
SVM			
ASVM+MCS [100]	2.15	0.93	89
Kriging			
AKMCS+U of [67]	2.23	-	96
metaAK-IS^2 of [127]	2.22	1.7	138
AKSS of [126]	2.23	0	45
meta-IS-AK of [166]	2.22	1.38	87
AKEE-SS of [162]	2.20	0.949	41.7

* estimate from different initial ED

** Reported as part of the variance of the P_f estimation

(See above-mentioned references for further information.)

* estimate from different initial ED.

** Reported as part of the variance of the P_f estimation

Other effective use of model construction and assumptions can be identified in the PC-Kriging [2,70], where PCE orthogonality performs as a global trend that supports the Kriging local interpolation with uncertainty.

4. Comparative application results

Reference examples of application of the methodologies discussed are presented in Tables 3–6. These cover, respectively, an example with multiple regions of failure (a series system), a high-dimensional example, and two engineering examples (non-linear oscillator and truss structure) with a medium number of random variables, and distinct probability of failure (relatively low and high P_f). These examples are only illustrative of literature comparison in adaptive metamodeling for reliability. In one hand, it is noted that real engineering examples can be significantly more complex than these (e.g. multiple failure regions in high d), and complementary analyses are necessary to understand applicability with extension to more involved examples. On the other hand, the efficiency and generalization for each case is bounded to the assumptions and algorithms used, which is a limitation. Nonetheless, comparative analyses such as this one have been an effective mean of understanding new developments in the field.

A series system is discussed in the first example in its variable q_a and q_b dependent-form. This function is many times described as the four-branch reliability function due to its four main regions of failure. It is a complex example that is globally non-linear, but with relatively weak local non-linearity; and that only involves two random variables.

It is common in the literature to report accuracy in relation to the number of evaluations of the true function $g(x)$ evaluations (g_{eval}) in order to evaluate the efficiency of an adaptive metamodel application in reliability analysis. Evaluation of $g(x)$ is expected to dominate the adaptive implementation efforts. Hence, efficiency in the present case is discussed in relation to the metrics of P_f prediction accuracy and g_{eval}.

Table 3 presents the results for the series system. For $q_a = 3,\ q_b = 6$ it is possible to infer that the adaptive methods using Kriging have proven to be highly efficient. The pioneer works using AK-MCS already produced a very efficient trade-off of accuracy with the number of g_{eval}. Works that use PCE, RS and SVM just recently had the breakthrough from this reference baseline set in AK. Efficient application of PCE have converged to the adaptive Kriging in the form of the PC-Kriging, which is closer to an AK implementation. [109] does not report the results for this particular example. Nonetheless, the authors show that by using a

Table 4
Comparative results for distinct adaptive metamodeling implementations for a high dimensional function with n normal variables ($\mu = 0$ and $\sigma = 0.2$). $a = 3$. For representation purposes, black horizontal bars are illustrative of a g_{eval} that is much larger than the other results.

$$G(x) = \left(n + a\sigma\sqrt{(n)}\right) - \sum_{i=1}^{n} x_i$$

Method/Reference	$P_f(10^{-3})$	$e_r(\%)$	g_{eval}
MCS (n = 20) [117]	2.23	-	10^7
MCS (n = 40) [67, 40]	[1.81, 1.98]	-	3×10^5
MCS (n = 100) [67, 40]	[1.65, 1.73]	-	3×10^5
MCS (n = 250) [40]	1.59	-	3×10^5
RS			
SSRM (n = 40) of [40]	1.93	2.53	198
SSRM (n = 100) of [40]	1.72	0.58	348
SSRM (n = 250) of [40]	1.53	3.77	734
SVM			
[2]SMART (n = 40) of [5]	1.95	-	3729
[2]SMART (n = 100) of [5]	1.74	-	6036
[2]SMART (n = 250) of [5]	1.61	-	10707
SVR (n = 100) of [101]	1.70	1.73	616
SVR (n = 250) of [101]	1.56	1.88	1264
ASVM-MCS (n=40) of [100]	1.78	2.20	341
ASVM-MCS (n=100) of [100]	1.72	0.58	810
ASVM-MCS (n=250) of [100]	1.57	0.63	2363
PCE			
Cubature PCE (n = 20) of [117]	2.11	5.38	463
Proposed (n = 20) in [120]	2.46 ($\beta = 2.8122$)	0.79 in β	258
Proposed (n = 100) in [120]	2.08 ($\beta = 2.8656$)	0.53 in β	1300
Kriging			
AK-MCS+U (n = 40) of [67]	1.813	*	112
AK-MCS+U (n = 100) of [67]	1.647	*	153
AK-MCS+EFF (n = 40) of [67]	1.813	*	112
AK-MCS+EFF (n = 100) of [67]	1.647	*	153

* Reported as part of the variance of the P_f estimation.

Table 5
Non-linear Oscillator results. For representation purposes, black horizontal bars are illustrative of a g_{eval} that is much larger than the other results.

Single degree-of-freedom non-linear oscillator presented in [68], [67], [100].

Method/Reference	$P_f\,(10^{-2})$	$e_r(\%)$ (reported)	g_{eval}
MCS [67, 40]	2.83	-	7×10^4
RS			
CQ2RS of [68]	3.32	-	40
P-LS of [69]	2.73	3.7	81
P-PLS of [69]	2.52	11.1	68
UD-BP-PLS [69]	2.85	0.6	42
iRS[1] of [36]	2.83	0.03	66
iRS[2] of [36]	2.82	0.55	52
SSRM of [40]	2.88	1.623	19
SVM			
ASVM-MCS (n=40) of [100]	2.79	2.78	56
Kriging			
AKMCS + U of [67]	2.834	*	58
AKMCS + EFF of [67]	2.851	*	45
AKSS of [126]	2.83	0.035	410
REAK of [132]	[2.84, 2.86]	[0.004, 0.16]	[30, 40]

[1] and [2] are results for respectively 0 and 20% thresholds of the dimension reduction relative measure of sensitivity.

Table 6
Truss structure with low probability of failure. For representation purposes, black horizontal bars are illustrative of a g_{eval} that is much larger than the other results.

Truss structure with serviceability limit state with reference to a maximum δ deflection of 14cm adopted in [65], [95], or [106].

Method/Reference	$P_f\,(10^{-5})$	$e_r(\%)$ (reported)	g_{eval}
MCS [70]	[3.6]	-	10^6
IS [65, 106, 36]	[3.30, 3.45]	-	5×10^5
RS			
ARSM of [95]	3.69	0.3 in β	142
iRS[1] of [36]	3.30	0.25	125
iRS[2] of [36]	3.06	7.32	81
PCE			
Full PCE of [65]	2.67	1.5 in β	443
Sparse PCE of [65]	3.93	2.3 in β	207
PC-Kriging of [70]	3.7	2.78	75
PC-Kriging (parallel enrichment with 6 centers) of [70]	3.2	11.1	78
Kriging			
AKMCS+LIF of [106]	3.31	4.06	121
Fast candidate AKMCS+LIF of [106]	[3.39, 3.55]*	[0.9, 2.9]	[135, 170]
AKEE-SS of [162]	3.25	3.67	80

[1] and [2] are results for respectively 0 and 20% thresholds of the dimension reduction relative measure of sensitivity.
* estimate from different initial ED

[1] and [2] are results for respectively 0 and 20% thresholds of the dimension reduction relative measure of sensitivity.
* estimate from different initial ED.

learning function, the BIP, in a sparse PCE implementation, accurate $G(x)$ can be attained with a relatively small number of function evaluations (≈ 40), in-line with some of the most efficient results presented.

For the case of $q_a = 3$ $q_b = 7$, [36] showed that a polynomial basis function RS could outperform other more complex methods. RS have the major advantage of a low implementation demand. However, their application to more involved examples is commonly achieved with multi-stage algorithms that are user-case-sensitive. In this case the weak local non-linearity may be related to the efficiency achieved by this multi-stage RS.

The errors reported were for all the cases lower than 4%, being, in most cases, of the same order of magnitude as the variance of the estimation (sample variance). It should be noted that newer methods in the four cases presented are expected to reduce the required g_{eval}, as they proceed to improve from a different reference value. The relative number of g_{eval} to produce accurate estimation decreased to such an extent with adaptive methods that more recent works discuss improvements in g_{eval} comparing small gains.

The following comparative problem is a failure reference function with a high number of random variables, Table 4. This problem involves a very large number of random variables but where each one has equal influence in a smooth $g(x)$. It is noted that such balanced division rarely occurs in real engineering problems. Results from this high-dimensional example show that AK outperforms the other adaptive metamodels by a large margin. Kriging computational demand is expected to increase in high dimensional spaces when $g(x)$ is more involved (optimization of parameters will occur in a non-linear space with different weights). [115] highlights the significant cost of computing the Kriging of [64] when studying a more involved problem in complexity, but with only 11 random variables. Recent works, such as [110,134] tackle the efficiency of AK implementations under the assumption that they may not be always negligible in comparison with the $g(x)$ evaluation. It is highlighted that with the increasingly complexity of computational codes, such as Finite-Element-Methods, the requirements to evaluate $g(x)$ are still expected to comprise most of the effort of a reliability analysis.

The PCE results are, even considering the limitations of PCE for large d, comparable to the other methods but being slightly less accurate.

SVM have synergy with large d problems, however, their accuracy in large d has been a concern [5]. Variant SVM approaches have pursued to tackle limitations of performance in high d spaces [167]. In the present example, SVM demands more g_{eval} than the other alternatives methodologies considered, and this may justify further investigation on the different assumptions used in their implementation (e.g., if the stopping condition is conservative). Due to the similarity with Kriging in kernels, results of SVM for reliability are expected to be able to approach those of AK in accuracy, see [63]. Methods such as the SVM can benefit to a large extent from their wider application as machine learning tools, which fomented the development of techniques that accelerate their characterization and accuracy for a specified ED size.

The third and fourth reference examples discuss the application of medium number of variables with distinct orders of P_f, which are more representative of engineering applications of structural reliability (a non-linear oscillating mass and a truss structure), Tables 5 and 6. Both $g(x)$ functions are smooth in the standard normal space, being initially introduced to research on simpler metamodeling approaches [11,86].

For the non-linear oscillator problem, all the metamodels produce comparable results. Apart from the SSRM of [40], the number of $g(x)$ evaluations is recurrently around 50. The SSRM of [40] in this particular example was seen to largely outperform the remaining metamodel alternatives. The AK-MCS performance was recently improved for this example in the REAK approach of [132]. When $g(x)$ is smooth further investigation on the assumed stopping conditions is of interest.

In the example of the truss structure, P_f was in the order of 10^{-5}. All the discussed metamodels were reported to accurately estimate this lower P_f with similar performance.

In general, when compared with traditional sampling techniques, all the methods in the presented examples have a significant impact in the reduction of g_{eval}. Moreover, application of adaptive metamodeling in a reliability analysis problem is generally robust, but it may be problem dependent in relation to the metamodel used.

In the presented discussion, the four examples presented did not cover the case of locally highly non-linear $g(x)$ that depend on more than one region of failure, such as the modified Rastringin function studied by Refs. [67,140], or the non-linear limit-state studied in [110,115]. This was due to the fact that only Kriging works were identified in the literature to tackle the reliability estimation for these types of functions. It is noted that real engineering examples are expected to be on the complex side, justifying further need to explicitly discuss the limits of application for different methods in future research; in particular when simpler models are used. Few applications to real engineering examples were identified in the review of applications, which indicates that further research needs to be performed in relation to issues such as, generalisation in applicability.

Nonetheless, it is important to highlight that despite this fact, all the metamodeling approaches are viable alternatives for reliability analysis. Examples can be found in; [62], where the authors showed that in general the Kriging as a metamodel is more robust than RS, however, when correctly applied, (*e.g.*, RS centred at x_D) RS produced more efficient results (similar accuracy and number of g_{eval}, but with lighter computational and analysis requirements); or in [108] where it was shown that bootstrapped PCE could perform significantly better than the AK-MCS in reliability analysis for a truss structure, emphasizing the relevance of model assumptions and algorithms. In the case of the Kriging and non-linear functions, it is highlighted that by construction it is expected for other interpolators with similar kernel, such as RS with RBF or SVM, to be able to at least produce comparable results in respect to robustness in relation to $g(x)$.

The following section discusses some of the ideas that are beyond the state-of-the-art and are of interest to exploit in future implementations.

5. Beyond the state of the art and areas of interest in research

The present section highlights areas of interest in the field of adaptive metamodeling to further enhance their applicability. It is noted that in adaptive metamodeling research for reliability the main concern in recent years has been the reduction of g_{eval} without compromising accuracy, which resulted in methodologies that have remarkable efficiency. However, when applied to other fields of knowledge (*e.g.*, nuclear, marine engineering), adaptive metamodeling is not always considered and its advantages not fully exploited (e.g. recent applications of [168,58,169]). There is a gap between application and method yet to be filled.

5.1. High dimensional problems and reduction of high-dimensional spaces

High dimensional problems are challenging. It was highlighted that metamodeling in reliability analysis could benefit from the usage of a combination of sensitivity and reliability analysis in order to reduce implementation efforts. [170] highlighted previously that in a high dimensional problem it is common for only a few random variables to enclose most of the sensitivity of an output. Works such as [171], showed that these considerations were held even in high-complexity problems. Therefore, it is of interest for future implementations to investigate to which extent using a method that performs for high dimensional spaces and pursues low ED should be balanced or prioritized in detriment of using dimension reduction techniques that may accelerate the application of adaptive metamodels; in particular when involved engineering examples are studied. PCE have proved to be efficient metamodels that suffer from the increase in d, but that enclose intrinsic measures of sensitivity [49]. The demand in time and memory for Kriging applications increases significantly when the ED and

d increases [172]. Moreover in the case where the ED encloses many input variables, the learning algorithms are more likely to spend time exploring and exploiting points that enclose limited relevance for the estimation of P_f [122]. The analysis presented in previous sections also indicates that some learning approaches rely on Euclidean distances, and as such, the effects of applying these in large d should be further discussed (their performance is expected to be affected by d, and other measure of distance may be of interest in large d). It is then of interest to use a metamodel for reliability implementations with the minimum number of input random variables possible, without loss of accuracy. Previous works successfully merged these ideas, *e.g.*, [172,36].

5.2. Hierarchical implementations, model assumptions and parameter consideration

With relation to the previous topic, the importance of model assumptions, which is a field largely unexploited in SVM [101], Kriging [142] or RS implementations [36], is a topic that needs to be further researched in the future. It is noted that some metamodels, such as PCE, have benefited from a larger discussion on this regard.

[142] highlighted the limited importance that is given to general model assumptions and parameters when discussing application of Kriging to replace multi-fidelity codes. This same disregard for model assumptions can be identified in many applications to reliability. Despite the range of adaptive SVM implementations, and their inherent sparsity, [101] also emphasised before the need for a comprehensively parameter selection to a further extent than what is currently performed. Previous research indicate that some model parameters are expected to have limited influence [62]. Nonetheless, for some metamodeling techniques, only limited research has been produced in improvements that use model assumptions, or model parameters with specific ties to the problem of reliability. The implementation of [142] is representative of the gains that can be achieved with a more detailed analysis and understanding of these.

SVM have synergy with high d problems, but reliability analysis implementations showed that this may not be always the case [5]. Nonetheless, by construction their performance should approach other models that use similar kernel, see [63]. SVC and its concept of margin is of relevance. Since SVR are informative, and SVC have this particularity, it may be of interest to exploit classification margin considerations in SVR enrichment.

[63,142] showed that gains could be achieved analysing the Kriging model assumptions (*e.g.*, correlation). If large ED are used, Kriging models can become unstable [2], and computing cost is still a limitation for these [172]. These characteristics are indicative that further research and guidance in their definition is of relevance.

For RS, usage of larger basis of polynomials functions and order should be further investigated in the future. [153] showed that higher order functions can be of interest, but little research was conducted in this regard in posterior works. Combining some measure of sparsity in the coefficients and higher order polynomials is expected to contribute for the improvement of the RS methods [36].

It was also seen that further gains in model implementation to reliability could be achieved by researching the potential of using feature spaces, such as in [69], and this may allow further research on model assumptions (*e.g.,* fitting simpler metamodels to more involved problems).

5.3. Application of alternative basis functions and hybrid models

In line with the previous topic, when different metamodeling techniques were established as fundamental viable alternatives for reliability analysis, research on adaptive metamodeling started to expand in order to tackle different limitations. New research in adaptive metamodeling indicates that the emergence of alternative basis functions, such as RBF [40,173] exponential RS [35], B-spline considerations [69], and hybrid models, such as the PC-Kriging [2], are areas of high added value for further assessment in adaptive metamodeling.

[174] shows that this improvement can be also attained when the metamodel adaptivity is coupled with $g(x)$ assumptions for involved problems, which is a field of potential interest in future research.

5.4. Initial ED

No comprehensive guidance was identified on the selection of the initial ED, and on its relation to $g(x)$. Even considering that initial ED commonly involves a low number of $g(x)$ evaluations, a well selected initial ED may contribute to alleviate the number of posterior adaptive $g(x)$ evaluations. Recent research in this regard in PCE can be found in [81], nevertheless, further research may contribute to enhance the understanding of the initial ED relevance in different metamodeling techniques.

5.5. ED Exploration and exploitation

In ED adaptivity, exploration and exploitation considerations are intrinsically enclosed in the implementation (e.g. learning function have a balance of both). Nonetheless, limited explicit discussion has been developed in relation to them. Recent works with Kriging show that explicitly discussing these concepts improves the performance of the adaptive implementations [162]. Both of them play an important role in the generalization of adaptive metamodeling implementations. The recent benefits attained with Kriging and innovative techniques, such as clustering the candidates in regions of interest with sensitivity measures in x [114], or the improvements of the U and EFF functions [161,132,115] are indicative of the interest on this explicit discussion, which is a field of implementation from which all adaptive methodologies, regardless of metamodel, could benefit from.

5.6. Non-deterministic ED

One of the fields of interest for future implementations is related to noise or non-deterministic responses of $g(x)$. This is the case where a single value of x, generates a random response. Models and adaptive metamodeling implementations for non-deterministic ED have been studied and proposed before [175–177]. Non-deterministic ED may be measured by means of a random variable, therefore it may depend on the reliability problem conceptualization, however, it may be of interest (as characterizing the non-deterministic responses is expensive, and may be even condition on x [178,179]) if the metamodel is able to also enclose and interpolate or predict it in the regions of interest when assessing reliability for multi-fidelity codes.

5.7. Adaptive metamodel selection

It was seen before that each metamodel has assumptions. If a RS is suitable for a certain problem, there is little justification to use a more complex model. At the same time, application of simpler metamodels or methods is expected to lack generalisation, which is a characteristic that is rarely addressed in the literature. No research was found to comprehensively discuss the concept of hierarchy or adaptivity in relation to different metamodels.

An example of the influence of the metamodel choice can be identified in the example used in Fig. 3, Section 2, where the PCE (in IV), for an equal size of ED, accurately approaches $g(x)$ when compared with the Kriging (in II). As metamodeling techniques progressively develop and new methods appear, an important demand can be identified in the need to set methodologies that allow an engineer to decide what type of metamodel is more suitable to his/her application. This may be achieved by adaptive selection of model with relation to a measure of complexity of $g(x)$ (as $g(x)$ knowledge increases) and is notoriously significant due to the black-box character of metamodeling. For example, when d increases it may be hard for the reliability engineer to understand what is the form of the reliability problem, and thus, selecting the most efficient metamodel without any *prior* information is challenging. Moreover if the lime function is implicit.

5.8. Ensembles

In the line of transversal implementations that consider different models, application of ensembles of metamodels is a field that only recently started to be studied in reliability. It addresses the need for a black-box hierarchy that uses or selects different models. [180] successfully improves the efficiency of adaptive metamodeling applying an ensemble. Ensembles take advantage of the best properties of each metamodel simultaneously. Their application to reliability can benefit from the extensive research performed up to date in the field of computational experiments [181,182]. However, [182] shows that further research is required to improve the extent that ensembles benefit the analysis when compared with the selection of an adequate metamodel.

Other models addressing the problem of classification or regression can be highlighted in the present case as alternative for the problem of metamodeling in reliability. In some cases application of these to reliability analysis is yet to be researched, *e.g.*, logistic regression [20].

6. Conclusions

The presented work reviewed adaptive metamodeling in reliability analysis. Adaptive metamodeling has gained significant leverage in reliability analysis in recent years. The idea of adaptive metamodeling is that of using some notion of improvement to sequentially increase the efficiency of static metamodel approaches. Due to their proved efficiency, research on adaptive metamodeling increased substantially in the last decade. This originated a diverse number of techniques and concepts that use different metamodels.

The objective of the developed discussion was then to address this diversity, in order to create a baseline for further development of adaptive metamodeling techniques. On one side it is challenging for new practitioners to cover the extensive literature already existing in the topic, and on the other, significant gains are expected with the crossing of information from traditionally individualised fields of research. [2] highlighted before this little interaction that exists between fields of metamodeling. The purpose of the present work is therefore the one of fomenting this type of transversal, whole picture, overview that is expected to foment further developments in the field.

Four main metamodels were discussed, response surfaces, polynomial chaos expansion, support vector machines and kriging models. Adaptivity in metamodeling with these appears in different forms, the most common and influential is related to the experimental design, but adaptivity can be also applied in model parameters, sampling schemes, or space definition. A description of these four metamodels was developed, and the adaptive implementations of each extensively discussed. This allowed to identify new areas of interest and unexploited areas for future research in adaptive metamodeling. In the light of the discussion presented, some of the features of each metamodel and its relation to reliability applications can be highlighted:

- RS that use polynomial basis functions may be suitable when the problem of reliability is local (failure is confined to a region of x) and $g(x)$ is weakly non-linear, being therefore implemented on low order polynomial basis. The recurrent application of RS, see Table 2, that estimate $\beta(x_D)$ is indicative of the synergy of RS with reliability problems where P_f is local. Higher complexity applications are possible, but with involved procedures. RS are fast to compute in their polynomial form. Other RS forms, such as RBF, allow tackling higher complexity problems and should have comparative interpolation performance with other models that use similar kernels (e.g. Kriging).
- PCE perform well in the capture of global behaviour of $g(x)$, and when $g(x)$ is globally smooth (which is common in reliability problems) [53]. They also perform locally. No evidence was found of methodologies applied to very complex $g(x)$ (e.g. non-linear $g(x)$ [67,110]). This may be possible, but with multi-stage algorithms. They are of interest to use higher-order basis with lower risk of producing ED specific models. Their challenging application in high d spaces is mitigated by sparsity, and PCE enclose by construction the potential to be merged with inherent sensitivity considerations.
- SVM and Kriging share similar properties (e.g. both of them are interpolation models and use similar kernels). SVM are of interest for involved problems in high d, however, further research needs to be performed in relation to their accuracy in high d [101]. The similarity between SVM and the Kriging (in kernel) indicates that they should at least achieve comparable accuracy, and this is an indicator of the potential of SVM for reliability. Kriging are overall performers in relation to the complexity or smoothness of $g(x)$, which makes them robust. They perform as interpolators and enclose an inherent measure of accuracy, but can be costly to use, in particular when d and the ED increase (it is noted that this cost should still be negligible in relation to the cost of evaluating multi-fidelity codes). Kriging were the only models that were identified to be researched for strongly non-linear $g(x)$ (locally and globally) applications.

Further discussion in relation to model assumptions, bridging the gap in methodology and application, hybrid models, or ensembles were some of the highlighted areas. One of the crucial topics to be addressed is the need for hierarchical techniques for a blind selection of the adequate metamodeling approach, and model assumptions. In the literature, model selection and assumptions are rarely addressed to the extent they should. There are various methods, and with comparative performances in certain circumstances. It is not uncommon to find the application of a complex technique when a simple model would equally suit the application, and vice versa. Finally, apart from creating a baseline, the present analysis was motivated by a need of unity in the field, to set a reference for discussion on relevant topics related to adaptive metamodeling in reliability.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] Rasmussen Carl Edward. Gaussian processes in machine learning. In: Summer school on machine learning. Springer; 2003. p. 63–71.
[2] Schobi Roland, Sudret Bruno, Wiart Joe. Polynomial-chaos-based kriging. Int J Uncertain Quant 2015;5(2).
[3] Dubourg Vincent. Adaptive surrogate models for reliability analysis and reliability-based design optimization. PhD thesis; 2011.
[4] Ditlevsen Ove, Madsen Henrik O. Structural reliability methods, volume 178.
[5] Bourinet Jean M, Deheeger François, Lemaire Maurice. Assessing small failure probabilities by combined subset simulation and support vector machines. Struct Saf 2011;33(6):343–53.
[6] Kahn Herman, Marshall Andy W. Methods of reducing sample size in monte carlo computations. J Oper Res Soc Am 1953;1(5):263–78.
[7] Hasofer Abraham M. An exact and invarient first order reliability format. J Eng Mech Div Proc ASCE 1974;100(1):111–21.
[8] Rackwitz Rüdiger, Flessler Bernd. Structural reliability under combined random load sequences. Comput Struct 1978;9(5):489–94.
[9] Siu-Kui Au, Beck James L. Estimation of small failure probabilities in high dimensions by subset simulation. Prob Eng Mech 2001;16(4):263–77.
[10] Box George EP, Wilson Kenneth B. On the experimental attainment of optimum conditions. J R Stat Soc Ser B (Methodological) 1951;13(1):1–38.
[11] Bucher Christian G, Bourgund Ulrich. A fast and efficient response surface approach for structural reliability problems. Struct Saf 1990;7(1):57–66.

[12] Rajashekhar Malur R, Ellingwood Bruce R. A new look at the response surface approach for reliability analysis. Struct Saf 1993;12(3):205–20.
[13] Hurtado Jorge E, Alvarez Diego A. Classification approach for reliability analysis with stochastic finite-element modeling. J Struct Eng 2003;129(8):1141–9.
[14] Hurtado Jorge E. Filtered importance sampling with support vector margin: a powerful method for structural reliability analysis. Struct Saf 2007;29(1):2–15.
[15] Kim Nam Ho, Wang Haoyu, Queipo Nestor. Adaptive reduction of design variables using global sensitivity in reliability-based optimization. In: 10th AIAA/ISSMO multidisciplinary analysis and optimization conference; 2004. p. 4515.
[16] Blatman Géraud, Sudret Bruno. Sparse polynomial chaos expansions and adaptive stochastic finite elements using a regression approach. C R Méc 2008;336(6):518–23.
[17] Kaymaz Irfan. Application of kriging method to structural reliability problems. Struct Saf 2005;27(2):133–51.
[18] Bichon Barron J, Eldred Michael S, Swiler Laura Painton, Mahadevan Sandaran, McFarland John M. Efficient global reliability analysis for nonlinear implicit performance functions. AIAA J 2008;46(10):2459–68.
[19] Chojaczyk Agnieszka A, Teixeira Angelo P, Neves Luís C, Cardoso Joao B, Soares Carlos Guedes. Review and application of artificial neural networks models in reliability analysis of steel structures. Struct Saf 2015;52:78–89.
[20] Chen Baojia, Chen Xuefeng, Li Bing, He Zhengjia, Cao Hongrui, Cai Gaigai. Reliability estimation for cutting tools based on logistic regression model using vibration signals. Mech Syst Signal Process 2011;25(7):2526–37.
[21] Herbert Martins (Gomes), Armando Miguel (Awruch). Comparison of response surface and neural network with other methods for structural reliability analysis. Struct Saf 2004;26(1):49–67.
[22] Hosni Elhewy A, Mesbahi Ehsan, YongChang Pu. Reliability analysis of structures using neural network method. Prob Eng Mech 2006;21(1):44–53.
[23] Shao Shaowen, Murotsu Yoshisada. Structural reliability analysis using a neural network. JSME Int J Ser A Solid Mech Mater Eng 1997;40(3):242–6.
[24] Dai Hongzhe, Zhang Hao, Wang Wei. A multiwavelet neural network-based response surface method for structural reliability analysis. Comput-Aid Civ Infrastruct Eng 2015;30(2):151–62.
[25] Mazumdar M, Marshall JA, Chay SC. Propagation of uncertainties in problems of structural reliability. Nucl Eng Des 1978;50(2):163–7.
[26] Richard Benjamin, Cremona Christian, Adelaide Lucas. A response surface method based on support vector machines trained with an adaptive experimental design. Struct Saf 2012;39:14–21.
[27] Cox Neil D. Comparison of two uncertainty analysis methods. Nucl Sci Eng 1977;64(1):258–65.
[28] Vaurio JK, Mueller C. Probabilistic analysis of liquid-metal fast breeder reactor accident consequences with response surface techniques. Nucl Sci Eng 1978;65(2):401–13.
[29] Lucia Alfredo C. Response surface methodology approach for structural reliability analysis: An outline of typical applications performed at cec-jrc, ispra. Nucl Eng Des 1982;71(3):281–6.
[30] Wong Felix S. Slope reliability and response surface method. J Geotech Eng 1985;111(1):32–53.
[31] Der Kiureghian Armen, Ke Jyh-Bin. The stochastic finite element method in structural reliability. Prob Eng Mech 1988;3(2):83–91.
[32] Faravelli Lucia. Response-surface approach for reliability analysis. J Eng Mech 1989;115(12):2763–81.
[33] Brandyberry Mark, Apostolakis George. Response surface approximation of a fire risk analysis computer code. Reliab Eng Syst Saf 1990;29(2):153–84.
[34] Liu Ying Wei, Moses Fred. A sequential response surface method and its application in the reliability analysis of aircraft structural systems. Struct Saf 1994;16(1–2):39–46.
[35] Hadidi Ali, Azar Bahman Farahmand, Rafiee Amin. Efficient response surface method for high-dimensional structural reliability analysis. Struct Saf 2017;68:15–27.
[36] Guimarães Hugo, Matos José C, Henriques António A. An innovative adaptive sparse response surface method for structural reliability analysis. Struct Saf 2018;73:12–28.
[37] Guan XL, Melchers RE. Effect of response surface parameter variation on structural reliability estimates. Struct Saf 2001;23(4):429–44.
[38] Hawkins Douglas M. The problem of overfitting. J Chem Inform Comput Sci 2004;44(1):1–12.
[39] Chatterjee Samprit, Hadi Ali S. Sensitivity analysis in linear regression, vol. 327. John Wiley & Sons 2009.
[40] Li Xu, Gong Chunlin, Liangxian Gu, Gao Wenkun, Jing Zhao, Hua Su. A sequential surrogate method for reliability analysis based on radial basis function. Struct Saf 2018;73:42–53.
[41] De Boor Carl, De Boor Carl, Mathématicien Etats-Unis, De Boor Carl, De Boor Carl, editors. A practical guide to splines, vol. 27. Springer-Verlag New York; 1978.
[42] Marsh Lawrence C, Cormier David R. Spline regression models, vol. 137. Sage; 2001.
[43] Schumaker Larry. Spline functions: basic theory. Cambridge University Press; 2007.
[44] Eilers Paul HC, Marx Brian D. Flexible smoothing with B-splines and penalties. Stat Sci 1996:89–102.
[45] Aggarwal Charu C, Hinneburg Alexander, Keim Daniel A. On the surprising behavior of distance metrics in high dimensional space. In: International conference on database theory. Springer; 2001. p. 420–34.
[46] Vapnik Vladimir. The nature of statistical learning theory. Springer science & business media; 2013.
[47] Smola Alex J, Schölkopf Bernhard. A tutorial on support vector regression. Stat Comput 2004;14(3):199–222.
[48] Villa-Vialaneix Nathalie, Follador Marco, Ratto Marco, Leip Adrian. A comparison of eight metamodeling techniques for the simulation of n2o fluxes and n leaching from corn crops. Environ Model Softw 2012;34:51–66.
[49] Sudret Bruno. Global sensitivity analysis using polynomial chaos expansions. Reliab Eng Syst Saf 2008;93(7):964–79.
[50] Marelli Stefano, Sudret Bruno. UQLab user manual–polynomial chaos expansions. Chair of Risk, Safety & Uncertainty Quantification, ETH Zürich, 0.9-104 edition; 2015. p. 97–110.
[51] Xiu Dongbin, Karniadakis George Em. The Wiener-Askey polynomial chaos for stochastic differential equations. SIAM J Sci Comput 2002;24(2):619–44.
[52] O'Hagan Anthony. Polynomial chaos: A tutorial and critique from a statistician's perspective; 2013.
[53] Lüthen Nora, Marelli Stefano, Sudret Bruno. Sparse polynomial chaos expansions: literature survey and benchmark. arXiv preprint arXiv:2002.01290; 2020.
[54] Rocco Claudio M, Moreno José Alí. Fast monte carlo reliability evaluation using support vector machine. Reliab Eng Syst Saf 2002;76(3):237–43.
[55] Li Hong-shuang, Lü Zhen-zhou, Yue Zhu-feng. Support vector machine for structural reliability analysis. Appl Math Mech 2006;27(10):1295–303.
[56] Aizerman Mark A. Theoretical foundations of the potential function method in pattern recognition learning. Autom Remote Control 1964;25:821–37.
[57] Tan Xiao-hui, Bi Wei-hua, Hou Xiao-liang, Wang Wei. Reliability analysis using radial basis function networks and support vector machines. Comput Geotech 2011;38(2):178–86.
[58] Ghosh Shyamal, Roy Atin, Chakraborty Subrata. Support vector regression based metamodeling for seismic reliability analysis of structures. Appl Math Model 2018;64:584–602.
[59] Hsu Chih-Wei, Chang Chih-Chung, Lin Chih-Jen, et al. A practical guide to support vector classification; 2003.
[60] Cherkassky Vladimir, Ma Yunqian. Practical selection of SVM parameters and noise estimation for svm regression. Neural Networks 2004;17(1):113–26.
[61] Roustant Olivier, Ginsbourger David, Deville Yves. DiceKriging, DiceOptim: two R packages for the analysis of computer experiments by kriging-based metamodeling and optimization; 2012.
[62] Gaspar Bruno, Teixeira Angelo P, Soares Carlos Guedes. Assessment of the efficiency of kriging surrogate models for structural reliability analysis. Prob Eng Mech 2014;37:24–34.
[63] Moustapha Maliki, Bourinet Jean-Marc, Guillaume Benoît, Sudret Bruno. Comparative study of kriging and support vector regression for structural engineering applications. ASCE-ASME J Risk Uncertain Eng Syst Part A Civ Eng 2018;4(2):04018005.
[64] Couckuyt Ivo, Dhaene Tom, Demeester Piet. ooDACE toolbox: a flexible object-oriented kriging implementation. J Mach Learn Res 2014;15(1):3183–6.
[65] Blatman Géraud, Sudret Bruno. An adaptive algorithm to build up sparse polynomial chaos expansions for stochastic finite element analysis. Prob Eng Mech 2010;25(2):183–97.
[66] Blatman Géraud, Sudret Bruno. Adaptive sparse polynomial chaos expansion based on least angle regression. J Comput Phys 2011;230(6):2345–67.
[67] Echard Benjamin, Gayton Nicolas, Lemaire Maurice. AK-MCS: an active learning reliability method combining kriging and monte carlo simulation. Struct Saf 2011;33(2):145–54.
[68] Gayton Nicolas, Bourinet Jean M, Lemaire Maurice. CQ2RS: a new statistical approach to the response surface method for reliability analysis. Struct Saf 2003;25(1):99–121.
[69] Zhao Wei, Fan Feng, Wang Wei. Non-linear partial least squares response surface method for structural reliability analysis. Reliab Eng Syst Saf 2017;161:69–77.
[70] Schöbi Roland, Sudret Bruno, Marelli Stefano. Rare event estimation using polynomial-chaos kriging. ASCE-ASME J Risk Uncertain Eng Syst Part A Civ Eng 2016;3(2):D4016002.
[71] Yang Xufeng, Mi Caiying, Deng Dingyuan, Liu Yongshou. A system reliability analysis method combining active learning kriging model with adaptive size of candidate points. Struct Multidiscip Optim 2019:1–14.
[72] Kleunen Jack PC. Regression metamodels for generalizing simulation results. IEEE Trans Syst Man Cybernet 1979;9:93–6.
[73] Box George EP, Hunter William Gordon, Stuart Hunter J, et al. Statistics for experimenters; 1978.
[74] Iman Ronald L, Helton Jon C. An investigation of uncertainty and sensitivity analysis techniques for computer models. Risk Anal 1988;8(1):71–90.
[75] McKay Michael D, Beckman Richard J, Conover William J. Comparison of three methods for selecting values of input variables in the analysis of output from a computer code. Technometrics 1979;21(2):239–45.
[76] Myers Raymond H, Montgomery Douglas C, Anderson-Cook Christine M. Response surface methodology: process and product optimization using designed experiments. John Wiley & Sons; 2016.
[77] Choi Seung-Kyum, Grandhi Ramana, Canfield Robert. Optimization of stochastic mechanical systems using polynomial chaos expansion. In: 10th AIAA/ISSMO multidisciplinary analysis and optimization conference; 2004. p. 4590.
[78] Choi Seung-Kyum, Grandhi Ramana V, Canfield Robert A. Structural reliability under non-gaussian stochastic behavior. Comput Struct 2004;82(13–14):1113–21.
[79] Jiang Youbao, Luo Jun, Liao Guoyu, Zhao Yulai, Zhang Jianren. An efficient method for generation of uniform support vector and its application in structural failure function fitting. Struct Saf 2015;54:1–9.

[80] Zhang Jinhao, Xiao Mi, Gao Liang, Chu Sheng. Probability and interval hybrid reliability analysis based on adaptive local approximation of projection outlines using support vector machine. Comput-Aid Civ Infrastruct Eng 2019.
[81] Fajraoui Noura, Marelli Stefano, Sudret Bruno. Sequential design of experiment for sparse polynomial chaos expansions. SIAM/ASA J Uncertain Quantification 2017;5(1):1061–85.
[82] Diaz Paul, Doostan Alireza, Hampton Jerrad. Sparse polynomial chaos expansions via compressed sensing and D-optimal design. Comput Methods Appl Mech Eng 2018;336:640–66.
[83] Zein Samih, Colson Benoît, Glineur François. An efficient sampling method for regression-based polynomial chaos expansion. Commun Comput Phys 2013;13(4):1173–88.
[84] Shin Yeonjong, Xiu Dongbin. Nonadaptive quasi-optimal points selection for least squares linear regression. SIAM J Sci Comput 2016;38(1):A385–411.
[85] Iman Ronald L, Helton Jon C. Comparison of uncertainty and sensitivity analysis techniques for computer models. Technical report, Sandia National Labs., Albuquerque, NM (USA); 1985.
[86] Kim Sang-Hyo, Na Seong-Won. Response surface method using vector projected sampling points. Struct Saf 1997;19(1):3–19.
[87] Das Purnendu K, Zheng Yunlong. Cumulative formation of response surface and its use in reliability analysis. Prob Eng Mech 2000;15(4):309–15.
[88] Gupta Sayan, Manohar CS. An improved response surface method for the determination of failure probability and importance measures. Struct Saf 2004;26(2):123–39.
[89] Duprat Frederic, Sellier Alain. Probabilistic approach to corrosion risk due to carbonation via an adaptive response surface method. Prob Eng Mech 2006;21(3):207–16.
[90] Muzeau JP, Lemaire Maurice. Reliability analysis with implicit formulations. In: Probabilistic methods for structural design. Springer; 1997. p. 141–60.
[91] Kaymaz Irfan, McMahon Chris A. A response surface method based on weighted regression for structural reliability analysis. Prob Eng Mech 2005;20(1):11–7.
[92] Wong Siew M, Hobbs Roger E, Onof Christian. An adaptive response surface method for reliability analysis of structures with multiple loading sequences. Struct Saf 2005;27(4):287–308.
[93] Zhao Hongbo, Zhongliang Ru, Chang Xu, Yin Shunde, Li Shaojun. Reliability analysis of tunnel using least square support vector machine. Tunnel Undergr Space Technol 2014;41:14–23.
[94] Goswami Somdatta, Ghosh Shyamal, Chakraborty Subrata. Reliability analysis of structures by iterative improved response surface method. Struct Saf 2016;60:56–66.
[95] Roussouly Nicolas, Petitjean Frank, Salaun M. A new adaptive response surface method for reliability analysis. Prob Eng Mech 2013;32:103–15.
[96] Gary G. Wang Adaptive response surface method using inherited latin hypercube design points. J Mech Des 2003;125(2):210–20.
[97] Basudhar Anirban, Missoum Samy. Adaptive explicit decision functions for probabilistic design and optimization using support vector machines. Comput Struct 2008;86(19–20):1904–17.
[98] Basudhar Anirban, Missoum Samy. An improved adaptive sampling scheme for the construction of explicit boundaries. Struct Multidiscip Optim 2010;42(4):517–29.
[99] Song Hyeongjin, Choi Kyung K, Lee Ikjin, Zhao Liang, Lamb David. Adaptive virtual support vector machine for reliability analysis of high-dimensional problems. Struct Multidiscip Optim 2013;47(4):479–91.
[100] Pan Qiujing, Dias Daniel. An efficient reliability method combining adaptive support vector machine and monte carlo simulation. Struct Saf 2017;67:85–95.
[101] Bourinet Jean M. Rare-event probability estimation with adaptive support vector regression surrogates. Reliab Eng Syst Saf 2016;150:210–21.
[102] Picheny Victor, Ginsbourger David, Roustant Olivier, Haftka Raphael T, Kim Nam-Ho. Adaptive designs of experiments for accurate approximation of a target region. J Mech Des 2010;132(7):071008 .
[103] Dubourg Vincent, Sudret Bruno, Bourinet Jean-Marc. Reliability-based design optimization using kriging surrogates and subset simulation. Struct Multidiscip Optim 2011;44(5):673–90.
[104] Vazquez Emmanuel, Bect Julien. A sequential bayesian algorithm to estimate a probability of failure. IFAC Proc Vol 2009;42(10):546–50.
[105] Lv Zhaoyan, Zhenzhou Lu, Wang Pan. A new learning function for kriging and its applications to solve reliability problems in engineering. Comput Math Appl 2015;70(5):1182–97.
[106] Sun Zhili, Wang Jian, Li Rui, Tong Cao. LIF: A new kriging based learning function and its application to structural reliability analysis. Reliab Eng Syst Saf 2017;157:152–65.
[107] Xiao Ning-Cong, Zuo Ming J, Zhou Chengning. A new adaptive sequential sampling method to construct surrogate models for efficient reliability analysis. Reliab Eng Syst Saf 2018;169:330–8.
[108] Marelli Stefano, Sudret Bruno. An active-learning algorithm that combines sparse polynomial chaos expansions and bootstrap for structural reliability analysis. Struct Saf 2018;75:67–74.
[109] Zhou Yicheng, Zhenzhou Lu. Active polynomial chaos expansion for reliability-based design optimization. AIAA J 2019:1–16.
[110] Zhang X, Wang L, Sørensen JD. A novel active-learning function towards adaptive kriging surrogate models for structural reliability analysis. Reliab Eng Syst Saf 2019.
[111] Jones Donald R, Schonlau Matthias, Welch William J. Efficient global optimization of expensive black-box functions. J Global Optim 1998;13(4):455–92.
[112] Meng Zeng, Zhang Zhuohui, Zhang Dequan, Yang Dixiong. An active learning method combining kriging and accelerated chaotic single loop approach (AK-ACSLA) for reliability-based design optimization. Comput Methods Appl Mech Eng 2019;357:112570 .
[113] Jian Wang, Zhili Sun, Qiang Yang, Rui Li. Two accuracy measures of the kriging model for structural reliability analysis. Reliab Eng Syst Saf 2017;167:494–505.
[114] Jiang Chen, Qiu Haobo, Yang Zan, Chen Liming, Gao Liang, Li Peigen. A general failure-pursuing sampling framework for surrogate-based reliability analysis. Reliab Eng Syst Saf 2019;183:47–59.
[115] Rui Teixeira, Maria Nogal, Alan O'Connor, Beatriz Martinez-Pastor. Reliability assessment with density scanned adaptive kriging. Reliab Eng Syst Saf 2020:106908. https://doi.org/10.1016/j.ress.2020.106908.
[116] Pan Qiujing, Dias Daniel. Sliced inverse regression-based sparse polynomial chaos expansions for reliability analysis in high dimensions. Reliab Eng Syst Saf 2017;167:484–93.
[117] Jun Xu, Kong Fan. A cubature collocation based sparse polynomial chaos expansion for efficient structural reliability analysis. Struct Saf 2018;74:24–31.
[118] Cheng Kai, Zhenzhou Lu. Adaptive sparse polynomial chaos expansions for global sensitivity analysis based on support vector regression. Comput Struct 2018;194:86–96.
[119] Fang Hai, Gong Chunlin, Hua Su, Zhang Yunwei, Li Chunna, Da Ronch Andrea. A gradient-based uncertainty optimization framework utilizing dimensional adaptive polynomial chaos expansion. Struct Multidiscip Optim 2019;59(4):1199–219.
[120] Jun Xu, Wang Ding. Structural reliability analysis based on polynomial chaos, Voronoi cells and dimension reduction technique. Reliab Eng Syst Saf 2019;185:329–40.
[121] Rosenblatt Murray. Remarks on a multivariate transformation. Ann Math Stat 1952;23(3):470–2.
[122] Gaspar Bruno, Teixeira Angelo P, Soares Carlos Guedes. Adaptive surrogate model with active refinement combining kriging and a trust region method. Reliab Eng Syst Saf 2017;165:277–91.
[123] Echard Benjamin, Gayton Nicolas, Lemaire Maurice, Relun Nicolas. A combined importance sampling and kriging reliability method for small failure probabilities with time-demanding numerical models. Reliab Eng Syst Saf 2013;111:232–40.
[124] Dai Hongzhe, Zhang Hao, Wang Wei, Xue Guofeng. Structural reliability assessment by local approximation of limit state functions using adaptive markov chain simulation and support vector regression. Comput-Aid Civ Infrastruct Eng 2012;27(9):676–86.
[125] Dubourg Vincent, Sudret Bruno, Deheeger Franois. Metamodel-based importance sampling for structural reliability analysis. Prob Eng Mech 2013;33:47–57.
[126] Huang Xiaoxu, Chen Jianqiao, Zhu Hongping. Assessing small failure probabilities by AK-SS: an active learning method combining kriging and subset simulation. Struct Saf 2016;59:86–95.
[127] Cadini Francesco, Santos Francisco, Zio Enrico. An improved adaptive kriging-based importance technique for sampling multiple failure regions of low probability. Reliab Eng Syst Saf 2014;131:109–17.
[128] Tong Cao, Sun Zhili, Zhao Qianli, Wang Qibin, Wang Shuang. A hybrid algorithm for reliability analysis combining kriging and subset simulation importance sampling. J Mech Sci Technol 2015;29(8):3183–93.
[129] Yun Wanying, Zhenzhou Lu, Jiang Xian. An efficient reliability analysis method combining adaptive kriging and modified importance sampling for small failure probability. Struct Multidiscip Optim 2018;58(4):1383–93.
[130] Wen Zhixun, Pei Haiqing, Liu Hai, Yue Zhufeng. A sequential kriging reliability analysis method with characteristics of adaptive sampling regions and parallelizability. Reliab Eng Syst Saf 2016;153:170–9.
[131] Chen Weidong, Chunlong Xu, Shi Yaqin, Ma Jingxin, Shengzhuo Lu. A hybrid kriging-based reliability method for small failure probabilities. Reliab Eng Syst Saf 2019;189:31–41.
[132] Wang Zeyu, Shafieezadeh Abdollah. REAK: Reliability analysis through error rate-based adaptive kriging. Reliab Eng Syst Saf 2019;182:33–45.
[133] Zhang Xufang, Wang Lei, Sørensen John Dalsgaard. AKOIS: an adaptive kriging oriented importance sampling method for structural system reliability analysis. Struct Saf 2020;82:101876 .
[134] Yun Wanying, Zhenzhou Lu, Jiang Xian, Zhang Leigang, He Pengfei. AK-ARBIS: An improved AK-MCS based on the adaptive radial-based importance sampling for small failure probability. Struct Saf 2020;82:101891 .
[135] Kim Jungho, Song Junho. Probability-adaptive kriging in n-ball (PAK-Bn) for reliability analysis. Struct Saf 2020;85:101924 .
[136] Teixeira Rui, O'Connor Alan, Nogal Maria. Adaptive kriging with biased randomisation for reliability analysis of complex limit state functions. In International Probabilistic Workshop 2019, Edinburgh, Sctoland, September 2019.
[137] Grasas Alex, Juan Angel A, Faulin Javier, de Armas Jesica, Ramalhinho Helena. Biased randomization of heuristics using skewed probability distributions: a survey and some applications. Comput Ind Eng 2017;110:216–28.
[138] Teixeira Rui, Nogal Maria, O'Connor Alan, Nichols James, Dumas Antoine. Stress-cycle fatigue design with kriging applied to offshore wind turbines. Int J Fatigue 2019;125:454–67.
[139] Ginsbourger David, Le Riche Rodolphe, Carraro Laurent. Kriging is well-suited to parallelize optimization. In: Computational intelligence in expensive optimization problems. Springer; 2010. p. 131–62.
[140] Lelièvre Nicolas, Beaurepaire Pierre, Mattrand Cécile, Gayton Nicolas. AK-MCSi: a kriging-based method to deal with small failure probabilities and time-consuming models. Struct Saf 2018;73:1–11.

[141] Cui Fengkun, Ghosn Michel. Implementation of machine learning techniques into the subset simulation method. Struct Saf 2019;79:12–25.
[142] Abdallah Imad, Lataniotis Christos, Sudret Bruno. Parametric hierarchical kriging for multi-fidelity aero-servo-elastic simulators: application to extreme loads on wind turbines. Prob Eng Mech 2019;55:67–77.
[143] Youn Byeng D, Choi Kyung K. A new response surface methodology for reliability-based design optimization. Comput Struct 2004;82(2–3):241–56.
[144] Nguyen Xuan Son, Sellier Alain, Duprat Frédéric, Pons Gérard. Adaptive response surface method based on a double weighted regression technique. Prob Eng Mech 2009;24(2):135–43.
[145] Kang Soo-Chang, Koh Hyun-Moo, Choo Jinkyo F. An efficient response surface method using moving least squares approximation for structural reliability analysis. Prob Eng Mech 2010;25(4):365–71.
[146] Zhang Leigang, Zhenzhou Lu, Wang Pan. Efficient structural reliability analysis method based on advanced kriging model. Appl Math Model 2015;39(2):781–93.
[147] Paffrath Meinhard, Wever Utz. Adapted polynomial chaos expansion for failure detection. J Comput Phys 2007;226(1):263–81.
[148] Li R, Ghanem Roger. Adaptive polynomial chaos expansions applied to statistics of extremes in nonlinear random vibration. Prob Eng Mech 1998;13(2):125–36.
[149] Choi Seung-Kyum, Grandhi Ramana V, Canfield Robert A, Pettit Chris L. Polynomial chaos expansion with latin hypercube sampling for estimating response variability. AIAA J 2004;42(6):1191–8.
[150] Notin Alban, Gayton Nicolas, Dulong Jean Luc, Lemaire Maurice, Villon Pierre, Jaffal Haidar. RPCM: a strategy to perform reliability analysis using polynomial chaos and resampling: Application to fatigue design. Eur J Comput Mech/Rev Eur Méc Numér 2010;19(8):795–830.
[151] Chao Hu, Youn Byeng D. Adaptive-sparse polynomial chaos expansion for reliability analysis and design of complex engineering systems. Struct Multidiscip Optim 2011;43(3):419–42.
[152] Ni Fei, Nguyen Phuong H, Cobben Joseph FG. Basis-adaptive sparse polynomial chaos expansion for probabilistic power flow. IEEE Trans Power Syst 2016;32(1):694–704.
[153] Gavin Henri P, Yau Siu Chung. High-order limit state functions in the response surface method for structural reliability analysis. Struct Saf 2008;30(2):162–79.
[154] Allaix Diego L, Carbone Vincenzo I. An improvement of the response surface method. Struct Saf 2011;33(2):165–72.
[155] Dai Hongzhe, Zhang Hao, Wang Wei. A support vector density-based importance sampling for reliability assessment. Reliab Eng Syst Saf 2012;106:86–93.
[156] Dai Hongzhe, Cao Zhenggang. A wavelet support vector machine-based neural network metamodel for structural reliability assessment. Comput-Aid Civ Infrastruct Eng 2017;32(4):344–57.
[157] Alibrandi Umberto, Alani Amir M, Ricciardi Giuseppe. A new sampling strategy for SVM-based response surface for structural reliability analysis. Prob Eng Mech 2015;41:1–12.
[158] Jiang Youbao, Zhao Linjie, Beer Michael, Patelli Edoardo, Broggi Matteo, Luo Jun, He Yihua, Zhang Jianren. Multiple response surfaces method with advanced classification of samples for structural failure function fitting. Struct Saf 2017;64:87–97.
[159] Pan Qiujing, Xingru Qu, Liu Leilei, Dias Daniel. A sequential sparse polynomial chaos expansion using bayesian regression for geotechnical reliability estimations. Int J Numer Anal Methods Geomech 2020.
[160] Fauriat William, Gayton Nicolas. AK-SYS: an adaptation of the ak-mcs method for system reliability. Reliab Eng Syst Saf 2014;123:137–44.
[161] Peijuan Zheng, Ming Wang Chien, Zhouhong Zong, Liqi Wang. A new active learning method based on the learning function U of the AK-MCS reliability analysis method. Eng Struct 2017;148:185–94.
[162] Zhang Jinhao, Xiao Mi, Gao Liang. An active learning reliability method combining kriging constructed with exploration and exploitation of failure region and subset simulation. Reliab Eng Syst Saf 2019.
[163] Wang Zeyu, Shafieezadeh Abdollah. Highly efficient bayesian updating using metamodels: an adaptive kriging-based approach. Struct Saf 2020;84:101915 .
[164] Xiao Mi, Zhang Jinhao, Gao Liang. A system active learning kriging method for system reliability-based design optimization with a multiple response model. Reliab Eng Syst Saf 2020;199:106935 .
[165] Marelli Stefano, Sudret Bruno. Bootstrap-polynomial chaos expansions and adaptive designs for reliability analysis. In Proceedings of the 6th Asian-Pacific Symposium on Structural Reliability and its Applications (APSSRA6), 28–30 May 2016, Shanghai, China. 6th Asian-Pacific Symposium on Structural Reliability and its Applications; 2016.
[166] Zhu Xianming, Lu Zhenzhou, Yun Wanying. An efficient method for estimating failure probability of the structure with multiple implicit failure domains by combining Meta-IS with IS-AK. Reliab Eng Syst Saf 2019:106644, .
[167] Alibrandi Umberto, Alani Amir, Koh CG. Implications of high-dimensional geometry for structural reliability analysis and a novel linear response surface method based on SVM. Int J Comput Methods 2015;12(04):1540016 .
[168] Hariri-Ardebili Mohammad Amin, Pourkamali-Anaraki Farhad. Support vector machine based reliability analysis of concrete dams. Soil Dyn Earthq Eng 2018;104:276–95.
[169] Roy Atin, Manna Ramkrishna, Chakraborty Subrata. Support vector regression based metamodeling for structural reliability analysis. Prob Eng Mech 2019;55:78–89.
[170] Saltelli Andrea, Ratto Marco, Andres Terry, Campolongo Francesca, Cariboni Jessica, Gatelli Debora, Saisana Michaela, Tarantola Stefano. Global sensitivity analysis: the primer. John Wiley & Sons; 2008.
[171] Teixeira Rui, O'Connor Alan, Nogal Maria. Probabilistic sensitivity analysis of offshore wind turbines using a transformed kullback-leibler divergence. Struct Saf 2019;81:101860 .
[172] Bouhlel Mohamed Amine, Bartoli Nathalie, Otsmane Abdelkader, Morlier Joseph. Improving kriging surrogates of high-dimensional design models by partial least squares dimension reduction. Struct Multidiscip Optim 2016;53(5):935–52.
[173] Wang Qian, Fang Hongbing. Reliability analysis of tunnels using an adaptive rbf and a first-order reliability method. Comput Geotech 2018;98:144–52.
[174] Menz Morgane, Gogu Christian, Dubreuil Sylvain, Bartoli Nathalie, Morio Jérôme. Adaptive coupling of reduced basis modeling and kriging based active learning methods for reliability analyses. Reliab Eng Syst Saf 2020;196:106771 .
[175] Ankenman Bruce, Nelson Barry L, Staum Jeremy. Stochastic kriging for simulation metamodeling. Oper Res 2010;58(2):371–82.
[176] Picheny Victor, Wagner Tobias, Ginsbourger David. A benchmark of kriging-based infill criteria for noisy optimization. Struct Multidiscip Optim 2013;48(3):607–26.
[177] Bae Harok, Clark Daniel L, Forster Edwin E. Nondeterministic kriging for engineering design exploration. AIAA J 2019;57(4):1659–70.
[178] Forrester Alexander IJ, Keane Andy J, Bressloff Neil W. Design and analysis of noisy computer experiments. AIAA J 2006;44(10):2331–9.
[179] Teixeira Rui, O'Connor Alan, Nogal Maria. Fatigue reliability using a multiple surface approach. In 13th International Conference on Applications of Statistics and Probability in Civil Engineering (ICASP13); 2019, https://doi.org/10.22725/ICASP13.438.
[180] Cheng Kai, Zhenzhou Lu. Structural reliability analysis based on ensemble learning of surrogate models. Struct Saf 2020;83:101905 .
[181] Goel Tushar, Haftka Raphael T, Shyy Wei, Queipo Nestor V. Ensemble of surrogates. Struct Multidiscip Optim 2007;33(3):199–216.
[182] Viana Felipe AC, Haftka Raphael T, Steffen Valder. Multiple surrogates: how cross-validation errors can help us to obtain the best predictor. Struct Multidiscip Optim 2009;39(4):439–57.