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Zhang, Lele; Slob, Evert

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Electromagnetic Marchenko scheme based internal multiple elimination for lossless media

Lele Zhang
Department of Geoscience and Engineering
Delft University of Technology
Delft, The Netherlands
L.Zhang-1@tudelft.nl

Evert Slob
Department of Geoscience and Engineering
Delft University of Technology
Delft, The Netherlands
E.C.Slob@tudelft.nl

***Abstract*—Iterative substitution of the Marchenko equation has been introduced recently to integrate internal multiple reflection in the seismic and electromagnetic imaging process. In the so-called Marchenko imaging, solving the Marchenko equation at each imaging point is required to meet this objective. It makes the scheme seriously expensive. Inspired by this limitation, we present an Electromagnetic Marchenko equation based one dimensional scheme to eliminate the internal multiples of the single-sided lossless surface ground penetrating radar data layer by layer, such that the conventional imaging schemes can be applied to get the internal multiple related artifacts free imaging result without the need of solving Marchenko equation at each imaging point. We show with an example that the method works well for a sample in a synthetic waveguide that could be used for measurements in laboratory and field.**

Keywords—Marchenko equation; iterative substitution; internal multiple elimination; lossless;

I. INTRODUCTION

The occurrence of internal multiple reflections is a long-standing issue in seismic reflection imaging for marine acquisition [1] and land acquisition [2]. It is in the same case for the surface ground penetrating radar (GPR) data. Various methods have been developed to predict and subtract internal multiples from recorded data, such as the internal multiple elimination (IME) and inverse scattering series (ISS). IME is done in a layer-stripping fashion and requires identification of the internal multiple generators in the input data [3]. The third term in ISS can be used to estimate the first order internal multiples. However, it requires accurate input data with a relatively broad frequency band.

Recently, it was shown that lossless medium internal multiple reflections can be eliminated by solving the Marchenko equation (lossless scheme) [4]. With this approach, up- and downgoing Green's functions are retrieved at an arbitrary level in the subsurface. By deconvolution of the retrieved upgoing Green's functions with the retrieved downgoing Green's functions at this level and evaluating the result at zero delay time, an image emerges without artifacts from internal multiple reflections[5]. Slob [6] derived a scheme for dissipative media and shown that it works well for dissipative seismic and electromagnetic case (lossy scheme). However, for both of the Marchenko schemes, solving the Marchenko equation at each imaging point is required to get the internal multiple related artifacts free image. It makes the scheme seriously expensive.

In this abstract, we present an Electromagnetic Marchenko equation based one-dimensional scheme to eliminate the internal multiples of single-sided lossless GPR data. Firstly, we solve the Marchenko equation with the focusing point at the bottom of the model to get the upgoing and downgoing focusing functions. For the upgoing focusing functions, which include physical primaries from the subsurface reflections and non-physical primaries caused by the downgoing focusing functions. Then we convolve and de-convolve the downgoing focusing functions with the physical primary layer by layer to predict and subtract the internal multiples. After the internal multiple elimination, conventional imaging schemes can be applied to get the image without artifacts from internal multiple reflections, such that it is not needed to solve Marchenko equation at each imaging point. Finally we give a synthetic example to illustrate this scheme and briefly discuss it's limitations and application for the 2D and field data.

II. THEORY

A. wavefield decomposition

In 1D we use z as spatial variable, the positive axis points downwards and t denotes time. Time and frequency can be interchanged through a Fourier transformation for which we use $E(z,\omega) = \int E(z,t)\exp(-j\omega t)dt$. The medium is assumed homogeneous for $z < z_0$ and $z > z_m$ and heterogeneous for $z_0 < z < z_m$. For this medium scattering data are assumed known from measurements taken at depth levels z_0 and z_m. At any location we can write electric $E(z,\omega)$ and magnetic $H(z,\omega)$ fields in the frequency domain as up- and downgoing wavefields according to Slob et al. [7]

$$E(z,\omega) = \left(\frac{\varsigma(z)}{\eta(z)}\right)^{\frac{1}{4}} \left(p^+(z,\omega) + p^-(z,\omega)\right), \qquad (1)$$

$$H(z,\omega) = \left(\frac{\eta(z)}{\varsigma(z)}\right)^{\frac{1}{4}} \left(p^+(z,\omega) - p^-(z,\omega)\right), \qquad (2)$$

In which $\eta = \sigma + j\omega\varepsilon$, σ and ε being the electric conductivity and permittivity, respectively, $\varsigma = j\omega\mu$, with μ being the magnetic permeability, and where p^+ denotes the downgoing and p^- denotes the upgoing wavefield. At any depth level z_i, the reflection response of the medium below that depth level can be written as a fraction combining the electric and magnetic fields

$$R(z_i,\omega)=\frac{\sqrt{\eta(z_i,\omega)}E(z_i,\omega)-\sqrt{\varsigma(z_i,\omega)}H(z_i,\omega)}{\sqrt{\eta(z_i,\omega)}E(z_i,\omega)+\sqrt{\varsigma(z_i,\omega)}H(z_i,\omega)},\quad (3)$$

where the depth levels for the medium parameters are taken in the limit of approaching z_i from above. By using the decompositions of equations (1) and (2) we find

$$R(z_i,\omega)=\frac{p^-(z_i,\omega)}{p^+(z_i,\omega)},\quad (4)$$

In this abstract, we just consider the lossless case, so the parameters of the medium should be frequency independent and real valued.

B. Coupled Marchenko equations

Following Slob et al. [7], we directly give the representations of the coupled Marchenko equations

$$f_1^-(z_0,z_i,t)=f_{1;0}^-(z_0,z_i,t)+\int_{-t_d}^{t_d}f_{1;m}^+(z_0,z_i,t')R(z_0,t-t')dt'\quad (5)$$

$$f_{1;m}^+(z_0,z_i,-t)=\int_{-t_d}^{t_d}f_1^-(z_0,z_i,-t')R(z_0,t-t')dt'\quad (6)$$

$$f_{1;0}^-(z_0,z_i,t)=\int_{-t_d}^{t_d}T_d^{-1}(z_i,z_0,t')R(z_0',t-t')dt'\quad (7)$$

where $R(z_0,t)$ is the single-sided GPR data of the lossless medium, $f_1^+(z_0,z_i,t)$ denotes the downgoing functions and $f_1^-(z_0,z_i,t)$ denotes the upgoing ones. $T_d^{-1}(z_i,z_0,t)$ denotes the inverse of first arrival of the transmission response, t_d is the traveltime of the first arrival.

Equations (5) and (6) are two coupled Marchenko equations that can be solved for the up- and downgoing focusing wavefields with the aid of equation (7). In 1D no estimate of the first arrival time is needed because it is given by half the recording time.

C. Internal multiple elimination

Based on equation (5) and (6) we give the focusing point at the bottom of the model to solve the downgoing focusing functions $f_1^+(z_0,z_i,t)$ and upgoing focusing functions $f_1^-(z_0,z_i,t)$ as shown in Figure 1, the red solid lines indicate the downgoing focusing functions and the black solid lines indicate physical primaries coming from subsurface reflectivity, the black dashed line indicates the non-physical primary caused by the downgoing focusing functions and blue dashed lines indicate the contribution of events in downgoing focusing functions to the physical primaries which would remove the transmission effect in primaries and make the amplitude equal to corresponding reflectivity, the red star shows the focusing point. At the beginning, we project the one-way focusing functions to the surface to be the two-way data by convolving the upgoing and downgoing focusing functions with the delta function which has positive t_d time shift

$$F_1^+(z_0,z_i,t)=f_{1;m}^+(z_0,z_i,t)*\delta(t-t_d),\quad (8)$$

$$F_1^-(z_0,z_i,t)=f_1^-(z_0,z_i,t)*\delta(t-t_d),\quad (9)$$

where $F_1^+(z_0,z_i,t)$ is the projected two-way downgoing focusing functions and $F_1^-(z_0,z_i,t)$ is the projected two-way upgoing focusing functions, $*$ indicates convolution.

Then we convolve the projected two-way downgoing focusing functions with the first reflector related primary and it's conjugated version

$$F_{1,1}^+(z_0,z_i,t)=F_1^+(z_0,z_i,t)*r_1\delta(t-t_1),\quad (10)$$

$$F_{1f}^+(z_0,z_i,t)=F_1^+(z_0,z_i,t)*\frac{\delta(t-t_1)}{r_1},\quad (11)$$

where r_1 is the reflectivity of the first reflector and t_1 is the two-way travel time from surface to the first reflector. Equation (10) predicts the first reflector related non-physical primaries and contributions to the physical primaries from the downgoing focusing functions. Equation (11) can offer some information for the elimination of first reflector related events in $F_1^+(z_0,z_i,t)$ which are redundant for following. We add $F_{1,1}^+(z_0,z_i,t)$ and $F_{1f}^+(z_0,z_i,t)$ to $F_1^-(z_0,z_i,t)$ to eliminate the first reflector related non-physical events in $F_1^-(z_0,z_i,t)$ and eliminate the events in $F_1^+(z_0,z_i,t)$ which need to be removed before the next step.

$$F_{1,2}^-(z_0,z_i,t)=F_1^-(z_0,z_i,t)+F_{1,1}^+(z_0,z_i,t),\quad (12)$$

$$F_{1F1}^+(z_0,z_i,t)=F_1^-(z_0,z_i,t)+F_{1f}^+(z_0,z_i,t),\quad (13)$$

for $F_{1,2}^-(z_0,z_i,t)$ in which the first reflector related non-physical events have been eliminated. In $F_{1F1}^+(z_0,z_i,t)$ the first reflector related events in $F_1^+(z_0,z_i,t)$ which need to be removed before next step have been totally removed but the phase and amplitude of the preserved events are different from which in $F_1^+(z_0,z_i,t)$, so we need to do the correction

$$F_{12}^+(z_0,z_i,t)=F_{1F1}^+(z_0,z_i,t)*\frac{\delta(t+t_1)}{r_1-\frac{1}{r_1}},\quad (14)$$

where $F_{12}^+(z_0,z_i,t)$ is the downgoing focusing functions in which the events which are redundant for following steps have been totally removed. So far we have finished the elimination of the first reflector related non-physical primaries in upgoing focusing functions and the events which would not be used for the next step in downgoing focusing functions. For the following reflectors, we just need to do the same steps as the first reflector based on $F_{1,2}^-(z_0,z_i,t)$ and $F_{12}^+(z_0,z_i,t)$.

We can conclude this scheme generally in the following form

$$F_{1,n}^+(z_0,z_i,t)=F_{1n}^+(z_0,z_i,t)*\tau_1^+\dots\tau_{n-1}^+r_n\tau_1^-\dots\tau_{n-1}^-\delta(t-t_n),\quad (15)$$

$$F_{1fn}^+(z_0,z_i,t)=F_{1n}^+(z_0,z_i,t)*\tau_1^+\dots\tau_{n-1}^+\tau_1^-\dots\tau_{n-1}^-$$

$$\frac{\delta(t-t_n)}{r_n}, (16)$$

$$F^-_{1,n+1}(z_0,z_i,t) = F^-_{1,n}(z_0,z_i,t) + F^+_{1,n}(z_0,z_i,t), \quad (17)$$

$$F^+_{1Fn}(z_0,z_i,t) = F^-_{1,n}(z_0,z_i,t) + F^+_{1fn}(z_0,z_i,t), \quad (18)$$

$$F^+_{1n+1}(z_0,z_i,t) = F^+_{1Fn}(z_0,z_i,t) * \delta(t+t_n)/[\tau^+_1 \ldots \tau^+_{n-1} \left(r_n - \frac{1}{r_n}\right)\tau^-_1 \ldots \tau^-_{n-1}], \quad (19)$$

where n indicates the n^{th}reflector and t_n is the two-way travel time from surface to the n^{th} reflector, τ_i^+ and τ_i^- are the downgoing and upgoing transmission effects of i^{th} reflector.

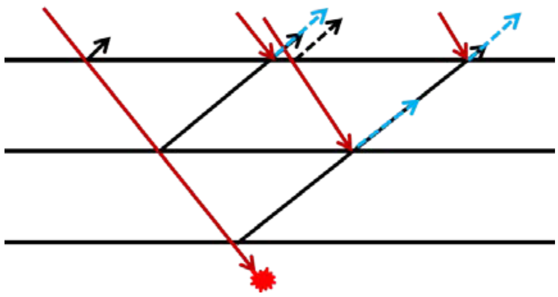

Fig 1. the red star gives the focusing point; the red solid line is the downgoing focusing functions and black solid lines are physical primaries from subsurface reflectivity; the black dashed line indicates the non-physical primary caused by the downgoing focusing functions; the blue dashed lines show the contribution to the amplitude of physical primaries from downgoing focusing functions.

III. NUMERICAL EXAMPLE

To demonstrate the effectiveness of the method in eliminating internal multiples in single-sided GPR data, we have performed a one-dimensional lossless modeling test. The model is shown in Table I. The modeled reflection data of the lossless medium is shown in Figure 2(a). We can see that many unexpected events appeared because of the multiple scattering. Then we gave the focusing point at the bottom of the model and solved the Marchenko equation for the downgoing and upgoing focusing functions, after the time shift we got the two-way downgoing and upgoing focusing functions shown in Figure 2(b) and 2(c). We can see events which would be used to eliminate the internal multiples appeared in original data (Figure 2(a)) in Figure 2(b) and in Figure 2(c) the multiple scattering existed in original data has been totally canceled but some non-physical primaries have been introduced when the events in downgoing focusing functions tried to eliminate deeper multiple scattering in the original data, at the same time, the physical primaries have been amplified because the transmission effects have been removed when we did the focusing. Then we convolved the downgoing focusing functions with corresponding events and it's conjugated version layer by layer to cancel non-physical primaries in upgoing focusing functions and the eliminated result is shown in Figure 2(d). We gave the comparison of the upgoing focusing functions (blue solid line) and the eliminated result (red dashed line) in Figure 2(e), we can see that most non-physical events have been effectively canceled even though some minor artifacts existed, which are caused by numerical errors. Compared to the upgoing focusing functions, the amplitude of primaries in eliminated result are lower because the transmission effects have been taken back during this elimination process. We also shown the comparison of the original data (blue solid line) and the eliminated result (red dashed line) in Figure 2(f). Compared to the original data, the amplitude of primaries in eliminated result is well preserved and most multiple scattering have been effectively canceled. It demonstrated that the scheme we presented can successfully eliminate most internal multiples appeared in original data, this would give big help to the following migration and image.

IV. CONCLUSIONS

We have shown a 1D theory and a numerical example to eliminate the internal multiples appeared in the original data based on lossless Marchenko scheme. Compared to the original data, this scheme can effectively eliminate the internal multiples existed in the original data and well preserve the waveform of the primaries even though some minor artifacts appeared which are caused by the numerical errors. This would make it possible that we can get internal multiples related artifacts free result after the migration and imaging, this would give a big help to following process. Especially, it is an affordable scheme because we just need to solve the Marchenko equation one time. We can predict that this scheme possibly works in 2D and 3D synthetic and field data when the related velocity model is available.

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TABLE I

Depth (m)*4	Velocity (m/s)	Density (kg/m^3)*1000
75	1700	1.53
90	1900	2.25
50	2100	1.82
110	1700	1.43
151	2100	1.75
71	3250	1.93
143	2100	1.5
151	2900	2.11
163	2500	2.11
221	2750	2.25
50	3000	2.30

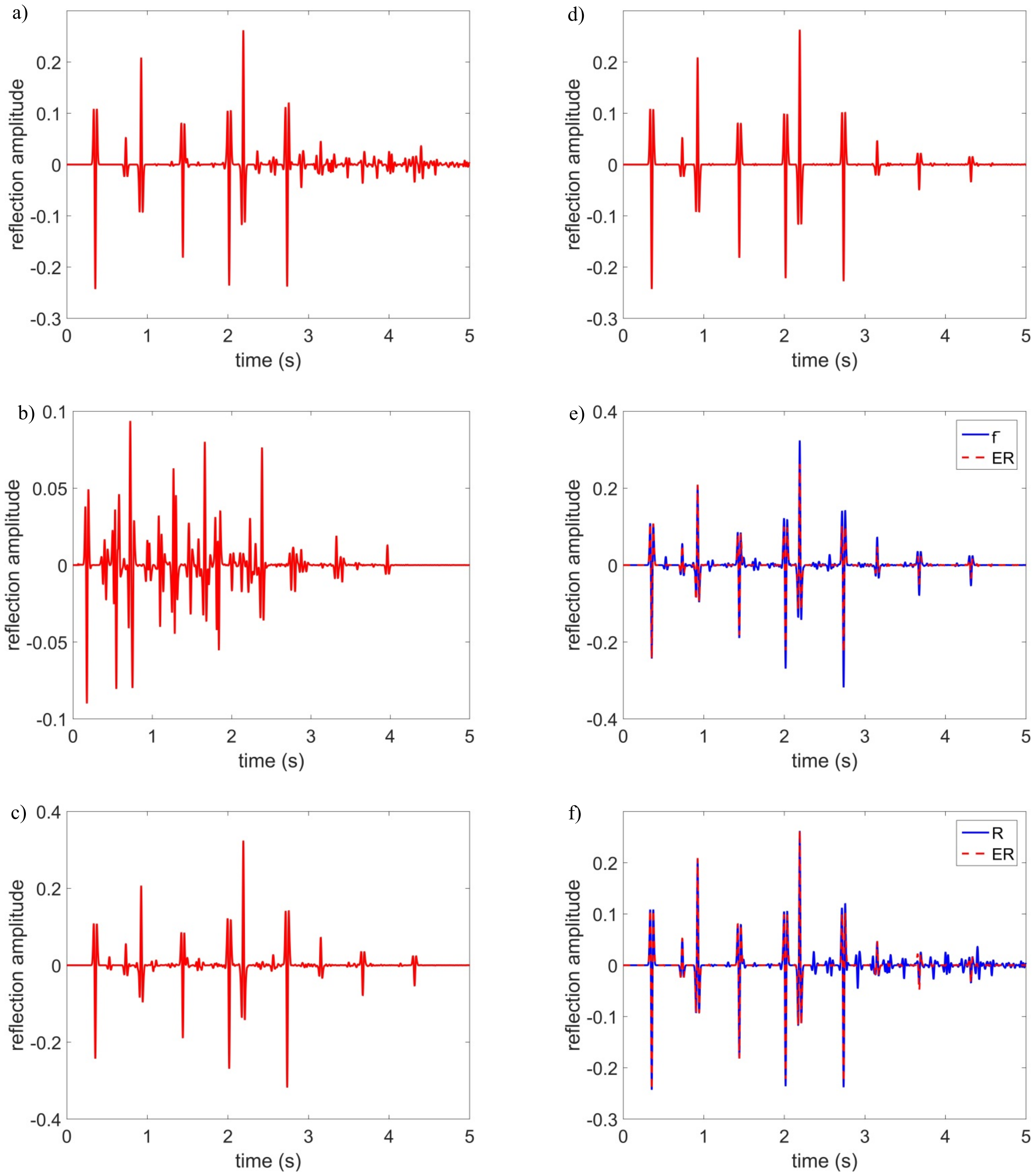

Fig 2: a) The modeled GPR data, there are many multiple reflections appeared. b) the two-way downgoing focusing functions. c) the two-way upgoing focusing functions, some non-physical events exist. d) the internal multiple eliminated result, most multiple reflections have been removed even though some minor errors existed. e) the comparison of the eliminated result (red dashed line) and the upgoing focusing functions (blue solid line), most non-physical events in upgoing focusing functions have been removed and transmission effects of physical primaries have been taken back. f) the comparison of the eliminated result (red dashed line) and the original data (blue solid line), most internal multiples have been removed and the waveform of primaries have been well preserved.

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