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An Overview of Marchenko Methods

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Summary

Since the introduction of the Marchenko method in geophysics, many variants have been developed. Using a compact unified notation, we review redatuming by multidimensional deconvolution and by double focusing, virtual seismology, double dereverberation and transmission-compensated Marchenko multiple elimination, and discuss the underlying assumptions, merits and limitations of these methods.

Introduction. Since the introduction of the Marchenko method in geophysics (Broggini et al., 2011; Wapenaar et al., 2011), many variants have been developed, ranging from data-driven redatuming by multidimensional deconvolution to model-independent Marchenko multiple elimination. We give a brief overview of methods developed in Delft, their underlying assumptions and their merits and limitations.

The focusing function. The central concept in the Marchenko method is the focusing function (Wapenaar et al., 2013; Slob et al., 2014), which is illustrated in Figure 1(a). The downgoing part of the focusing function f_1^+ (indicated by yellow rays), when emitted from the surface into a truncated version of the actual medium, focuses at a predefined location $\mathbf{x}_F$, without artefacts due to multiple scattering. The upgoing response (indicated by blue rays) is called f_1^-. The focusing functions can be retrieved from the reflection response R at the surface and an estimate of the direct focusing function f_{1d}^+ (the latter is equivalent to the standard focusing function for primaries). In the compact notation of Van der Neut et al. (2015), the algorithm reads

$$f_1^+ = \sum_{k=0}^{\infty} (\Theta R^\star \Theta R)^k f_{1d}^+, \qquad f_1^- = \Theta R f_1^+. \tag{1}$$

Rf stands for a multidimensional convolution of the reflection data with a function f, the star denotes time-reversal and Θ stands for a symmetric time window $\Theta_{-t_d+\varepsilon}^{t_d-\varepsilon}$ that removes all events after the direct wave at t_d (including the direct wave itself; ε is a small value to account for the finite duration of the seismic wavelet). The scheme requires a macro model to define the initial focusing function f_{1d}^+.

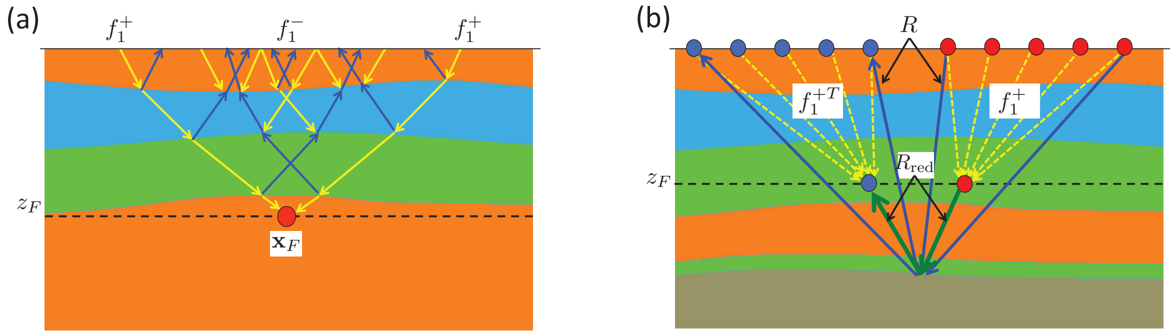

Figure 1 *(a) Focusing function. (b) Redatuming by double focusing. The 'rays' represent primaries and multiples.*

Redatuming by multidimensional deconvolution (MDD). Once the focusing functions are found, the downgoing and upgoing Green's functions at the focal depth level z_F follow from (Wapenaar et al., 2014)

$$G^+ = f_{1d}^{+\star} - \Psi R f_1^{-\star}, \qquad G^- = \Psi R f_1^+. \tag{2}$$

Here Ψ is the complement of time window $\Theta_{-t_d+\varepsilon}^{t_d-\varepsilon}$, hence, it passes the direct wave and all events after it. The Green's functions are mutually related via $G^- = R_{\rm red} G^+$, where $R_{\rm red}$ is the redatumed reflection response at z_F of the medium below z_F. Hence, $R_{\rm red}$ follows from MDD, as follows

$$R_{\rm red} = G^-(G^+)^{-1}. \tag{3}$$

$R_{\rm red}$ is free of multiples related to the overburden and can be used for imaging the medium below the focal depth level. The method relies on a macro model to estimate f_{1d}^+. Possible amplitude errors in f_{1d}^+ are transferred to $f_1^\pm$ and $G^\pm$, but they are largely annihilated in the MDD step. A complication of the method is that the MDD process requires a careful stabilised matrix inversion.

Redatuming by double focusing. An alternative method to obtain $R_{\rm red}$ is redatuming by double focusing (Figure 1(b)), formulated as

$$R_{\rm red} = f_1^{+T} \Psi R f_1^+, \tag{4}$$

where superscript T denotes transposition. Equation (4) is stable and can easily be applied in an adaptive way (Staring et al., 2018). The retrieved response $R_{\rm red}$ contains some interactions with the overburden and amplitude errors in f_1^+ are not annihilated.

Virtual seismology. The full Green's function between any two points in the subsurface can be obtained by the following variant of double focusing

$$G = \Psi f_2^T \Psi R f_2, \tag{5}$$

where $f_2 = f_1^+ - f_1^{-\star}$. This method can be used to forecast the response of induced earthquakes or to measure the response of earthquakes with virtual receivers in the subsurface (Brackenhoff et al., 2019).

Double dereverberation. To reduce the sensitivity for a macro model, Van der Neut and Wapenaar (2016) proposed to project the focusing functions to the surface, according to $v^+ = f_1^+ T_d$, where v^+ is the projected focusing function and T_d is the direct arrival of the transmission response. Since the direct focusing function is the inverse of T_d, according to $f_{1d}^+ T_d = \delta$ (where δ is a space-time delta function), we obtain from equation (1)

$$v^+ = f_1^+ T_d = \sum_{k=0}^{\infty} (\Theta R^\star \Theta R)^k \delta, \qquad v^- = \Theta R v^+, \tag{6}$$

where Θ stands now for an asymmetric time window $\Theta_\varepsilon^{t_{d2}-\varepsilon}$ that removes all events at and after the two-way traveltime t_{d2} of a fictitious reflector at the focal depth z_F. Since this equation does not require an estimate of f_{1d}^+ (unlike equation (1)) it is significantly less sensitive to the macro model (only Θ depends on it). Applying T_d^T and T_d to the left and right of the double focusing equation (4), we obtain

$$R_{\text{tar}} = T_d^T R_{\text{red}} T_d = v^{+T} \Psi R v^+, \tag{7}$$

where Ψ is now the complement of $\Theta_\varepsilon^{t_{d2}-\varepsilon}$. The response R_{tar} is the redatumed response projected to the surface. It can be seen as the reflection response at the surface of the target below z_F without the internal multiples related to the overburden. Therefore the right-hand side of equation (7) is a double dereverberation method (Staring et al., 2020). Like double focusing, it can be applied in an adaptive way, but it is significantly less sensitive to the macro model.

Transmission-compensated Marchenko multiple elimination (T-MME). By replacing the asymmetric window $\Theta_\varepsilon^{t_{d2}-\varepsilon}$ in equation (6) by $\Theta_\varepsilon^{t_{d2}+\varepsilon}$, the event in v^- at the two-way traveltime t_{d2} is retained. It can be shown that the last event of v^- can be written as

$$v_{\text{last}}^- = T_d^T r (T_d^{-1})^\star. \tag{8}$$

Here r is the reflectivity of the deepest reflector above the focal depth z_F. The right-hand side can be interpreted as the primary reflection response of that reflector, observed at the surface and compensated for transmission losses. To obtain the complete primary reflection response, Zhang et al. (2019) propose the following procedure: apply equation (6) with the modified window for all possible two-way traveltimes t_{d2} (instead of focal depths z_F), select the sample $v^-(t = t_{d2})$ and store this to $R_t(t_{d2})$. The resulting response $R_t(t)$ for all t is the transmission-compensated primary reflection response at the surface. This method uses no subsurface information at all.

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