Design and Analysis of Macro-Economic Models in the Laplace Domain

An Economic-Engineering Approach

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Master of Science Thesis

Delft University of Technology

Delft Center for Systems and Control

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For the degree of Master of Science in Mechanical Engineering at Delft University of Technology

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January 29, 2024

Faculty of Mechanical, Maritime and Materials Engineering (3mE) · Delft University of Technology

The work in this thesis was supported by Rabobank. Their cooperation is hereby gratefully acknowledged.

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Design and Analysis of Macro-Economic Models in the Laplace Domain

by

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in partial fulfillment of the requirements for the degree of

Master of Science Mechanical Engineering

Dated: January 29, 2024

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Abstract

In this thesis, we demonstrate the efficiency of Laplace domain techniques for the design and analysis of economic systems. To make the techniques applicable to economic modeling, we establish the economic analogs to the various tools and nomenclature in the engineering literature. We show that the Laplace domain provides an alternative description of economic systems, offering insights into behavior not apparent in the time domain. This allows economic discounting and cycles to be efficiently analyzed using pole-zero maps, Bode plots, and similar techniques. In addition, we demonstrate that transforming the linear differential equations of economic engineering into algebraic equations in the Laplace domain simplifies the design of economic systems.

We use the Laplace domain techniques to design and analyze a macroeconomic model. By designing the model in the Laplace domain, we are able to integrate supply chain dynamics and the housing market using two-port network theory. By analyzing the model using a pole-zero map, we show that the economy's discount rates and business cycles are represented by complex poles and the economy's transmission blocking rates by complex zeros. Additionally, we demonstrate that the Bullwhip effect, a supply chain phenomenon, can be intuitively visualized using a Bode plot. These applications illustrate how Laplace-domain techniques enable the efficient design and analysis of economic systems.

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Preface

This thesis is the result of a lot of hard work and led to many unexpected revelations. It was an experience I thoroughly enjoyed, as it allowed me to combine two of my great interests: engineering and economics. What contributed to this experience is that I could break new ground, as economic engineering is constantly evolving and pushing the boundaries of what is known. Despite the sound analysis that underpins the ideas described in my thesis, it sometimes baffles me how well it all "works". However, I can say nothing other than that it works as it simply is engineering.

I would like to thank my supervisors dr.ir. M.B. Mendel and ir. C. Hutters for their assistance during the writing of this thesis. Their guidance, suggestions, and research have allowed me to consolidate my research and also helped to develop my skills as a presenter and communicator.

Furthermore, I would also like to thank my friends and family, who have endured my many thoughts and reflections in numerous conversations. Just by asking questions, they have often helped me more than they realized, for which I am grateful. A similar gratitude goes out to the economic engineering group for their questions and feedback during our many discussions.

Finally, I would like to thank Rabobank for showing their interest in this project and providing valuable input in developing the applications presented in this thesis. The collaboration has allowed us to link our academic research with relevant socio-economic issues, which has been an additional motivation for me to complete this work.

Delft, University of Technology G.A.M. van Rijen
January 29, 2024

"It is only afterward that a new idea seems reasonable. To begin with, it usually seems unreasonable."

— *Isaac Asimov*

Chapter 1

Introduction

Laplace domain techniques are used in engineering to design and analyze linear time-invariant (LTI) systems. By transforming differential equations into algebraic equations, the mathematical analysis of a system is simplified in the Laplace domain. Furthermore, the Laplace domain provides an alternative representation of a system, offering insight into behavior that is not apparent in the time domain. To visualize and analyze this behavior, the corresponding tools and nomenclature have been developed in the engineering literature.[2][23][51][52][64][66]

In this thesis, we establish the economic analogs to Laplace domain tools and nomenclature. We show that the algebraic simplification of differential equations allows us to efficiently design economic models. Additionally, we demonstrate that economic discounting and cycles can be analyzed using pole-zero maps and Bode plots. This allows us to study economic phenomena such as the business cycle and the Bullwhip effect.

To model economic systems in the Laplace domain, we use economic engineering. By recognizing that demand is analogous to inertia, economic engineering defines an economic force as the driver of a price change. Extending this analogy yields a price-dynamic framework that allows economic systems to be modeled as LTI systems. Readers unfamiliar with economic engineering are recommended to read *"The Newtonian Mechanics of Demand"* by M.B Mendel, which lays down the foundations of economic engineering.[48]

Having developed the economic analogs to Laplace domain techniques in Chapter 2, we demonstrate their efficiency by designing and analyzing macroeconomic models. We review the economic engineering analogs to macroeconomic principles in Chapter 3. Readers familiar with economic engineering can reference this chapter or use it to read up on macroeconomic theory. For readers seeking to expand their knowledge of macroeconomics, I recommend textbooks authored by Mankiw or Samuelson.[43][62]

Moreover, the economic-engineering approach allows us to overcome important theoretical limitations of macroeconomic models. To motivate this, we briefly discuss the theoretical limitations of existing macroeconomic models:

Econometric models, such as vector auto-regression (VAR) models, are statistical models that describe correlations between economic variables. While they provide strong short-term predictive performance, their model structure is based on economic data. Shifts in economic policies, therefore, cause the model structure to change. This is known in macroeconomic literature as the Lucas critique. As a result, VAR models are fundamentally limited as they are susceptible to underlying data trends.[1][3][11][12][14][40][55][69]

In response to the Lucas critique, economists developed structural models, such as dynamic stochastic general equilibrium (DSGE) models. By using behavioral equations based on microeconomic theory to model the decision-making processes of representative agents, structural models proved more robust to the Lucas critique. However, to accommodate this structure, multiple strong generalizations and assumptions have to be made, resulting in top-down models that presume the economy is always in a state of general equilibrium.[33][50][57][59][70]

The limitations of top-down structural models led to the recognition of individual decision-making processes. By modeling the economy for the bottom-up, agent-based modeling (ABM) can capture nonlinear and emergent behavior within macroeconomic models. However, agent-based modeling lacks a systematic modeling framework, making it a major challenge to efficiently analyze, interpret, and validate simulation results because of the inherent complexity of a bottom-up modeling approach. As a result, ABM tends to overfit economic data, exacerbating existing trends, limiting their use to purely theoretical models that illustrate stylized facts.[19][20][21][73]

In addition, none of the above macroeconomic models can be used to perform Laplace domain analysis. The statistical methods of VAR models and the ad hoc modeling approach of ABM make these models lack the economic interpretation of model parameters, restricting the insights of the Laplace domain. Moreover, these models lack the causal and dynamic structure of linear time-invariant systems. This also applies to structural models, which kinematically describe macroeconomic behavior as opposed to economic-engineering models, where economic forces drive changes in economic variables.[8][29][42]

To overcome these limitations, Chapter 4 demonstrates the algebraic simplification of the Laplace domain can be used to design economic networks from the bottom up. Using Laplace domain analysis, we show this modeling approach allows us to capture emergent behavior. As a systematic modeling framework, we use electrical network theory, which has been proposed as an extension of the economic-engineering approach by C.Hutters et al.[31][32]

To develop a bottom-up macroeconomic model, we first design a top-down model of the economy in the time domain. Subsequently, we integrate the bottom-up economic networks into the top-down model. Chapter 5 derives the top-down macroeconomic model by transforming a circular flow model of a four-market economy into an analogous circuit diagram. To demonstrate its workings, we analyze the model in the time domain. Using Laplace domain analysis, we demonstrate how Laplace domain techniques contribute to macroeconomic modeling by providing insight into the economy's discounting behavior and economic cycles.

In Chapter 6, we show how bottom-up economic networks can be integrated into the top-down macroeconomic model to derive a bottom-up macroeconomic model. Following our collaboration with Rabobank, we design the bottom-up model for two specific applications: Integrating supply chain dynamics and modeling the housing market. Their interest in these applications arises from their significant challenges in modeling these aspects within their macroeconomic models. Using Laplace domain analysis, we show that we can explain emergent behavior, such as the Bullwhip effect, as well as macroeconomic phenomena, such as the business cycle.

We conclude with Chapter 7, which summarizes the key contributions of this thesis, followed by Chapter 8, which provides recommendations for further research.

Chapter 2

Economic Analysis using Laplace Domain Analysis

2-1 Introduction

To design and analyze the macroeconomic models developed in Chapters 4, 5, and 6, we establish the economic analogs of Laplace domain tools and nomenclature. For readers looking to read up on Laplace domain theory, I recommend textbooks authored by Oppenheim or Siebert.[52][63] We show that Laplace domain techniques simplify the mathematical analysis of economic systems by transforming the linear differential equations of economic engineering into algebraic equations in the Laplace domain. Additionally, we demonstrate that the Laplace domain provides an alternative representation of economic systems, providing further insight into economic behavior not apparent in the time domain.

This chapter is split into three sections: Section 2-2 shows that the Laplace transform allows us to transform an economic time series to a complex present value function and can be interpreted as complex detrending. Thereafter, we establish the economic analogs to various interpretations of complex present value functions. In addition, we address a common misconception within economic models, which can be traced back to the generalized uncertainty principle, and discuss its implications regarding economic analysis.

Section 2-3 develops the techniques needed to analyze economic systems in the Laplace domain. Using two examples of economic systems, we demonstrate that a system's poles can be interpreted as the complex discount rates and the system's zeros as the transmission-blocking rates of an economic system. Thereafter, we establish the economic interpretation of system properties, such as resonance and stability, and visualization tools, such as pole-zero maps and Bode plots.

We conclude with Section 2-4 by discussing the critical insights of Laplace domain analysis for macroeconomic modeling. However, the results in this chapter are independent of macroeconomic modeling and applicable to all economic-engineering models.

2-2 Laplace Domain Analysis of Economic Time Series

2-2-1 The Laplace Transform as Complex Detrending

In engineering, a signal is defined as any physical quantity that changes over time.[52] In economics, an economic time series $f(t)$ can describe arbitrary economic quantities that change over time. Economic engineering allows economic time series to be modeled analogous to physical quantities. Therefore, we interpret an economic time series analogous to a signal.

Signals can be represented in a transform domain using an integral transform. Conversely, a signal can be transformed back to the time domain using an inverse transform. This has been illustrated for the Laplace transform in Figure 2-1, which is the preeminent transform in engineering that is used for signal processing, system analysis, controller design, and more.[52][64] Therefore, this chapter demonstrates how the Laplace transform can be leveraged for economic analysis.

$$f(t) \underset{\mathcal{L}^{-1}}{\overset{\mathcal{L}}{\rightleftarrows}} F(s)$$

Figure 2-1: Schematic visualization of the Laplace transform $\mathcal{L}$, where $f(t)$ denotes an economic time series and $F(s)$ a complex present value function.

Mathematically, the Laplace transform converts a real function $f(t)$ of time t to a complex function $F(s)$ of the Laplace variable s:

$$F(s) = \int_0^\infty f(t)e^{-st}dt \quad \in \mathbb{C} \tag{2-1}$$

$$\text{where} \quad s = \sigma + i\omega \quad \in \mathbb{C} \tag{2-2}$$

Analogously, the Laplace transform converts an economic time series $f(t)$ of time t to a complex present value function $F(s)$ of the complex trend s.

Discounted Cash Flow Method

To motivate why we can interpret a complex function as a complex present value function, we show that the Laplace transform is equivalent to the discounted cash flow (DCF) method.[9][28] The DCF method is used to determine the *net present value (NPV)* of a capital good. It is calculated by discounting all future cash flows $f(n)$ to their present value. This is done for a discount rate σ that quantifies the time preference:

$$NPV = \sum_{n=0}^{N} \frac{f(n)}{(1+\sigma)^n} \tag{2-3}$$

From the discrete time DCF formula we can derive the equivalent continuous-time formula[1]:

$$NPV = \int_0^\infty f(t)e^{-\sigma t}dt \tag{2-4}$$

We observe that Equation 2-4 equals the Laplace transform for strictly real values of $s = \sigma$:

$$F(\sigma) = \int_0^\infty f(t)e^{-\sigma t}dt \quad \in \mathbb{R} \tag{2-5}$$

Therefore, we conclude that the Laplace transform of an economic time series $f(t)$ is a generalization of the DCF method. This implies we interpret the $NPV(\sigma)$ as the transform of an economic time series $f(t)$. As a result, the NPV is a function $F(\sigma)$ of the discount rate. This is a noteworthy result as economists usually calculate the NPV for a single discount rate, treating it as a single value. Sections 2-2-3 and 8-1-1 further discuss the implications that arise from this insight.

Because we consider the Laplace transform as a generalization of the DCF method, we can apply the DCF method to economic time series other than cash flows, as the Laplace transform is applicable to arbitrary signals. This allows us to calculate the present value of any economic time series as a function of the discount rate. Accordingly, we interpret $F(\sigma)$ as a present value function. In the subsequent section, we show that the present value of an economic time series is complex for complex values of s. Therefore, we interpret $F(s)$ as a complex present value function.

Fourier Transform

For strictly imaginary values of $s = i\omega$, the Laplace transform reduces to the one-sided Fourier transform. The Fourier transform is another integral transform that converts an economic time series $f(t)$ to a complex function $F(i\omega)$:

$$F(i\omega) = \int_0^\infty f(t)e^{-i\omega t}dt \quad \in \mathbb{C} \tag{2-6}$$

In economics, the Fourier transform is used to identify cyclical trends ω in an economic time series, as it decomposes a time series into an infinite sum of sinusoids at different frequencies.[45] Examples of cyclical trends in economics are, for instance, daily and seasonal trends in the electricity market due to increased demand during the day and winter.[65] Similarly, recurrent patterns of expansion and contraction are observed in an economy's output, known as the business cycle.[2].[35]

[1] For a detailed proof, readers are referred to *"Advanced Valuation: Modelling DCF in Continuous Times"*.[49]

[2] The business cycle consists of multiple cycles linked to different economic drivers, respectively known as the Kitchen, Juglar, Kuznets, and Kondratiev cycles

Complex Detrending

Both the DCF method as the Fourier transform can be interpreted as a form of detrending. The DCF method detrends an economic time series into real trends that represent economic discounting. Therefore, we define the real part σ of the Laplace variable s as the real trend.

The Fourier transform detrends an economic time series into economic cycles. As a result, we define the imaginary part ω of the Laplace variable s as the cyclical trend. In engineering, the cyclical trend is defined as the frequency. We can easily derive a cycle's period, which economists are more accustomed to, as it is equal to the reciprocal of the cyclical trend.[35]

For complex values of s, the Laplace transform combines the Fourier transform and the DCF method into one integral transform. We can thus interpret the Laplace transform as the complex detrending of an economic time series into real and cyclical trends. By extension, we define the Laplace variable s as the complex trend.

To explain this further, we visualize the process of complex detrending in Figure 2-2 by breaking it down into three steps:

1. Consider an economic time series $f(t)$, such as a decaying sinusoid (top figure). We detrend the economic time series by multiplying it with an exponential $e^{-\sigma t}$ for each possible real trend σ. Note that because we multiply with minus the complex trend s, increasing trends are located in the left half of the complex plane, and decreasing trends are located in the right half of the complex plane.

2. The Fourier transform decomposes each detrended time series along the vertical line $s = \sigma$ in the complex plane (bottom middle figure) that corresponds to the real trend with which we detrended the economic time series. Thereby, each detrended time series is decomposed into cyclical trends, weighing the complex trends that constitute the signal.

3. Since a complex present value function $F(s)$ is a mapping from a complex plane to another complex plane, a four-dimensional space is needed to visualize $F(s)$. Therefore, we visualize $F(s)$ by plotting the magnitude (bottom left figure) and phase (bottom right figure) for each complex trend s, allowing us to represent the function as two three-dimensional surface plots[3].

The magnitude of a complex present value function $|F(s)|$ shows the weighting of a respective complex trend s in the Laplace response. A large magnitude (e.g., a peak) indicates a strong trend, and a small magnitude indicates a weak trend.

The real part of the function's magnitude $|F(\sigma)|$ indicates a real trend σ in the Laplace response. For a positive real trend, the time series grows exponentially. For a negative real trend, the time series decays exponentially. The growth or decay rate is determined by the magnitude of the real trend $|\sigma|$.

[3] In Equations 2-29 and phase 2-31 are defined as a function of the complex value function $F(s)$. Alternatively, the magnitudes of the real and imaginary components of a complex present value function are plotted.

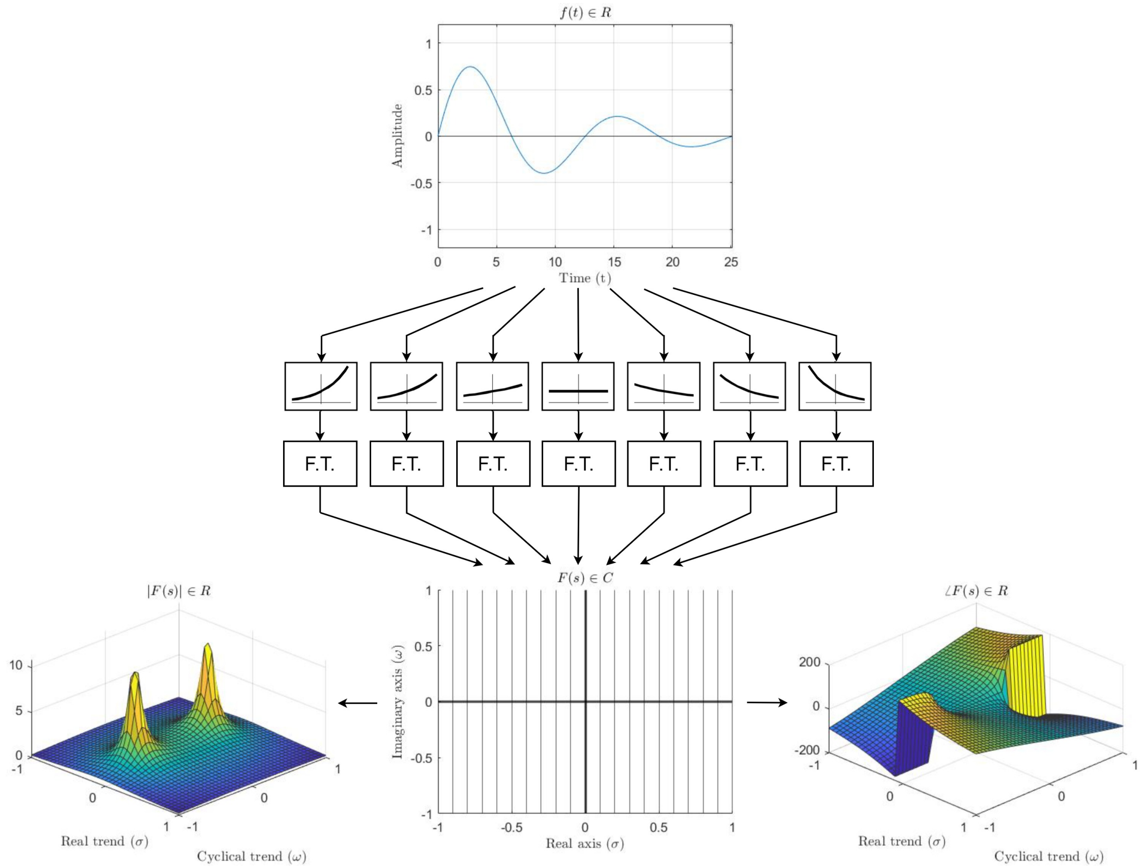


Figure 2-2: Breakdown of complex detrending: For each possible real trend σ, we detrend an economic time series $f(t)$ (top figure) by multiplying it with an exponential $e^{-\sigma t}$. Next, we transform the detrended time series using the Fourier transform (F.T.) and arrange it along the vertical line $s = \sigma$ in the complex plane (bottom middle figure). Last, we visualize the complex present value function $F(s)$ by plotting the magnitude (bottom left figure) and phase (bottom right figure) for each complex trend s. This figure is reproduced from *"Digital Signal Processing"* by *S.W. Smith*.[66]

The imaginary part of the function's magnitude $|F(i\omega)|$ indicates the Laplace response's cyclical trend ω. Because an economic time series $f(t)$ is always a real function, the Laplace response is conjugate symmetric, which is a consequence of the properties of complex numbers. As a result, cyclical trends always occur as a pair of complex conjugate peaks. The magnitude of the cyclical trend $|\omega|$ determines the cyclical trend's frequency.

The phase plot indicates a phase lead (positive phase) or lag (negative phase) of the complex present value function $F(s)$ with respect to a complex trend s. Economically, we associate a phase lead with speculative behavior and a phase lag with reactive behavior. We discuss this in more detail in Section 2-3-7.

Now consider the economic time series $f(t)$ in Figure 2-2. It consists of a cyclical trend and a downwards-sloping real trend. Therefore, we expect to observe a pair of complex conjugate peaks with a real and complex part. Since the real trend is downwards-sloping, the peaks should be located in the left half of the complex plane. We can confirm this is the case from the magnitude plot (bottom left figure) in Figure 2-2. Thus, the Laplace transform effectively detrends an economic time series into its real and cyclical trends, which we can visualize using a magnitude plot in the Laplace domain.

2-2-2 Economic Interpretations of Complex Present Value Functions

Section 2-2-1 demonstrated that DCF method transforms a cash flow into NPV. This proves that the interpretation of an economic time series $f(t)$ changes when transformed to a complex present value function $F(s)$. Also, we argued that the Laplace transform can be applied to arbitrary signals. Therefore, this section establishes the economic interpretations of complex present value functions.

As we design the economic models in this thesis using electrical network theory, we establish the economic interpretations of the complex present value function of a desire and a flow of goods (i.e., a voltage and a current). Moreover, the complex present value functions of a desire and a flow of goods have the same units as a price and a stock, making their economic interpretation intuitive.

The complex present value function of a flow of goods $I(s)$ is equal to the integral of future flows of goods discounted to the present. Because the integral of a flow of goods equals a quantity, we can interpret the complex present value function of a flow of goods as a net present quantity (NPQ). In case the flow of goods equals a cash flow, the Laplace transform is interpreted as the time-adjusted quantity of money (i.e., net present value). This illustrates the intuition behind NPV.[9]

Similarly, the complex present value function of a desire is equal to the integral of the present value of all future desires discounted to the present. The integral of a desire equals a price. In economics, the willingness to pay is used to indicate an agent's reservation price.[71] However, because willingness suggests a future action, we interpret the complex present value function of a desire as the willingness $V(s)$.

Table 2-1 summarizes the economic interpretations of the complex present value functions of a desire $V(s)$ and a flow of goods $I(s)$.[4]

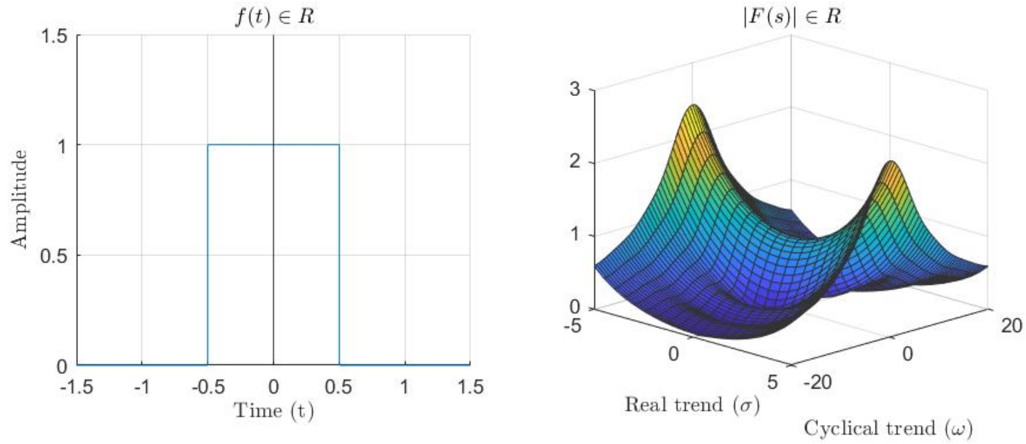
Laplace transform	**Economics**	**Unit**
$\mathrm{I}(s) = \int_0^\infty f(t)e^{-st}\,\mathrm{d}t$	NPQ	$[\frac{\$}{\#}]$
$\mathrm{V}(s) = \int_0^\infty e(t)e^{-st}\,\mathrm{d}t$	Willingness	$[\#]$

Table 2-1: Economic interpretation of the complex present value functions of a desire $V(s)$ and a flow of goods $I(s)$.

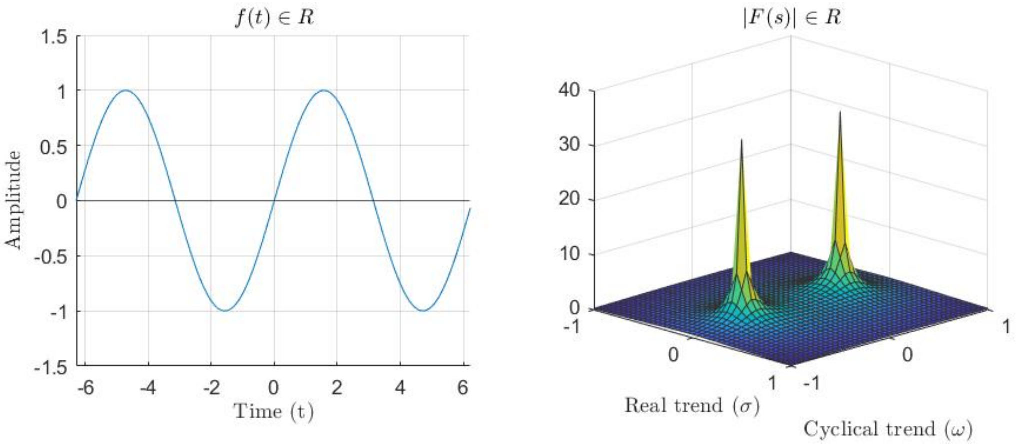
[4]In general, engineers denote Laplace domain variables with the capital letter of the time domain variable. However, this notation can become cumbersome as electrical engineers also use a capital letter for voltage and current. Therefore, we denote a desire using $e(t)$ (analogous to a voltage) and a flow of goods using $f(t)$ (analogous to a current), allowing us to denote the willingness using $V(s)$ and the net present quantity using $I(s)$.

2-2-3 Generalized Uncertainty Principle

In Section 2-2-1, we stated that a signal can be represented in the time and Laplace domain. Because these are alternative representations of the same signal, we cannot possibly visualize the information from both domains at the same time. As a result, a signal cannot be simultaneously localized in the time and Laplace domain with unlimited precision. In signal processing, this limit is known as the Gabor limit[5].[63]



(a) Amplitude/Magnitude plot of an economic shock (unit pulse) in the time and Laplace domain. Since the shock is time-limited, it comprises infinite complex trends.



(b) Amplitude/Magnitude plot of a simplified business cycle (sinusoid) in the time and Laplace domain. As the business cycle is time-unlimited, its Laplace response is band-limited. The Laplace domain efficiently shows that the business cycle comprises a single strictly cyclical trend ω.

Figure 2-3: Graphical example demonstrating the generalized uncertainty principle. The figure demonstrates that no signal can be simultaneously localized in the time and Laplace domain. I.e., if a signal is time-limited, it is band-unlimited, and if a signal is time-unlimited, it is band-limited.

Consider the economic time series $f(t)$ and its complex present value function $F(s)$. We can then determine a duration T_D that completely contains the economic time series $f(t)$. Similarly, we can determine a bandwidth W_B that completely contains the complex trend of

[5] Mathematically, this follows from the Pontryagin duality of the Fourier transform.[60]

the complex present value function $F(s)$. It then follows from Benedick's theorem that:

$$T_D W_B \geq 1 \tag{2-7}$$

More generally, this result is known as the generalized uncertainty principle.[5]

We illustrate the economic implication of this limit using an example. Figure 2-3a visualizes a large shock that describes a hypothetical market crash. The signal has a limited duration (i.e., time-limited), causing the signal's Laplace response to be band-unlimited. I.e., an infinite number of complex trends are needed to describe the signal. As a result, we cannot obtain any information on the signal's trends using Laplace domain analysis.

Now consider Figure 2-3b, which shows a simplified representation of the business cycle. Because the sinusoidal signal consists of a single cyclical trend ω, it is time-unlimited. In the Laplace domain, however, the signal is represented as a complex conjugate peak, demonstrating that the signal comprises a single cyclical trend.

Thus, Laplace domain analysis does not provide insight into time-limited signals, as Laplace response is band-unlimited. Conversely, complex trends can be difficult to identify, especially during limited time intervals, as these are unlimited in time. As a result, we can use Laplace domain analysis to gain further insight into band-limited signals.

2-3 Laplace Domain Analysis of Economic Systems

2-3-1 Economic Systems in the Laplace Domain

In engineering, systems are viewed in terms of their input-output relations. Figure 2-4 illustrates this, where a block diagram of an arbitrary system $h(t)$ is shown with an input signal $u(t)$ and an output signal $y(t)$.

In economics, a system is generally viewed in terms of its exogenous shocks and endogenous variables. The system's input-output relation is referred to as a "transmission", as it describes how a shock is transmitted through an economic system.[41]

In essence, the economic and engineering viewpoints are identical. Therefore, we model an economic system as an engineering system, where input signals are determined exogenously, and output signals are endogenous to the system.

In the time domain, a system's output $y(t)$ can be calculated from the system's input-output relation, given by the impulse response $h(t)$, and input $u(t)$ using the convolution integral:

$$y(t) = \int_0^t h(t-\tau)u(t)d\tau \tag{2-8}$$

However, this operation is mathematically cumbersome as it involves an integral. The previous section demonstrated that signals can be represented in the time and Laplace domain. By transforming the input and output signal using the Laplace transform, we can also represent an economic system in the Laplace domain (see Figure 2-4).

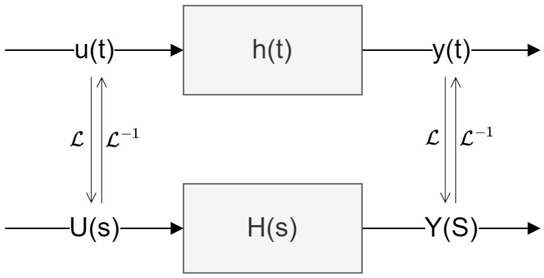


Figure 2-4: Block diagram of an arbitrary system in the time and Laplace domain, where $u(t)$ and $U(s)$ indicate the exogenous input signal, $y(t)$ and $Y(s)$ the endogenous output signal, $h(t)$ the impulse response, and $H(s)$ the transfer function.

Subsequently, we can calculate the system's output $Y(s)$ by multiplying[6] the transfer function $H(s)$ with the input $U(s)$:

$$Y(s) = H(s)U(s) \tag{2-9}$$

By making Laplace domain techniques applicable to economic analysis, we are able to transform the linear differential equations of economic engineering systems into algebraic equations. This simplifies the design and analysis of economic systems.

2-3-2 Economic Interpretations of Transfer Functions

The economic interpretation of a system's transfer function depends on the input-output relation that is modeled. As the economic models in this thesis are designed using electrical network theory, we typically express the system's transfer function in terms of impedance:

$$Z(s) = \frac{V(s)}{I(s)} \tag{2-10}$$

Because willingness $V(s)$ has the units of a price and NPQ $I(s)$ the units of stock, impedance has the same units, except for a factor $[yr]$, as the price inelasticity m. The price inelasticity quantifies the level of demand, which M.B. Mendel interpreted analogous to inductance.[48] Similar to inductance, impedance is a measure of the opposition to electrical flow since impedance is the Laplace domain equivalent of an ideal inductor's inductance.[2] Therefore, we interpret the impedance as a system's inelasticity.

Typically, economists consider a price change to arise from a change in the flow of goods. I.e., the price is the dependent variable, and the flow of goods is the independent variable. Consequently, economists generally consider a system's elasticity instead of its inelasticity. Hence, it is economically more intuitive to analyze a system's admittance $Y(s)= Z(s)^{-1}$, which we interpret as a system's elasticity.

Table 2-2 summarizes the economic interpretations of the impedance $Z(s)$ and admittance $Y(s)$.

[6] This follows from the convolution theorem.[52]

Transfer function	Economics	Unit
$Z(s) = \frac{V(s)}{I(s)}$	Inelasticity	$[\frac{\#^2}{\$}]$
$Y(s) = \frac{I(s)}{V(s)}$	Elasticity	$[\frac{\$}{\#^2}]$

Table 2-2: Economic interpretation of impedance $Z(s)$ and admittance $Y(s)$.

2-3-3 Discount Rates

First-Order System

Consider the system pictured in Figure 2-5. Economically, this system represents an agent that can trade freely with a perfectly inelastic market. I.e., when subjected to an exogenous desire, the agent adjusts its price without affecting the market price. In addition, the agent is subjected to some trade friction in the form of handling costs. As a result of the trade friction, the agent dissipates economic surplus, causing it to reach a state of equilibrium in the long run.[48]

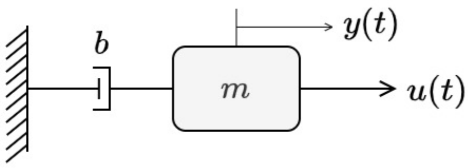


Figure 2-5: Graphic representation of an agent that can trade with a perfectly inelastic market while subjected to an exogenous desire $u(t)$ and trade friction in the form of handling cost. b Specifies the agent's handling cost, m represents the agent's price inelasticity, and $y(t)$ denotes the agent's price.

Because the input is the exogenous desire $u(t)$ and the output is the agent's price $y(t)$, the input-output relation $h(t)$ describes the agent's price dynamics. We identify the agent's price dynamics by simulating an instantaneous impulse input $\delta(t)$, known as an impulse response, that perturbs all complex trends s. As a result, an impulse response completely characterizes the agent's price $y(t)$.

Figure 2-6a plots the agent's price $y(t)$ for varying γ. We observe that the agent's output response is shaped by a discount factor $e^{\gamma t}$, which quantifies the agent's time preference:

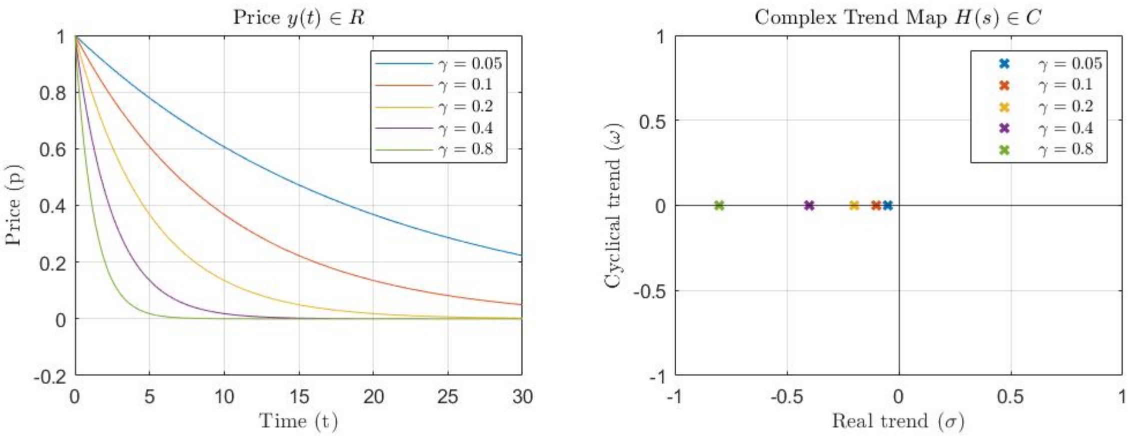
$$y(t) = e^{\gamma t}u(t) \qquad \text{where} \qquad \gamma = \frac{b}{m} \tag{2-11}$$

Note that, in this case, $u(t)$ denotes a unit step instead of the exogenous input. We deduce that for large γ, the agent prefers to receive goods earlier, and for small γ, the agent prefers to receive goods later. Therefore, we interpret γ as the discount rate.

The system's transfer function $H(s)$ can then be derived by dividing the Laplace transform of the agent's price $y(t)$ by the Laplace transform of impulse input $\delta(t)$:

$$H(s) = \frac{Y(s)}{U(s)} = \frac{\mathcal{L}\{e^{\gamma t}u(t)\}(s)}{\mathcal{L}\{\delta(t)\}(s)} = \frac{1}{s+\gamma} \tag{2-12}$$

We observe that for $s = -\gamma$, the transfer function's denominator becomes zero, thus unbounded. In engineering, such complex trends are known as poles. Since we interpreted γ as the agent's discount rate, we can also interpret the agent's pole as the discount rate. Figure 2-6b visualizes the agent's poles in the complex plane for varying discount rates, summarizing the agent's dynamic response using a single point.



(a) The agent's price $y(t)$ for varying discount rates γ.

(b) The agent's complex discount rate for varying discount rates γ.

Figure 2-6: The agent's response, visualized in the time and Laplace domain for varying discount rates γ. The agent's time preference increases as the discount rate increases.

The agent's output response is exponential because the system has a strictly real discount rate. The following example demonstrates that a system with strictly imaginary discount rates displays cyclical behavior.

Undamped Second-Order System

The system in Figure 2-7 again represents an agent that can trade freely with a perfectly inelastic market. Compared to the previous example, this agent can store goods and experiences no trade friction. Therefore, the system never reaches a state of equilibrium but continuously over- and under-stocks its inventory. Economically, this type of agent is analogous to a market maker.[48]

The exogenous desire $u(t)$ is the input, and the agent's inventory stock $y(t)$ is the output. As a result, the input-output relation describes the agent's inventory dynamics. We again simulate an impulse response to identify the agent's inventory dynamics.

Figure 2-8a shows the agent's inventory stock $y(t)$ for varying ω_n. As expected, the agent's output response has a sinusoidal shape, implying the agent never reaches a state of equilibrium.

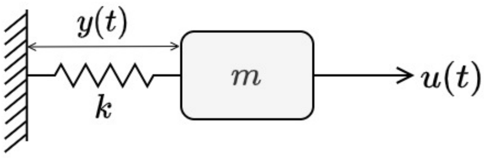


Figure 2-7: Graphic representation of an agent that trades with a perfectly inelastic market while subjected to an exogenous desire $u(t)$ and a convenience arising from the agent's storage. k Specifies the agent's inventory elasticity, m represents the agent's price inelasticity, and $y(t)$ denotes the agent's inventory stock.

$$y(t) = \frac{\sin(\omega_n t)}{m\omega_n} u(t) \qquad \text{where} \qquad \omega_n = \sqrt{\frac{k}{m}} \tag{2-13}$$

From Equation 2-13, we can infer that ω_n determines the amplitude and period of the agent's economic cycles, i.e., for small ω_n the cyclical trend has a long period and a large magnitude, and a large for large ω_n the cyclical trend has a short period and a small magnitude. Since the agent's inventory causes the cyclical behavior, we interpret ω_n as the agent's inventory cycle.

Again, taking the Laplace transform of the agent's inventory stock $y(t)$ divided by the Laplace transform of impulse input $\delta(t)$, we derive the system's transfer function $H(s)$:

$$H(s) = \frac{\mathcal{L}\left\{\frac{\sin(\omega_n t)}{m\omega_n} u(t)\right\}(s)}{\mathcal{L}\{\delta(t)\}(s)} = \frac{1}{m\left(s^2 + {\omega_n}^2\right)} \tag{2-14}$$

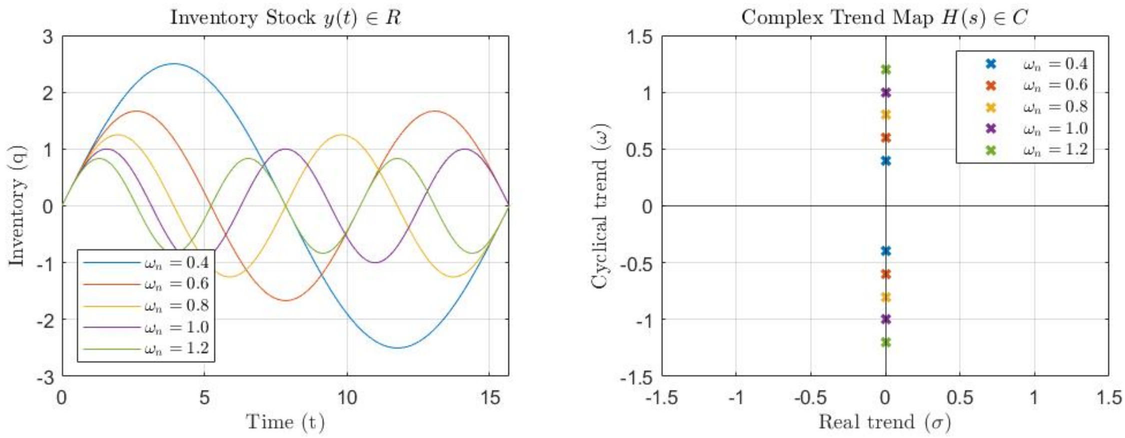
From Equation 2-14, we deduce that the system has two complex conjugate poles at $s = \pm\omega_n i$, which are the agent's complex discount rates. In Figure 2-8b, the agent's discount rates are plotted for varying inventory cycles ω_n. The output response is purely sinusoidal since all complex discount rates are strictly imaginary.

Higher-Order Systems

In general, an economic system can be of an arbitrary order of magnitude, having multiple complex discount rates. Moreover, a system can have multiple inputs and outputs, in which case, $m \times n$ transfer functions $H(s)$ are needed to describe a system with m inputs and n outputs.

In engineering, we typically study LTI systems, such as the previous two examples. These are generally described using a state-space representation. We can easily obtain the system's transfer function from a state-space representation using the following equation:

$$H(s) = C(sI - A)^{-1}B + D \tag{2-15}$$



(a) The agent's inventory stock $y(t)$ for varying inventory cycles ω_n.

(b) The agent's complex discount rates for varying inventory cycles ω_n.

Figure 2-8: The agent's response, visualized in the time and Laplace domain for varying inventory cycles ω_n. The period and amplitude of the agent's output response decrease as the inventory cycle increases.

In case the LTI system is a MIMO system, $H(s)$ becomes a matrix of transfer functions. Each transfer function $H(s)$ can be written as a fraction of two polynomials of the complex trend s.

$$H(s) = \frac{N(s)}{D(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0} \tag{2-16}$$

If we then factor the polynomials in the numerator and denominator of the transfer function, we can easily identify the system's complex discount rates p_i (for $i \in 1, \ldots, n$):

$$H(s) = \frac{b_m}{a_n} \frac{(s - z_1)(s - z_2) \ldots (s - z_{m-1})(s - z_m)}{(s - p_1)(s - p_2) \ldots (s - p_{n-1})(s - p_n)} \tag{2-17}$$

Using a partial fractional expansion, we can then rewrite Equation 2-17, a sum of first-order systems for each unique pole and second-order systems for each repeated pole. Note that a partial fractional expansion can be applied to a proper transfer function, which is generally the case for physical systems. Thus, any higher-order LTI system can be decomposed into a combination of first- and second-order systems.

These examples show that a system's poles are analogous to an economic system's complex discount rates. Moreover, we demonstrated that similar to the economic time series in Section 2-2, the real part σ of a pole gives a system's economic discounting, and the imaginary part ω of a pole gives a system's economic cycles.

2-3-4 Transmission-blocking Rates

In engineering, the complex trends s for which the transfer function's nominator (see Equation 2-17) becomes zero are defined as zeros z_j (for $j \in 1, \ldots, m$). By extension, a system's transfer function is also zero:

$$\lim_{s \to z_i} H(s) = 0 \tag{2-18}$$

Economically, a system does not transfer any economic surplus for complex trends $s = z_j$, which means the system is infinitely inelastic. Therefore, we interpret a zero as a transmission-blocking rate.[37][41]

If a complex discount rate and transmission-blocking rate overlap, the transmission-blocking rate cancels the effects of the complex discount rate. This is known in engineering as pole-zero cancellation.[64] In Section 6-5-2, we see an example of pole-zero cancellation.

2-3-5 Complex Trend Map

In engineering, a system's poles and zeros are visualized using a pole-zero map in which the poles are indicated using an "x" and the zeros using an "o". Similarly, we can visualize the economic analogs of a pole and zero, which we introduced in Sections 2-3-3 and 2-3-4, using a complex trend map[7]. This section establishes the economic insights we can obtain using a complex trend map.

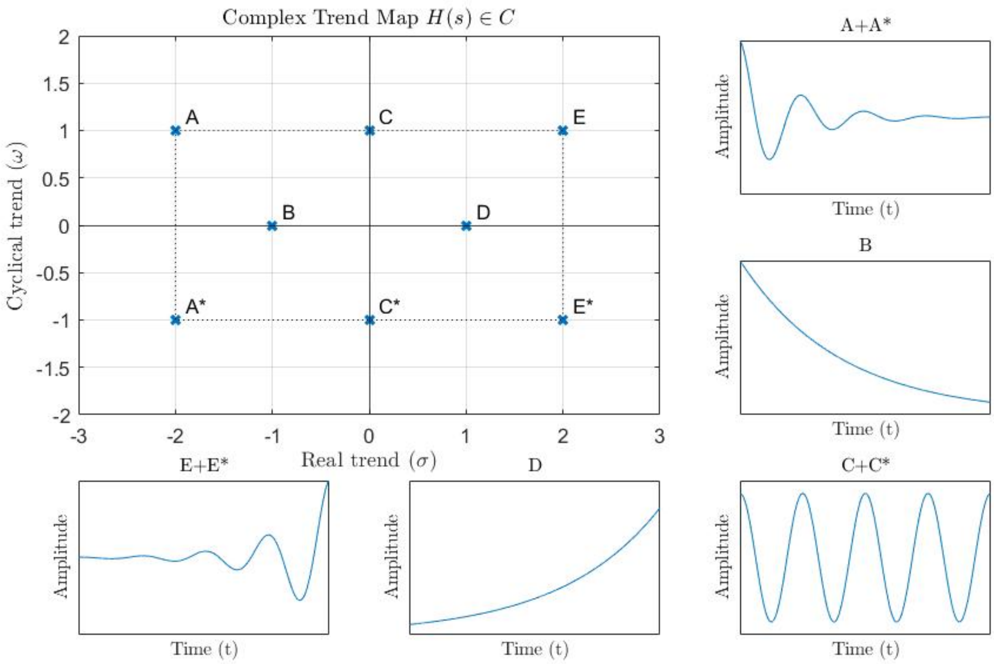


Figure 2-9: The natural response of three pairs of complex conjugate discount rates and two single complex discount rates. The real part σ of the complex discount rate determines the system's economic discounting, and the imaginary part ω of the complex discount rate determines the system's economic cycles.

[7]Examples of complex trend maps can be found in Figures 2-6b and 2-8b.

Natural Response

We can determine the system's unforced time domain response from a complex trend map. This is known in engineering as a system's natural response, which can be interpreted in economics as a system's short-run (i.e., transient) response independent of any exogenous shocks. The natural response can be written as a function system's complex discount rates p_i for initial conditions C_i:

$$y_h(t) = \sum_{i=1}^{n} C_i e^{p_i t} \tag{2-19}$$

Figure 2-9 plots the natural response of three pairs of complex conjugate discount rates and two single complex discount rates. In correspondence with the examples in Section 2-3-3, we observe that the natural response of a system with strictly real discount rates (e.g., B and D) comprises only real trends, and the natural response of a system with strictly imaginary discount rates (e.g., C+C*) comprises only cyclical trends. A system with complex discount rates (e.g., A+A* and E+E*) comprises real and cyclical trends.

The natural response of a higher-order system is equal to the sum of the natural responses of all unique and repeated complex discount rates, i.e., first- and second-order systems. Using a complex trend map, we can thus efficiently determine the natural response of higher-order economic systems.

Stability

From Figure 2-9, we observe that a complex discount rate's natural response in the right half of the complex plane grows exponentially. Conversely, the natural response of a complex discount rate located in the left half of the complex plane decays exponentially. Since the natural response of a system is equal to the sum of the natural response of all complex discount rates (see Equation 2-19), only the response of a system with all complex discount rates located in the left half of the complex plane can be bounded. Given that the input is also bounded, such systems are defined as stable. All other systems are defined as unstable, with the exception of systems with strictly imaginary discount rates, which are defined as marginally stable.

Economically, stability guarantees the existence of a competitive equilibrium. We will substantiate this analogy in Section 3-4-2. Moreover, the decay rate of the natural response increases for complex discount rates located further into the left half of the complex plane. This implies that complex discount rates located close to the imaginary axis cause prices to be sticky. Conversely, the prices of a system with all complex discount rates located near the point $s = -\infty$ are flexible.

Forced Response

If an exogenous input acts on a system, a system is said to be forced. The forced response $y_f(t)$ of an LTI system is defined as the sum of the natural response $y_h(t)$ and the particular solution

$y_p(t)$ (see Equation 2-20). The particular solution $y_p(t)$ is associated with the system's long run (i.e., steady state) response and depends on the exogenous input acting on the system.

$$y_f(t) = y_h(t) + y_p(t) \tag{2-20}$$

A system's forced response is influenced both by a system's complex discount rates and a system's transmission-blocking rates. If we substitute Equation 2-17 into Equation 2-9, we find:

$$Y(s) = \frac{1}{D(s)} N(s)U(s) \tag{2-21}$$

where,

$$N(s) = b_m (s - p_1)(s - p_2) \ldots (s - p_{n-1})(s - p_n) \tag{2-22}$$
$$D(s) = a_n (s - z_1)(s - z_2) \ldots (s - z_{m-1})(s - z_m) \tag{2-23}$$

Hence, the system's transmission-blocking rates can be interpreted as a pre-filter with which we multiply the input signal. As a result, two systems with identical complex discount rates can have different forced responses if the system's transmission-blocking rates differ. Therefore, both the complex discount rates and transmission-blocking rates must be taken into account when determining the forced response of a complex trend map.

2-3-6 Bullwhip Effect

In this section, we establish the economic analog to resonance. To achieve this, we introduce a model of a damped second-order system. The agent, visualized in Figure 2-10, is identical to the undamped second-order system presented in Figure 2-7 apart from the addition of trade friction (damper). As a result of the trade friction, the agent reaches a state of equilibrium in the long run while it continuously over- and under-stocks its inventory.[48]

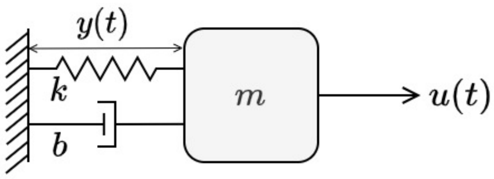


Figure 2-10: Graphic representation of an agent that trades with a perfectly inelastic market while subjected to an exogenous desire $u(t)$, trade friction in the form of handling cost, and convenience arising from the agent's storage. b Specifies the agent's handling cost, k represents the agent's inventory elasticity, m determines the agent's price inelasticity, and $y(t)$ denotes the agent's inventory stock.

Following the same procedure to determine as in Section 2-3-3, we derive the agent's inventory dynamics, which is given by the following transfer function:

$$H(s) = \frac{1}{m\left(s^2 + \frac{b}{m}s + \frac{k}{m}\right)} \tag{2-24}$$

We can rewrite the above transfer function (except for a scaling factor k) as a function of the inventory cycle ω_n and discount propensity[8] ζ:

$$\frac{H(s)}{k} = \frac{{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2} \qquad \text{where} \qquad \zeta = \frac{b}{2\sqrt{mk}} \tag{2-25}$$

By rewriting the system's transfer function as in Equation 2-25, we essentially performed a coordinate transformation from Cartesian to polar coordinates. This is illustrated by Figure 2-11a, which plots the system's complex discount rates for varying discount propensities. The distance of the complex discount rate to the origin is determined by the inventory cycle ω_n and the angle with the negative real axis by the discount propensity ζ.

The discount propensity ζ allows us to classify this agent into two categories. Systems with a discount propensity between $1 > \zeta \geq 0$ are classified as under-damped, and systems with a discount propensity between $\zeta \geq 1$ are over-damped. The associated complex discount rates of under- and over-damped systems are given by:

$$p_1, p_2 = \begin{cases} -\zeta\omega_n \pm \omega_n\sqrt{1-\zeta^2} & \text{for} \quad 1 > \zeta \geq 0 \\ -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1} & \text{for} \quad \zeta \geq 1 \end{cases} \tag{2-26}$$

For a discount propensity $\zeta = 0$, the system is identical to the undamped second-order system in Section 2-3-3. For $\zeta > 0$, the system is under-damped and discounts cyclically, analogous to a broker-dealer in economics. As the discount propensity increases, the system is the system's economic discounting increases while the system's cyclical behavior decreases. For $\zeta \geq 1$, the system is over-damped and analogous to a hyperbolic discounter as the system's complex discount rates are strictly real.[48]

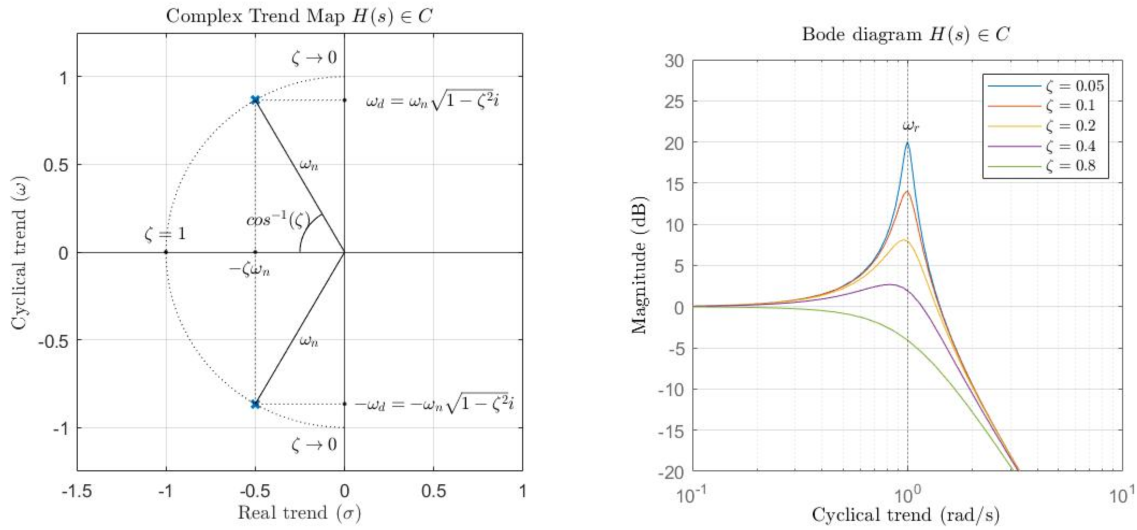
We have shown that the inventory levels of a second-order system can oscillate as the agent continuously under and over-stocks their inventory. If the agent is under-damped, the amplitude of oscillations can increase significantly when subjected to a periodic exogenous desire. This dynamic phenomenon is known as resonance. Economically, resonance can be interpreted as the Bullwhip effect, which is observed in supply chains.[22][54] In Section 4-4-4, we develop a model of a supply chain network that substantiates this analogy.

The cyclical trend ω around which resonance occurs is defined in engineering as the resonance frequency ω_r [51]:

$$\omega_r = \omega_n\sqrt{1 - 2\zeta^2} \tag{2-27}$$

We interpret the resonance frequency as the Bullwhip frequency. We can deduce that only systems with discount propensities larger than $\zeta < 1/\sqrt{2}$ cause the Bullwhip effect. In the complex trend map, these are the complex discount rates with $\cos^{-1}(\zeta) > \frac{\pi}{4}$.

[8] M.B. Mendel introduced this terminology in his paper *"The Newtonian Mechanics of Demand"*.



(a) The agent's complex trend map for varying discount propensities ζ.

(b) The agent's magnitude plot for varying discount propensities ζ.

Figure 2-11: The agent's Laplace response, visualized using a complex trend map and Bode plot for varying discount propensities ζ. In this case, the inventory cycle ω_n was chosen equal to one. This figure is reproduced from *"Feedback Control of Dynamic Systems"* by *G.F. Franklin, J.D. Powell, and A. Emami-Naeini.*[23]

Approximately, the Bullwhip frequency is equal to the imaginary part of an under-damped system's complex discount rate, which is known as the damped natural frequency ω_d in engineering:

$$\omega_d = \omega_n\sqrt{1-\zeta^2} \tag{2-28}$$

Therefore, we can use the damped natural frequency ω_d as an indication of the Bullwhip frequency ω_r in a complex trend map.

2-3-7 Bode Plot

Instead of visualizing as system's complex discount rates and transmission-blocking rates, we can represent a transfer function $H(s)$ as the function of the magnitude $|H(s)|$ and phase $\phi(s)$:

$$H(s) = |H(s)|e^{j\phi(s)} \tag{2-29}$$

where,

$$|H(s)| = \sqrt{\mathfrak{Re}\{H(s)\}^2 + \mathfrak{Im}\{H(s)\}^2} \tag{2-30}$$

$$\phi(s) = \tan^{-1}\left(\frac{\mathfrak{Im}\{H(s)\}}{\mathfrak{Re}\{H(s)\}}\right) \tag{2-31}$$

A Bode plot visualizes a system's magnitude $|H(s)|$ and phase $\phi(s)$ responses to a sinusoidal input signal for all cyclical trends (i.e., $\omega \in [-\infty, \infty]$). The magnitude response indicates how much the input signal is attenuated or amplified, generally denoted in dB. The phase response indicates how much the output signal leads or lags the input signal. In the case of a phase lead, an economic system preemptively reacts to an exogenous input. This behavior is similar to speculators in economic markets. Therefore, a phase lead can be interpreted as speculative or anticipatory behavior. In the case of a phase lag, an economic system reacts with a delay to an exogenous input. Therefore, a phase lag can be interpreted as reactive behavior.

The specific interpretation of the magnitude and phase, however, varies depending on the economic interpretation of the transfer function. If, for instance, a transfer function is expressed as impedance $Z(s)$, the Bode plot indicates the elasticity's magnitude and phase response as a function of the cyclic trend ω.

Bode plots are especially suited for visualizing the Bullwhip effect. For example, Figure 2-11b depicts the magnitude plot of the damped second-order system presented in Section 2-3-6 for varying discount properties ζ. A peak emerges, which increases in magnitude as the discount propensity ζ approaches zero, indicating the amount of goods the agent stores.

2-4 Discussion

The algebraic simplification of differential equations enables the design of bottom-up economic models in the Laplace domain. Chapter 4 takes advantage of this by modeling large interconnections of two-port networks using only multiplications and additions. This provides a agent-based modeling framework for designing bottom-up macroeconomic models. Essential, however, is that the system dynamics are linear. Therefore, this chapter motivates the modeling of macroeconomic systems as LTI systems, enabling the efficient design and analysis of macroeconomic systems.

The complex trend map demonstrated our ability to analyze discounting and economic cycles within an economic system. Chapters 5 and 6 utilize complex trend maps to analyze the economy, allowing us to identify market-specific discount rates and the economy's business cycles.

Furthermore, Bode plots prove to be a valuable tool for visualizing a system's elasticity function $Z(s)$. Chapter 4 uses Bode plots to analyze the elasticities of various network topologies, allowing the identification of the Bullwhip effect in supply chain topologies.

Despite the recognition of the generalized uncertainty principle in econometrics, its importance is poorly understood.[67] As a result, many errors are still made in practice. By extending the generalized uncertainty principle to the analysis of economic models, we are able to motivate, depending on the scenario, whether to analyze a system in the time or Laplace domain. We demonstrate the practical applicability of this limit using the macroeconomic model in Chapter 6.

In conclusion, these applications demonstrate how Laplace domain techniques make the analysis and design of macroeconomic models more efficient. Moreover, they demonstrate that Laplace domain analysis provides further insight into economic behavior not apparent in the time domain.

Chapter 3

Economic Engineering for Macroeconomics

3-1 Introduction

This chapter establishes the economic-engineering analogs to key macroeconomic principles. Conversely, the matching of macroeconomic principles helps comprehend what limitations we implicitly introduce when modeling macroeconomic systems. To achieve this, we build on the economic-engineering foundations laid down by M.B. Mendel in his paper *"The Newtonian Mechanics of Demand"*. This results in a price-dynamic framework analogous to Newtonian mechanics, which we can use to design macroeconomic models in subsequent chapters.

The chapter is structured as follows: Section 3-2 defines a stock space analogous to a physical space. Using a stock space, we can kinematically describe agents. Section 3-3 explains how agents are modeled in macroeconomics, introducing principles such as aggregate demand, homogeneity, and price takers. We discuss the economic engineering analogs to each of these principles. Having kinematically defined an agent, Section 3-4 formulates an agent's price dynamics. We address the difference between sticky and flexible prices and examine the conditions for equilibrium. Section 3-5 presents how we can model important macroeconomic principles, such as the substitution effect, production, and exogenous effects, using economic engineering. We conclude with Section 3-6 that discusses the benefits of the economic-engineering approach for macroeconomic modeling.

3-2 Defining a Macroeconomic Space

Economic engineering defines a physical space analogous to a stock space, where each dimension corresponds to an account in economics. An account keeps track of one type of fungible good, referred to as a commodity. Since an economy comprises many types of commodities, the stock space describing the economy consists of equal accounts as fungible goods.[48]

For obvious reasons, keeping track of all goods within an economy is unrealistic. To solve this dimensionality problem, macro-economists often define generalized types of goods to limit the total number of goods in the economy. For instance, to keep track of the price level in an economy, the consumer price index (CPI) has been defined. It is calculated by averaging the price of a basket of goods over several years, quantifying the approximate price level for consumer goods in an economy. Likewise, economists use the interest rate spread between 10-year government and corporate bonds as a generalized market price for financial goods.[43] Using this approach, the stock space can be reduced to only a small number of accounts.

Alternatively, the stock space of an economy could be modeled using $\mathcal{R}^{n+1}$ accounts. Where n accounts span all relevant baskets of goods essential to modeling an economy, the last account represents an aggregated variable spanning all unmodeled accounts.[48]

3-2-1 Types of Goods

Commodity vs Capital Goods

In this section, we have established a stock space for fungible goods. However, in macroeconomics, many goods, such as labor, are defined as capital goods. Capital goods differ from commodity goods, as capital goods produce or yield commodity goods. The production of commodity goods is analogous to a rotation, which is discussed in greater detail in Section 3-5-2. While rotations are kinematically described differently from linear motion, we can extend the stock space by including one account for each capital good to model a rotation. This extension allows us to capture the interactions between capital and commodity goods.

Substitution vs Complementary Goods

In engineering, physical spaces are typically defined as orthonormal. For an orthonormal space, the mass values are the same in all directions. In economic engineering, this would result in a stock space for which all price cross-elasticities would be zero since a mass is analogous to price inelasticity.

In economics, the price elasticities of goods are typically not zero. A distinction is made between substitution goods and complementary goods. Consumers regard substitution goods as interchangeable, e.g., pencils and pens. If the price of pencils increases, consumers will buy a pen instead. As a result, the price cross-elasticity of two substitution goods is positive. Complementary goods are consumed in combination and are thus considered interdependent goods. An example of two complementary goods is a pencil and an eraser. The demand for erasers will decrease as the price of pencils increases. Therefore, the price cross-elasticity of two complementary goods is negative.[62]

Therefore, contrary to physical spaces, stock spaces are generally not orthonormal but mathematically equivalent to a general differentiable manifold.[48] Using economic engineering, we can model the substitution goods and complementary goods analogous to gyroscopic effects in mechanical systems.

3-3 Macroeconomic Agents

3-3-1 Lumped Agents

A notable difference when comparing macroeconomic to microeconomic theory is that macroeconomists consider agents as representative agents. Typically, macroeconomic models are populated by only a few agents, such as households or firms, that are lumped representations of agents performing an identical role. Because of this simplification, macroeconomists inherently introduce various generalizations and assumptions to their models. However, given the number of agents is large enough, we can argue these are reasonable from an economic engineering viewpoint.

Aggregate Demand

In his paper, M.B. Mendel describes how aggregating the demand of multiple agents results in the aggregate demand of a multi-agent system while preserving the kinematic description for a single particle.[48] Hence, we can model the price elasticity of a lumped agent as the inverse reduced mass of all agents. Respectively, mass and mass velocity centers become the center of demand and aggregate quantity demanded. Thus, analogous to macroeconomic literature, we can simplify a multi-agent system to a single lumped agent using the economic-engineering approach.[71]

Homogeneity

Lumped agents are generally assumed to be homogeneous, implying they are all identical and have the same behavior. In reality, individual agents display heterogeneous behavior while appearing homogeneous on a macro level. In economic engineering, this is equivalent to how large numbers of quantum particles can be approximated by classical mechanics on a macro scale.[47] The homogeneity assumption is a valid approximation in macroeconomic analysis, as it allows one to focus on macro-phenomena without modeling individual agents, reducing the computational complexity.

Price Takers

A price taker is an agent who accepts the price offered in the market as it cannot influence the market price through its actions.[71] Given that the number of agents is sufficiently large, each agent makes up only a tiny portion of the market share. Consequently, by offering goods at a discount or buying goods at a premium, the effect on the market as a whole is negligible. Therefore, we conclude that all agents are approximately price takers, given that the market consists of many agents.[26] By extension, a lumped agent can also be modeled as a price taker. Following economic-engineering theory, a price taker can be modeled using strictly passive elements. This substantial result dramatically reduces the complexity of economic-engineering models.

3-3-2 Rational Expectations

In macroeconomics, rational expectations model how agents' expectation formation impacts decision-making, such as investing, saving, and spending. The theory stipulates that agents form expectations consistent with a model's predictions.[43] Given that all agents have perfect information, this implies that agents behave rationally since their expectations are based on unbiased and accurate predictions of future events. Therefore, agents maximize their utility function. In the case of a producer, this leads to profit maximization, and in the case of a consumer, utility maximization.[71]

Since lumped agents are modeled using strictly passive energy-storing elements, we can formulate a Lagrangian specifying the agent's disutility function. The disutility function, known as an agent's running cost, is defined as the direct costs minus the potential surplus. Classical mechanics then stipulates that the trade activity of an agent minimizes the time integral of its running cost, i.e., minimizing an agent's period cost. This is known as the stationary action principle or Hamilton's principle.[47]

Since minimizing disutility is equivalent to maximizing utility, we induce that the stationary action principle is analogous to the principle of utility maximization. Economically, utility maximization can only transpire if an agent behaves rationally. Therefore, the hypothesis of rational expectations permits us to model agents having a Lagrangian disutility function, which they minimize.

The traditional Lagrangian formulation is only defined for conservative systems. However, alternative economic engineering approaches extend the Lagrangian framework to incorporate energy-dissipating elements while adhering to the stationary action principle.[39] Consequently, the theory of rational expectations supports modeling economic agents using a Lagrangian disutility function.

3-4 Price Dynamics

3-4-1 Sticky vs Flexible Prices

The distinction between flexible and sticky prices is an essential theoretical divide between the neoclassical and Keynesian schools of thought. Neoclassical economists argue that prices are flexible and change instantaneously in response to market conditions. Keynesians argue that prices can be sticky and slow to adjust to changes in supply and demand.[58] As a result, the distinction between sticky and flexible prices has led to a dichotomy in the development of macroeconomic models for the short and long run.[43]

In macroeconomic literature, the consensus is that prices are generally stickier in the short run and more flexible in the long run.[43][62] Economic engineering shows by deriving a price-dynamic framework that prices are both sticky and flexible. In the short run, agents are subjected to inefficiencies such as trade friction, resulting in prices adjusting slowly. In the long run, prices are flexible as the forces of demand and supply adjust to reach a new equilibrium price. As a result, economic engineering presents an approach to unify macroeconomic models for both the short and long run. In addition, it offers insight into what factors contribute to prices adjusting slowly in the short run and price stabilizing in the long run.

3-4-2 Equilibrium Conditions

As M.B. Mendel established in his paper, various equilibrium forms can be derived using economic-engineering theory. Because economic-engineering models are spanned by two power conjugate variables, the stock q and price p, we can define two types of clearing: physical clearing and price clearing.[48]

In an analogous electrical circuit, physical clearing occurs in a node and price clearing in a mesh. However, electrical circuits produce a dynamic response regardless. Therefore, contrary to some economic theories, either price clearing or physical goods clearing are insufficient conditions to guarantee an equilibrium.[31]

Logically, both clearing conditions must be satisfied, resulting in the net change in price $\dot{p}$ and stock $\dot{q}$ being zero. This can only occur if all available surplus is dissipated since otherwise, any imbalances would be absorbed by a change in the price or stock. In mechanics, the dissipation of surplus is postulated by the second law of thermodynamics, analogous to Adam Smith's invisible hand.[48] As a result, the equilibria of economic-engineering models are, by definition, stable and analogous to a competitive equilibrium in economics.

If an equilibrium exists, all price and stock variables, known as state variables, approach zero as time goes to infinity. In dynamical systems theory, this is known as the steady-state response. Therefore, all dissipative economic-engineering models converge to an equilibrium in the long run. The short run, known as the transient response, describes out-of-equilibrium behavior. As a result, the equilibrium conditions for economic-engineering models allow prices to be sticky in the short run and flexible in the long run. [48]

In macroeconomic literature, economists distinguish between partial and general equilibrium.[44] Partial equilibrium implies a distinct market satisfies the equilibrium conditions, whereas general equilibrium implies that all markets are in a state of partial equilibrium. In Section 3-2, we established how different economic accounts can be coupled because of the substitution effect. Therefore, a disequilibrium in one market excludes the whole economy from reaching equilibrium. Conversely, if all markets are in a state of competitive equilibrium, the entire economy is in a state of general equilibrium. Hence, similar to a single market, general equilibrium is postulated by the invisible hand, which guarantees both clearing conditions are satisfied and all state variables approach zero as time goes to infinity.[48]

Perfect Competition

In the case of physical goods clearing, economic-engineering theory stipulates a clearing price p^* can be obtained that is the same for all goods supplied and demanded.[31] Interestingly, if we associate each flow of goods supplied or demanded with an agent, clearing physical goods is analogous to agents operating in perfect competition.

Economically, perfect competition can only exist given several assumptions: all agents are price takers, goods are homogeneous, and all agents have perfect information.[26] In Section 3-3-1, we specified under what conditions agents are considered price takers. The homogeneity of goods is guaranteed since we defined all commodity goods as fungible goods (see Section 3-2). Last, all information is perfect because of the no-arbitrage condition, which is analogous to the Kirchhoff mesh law.[31] Hence, given that all agents are price takers, physical goods clearing guarantees perfect competition.

3-5 Macroeconomic Theory

3-5-1 Substitution Effect

C. Hutters shows how mutual inductance can be used to model the substitution effect. While we previously exemplified the substitution effect using a gyroscope, mutual inductors can transfer energy between topologically disconnected circuits due to the magnetic coupling between two inductors. This magnetic coupling is analogous to price coupling in economics. Hence, the analogy to mutual inductance provides more intuitive reasoning as to why a price change can adjust the flow of goods in topologically disconnected markets. Given the significance of the substitution effect in macroeconomics, electrical network theory provides a practical framework for modeling macroeconomic systems.[31]

3-5-2 Production

Macroeconomic theory emphasizes the role of capital in modeling an economy's growth. Capital produces goods and services, determining the total economic output. This process is described by a production function. Although many production functions exist, e.g., Cobb-Douglas production function, general production theory states that firms use labor and capital (and sometimes land) to produce goods and services.[68][43] In this case, labor and capital are considered capital goods and goods and services commodity goods. Both the amount and productivity of capital influence the total economic output. Economic growth is, therefore, attributed to an increase in the capital stock or capital productivity.[27]

In economic engineering, we model production as a rotation around a fixed axis. The configuration space of a rotational motion is the plane that is normal to the axis of rotation. Therefore, a physical space of dimension $\mathcal{R}^2$ is needed to describe a rotational motion. Following Section 3-2, one account corresponds to the commodity good, and one corresponds to the capital good. The rotational mapping then is analogous to producing commodity goods through capital goods.[47]

A rotation has been visualized in Figure 3-1. The q_0 axis represents the capital account, and the q_1 axis the commodity account. The flow of commodity goods f is a function of the capital stock K and the yield ω:

$$f = K\omega \tag{3-1}$$

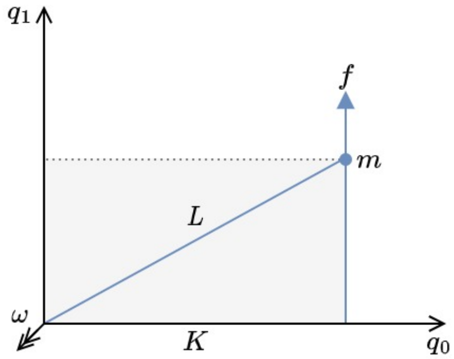


Figure 3-1: Visualization of a shear rotation. The mass m rotates perpendicular to the capital axis q_0, conserving the capital stock K. The process yields ω a flow of commodities f. The shaded surface gives the value of capital L.

Analogous to economic theory, the flow of commodity goods increases by either raising the capital stock or the yield of the process. Moreover, economic theory states that capital derives value from the goods it produces. Therefore, the value of capital should be zero if the capital stock or the capital yield were zero. To show this is the case, we write the value of capital, given by the angular momentum L, as a function of the capital stock K and the yield ω:

$$L = Kp \tag{3-2}$$
$$= Kmf \tag{3-3}$$
$$= K^2 m\omega \tag{3-4}$$
$$= I\omega \tag{3-5}$$

From Equation 3-4, we can see that the value of capital is indeed zero if either the capital stock or the yield is zero. More generally, however, the value of capital is written as a product of the yield ω and yield sensitivity I analogous to the moment of inertia. The yield sensitivity is, in turn, a function of the price inelasticity m and capital stock K.

There exist three forms of rotations, namely, Elliptical, hyperbolic, and shear rotations.[47] Most capital goods used in macroeconomics are classified as shear rotations since the capital stock K is preserved. For instance, the labor force is invariant in the short run and only provides labor services. Only in the very long run can the labor force increase due to, for instance, immigration or population growth. These effects are, however, independent of the labor services provided.

Adiabatic Invariance

Euler's second law of motion (see Equation 3-6) states that torque equals the time rate of change in angular momentum. Thereby, it establishes a price-dynamic framework for capital goods. Contrary to Newton's second law of motion, which applies to rectilinear motion, rotations are nonlinear if the capital stock changes over time ($\dot{K} \neq 0$):

$$\begin{aligned}
\tau &= \dot{L} && \text{(3-6)}\\
&= 2mK\dot{K}\omega + mK^2\dot{\omega} && \text{(3-7)}
\end{aligned}$$

Maintaining the linearity of the price-dynamic framework is desirable as it allows us to model macroeconomic systems using linear systems theory, which is more widely applicable and provides access to a wide range of analytical techniques. As we established in the previous section, however, the increase in capital goods is a critical factor in driving economic growth. Hence, modeling growth dynamics is challenging using economic engineering.

If, however, the change in capital is small with respect to time, the time rate of change of capital is approximately zero ($\dot{K} \approx 0$). This is known as adiabatic invariance in engineering. As a result, Equation 3-7 simplifies to linear relation:

$$\tau \approx mK^2\dot{\omega} = I\dot{\omega} \tag{3-8}$$

Such as in the above example for the labor market, the capital stock changes relatively slowly compared to the price dynamics. Therefore, we assume that macroeconomic systems are adiabatically invariant systems. This allows us to determine the value of capital goods and model production while conserving the linearity of the price-dynamic framework. Moreover, we could extend the model by varying the capital stock over time. As a result, we could potentially use economic engineering to develop a macroeconomic model in the short, long, and very long run (growth dynamics).

3-5-3 Exogenous Effects

In macroeconomics, economic shocks or stochastic effects are modeled as external inputs. In engineering, external inputs are used to model all exogenous behavior. Consequently, economic engineering uses external sources to model the exogenous effects of macroeconomic models. Since a desire (i.e., economic force) and a flow of goods are defined as a power conjugate pair, one can model two types of external sources. A flow source provides a constant flow of goods and behaves as a perfectly inelastic supply. An effort source provides a constant desire and behaves as a perfectly elastic supply. Likewise, a sink[1] can be used to model a demander. Additionally, one can use stochastic processes to model an external source to introduce stochastic effects.[31]

3-6 Discussion

The price-dynamic framework of economic engineering, along with the various economic-engineering analogs to macroeconomic principles, enables us to design the macroeconomic

[1] A sink is identical to a negative source. More generally, however, we model an external input as a source which can also take on negative values.

systems as linear time-invariant (LTI) systems. Crucially, we can use the numerous techniques in the engineering literature for the analysis, design, control, and visualization of LTI systems for economic systems. Specifically, it allows us to design and analyze economic systems using the Laplace domain techniques presented in Chapter 2.

Particularly for macroeconomic modeling, an advantage of the price-dynamic framework is that it presents a unified approach for modeling the economy in the short and long run. Additionally, we can include growth dynamics by modeling production using rotations, allowing us to simulate the economy in the very long run. This allows us to unify multiple economic time horizons into a single macroeconomic model.

The extension of the economic engineering analog to the electrical domain enables the use of circular flow models as a basis for macroeconomic models. This provides a structural modeling framework that facilitates the applicability of the economic-engineering approach for macroeconomic modeling. Also, economic engineering facilitates the design of large-scale macroeconomic models by leveraging electrical network theory.[31]

However, building upon underlying macroeconomic principles exposes the model to criticisms from macroeconomic literature. Engineers, tend to judge a model by its practical applicability rather than the exactness of the physical approximation. As a result, it is common engineering practice to simplify complex systems. For example, maintaining the linearity of the price-dynamic framework during the modeling of production is a result of this engineering practice.

Chapter 4

Bottom-up Economic Networks

4-1 Introduction

The many assumptions that underlie the top-down structural models motivate a bottom-up agent-based modeling (ABM) approach. By modeling individual agents and agent interactions within macroeconomic models, ABM is able to reconstruct macro-phenomena from microeconomic interactions. Therefore, ABM does not rely on the generalizations and assumptions of top-down structural models, allowing them to non-linear and emergent behavior in economic models.

In this chapter, we use two-port network theory to efficiently design and analyze bottom-up economic networks in the Laplace domain. By analyzing the various networks in the Laplace domain, we are able to identify how different topologies affect an economic network's dynamic response. This ABM framework was developed by C. Hutters in the paper *"Modular Economic-Engineering Agent-Based Models as N-port Networks"*. For a more detailed description of the approach, readers are referred to this paper.[32]

To emphasize the versatility of the ABM framework, rather than deriving a specific model, the chapter outline is more general than in Chapters 5 and 6. The chapter is structured as follows: Section 4-2 motivates how two-port network theory, in combination with a Laplace domain representation, simplifies the design of economic models. Section 4-3 models an economic agent using a two-port network and analyzes its response using a Bode plot. Section 4-4 proceeds by discussing five different types of agent interconnections, introducing their economic analogs, and analyzing their Laplace response. Section 4-5 demonstrates, using two examples, how we can design complex economic networks by interconnecting multiple agents. Section 4-6 concludes by discussing the key insights we have attained using the ABM framework.

4-2 Two-Port Networks

In electrical network theory, two-port networks are used to simplify parts of electrical circuits into a black-box model. By doing so, one can analyze the input-output behavior of a network without having to solve for every internal voltage and current. Therefore, two-port networks are used to model the macro-level behavior of a system. This section presents two-port network theory as a ABM framework for bottom-up macroeconomic models. Since macroeconomics also studies the macro-level behavior of the economy, the loss of interpretability caused by using two-port networks is irrelevant.

Bottom-up modeling approaches typically result in complex macroeconomic models, which makes the complexity of designing these models a considerable challenge. As discussed in Section 2-3, transfer functions simplify the analysis of linear systems since linear differential equations reduce to algebraic equations in the Laplace domain. Figure 4-1 shows an example of a two-port network. By defining four transfer functions that describe the input-output relations between the two input and output terminals, we are able to model a two-port network as a multiple-input multiple-output transfer function (see Equation 4-1). Subsequently, we can use matrix algebra to design large interconnections of two-port networks. This motivates the use of two-port network theory for a ABM framework.

There are multiple sets of two-port network parameters, which can be found in Appendix C. Admittance parameters, also known as Y-parameters, are most intuitive as the two-port network can be interpreted as an elasticity tensor, with each transfer function expressing the elasticity between two terminals[1] [31][48]:

$$\begin{bmatrix} I_I \\ I_O \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_I \\ V_O \end{bmatrix} \tag{4-1}$$

$$\begin{aligned} y_{11} &:= \left.\frac{I_I}{V_I}\right|_{V_O=0}, \quad & y_{12} &:= \left.\frac{I_I}{V_O}\right|_{V_I=0}, \\ y_{21} &:= \left.\frac{I_O}{V_I}\right|_{V_O=0}, \quad & y_{22} &:= \left.\frac{I_O}{V_O}\right|_{V_I=0} \end{aligned} \tag{4-2}$$

Respectively, transfer function y_{11} specifies the elasticity of the inputs, transfer functions y_{22} the elasticity of the orders, and transfer functions y_{12} and y_{21} the cross-elasticities between the inputs and orders.

Another advantage of interpreting admittance as the analog of elasticity is that a two-port network must be power-conserving, given that the admittance matrix is symmetric. Therefore, only reciprocal two-port networks are power-conserving[2]. This is consistent with economic theory, as one can read in *"Intermediate Microeconomics"* by H.R. Varian.[71]

[1] Reader are referred to Section 2-3-2 for the justification of this analogy.

[2] Interestingly, this implies that the production network presented in *"Modular Economic-Engineering Agent-Based Models as N-port Networks"*[32] is not power conserving, i.e., the surplus transfer from the input to the output port does not equal the surplus transfer from the output to the input port.

4-3 Economic Agent

An agent can be defined arbitrarily, regardless of its scale. In Chapter 5, for instance, we model the aggregate of all households as a single agent. In this case, our goal is to design an economy from the bottom up. Therefore, we start by modeling a simple economic agent.

Figure 4-1 depicts a two-port network representing a trader. The basic building blocks of the model are the same electrical elements as we used in the time domain. However, now we express these using their generalized s-plane impedance. The topology is determined based on the economic processes carried out by a trader. We assume a trader either demands or supplies goods[3], stores goods, pays a handling cost, and experiences depreciation of stored goods. Respectively, we model demand using an inductor L, storage using a capacitor C, the handling costs using a resistor R, and depreciation using a conductor[4] G.

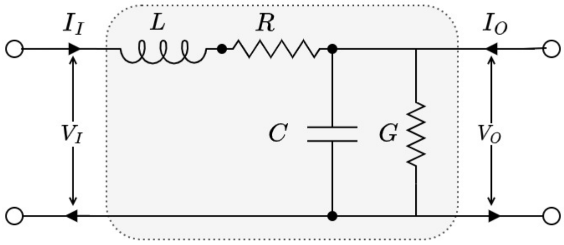


Figure 4-1: Two-port network of an economic agent representing a trader, where the input current I_I denotes the flow of inputs, output current I_O the flow of orders, input voltage V_I the input incentive, and output voltage V_O order incentive.

4-3-1 Trader Dynamics

The trader's dynamics[5] can be derived from Figure 4-1 using Equations 4-2:

$$\begin{bmatrix} I_I \\ I_O \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_I \\ V_O \end{bmatrix} = \begin{bmatrix} \frac{1}{Ls+R} & -\frac{1}{Ls+R} \\ -\frac{1}{Ls+R} & Cs+G+\frac{1}{Ls+R} \end{bmatrix} \begin{bmatrix} V_I \\ V_O \end{bmatrix} \tag{4-3}$$

The trader's input elasticity y_{11} is calculated by shorting the output port and looking at the ratio between I_I and V_I. Therefore, the two-port network reduces to a circuit consisting of an inductor and resistor in series. The resulting one-port network is analogous to a trader that adjusts its flow of inputs by balancing incentives[6] with its handling costs.

Economic engineering defines demand analogous to inductance. Inductance is a measure of an electrical conductor's tendency to oppose a change in the electric current. From Faraday's law, we can derive that an electrical current going through an inductor is proportional to

[3]Because of the unified demand schedule, the same topology can be used to model a trader as a supplier or demander.

[4]Conductivity is defined as the reciprocal of resistivity.

[5]The model parameters were chosen such that trader is under-damped and can be found in Appendix B, Table B-1.

[6]Since we consider a voltage drop in this case, we refer to an incentive instead of a desire.

the time integral of an electromotive force. Analogously, the flow of inputs of the trader is proportional to the time integral of a desire. Because duration is equal to the reciprocal of frequency, the trader's demand decreases as a function of the frequency.[48]

At low cyclical trends ω, therefore, the trader's input elasticity y_{11} is more elastic. At high cyclical trends, the trader's input elasticity y_{11} is more inelastic, resulting in the trader handling a flow of input at the cost of a handling fee. The upper left Bode plot in Figure 4-2 shows the Laplace response of the trader's input elasticity y_{11}. As expected, the elasticity decreases as the cyclical trend increases. The roll-off in magnitude coincides with a phase lag of 90 degrees as the cyclical trend approaches infinity.

The trader's cross-elasticity y_{12} is calculated by shorting the input port and looking at the ratio between I_I and V_O. The resulting one-port network also reduces to an inductor and resistor in series, with the electrical current flowing opposite to input elasticity y_{11}. Therefore, cross-elasticity y_{12} is dynamically equivalent to input elasticity y_{11} besides a phase lead of 180 degrees arising from the opposing flow of inputs. Since the two-port network is reciprocal, cross-elasticity y_{21} is identical to cross-elasticity y_{12}. The upper right and lower left Bode plots in Figure 4-2 indicate the Laplace responses of the trader's cross-elasticities y_{12} and y_{21} and are almost identical to the Laplace response of input elasticity y_{11}.

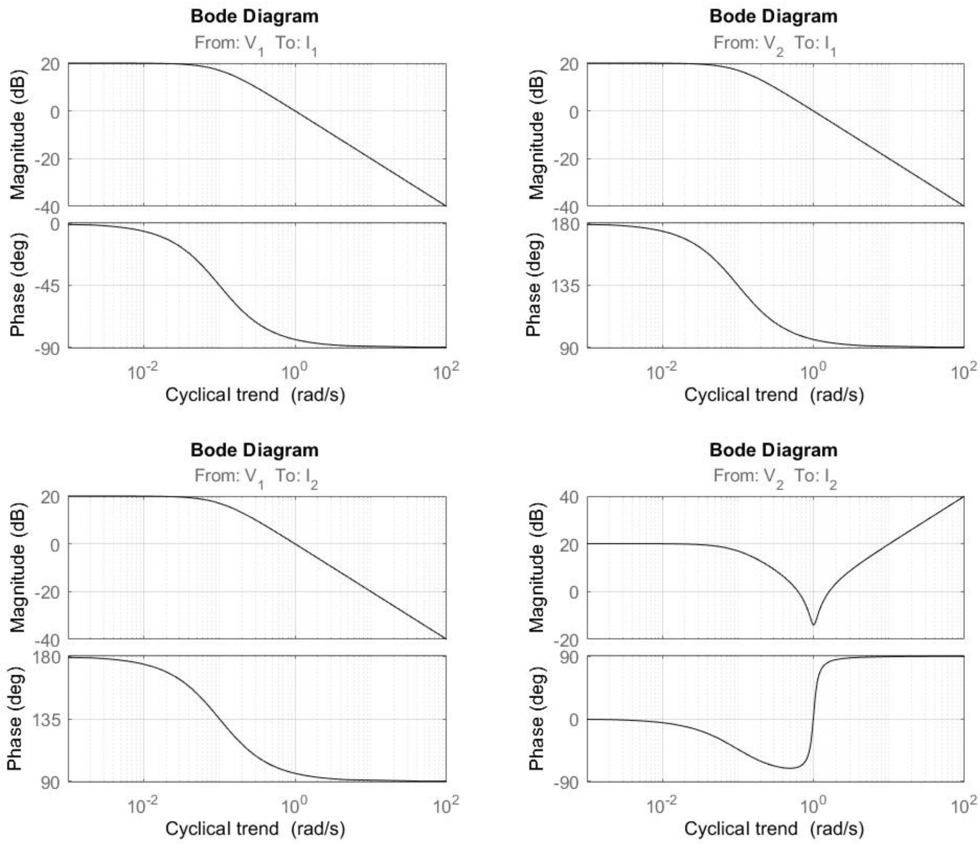


Figure 4-2: Bode plot (Y-parameters) of a trader's input (y_{11}), cross- (y_{12}, y_{21}), and output (y_{22}) elasticities.

The trader's output elasticity y_{22} is calculated by shorting the input port and looking at the ratio between the I_O and V_O. The resulting one-port network reduces to an inductor and resistance in series that are parallelly connected to a capacitor and conductance. A trader's storage is inversely related to its demand, e.g., when a trader's demand for a good is high, its convenience is low. Analogously, the dynamics of a capacitor are inversely related to an inductor. As a result, a trader's storage is more elastic at high cyclical trends and more inelastic at low cyclical trends.

The lower right Bode plot in Figure 4-2 depicts the Laplace response of the trader's output elasticity y_{22}. At low cyclical trends, the elasticity is constant, with a slight phase lag appearing as the cyclical trend increases. At high cyclical trends, we observe that the trader's output elasticity increases as the trader uses its storage to react to a change in demand.

In addition, we observe the trader's output elasticity y_{22} (see Equation 4-3) is an improper under-damped second-order system that contains a pair of complex conjugate transmission-blocking rates. The pair of complex conjugate transmission-blocking rates causes the opposite effect of a pair of complex conjugate discount rates, resulting in an anti-resonance peak at a corresponding anti-resonance frequency. Economically, the trader's storage counteracts any cyclical desire at the anti-resonance frequency. Accordingly, we interpret an anti-resonance peak as an inelasticity peak and an anti-resonance frequency as the inelasticity frequency ω_a.

4-3-2 Trader Characteristics

In this section, we establish the economic interpretation of the trader's dynamics using the Laplace domain techniques introduced in Section 2-3. The transfer functions representing the input elasticity y_{11} and cross-elasticities y_{12} and y_{21} are the electrical equivalent of the agent introduced in Figure 2-5. Therefore, these transfer functions specify the trader's price dynamics and are characterized as a function of the discount rate: $\gamma = R/L$.[48]

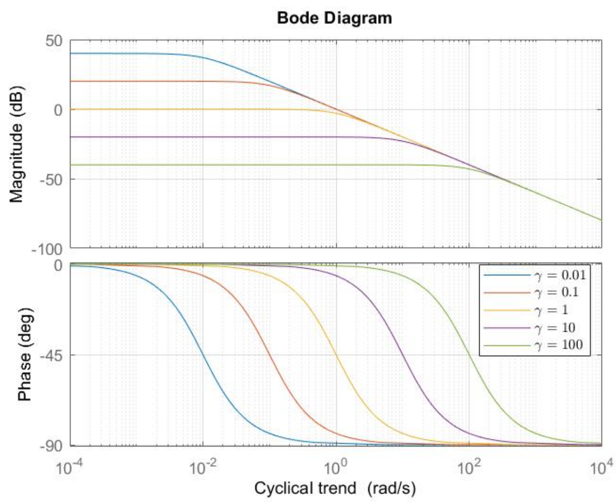


Figure 4-3: Bode plot (Y-parameters) depicting the trader's input (y_{11}) and cross-elasticities (y_{12}, y_{21}) for varying discount rates γ.

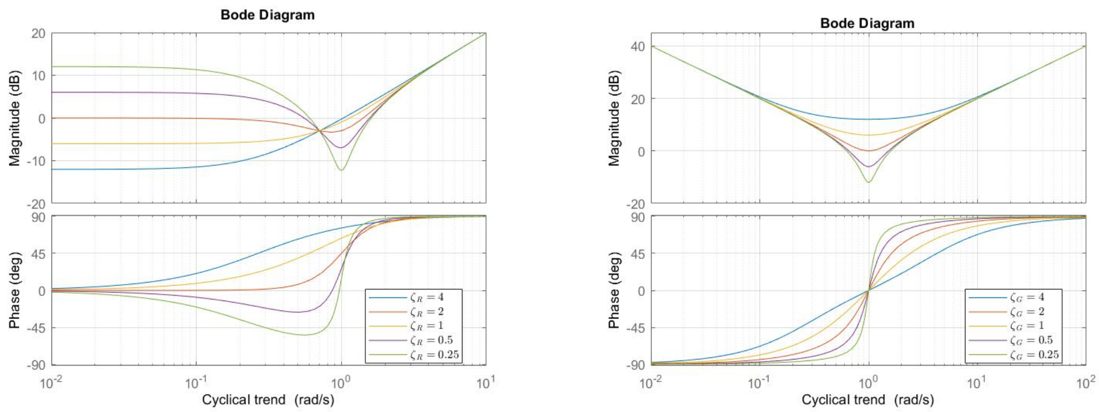
Figure 4-3 plots the Laplace response of the trader's input and cross-elasticities for varying discount rates γ. A trader with a high discount rate has a strong time preference and, thus, prefers to receive goods immediately. Therefore, such a trader reacts little to a change in the cyclical desires, i.e., is relatively inelastic. Conversely, a trader with a low discount rate has a relatively higher elasticity that decreases as the cyclical trend ω increases.

Characterizing the output elasticity y_{22} is less straightforward as the transfer function describes the trader's price and inventory dynamics. To simplify the analysis, we analyze two versions of the system, either neglecting the depreciation G or the handling cost R. This allows us to formulate the discount propensity ζ_R and the propensity to carry ζ_G [48]:

$$\zeta_R|_{G=0} = \frac{R}{2}\sqrt{\frac{C}{L}}, \qquad \zeta_G|_{R=0} = \frac{G}{2}\sqrt{\frac{L}{C}} \tag{4-4}$$

Figure 4-4a plots the Laplace response of the trader's output elasticity for varying discount propensities ζ_R and Figure 4-4b plots the Laplace response of the trader's output elasticity for varying propensities to carry ζ_G. As we have seen in Section 2-3, the inelasticity peak only appears if the system is under-damped. Economically, we can interpret an under-damped trader as a dealer-broker that changes its stock level in response to cyclical desires. An over-damped trader acts more as a buy and holder that hedges its trades by consuming portions of its inventory. In both cases, the trader displays speculative behavior.[48]

Additionally, we observe that, similar to the trader's input elasticity, the trader's output elasticity increases as its discount propensity decreases. Economically, this behavior is expected as a trader with a high discount propensity is not interested in long-run returns and, therefore, less reactive to a change in cyclical desires. Conversely, a trader with a low discount propensity is interested in long-run returns, making it more reactive to a change in cyclical desires.



(a) Varying discount propensities ζ_R.

(b) Varying propensities to carry ζ_G

Figure 4-4: Bode plot (Y-parameters) depicting the trader's output elasticity (y_{22}) for varying propensities.

Note that the above analysis of the output elasticity y_{22} is a twofold analysis of what can be considered as one system. When varying the trader's discount propensity ζ_R or propensity to carry ζ_G simultaneously, more subtle differences in the trader's characteristics can be observed.

4-4 Agent Interconnections

This section analyzes the economic behavior of different types of agent interconnections. The agents populating each network are all traders we modeled in Section 4-3. Five interconnections exist: series-series, parallel-parallel, series-parallel, parallel-series, and cascade interconnections. These are visualized in Figure 4-5. Each interconnection yields an alternative network topology and has a distinct economic interpretation.[7] In Appendix C, an overview of the mathematical relations describing each type of interconnection can be found.

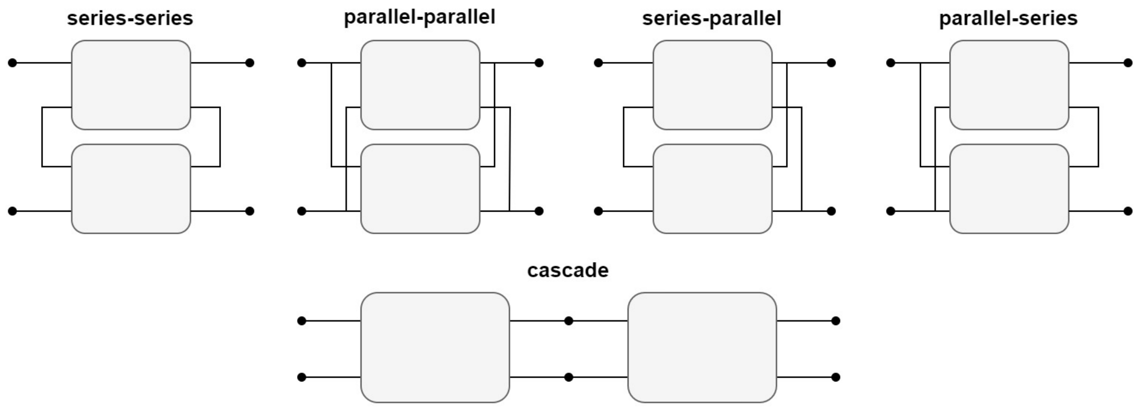


Figure 4-5: Five possible types of agent interconnections.

4-4-1 Cooperative Network

Agents operating on a shared flow of inputs and orders are interpreted as a cooperative network. Cooperative agents are analogous to a series-series interconnection in electrical network theory (see Figure 4-5).[32] The incentive of a cooperative network equals the sum of the individual agent's incentives. Hence, a cooperative interconnection decreases the collective elasticity. Analogous to cooperative topologies in economics, a cooperative network's liquidity reduces since a price change results in relatively larger oscillations in the flow of goods.

Figure 4-6 visualizes the Laplace response of a cooperative network's elasticities consisting of four identical traders. Compared to a single agent, the cooperative network's magnitude decreases across all cyclical trends ω. Consequently, the elasticity of the cooperative network must be lower. The phase plot and shape of the magnitude plot remain unchanged.

4-4-2 Competitive Network

Agents competing for a share of the total flow of inputs and orders are interpreted as a competitive network. Competitive agents are analogous to a parallel-parallel interconnection (see Figure 4-5).[32] The total flow of inputs/orders in a competitive network is equal to the sum of the inputs/orders of each agent. As a result, the collective elasticity of a competitive

[7] A detailed discussion of the derivation and interpretation of each type of interconnection is provided by C.Hutters.[32].

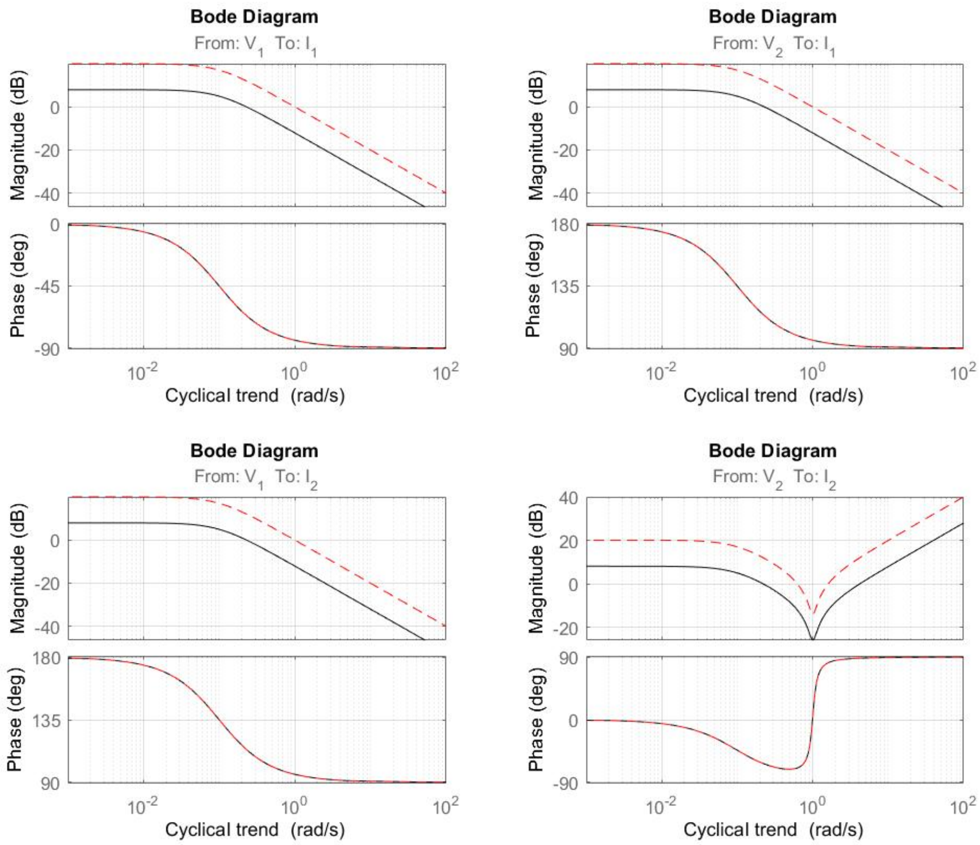


Figure 4-6: Bode plot (Y-parameters) of a cooperative network's input (y_{11}), cross- (y_{12}, y_{21}), and output (y_{22}) elasticities. For reference, the Laplace response of a single agent is included using a red dotted line. Compared to a single agent, the collective elasticity of cooperative agents decreases across all cyclical trends ω.

network increases. I.e., the network's liquidity increases since oscillations in the price have less effect on the total flow of goods.

We model a competitive network by interconnecting four identical traders parallelly. Figure 4-7 plots the Laplace response of the competitive network's elasticities. Consistent with the cooperative network, the phase plot and shape of the magnitude plot remain unchanged compared to a single agent. Conversely, the elasticity of a competitive network of agents increases across all cyclical trends ω.

4-4-3 Hybrid Network

In the previous two sections, we discussed cooperative and competitive agents. Both featured a series or a parallel interconnection at the input and output ports of the two agents. Some topologies operate cooperatively for inputs but compete for orders (e.g., franchises). Conversely, there exist topologies that operate competitively for inputs but cooperate for orders (e.g., cartels). In electrical network theory, these topologies are known as series-parallel and

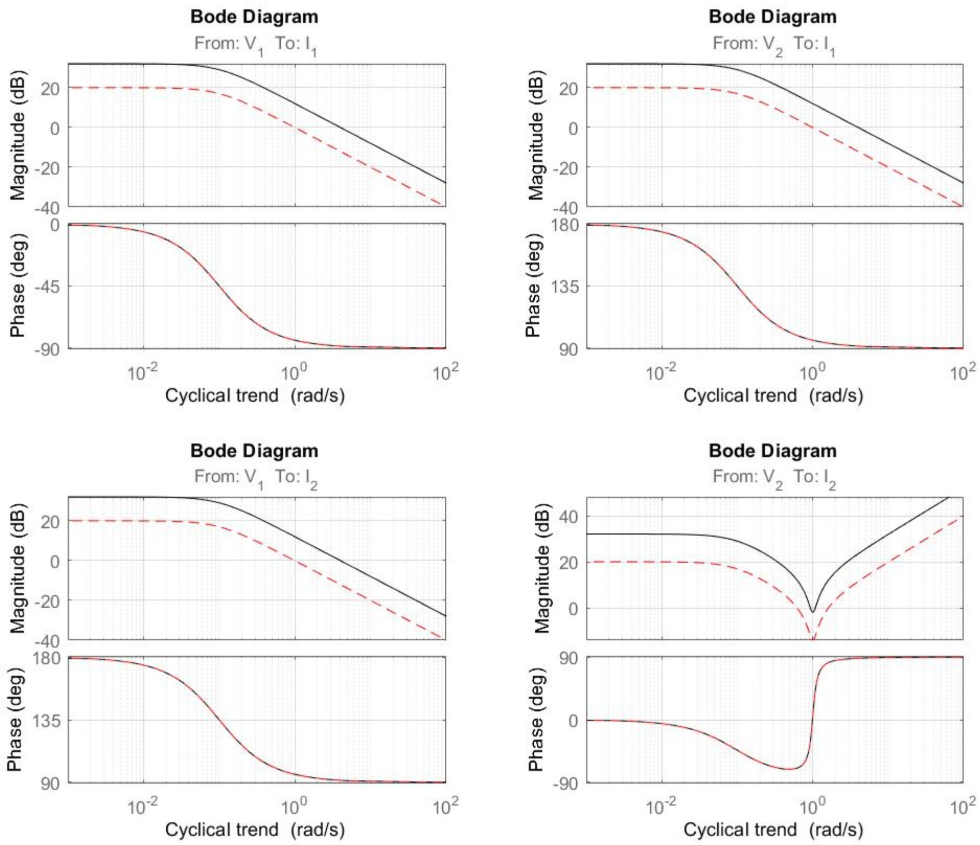


Figure 4-7: Bode plot (Y-parameters) of a competitive network's input (y_{11}), cross- (y_{12}, y_{21}), and output (y_{22}) elasticities. For reference, the Laplace response of a single agent is included using a red dotted line. Compared to a single agent, the collective elasticity of competitive agents increases across all cyclical trends ω.

parallel-series interconnections. In general, however, we refer to both types of interconnections as hybrid agents.[32]

The economic behavior of hybrid agents can be explained by combining the insights of cooperative and competitive agents. A series-parallel network has a reduced input elasticity y_{11} since its input ports are interconnected cooperatively. The output elasticity y_{22} increases since its output ports are interconnected competitively. The cross-elasticities y_{12} and y_{21} are impacted by the competitive and cooperative topologies, thus canceling each other out. As a result, the cross-elasticities are identical to that of a single agent, as are the phase graph and the shape of the magnitude graph. In Figure 4-8, the Laplace response of the series-parallel network's elasticities is visualized.

The economic behavior of a parallel-series network is almost identical to that of a series-parallel network. However, the input elasticity y_{11} increases for a parallel-series network since its input ports are interconnected competitively. Its output elasticity y_{22} decreases since its output ports are interconnected cooperatively. The Laplace response of a parallel-series network's elasticities is plotted in Figure 4-9.

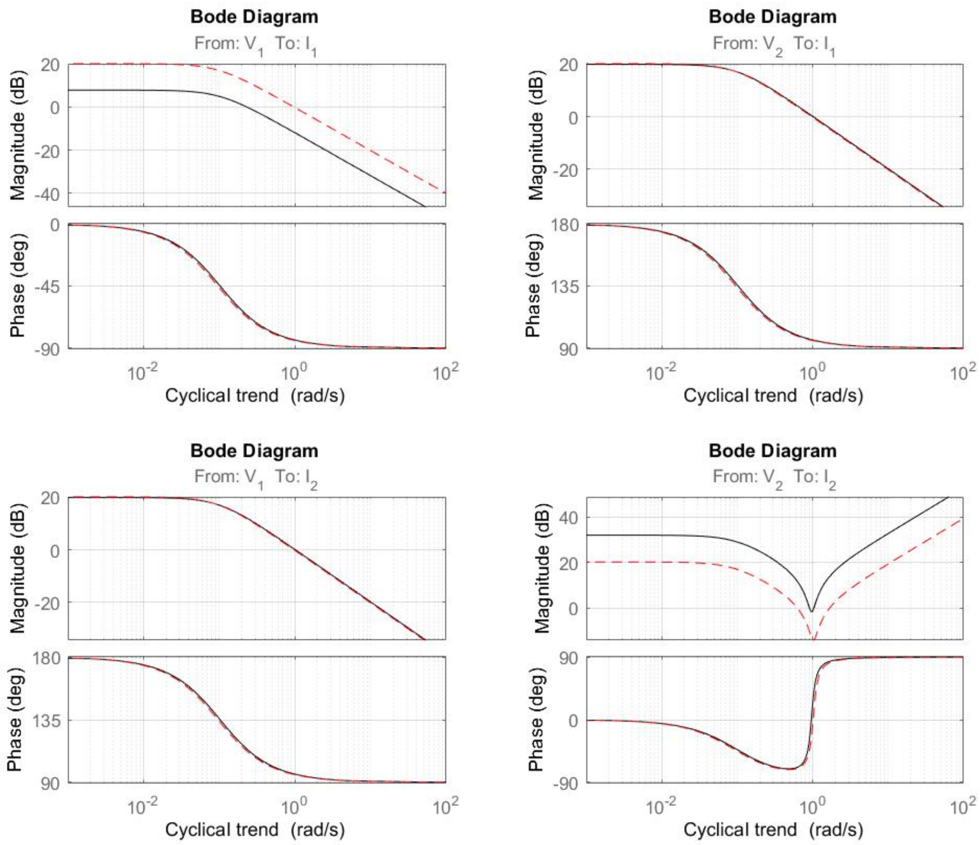


Figure 4-8: Bode plot (Y-parameters) of a series-parallel network's input (y_{11}), cross- (y_{12}, y_{21}), and output (y_{22}) elasticities. For reference, the Laplace response of a single agent is included using a red dotted line. Compared to a single agent, the input elasticity decreases, and the output elasticity increases across all cyclical trends ω.

4-4-4 Supply Chain Network

Cascade interconnections are an alternative topology for modeling agents cooperatively. Whereas the agents in Section 4-4-1 shared the same flow of inputs and orders, a cascade interconnection models a flow of orders of a preceding agent as the flow of inputs of a subsequent agent (see Figure 4-5). The resulting network is analogous to a supply chain network in economics.[32]

The Bullwhip effect describes a dynamic phenomenon where small fluctuations in demand are amplified as they propagate through the supply chain network. As a result, a small fluctuation in consumer demand can cause significant fluctuations in producer demand.[22][38] This phenomenon is similar to the movement of a whip, explaining why it is named the Bullwhip Effect. Because a quantity demanded is analogous to an electrical current, the Bullwhip effect is analogous to resonance. Since we stated in Section 4-3 that the trader is modeled as an under-damped second-order system, this also follows from Section 2-3-6.

Figure 4-10 shows the Laplace response of a supply chain network's elasticities consisting of

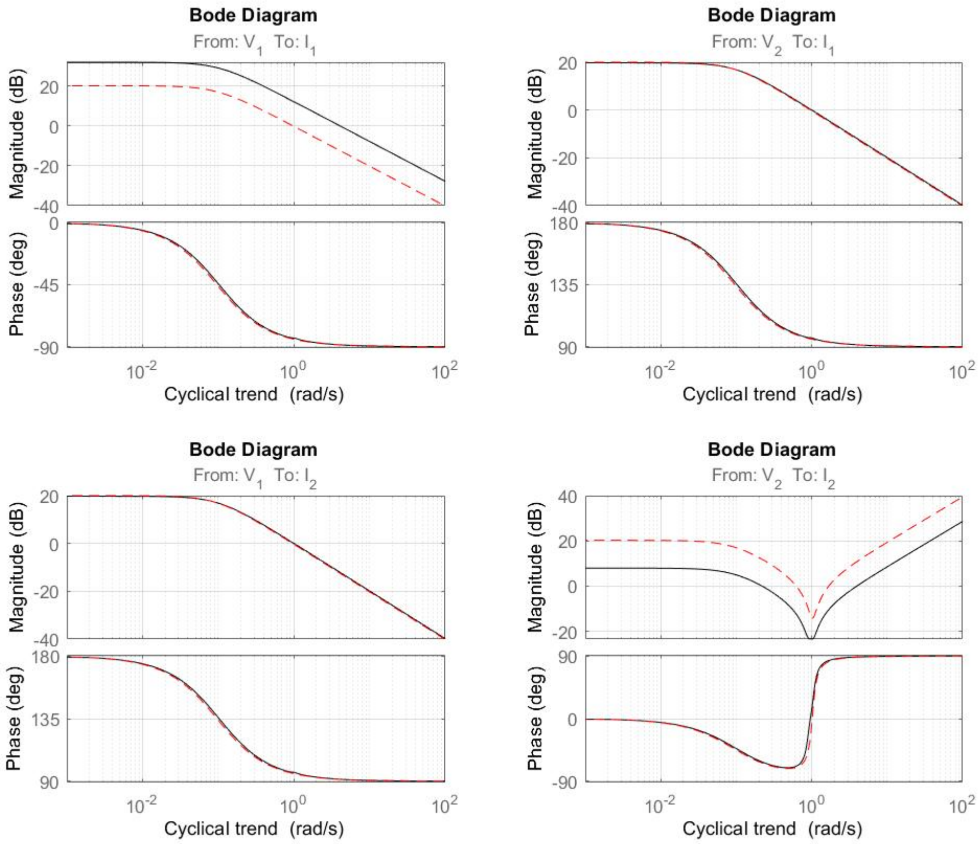


Figure 4-9: Bode plot (Y-parameters) of a parallel series network's input (y_{11}), cross- (y_{12}, y_{21}), and output (y_{22}) elasticities. For reference, the Laplace response of a single agent is included using a red dotted line. Compared to a single agent, the input elasticity increases, and the output elasticity decreases across all cyclical trends ω.

four identical traders. The magnitude plots of input elasticity y_{11} and output elasticity y_{22} clearly show multiple peaks, causing the total flow of inputs/orders to be amplified.

Because of the cascade interconnection, the Bullwhip frequencies ω_r of the individual traders coincide. This causes the magnitude of the elasticity peak to increase. Additionally, the system's degrees of freedom (DoF)[8] increase, causing additional Bullwhip frequencies ω_r. Thus, the cascade interconnection reinforces the Bullwhip characteristics of the individual traders.

In addition, we observe an increase in the phase at each elasticity peak. This is analogous to a trader preemptively reacting to an expected increase in an incentive by increasing the flow of inputs/orders. This behavior is similar to speculators in economic markets and can, therefore, be interpreted as speculative behavior. Interestingly, speculative behavior is considered one of the contributing factors causing the Bullwhip effect in supply chain literature.[38]

The Bode plots of the cross-elasticities y_{12} and y_{21} also include elasticity peaks, meaning that

[8]The DoF are equal to the number of agents minus one.

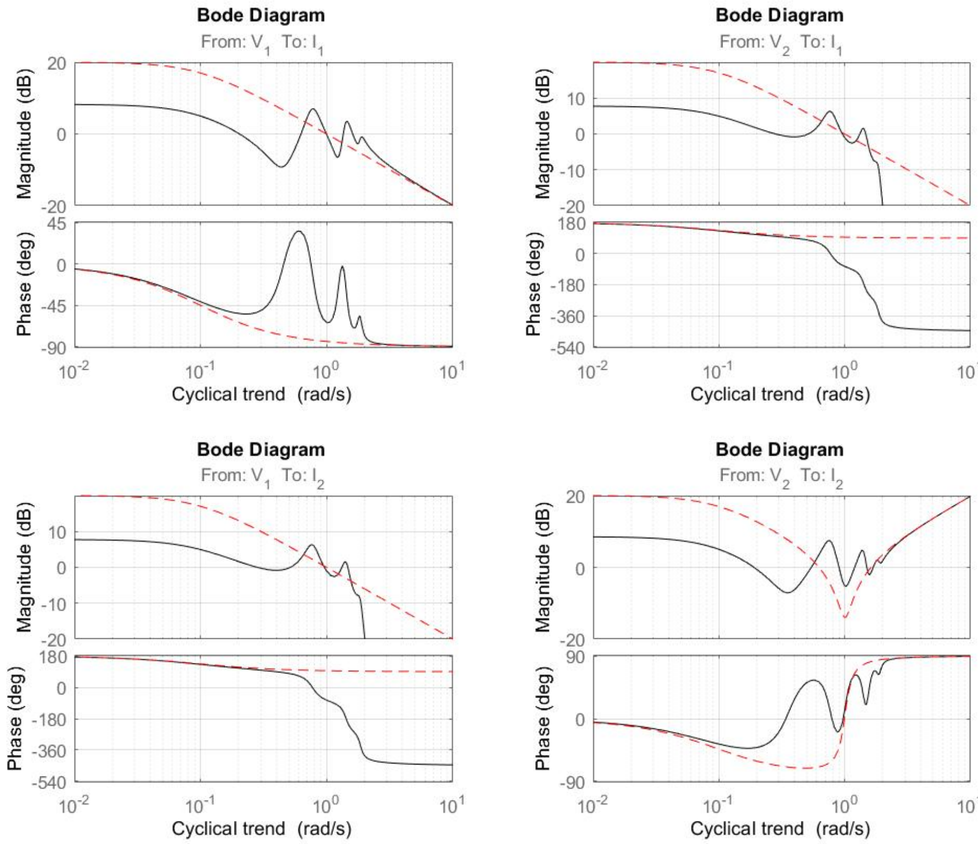


Figure 4-10: Bode plot (Y-parameters) of a supply chain network's input (y_{11}), cross- (y_{12}, y_{21}), and output (y_{22}) elasticities. For reference, the Laplace response of a single agent is included using a red dotted line. Compared to a single agent, we observe several elasticity peaks. In supply chain literature, this dynamic phenomenon is known as the Bullwhip effect.

oscillations in the flow of inputs/orders are propagated through the supply chain network. At high cyclical trends ω, we observe a sharp roll-off of the elasticity, suggesting that cyclical desires of a high frequency take longer to propagate through the supply chain network. Additionally, we notice the total phase lag between the input and output scales with 180 degrees for every elasticity peak, i.e., DoF. In the case of Figure 4-10, three elasticity peaks are observed, resulting in a total phase lag of 540 degrees.

Moreover, the supply chain network's elasticity decreases relative to a single agent at low cyclical trends. At high cyclical trends, however, a supply chain network's elasticity is identical to that of a single agent. Economically, this makes sense, as traders located in the last tiers of the supply chain network use their storage to counteract high-frequency fluctuations in cyclical desires.

4-5 Modeling Complex Economic Networks

In the previous section, we have seen how various interconnections of agents can be used to model different types of economic networks. This section combines the different types of interconnection to create two examples of complex economic networks. First, we present an hourglass-shaped supply chain network, which is a commonly observed supply chain topology. Second, we model a competitive market that contains multiple supply chain networks. In addition, we extend the models by including heterogeneous agents.

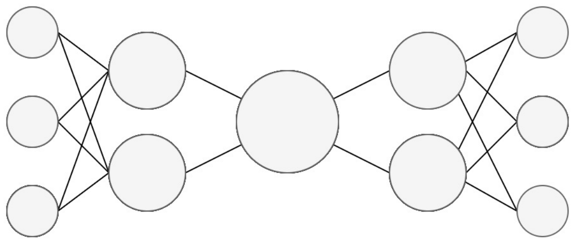


Figure 4-11: Schematic representation of an hourglass-shaped supply chain network. Respectively, a node represents an agent, a line represents a flow of inputs/orders, and a vertical grouping of agents represents a tier.

4-5-1 Hourglass-Shaped Supply Chain Network

In practice, supply chain networks take many different shapes. A commonly observed supply chain typology is an hourglass-shaped supply chain network, which is schematically visualized in Figure 4-11. Compared to the supply chain network presented in Section 4-4-4, the hourglass-shaped supply chain network features a decreasing number of competitive agents closer to the middle tiers of a supply chain network. The converging tiers of the supply chain network represent the intermediate production steps required to assemble a product, with the single node representing the final assembly step. The diverging supply chain network tiers represent the distribution steps needed to deliver a product to a consumer.

We model the hourglass-shaped supply chain network using a combination of parallel-parallel interconnections alternated with cascade interconnections. The resulting supply chain network contains eleven agents divided over five tiers. Since, in practice, no economic system exclusively consists of homogeneous agents, we recreate heterogeneity among the traders by sampling trader's parameters from a normal distribution[9]. As a result, the Bullwhip frequencies ω_r and the magnitude of the elasticity peaks change slightly compared to a supply chain network with homogeneous agents.

Figure 4-12 visualizes the Laplace response of the hourglass-shaped supply chain network's elasticities. From the magnitude plots, we observe that the number of elasticity peaks has increased due to the increase in DoF. We also note that the elasticity across all cyclical trends ω has increased compared to the linear supply chain network presented in Section 4-4-4 since we added additional competitive agents to the supply chain network.

[9]See Appendix B, Table B-1.

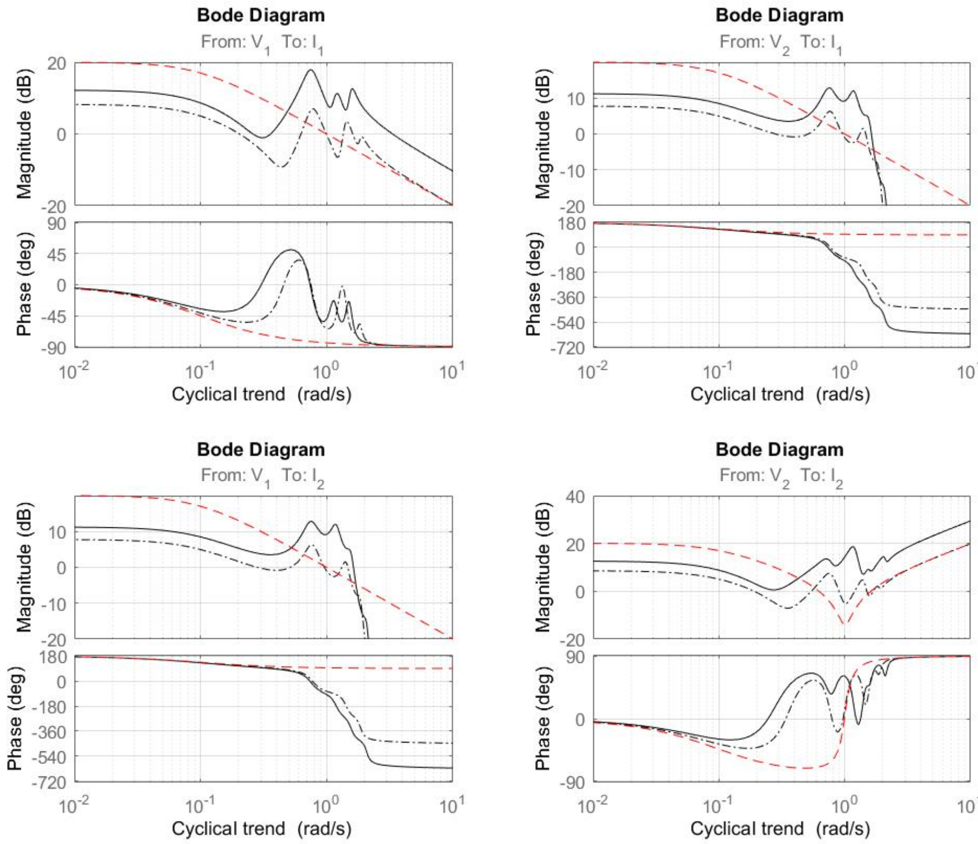


Figure 4-12: Bode plot (Y-parameters) of an hourglass-shaped supply chain network's input (y_{11}), cross- (y_{12}, y_{21}), and output (y_{22}) elasticities. The Laplace responses of a single agent (red dotted) and a supply chain network (black dotted) are included for reference. Compared to a linear supply chain network, the magnitude of the hourglass-shaped supply chain network's elasticity peaks has increased across all cyclical trends ω.

In economics, competitive topologies are more price-stable as competitive markets are more elastic.[13] By extension, we would expect the hourglass-shaped supply chain network to be less reactive to cyclical desires compared to a linear supply chain network. Interestingly, however, the elasticity peaks have increased in amplitude relative to the linear supply chain network. As a result, the Bullwhip effect is even more substantial in the hourglass-shaped supply chain network. This is presumably a result of the single agent located at the mid-tier of the supply chain network, known in graph theory as a critical node.[10][13]

The phase plots are comparable to a linear supply chain network for all elasticities. This again suggests that the total phase lag between the input and output of a supply chain network equals the DoF of the system.[10]

[10] The supply chain network was extended to five cascaded traders to equal the number of tiers of the hourglass-shaped supply chain network visualized in Figure 4-11.

4-5-2 Competitive Market Network

In this section, we model a hypothetical competitive market by interconnecting multiple supply chain networks competitively. Again, for the market to operate in perfect competition, three assumptions must be satisfied. First, all agents are price-takers since a trader only consists of passive elements. Secondly, all agents operate on the same flow, so all goods traded are homogeneous. Last, because of the no-arbitrage condition, all agents have access to perfect information. Hence, the market operates in perfect competition.[31]

Similar to the hourglass-shaped supply chain network, we model heterogeneity among traders by sampling the trader's parameters from a normal distribution. In addition, we sample the inventory elasticity (C) from two normal distributions[11]. As a result, the market comprises two types of traders with a large or small inventory elasticity.

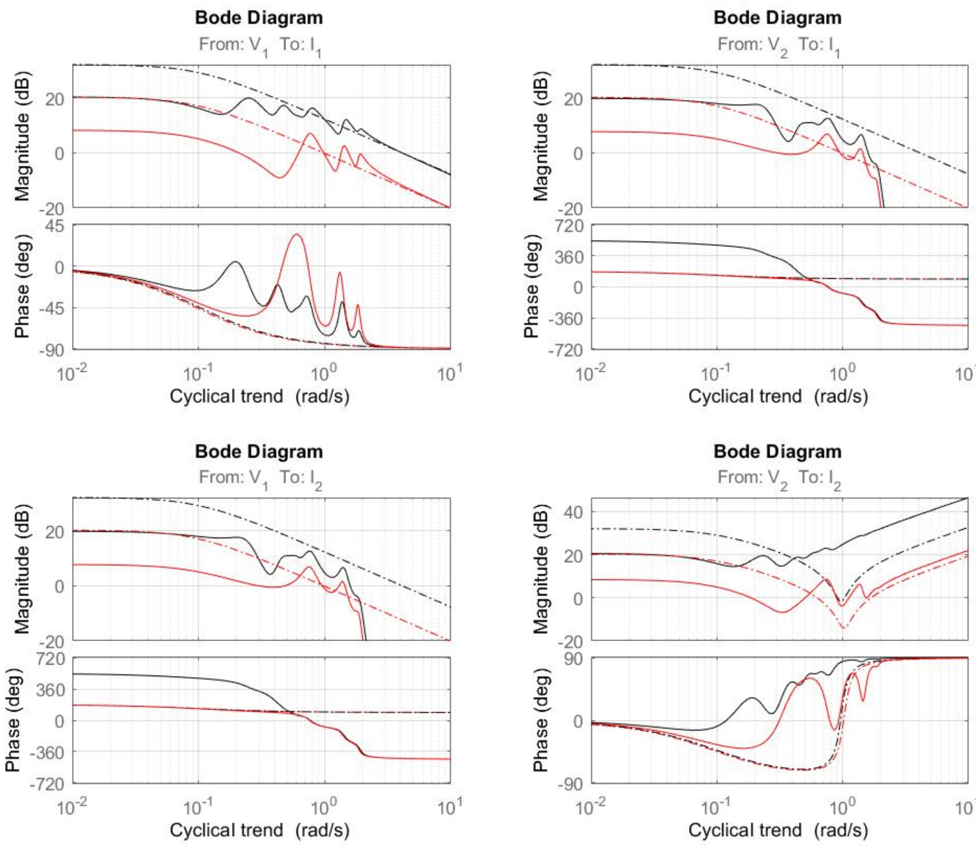


Figure 4-13: Bode plot (Y-parameters) of a competitive market's input (y_{11}), cross- (y_{12}, y_{21}), and output (y_{22}) elasticities. For reference, the Laplace responses of a single agent (red dotted), a linear supply chain network (red), and a competitive network (black dotted) are included. Analogous to competitive networks, the elasticity of the competitive market has increased across all cyclical trends ω compared to a single supply chain network. The additional elasticity peaks are a result of traders having variable storage elasticities.

[11] See Appendix B, Table B-1.

The Laplace response of the competitive market's elasticities is plotted in Figure 4-13. For reference, the Laplace response of a single agent's elasticity (red dotted), a single supply chain network's elasticity (red), and a competitive network's elasticity (black dotted) have been included.

We notice similar differences when comparing the elasticity of a single agent to the elasticity of a linear supply chain network and the elasticity of a competitive market to the elasticity of a competitive network. For instance, the supply chain network's and competitive market's elasticities both include elasticity peaks accompanied by an increase in the phase, compared to no peaks in the elasticities of a single agent and competitive network. These similarities can be explained. Considering the supply chain network as a lumped agent, the competitive market is nothing more than a competitive interconnection identical to the competitive interconnection presented in Section 4-4-2 using a different agent.

The minor differences result from the heterogeneity we modeled among the traders. Foremost, we observe that the number of elasticity peaks has increased from three to five and shifted to a lower Bullwhip frequency ω_r. Accordingly, the total phase lag between the input and output has increased with 360 degrees. The larger inventory elasticity makes traders react to cyclical desires of a lower frequency. Hence, the variable inventory elasticity results in two additional elasticity peaks at lower Bullwhip frequencies ω_r.

4-6 Discussion

This chapter introduces an agent-based modeling framework for bottom-up macroeconomic models. Crucially, the Laplace domain techniques developed in Chapter 2 facilitate the efficient design and analysis of economic systems, thereby enabling the use of two-port network theory to interconnect individual agents into complex economic networks.

As a result of analyzing the economic networks using Bode plots, we were able to identify the Bullwhip effect in supply chain topologies. This phenomenon is, however, not unique to supply chain networks but is observed in any network that contains a cascade interconnection. This suggests that the Bullwhip effect should be observed in other sectors of the economy. Numerous implications in economic literature suggest that this is indeed the case. For example, in financial markets, the Bullwhip effect has been linked to mass bankruptcies.[54]

Moreover, resonance is a well-studied phenomenon in engineering, with a large body of literature dedicated to analyzing and controlling this dynamic behavior.[51][23] Therefore, economic engineering could, for instance, be used to mitigate the Bullwhip effect in supply chain networks.[16]

This chapter exclusively used bode plots for analyzing the economic network's Laplace response. Section 2-3 presented alternative visualization tools that can provide additional insights into a system's economic behavior. For instance, the complex trend map of the supply chain network illustrates that the system's complex discount rates increase.[12]. This explains the additional elasticity peaks and phase lag, providing an alternative rationale for the causes of the Bullwhip effect.

[12]Given that all traders have identical parameters.

A key contribution of the ABM framework is to capture emergent behavior such as the bullwhip effect (i.e., unexpected behavior arising from modeling the interactions and behavior of individual agents within a system). Moreover, we demonstrated that the ABM framework can be used to model heterogeneity among individual agents. This contrasts bottom-up modeling approaches with top-down modeling approaches, whose underlying generalizations and assumptions limit the ability to model emergent behavior.[20]

Nonetheless, we modeled each trader exclusively from passive elements. Economically, it therefore follows from Section 3-3-1 that the trader is a price taker. In theory, one could introduce an active element to the model of the trader to model these as price setters. However, this is not possible within the restrictions of the two-port network theory.[2] Also, the no-arbitrage condition implies all agents have perfect information, and all goods are homogeneous since one current flows through all agents. As a result, all two-port network models satisfy the conditions for perfect competition.

Important, however, is that this generalization is a result of modeling preferences and, as such, not fundamental to the ABM framework. Hence, we can further develop the modeling framework to transcend this behavior.

Chapter 5

Top-down Model of the Economy

5-1 Introduction

In this chapter, we develop a price-dynamic top-down model of the economy. The reasons for this are twofold: First, economic-engineering models are able to simulate the economy in the short- and long-run. Secondly, economic-engineering models are causal because of the definition of an economic force. Therefore, an economic engineering model improves the predictive performance and interpretability of macroeconomic models.

The chapter is split into five sections: Section 5-2 introduces a circular flow model of a four-market economy, providing a structural framework for an economic engineering model. Because the circular flow model is a top-down structural model, it describes the aggregated behavior of lumped agents.

Section 5-3 uses the macroeconomic principles presented in Chapter 3 to design an electrical circuit analogous to the circular flow model of the economy. We start by listing the model assumptions in Section 5-3-1, followed by a market-by-market derivation of the economy's market dynamics in Sections 5-3-2, 5-3-3 and 5-3-4. The section concludes by discussing the exogenous effects in Section 5-3-5 and motivating how the model parameters are determined in Section 5-3-6.

Section 5-4 analyzes the models of the commodity market and the four-market economy in the time domain. We demonstrate how the price-dynamic framework can be used to explain the dynamic behavior of economic markets and incorporate sticky and flexible prices.

Section 5-5 analyzes the model of the four-market economy using the Laplace domain techniques developed in Chapter 2. We show that the system's complex trend map uncovers the economy's market-specific discount rates and inventory cycles.

Section 5-6 concludes by comparing the price-dynamic top-down model to existing macroeconomic models.

5-2 Circular Flow Model of the Economy

Circular flow models are frequently used in macroeconomics because of their effectiveness in visualizing the goods and money flows within an economy. Underpinning circular flow models is the principle of national accounting. Because an economy is circular, all cash flows within the economy have to "balance out". This allows economists to keep track of the cash flows over a period, analogous to a balance sheet. The sum of all income or expenditure accounts equals the gross domestic product (GDP), which is accordingly defined as the total income or expenditure of the economy within one year.[43]

Figure 5-1 depicts an example of a two-sector circular flow model of the economy. Each cash flow (green) is opposed by a flow of goods (red). Commonly, economists only portray the cash flows as these are easier to keep track of and compare. Moreover, an economic market intermediates each transaction between two economic agents. In this case, the economic agents are lumped agents representing the households and firms. The economic markets represent the market for goods and services and the markets for factors of production.

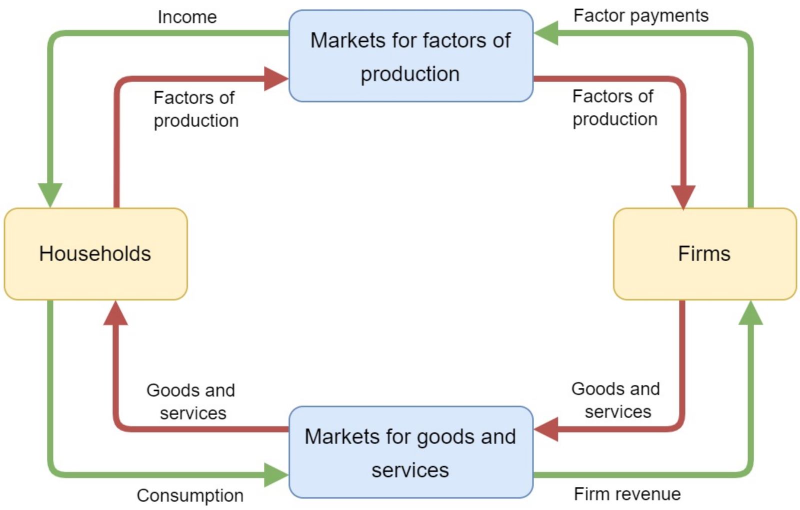


Figure 5-1: The two-sector model of an economy, where economic agents are labeled in yellow, economic markets in blue, the flows of goods in red, and cash flows in green.

The two-sector model, given in Figure 5-1, describes the most basic structure of an economy. Additional sectors can be incorporated to formulate a more comprehensive model of the economy. First, we introduce the government, which interacts with households by raising taxes and with the markets for goods and services by making government purchases. Secondly, we include the financial market, which introduces the cash flows for public savings, private savings, and investments. Last, we introduce the rest of the world, which interacts by lending or borrowing in foreign financial markets and importing or exporting goods and services. Combining these three additional sectors yields a five-sector model, as shown in Figure 5-2.

Important to note is that not all cash flows need to be positive. For instance, public saving can be negative, translating to the government borrowing capital from the financial market. Alternatively, negative net exports result in an economy that imports goods and services.

The markets for factors of production consist of several smaller markets, each corresponding to a factor of production. Hence, we can extend the model by including these individual smaller markets. Classical economic textbooks define three factors of production: [62]

- *Labor:* The time spent by the households working. Households provide labor services in exchange for wages provided by the firms.
- *Capital:* All forms of durable goods, i.e., physical capital such as machines or buildings.
- *Land:* All forms of natural resources, i.e., land for farming but also natural resources such as oil, coal, or water.

Of the three factors of production, labor and capital are the most important, as these account for the majority of the markets for factors of production. Therefore, labor and capital are included as the primary drivers of most production functions.

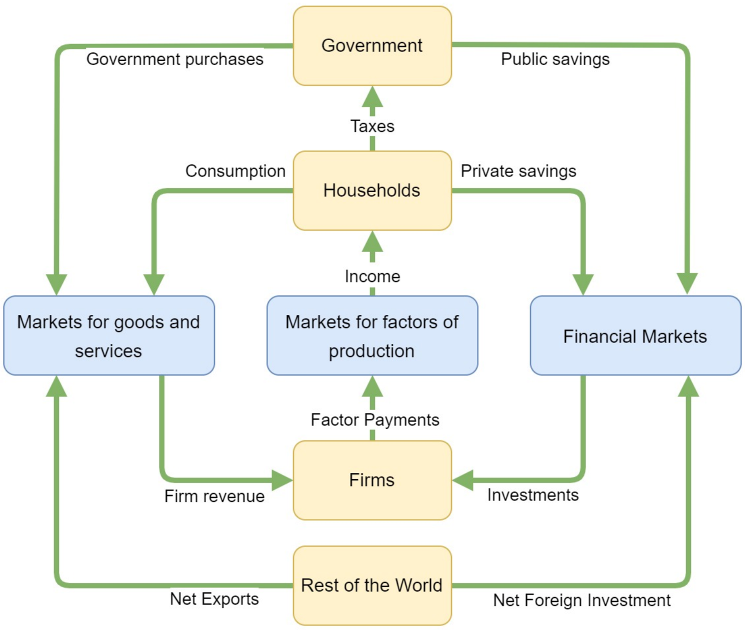


Figure 5-2: The five-sector circular flow model of an open economy. Note that only the cash flows have been depicted and that the flows of goods have been omitted. Again, the different economic agents are labeled in yellow, and the different economic markets in blue.

5-3 Deriving the Model

Circular flow models are used to establish an accounting framework for an economy. As a result, these models lack insight into the causal relations driving price changes. Thus, an additional framework is necessary when using circular flow models to gain insights into the price dynamics of the economy. In this section, we demonstrate the effectiveness of the economic-engineering approach by deriving a price-dynamic top-down model of the economy.

To achieve this, we use the electrical network theory developed by C. Hutters et al.[31] By recognizing that a flow of goods can be interpreted as an electrical current, we can use a circular flow model as the structure for an analogous electrical circuit. We model each economic process analogous to an electrical element, whose constitutive relations relate the flow of goods to economic desires. As a result, each economic process is associated with a voltage e and current f, forming a power conjugate pair analogous to the transfer of economic surplus. [15]

We begin, however, by modifying the five-sector circular flow model presented in Figure 5-2. First, we exclude the rest of the world, simplifying the model to a four-sector circular flow model. Any interactions with the rest of the world we model using external inputs.[1] Secondly, we rename the market for goods and services to the commodity market to emphasize that exclusively non-durable goods, i.e., commodity goods, are traded in this market. Thirdly, the markets for factors of production are subdivided into the labor and capital markets. Accordingly, we split the taxes into labor and capital taxes, which act on their respective markets. Last, we model an external input for each economic market.

The resulting circular flow model is visualized in Figure 5-3, where the expenditure flows are represented by green lines and flows of goods arising from the external inputs are represented in red. We also distinguish between capital markets and commodity markets. The labor, capital, and financial goods are all capital markets and represented in light blue. The commodity market is indicated in dark blue.

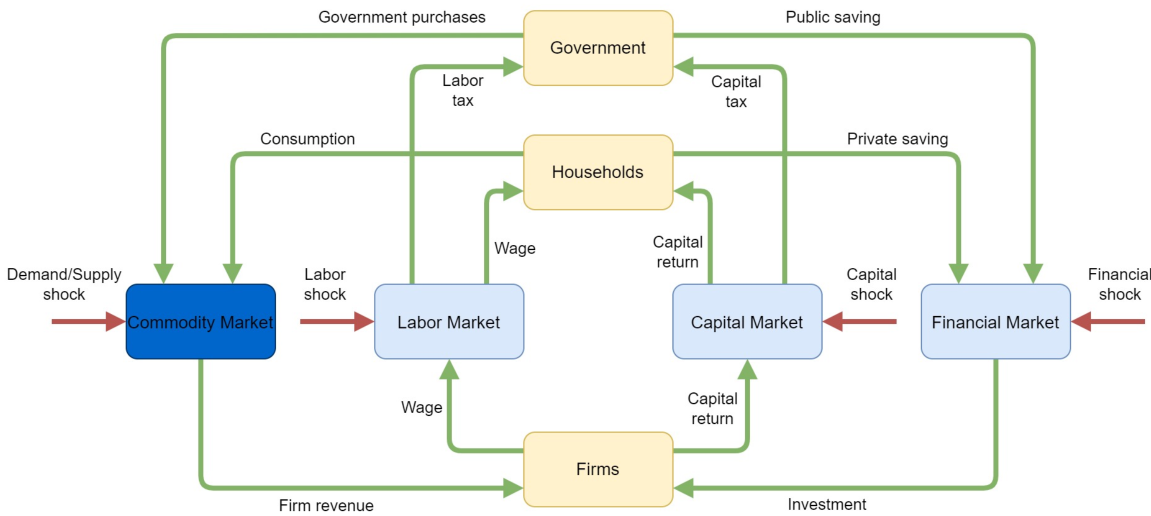


Figure 5-3: The adapted four-sector circular flow model of an open economy. Note that only the cash flows have been depicted and that the flows of goods have been omitted. The different economic agents are again labeled in yellow, the commodity markets in dark blue, and the capital markets in light blue.

[1] See Section 3-5-3.

5-3-1 Model Assumptions

Since we use the circular flow model as a foundation of our price-dynamic model of the economy, it is important to be mindful of the implicit assumptions we might introduce to the model. In Chapter 3, we discussed various macroeconomic principles and their economic-engineering analogs in detail. This section examines the economic approximations underlying the model, followed by the approximations required to implement the economic-engineering approach.

Economic Agents

In Figure 5-3, three economic agents have been introduced: the government, households, and firms. All three agents are modeled as lumped agents.[2] This implies that we assume all subsidiary agents are homogeneous, and price-takers. As a result, we can model the demand of each lumped agent as the aggregate demand of their subsidiaries.

In Section 3-3-1, it was argued that the approximation of lumped agents is justifiable under the condition of a sufficiently large number of agents. Since we model a complete economy, we can justify modeling the households and firms as lumped agents as the number of subsidiary agents is sufficiently large. In the case of the government, this argument is considerably weaker. The government acts as a holistic agent, allowing it to actively determine fiscal policy. Therefore, the government should ideally be modeled as a price maker. In Section 8-2-1, we discuss how this can be done using economic engineering. However, to initially reduce the complexity of the model, we model government as a lumped agent.

Moreover, we assume all agents form their expectations rationally, i.e., a Lagrangian disutility function exists. Since all agents are price takers and rational, which implies all agents will behave deterministically, no active elements are required. As a result, we can thus model each agent using strictly passive elements.

Linearity and Time Invariance

In Chapter 3, we introduced the model class of LTI systems. To harness the various techniques developed for LTI systems, we require all elements to be linear and time-invariant. Requiring all elements to be linear might seem like a stringent limitation since practically all mechanical and electrical elements are non-linear. However, for small perturbations, elements behave approximately linearly, motivating the use of linear elements. Under this hypothesis, we also motivate our assumption of adiabatically invariant rotations. Note that this does not imply the overall system response behaves linearly.[15]

Secondly, we assume all elements to be time-invariant. The elements in most physical systems are often time-invariant by definition. However, this is less straightforward for most economic systems as these often change over time. For instance, a change in economic policies might cause agents to behave differently, translating to a change in a model parameter.[40] Macroeconomists solved this by designing models with various simulation intervals (e.g., short run,

[2] We consider the government as a lumped agent of all ministerial and regional bodies of the government apparatus.

long run, or very long run.[43]) that are approximately time-invariant with respect to their simulation interval. Since we use a circular flow model as a foundation for a price-dynamic model, we assume the model to be time-invariant.

In Section 3-5-2, we claimed that economic growth could be modeled by increasing the capital stock. Although this is the case, this would result in the system no longer being time-invariant because the capital stock is a model parameter. For this reason, we have limited the scope of this thesis to time-invariant systems. However, there are various possibilities for modeling time-variant systems within the field of adaptive systems.[24]

5-3-2 Modeling Commodity Markets

We begin with modeling the commodity market. Afterward, we generalize the commodity market model to represent a capital market. In the commodity market, households and the government demand commodities to satisfy their needs, while firms produce commodities. Hence, the firms are modeled as suppliers, and the households and government are demanders. Economic engineering defines a unified demand schedule for a demander and a supplier, with the direction of the flow of goods determining its characteristics. Therefore, all agents are modeled using an inductor. We model any exogenous supply or demand shocks using a voltage source.

Dealers and brokers play an essential role in shaping the market dynamics. Dealers maintain a small inventory to meet orders efficiently, while brokers facilitate deals at the cost of a transaction fee.[31] Therefore, the market is modeled using a capacitor, representing the combined inventory of all dealers, and a resistor, representing the trade friction induced by brokers. Figure 5-4 depicts a circuit diagram of the commodity market.

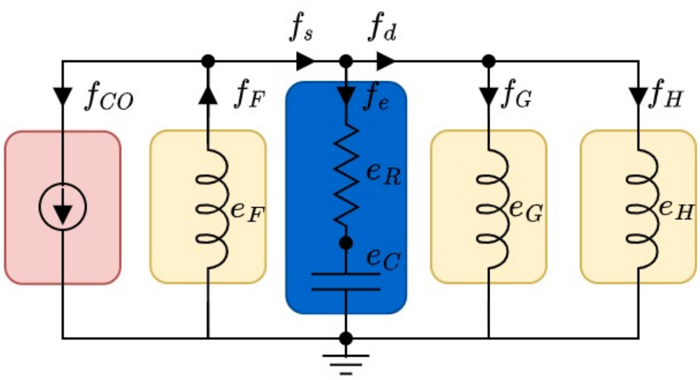


Figure 5-4: Circuit diagram analogous to the commodity market visualized in Figure 5-3. From left to right: the external input is modeled as a current source (red), the firms (yellow) as an inductor, the market forces (blue) using a series resistor and capacitor, the government (yellow) also as an inductor, and the households (yellow) as an inductor. The arrows indicate the direction of the flow of goods, analogous to the electric current.

Market Dynamics

From the circuit diagram, we can immediately induce that all demanders and suppliers are connected using a competitive topology, i.e., all agents maintain an identical market price:

$$p = p_H = p_G = p_F \tag{5-1}$$

Therefore, we can derive the market price dynamics using Kirchhoff's voltage law:

$$\dot{p} = e_R + e_C \tag{5-2}$$

Where e_R and e_C represent the incentives arising from the dealer's inventory and brokers' trade friction and are defined by their respective constitutive relationships:

$$e_R = R_{CO} f_e \tag{5-3}$$

$$e_C = C_{CO} q \tag{5-4}$$

Where R_{CO} and C_{CO} are model parameters, defined as the broker elasticity and the inventory elasticity. To determine the excess flow of goods f_e, we must apply Kirchhoff's current law to the upper node. The commodities supplied f_s are equal to the sum of the commodity flows of all suppliers, i.e., the firms and external input, and the commodities demanded f_d as the sum of the commodity flows of all demanders, i.e., the households and government. Mathematically, we can summarize the flows as follows:

$$f_s = f_F - f_{CO} \tag{5-5}$$

$$f_d = f_H + f_G \tag{5-6}$$

$$f_e = f_d - f_s \tag{5-7}$$

Subsequently, we define a flow f_i as a function of the price p using the constitutive relationship of an inductor:[3]

$$f_i = \varepsilon_i p \tag{5-8}$$

If we now substitute Equations 5-8 to 5-3 in Equation 5-2, we find the differential equation describing the market's price dynamics. Next, we determine the market's inventory dynamics, which have conveniently already been defined when deriving the market's price dynamics:

$$\dot{q} = f_e \tag{5-9}$$

By combining Equations 5-2 and 5-9, we can then write the market dynamics as an LTI system:

[3] Note that this equation is not applicable for f_{CO}, as its constitutive relationship is not given by an inductor.

$$\begin{bmatrix} \dot{p} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} R_{CO}(\varepsilon_H + \varepsilon_G - \varepsilon_F) & C_{CO} \\ (\varepsilon_H + \varepsilon_G - \varepsilon_F) & 0 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} + \begin{bmatrix} R_{CO} \\ 1 \end{bmatrix} f_{CO} \tag{5-10}$$

The resulting LTI system is described by two state variables: the market price p and the backlog q. Because the market is modeled competitively, each agent's reservation price equals the market price. Because of our adopted sign convention, the inventory dynamics are expressed as a backlog, the inverse of an inventory. Furthermore, we assume that all the states of the model are measurable. As a result, the output equation is given by:

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} f_{CO} \tag{5-11}$$

For completeness, table 5-1 provides the units and description of all state variables and their derivatives.

Variable	Description	Unit
q	Commodity stock	$[\#]$
$\dot{q} = f$	Commodity flow	$\left[\frac{\#}{yr}\right]$
p	Commodity price (CPI)	$\left[\frac{\$}{\#}\right]$
$\dot{p} = e$	Commodity desire	$\left[\frac{\$}{\#\cdot yr}\right]$

Table 5-1: State variables and their derivatives within the commodity market.

5-3-3 Modeling Capital Markets

Now that we have derived a model for a commodity market, we extend this model to describe a capital market. From Section 3-5-2, we know that commodity goods are produced by capital goods, which we can model using a rotation. We thus revise the supply of commodity goods in the circuit diagram (see Figure 5-4) to include the production of commodity goods. Since we assume the system is adiabatically invariant, we can do this using a transformer.

Like a mutual inductor, a transformer transfers electrical energy while changing the voltage-current ratio between two physically disconnected circuits using electromagnetic induction. Mechanically, this is analogous to a rotation with a constant radius, and economically, a transformer is analogous to a fixed amount of capital stock producing a flow of commodity goods. Identically to a transformer, the total transfer of surplus, i.e., power, during production is conserved.[15]

In Figure 5-5, an electrical circuit of the labor markets is visualized. In the labor market, households form a labor force $K_{H,L}$ that produces labor services q. To model this, we replace the inductor representing the households with a transformer and a series inductor. Also, because the firms are defined as demanders and the households and government as suppliers, the direction of the labor services flow f is opposite to the commodity flow in Figure 5-4.

Since no other desires act on the capital goods, it follows from Kirchhoff's voltage law that the labor value L_H is equal for the inductor and transformer. As a result, the labor market

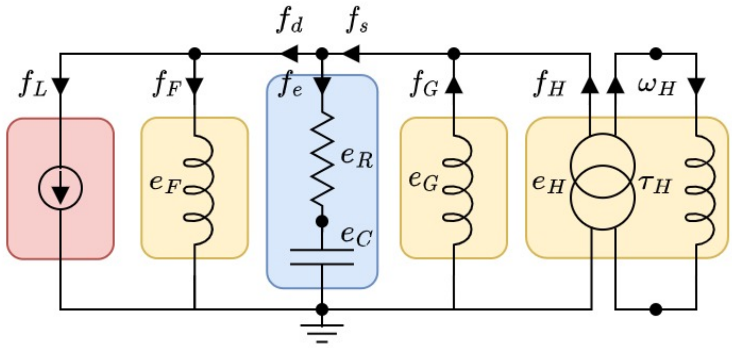


Figure 5-5: Circuit diagram analogous to the labor market visualized in Figure 5-3. From left to right: the external input is modeled as a current source (red), the firms (yellow) as an inductor, the market forces (blue) using a series resistor and capacitor, the government (yellow) also as an inductor, and the households (yellow) as a series transformer and inductor. The arrows indicate the direction of the flow of goods, analogous to the electric current.

dynamics are the same as those for a commodity market. If, however, we include additional price-driving mechanisms that act on capital goods, the market dynamics will change with respect to the commodity market.

Market Dynamics

We already established that the derivation of the market dynamics is almost identical to the commodity market. A subtle difference is that the firms are defined as demanders, and the households and government as suppliers in the labor market. As a result, the signs of the price elasticities ε switch. This should not be confused with the difference between complementary and substitution goods. In addition, we rename the subscripts of model parameters and the external input so these correspond to the labor market. Following these changes, the market dynamics can be written as follows:

$$\begin{bmatrix} \dot{p} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} R_L(\varepsilon_F - \varepsilon_H - \varepsilon_G) & C_L \\ (\varepsilon_F - \varepsilon_H - \varepsilon_G) & 0 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} + \begin{bmatrix} R_L \\ 1 \end{bmatrix} f_L \tag{5-12}$$

As one can see, the introduction of capital goods does not impact the price of labor services. The yield ω_H and labor value L_H are a function of the labor force $K_{H,L}$ and can be defined using the constitutive relationships of a transformer:

$$\omega_H = \frac{f_H}{K_{H,L}} \tag{5-13}$$

$$\tau_H = K_{H,L}\, e_H \tag{5-14}$$

Because we assume that the labor force $K_{H,L}$ is constant and the market is competitive ($p = p_H$), we can write the labor value L_H as a function of the labor services price p using Euler's second law of motion:

$$\begin{aligned} L_H &= \dot{\tau}_H && (5\text{-}15)\\ &= K_{H,L}\,\dot{e}_H && (5\text{-}16)\\ &= K_{H,L}\,p && (5\text{-}17)\end{aligned}$$

Similar to the commodity market, we assume all the model states are measurable. Using Equation 3-4, we can consequently augment the output equation to include the labor value:

$$\begin{bmatrix} p \\ L_H \\ q \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ K_{H,L} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} f_L \tag{5-18}$$

Table 5-2 again gives the units and description of all state variables and their derivatives.

The above derivation was explicitly made for the labor market. However, to derive the dynamics of the capital or financial market, only a few changes have to be made. First, all the parameters and variables must be revised depending on the market. Second, the signs of the price elasticity switch depending on which market. In the capital market, the firms are defined as demanders, and the households and government as suppliers. Hence, the state equation is the same as for the labor market (see Equation 5-12). In the financial market, the firms are defined as suppliers, and the households and government as demanders. Accordingly, the state equation is the same as that of the commodity market (see Equation 5-10). For both markets, the output equations equal that of the labor market (see Equation 5-18).

In Appendix A, readers can find a complete mathematical description of the capital and financial market, including a description of their respective state variables and derivatives.

Variable	Description	Unit
q	Labor services stock	$[ps]$
$\dot{q} = f$	Labor services	$\left[\frac{ps}{yr}\right]$
p	Labor services price (Wage)	$\left[\frac{\$}{ps}\right]$
$\dot{p} = e$	Labor services desire	$\left[\frac{\$}{ps\cdot yr}\right]$
θ	Accumulated yield	$[\%]$
$\dot{\theta} = \omega$	Labor yield	$\left[\frac{\%}{yr}\right]$
L	Labor value	$[\$]$
$\dot{L} = \tau$	Labor yield desire	$\left[\frac{\$}{yr}\right]$

Table 5-2: State variables and their derivatives within the labor market.

5-3-4 Modeling Market Interconnections

In this section, we combine the models of the commodity and capital markets to derive a model of a four-market economy. Central to a multi-market economy is the substitution effect. As discussed in Section 3-5-1, we use mutual inductors to model this phenomenon within electrical network theory. Therefore, we replace the inductors used to model an agent in a single market with a mutual inductor spanning multiple markets.

In Figure 5-6, a circuit diagram of a two-market model of the economy has been visualized, consisting of the commodity and labor markets. The subscript CO indicates the variables in the commodity market, and the subscript L indicates the variables in the labor market. The circuit diagram combines the circuits visualized in Figures 5-4 and 5-5. Note that the role of supply and demand varies in the two markets for each agent, e.g., the firms supply commodities but demand labor, resulting in an opposite flow of goods in the two markets.

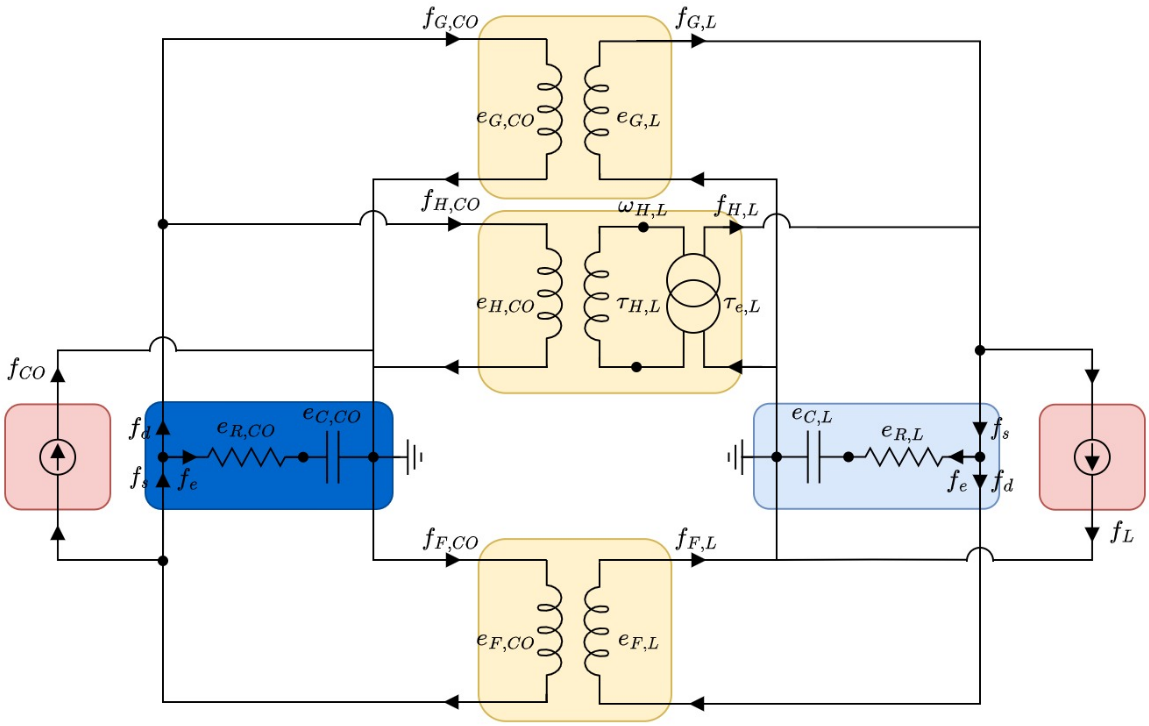


Figure 5-6: Circuit diagram analogous to a two-market model of the economy, consisting of the commodity and labor markets. Each agent (yellow) is modeled as a mutual inductor, the market forces (blue) as a series capacitor and resistor, and the external inputs as voltage sources (red). The arrows indicate the direction of the flow of goods, analogous to the electric current.

Economy Dynamics

Mathematically, the derivation of the market dynamics for a two-market model is almost the same as for a single-market model. The only difference is that the constitutive relation for an inductor should be changed to that of a mutual inductor, given by the following equation:

$$\begin{bmatrix} f_{CO} \\ f_L \end{bmatrix} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} \\ \varepsilon_{21} & \varepsilon_{22} \end{bmatrix} \begin{bmatrix} p_{CO} \\ p_L \end{bmatrix} \tag{5-19}$$

If we then follow the derivation performed in Sections 5-3-2 and 5-3-3 for the commodity and labor market, we find the following LTI system representation for the two-market model:

$$\begin{bmatrix} \dot{p}_{CO} \\ \dot{q}_{CO} \\ \dot{p}_L \\ \dot{q}_L \end{bmatrix} = \begin{bmatrix} R_{CO}(\varepsilon_{H11}+\varepsilon_{G11}-\varepsilon_{F11}) & C_{CO} & R_{CO}(\varepsilon_{H12}+\varepsilon_{G12}-\varepsilon_{F12}) & 0 \\ (\varepsilon_{H11}+\varepsilon_{G11}-\varepsilon_{F11}) & 0 & (\varepsilon_{H12}+\varepsilon_{G12}-\varepsilon_{F12}) & 0 \\ R_L(\varepsilon_{F21}-\varepsilon_{H21}-\varepsilon_{G21}) & 0 & R_L(\varepsilon_{F22}-\varepsilon_{H22}-\varepsilon_{G22}) & C_L \\ (\varepsilon_{F21}-\varepsilon_{H21}-\varepsilon_{G21}) & 0 & (\varepsilon_{F22}-\varepsilon_{H22}-\varepsilon_{G22}) & 0 \end{bmatrix} \begin{bmatrix} p_{CO} \\ q_{CO} \\ p_L \\ q_L \end{bmatrix} \dots$$
$$+ \begin{bmatrix} R_{CO} & 0 \\ 1 & 0 \\ 0 & R_L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} f_{CO} \\ f_L \end{bmatrix} \tag{5-20}$$

$$\begin{bmatrix} p_{CO} \\ q_{CO} \\ p_L \\ L_{H,L} \\ q_L \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & K_{H,L} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_{CO} \\ q_{CO} \\ p_L \\ q_L \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} f_{CO} \\ f_L \end{bmatrix} \tag{5-21}$$

In the previous sections, we saw that to model a competitive market, one account[4] and two state variables are required to describe its dynamics. The two-market model is represented by two accounts and four state variables. Consequently, a multi-market economy consisting of n markets will be spanned by $\mathcal{R}^n$ accounts and yield a price-dynamic model comprising of $\mathcal{R}^n$ power conjugate price p and backlog q variables.

To complete the extension of the two-market model to the four-sector circular flow model visualized in Figure 5-3, we must include the capital and financial markets. This can be done using the same approach as presented in this section. The only difference is that the elasticity tensor will become a 4×4 matrix for the four-market economy. Because of the lengthy calculations and complexity of the circuit diagram, we chose not to include the derivation of the four-market economy. In Appendix A, readers can find a complete mathematical description of the model. Moreover, we can obtain the system's transfer function matrix $H(s)$ from the system's state-space representation using Equation 2-15.

5-3-5 Modeling Exogenous Effects

As mentioned in Section 3-5-3, we can use external inputs to model exogenous effects. These can be used to bring about two effects. First, we can use external inputs to perform scenario analysis. In that case, we apply a fictitious shock to the economy to study the model's response. We demonstrate this in Section 5-4 by simulating various external shocks to perform scenario analysis.

Alternatively, we can use external inputs to include unmodeled dynamics. For instance, the four-sector circular flow model does not model any interaction with the rest of the world. We could model the net exports and foreign investments using an external input. Interestingly, this need not be a model simplification. Macroeconomists distinguish between large and small open economies.[5] In a small economy, domestic supply and demand have a negligible effect on the rest of the world. Consequently, we must model the net exports as an inelastic supply

[4]If the capital stock is time-variant, one additional account is needed to keep track of the capital goods.

[5]For more details, readers are referred to *"Macroeconomics"* by *N.G. Mankiw.*[43]

or demand. Since a flow source represents a perfectly inelastic demander or supplier, it would be best to model the net exports as an external input.

External inputs can originate from a wide range of causes. Table 5-3 lists possible causes for external inputs specific to each economic market. These can either be attributed to unmodeled dynamics or are used to simulate typical scenarios. Furthermore, in the case of capital markets, some of these external inputs may result from an increase in the capital stock, which can be modeled more accurately in alternative ways.

Commodity market	**External input**	**Possible cause**
	Supply shock	Supply chain disruptions
	Demand shock	Behavioral change consumers
	Net exports	Implementation trade barriers
	Fiscal policy input	Industrial policy/Tax reform
Labor market		
	Supply shock	Immigration reform
	Demand shock	Covid-19 pandemic
Capital market		
	Demand shock	Natural disaster
	Supply shock	Technological innovation
Financial market		
	Supply shock	Currency devaluation
	Demand shock	Stock market bubble
	Net foreign investments	Improved market access
	Monetary policy input	Inflationary episode

Table 5-3: Possible causes for external inputs to each economic market.

5-3-6 Parameter Estimation

In distinguishing between model parameters and model variables, economists tend to be less formal than engineers. The state variables describe a model's dynamic response and change over time. The model parameters, however, characterize the system's dynamics and are time-invariant.

Ideally, model parameters are determined through a process known as system identification. This process optimizes the model parameters so that the system recreates a measured input-output response as accurately as possible. Once the model's parameters have been determined, predictive simulations can be performed for a fictitious input, i.e., scenario analysis. Crucially, the input-output response should be causally related since the model is a causal reflection of systems. Therefore, accurate measurements are imperative for performing system identification. If not, the estimated model will not match the dynamic behavior of the actual system.

For the scope of this thesis, we limited ourselves to estimating the model parameters instead of performing system identification. As a result, the model will not be able to describe the economy quantitatively. However, provided we can estimate the model's parameters to some degree of accuracy, the model can qualitatively describe the dynamic behavior of the economy. To achieve this, we first establish how the different economic markets interact.

Cross-Market Effects

As discussed extensively, cross-market effects are modeled using mutual inductors, characterized by their elasticity tensor ε. From Section 3-2, we know that the cross elasticity of two substitution goods is positive and two complementary goods negative. Therefore, we can model the interactions between different economic markets by specifying the signs of the elasticity tensor. Note that the elasticity tensor is defined independently from an agent.

The economy in Figure 5-3 comprises four types of goods: goods and services (commodity goods), labor, capital, and financial goods. Labor and capital produce both goods and services and financial goods. Therefore, we model labor and capital complementary to commodity and financial goods. Household divides their disposable income between consumption and saving. Therefore, if the price of commodities increases, households have less disposable income for saving. Consequently, the demand for financial goods will weaken, lowering prices. Accordingly, commodity goods and capital goods are also modeled as complementary goods.

Since firms can choose whether to take on more labor or capital to increase production, we see labor and capital as substitution goods. As a result, the price of labor increases if the demand for capital increases or vice versa. Combining the above conclusions, we specify the signs of the elasticity tensors as follows:

$$\begin{bmatrix} f_{CO} \\ f_L \\ f_{CA} \\ f_F \end{bmatrix} = \begin{bmatrix} + & - & - & - \\ - & + & + & - \\ - & + & + & - \\ - & - & - & + \end{bmatrix} \begin{bmatrix} p_{CO} \\ p_L \\ p_{CA} \\ p_F \end{bmatrix} \tag{5-22}$$

General Equilibrium

Engineering systems and, by extension, engineering models are generally designed to be stable. Since we included dissipative elements in our four-market economy, the second law of thermodynamics ensures a stable equilibrium exists, given that we choose a stable set of model parameters. Following Section 3-4-2, the four-market economy, therefore, satisfies the conditions for general equilibrium. However, in economics, the conditions for general equilibrium are particularly stringent and are substantiated by many assumptions.

Generally, the conditions for general equilibrium come down to the rationality of agents, perfect competition, market clearing, and perfect information.[46] Rationality is guaranteed through the theory of rational expectation. Since all markets are modeled competitively, and all agents are passive agents, the conditions for perfect competition are also satisfied. This also implies that all markets clear (i.e., physical clearing) and all agents have access to perfect information. Therefore, we induce the conditions for general equilibrium to be satisfied.

For LTI systems, we can quickly check if the conditions for general equilibrium are satisfied, namely, if all real parts of the eigenvalues of state matrix A are strictly negative. Hence, using a simple brute force algorithm, which can be found in Appendix B, we determine a combination of model parameters that produce strictly negative eigenvalues. By scaling our initial estimates of the model parameters, we can ensure the model response is of the proper order of magnitude.

5-4 Time Domain Analysis

In this section, we perform several simulations for the commodity market model described in Section 5-3-2 and the four-market economy described in Section 5-3-4. Through analysis, we demonstrate how we can use the causal nature of economic-engineering models to explain the model's dynamic response. Note that the model parameters in Sections 5-4-1 and 5-4-2 are varied for demonstrative purposes[6].

5-4-1 Commodity Market Dynamics

We begin by simulating the commodity market visualized in Figure 5-4 in the time domain. As a scenario, we simulate a temporary increase in demand for commodities. We expect the price of commodities to increase due to the increased demand. When demand drops to its original level again, the price of commodities should also return to its initial level.

The above scenario has been visualized in Figure 5-7. The upper left plot shows the demand shock from t= 1 to $t = 3$. Immediately, we observe an increase in the excess flow of orders f_e, which is visualized in the middle left plot. The dealers in the market respond by going short on their inventory, and the brokers facilitate trades at a transaction fee, which results in market friction. As a result, the incentives (middle right plot) arising from storage e_C and trade friction e_C drive up the price of commodities. This can be ascertained in the bottom two plots, which indicates an increase in the market price p and backlog q of commodities.

Because of the price increase, the households, firms, and government adjust their supply and demand for commodities (see upper right plot). This leads to a reduction in the excess flow of orders f_e, lowering the economic incentives and resulting in a declining increase in the price of commodities. Ultimately, the market approaches a new price equilibrium.

Once the demand falls again, we observe the exact opposite. The flow of orders f_e drops sharply. The dealers use this opportunity to go long, and the brokers facilitate trades again. This leads to a negative market incentive, resulting in the price of commodities to drop. Consequently, all agents adjust their supply and demand, with the market eventually reverting to its prior equilibrium.

Economists visualize such a scenario using a supply-demand graph, which can be seen in Figure 5-8.[43] The demand shock, corresponding to an inelastic increase in demand, leads to a horizontal shift in the demand curve ($D \rightarrow D'$). Consequently, the market adjusts to a new long-run equilibrium ($E \rightarrow E'$), resulting in an increased price ($p \rightarrow p'$) and commodity flow ($f \rightarrow f'$). After the demand shock, the market will return to its prior equilibrium.

When comparing the two models, the economic-engineering approach stands out due to its ability to simulate short-run and long-run dynamics. Hence, this simulation serves as

[6] See Appendix B, Tables B-2 and B-3.

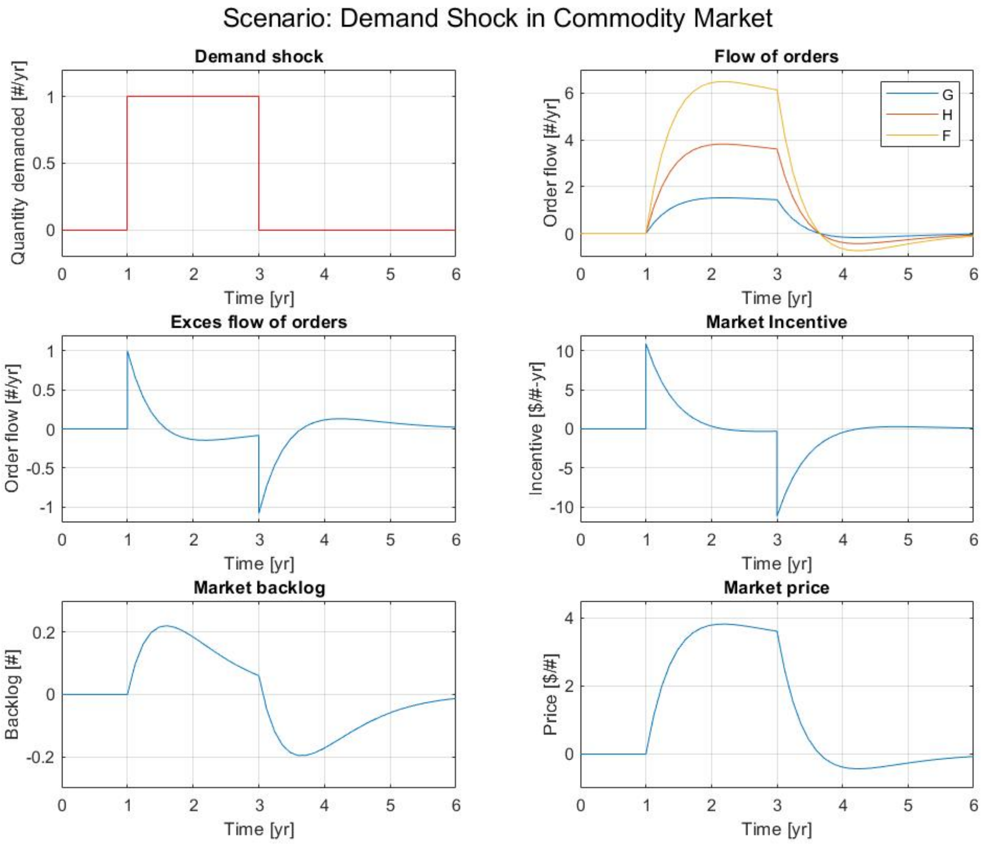


Figure 5-7: Simulation of a demand shock in the commodity market. Following the demand shock, the market converges to a new long-run equilibrium, demonstrating that prices are sticky in the short run but flexible in the long run. Once the demand shock has passed, the market reverts to its prior equilibrium.

empirical proof of the price-dynamic framework that economic-engineering models provide. Moreover, the economic-engineering approach maintains a causal framework, enabling the explanation of economic behavior.

5-4-2 Four-Market Economy Dynamics

We extend our analysis to the model of the four-market economy. To do this, we analyze two scenarios. The first scenario is identical to the demand shock discussed in the previous section. The second scenario simulates a monetary policy input in addition to a demand shock. Both these scenarios are simulated in the time domain.

Demand Shock

We simulate a demand shock in the goods and services market. Similar to the simulation in the previous section, we expect the price of commodities to increase only to revert to its prior

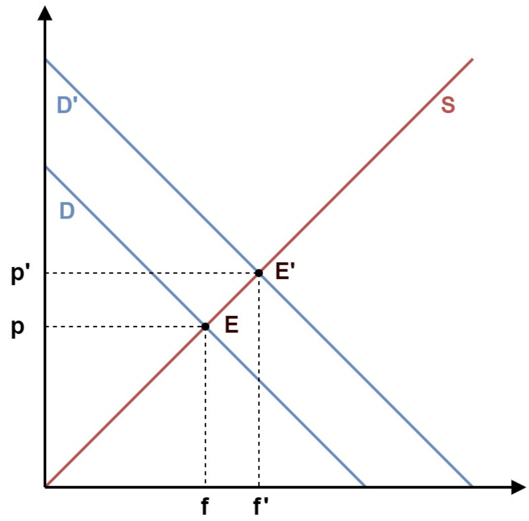


Figure 5-8: Supply-Demand graph following a demand shock ($D \rightarrow D'$). Consequently, the market adjusts to a new long-run equilibrium ($E \rightarrow E'$), resulting in a higher price ($p \rightarrow p'$) and commodity flow ($f \rightarrow f'$).

equilibrium after the demand shock has faded. In addition, the demand shock should feed through to the labor, capital, and financial markets because of the substitution effect.

The scenario has been visualized in Figure 5-9. The upper plot shows the demand shock, the middle plot shows the market backlogs and the bottom plot shows the market prices. As we expected, the price of commodities increases. Since commodities and financial goods are modeled as complementary goods, their prices are inversely related. Similarly, we would expect the price of labor and capital to decrease since these are also modeled as complementary goods. This is, however, not the case. Likely, this results from the more complex dynamic interaction between the four markets, resulting in a net price increase for labor and capital. Economically, however, this result is expected because an increase in commodity demand fuels investments in labor and capital, driving up the price. Moreover, we observe that prices are sticky in the short run, and in the long run, the economy converges to an equilibrium identical to the commodity market response.

When studying the market backlogs, we observe the backlogs are positively correlated to a change in the price. Since an increase or decrease in the price is proportional to a change in supply and demand, the backlogs indicate if a market is short or long. Hence, this dynamic behavior is expected and in line with economic theory.

We also notice some differences compared to the commodity market response in the previous section. First, the settling time, the time it takes for the economy to reach an equilibrium state, has more than doubled. As a result, the economy does not have time to stabilize in its new long-run equilibrium E', which was the case for the commodity market response. However, since the four markets continuously affect each other through the substitution effect, an increase in settling time is to be expected.

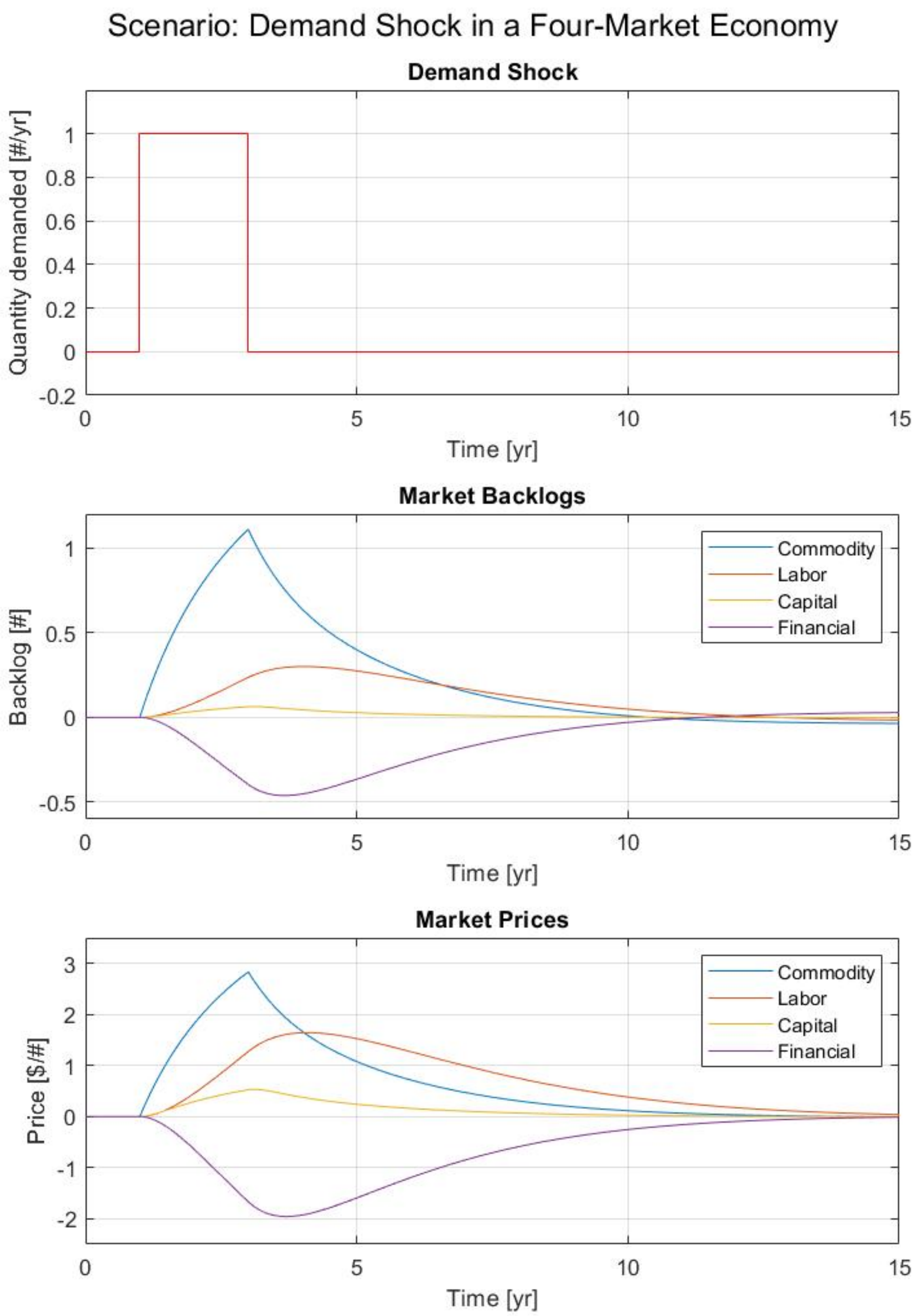


Figure 5-9: Simulation of a demand shock in the commodity market. Following the demand shock, the prices of commodities increase. Because of the substitution effect, the prices of labor, capital, and financial goods also increase. After the demand shock has passed, the economy reverts to its prior equilibrium.

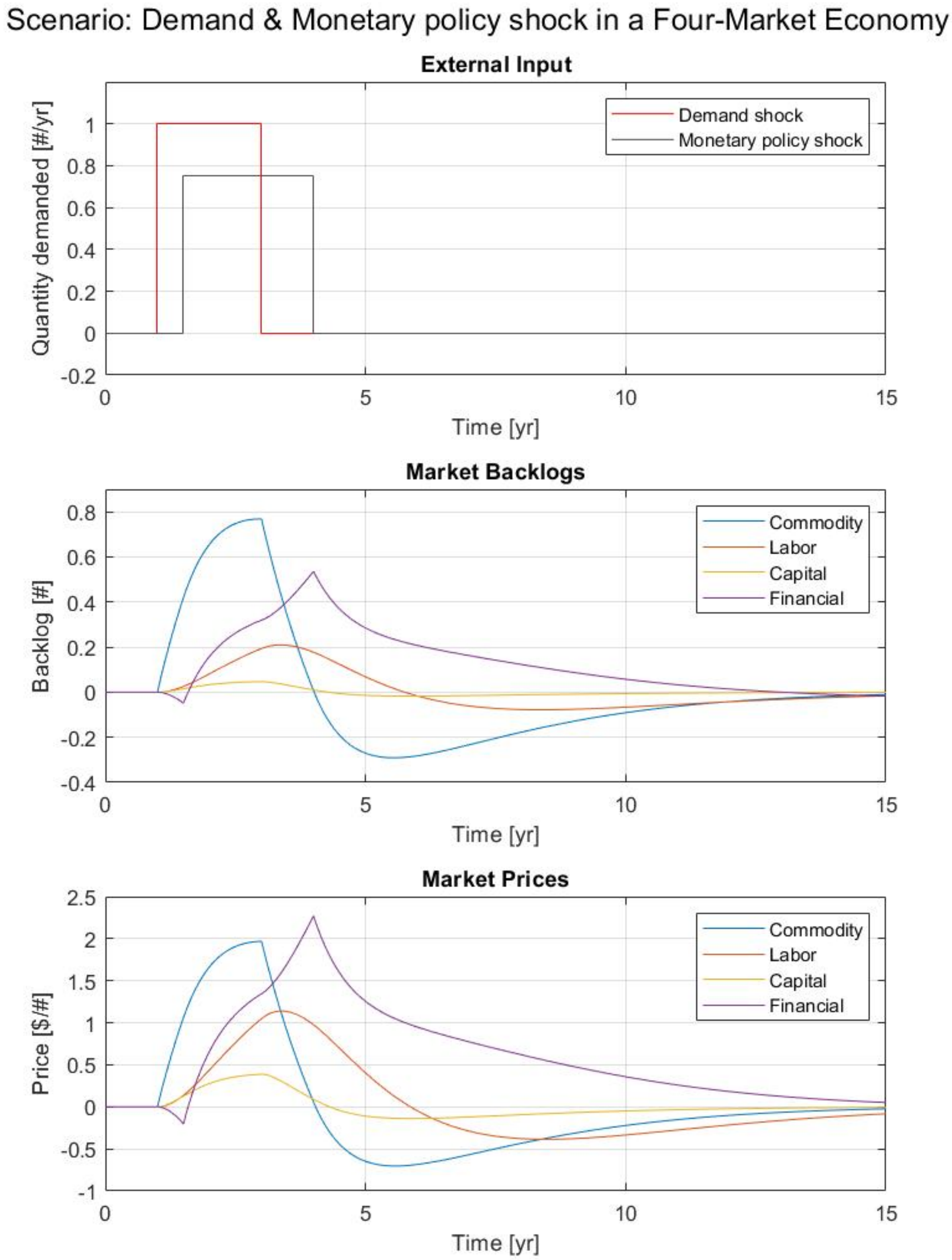


Figure 5-10: Simulation of a demand shock and monetary policy input. Following the demand shock, we observe an increase in inflation, expressed by a rise in the price of commodities. In response, monetary theory stipulates a monetary tightening cycle must proceed. Therefore, the central bank increases its demand for financial goods, counteracting the price increase in commodities induced by the demand shock. Following the two shocks, the economy reverts to its prior equilibrium.

Inflationary Episode

For the second scenario, we simulate an inflationary episode. To achieve this, we apply an identical demand shock in combination with a monetary policy input in the financial market that imitates the central bank's response in reaction to the increase in inflation.[7] Because of the central bank's monetary policy input, the demand for financial goods increases, leading to an increase in the price. Because the price of financial goods and demand for commodities are inversely related, this action offsets an increase in the price of commodities, limiting inflation.

In Figure 5-10, the above scenario is plotted, where the upper plot again depicts the external inputs, the middle plot the market backlogs, and the lower plot the market prices. Once the monetary policy input hits, we see an immediate increase in the price of financial goods and a decrease in the price of commodities. Moreover, the monetary policy input offsets the price increases in labor and capital. Economically, a monetary tightening cycle induces a period of decreased demand, resulting in a decrease in labor and capital investments. Accordingly, we can verify that the price of labor and capital drops.

If we compare the simulation in Figure 5-9 to Figure 5-10, we see that because of the monetary policy input, the amplitude of the price increase in commodities has around halved. As a result, this scenario provides two insights: First, it shows how external shocks in different markets can reinforce or weaken the price and demand in other markets through the substitution effect. Second, it demonstrates the effects of monetary policy.

5-5 Laplace Domain Analysis

If we compare the simulations of the commodity market and the four-market economy, we notice a key difference. In Figure 5-7, both the price p_{CO} and the backlog q_{CO} of commodities initially overshoot the long-run equilibrium. This is a typical feature of an under-damped system. In Figures 5-9 and 5-10, the prices and backlogs converge exponentially to the long-run equilibrium, which is a typical feature of an over-damped system. From Section 2-3, we know this behavior is determined by the discount propensity ζ, which depends on the parameters of economic processes. As a result, we can describe markets with varying types of dynamic market responses simply by changing the parameters describing an economic process.

In case a market is under-damped, both complex discount rates have the same strictly real part σ that determines the discounting of the market. In case a market is over-damped, the market consists of two strictly real discount rates. The discount rate located further from the imaginary axis decays faster than the discount rate located closer to the imaginary axis. In the short run, both complex discount rates influence the market's behavior. In the long run, however, the market's behavior is dominated by the slower complex discount rate, as the effects of the faster complex discount rate are negligible. The market's discounting is, therefore, approximately determined by the slower complex discount rate. Interestingly, this implies that in each market, the discounting is dominated by the strictly real part σ of the

[7] The price of commodities, better known as the CPI, is an important indicator for inflation. Therefore, we interpret an increase in the price of commodities as an increase in inflation. In response, monetary theory stipulates a monetary tightening cycle must proceed. Therefore, the central bank increases its demand for financial goods, driving up the price, i.e., increasing interest rates.[72]

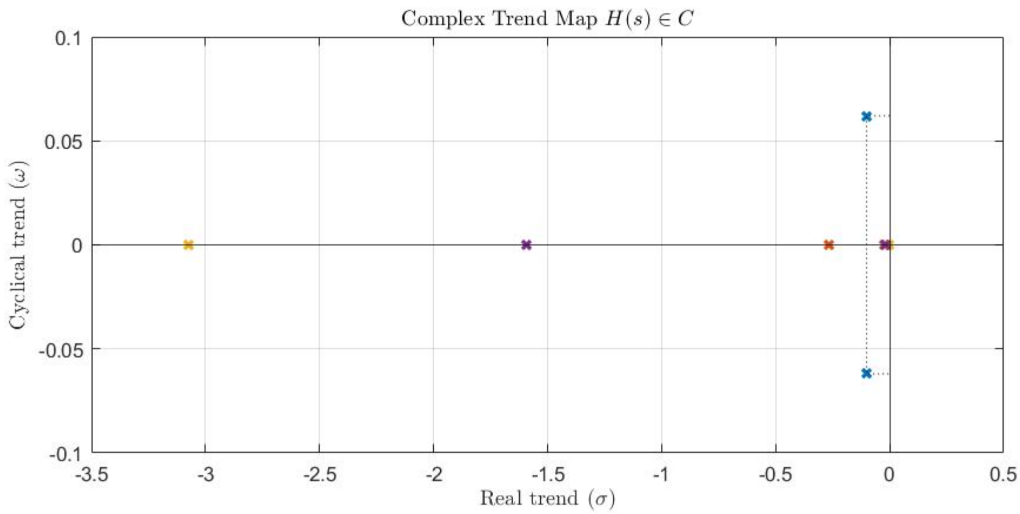


Figure 5-11: Complex trend map of the four-market economy. The four pairs of complex discount rates correspond to the complex discount rates of the respective markets. The real σ and imaginary ω parts of the complex discount rates determine the economy's discounting and inventory cycle.

market's discount rates. This is consistent with Keynes's definition of an own rate of interest, which postulates that an economy has a unique discount rate for each type of good.[34]

The complex discount rates of the under-damped market also have an imaginary part ω_d that determines the economy's cycles. From Equation 2-28, we can infer that cyclical trend ω_d species the inventory cycle of the economy adjusted for the discount propensity ζ. The true inventory cycle ω_n is given by the magnitude of the complex discount rate. Laplace domain analysis shows that cyclical behavior, such as the business cycle, is inherent to macroeconomic systems. In economics, cycles of various periodicities have been identified. Therefore, we would likely find multiple inventory cycles if we would fit the four-market economy to actual economic data.[35]

5-6 Discussion

This chapter derives a price-dynamic top-down model of a four-market economy. A limitation of the top-down modeling approach is that it builds on many of the macroeconomic principles outlined in Chapter 3. Notably, the model incorporates multiple assumptions also used for dynamic stochastic general equilibrium (DSGE) models, e.g., all agents are price takers, agents are homogeneous, rational expectations, and more. DSGE models have been heavily criticized for their strong reliance on assumptions.[73] As a result, both DSGE models as the model of the four-market economy presented behave as dynamic general equilibrium models, which exaggerate the macro-optimal behavior of the economy. Therefore, we should be critical of the many assumptions built into the four-market economy.

The assumptions underlying DSGE models are fundamental to the structure of the model.[59] The assumptions underlying the four-market economy are, in contrast, a result of design preferences. The economic-engineering approach can be used to model various economic processes, with the combination of these economic processes resulting in a distinctive dynamic market response. This demonstrates that economic-engineering models can approximate the over- and under-shoot dynamics typically observed in the short run. As a result, economic-engineering models can predict out-of-equilibrium behavior while simultaneously incorporating the long-run equilibrium dynamics.

In comparison, static equilibrium models, such as in Figure 5-8, or dynamic general equilibrium models, such as DSGE models, can only describe the economy when in equilibrium.[43] Moreover, the price-dynamic framework distinguishes economic-engineering models from existing macroeconomic models as these lack a definition of an economic force.

Using Laplace domain techniques, we were able to analyze the economy's discounting and cycles. Interestingly, we were able to link our observations of the economy's discount rates to economic phenomena, such as the business cycle and Keynes' own rates of interest, substantiating our findings. This shows how Laplace domain techniques complement the existing economic-engineering approach and can thus enhance our understanding of macroeconomic models.

Chapter 6

Bottom-up Model of the Economy

6-1 Introduction

This chapter integrates the ABM framework into the price-dynamic top-down model of the economy. We have seen that a bottom-up modeling approach can be used to transcend the generalizations and assumptions of top-down structural models. A top-down modeling approach, however, provides a structural framework that limits the complexity of the macroeconomic model. By combining both approaches, we are able to design a macroeconomic model from the bottom up while surpassing the limitations of ABM.

This chapter is split into five sections: Section 6-2 shows under which conditions we can integrate the economic networks of chapter 4 into the top-down model of chapter 5.

Section 6-3 integrates the supply chain network we designed in Section 4-11 and a housing market model into the existing four-market economy to derive a price-dynamic bottom-up model of the economy. In addition, we list the model assumptions in Section 5-3-1 and motivate how the model parameters are determined.

Section 6-4 performs time domain analysis of the bottom-up model of the economy. We show that microeconomic effects, such as the Bullwhip effect, can affect the economy on a macro-level.

Section 6-5 extends this analysis to the Laplace domain using the Laplace domain techniques developed in chapter 2. We demonstrate that Laplace domain analysis can be leveraged to gain intuitive insights into the dynamic behavior of macroeconomic models.

Section 6-6 concludes by discussing the insights of the bottom-up model of the economy and how these can contribute to existing macroeconomic models.

6-2 Integrating Bottom-up Networks in Top-down Macroeconomic Models

The four-market economy was modeled using a state-space model, which is a time-domain representation. Meanwhile, we modeled the two-port networks as transfer functions in the Laplace domain, which is a Laplace domain representation. To integrate the two models, we must either work using state-space models or transfer functions. Since a time domain representation is most intuitive to economists, we attempt to transform the two-port network into a state-space representation.

However, this can only be done under specific circumstances. Transfer functions give a unique representation of a system, whereas an infinite amount of state-space representations exist. Therefore, transforming a transfer function to a state-space representation may result in a loss of information[1].

The most practical technique for converting a transfer function to state-space realization is using the controllable canonical form (CCF) and observable canonical form (OCF). A transfer function must meet certain conditions to apply the CCF. First, the transfer function should represent a controllable system, i.e., the inputs can drive the system to any desired state, and second, the transfer function should be proper. Since single-input multiple-output (SIMO) systems are by definition controllable, the CCF can always be applied to a proper SIMO system. If a system is not controllable but observable, the OCF can be used, given that the transfer function is also proper. Similarly, multiple-input single-output (MISO) systems are, by definition, observable. Hence, the OCF can always be applied to a proper MISO system. The drawback of this approach is that the states of both forms lack a direct physical or economic interpretation.

The two-port networks in chapter 4 are multiple-input multiple-output (MIMO) transfer functions. Hence, to apply a canonical form, we first convert the MIMO system into a single-input single-output (SISO) system, which is both a SIMO and a MISO system. This can be done by introducing a constraint and shorting either the input port with a source z_S or the output port with a load z_L. The following two equations give the constraint equations:

$$V_O = -z_L I_O \tag{6-1}$$
$$V_I = -z_S I_I \tag{6-2}$$

If we then substitute the above equations into the impedance-parameter representation of a two-port:

$$\begin{bmatrix} V_I \\ V_O \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_I \\ I_O \end{bmatrix} \tag{6-3}$$

$$\begin{aligned} z_{11} &:= \left.\frac{V_I}{I_I}\right|_{I_O=0}, \quad z_{12} := \left.\frac{V_I}{I_O}\right|_{I_I=0}, \\ z_{21} &:= \left.\frac{V_O}{I_I}\right|_{I_O=0}, \quad z_{22} := \left.\frac{V_O}{I_O}\right|_{I_I=0} \end{aligned} \tag{6-4}$$

[1]E.g., pole-zero cancellation in a transfer function cancellation can lead to an order reduction of the state space.

We find the following transfer functions:

$$\frac{V_I}{I_I} = z_{11} - \frac{z_{12}z_{21}}{z_{22} + z_L} \tag{6-5}$$
$$\frac{V_O}{I_O} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + z_S} \tag{6-6}$$

As a result, the two-port network reduces to a one-port, a SISO system and, therefore, controllable and observable by definition. Given that the transfer function is proper, we can thus derive a state-space realization of an arbitrary network using the Controllable or Observable Canonical Form.

6-3 Deriving the Model

In this section, we derive a price-dynamic model of the economy that includes a supply chain network and a housing market model. This allows us to model supply chain dynamics within a macroeconomic model and simulate the interaction between macroeconomic systems and the housing market. These two applications result from our collaboration with Rabobank, as they face significant challenges in modeling these aspects within their existing macroeconomic models.

6-3-1 Model Assumptions

We have argued that the ABM framework allows us to move away from macroeconomic generalizations and assumptions. Nevertheless, several assumptions are made in practice to limit the model complexity, as we discussed for the bottom-up networks in Section 4-6. Therefore, as in chapter 4, we assume that all traders are rational and operate in perfect competition. Note, however, that these assumptions are not fundamental to the model.

Because we take the four-market economy as a basis, the resulting model inherits all assumptions outlined in Section 5-3-1, i.e., all subsidiary agents are homogeneous, price takers, and rational. Using the ABM framework, we could, in theory, continue to develop the model to mitigate these assumptions. In addition, we again introduce the two same assumptions to ensure the resulting model is a LTI system, namely that all model parameters are time-invariant and the dynamics linear. The latter is guaranteed by the fact that we only use linear elements.

6-3-2 Integrating the Housing Market

To quantify the interaction between macroeconomic systems and the housing market, we model the housing market and integrate it into the four-market economy presented in chapter 5 using the substitution effect. Since this adds another market to the model, we will refer to the model as the five-market economy.

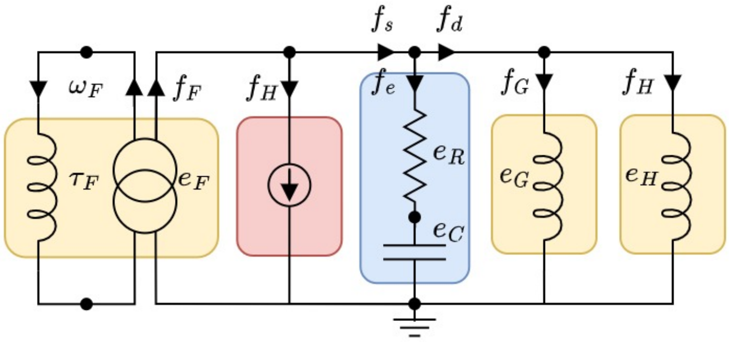


Figure 6-1: Circuit diagram analogous to the housing market. From left to right: the firms (yellow) are modeled as a series inductor and transformer, the external input as a current source (red), the firms (yellow) as an inductor, the market frictions (blue) using a series resistor and capacitor, the government (yellow) also as an inductor, and the households (yellow) also as an inductor. The arrows indicate the direction of the flow of goods, analogous to the electric current.

Housing is a capital good that provides housing services.[2] The argument for this comes from the fact that housing value is determined by the housing stock and the occupancy rate[3].[27] As a result, we model the housing market almost identically to the capital market model provided in Section 5-3-3. In this case, the firms are defined as suppliers, and the households and government as demanders. Therefore, the signs of the price elasticities ε switch. By tuning the model parameters, we can subsequently model the dynamic behavior of the housing market.

Market Dynamics

In Figure 6-1, a circuit diagram analogous to the housing market is given. Following the mathematical derivation in Section 5-3-2, the market dynamics can be written as follows:

$$\begin{bmatrix} \dot{p} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} R_H(\varepsilon_H + \varepsilon_G - \varepsilon_F) & C_H \\ (\varepsilon_H + \varepsilon_G - \varepsilon_F) & 0 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} + \begin{bmatrix} R_H \\ 1 \end{bmatrix} f_H \tag{6-7}$$

$$\begin{bmatrix} p \\ L_F \\ q \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ K_F & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} f_H \tag{6-8}$$

Because no desires act on the capital goods, the market dynamics are almost the same as given by Equations 5-10 and 5-11. If this is not the case, the market dynamics change with respect to the commodity market. Table 6-1 provides the units and description of all state variables and their derivatives.

[2]In reality, the housing market can be subdivided into homes that are rented or bought. Economists, however, assume that homeowners pay themselves a notional rent. The result is that all houses provide homogeneous housing services, enabling economists to model the housing market as a uniform market.

[3]See Section 3-5-2.

Variable	Description	Unit
q	Housing services stock	$[m^2]$
$\dot{q} = f$	Housing services	$\left[\frac{m^2}{yr}\right]$
p	Housing services price	$\left[\frac{\$}{m^2}\right]$
$\dot{p} = e$	Housing services desire	$\left[\frac{\$}{m^2 \cdot yr}\right]$
θ	Occupancy	$[\%]$
$\dot{\theta} = \omega$	Housing occupancy	$\left[\frac{\%}{yr}\right]$
L	Housing value	$[\$]$
$\dot{L} = \tau$	Housing occupancy desire	$\left[\frac{\$}{yr}\right]$

Table 6-1: State variables and their derivatives within the housing market.

We now use the substitution effect to integrate the housing market into the four-market economy. Following a similar approach as in Section 5-3-4, we replace the agents previously represented by inductors with mutual inductors. Therefore, we must define the cross-elasticities between the five markets, comprising the elasticity tensor ε.

Households consume labor services just as they consume commodity goods. Therefore, labor and commodities are modeled as substitution goods. Many houses are financed using mortgages, a type of financial good. If the cost of a mortgage increases, the demand for housing services weakens, resulting in a lower price. I.e., the prices of houses and financial goods are inversely related. Therefore, the housing and financial markets are modeled as substitution goods. The cross-elasticities between the housing, labor, and capital markets are assumed to be negligible. These cross-elasticities are summarized by Equation 6-9, which specifies the signs of all cross-elasticities:

$$\begin{bmatrix} f_{CO} \\ f_H \\ f_L \\ f_{CA} \\ f_F \end{bmatrix} = \begin{bmatrix} + & + & - & - & - \\ + & + & 0 & 0 & - \\ - & 0 & + & + & - \\ - & 0 & + & + & - \\ - & - & - & - & + \end{bmatrix} \begin{bmatrix} p_{CO} \\ p_H \\ p_L \\ p_{CA} \\ p_F \end{bmatrix} \tag{6-9}$$

To keep it concise, we chose not to include the complete derivation of the model as it is practically the same as in Section 5-4-2. Readers can find a complete mathematical description of the model in Appendix A.

6-3-3 Integrating Supply Chain Networks

We hypothesize that supply chain networks predominantly affect the commodity market. Therefore, we integrate a supply chain network in the five-market economy derived in the previous section. In reality, the supply chain topologies vary depending on the type of good. However, because the commodity market comprises all fungible goods, we use a generic hourglass-shaped supply chain network as specified in Section 4-5-1 to model the effects of supply chain dynamics.

As described in Section 6-2, we integrate the supply chain network by transforming the two-port into a one-port network. To accomplish this, we model the brokers and dealers, represented by a capacitor and resistor, as a source. Therefore, the excess flow of commodities enters the input port of the supply chain network. The flow of commodities then passes through the supply chain network, reaching the brokers and dealers. The brokers and the dealers handle the excess flow of orders f_e, which results in a corresponding incentive to the market. In Figure 6-2, we have graphically indicated this using a circuit diagram.

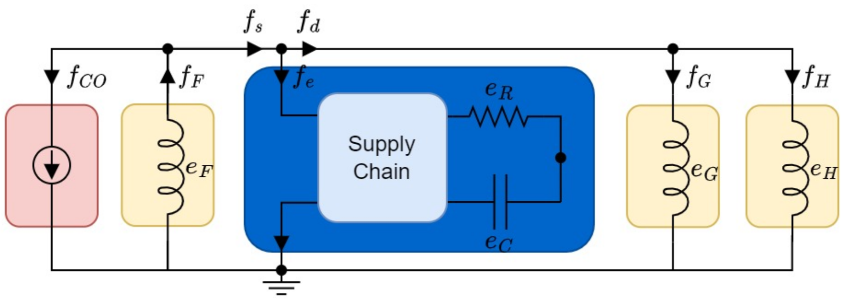


Figure 6-2: Circuit diagram of the commodity market, which includes a supply chain network. From left to right: the external input is modeled as a current source (red), the firms (yellow) as an inductor, the market frictions (blue) as a two-port shorted with a series resistor and capacitor, the households (yellow) as an inductor, and the government (yellow) also as an inductor.

The resulting one-port network is enclosed in the blue box, comprising a two-port shorted with a resistor and capacitor. Identically to the commodity market without a supply chain network, an incentive arises from the dealer's inventory and broker's trade friction. Accordingly, we model the one-port in impedance parameters as the flow of orders is the independent variable, and the incentive for orders is the dependent variable.

Market Dynamics

We begin by defining the generalized z-plane impedance of the source, which is the sum of the elasticities of the brokers and dealers:

$$z_L = \frac{1}{C_{CO}s} + R_{CO} \tag{6-10}$$

If we then substitute Equation 6-10 in Equation 6-6, we find an expression for the transfer function of the one-port:

$$\frac{V_{CO}}{I_{CO}} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + \frac{1}{C_{CO}s} + R_{CO}} \tag{6-11}$$

Next, we use the CCF, specified in Appendix D, to derive a state-space representation of the supply chain network. Figure 6-3 again shows the commodity market. In the block dock diagram, an inductor has been replaced with an integrator and gain, which gives the

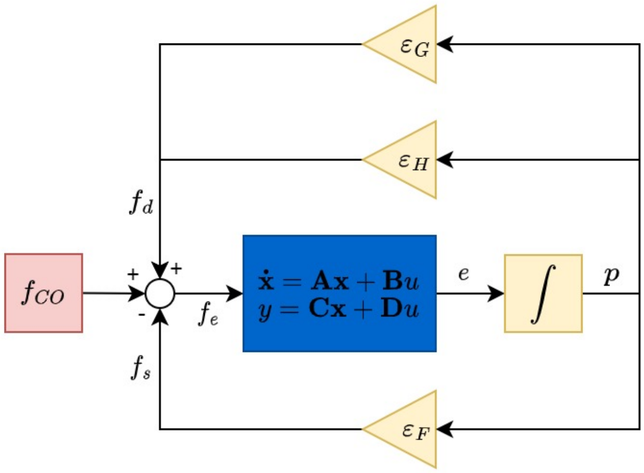


Figure 6-3: Block Diagram of the commodity market includes a supply chain network, where the one-port network has been substituted with a state space representation.

equivalent constitutive relationship of an inductor (see Equation 5-8). The one-port has been replaced with the equivalent state space representation.

Using this block diagram, we can easily derive the state-space realization of the entire commodity market. From the integral, we can derive the following relation:

$$\dot{p} = e \tag{6-12}$$

Since the market incentive e is equal to the output of the state space, we can substitute the output equation to find:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \tag{6-13}$$
$$\dot{p} = \mathbf{C}\mathbf{x} + \mathbf{D}u \tag{6-14}$$

The excess flow of orders f_e is equal to the input of the state space. From the block diagram, we can derive the following relation for the excess flow of orders:

$$f_e = (\varepsilon_H + \varepsilon_G - \varepsilon_F)p + f_{CO} \tag{6-15}$$

If we then substitute Equation 6-15 in Equations 6-13 and 6-14 we find the that:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}(\varepsilon_H + \varepsilon_G - \varepsilon_F)p + \mathbf{B}f_{CO} \tag{6-16}$$
$$\dot{p} = \mathbf{C}\mathbf{x} + \mathbf{D}(\varepsilon_H + \varepsilon_G - \varepsilon_F)p + \mathbf{D}f_{CO} \tag{6-17}$$

If we then define an augmented state vector, we find the following state equation for state space realization of the commodity market, including the supply chain network:

$$\begin{bmatrix} \dot{p} \\ \dot{\mathbf{x}} \end{bmatrix} = \begin{bmatrix} \mathbf{D}(\varepsilon_H + \varepsilon_G - \varepsilon_F) & \mathbf{C} \\ \mathbf{B}(\varepsilon_H + \varepsilon_G - \varepsilon_F) & \mathbf{A} \end{bmatrix} \begin{bmatrix} p \\ \mathbf{x} \end{bmatrix} + \begin{bmatrix} \mathbf{D} \\ \mathbf{B} \end{bmatrix} f_{CO} \tag{6-18}$$

If we then also substitute Equation 5-7 for the output equation of the state space and augment the output vector with the price p, we find the following output equation:

$$\begin{bmatrix} p \\ e \end{bmatrix} = \begin{bmatrix} 1 & \mathbf{0} \\ \mathbf{D}(\varepsilon_H + \varepsilon_G - \varepsilon_F) & \mathbf{C} \end{bmatrix} \begin{bmatrix} p \\ \mathbf{x} \end{bmatrix} + \begin{bmatrix} 0 \\ \mathbf{D} \end{bmatrix} f_{CO} \tag{6-19}$$

Note that since the states of the transfer function have no economic interpretation, we cannot determine the backlog in the market. We can, however, determine the market incentive e as it equals the output of the transfer function.

In Figure 6-4, the circuit diagram of a two-market model is displayed where again the CO subscript denotes the commodity market and the L subscript the labor market. For the same reasons as in Section 6-3-2, we only demonstrate how to derive the dynamics of a two-market economy in this section. In Appendix A, readers can find the complete mathematical descriptions of the five-market economy, including supply chain dynamics.

If we substitute Equation 5-19 for the equation of the inductor, we find the following LTI system representation for the two-market model:

$$\begin{aligned} \begin{bmatrix} \dot{p}_{CO} \\ \dot{\mathbf{x}} \\ \dot{p}_L \\ \dot{q}_L \end{bmatrix} &= \begin{bmatrix} \mathbf{D}(\varepsilon_{H11} + \varepsilon_{G11} - \varepsilon_{F11}) & \mathbf{C} & \mathbf{D}(\varepsilon_{H12} + \varepsilon_{G12} - \varepsilon_{F12}) & 0 \\ \mathbf{B}(\varepsilon_{H11} + \varepsilon_{G11} - \varepsilon_{F11}) & \mathbf{A} & \mathbf{B}(\varepsilon_{H12} + \varepsilon_{G12} - \varepsilon_{F12}) & \mathbf{0} \\ R_L(\varepsilon_{F21} - \varepsilon_{H21} - \varepsilon_{G21}) & \mathbf{0} & R_L(\varepsilon_{F22} - \varepsilon_{H22} - \varepsilon_{G22}) & C_L \\ (\varepsilon_{F21} - \varepsilon_{H21} - \varepsilon_{G21}) & \mathbf{0} & (\varepsilon_{F22} - \varepsilon_{H22} - \varepsilon_{G22}) & 0 \end{bmatrix} \begin{bmatrix} p_{CO} \\ \mathbf{x} \\ p_L \\ q_L \end{bmatrix} \cdots \\ &+ \begin{bmatrix} \mathbf{D} & 0 \\ \mathbf{B} & \mathbf{0} \\ 0 & R_L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} f_{CO} \\ f_L \end{bmatrix} \end{aligned} \tag{6-20}$$

$$\begin{aligned} \begin{bmatrix} p_{CO} \\ \mathbf{x} \\ p_L \\ L_{H,L} \\ q_L \end{bmatrix} &= \begin{bmatrix} 1 & \mathbf{0} & 0 & 0 \\ \mathbf{D}(\varepsilon_{H11} + \varepsilon_{G11} - \varepsilon_{F11}) & \mathbf{C} & \mathbf{D}(\varepsilon_{H12} + \varepsilon_{G12} - \varepsilon_{F12}) & 0 \\ 0 & \mathbf{0} & 1 & 0 \\ 0 & \mathbf{0} & K_{H,L} & 0 \\ 0 & \mathbf{0} & 0 & 1 \end{bmatrix} \begin{bmatrix} p_{CO} \\ e_{CO} \\ p_L \\ q_L \end{bmatrix} \cdots \\ &+ \begin{bmatrix} 0 & 0 \\ \mathbf{D} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} f_{CO} \\ f_L \end{bmatrix} \end{aligned} \tag{6-21}$$

Using Equation 2-15, we can subsequently obtain the transfer function matrix $H(s)$ from the system's state-space representation.

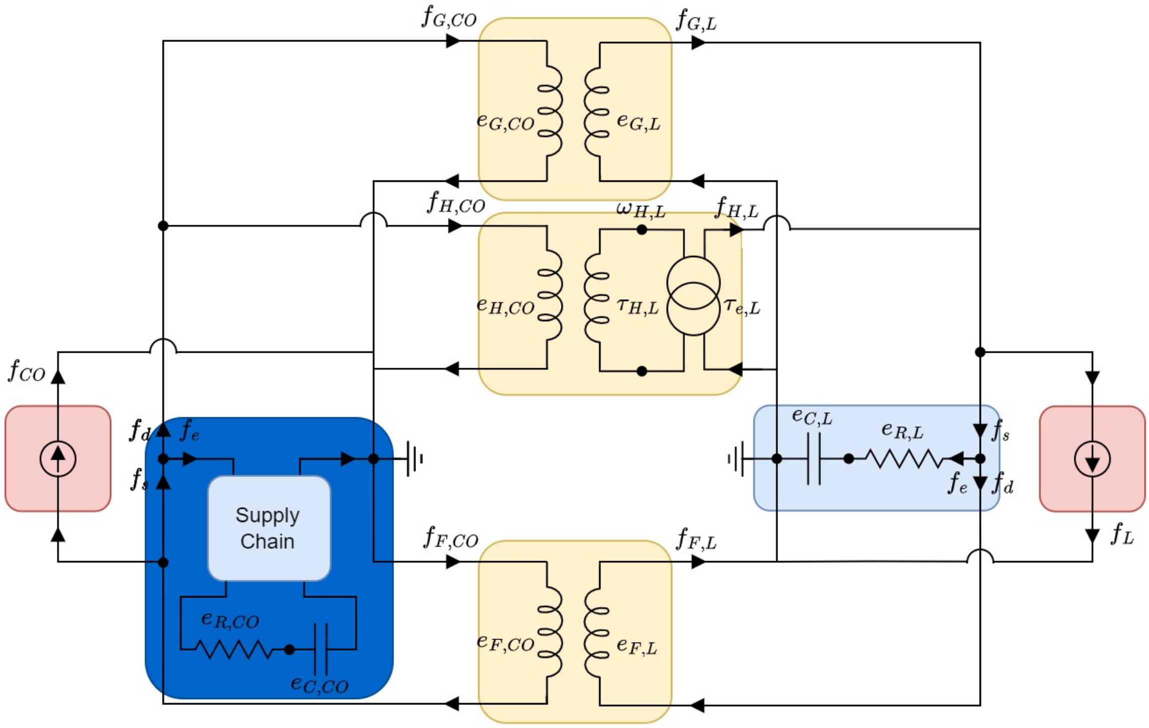


Figure 6-4: Circuit diagram analogous to a two-market economy, consisting of the labor and commodity market, including a supply chain network. Each agent (yellow) is modeled as a mutual inductor, the market frictions (blue) as (a two-port shorted with) a series capacitor and resistor, and the external inputs as voltage sources (red).

6-3-4 Parameter Estimation

In estimating the model parameters, we follow the same procedure as Section 5-3-6. That means we use the elasticity tensor as specified in equitation 6-9. In addition, we must check if all the assumptions for general equilibrium are still satisfied. We already stated in Section 6-3-1 that all traders operate in perfect competition and are rational. Since the underlying model is the same as the four-market economy from chapter 5, all markets are modeled competitively and, therefore, clear. Thus, all the economic assumptions for general equilibrium are satisfied.

Again, the algorithm specified in Appendix B estimates a stable combination of model parameters. We use comparable initial estimates for the five-market economy's parameters as in Section 5-3-6. The model parameters of the supply chain network were chosen such that the transfer function was of the same magnitude as the transfer function of the commodity market without supply chain network (see Figure 6-9). As a result, the dynamic response of the two models is of the same order of magnitude, making it easier to compare the two models in performance.

6-4 Time Domain Analysis

In this section, we analyze the five-market economy.[4] This section focuses exclusively on simulations in the time domain, with Section 6-5 analyzing the model in the Laplace domain. Section 6-4-1 studies the effects of supply chain dynamics, and Section 6-4-2 the effects of macroeconomic dynamics on the housing market.

6-4-1 Supply Chain Dynamics

To analyze the effects of the supply chain dynamics on the macroeconomic model, we simulate three scenarios: The first scenario simulates a demand shock identically to the scenario performed in chapter 5. For the second scenario, we simulate a cyclical demand input that imitates the business cycle. In the third scenario, we simulate a fictitious cyclical demand input that exemplifies the model's behavior.

Demand Shock

We begin by simulating a demand shock in the commodity market. We anticipate that the five-market economy with supply chain network roughly exhibits the same behavior as the five-market economy without supply chain network, i.e., a temporary increase in the price of commodities. Because of the substitution effect, we also expect to observe an increase in the prices of housing services, labor services, and capital goods and a decrease in the price of financial goods. Eventually, all markets revert to their prior equilibrium after the impact of the demand shock dissipates.

Figure 6-5 plots the above scenario. The upper plot depicts the demand shock f_{CO}, the middle plot shows the market prices p of the five-market economy without supply chain network, and the bottom plot shows the market prices p of the five-market economy with supply chain network. Comparing the two models, we observe that the model's dynamic responses are roughly identical. However, for the five-market economy with supply chain network, we notice a significant increase in price volatility and a slight increase in the settling time.

In Section 4-4-4, we have seen that supply chain networks can cause demand amplification and phase shifts because of the Bullwhip effect. The demand shock must excite some of the Bullwhip frequencies ω_r, which translates into additional price volatility of the commodity price. Because of the substitution effect, this price volatility transfers into the interconnected economic markets. As a result, we observe an increase in price volatility across all markets, ultimately resulting in a longer settling time.

Business Cycle

For the second scenario, we simulate a cyclical demand input consisting of multiple sinusoidal signals of different frequencies in this scenario. Both endogenous and exogenous effects have

[4] The following link provides access to an online version of the five-market economy with supply chain network: https://drive.matlab.com/sharing/f480eaa5-2480-41f2-882f-f18226f350da

been suggested to cause the business cycle. With this cyclical demand input, we imitate an endogenously driven business cycle consisting of multiple cycles, e.g., Kitchen, Juglar, Kuznets, or Kondratiev cycles.[35]

The simulation has been visualized in Figure 6-6. The upper plot shows the cyclical demand input f_{CO}, the middle plot again shows the market prices of the five-market economy without supply chain network, and the bottom plot shows the market prices of the five-market economy with supply chain network. We observe that the commodity price roughly follows the cyclical demand input. As before, the prices for housing services, labor services, capital goods, and financial goods also change because of the substitution effect. Since the external input continuously perturbed the system, the economy never reaches a state of equilibrium.

If we compare the two models, we see that similar to the previous scenario, the dynamic responses are almost identical. However, the supply chain network tends to smooth out some price volatility, especially for the higher-frequency price fluctuations (e.g., $t = 35$ yr or $t = 65$ yr). We know that the dynamic behavior of the five-market economy with supply chain network varies depending on the cyclical trend ω. Consequently, the supply chain network attenuates some of the higher-frequency cyclical trends of the business cycle. Economically, this result can be explained by traders within the supply chain network using their storage to counteract small changes in demand. We explore this behavior further in the following scenario.

Frequency Sweep

We simulate a fictitious demand input known in engineering as a frequency sweep. A frequency sweep is a sinusoidal signal whose frequency increases with time. This fictitious input allows us to study the relationship between the cyclical trend ω of a demand input f_{CO} and the price of commodities p_{CO}. As the frequency of a signal increases, its duration decreases. Economically, we would expect the price of commodities to be less reactive to a demand shock of a shorter duration. Consequently, we hypothesize that as the cyclical trend increases, the price of commodities amplitude must decrease.

Figure 6-7 plots the above scenario. The upper plot shows the frequency sweep of the cyclical demand input f_{CO}, the middle plot depicts the market prices p of the five-market economy without supply chain network, and the bottom plot shows the market prices p of the five-market economy with supply chain network. For the five-market economy without supply chain network, we observe exactly the behavior we expected, namely an inverse relationship between the cyclical trend ω of the cyclical demand input f_{CO} and the amplitude of the price of commodities p_{CO}. However, in the five-market economy with supply chain network, the price of commodities is amplified depending on the cyclical trend of the input. We recognize this phenomenon as the Bullwhip effect.

6-4-2 Housing Market Dynamics

Since the housing market is structurally identical to the commodity market, its response to a demand shock is comparable to the simulation presented in Figure 5-7. Therefore, readers are referred to Section 5-4-1 for a more detailed discussion of the housing market dynamics. As mentioned in Section 2-3, we can choose our model parameters such that the market is under or over-damped. In addition, the cross-elasticities of each agent's elasticity tensor determine to what degree price changes in other markets influence the housing market or vice versa.

Inflationary Episode

The integration of the housing market aims to gain insight into how macroeconomic phenomena affect the housing market. Historically, inflationary episodes are associated with monetary and fiscal intervention. This often significantly impacts the economy, leading to structural changes and a reversal of economic trends. Therefore, we simulate an inflation episode, identical to the simulation presented in Section 5-9, to analyze the effects on the housing market.

The simulation has been visualized in Figure 6-8. The upper plot depicts the external inputs, the middle plot again shows the market prices of the five-market economy without supply chain network, and the bottom plot shows the market prices of the five-market economy with supply chain network. Initially, the model's response is identical to that of the demand shock. Specifically, we note that the increased demand for commodities increases the price of housing services. As we already explained in Section 5-9, the increase in commodity prices triggers a response by the central bank, which increases its demand for financial goods to counteract the price increase in commodities. The combination of an increase in the price of financial goods and a decrease in the price of commodities results in the price of housing services falling. Following the two shocks, the five-market economy reverts to its prior equilibrium.

Interestingly, the price of housing services fluctuates relatively little across the inflationary episode. This implies that the housing market is more price-rigid, i.e., more inelastic than other markets. Economically, this is often the case, as homes are relatively illiquid assets. The differences between the five-market economy with and without supply chain network correspond with our observations in the previous section.

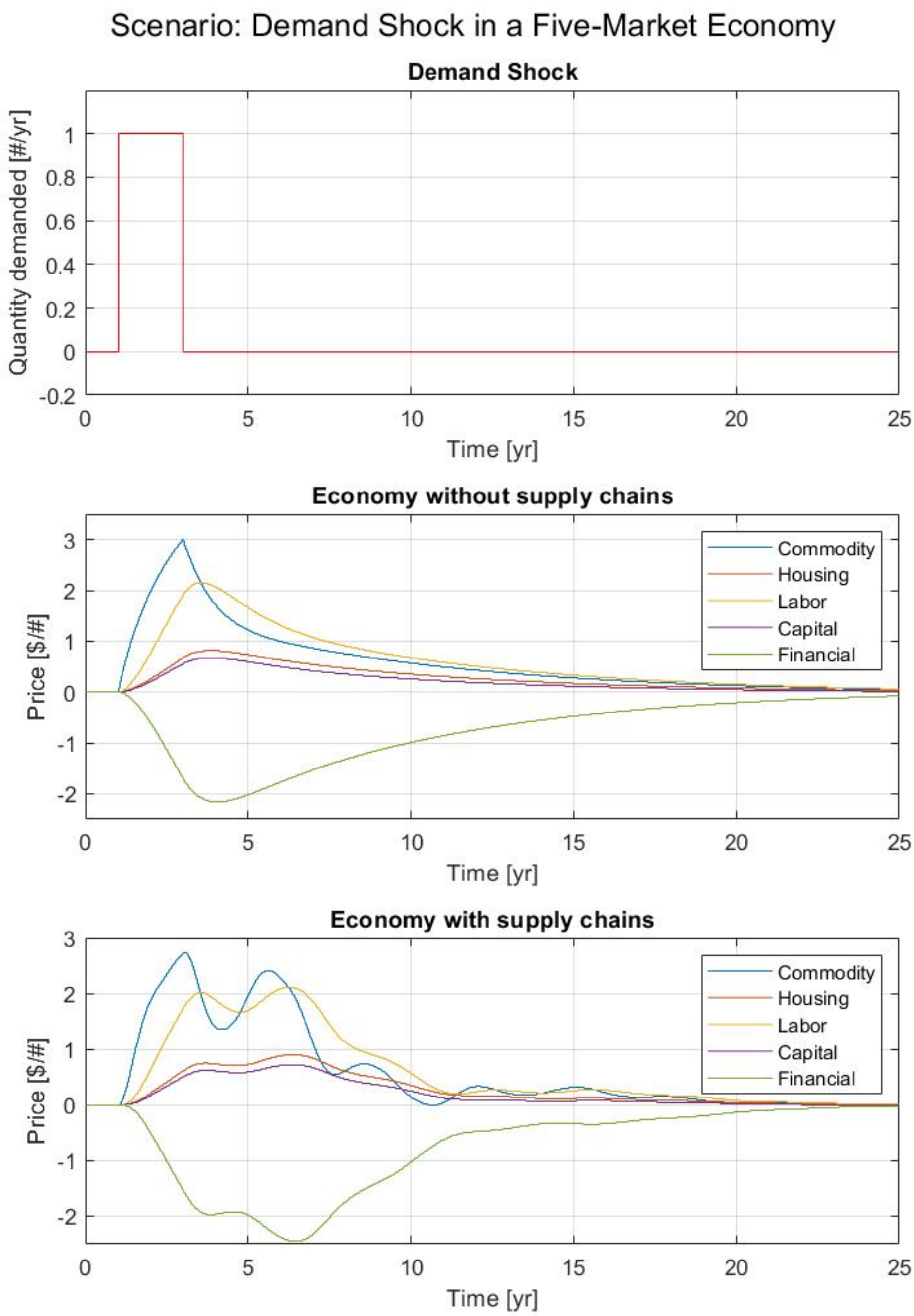


Figure 6-5: Simulation of a demand shock in the commodity market. Following the demand shock, we observe an increase in the price of commodities. The supply chain network causes an increase in price volatility, which results in a longer settling time. After the demand shock has passed, the five-market economy reverts to its prior equilibrium.

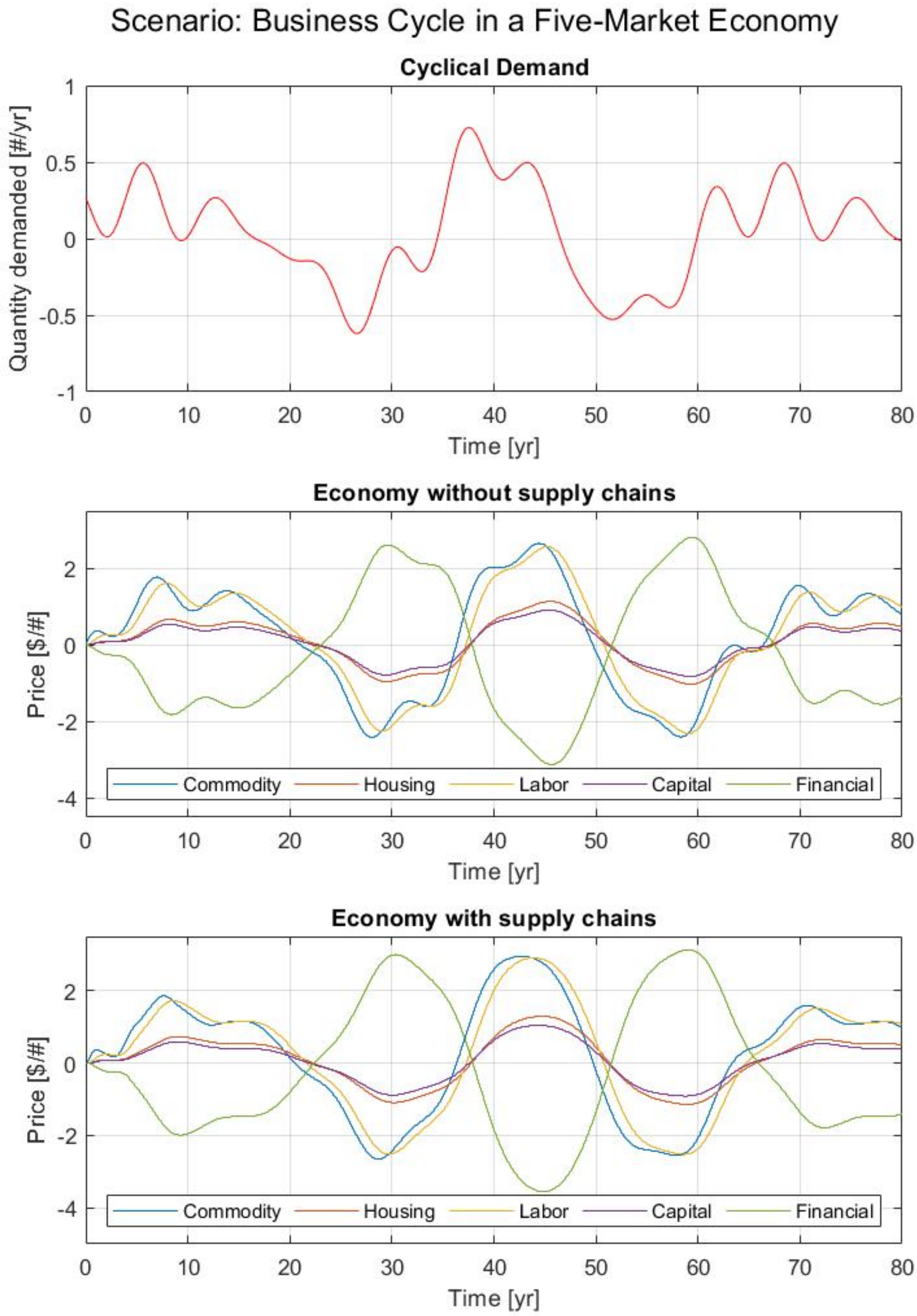


Figure 6-6: Simulation of a cyclical demand input imitating the business cycle. The commodity price roughly follows the cyclical demand input. For higher-frequency demand fluctuations (e.g., $t = 35$ yr or $t = 65$ yr), the five-market economy with supply chain network tends to smooth out price volatility. Since the external input continuously perturbs the system, the five-market economy never reaches a state of equilibrium.

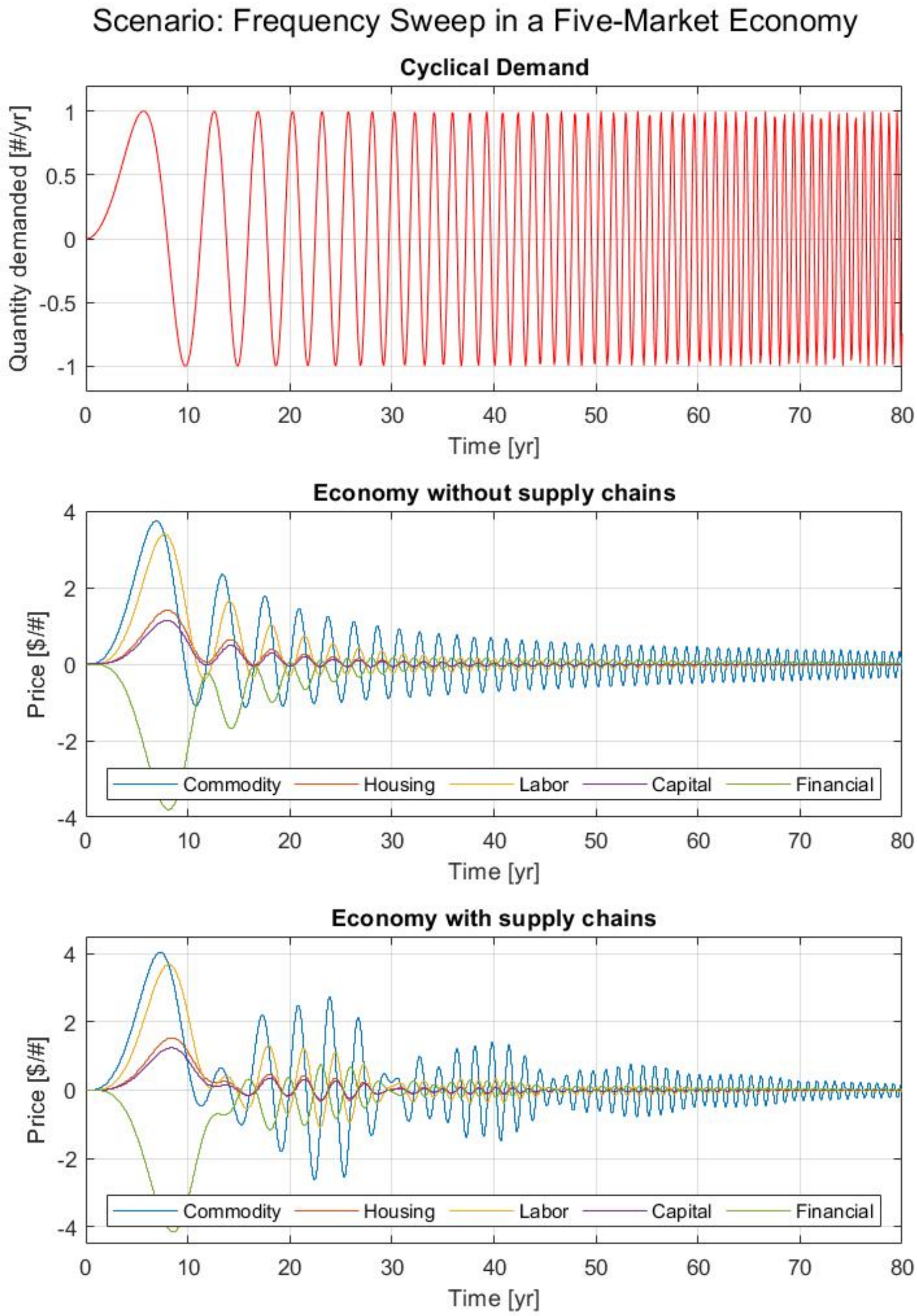


Figure 6-7: Simulation of a frequency sweep as cyclical demand input. In the five-market economy without supply chain network, the amplitude of the price of commodities decreases as the cyclical demand input's frequency increases. In the five-market economy with supply chain network, the commodity price's amplitude varies as the frequency of the cyclical demand input increases. This phenomenon is known as the Bullwhip effect.

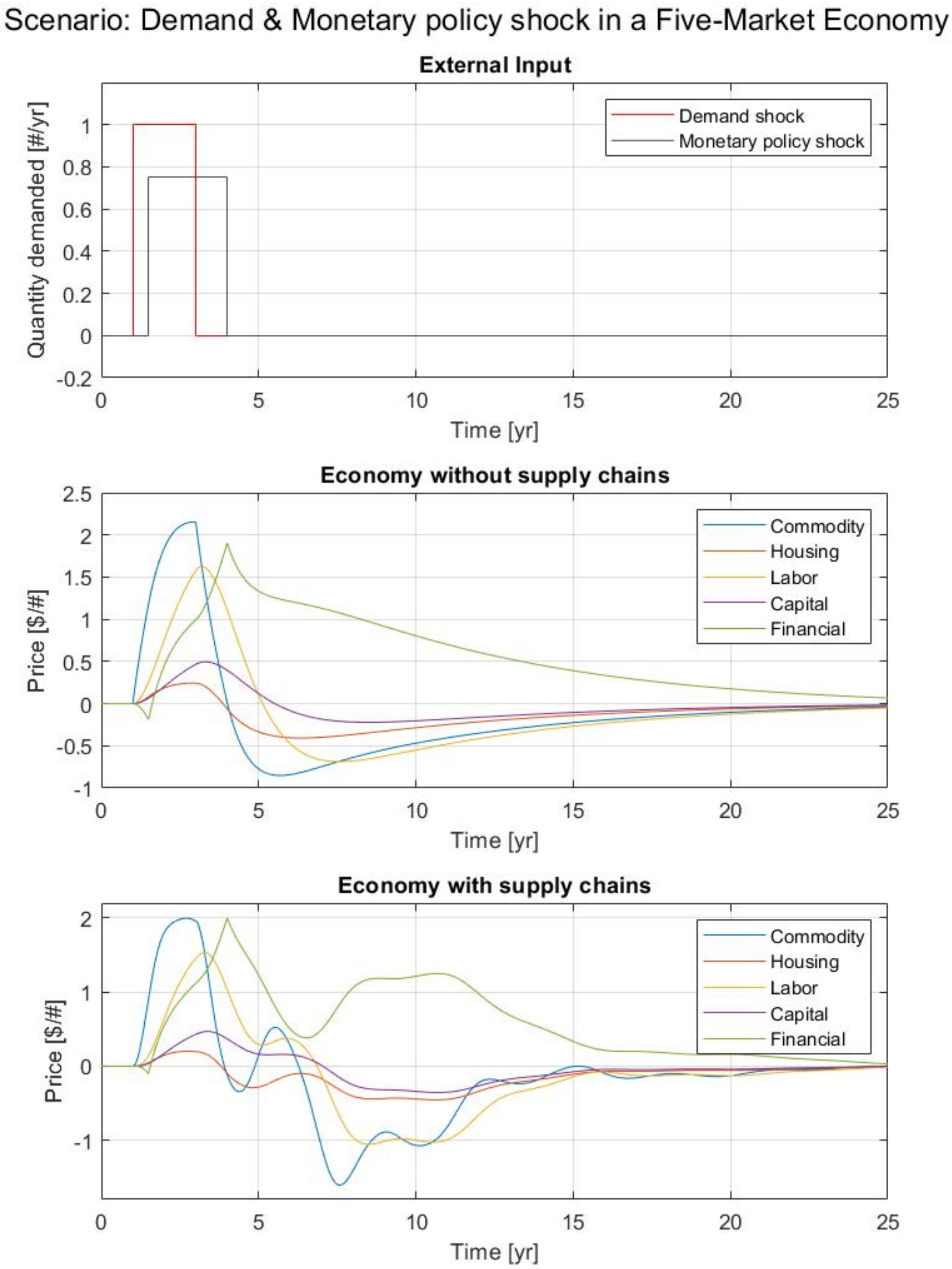


Figure 6-8: Simulation of a demand shock and monetary policy input. Following the demand shock, we observe an increase in inflation, expressed by a rise in the price of commodities. As a result, the price of housing services initially increases. In response, the central bank increases its demand for financial goods, counteracting the commodity price increase. As a result of the rate hikes, the price of housing services falls. Following the two shocks, the five-market economy reverts to its prior equilibrium.

6-5 Laplace Domain Analysis

This section extends our analysis of the five-market economy to the Laplace domain. Section 6-5-1 analyzes the effects of supply chain dynamics, and Section 6-5-2 analyzes the effects of macroeconomic dynamics on the housing market.

6-5-1 Supply Chain Dynamics

We visualize the last scenario presented in Section 6-4-1 using a Bode plot. Subsequently, we visualize the same scenario using a complex trend map to highlight the differences between the five-market economy with and without supply chain network. Last, we visualize the first two scenarios presented in Section 6-4-1 in the Laplace domain, demonstrating how the generalized uncertainty principle affects scenario analysis.

The Bullwhip Effect

A Bode plot visualizes the same relation as we simulated using the frequency sweep in Section 6-4-1. By comparing the price of commodities (output) to the cyclical demand (input) in Figure 6-7, we can reconstruct the model's amplitude and phase response.

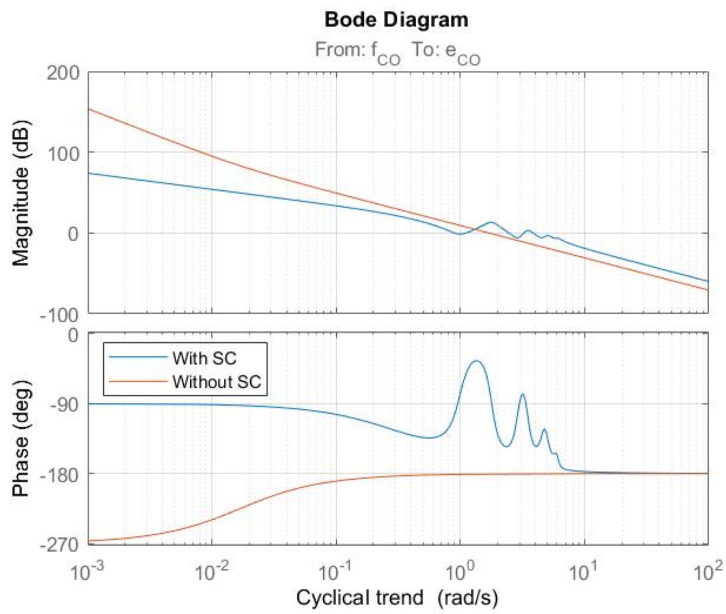


Figure 6-9: Bode diagram (Z-parameters) of the transfer function between the external input f_{CO} and the incentive e_{CO} of the commodity market with and without supply chain network. Broadly, the two transfer functions are comparable in magnitude. However, the five-market economy with supply chain network has several elasticity peaks corresponding to a locally increased inelasticity. This dynamic phenomenon is known as the Bullwhip effect in supply chain literature.

Figure 6-9 depicts the Bode plot of the transfer function between the external input f_{CO} and the incentive e_{CO} of the commodity market with and without supply chain network. Note that we have chosen to visualize the transfer function between the flow of goods and the incentive instead of the price, as this gives us the elasticity $Y(s)$.[5].

The magnitude plot confirms what we already observed in the time domain, i.e., in the five-market economy without supply chain network, the incentive for commodities e_{CO} is inversely related to the cyclical trend s, and in the five-market economy with supply chain network, the incentive for commodities is amplified at Bullwhip frequencies ω_r. Moreover, we see a strong resemblance between Figure 6-9 and Figure 4-11. In the latter's case, we established that the phase lead corresponds to the speculative behavior of agents, explaining the demand amplification. Similarly, we observe in Figure 6-9 that the phase lead coincides with the elasticity peaks in the magnitude plot. This implies the Bullwhip effect observed in the five-market economy is similarly the result of speculative behavior.

Critical, however, is that the Bode plot in Figure 6-9 simplifies the scenario we attempted to visualize in Figure 6-7. We can instantly identify system properties, such as the relationship between the price of commodities p_{CO} and cyclical trend ω, the Bullwhip frequencies ω_r, demand amplification, and the trader's speculative behavior, demonstrating how Laplace domain techniques generate insights into complex economic phenomena.

Complex Trend Map

Alternatively, we can visualize the transfer function between the external input f_{CO} and the incentive e_{CO} of the commodity market using a complex trend map. Figure 6-10 visualizes the complex discount rates and transmission-blocking rates of the five-market economy with and without supply chain network.

The complex trend map of the five-market economy without supply chain network is comparable to that of the four-market economy in Section 5-5. I.e., the real parts σ of the complex discount rates determine the economy's discounting, and the imaginary parts ω_d of the complex discount rates determine the economy's inventory cycles ω_n adjusted for the discount propensity ζ. Since we included the housing market, the five-market economy now consists of five pairs of complex discount rates. As a result, the economy's business cycle comprises two inventory cycles and contains an additional discount rate for housing.

In addition, we observe an increase in primarily complex conjugate discount rates and transmission-blocking rates. We demonstrated in Section 2-3 that these additional complex conjugate discount rates cause the Bullwhip effect. These additional complex conjugate discount rates and transmission-blocking rates result from integrating the supply chain network. Interestingly, while the economy with and without supply chain network contains imaginary valued complex discount rates, we exclusively observe the bullwhip effect in the five-market economy with supply chain network.

From Equation 2-27, we deduced that only complex discount rates that correspond to discount propensities larger than $\zeta < 1/\sqrt{2}$ cause the bullwhip effect. These are all the complex discount rates that lie outside the black dotted cone indicated in Figure 6-10. Because the discount propensity of the complex discount rates of the five-market economy without supply

[5]Since an incentive is equal to the change in the price, we can obtain the transfer function of the commodity price by multiplying with a differentiator. I.e., an increase of 20 dB in the magnitude's slope and 90 degrees phase lead.

chain network is relatively large, the bullwhip effect is not observed in the five-market economy without supply. Conversely, the discount propensity of the complex discount rates of the five-market economy with supply chain network is relatively small, causing the Bullwhip effect.

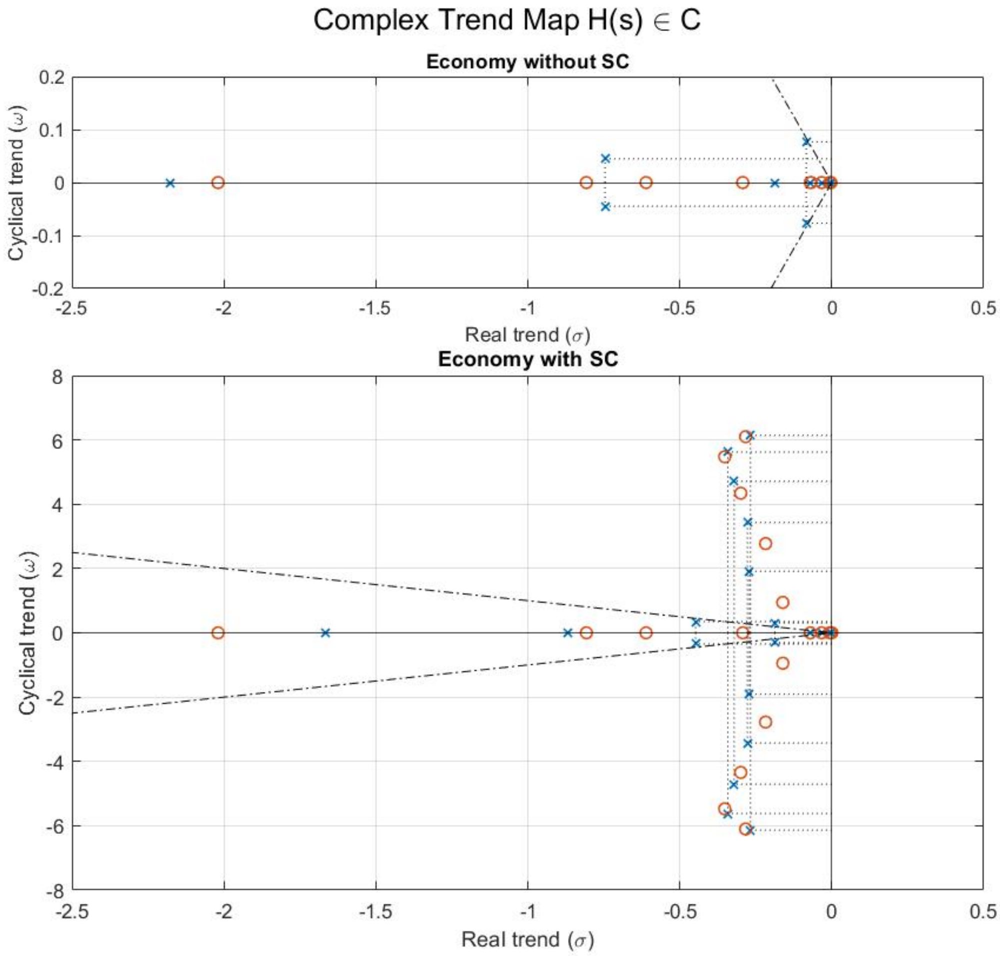


Figure 6-10: Complex trend map of the transfer function between the external input f_{CO} and the incentive e_{CO} of the commodity market with and without supply chain network. The complex discount rates' real σ and imaginary ω parts determine the economy's discounting and inventory cycles. The transmission-blocking rates indicate no economic surplus is transferred.

Generalized Uncertainty Principle

In the previous two scenarios, we looked at a transfer function between the external input f_{CO} and the incentive e_{CO} of the commodity market. In this scenario, we again analyze the commodity market, but now visualize the willingness for commodities V_{CO} (i.e., output signal) as a result of two input signals input I_{CO}: a demand shock and a cyclical demand input imitating the business cycle. These input signals are chosen identically to the demand shock and business cycle simulated in Section 6-4-1.

The Laplace response of the willingness for commodities V_{CO} following a demand shock I_{CO} is plotted in Figure 6-11. Since the demand shock is time-limited, the Laplace response of the willingness for commodities must be band-unlimited because of the generalized uncertainty principle. If we study the magnitude plot, we see that this is the case. As a result, the Laplace response provides little insight into the economy's behavior. This is expected as the Laplace transform is best suited for analyzing signals comprising sinusoids and exponentials, which the demand shock (pulse) is not.[66]

In addition, we observe several peaks in the magnitude plot whose locations correspond to the complex discount rates presented in the previous simulation. This is because the impulse response of the commodity market has a peak at each complex discount rate location, whose magnitude approaches infinity. Also, we observe that for complex trends s that exceed the discount rate of the commodity market, the willingness for commodities V_{CO} becomes unbounded. This is because the detrended incentives for commodities $e^{-\sigma t}$ increase exponentially in time, causing the Laplace transform to be undefined. Therefore, these complex trends lie outside the region of convergence (RoC) of the Laplace transform.

The Laplace response of the willingness for commodities V_{CO} following a cyclical demand input I_{CO} is plotted in Figure 6-6. As discussed in Section 6-4-1, the cyclical demand input consists of four sinusoidal frequencies. Consequently, the input is time-unlimited, causing the Laplace response of the willingness for commodities to be band-limited. The magnitude plot is dominated by four complex conjugate peaks corresponding to the sinusoidal frequencies of the cyclical demand input. As a result, the willingness for commodities closely follows the cyclical demand input. We also observe a smaller complex conjugate peak with a nonzero real trend σ and a comparatively high cyclical trend ω. As a result, higher-frequency fluctuations in the cyclical demand input are damped out.

These Laplace domain observations can explain the time domain response of the five-market economy with supply chain network in Figure 6-6: First, it explains why commodity prices closely follow the cyclical demand input. Second, it explains why the five-market economy with supply chain network smooths out some of the higher-frequency price volatility. This again shows how Laplace domain techniques can provide insights into complex economic phenomena. This section also demonstrates that the generalized uncertainty principle motivates analyzing a signal in the time or Laplace domain.

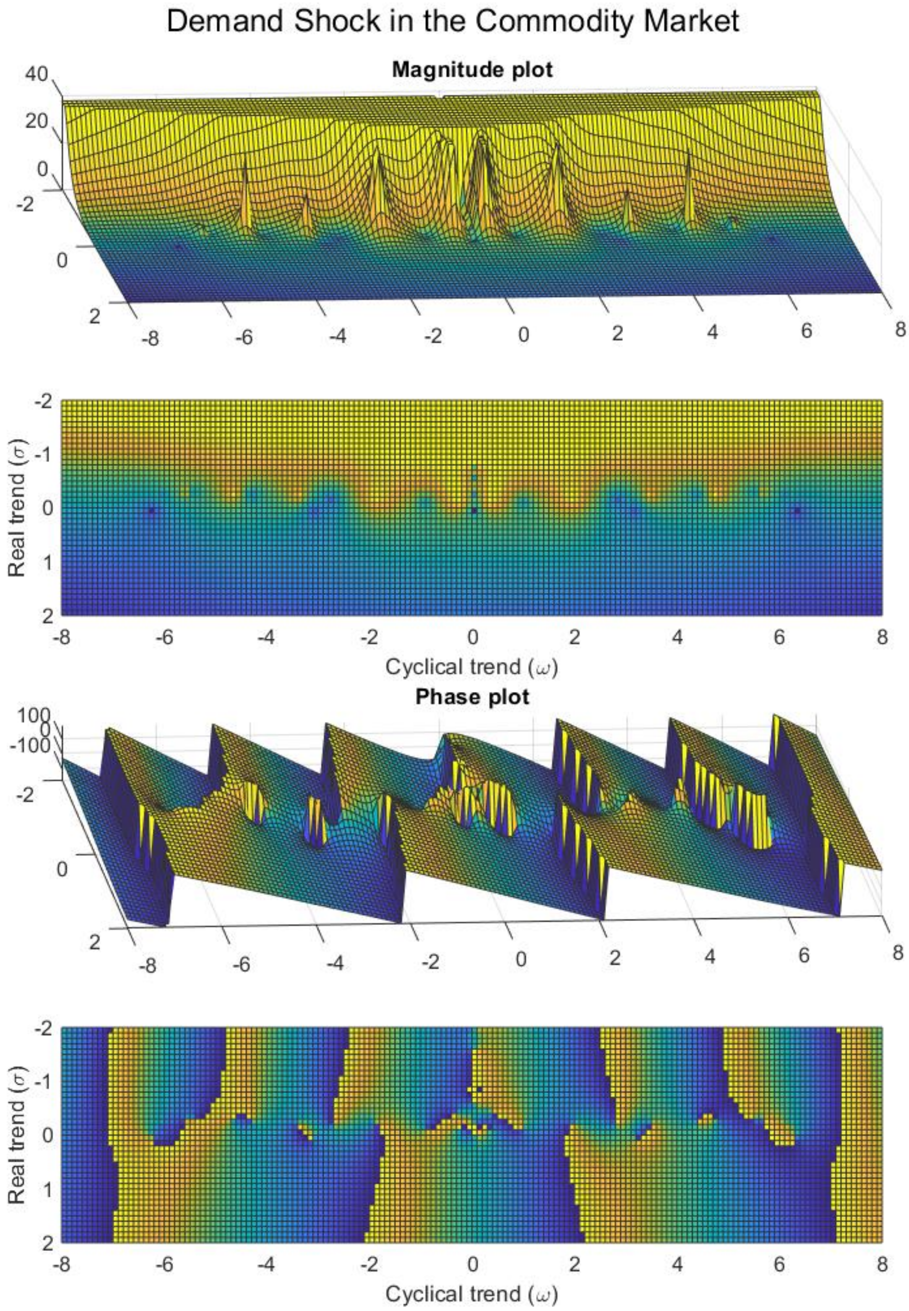


Figure 6-11: The Laplace response of the willingness for commodities V_{CO} following a demand shock I_{CO} (pulse). Since the demand shock is time-limited, the Laplace response is band-unlimited because of the generalized uncertainty principle. As a result, the Laplace response provides little insight into the economy's dynamic behavior.

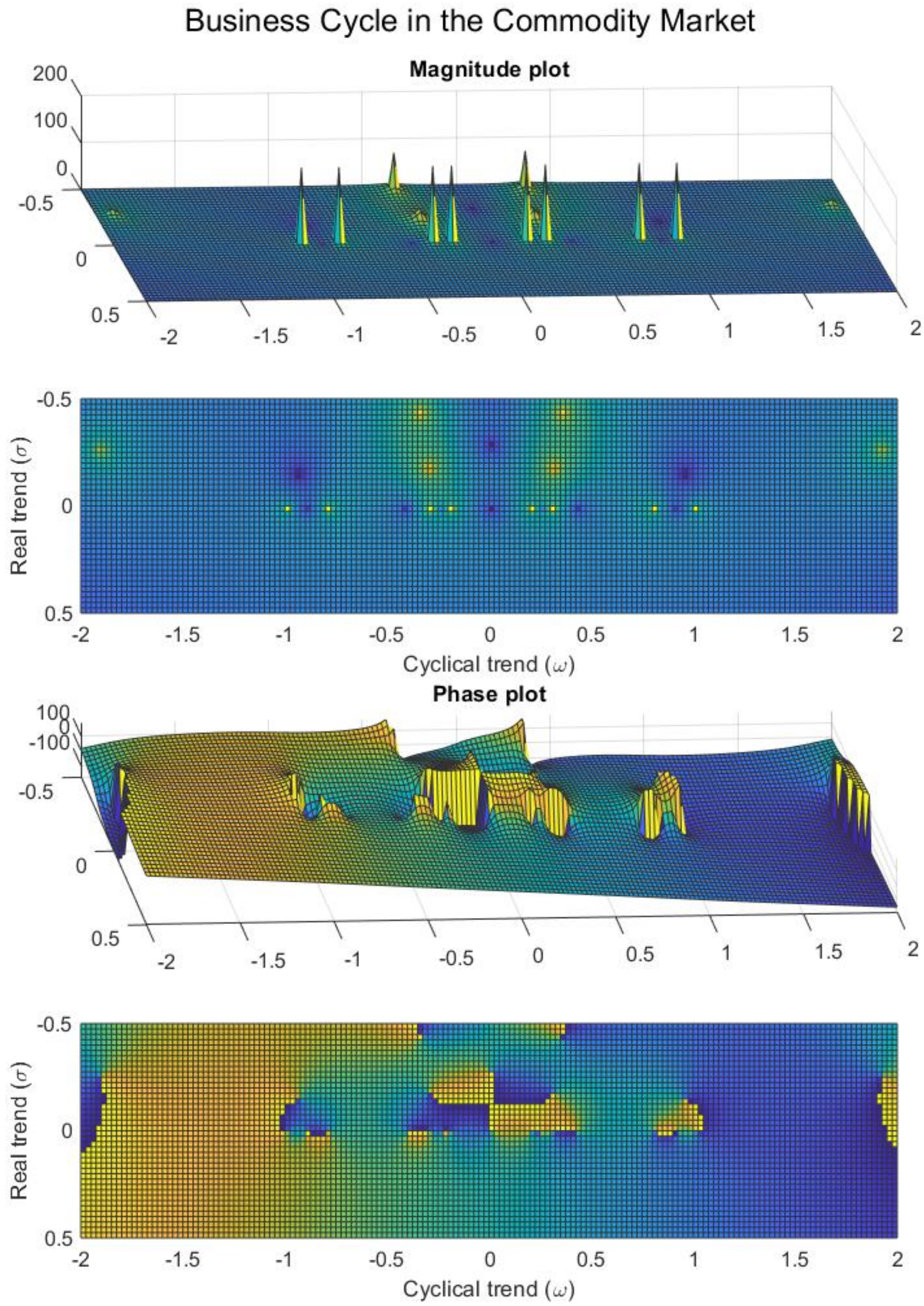


Figure 6-12: The Laplace response of the willingness for commodities V_{CO} following a cyclical demand input I_{CO} imitating the business cycle. The input is time-unlimited, causing the Laplace response to be band-limited. Because the magnitude plot is dominated by four complex conjugate peaks corresponding to the cyclical demand input, the willingness for commodities V_{CO} closely follows the cyclical demand input I_{CO}.

6-5-2 Housing Market Dynamics

In this section, we analyze the effects of macroeconomic dynamics on the housing market. We visualize the transfer function between the external input f_H and the incentive e_H of the housing market to demonstrate this. Similar to the commodity market, this transfer function describes the housing market's elasticity $Y(s)$. Figure 6-13 visualizes the complex discount rates and transmission-blocking rates of the five-market economy with and without supply chain network.

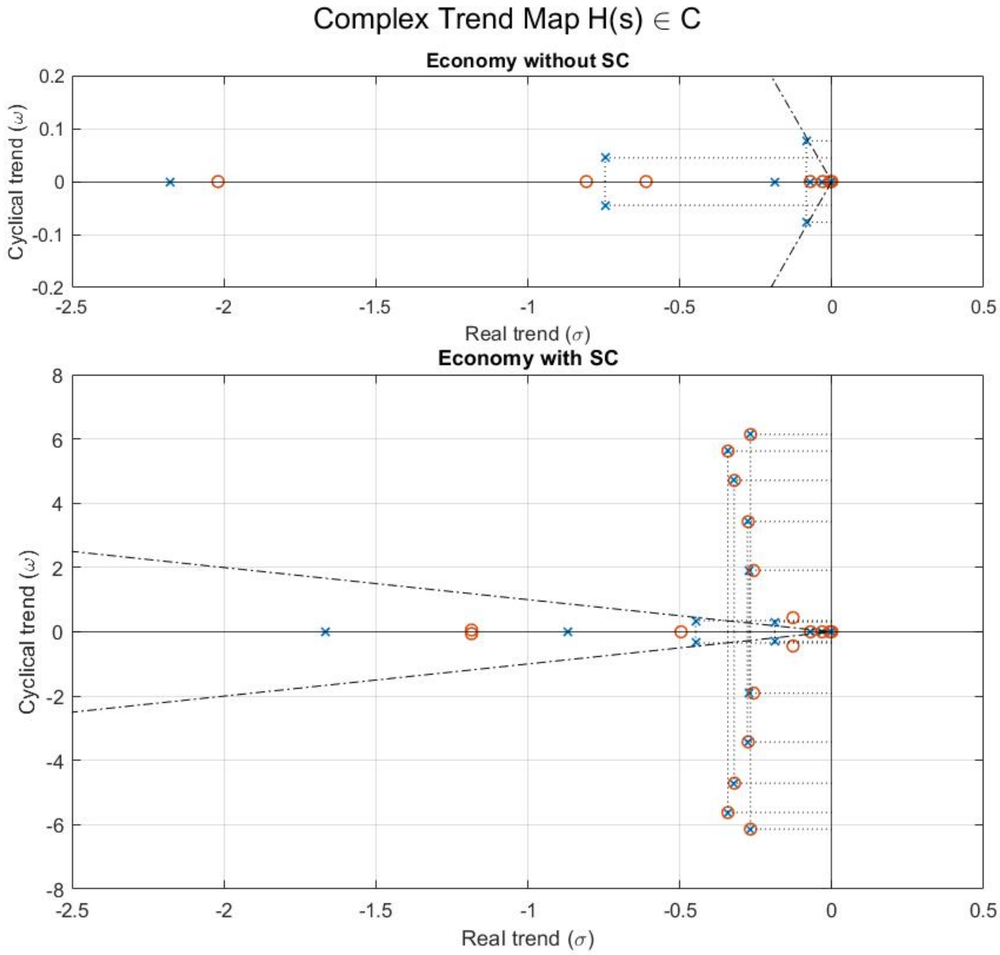


Figure 6-13: Complex trend map of the transfer function between the external input f_H and the incentive e_H of the housing market with and without supply chain network. The complex discount rates' real σ and imaginary ω parts determine the economy's discounting and inventory cycles. The transmission-blocking rates indicate no economic surplus is transferred.

Comparing Figure 6-13 to Figure 6-10, we observe that the complex discount rates are identical. Therefore, the observations based on the complex discount rates of Section 6-5-1 also extend to the housing market. This might seem counter-intuitive as the time domain responses of the two markets differ. However, the transmission-blocking rates are unique for both transfer functions, which explains why the time-domain responses differ. Economically, this is intuitive as Keynes considered the complex discount rates "own" to an economy. Therefore, an economy's complex discount rates should not depend on what input-output relation is considered.

In addition, we observe that multiple complex discount rates coincide with transmission-blocking rates, causing the dynamic effects of the respective complex discount rate to be canceled. Since especially the complex discount rates causing the Bullwhip effect (i.e., the complex discount rates located further away from the real axis) are canceled, its effects are reduced compared to the dynamic response of the commodity market. This is consistent with our time domain observations in Section 6-4-2.

6-6 Discussion

From a design perspective, we used Laplace domain techniques to model specific subsystems from the bottom up and integrate these into a top-down model of the economy. With this, we are able to systematically design bottom-up macroeconomic models to overcome an essential limitation of ABM. As a result, we can enhance specific parts of a model while limiting the overall model complexity. This contrasts the economic-engineering approach from agent-based modeling, which requires the entire model to be designed from the bottom up, resulting in overly complex models.

Simultaneously, we can model the individual decision-making processes of agents because of the ABM framework. This reduces the dependence of the model on assumptions, such as lumped agents, which are fundamental to top-down structural models. As a result, the price dynamic bottom-up model exhibits emergent behavior like the Bullwhip effect. This also shows that the bottom-up macroeconomic models surpass the top-down structural models, such as DSGE models.

Building upon the five-market economy also presents challenges as certain parts of the model are, to some extent, dependent on the macroeconomic assumptions inherent to a top-down modeling approach. However, as we argued in chapter 4, we can further extend our models to overcome these limitations. On the other hand, some assumptions may be justified using microeconomic analyses as assumptions are inherent to macroeconomic modeling.

From an analysis perspective, we were able to explain the effects of integrating the supply chain network on the five-market economy using Laplace domain techniques. By visualizing the macroeconomic system's complex discount rates, we could analyze the economy's discounting and cycles. Also, we demonstrated that the complex conjugate discount rates of the supply chain network cause the Bullwhip effect, which could more intuitively be visualized using a Bode plot. However, Laplace domain analysis of a complex present value function does not always provide further insights, as we are limited by the generalized uncertainty principle in time-limited scenarios.

This demonstrates how Laplace domain techniques make the analysis of macroeconomic models more efficient, in addition to enabling the systematic design of bottom-up macroeconomic models.

Chapter 7

Conclusion

This thesis develops the techniques to design and analyze economic systems in the Laplace domain. The critical insight is that the Laplace transform is analogous to complex detrending, allowing the analysis of economic time series in the Laplace domain. An important limitation of Laplace domain analysis follows from the generalized uncertainty principle. Based on this limit, it can be justified that an economic time series must be analyzed in the domain in which the information is localized.

By taking an input-output perspective, we can also analyze economic systems in the Laplace domain. Using the economic analogs to Laplace-domain nomenclature, we were able to explain economic economic discounting and cycles. Also, we developed tools, such as Bode plots or complex trend maps, to visualize and analyze economic systems. By providing insight into behavior that is otherwise not apparent in the time domain, Laplace domain techniques can contribute to economic modeling.

We demonstrated the efficiency of the Laplace domain techniques by developing a bottom-up macroeconomic model. The algebraic simplification of the Laplace domain enables the efficient design of economic networks from the bottom up. Integrating the bottom-up economic networks into a top-down macroeconomic model, resulted in a bottom-up model of the economy. This modeling approach significantly limits the model complexity. This contrasts the economic-engineering approach from agent-based modeling, which requires the entire model to be designed from the bottom up, resulting in overly complex models.

Simultaneously, adopting a bottom-up modeling approach enables the integration of individual decision-making processes, reducing the model's dependence on assumptions and generalizations. As a result, the bottom-up models in this thesis exhibit emergent behavior in the form of the Bullwhip effect. With this, the bottom-up macroeconomic model transcends the macro-optimal behavior of dynamic stochastic general equilibrium models.

Analyzing the model's discount rates using a complex trend map reveals the economy's own rates of interest and business cycle. By doing so, we also strengthened the economic underpinnings of Laplace domain techniques. Additionally, we showed using Bode plots that supply chain topologies cause demand amplification, which results in the Bullwhip effect. These insights allowed us to explain the effects of supply chain dynamics on the economy.

Economic engineering provides the theoretical foundation for the economic analogs to Laplace domain techniques, as it allows us to define economic time series as signals and model economic systems as LTI systems. Moreover, it establishes a price-dynamic framework for modeling the economy in the short and long run. As a result, we can predict out-of-equilibrium behavior and offer insights into the economic forces driving dynamic behavior, expanding on traditional static equilibrium models and econometric models.

In conclusion, Laplace-domain techniques enable the efficient design and analysis of economic systems. Therefore, this thesis contributes to macroeconomic modeling and the development and application of economic engineering theory.

Chapter 8

Recommendations

8-1 Theoretical Extensions

By establishing the economic interpretation of the Laplace transform, we have developed the economic analogs to Laplace domain nomenclature and a number of tools for economic analysis. However, there are many other techniques, especially within control engineering, that we can make applicable to economic systems. This section examines some promising applications for further research.

8-1-1 Valuation Methods

In Section 2-2-1, we demonstrated that the Laplace transform is a generalization of the DCF method. In economics, the DCF method is used to value capital. Economists typically discount using a single real discount rate (e.g., the risk-free rate) that accounts for the time value of money. However, since we have shown that the *net present value (NPV)* is a function of the complex trend s, all complex trends of an economic time series should be taken into account when calculating the NPV[1]. This also implies that by discounting using complex trends, the Laplace transform could be used to take into account value changes as a result of cyclical trends, such as the business cycle.

In addition, the generalized uncertainty principle applies to capital valuation. By discounting using a limited number of discount rates, economists limit the applicability of the DCF method to valuing time-unlimited economic time series. This is mathematically consistent as the DCF method integrates over time. Consequently, the NPV should be recalculated for any change in the economic time series. Nevertheless, economists sometimes use the time derivative of a NPV to determine a change in value, suggesting that this is not well understood in economics.[27]

[1] In practice, it can be argued that it is sufficient to detrend using only the dominant trends, given that the number of trends in the Laplace transform is limited.

To value time-limited economic time series, economists use the multiple of the asset's earnings, known as the multiples method.[30] In economic engineering, we achieve this by modeling a capital good as rotating around a fixed axis, as demonstrated in Section 3-5-2, where the capital value is equal to the angular momentum L. Since the value depends entirely on a time-domain description of the earnings, this method can be used to value time-limited signals. In conclusion, the generalized uncertainty principle formalizes whether the DCF method or the multiples method is more applicable for valuing economic time series.

8-1-2 Linear Control Theory

An important application of Laplace domain techniques that this thesis did not touch upon is in linear control theory. Linear control theory uses the algebraic simplification of the Laplace domain to design controllers efficiently. In economics, controllers can be used to model price makers. Price makers are agents that can influence the market price by manipulating the aggregate supply or demand.[71]

Contrary to price takers, we need active elements, in addition to passive elements, to model price makers (see Section 3-3-1). Increasing the number of exogenous inputs, however, complicates the model. Using a controller, we can determine an agent's exogenous input endogenously. We illustrate this using an example:

By managing the money supply, the central bank controls the economy's price level. Therefore, the central bank is a price maker and can be modeled as a controller. The controller is characterized by a control objective. In control theory, common control objectives are reference tracking or disturbance rejection. In the central bank's case, disturbance rejection could be used to reject, for instance, supply and demand shocks and reference tracking to maintain an inflation target.[6] Based on the control objective, the controller determines what control input, or in this case, open market operations, to provide.

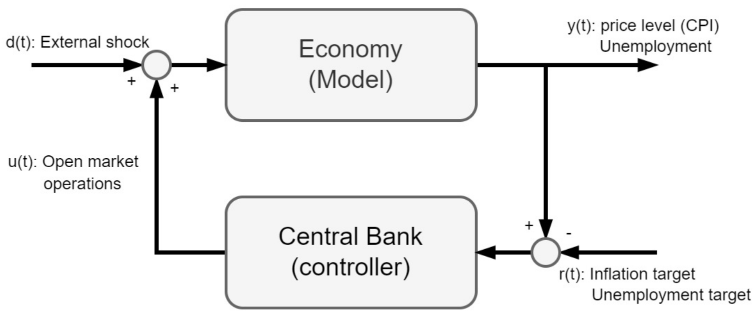


Figure 8-1: Block diagram illustrating how the central bank could be modeled as a controller. The central bank can reject disturbances $d(t)$, such as supply and demand shocks, and track a reference $r(t)$, such as an inflation target, by conducting open market operations $u(t)$.

Figure 8-1 conceptualizes the control scheme of the central bank. In control engineering, this control scheme is known as a feedback loop. Using Laplace domain techniques, we can determine the transfer functions of the model and controller. Subsequently, the closed-loop transfer function (i.e., the transfer function of the feedback loop) can be determined using only additions and multiplications.[64]

We can shape the closed-loop transfer function by tuning the controller's parameters. This controller design technique is known as loop shaping and is typically done using Bode plots. By shaping the closed-loop transfer function, we can, for instance, improve the system's disturbance rejection or reference tracking characteristics. Another controller design technique is known as pole placement. It allows to "place" a system's poles in a desired location of the complex plane, given that the system's state-space representation is known and the system is controllable and observable. Then, there are robust control strategies that aim for robust stability and performance in the presence of disturbances and model uncertainty.[64]

Economically, we can use controller design techniques to influence an economic system's dynamic behavior. Loop shaping could, for instance, be used to remove an elasticity peak from a supply chain network's elasticity function. This allows us to mitigate the bullwhip effect, in which the controller represents a supply chain manager.[16] Alternatively, we could use pole placement to design a controller representing the central bank or government that counteracts the business cycle.[17] There are numerous other applications to consider. However, the important insight is that controller design techniques add new possibilities for the design of economic systems.

8-2 Macroeconomic Modeling Extensions

The price-dynamic framework of economic-engineering models allows us to integrate subsystems at any level, accommodating distinctive types of agents, networks, or markets. We have demonstrated this by integrating supply chain networks and the housing market. In general, however, the economic-engineering approach can be used to rapidly prototype macroeconomic models. This section discusses possible model extensions specific to the economic-engineering.

8-2-1 Intelligent Agents

In Section 8-1-2, we stated that price makers can be modeled using controllers. Price makers do not necessarily maximize their utility but can pursue different economic policies. To achieve this, price makers generally use predictions to determine their policy optimally. Therefore, such agents are also known as intelligent agents.[61]

Depending on the characteristics of the intelligent agent, we can select and tune from various control strategies. As discussed in Section 8-1-2, traditional linear control techniques typically use simple proportional integral derivative (PID) controllers to perform reference tracking of disturbance rejection. Alternatively, we can use optimal controllers, such as model predictive control (MPC), that use internal predictions of the model to optimize the control input. Moreover, this enables the design of custom control objectives and constraints on the system and controller.[56]

Economically, optimal control can be used to model the objectives and constraints of intelligent agents more accurately. In the example of the central bank, we could design a dual mandate monetary policy, which amounts to stabilizing prices and reducing unemployment.[4][7]. Additionally, we could include practical limitations of monetary policy, such as the liquidity trap.[36][18]

From a macroeconomic modeling perspective, introducing intelligent agents has interesting applications. Firstly, we can model price takers endogenously increasing, which enables us to reduce a model's dependence on assumptions and generalizations. Secondly, we could use controllers to increase a model's performance under the influence of regime changes. By endogenously modeling the economic policies of, for instance, the central bank and government, we can simulate their response to shifting regimes.[53] By extension, this allows us to design macroeconomic models that are more robust with respect to the influential Lucas critique.[40]

8-2-2 Model Validation

A critical step for further research is to validate the macroeconomic models in this thesis. So far, we have qualitatively validated the models by comparing the simulations with macroeconomic literature. Next, the models should be quantitatively validated using economic data. To begin, this can be done for subsystems, such as an economic market or supply chain network. Subsequently, this can be extended to the validation of, for instance, the price-dynamic bottom-up model of the economy.

To achieve this, we must perform system identification, as briefly discussed in Section 5-3-6. Essential for system identification to succeed is that the economic data should be causally related. This means that historical data must be carefully examined to make sure the data is causal. Potentially, well-studied historic events, such as the supply shocks following the oil crisis of the 1970s, can be considered.[43]

However, initial attempts to perform system identification demonstrated that the model tends to overfit data. This is mainly a consequence of the many model parameters that arise from the price-elasticity matrices e_H, e_G, and e_F. A possible solution would be using statistical methods, such as the Granger causality test, to eliminate some model parameters. Further research must show whether this improves the fit of the model. Once a qualitative validation of the model has been completed, we should re-evaluate whether the underlying assumptions and generalizations of the models are permissible.[25]

Appendix A

Mathematical Description Models

A-1 Capital Market

The derivation of the market's state (A-1) and output (A-2) equations is identical to the derivation of the labor market outlined in Section 5-3-3:

$$\begin{bmatrix} \dot{p} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} R_{CA}(\varepsilon_F - \varepsilon_H - \varepsilon_G) & C_{CA} \\ (\varepsilon_F - \varepsilon_H - \varepsilon_G) & 0 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} + \begin{bmatrix} R_{CA} \\ 1 \end{bmatrix} f_{CA} \tag{A-1}$$

$$\begin{bmatrix} p \\ L_{H,CA} \\ q \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ K_{H,CA} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} f_C \tag{A-2}$$

Table A-1 gives the units and description of all state variables and their derivatives.

A-2 Financial Market

The derivation of the market's state (A-3) and output (A-4) equations is identical to the derivation of the labor market outlined in Section 5-3-3:

$$\begin{bmatrix} \dot{p} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} R_F(\varepsilon_H + \varepsilon_G - \varepsilon_F) & C_F \\ (\varepsilon_H + \varepsilon_G - \varepsilon_F) & 0 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} + \begin{bmatrix} R_F \\ 1 \end{bmatrix} f_F \tag{A-3}$$

$$\begin{bmatrix} p \\ L_{F,F} \\ q \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ K_{F,F} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} f_F \tag{A-4}$$

Table A-2 gives the units and description of all state variables and their derivatives.

Variable	Description	Unit
q	Product stock	$[kg]$
$\dot{q} = f$	Product flow	$\left[\frac{kg}{yr}\right]$
p	Product price	$\left[\frac{\$}{kg}\right]$
$\dot{p} = e$	Product desire	$\left[\frac{\$}{kg \cdot yr}\right]$
θ	Accumulated yield	$\left[\frac{kg}{ppe}\right]$
$\dot{\theta} = \omega$	Production yield	$\left[\frac{kg}{ppe \cdot yr}\right]$
L	Capital Value	$\left[\frac{\$ \cdot ppe}{kg}\right]$
$\dot{L} = \tau$	Capital value desire	$\left[\frac{\$ \cdot ppe}{kg \cdot yr}\right]$

Table A-1: State variables and their derivatives within the capital market.

Variable	Description	Unit
q	Accumulated interest	$[\$]$
$\dot{q} = f$	Interest income	$\left[\frac{\$}{yr}\right]$
p	Risk adjustment factor	$[-]$
$\dot{p} = e$	Risk appetite	$\left[\frac{1}{yr}\right]$
θ	Return on investment	$[\%]$
$\dot{\theta} = \omega$	Interest rate	$\left[\frac{\%}{yr}\right]$
L	Asset value	$[\$]$
$\dot{L} = \tau$	Interest rate desire	$\left[\frac{\$}{yr}\right]$

Table A-2: State variables and their derivatives within the financial market.

A-3 Four-market Economy

The derivation of the economy's state (A-5) and output (A-6) equations is outlined in Section 5-3-4:

$$\begin{bmatrix} \dot{p}_{CO} \\ \dot{q}_{CO} \\ \dot{p}_{L} \\ \dot{q}_{L} \\ \dot{p}_{CA} \\ \dot{q}_{CA} \\ \dot{p}_{F} \\ \dot{q}_{F} \end{bmatrix} = \begin{bmatrix} R_{CO}(\varepsilon_{H11}+\varepsilon_{G11}-\varepsilon_{F11}) & C_{CO} & R_{CO}(\varepsilon_{H12}+\varepsilon_{G12}-\varepsilon_{F12}) & 0 & R_{CO}(\varepsilon_{H13}+\varepsilon_{G13}-\varepsilon_{F13}) & 0 & R_{CO}(\varepsilon_{H14}+\varepsilon_{G14}-\varepsilon_{F14}) & 0 \\ (\varepsilon_{H11}+\varepsilon_{G11}-\varepsilon_{F11}) & 0 & (\varepsilon_{H12}+\varepsilon_{G12}-\varepsilon_{F12}) & 0 & (\varepsilon_{H13}+\varepsilon_{G13}-\varepsilon_{F13}) & 0 & (\varepsilon_{H14}+\varepsilon_{G14}-\varepsilon_{F14}) & 0 \\ R_{L}(\varepsilon_{F21}-\varepsilon_{H21}-\varepsilon_{G21}) & 0 & R_{L}(\varepsilon_{F22}-\varepsilon_{H22}-\varepsilon_{G22}) & C_{L} & R_{L}(\varepsilon_{F23}-\varepsilon_{H23}-\varepsilon_{G23}) & 0 & R_{L}(\varepsilon_{F24}-\varepsilon_{H24}-\varepsilon_{G24}) & 0 \\ (\varepsilon_{F21}-\varepsilon_{H21}-\varepsilon_{G21}) & 0 & (\varepsilon_{F22}-\varepsilon_{H22}-\varepsilon_{G22}) & 0 & (\varepsilon_{F23}-\varepsilon_{H23}-\varepsilon_{G23}) & 0 & (\varepsilon_{F24}-\varepsilon_{H24}-\varepsilon_{G24}) & 0 \\ R_{CA}(\varepsilon_{F31}-\varepsilon_{H31}-\varepsilon_{G31}) & 0 & R_{CA}(\varepsilon_{F32}-\varepsilon_{H32}-\varepsilon_{G32}) & 0 & R_{CA}(\varepsilon_{F33}-\varepsilon_{H33}-\varepsilon_{G33}) & C_{CA} & R_{CA}(\varepsilon_{F34}-\varepsilon_{H34}-\varepsilon_{G34}) & 0 \\ (\varepsilon_{F31}-\varepsilon_{H31}-\varepsilon_{G31}) & 0 & (\varepsilon_{F32}-\varepsilon_{H32}-\varepsilon_{G32}) & 0 & (\varepsilon_{F33}-\varepsilon_{H33}-\varepsilon_{G33}) & 0 & (\varepsilon_{F34}-\varepsilon_{H34}-\varepsilon_{G34}) & 0 \\ R_{F}(\varepsilon_{H41}+\varepsilon_{G41}-\varepsilon_{F41}) & 0 & R_{F}(\varepsilon_{H42}+\varepsilon_{G42}-\varepsilon_{F42}) & 0 & R_{F}(\varepsilon_{H43}+\varepsilon_{G43}-\varepsilon_{F43}) & 0 & R_{F}(\varepsilon_{H44}+\varepsilon_{G44}-\varepsilon_{F44}) & C_{F} \\ (\varepsilon_{H41}+\varepsilon_{G41}-\varepsilon_{F41}) & 0 & (\varepsilon_{H42}+\varepsilon_{G42}-\varepsilon_{F42}) & 0 & (\varepsilon_{H43}+\varepsilon_{G43}-\varepsilon_{F43}) & 0 & (\varepsilon_{H44}+\varepsilon_{G44}-\varepsilon_{F44}) & 0 \end{bmatrix} \begin{bmatrix} p_{CO} \\ q_{CO} \\ p_{L} \\ q_{L} \\ p_{CA} \\ q_{CA} \\ p_{F} \\ q_{F} \end{bmatrix} + \begin{bmatrix} R_{CO} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & R_{L} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & R_{CA} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & R_{F} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_{CO} \\ f_{L} \\ f_{CA} \\ f_{F} \end{bmatrix} \quad \text{(A-5)}$$

$$\begin{bmatrix} p_{CO} \\ q_{CO} \\ p_{L} \\ L_{H,L} \\ q_{L} \\ p_{CA} \\ L_{H,CA} \\ q_{CA} \\ p_{F} \\ L_{F,F} \\ q_{F} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{H,L} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & K_{H,CA} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & K_{F,F} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_{CO} \\ q_{CO} \\ p_{L} \\ q_{L} \\ p_{CA} \\ q_{CA} \\ p_{F} \\ q_{F} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} f_{CO} \\ f_{L} \\ f_{CA} \\ f_{F} \end{bmatrix} \quad \text{(A-6)}$$

A-4 Five-market Economy

The derivation of the economy's state (A-7) and output (A-8) equations is outlined in Section 6-3-3:

$$
\begin{bmatrix} \dot{p}_{CO} \\ \dot{q}_{CO} \\ \dot{p}_{H} \\ \dot{q}_{H} \\ \dot{p}_{L} \\ \dot{q}_{L} \\ \dot{p}_{CA} \\ \dot{q}_{CA} \\ \dot{p}_{F} \\ \dot{q}_{F} \end{bmatrix}
=
\left[\begin{array}{rcrcrc}
\mathbf{D}(\varepsilon_{H11}+\varepsilon_{G11}-\varepsilon_{F11}) & \mathbf{C} & \mathbf{D}(\varepsilon_{H12}+\varepsilon_{G12}-\varepsilon_{F12}) & 0 & \mathbf{D}(\varepsilon_{H13}+\varepsilon_{G13}-\varepsilon_{F13}) & 0 & \cdots \\
\mathbf{B}(\varepsilon_{H11}+\varepsilon_{G11}-\varepsilon_{F11}) & \mathbf{0} & \mathbf{B}(\varepsilon_{H12}+\varepsilon_{G12}-\varepsilon_{F12}) & \mathbf{0} & \mathbf{B}(\varepsilon_{H13}+\varepsilon_{G13}-\varepsilon_{F13}) & \mathbf{0} & \cdots \\
R_H(\varepsilon_{H21}+\varepsilon_{G21}-\varepsilon_{F21}) & \mathbf{0} & R_H(\varepsilon_{H22}+\varepsilon_{G22}-\varepsilon_{F22}) & C_H & R_H(\varepsilon_{H23}+\varepsilon_{G23}-\varepsilon_{F23}) & 0 & \cdots \\
(\varepsilon_{H21}+\varepsilon_{G21}-\varepsilon_{F21}) & \mathbf{0} & (\varepsilon_{H22}+\varepsilon_{G22}-\varepsilon_{F22}) & 0 & (\varepsilon_{H23}+\varepsilon_{G23}-\varepsilon_{F23}) & 0 & \cdots \\
R_L(\varepsilon_{F31}-\varepsilon_{H31}-\varepsilon_{G31}) & \mathbf{0} & R_L(\varepsilon_{F32}-\varepsilon_{H32}-\varepsilon_{G32}) & 0 & R_L(\varepsilon_{F33}-\varepsilon_{H33}-\varepsilon_{G33}) & C_L & \cdots \\
(\varepsilon_{F31}-\varepsilon_{H31}-\varepsilon_{G31}) & \mathbf{0} & (\varepsilon_{F32}-\varepsilon_{H32}-\varepsilon_{G32}) & 0 & (\varepsilon_{F33}-\varepsilon_{H33}-\varepsilon_{G33}) & 0 & \cdots \\
R_{CA}(\varepsilon_{F41}-\varepsilon_{H41}-\varepsilon_{G41}) & \mathbf{0} & R_{CA}(\varepsilon_{F42}-\varepsilon_{H42}-\varepsilon_{G42}) & 0 & R_{CA}(\varepsilon_{F43}-\varepsilon_{H43}-\varepsilon_{G43}) & 0 & \cdots \\
(\varepsilon_{F41}-\varepsilon_{H41}-\varepsilon_{G41}) & \mathbf{0} & (\varepsilon_{F42}-\varepsilon_{H42}-\varepsilon_{G42}) & 0 & (\varepsilon_{F43}-\varepsilon_{H43}-\varepsilon_{G43}) & 0 & \cdots \\
R_F(\varepsilon_{H51}+\varepsilon_{G51}-\varepsilon_{F51}) & \mathbf{0} & R_F(\varepsilon_{H52}+\varepsilon_{G52}-\varepsilon_{F52}) & 0 & R_F(\varepsilon_{H53}+\varepsilon_{G53}-\varepsilon_{F53}) & 0 & \cdots \\
(\varepsilon_{H51}+\varepsilon_{G51}-\varepsilon_{F51}) & \mathbf{0} & (\varepsilon_{H52}+\varepsilon_{G52}-\varepsilon_{F52}) & 0 & (\varepsilon_{H53}+\varepsilon_{G53}-\varepsilon_{F53}) & 0 & \cdots
\end{array} \right.
$$

$$
\left. \begin{array}{rcrc}
\mathbf{D}(\varepsilon_{H14}+\varepsilon_{G14}-\varepsilon_{F14}) & 0 & \mathbf{D}(\varepsilon_{H15}+\varepsilon_{G15}-\varepsilon_{F15}) & 0 \\
\mathbf{B}(\varepsilon_{H14}+\varepsilon_{G14}-\varepsilon_{F14}) & \mathbf{0} & \mathbf{B}(\varepsilon_{H15}+\varepsilon_{G15}-\varepsilon_{F15}) & \mathbf{0} \\
R_H(\varepsilon_{H24}+\varepsilon_{G24}-\varepsilon_{F24}) & 0 & R_H(\varepsilon_{H25}+\varepsilon_{G25}-\varepsilon_{F25}) & 0 \\
(\varepsilon_{H24}+\varepsilon_{G24}-\varepsilon_{F24}) & 0 & (\varepsilon_{H25}+\varepsilon_{G25}-\varepsilon_{F25}) & 0 \\
R_L(\varepsilon_{F34}-\varepsilon_{H34}-\varepsilon_{G34}) & 0 & R_L(\varepsilon_{F35}-\varepsilon_{H35}-\varepsilon_{G35}) & 0 \\
(\varepsilon_{F34}-\varepsilon_{H34}-\varepsilon_{G34}) & 0 & (\varepsilon_{F35}-\varepsilon_{H35}-\varepsilon_{G35}) & 0 \\
R_{CA}(\varepsilon_{F44}-\varepsilon_{H44}-\varepsilon_{G44}) & C_{CA} & R_{CA}(\varepsilon_{F45}-\varepsilon_{H45}-\varepsilon_{G45}) & 0 \\
(\varepsilon_{F44}-\varepsilon_{H44}-\varepsilon_{G44}) & 0 & (\varepsilon_{F45}-\varepsilon_{H45}-\varepsilon_{G45}) & 0 \\
R_F(\varepsilon_{H54}+\varepsilon_{G54}-\varepsilon_{F54}) & 0 & R_F(\varepsilon_{H55}+\varepsilon_{G55}-\varepsilon_{F55}) & C_F \\
(\varepsilon_{H54}+\varepsilon_{G54}-\varepsilon_{F54}) & 0 & (\varepsilon_{H55}+\varepsilon_{G55}-\varepsilon_{F55}) & 0
\end{array} \right]
\begin{bmatrix} p_{CO} \\ q_{CO} \\ p_{H} \\ q_{H} \\ p_{L} \\ q_{L} \\ p_{CA} \\ q_{CA} \\ p_{F} \\ q_{F} \end{bmatrix}
+
\begin{bmatrix}
\mathbf{D} & 0 & 0 & 0 & 0 \\
\mathbf{B} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
0 & R_H & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & R_L & 0 & \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & R_{CA} & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & R_F \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix} f_{CO} \\ f_{H} \\ f_{L} \\ f_{CA} \\ f_{F} \end{bmatrix}
\qquad \text{(A-7)}
$$

$$
\begin{bmatrix} p_{CO} \\ e_{CO} \\ p_{H} \\ L_{F,H} \\ q_{H} \\ p_{L} \\ L_{H,L} \\ q_{L} \\ p_{CA} \\ L_{H,CA} \\ q_{CA} \\ p_{F} \\ L_{F,F} \\ q_{F} \end{bmatrix}
=
\left[\begin{array}{rcrcrcc}
1 & \mathbf{0} & 0 & 0 & 0 & 0 & \cdots \\
\mathbf{D}(\varepsilon_{H11}+\varepsilon_{G11}-\varepsilon_{F11}) & \mathbf{C} & \mathbf{D}(\varepsilon_{H12}+\varepsilon_{G12}-\varepsilon_{F12}) & 0 & \mathbf{D}(\varepsilon_{H13}+\varepsilon_{G13}-\varepsilon_{F13}) & 0 & \cdots \\
0 & \mathbf{0} & 1 & 0 & 0 & 0 & \cdots \\
0 & \mathbf{0} & K_{F,H} & 0 & 0 & 0 & \cdots \\
0 & \mathbf{0} & 0 & 1 & 0 & 0 & \cdots \\
0 & \mathbf{0} & 0 & 0 & 1 & 0 & \cdots \\
0 & \mathbf{0} & 0 & 0 & K_{H,L} & 0 & \cdots \\
0 & \mathbf{0} & 0 & 0 & 0 & 1 & \cdots \\
0 & \mathbf{0} & 1 & 0 & 0 & 0 & \cdots \\
0 & \mathbf{0} & 0 & 0 & 0 & 0 & \cdots \\
0 & \mathbf{0} & 0 & 0 & 0 & 0 & \cdots \\
0 & \mathbf{0} & 0 & 0 & 0 & 0 & \cdots \\
0 & \mathbf{0} & 0 & 0 & 0 & 0 & \cdots \\
0 & \mathbf{0} & 0 & 0 & 0 & 0 & \cdots
\end{array} \right.
$$

$$
\left. \begin{array}{rcrc}
0 & 0 & 0 & 0 \\
\mathbf{D}(\varepsilon_{H14}+\varepsilon_{G14}-\varepsilon_{F14}) & 0 & \mathbf{D}(\varepsilon_{H15}+\varepsilon_{G15}-\varepsilon_{F15}) & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
K_{H,CA} & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & K_{F,F} & 0 \\
0 & 0 & 0 & 1
\end{array} \right]
\begin{bmatrix} p_{CO} \\ q_{CO} \\ p_{H} \\ q_{H} \\ p_{L} \\ q_{L} \\ p_{CA} \\ q_{CA} \\ p_{F} \\ q_{F} \end{bmatrix}
+
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
\mathbf{D} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} f_{CO} \\ f_{H} \\ f_{L} \\ f_{CA} \\ f_{F} \end{bmatrix}
\qquad \text{(A-8)}
$$

Appendix B

Model Parameters

B-1 Parameter Estimation Algorithm

Algorithm 1 Parameter Estimation

Require: A,B,C,D,e_H,e_G,e_H ▷ Import state-space and price-elasticity matrices

while maximum(real(eigenvalues(A))) > 0 **do**
  $C_{CO} = 0.1 * \text{randn}(1)$
  ⋮ ▷ Randomly initialize inventory elasticity parameters
  $C_F = 0.1 * \text{randn}(1)$
  $R_{CO} = 8 * \text{randn}(1)$
  ⋮ ▷ Randomly initialize friction parameters
  $R_F = 8 * \text{randn}(1)$
  Update A,B,C,D
end while

B-2 Model Parameters

	Homogeneous	**Heterogeneous** ($C \downarrow$)	**Heterogeneous** ($C \uparrow$)
L	1.0	$1.0 + 0.1 \cdot \mathcal{N}(0,1)$	$1.0 + 0.1 \cdot \mathcal{N}(0,1)$
C	1.0	$1.0 + 0.1 \cdot \mathcal{N}(0,1)$	$10 + 1.0 \cdot \mathcal{N}(0,1)$
R	0.1	$0.1 + 0.01 \cdot \mathcal{N}(0,1)$	$0.1 + 0.01 \cdot \mathcal{N}(0,1)$
G	0.1	$0.1 + 0.01 \cdot \mathcal{N}(0,1)$	$0.1 + 0.01 \cdot \mathcal{N}(0,1)$

Table B-1: Model parameters of the trader (Chapter 4).

Commodity Market	
C_{CO}	10.000
R_{CO}	20.739
e_H	1.0
e_G	0.4
e_F	1.7

Table B-2: Model parameters of the commodity market (Section 5-7)

Commodity Market		**Labor Market**			
C_{CO}	0.0180	C_L	0.0551		
R_{CO}	6.913	R_L	0.907		
		$K_{H,L}$	1.0		
Capital Market		**Financial Market**			
C_{CA}	0.0683	C_F	0.117		
R_{CA}	3.187	R_F	7.072		
$K_{H,CA}$	1.0	$K_{F,F}$	1.0		
Households		**Government**		**Firms**	
ε_{H11}	1.0	ε_{G11}	0.4	ε_{F11}	1.7
ε_{H12}	-0.3	ε_{G12}	-0.2	ε_{F12}	-0.4
ε_{H13}	-0.3	ε_{G13}	-0.2	ε_{F13}	-0.4
ε_{H14}	-0.4	ε_{G14}	-0.4	ε_{F14}	-0.6
ε_{H21}	-0.3	ε_{G21}	-0.2	ε_{F21}	-0.4
ε_{H22}	0.8	ε_{G22}	0.3	ε_{F22}	1.0
ε_{H23}	0.3	ε_{G23}	0.1	ε_{F23}	0.4
ε_{H24}	-0.1	ε_{G24}	-0.1	ε_{F24}	-0.2
ε_{H31}	-0.3	ε_{G31}	-0.2	ε_{F31}	-0.4
ε_{H32}	0.3	ε_{G32}	0.1	ε_{F32}	0.4
ε_{H33}	0.7	ε_{G33}	0.3	ε_{F33}	0.5
ε_{H34}	-0.1	ε_{G34}	-0.1	ε_{F34}	-0.2
ε_{H41}	-0.4	ε_{G41}	-0.4	ε_{F41}	-0.6
ε_{H42}	-0.1	ε_{G42}	-0.1	ε_{F42}	-0.2
ε_{H43}	-0.1	ε_{G43}	-0.1	ε_{F43}	-0.2
ε_{H44}	0.8	ε_{G44}	0.2	ε_{F44}	1.2

Table B-3: Model parameters of the four-market economy (Chapter 5).

Commodity Market		Housing Market		Labor Market	
C_{CO}	0.0663	C_H	0.0870	C_L	0.0979
R_{CO}	0.977	R_H	1.447	R_L	3.263
		$K_{F,H}$	1.0	$K_{H,L}$	1.0
Capital Market		**Financial Market**		**Homogeneous Trader**	
C_{CA}	0.0190	C_F	0.0252	L	1.0
R_{CA}	2.927	R_F	9.577	C	0.1
$K_{H,CA}$	1.0	$K_{F,F}$	1.0	R	0.01
				G	0.04
Households		**Government**		**Firms**	
ε_{H11}	1.0	ε_{G11}	0.4	ε_{F11}	1.7
ε_{H12}	0.2	ε_{G12}	0.1	ε_{F12}	0.2
ε_{H13}	-0.3	ε_{G13}	-0.2	ε_{F13}	-0.4
ε_{H14}	-0.3	ε_{G14}	-0.2	ε_{F14}	-0.4
ε_{H15}	-0.4	ε_{G15}	-0.4	ε_{F15}	-0.6
ε_{H21}	0.2	ε_{G21}	0.1	ε_{F21}	0.2
ε_{H22}	0.4	ε_{G22}	0.3	ε_{F22}	1.2
ε_{H23}	0.0	ε_{G23}	0.0	ε_{F23}	0.0
ε_{H24}	0.0	ε_{G24}	0.0	ε_{F24}	0.0
ε_{H25}	-0.2	ε_{G25}	-0.1	ε_{F25}	-0.2
ε_{H31}	-0.3	ε_{G31}	-0.2	ε_{F31}	-0.4
ε_{H32}	0.0	ε_{G32}	0.0	ε_{F32}	0.0
ε_{H33}	0.8	ε_{G33}	0.3	ε_{F33}	1.0
ε_{H34}	0.3	ε_{G34}	0.1	ε_{F34}	0.4
ε_{H35}	-0.1	ε_{G35}	-0.1	ε_{F35}	-0.2
ε_{H41}	-0.3	ε_{G41}	-0.2	ε_{F41}	-0.4
ε_{H42}	0.0	ε_{G42}	0.0	ε_{F42}	0.0
ε_{H43}	0.3	ε_{G43}	0.1	ε_{F43}	0.4
ε_{H44}	0.6	ε_{G44}	0.2	ε_{F44}	0.5
ε_{H45}	-0.1	ε_{G45}	-0.1	ε_{F45}	-0.2
ε_{H51}	-0.4	ε_{G51}	-0.4	ε_{F51}	-0.6
ε_{H52}	-0.2	ε_{G52}	-0.1	ε_{F52}	-0.2
ε_{H53}	-0.1	ε_{G53}	-0.1	ε_{F53}	-0.2
ε_{H54}	-0.1	ε_{G54}	-0.1	ε_{F54}	-0.2
ε_{H55}	0.8	ε_{G55}	0.2	ε_{F55}	1.2

Table B-4: Model parameters of the five-market economy (Chapter 6).

Appendix C

Two-Port Networks

In Section 4-4, we showed that two-port networks can be combined using five types of interconnections. By selecting an appropriate set of two-port network parameters, two-port network theory allows us to interconnect agents using only additions and multiplications. Subsequently, we can switch between the various two-port network parameters to construct more complex two-port networks.

C-1 Two-port Network Parameters

Section 4-2 already introduced admittance parameters, and Section 6-2 impedance parameters [$\mathbf{z}$]. Three other types of parameters exist, namely hybrid, inverse-hybrid, and ABCD parameters. Below we list all five two-port network parameters:

Impedance Parameters

$$\begin{gathered}\begin{bmatrix} V_I \\ V_O \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_I \\ I_O \end{bmatrix} \\ z_{11} := \left.\frac{V_I}{I_I}\right|_{I_O=0}, \quad z_{12} := \left.\frac{V_I}{I_O}\right|_{I_I=0}, \\ z_{21} := \left.\frac{V_O}{I_I}\right|_{I_O=0}, \quad z_{22} := \left.\frac{V_O}{I_O}\right|_{I_I=0} \end{gathered} \tag{C-1}$$

Admittance Parameters

$$\begin{gathered}\begin{bmatrix} I_I \\ I_O \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_I \\ V_O \end{bmatrix} \\ y_{11} := \left.\frac{I_I}{V_I}\right|_{V_O=0}, \quad y_{12} := \left.\frac{I_I}{V_O}\right|_{V_I=0}, \\ y_{21} := \left.\frac{I_O}{V_I}\right|_{V_O=0}, \quad y_{22} := \left.\frac{I_O}{V_O}\right|_{V_I=0} \end{gathered} \tag{C-2}$$

Hybrid Parameters

$$\begin{bmatrix} V_I \\ I_O \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_I \\ V_O \end{bmatrix}$$

$$\begin{aligned} h_{11} &:= \left.\frac{V_I}{I_I}\right|_{V_O=0}, \quad h_{12} := \left.\frac{V_I}{V_O}\right|_{I_I=0}, \\ h_{21} &:= \left.\frac{I_O}{I_I}\right|_{V_O=0}, \quad h_{22} := \left.\frac{I_O}{V_O}\right|_{I_I=0} \end{aligned} \tag{C-3}$$

Inverse-Hybrid Parameters

$$\begin{bmatrix} I_I \\ V_O \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_I \\ I_O \end{bmatrix}$$

$$\begin{aligned} g_{11} &:= \left.\frac{I_I}{V_I}\right|_{I_O=0}, \quad g_{12} := \left.\frac{I_I}{I_O}\right|_{V_I=0}, \\ g_{21} &:= \left.\frac{V_O}{V_I}\right|_{I_O=0}, \quad g_{22} := \left.\frac{V_O}{I_O}\right|_{V_I=0} \end{aligned} \tag{C-4}$$

ABCD Parameters

$$\begin{bmatrix} V_I \\ I_I \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_O \\ -I_O \end{bmatrix}$$

$$\begin{aligned} A &:= \left.\frac{V_I}{V_O}\right|_{I_O=0}, \quad B := \left.\frac{V_I}{I_O}\right|_{V_O=0}, \\ C &:= \left.\frac{I_I}{V_O}\right|_{I_O=0}, \quad D := \left.\frac{I_I}{I_O}\right|_{V_O=0} \end{aligned} \tag{C-5}$$

C-2 Interconnecting of Two-Port Networks

Each of the five interconnections illustrated in Figure 4-5 can be simplified by choosing the appropriate set of two port network parameters:

Series-Series Interconnection

$$[\mathbf{z}] = [\mathbf{z}]_1 + [\mathbf{z}]_2 \tag{C-6}$$

Parallel-Parallel Interconnection

$$[\mathbf{y}] = [\mathbf{y}]_1 + [\mathbf{y}]_2 \tag{C-7}$$

Parallel-Series Interconnection

$$[\mathbf{h}] = [\mathbf{h}]_1 + [\mathbf{h}]_2 \tag{C-8}$$

Series-Parallel Interconnection

$$[\mathbf{g}] = [\mathbf{g}]_1 + [\mathbf{g}]_2 \tag{C-9}$$

Cascade Interconnection

$$[\mathbf{a}] = [\mathbf{a}]_1 \cdot [\mathbf{a}]_2 \tag{C-10}$$

C-3 Interrelation of Two-Port Parameters

We can transform any set of two-port parameters into another set of two-port parameters as indicated in Table C-1:

	[**z**]	[**y**]	[**h**]	[**g**]	[**a**]
[**z**]	$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$\frac{1}{\Delta[y]}\begin{bmatrix} y_{22} & -y_{12} \\ -y_{21} & y_{11} \end{bmatrix}$	$\frac{1}{h_{22}}\begin{bmatrix} \Delta[h] & h_{12} \\ -h_{21} & 1 \end{bmatrix}$	$\frac{1}{g_{11}}\begin{bmatrix} 1 & -g_{12} \\ g_{21} & \Delta[g] \end{bmatrix}$	$\frac{1}{a_{21}}\begin{bmatrix} a_{11} & \Delta[a] \\ 1 & a_{22} \end{bmatrix}$
[**y**]	$\frac{1}{\Delta[z]}\begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix}$	$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$\frac{1}{h_{11}}\begin{bmatrix} 1 & -h_{12} \\ h_{21} & \Delta[h] \end{bmatrix}$	$\frac{1}{g_{22}}\begin{bmatrix} \Delta[g] & g_{12} \\ -g_{21} & 1 \end{bmatrix}$	$\frac{1}{a_{12}}\begin{bmatrix} a_{22} & -\Delta[a] \\ -1 & a_{11} \end{bmatrix}$
[**h**]	$\frac{1}{z_{22}}\begin{bmatrix} \Delta[z] & z_{12} \\ -z_{21} & 1 \end{bmatrix}$	$\frac{1}{y_{11}}\begin{bmatrix} 1 & -y_{12} \\ y_{21} & \Delta[y] \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$	$\frac{1}{\Delta[g]}\begin{bmatrix} g_{22} & -g_{12} \\ -g_{21} & g_{11} \end{bmatrix}$	$\frac{1}{a_{22}}\begin{bmatrix} a_{12} & \Delta[a] \\ -1 & a_{21} \end{bmatrix}$
[**g**]	$\frac{1}{z_{11}}\begin{bmatrix} 1 & -z_{12} \\ z_{21} & \Delta[z] \end{bmatrix}$	$\frac{1}{y_{22}}\begin{bmatrix} \Delta[y] & y_{12} \\ -y_{21} & 1 \end{bmatrix}$	$\frac{1}{\Delta[h]}\begin{bmatrix} h_{22} & -h_{12} \\ -h_{21} & h_{11} \end{bmatrix}$	$\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$	$\frac{1}{a_{22}}\begin{bmatrix} a_{21} & -\Delta[a] \\ 1 & a_{12} \end{bmatrix}$
[**a**]	$\frac{1}{z_{21}}\begin{bmatrix} z_{11} & \Delta[z] \\ 1 & z_{22} \end{bmatrix}$	$\frac{1}{y_{12}}\begin{bmatrix} -y_{22} & -1 \\ -\Delta[y] & -y_{22} \end{bmatrix}$	$\frac{1}{h_{12}}\begin{bmatrix} -\Delta[h] & -h_{11} \\ -h_{22} & -1 \end{bmatrix}$	$\frac{1}{g_{21}}\begin{bmatrix} 1 & g_{22} \\ g_{11} & \Delta[g] \end{bmatrix}$	$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

Table C-1: Interrelation of impedance [**z**], admittance [**y**], hybrid [**h**], inverse hybrid [**g**], and ABCD [**a**] parameters.

Appendix D

Canonical Forms

D-1 (Proper) Transfer Function

$$H(s) = \frac{Y(s)}{U(s)} = \frac{b_0 s^n + b_1 s^{n-1} + \cdots + b_{n-1}s + b_n}{s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n} \tag{D-1}$$

D-2 Controllable Canonical Form (CCF)

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}; \quad \mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} \tag{D-2}$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \tag{D-3}$$

$$\mathbf{C} = \begin{bmatrix} (b_n - a_n b_0) & (b_{n-1} - a_{n-1} b_0) & \cdots & (b_2 - a_2 b_0) & (b_1 - a_1 b_0) \end{bmatrix}; \quad \mathbf{D} = b_0 \tag{D-4}$$

D-3 Observable Canonical Form (OCF)

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}; \quad \mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} \tag{D-5}$$

$$\mathbf{A} = \begin{bmatrix} -a_1 & 1 & 0 & \cdots & 0 \\ -a_2 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_{n-1} & 0 & 0 & \cdots & 1 \\ -a_n & 0 & 0 & \cdots & 0 \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} b_1 - a_1 b_0 \\ b_2 - a_2 b_0 \\ \vdots \\ b_{n-1} - a_{n-1} b_0 \\ b_n - a_n b_0 \end{bmatrix} \tag{D-6}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \end{bmatrix}; \quad \mathbf{D} = b_0 \tag{D-7}$$

Appendix E

MATLAB Code

E-1 Two-Port Networks

E-1-1 Economic Networks

```
%% Define Basic Trader
% System parameters
syms s                  % frequency domain variable
I = tf(1,[1  0]);       % integrator
N = 16;               % # of agents, needs to be larger or equal to Nc*Np
rng(100)                % set the seed of the random number generator

% Define inputs and outputs of different parameters
in_Y  = {'V_1','V_2'};
out_Y = {'I_1','I_2'};

% Generate N agents
for i = 1:N
     if i <= N/2
        L = 1 + 0.1*round(randn(1),1);        % inductor
        C = 1 + 0.1*round(randn(1),1);        % capacitor
        R = 0.1 + 0.01*round(randn(1),1);     % series resistor
        G = 0.1 + 0.01*round(randn(1),1);     % parallel resistor
    else
        L = 1 + 0.1*round(randn(1),1);        % inductor
        C = 10 + 1.0*round(randn(1),1);       % capacitor
        R = 0.1 + 0.01*round(randn(1),1);     % series resistor
        G = 0.1 + 0.01*round(randn(1),1);     % parallel resistor
    end

    % Modeling trader using admittance parameters
    z11 = L*s + R + 1/(s*C + G);
    z12 = 1/(s*C + G);
```

```
    z21 = 1/(s*C + G);
    z22 = 1/(s*C + G);
    eval(['Z' num2str(i) '= [z11 z12; z21 z22];']);

    % Agent using abcd parameters
    A = z2ABCD(eval(['Z' num2str(i)]));
    eval(['A' num2str(i) '= A;']);

    % Agent using abcd parameters
    Y = z2y(eval(['Z' num2str(i)]));
    eval(['Y' num2str(i) '= Y;']);

    % Agent using H parameters
    H = z2h(eval(['Z' num2str(i)]));
    eval(['H' num2str(i) '= H;']);

    % Agent using G parameters
    G = z2g(eval(['Z' num2str(i)]));
    eval(['G' num2str(i) '= G;']);
end

%% System 1: Simple agent (A)
% admittance parameters
TF1 = symb2tf(Y1,2);
TF1.InputName = in_Y; TF1.OutputName = out_Y;

% visualization
figure
pzmap(TF1); grid on; title('Pole-zero map: 2-port of single agent (Y)');
figure
bode(TF1); grid on; title('Bode plot: 2-port of single agent (Y)');

%% System 2: Supply chain (SC)
% cascade interconnection of Nc agents
Nc = 4;          % number of cascaded agents
A_SC = eye(2);   % initialize

for i = 1:Nc
    A_SC = cascade_interconnection(A_SC, eval(['A' num2str(i)]));
end

Y_SC = ABCD2y(A_SC);

% admittance paramters
TF2 = symb2tf(Y_SC,2);
TF2.InputName = in_Y; TF2.OutputName = out_Y;

% visualization
figure
pzmap(TF2); grid on; title('Pole-zero map: 2-port of supply chain (Y)');
figure
bode(TF2); grid on; title('Bode plot: 2-port of supply chain (Y)');

```

```
%% System 3: Competative agents (SM)
%  parallel interconnection of Np agents
Np = 4;                 % number of parallel agents
Y_SM = zeros(2);        % initialize

for j = 1:Np
    Y_SM = parallel_interconnection(Y_SM, eval(['Y' num2str(j)]));
end

% admittance parameters
TF3 = symb2tf(Y_SM,2);
TF3.InputName = in_Y; TF3.OutputName = out_Y;

% visualization
figure
pzmap(TF3); grid on; title('Pole-zero map: 2-port of simple market (Y)');
figure
bode(TF3); grid on; title('Bode plot: 2-port of simple market (Y)');

%% System 4: Cooperative agents (CO)
% series interconnection of Ns agents
Ns = 4;                  % number of series agents
Z_CO = zeros(2);         % initialize

for k = 1:Ns
    Z_CO = series_interconnection(Z_CO, eval(['Z' num2str(k)]));
end

Y_CO = z2y(Z_CO);

% admittance parameters
TF4 = symb2tf(Y_CO,2);
TF4.InputName = in_Y; TF4.OutputName = out_Y;

% visualization
figure
pzmap(TF4); grid on; title('Pole-zero map: 2-port of cooperative agents (
    Y)');
figure
bode(TF4); grid on; title('Bode plot: 2-port of cooperative agents (Y)');

%% System 5: Hybrid Interconnection (H)
% Hybric interconnection of Nh agents
Nh = 4;                   % number of series agents
H_hyb = zeros(2);         % initialize

for i = 1:Nh
    H_hyb = hybrid_interconnection(H_hyb, eval(['H' num2str(i)]));
end

Y_hyb = h2y(H_hyb);

% admittance paramters
```

```
TF5 = symb2tf(Y_hyb,2);
TF5.InputName = in_Y; TF5.OutputName = out_Y;

% visualization
figure
pzmap(TF5); grid on; title('Pole-zero map: 2-port of hybrid interconnection (Y)');
figure
bode(TF5); grid on; title('Bode plot: 2-port of hybrid interconnection (Y)');

%% System 6: Inverse Hybrid Interconnection (IH)
% Inverse hybric interconnection of Nh agents
G_inv = zeros(2);      % initialize

for i = 1:Nh
    G_inv = inverse_hybrid_interconnection(G_inv, eval(['G' num2str(i)]));
end

Y_inv = g2y(G_inv);

% admittance paramters
TF6 = symb2tf(Y_inv,2);
TF6.InputName = in_Y; TF6.OutputName = out_Y;

% visualization
figure
pzmap(TF6); grid on; title('Pole-zero map: 2-port of hybrid interconnection (Y)');
figure
bode(TF6); grid on; title('Bode plot: 2-port of hybrid interconnection (Y)');

%% System 7: Diamond supply chain
SC_1_Y = Y1;
SC_1_A = y2ABCD(SC_1_Y);

SC_2_Y = Y2 + Y3;
SC_2_A = y2ABCD(SC_2_Y);

SC_3_Y = Y4 + Y5 + Y6;
SC_3_A = y2ABCD(SC_3_Y);

SC_4_Y = Y7 + Y8;
SC_4_A = y2ABCD(SC_4_Y);

SC_5_Y = Y9;
SC_5_A = y2ABCD(SC_5_Y);

SC_A = SC_1_A * SC_2_A * SC_3_A * SC_4_A * SC_5_A;
SC_Y = ABCD2y(SC_A);

```

```
% admittance parameters
TF7 = symb2tf(SC_Y,2);
TF7.InputName = in_Y; TF7.OutputName = out_Y;

% visualization
figure
pzmap(TF7); grid on; title('Pole-zero map: 2-port of diamond SC (Y)');
figure
bode(TF7); grid on; title('Bode plot: 2-port of diamond SC (Y)');

%% System 8: Complex competative market (SC&M)
% Np parallel interconnected supply chains, each consisting out of Nc agents
Nc = 4;              % number of cascaded agents
Np = 4;              % number of parallel agents
Y_M = zeros(2);      % initialize

for j = 1:Np
    A_SC = eye(2);   % initialize
    for i = 1:Nc
        A_SC = cascade_interconnection(A_SC, eval(['A' num2str((j-1)*Nc+i)]));
    end
    Y_SC = ABCD2y(A_SC);
    eval(['Y_SC_' num2str(j) '= Y_SC;']);
    Y_M = parallel_interconnection(Y_M, eval(['Y_SC_' num2str(j)]));
end

% admittance parameters
TF8 = symb2tf(Y_M,2);
TF8.InputName = in_Y; TF8.OutputName = out_Y;

% visualization
figure
pzmap(TF8); grid on; title('Pole-zero map: 2-port of complex market (Y)');
figure
bode(TF8); grid on; title('Bode plot: 2-port of complex market (Y)');
```

E-1-2 Interconnection Functions

```
%% Function Library
% interconnections
function [Y] = parallel_interconnection(Y_1,Y_2)
    % pararallel agent interconnection (admittance parameters required)
    Y = Y_1 + Y_2;
end

function [Z] = series_interconnection(Z_1,Z_2)
    % series agent interconnection (impedance parameters required)
    Z = Z_1 + Z_2;
end

```

```
function [A] = cascade_interconnection(A_1,A_2)
    % pararallel agent interconnection (ABCD parameters required)
    A = A_1 * A_2;
end

function [H] = hybrid_interconnection(H_1,H_2)
    % hybrid agent interconnection (hybrid parameters required)
    H = H_1 + H_2;
end

function [IH] = inverse_hybrid_interconnection(IH_1,IH_2)
    % inverse hybrid agent interconnection (inverse hybrid parameters
        required)
    IH = IH_1 + IH_2;
end

% transformations
function [A] = y2ABCD(Y)
    % transforms admittance parameters into ABCD parameters
    y11 = Y(1,1);
    y12 = Y(1,2);
    y21 = Y(2,1);
    y22 = Y(2,2);
    delta_y = y11*y22-y12*y21;

    A = 1/y21 * [    -y22    -1;
                 -delta_y -y11];
    A = simplify(A);
end

function [Z] = y2z(Y)
    % transforms admittance parameters into impedance parameters
    Z = simplify(eye(2)/Y);
end

function [H] = y2h(Y)
    % transforms admittance parameters into hybrid parameters
    y11 = Y(1,1);
    y12 = Y(1,2);
    y21 = Y(2,1);
    y22 = Y(2,2);
    delta_y = y11*y22-y12*y21;

    H = 1/y11 * [  1    -y12;
                 y21 delta_y];
    H = simplify(H);
end

function [G] = y2g(Y)
    % transforms admittance parameters into inverse hybrid parameters
    y11 = Y(1,1);
    y12 = Y(1,2);
    y21 = Y(2,1);
```

```
    y22 = Y(2,2);
    delta_y = y11*y22-y12*y21;

    G = 1/y22 * [delta_y y12;
                -y21    1];
    G = simplify(G);
end

function [A] = z2ABCD(Z)
    % transforms impedance parameters into ABCD parameters
    z11 = Z(1,1);
    z12 = Z(1,2);
    z21 = Z(2,1);
    z22 = Z(2,2);
    delta_z = z11*z22-z12*z21;

    A = 1/z21 * [z11 delta_z;
               1      z22];
    A = simplify(A);
end

function [Y] = z2y(Z)
    % transforms impedance parameters into admittance parameters
    Y = simplify(eye(2)/Z);
end

function [H] = z2h(Z)
    % transforms impedance parameters into hybrid parameters
    z11 = Z(1,1);
    z12 = Z(1,2);
    z21 = Z(2,1);
    z22 = Z(2,2);
    delta_z = z11*z22-z12*z21;

    H = 1/z22 * [delta_z z12;
             -z21       1];
    H = simplify(H);
end

function [G] = z2g(Z)
    % transforms impedance parameters into hybrid parameters
    z11 = Z(1,1);
    z12 = Z(1,2);
    z21 = Z(2,1);
    z22 = Z(2,2);
    delta_z = z11*z22-z12*z21;

    G = 1/z11 * [   1     -z12;
             z21 delta_z];
    G = simplify(G);
end

function [Y] = ABCD2y(A)
```

```
    % transforms ABDC parameters into admittance parameters
    a11 = A(1,1);
    a12 = A(1,2);
    a21 = A(2,1);
    a22 = A(2,2);
    delta_a = a11*a22-a12*a21;

    Y = 1/a12 * [a22 -delta_a;
                 -1       a11];
    Y = simplify(Y);
end

function [Z] = ABCD2z(A)
    % transforms ABDC parameters into impedance parameters
    a11 = A(1,1);
    a12 = A(1,2);
    a21 = A(2,1);
    a22 = A(2,2);
    delta_a = a11*a22-a12*a21;

    Z = 1/a21 * [a11  delta_a;
                  1       a22];
    Z = simplify(Z);
end

function [H] = ABCD2h(A)
    % transforms ABCD parameters into hybrid parameters
    a11 = A(1,1);
    a12 = A(1,2);
    a21 = A(2,1);
    a22 = A(2,2);
    delta_a = a11*a22-a12*a21;

    H = 1/a22 * [a12 delta_a;
                 -1      a21];
    H = simplify(H);
end

function [G] = ABCD2g(A)
    % transforms ABCD parameters into inverse hybrid parameters
    a11 = A(1,1);
    a12 = A(1,2);
    a21 = A(2,1);
    a22 = A(2,2);
    delta_a = a11*a22-a12*a21;

    G = 1/a11 * [a21 -delta_a;
                 1       a12];
    G = simplify(G);
end

function [A] = h2ABCD(H)
    % transforms hybrid parameters into ABCD parameters
```

```
    h11 = H(1,1);
    h12 = H(1,2);
    h21 = H(2,1);
    h22 = H(2,2);
    delta_h = h11*h22-h12*h21;

    A = 1/h21 * [-delta_h -h11;
                    -h22    -1];
    A = simplify(A);
end

function [Y] = h2y(H)
    % transforms hybrid parameters into admittance parameters
    h11 = H(1,1);
    h12 = H(1,2);
    h21 = H(2,1);
    h22 = H(2,2);
    delta_h = h11*h22-h12*h21;

    Y = 1/h11 * [  1    -h12;
                 h21 delta_h];
    Y = simplify(Y);
end

function [Z] = h2z(H)
    % transforms hybrid parameters into impedance parameters
    h11 = H(1,1);
    h12 = H(1,2);
    h21 = H(2,1);
    h22 = H(2,2);
    delta_h = h11*h22-h12*h21;

    Z = 1/h22 * [delta_h h12;
                 -h21      1];
    Z = simplify(Z);
end

function [G] = h2g(H)
    % transforms inverse hybrid parameters into hybrid parameters
    G = simplify(eye(2)/H);
end

function [A] = g2ABCD(G)
    % transforms inverse hybrid parameters into ABCD parameters
    g11 = G(1,1);
    g12 = G(1,2);
    g21 = G(2,1);
    g22 = G(2,2);
    delta_g = g11*g22-g12*g21;

    A = 1/g21 * [  1     g22;
                 g11 delta_g];
    A = simplify(A);
```

```
end

function [Z] = g2z(G)
    % transforms inverse hybrid parameters into impedance parameters
    g11 = G(1,1);
    g12 = G(1,2);
    g21 = G(2,1);
    g22 = G(2,2);
    delta_g = g11*g22-g12*g21;

    Z = 1/g11 * [  1     -g12;
                 g21  delta_g];
    Z = simplify(Z);
end

function [Y] = g2y(G)
    % transforms inverse hybrid parameters into admittance parameters
    g11 = G(1,1);
    g12 = G(1,2);
    g21 = G(2,1);
    g22 = G(2,2);
    delta_g = g11*g22-g12*g21;

    Y = 1/g22 * [delta_g g12;
                    -g21    1];
    Y = simplify(Y);
end

function [H] = g2h(G)
    % transforms hybrid parameters into inverse hybrid parameters
    H = simplify(eye(2)/G);
end

% miscelanious
function [Y_op] = tp2op_L(Y_tp,Y_L)
    % transform 2-port network in a 1-port network using an admittance
       load
    Y_op = Y_tp(1,1) - (Y_tp(1,2)*Y_tp(2,1))/(Y_L + Y_tp(2,2));
    Y_op = simplify(Y_op);
end

function [Y_op] = tp2op_S(Y_tp,Y_S)
    % transform 2-port network in a 1-port network using an admittance
       source
    Y_op = Y_tp(2,2) - (Y_tp(1,2)*Y_tp(2,1))/(Y_S + Y_tp(1,1));
    Y_op = simplify(Y_op);
end

function [TF] = symb2tf(Y,n)
    % transforms a symbolic expression of a transfer funtion to a matlab
       transfer function
    % n gives the number of ports of the system
    syms s
```

```
    Y = vpa(Y,4);                       % evaluate fractions
    for i = 1:n
        for j = 1:n
            symExp = Y(i,j);
            if has(symExp,s) == 1   % check if symbolic expression is a contant
                ExpFun = matlabFunction(symExp);
                ExpFun = str2func(regexprep(func2str(ExpFun), '\.([/^\\*])', '$1'));
                TF(i,j) = ExpFun(tf('s'));
            else                        % in case symbolic expression is a constant
                TF(i,j) = tf(double(symExp),1);
            end
        end
    end
end
```

E-2 Five-Market Economy

```
%% Define Model Parameters
load('IMparam')

%% Define Basic Blocks (simple agents, transformers, ...)
% System parameters
syms s                  % frequency domain variable
I = tf(1,[1 0]);        % integrator
N = 11;                 % # of agents
rng(100)                % set the seed of the random number generator

for i = 1:N
    L = 1;          % inductor
    C = 0.1;        % capacitor
    R = 0.01;       % series resistor
    G = 0.04;       % parallel resistor

    % Modeling trader using admittance parameters
    z11 = L*s + R + 1/(s*C + G);
    z12 = 1/(s*C + G);
    z21 = 1/(s*C + G);
    z22 = 1/(s*C + G);

    eval(['Z' num2str(i) '= [z11 z12; z21 z22];']);

    % Agent using abcd parameters
    A0 = z2ABCD(eval(['Z' num2str(i)]));
    eval(['A' num2str(i) '= A0;']);

    % Agent using y parameters
    Y = z2y(eval(['Z' num2str(i)]));
    eval(['Y' num2str(i) '= Y;']);
end
```

```
%% Hourglass supply chain
SC_1_Y = Y1 + Y2 + Y3;
SC_1_A = y2ABCD(SC_1_Y);

SC_2_Y = Y4 + Y5;
SC_2_A = y2ABCD(SC_2_Y);

SC_3_Y = Y6;
SC_3_A = y2ABCD(SC_3_Y);

SC_4_Y = Y7 + Y8;
SC_4_A = y2ABCD(SC_4_Y);

SC_5_Y = Y9 + Y10 + Y11;
SC_5_A = y2ABCD(SC_5_Y);

SC_A = SC_1_A * SC_2_A * SC_3_A * SC_4_A * SC_5_A;
SC_Z = ABCD2z(SC_A);

%% Collapsing two-port to one-port
Z_L_CO = R_CO + 1/(s*C_CO);

Z_CO = tp2op_S(SC_Z,Z_L_CO);
TF_CO = symb2tf(Z_CO,1);

num_CO = cell2mat(TF_CO.Numerator);
den_CO = cell2mat(TF_CO.Denominator);

[A_tf, B_tf, C_tf, D_tf] = tf2ss(num_CO,den_CO);

%% Deriving State-Space Model
A_IM = [D_tf*(e_H11+e_G11-e_F11),              C_tf, ...
        D_tf*(e_H12+e_G12-e_F12),                 0, ...
        D_tf*(e_H13+e_G13-e_F13),                 0, ...
        D_tf*(e_H14+e_G14-e_F14),                 0, ...
        D_tf*(e_H15+e_G15-e_F15),                 0;
        B_tf*(e_H11+e_G11-e_F11),              A_tf, ...
        B_tf*(e_H12+e_G12-e_F12), zeros(size(B_tf)), ...
        B_tf*(e_H13+e_G13-e_F13), zeros(size(B_tf)), ...
        B_tf*(e_H14+e_G14-e_F14), zeros(size(B_tf)), ...
        B_tf*(e_H15+e_G15-e_F15), zeros(size(B_tf));
        R_H*(e_H21+e_G21-e_F21),  zeros(size(C_tf)), ...
        R_H*(e_H22+e_G22-e_F22),                C_H, ...
        R_H*(e_H23+e_G23-e_F23),                  0, ...
        R_H*(e_H24+e_G24-e_F24),                  0, ...
        R_H*(e_H25+e_G25-e_F25),                  0;
             (e_H21+e_G21-e_F21), zeros(size(C_tf)), ...
             (e_H22+e_G22-e_F22),                 0, ...
             (e_H23+e_G23-e_F23),                 0, ...
             (e_H24+e_G24-e_F24),                 0, ...
             (e_H25+e_G25-e_F25),                 0;
         R_L*(e_F31-e_G31-e_H31), zeros(size(C_tf)), ...
```

```
          R_L*(e_F32-e_G32-e_H32),                 0, ...
          R_L*(e_F33-e_G33-e_H33),               C_L, ...
          R_L*(e_F34-e_G34-e_H34),                 0, ...
          R_L*(e_F35-e_G35-e_H35),                 0;
              (e_F31-e_G31-e_H31), zeros(size(C_tf)), ...
              (e_F32-e_G32-e_H32),                 0, ...
              (e_F33-e_G33-e_H33),                 0, ...
              (e_F34-e_G34-e_H34),                 0, ...
              (e_F35-e_G35-e_H35),                 0;
          R_C*(e_F41-e_G41-e_H41), zeros(size(C_tf)), ...
          R_C*(e_F42-e_G42-e_H42),                 0, ...
          R_C*(e_F43-e_G43-e_H43),                 0, ...
          R_C*(e_F44-e_G44-e_H44),               C_C, ...
          R_C*(e_F45-e_G45-e_H45),                 0;
              (e_F41-e_G41-e_H41), zeros(size(C_tf)), ...
              (e_F42-e_G42-e_H42),                 0, ...
              (e_F43-e_G43-e_H43),                 0, ...
              (e_F44-e_G44-e_H44),                 0, ...
              (e_F45-e_G45-e_H45),                 0;
          R_F*(e_H51+e_G51-e_F51), zeros(size(C_tf)), ...
          R_F*(e_H52+e_G52-e_F52),                 0, ...
          R_F*(e_H53+e_G53-e_F53),                 0, ...
          R_F*(e_H54+e_G54-e_F54),                 0, ...
          R_F*(e_H55+e_G55-e_F55),               C_F;
              (e_H51+e_G51-e_F51), zeros(size(C_tf)), ...
              (e_H52+e_G52-e_F52),                 0, ...
              (e_H53+e_G53-e_F53),                 0, ...
              (e_H54+e_G54-e_F54),                 0, ...
              (e_H55+e_G55-e_F55),                 0];
B_IM = [             D_tf,                  0,                  0, ...
                        0,                  0;
                     B_tf, zeros(size(B_tf)), zeros(size(B_tf)), ...
        zeros(size(B_tf)), zeros(size(B_tf));
                        0,                R_H,                  0, ...
                        0,                  0;
                        0,                  1,                  0, ...
                        0,                  0;
                        0,                  0,                R_L, ...
                        0,                  0;
                        0,                  0,                  1, ...
                        0,                  0;
                        0,                  0,                  0, ...
                      R_C,                  0;
                        0,                  0,                  0, ...
                        1,                  0;
                        0,                  0,                  0, ...
                        0,                R_F;
                        0,                  0,                  0, ...
                        0,                  1];
C_IM = [1, zeros(size(C_tf)), 0, 0, 0, 0, 0, 0, 0, 0;
        0, zeros(size(C_tf)), 1, 0, 0, 0, 0, 0, 0, 0;
        0, zeros(size(C_tf)), 0, 0, 1, 0, 0, 0, 0, 0;
        0, zeros(size(C_tf)), 0, 0, 0, 0, 1, 0, 0, 0;
```

```
        0, zeros(size(C_tf)), 0, 0, 0, 0, 0, 0, 1, 0];
D_IM = zeros(width(B_IM),height(C_IM));

IM = ss(A_IM, B_IM, C_IM, D_IM);

%% Transform SS to TF
% select input
in = 2;             % 1: Demand/Supply shock
                    % 2: Housing shock
                    % 3: Labor shock
                    % 4: Capital shock
                    % 5: Monetary policy shock
A_siso = A_IM;
B_siso = B_IM(:,in);

% select output
out = 33;           % 1:  Price goods and services
                    % 13: Price housing services
                    % 14: Backlog housing services
                    % 15: Wage
                    % 16: Backlog labor services
                    % 17: Price capital goods
                    % 18: Backlog capital goods
                    % 19: Price financial goods
                    % 20: Backlog financial goods

                    % 21: Effort goods and services
                    % 33: Effort housing services
                    % 34: Flow housing services
                    % 35: Effort labor services
                    % 36: Flow labor services
                    % 37: Effort capital goods
                    % 38: Flow capital goods
                    % 39: Effort financial goods
                    % 40: Flow financial goods
if out <= 20
    I = eye(20);
    C_siso = I(out,:);
    D_siso = B_siso(out,:);
elseif out > 20
    C_siso = A_IM(out-20,:);
    D_siso = B_siso(out-20,:);
end

% Convert to laplace domain
[num,den] = ss2tf(A_siso,B_siso,C_siso,D_siso,1);
syms s
H = poly2sym(num,s)/poly2sym(den,s);
```

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Glossary

List of Acronyms

SISO	single-input single-output
SIMO	single-input multiple-output
MISO	multiple-input single-output
MIMO	multiple-input multiple-output
CCF	controllable canonical form
OCF	observable canonical form
LTI	linear time-invariant
DoF	degrees of freedom
RoC	region of convergence
CPI	consumer price index
GDP	gross domestic product
VAR	vector auto-regression
DSGE	dynamic stochastic general equilibrium
ABM	agent-based modeling
NPV	net present value
DCF	discounted cash flow
NPQ	net present quantity
PID	proportional integral derivative
MPC	model predictive control

List of Symbols

γ	Damping rate/Discount rate
ω	Angular velocity/Yield
ω	Imaginary part/Cyclical trend
ω_a	Anti-resonance frequency/Inelasticity frequency
ω_n	Natural frequency/Inventory cycle
ω_r	Resonance frequency/Bullwhip frequency
σ	Real part/Real trend
τ	Torque/Capital yield desire
θ	Angle/Accumulated yield
ε	Inverse mass/Price elasticity
ζ	Damping ratio/Propensity
C	Capacitance/Inventory elasticity
e	Force/Voltage/Desire
f	Velocity/Current/Flow of goods
G	Conductance/Friction
I	Moment of inertia/Yield sensitivity
$I(s)$	Velocity/Current/Net present quantity
K	Radius/ Capital stock
k	Inventory elasticity
L	Angular momentum/Capital value
L	Inductance/Price elasticity
m	Mass/Price inelasticity
p	Momentum/Price
p_i	Pole/Complex discount rate
q	Position/Commodity stock
q_0	Coordinate/Capital account
q_1	Coordinate/Commodity account
R	Resistance/Friction
s	Laplace variable/Complex trend
t	Time
$V(s)$	Force/Voltage/Willingness
$Y(s)$	Admittance/Elasticity
z_j	Zero/Transmission-blocking rate
f(n)	Digital Signal/Economic time series
F(s)	Signal/Complex present value function
f(t)	Analog signal/Economic time series
i	Imaginary unit
Z(s)	Impedance/Inelasticity

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