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A combined probabilistic-fuzzy approach for dynamic modeling of traffic in smart cities: Handling imprecise and uncertain traffic data

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ABSTRACT

Humans and autonomous vehicles will jointly use the roads in smart cities. Therefore, it is a requirement for autonomous vehicles to properly handle the information and uncertainties that are introduced by humans (e.g., drivers, pedestrians, traffic managers) into the traffic, to accordingly make proper decisions. Such information is commonly available as linguistic, fuzzy (non-quantified) terms. Thus, we need mathematical modeling approaches that, at the same time, handle mixed (i.e., quantified and non-quantified) data. For this, we introduce novel type-2 sets and membership functions to translate such mixed traffic data into mathematical concepts that handle different levels and types of uncertainties and that can undergo mathematical operations. Next, we propose rule-based data processing and modeling approaches to exploit the advantages of these sets. This is inspired by the rule-based reasoning of humans, which has proven to be very effective and efficient in various applications, especially in traffic. The resulting models, hence, handle more than one level and type of uncertainty, which results in precise estimations of traffic dynamics that are comparable in accuracy with similar analyses if only one level of uncertainty (either probabilistic or fuzzy) would exist in the dataset. This will significantly improve the analysis, prediction, management, and safety of traffic in future smart cities.

1. Introduction

Autonomous vehicles are key elements of future transportation systems [1]. These vehicles must navigate efficiently and safely in smart cities, interacting with pedestrians, cyclists, and other (autonomous or human-driven) vehicles. In fact, safety is crucial to ensure that autonomous vehicles will be deployed, accepted, and used [2,3]. Understanding human users of the roads is essential for safety of autonomous vehicles [4–6]. Humans introduce uncertainties to the traffic dynamics, especially based on their cognitive states [7]. Human drivers intuitively perceive and take into account such psychologically-driven variations in the behavior of other drivers and pedestrians [8,9].

Model-based control methods [10] can potentially steer autonomous vehicles based on predictions of the behavior of other people on the road, so that safety is guaranteed and various objectives (e.g., reduced congestion and emissions [11,12]) are obtained.

When reliable, large-scale statistical data from systems or processes is available, generating probability density functions and employing them in the modeling of those systems and processes have effectively been considered for a long time in different fields. Modeling based on statistically sound data, however, requires to perform a large number of controlled tests and to collect and statistically analyze the resulting large datasets [13]. Moreover, the resulting models may still suffer from lack of robustness to

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Table 1
Mathematical symbols and notations frequently used throughout the paper.

$\boldsymbol{u}_s(k)$	Vector of all control inputs to system s for time step k
$u_{s,i}(k)$	Element i of control input vector $\boldsymbol{u}_s(k)$
$\boldsymbol{x}_s^{\text{eval}}(k)$	Vector of all evaluated (e.g., based on observations and/or measurements) state variables of system s for time step k
$x_{s,i}^{\text{eval}}(k)$	Element i of the evaluated state variable vector $\boldsymbol{x}_s^{\text{eval}}(k)$
$\boldsymbol{x}_s^{\text{est}}(k)$	Vector of all state variables of system s estimated (via a model) for time step k
$x_{s,i}^{\text{est}}(k)$	Element i of the estimated state variable vector $\boldsymbol{x}_s^{\text{est}}(k)$
$\boldsymbol{v}_s(k)$	Vector of all uncontrolled inputs (i.e., external disturbances) to system s for time step k
$v_{s,i}(k)$	Element i of the uncontrolled input vector $\boldsymbol{v}_s(k)$
$\lvert\boldsymbol{x}_s\rvert$	Size of the state variable vector for system s
$\lvert\boldsymbol{u}_s\rvert$	Size of the control input vector for system s
$\mathbb{K}$	Set of all discrete time steps
$\mathbb{K}_s^{\text{eval}}$	Set of all time steps when an evaluation of the state variable vector is available for system s
$\mathbb{K}_s^{\text{id}}$	Set of all identification time steps for system s
$\pi_s(k)$	The time step before time step k when the most recent information about the evaluated state variable vector of system s exists

uncertainties due to unmodeled scenarios. Crucially, such models may face the following issues: First, they are not capable of interpreting newly received information/data that involves fuzziness and that has not been (extensively) calibrated or filtered yet, i.e., is given in vague human terms. An example of this is: Driver A is **very frustrated**. Second, these models are not capable of interpreting information/data that does not (in a probabilistic sense, where the likelihoods of complementary events add up to 1) comply with complementary events. An example of this is: Pedestrian B reports that *all* vehicles in area C are driving **high** speed, whereas driver D reports that *some* vehicles in area C drive with an **average** speed.

These excellently motivate the use of fuzzy logic, which has been proposed to handle such lack of precision and consistency in the data that is collected from humans, and the integration of heuristic and statistical knowledge, especially for an application like traffic, where a large number of effective heuristic rules for interpretation and modeling of the processes exists. Moreover, novel approaches that incorporate human-like (logical) reasoning into traffic contexts, especially those that have not been reproduced in controlled test environments, are of significant value [14–16]. Fuzzy logic has proven to be one of the strongest mathematical tools for modeling human perception, cognition, and decision making [17]. While fuzzy sets of type-1 and type-2 handle, respectively, one and two levels of fuzzy uncertainties, systematic incorporation of both probabilistic and fuzzy uncertainties in one model is a challenge. Thus, in this paper, we will close this gap by introducing type-2 sets that incorporate both probabilistic and fuzzy uncertainties, and by introducing rule-based models that apply such combined sets for dynamical systems, e.g., for traffic.

This paper is organized as follows: The rest of this section covers the main contributions of the paper and provides a brief preliminary discussion about fuzzy sets. Section 2 introduces the novel concepts of probabilistic-fuzzy and fuzzy-probabilistic sets for representing data with multiple levels and types of uncertainties. Section 3 proposes a dynamic model that processes such data to estimate and predict the dynamics of the system. We also explain an identification procedure for these type-2 models. Section 5 gives the results of a case study. Finally, Section 6 concludes the paper and suggests some topics for future research.

Table 1 gives the most frequently used mathematical symbols and notations used throughout the paper. Scalar variables have been shown by a regular italic font, whereas for vector variables a bold italic font has been used. Indices of the variables (e.g., the position of that variable in a vector or the index of the system that the variable belongs to) have been indicated as subscripts for those variables. Linguistic terms that may specify or distinguish these variables from one another have been included with a non-italic font, as superscripts for the variables. For instance, the superscript "eval" and "est" have been used to distinguish the evaluated and estimated state variable vectors.

1.1. Road-map and main contributions

To summarize the core findings of the previous section and to provide a clear road-map for the paper, note the following recapped information:

State-of-the-art of online traffic modeling. Analysis and modeling of traffic based on data that is received online is mainly based on quantified data. In future smart cities, however, traffic control centers and connected users will receive various, heterogeneous data from different sources and with different representations. For instance, users of the roads may send a voice or text message about their perception (in vague linguistic terms) of the neighboring traffic, whereas historical data may be obtained from the cameras and other traffic sensors.

Open challenges. Data that is provided by humans in linguistic, fuzzy terms can provide valuable insights about the current and expected states of the traffic, and will thus contribute significantly to effective and efficient control of traffic and to safety in autonomous driving. In most situations, received traffic data involves both quantified and qualified information and uncertainties, whereas current mathematical tools handle one type of uncertainty at a time.

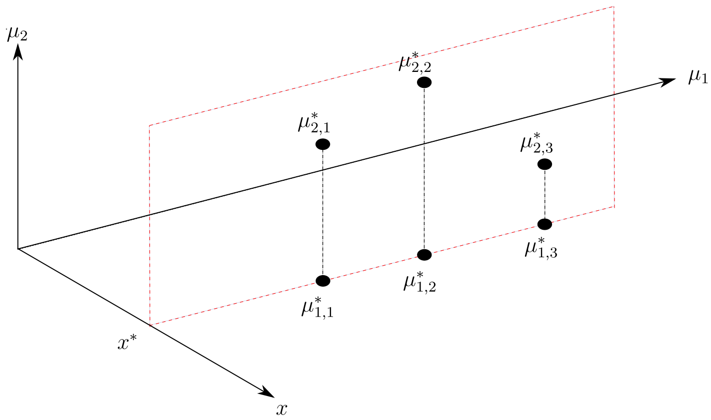

Fig. 1. Type-2 membership function in a discrete domain (for a detailed definition of the mathematical notations see Table 2).

Main contributions. Accordingly, the main contributions of the current paper are:

- we propose a generalization of type-2 fuzzy sets and membership functions, via introducing probabilistic-fuzzy and fuzzy-probabilistic sets and membership functions, for precise representation and analysis of real-life traffic data with both probabilistic and human-based (fuzzy) uncertainties;
- we formulate type-2 rules for human-centered modeling of road traffic network, with (delayed and/or asynchronous) traffic measurements and observations, where the resulting rule-based type-2 models can process such combined probabilistic and human-based uncertainties;
- we provide an assessment of the proposed theoretical approaches via numerical simulations for an urban traffic network.

Expected impact. The resulting analysis and modeling approaches will allow to incorporate more than one level and type of uncertainty, resulting in precise estimations that are comparable in accuracy with similar analyses if only one level of uncertainty (either probabilistic or fuzzy) would exist in the dataset. This not only improves the analysis of traffic, but will significantly contribute to safety and autonomous control of traffic systems in smart cities.

1.2. Background of human behavior modeling in autonomous driving

End-to-end learning-based control, including (fully) convolutional neural networks, with or without long-short-term memory [18–21], has been used for autonomous driving. This approach, which receives an input image to generate a control signal for autonomous driving, requires a large dataset for training its inner neural network.

In [22], a Bayesian neural network has been used that provides predictions about navigation and localization based on GPS and image inputs. Bayesian neural networks generally work based on the concept of probabilistic safety, i.e., the probability that the generated control signal from the Bayesian neural network keeps the vehicle safe [23–25]. Such predictions of safety, however, are still prone to uncertainties, since autonomous driving occurs in highly dynamic environments that cannot comprehensively be modeled according to only one level of quantified (i.e., probabilistic) uncertainties. In addition to noisy measurements and a priori unknown scenarios, the mental states, particularly, intentions of human users of the road cannot be measured (directly) [26]. Thus, various alternative approaches have been proposed, in order to incorporate such non-measurable or non-quantifiable human factors into the decision making, thus traffic modeling, by autonomous vehicles. One of the most promising frameworks includes interaction models based on fuzzy logic [27–29].

Fuzzy-logic-based modeling and control systems are capable of emulating, respectively, the cognitive procedure of humans (including drivers and pedestrians) in processing the environmental data for analyzing the environmental states and changes, and the heuristic knowledge processing and decision making of experts/experienced drivers. Therefore, fuzzy-logic-based methods are able to steer autonomous vehicles similarly to experts/experienced humans [30–36]. A significant advantage of using fuzzy-logic-based models in autonomous driving is the ability of accurately predicting the behavior of human drivers, e.g., in lane shifting [37–39].

1.3. Preliminary discussions

In fuzzy logic [40,41], the concept of sets is generalized. More specifically, while in classical logic an element either belongs to or does not belong to a set, in fuzzy logic, partial membership to a set is allowed, i.e., the membership degree to a set can be a value within $[0,1]$. Thus, each fuzzy set is defined by a membership function. Fuzzy sets are the most proper mathematical tools for representing human-inspired uncertain perceptions and for processing non-quantified (linguistic) data that has more than a unique possible quantification.

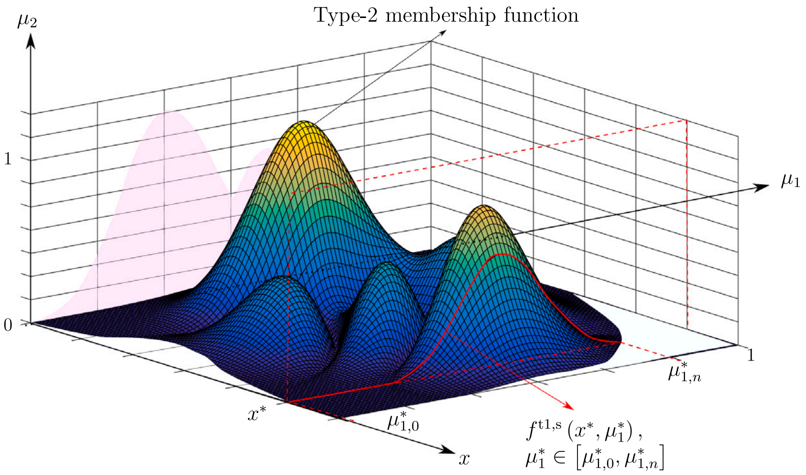

Fig. 2. Type-2 membership function in a continuous domain (for a detailed definition of the mathematical notations see Table 2).

Table 2
Mathematical notations used in Section 1.3.

x	A (fuzzy) variable with domain $\mathbb{X}$
$f^{t1,p}(\cdot)$	A primary type-1 membership function defined on the domain $\mathbb{X}$ and returning a value from $[0,1]$
$f^{t1,s}(\cdot,\cdot)$	A secondary type-1 membership function defined on the domain $\mathbb{X} \times [0,1]$ and returning a value from $[0,1]$
x^*	A (fuzzy) element, i.e., a realization for x
$\mu_{i,j}$	Primary ($i = 1$) or secondary ($i = 2$) membership degree for element j
$\mu^*_{i,j}$	A realization for $\mu_{i,j}$

A fuzzy set that allows partial membership of elements, with certain degrees of (partial) membership, is a type-1 fuzzy set. Type-2 fuzzy sets [42] additionally handle multiple levels of uncertainties, i.e., similarly to type-1 fuzzy sets there is uncertainty about whether or not an element belongs to the type-2 fuzzy set (thus a primary membership degree in $[0,1]$ is assigned to each element) and also this degree of membership is uncertain, i.e., there is a degree of certainty within $[0,1]$ about the primary membership degree. Mathematically, with n fuzzy type-2 sets defining the domain of element x^*, each primary membership degree $f_i^{t1,p}(x^*) \in [0,1]$ for element x^*, with $i \in \{1,\ldots,n\}$, corresponds to a secondary membership degree $f_i^{t1,s}\left(x^*, f_i^{t1,p}(x^*)\right) \in [0,1]$, which represents the level of certainty about the primary membership degree.

Membership functions of type-2 fuzzy sets are represented in a 3D space: Fig. 1 shows a discrete-time type-2 membership function, where three primary membership degrees $\mu_{1,1}$, $\mu_{1,2}$, and $\mu_{1,3}$ are proposed for x^*, with secondary membership degrees $\mu_{2,1}$, $\mu_{2,2}$, and $\mu_{2,3}$, respectively. An illustration of a continuous-domain type-2 membership function with its secondary type-1 membership functions $f^{t1,s}(x^*,\cdot)$ defined for an arbitrary value x^* is provided in Fig. 2, where the secondary type-1 membership function corresponds to the intersection of the given type-2 fuzzy membership function with the plane parallel to the μ_1–μ_2 plane through the point x^*.

In general, uncertainties may be probabilistic or fuzzy. When various possible realizations of an event or variable are complementary, i.e., their degrees of certainty add up to 1, the uncertainty is probabilistic and these probabilities are represented via probability density functions [43]. With fuzzy uncertainties, different realizations of an event or variable do not necessarily add up to 1. In other words, fuzzy interpretation of data allows to carry the uncertainties through analyzing and processing a data, without limiting the possible realization to a unique value. This is particularly important when associating a certain interpretation to a human-based concept is either impossible or likely erroneous. Fuzzy logic, thus, allows analysis and mathematical operations on such data, without the need to assign certainty to the realizations initially.

2. Probabilistic-fuzzy & fuzzy-probabilistic sets for traffic data

In this section, we expand the concept of type-2 fuzzy sets, in order to represent various traffic uncertainties that should autonomously be processed and modeled in smart cities. In other words, we focus on data that involve both primary and secondary uncertainties, due to combined probabilistic and human-based information that is involved in traffic phenomena.

Consider a vehicle at an intersection (see the red ellipse in Fig. 3), where the vehicle has a known destination (indicated by "D" in Fig. 3), which is feasible both if the vehicle turns right at the intersection and if it moves straight ahead. An autonomous vehicle (see the yellow ellipse in Fig. 3) behind this vehicle may assign a probability to each of these events, based on the historical observations about the behavior of vehicles with the same destination (e.g., 75% to turning right and 25% to moving straight ahead). In case

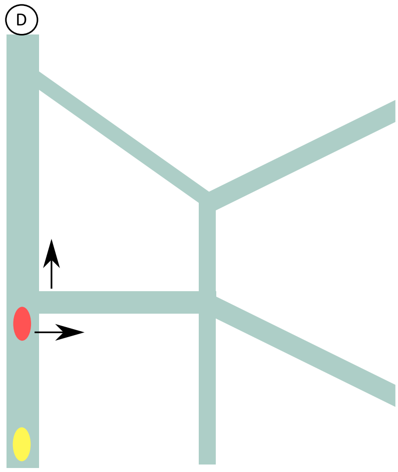

Fig. 3. Illustration of a part of a traffic network: The yellow ellipse represents an autonomous vehicle that should model the behavior of the front vehicle shown via a red ellipse, where the front vehicle has destination "D". The front vehicle may choose to continue its trip in the direction of either of the black arrows shown in the figure.

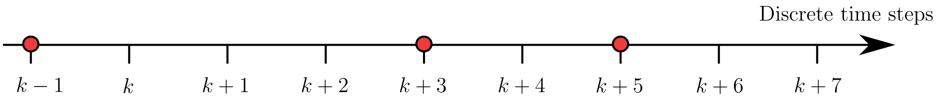

Fig. 4. Illustration of the discrete time steps, when the dynamics of the traffic network is updated using a discrete-time modeling approach. The small red circles imply that information (based on measurements or observations) about the state variable vector is available at that time step. Note that collecting such information is not necessarily done periodically.

such historical data is not available, or when incorporation of a human-centered approach is of interest (e.g., including the mental states of the front driver in their decision making) an intuitive (i.e., quantitative) approach may be used: For instance, the driver is **likely** to change the destination due to being tired, or is **very unlikely** to turn right due to the large number of traffic lights on its way. While this data does not correspond to the historical observations, they reflect a more human-inspired and human-centered interpretation. For efficient and safe route planning, autonomous vehicles should deal with both historical and human-inspired interpretation of data.

In particular, representing the environmental data, such that they incorporate both levels of uncertainty, i.e., probabilistic and fuzzy, and developing models that handle such data, allow for analysis and decision making that simultaneously benefit from both historical knowledge and human-inspired intuition in traffic. For instance, the autonomous vehicle can use the following data to analyze the behavior of the front vehicle: "54% of drivers are **very likely** to change their destination when traffic is slow", whereas "46% of drivers are **likely** to move straight ahead". In these statements, the primary uncertainty (expressed via **very likely** and **likely**) is fuzzy (i.e., is based on a human-inspired approach), whereas the secondary uncertainty about the primary uncertainty is probabilistic (i.e., is gathered via historical observations). This may be swapped by having a probabilistic (historically known) primary uncertainty and a fuzzy (human-inspired) secondary uncertainty. For instance, "**many** observations indicate that 23% of the drivers change their destination and the rest turn right", whereas "**some** observations indicate that 77% of the drivers go straight ahead and the rest turn right". We mathematically represent such analyses of environmental data in smart cities via, respectively, probabilistic-fuzzy (i.e., a primary type-1 membership function and a secondary probability density function) and fuzzy-probabilistic (i.e., a primary probability density function and a secondary type-1 membership function) membership functions.

3. Type-2 knowledge-based dynamical models

Next, we formulate type-2 rules that can effectively be used as the basis of dynamic mathematical models, to autonomously process uncertain (combined probabilistic and fuzzy) data in smart cities and to make decisions based on such data. We consider discrete-time models throughout the paper, as such discretization is common in both collecting traffic data and modeling the evolution of the traffic states (see, e.g., [11,44–46]). A discrete-time traffic model can also perfectly match and feed a decision-making system for traffic that performs based on a discrete-time framework.

We will consider various types of traffic data that may be collected within smart cities, from both users and managers of the roads and the sensor and measurement devices: for instance, historical and statistical data based on the cameras and traffic detectors on the roads; fuzzy data and information that is provided by large groups of participants about their regular driving habits and their culture of using the roads (this procedure is performed in advance and offline, and through, e.g., questionnaires and interviews);

online data and information (qualitative, quantitative, or combined) that sensors and humans send out (to, e.g., a central control station) via their mobile phone, smartwatch, or any other (simple) device or app that may be installed in their cars.

To provide a more realistic modeling approach for traffic, nonlinear dynamics are modeled considering measurements and observations that are possibly delayed. In other words, at time step k the state variable vector $\boldsymbol{x}_s^{\text{eval}}\left(\pi_s(k)\right)$ that has been evaluated (i.e., has been described quantitatively, qualitatively, or in a combined way) based on the most recent information captured at time step $\pi_s(k)$ is used for system s, with $\pi_s(k)$ an integer, for which we have $\pi_s(k) < k$. In general, $\pi_s(k)$ may change for each time step. For instance, if at time step 10 the most recent information about the evaluated state variable vector corresponds to time step 8 (i.e., $\pi_s(k) = k-2$ for $k = 10$), and then at time step 11 no new information is received, then $\pi_s(11) = 8$, i.e., $\pi(k) = k-3$ for $k = 11$. Fig. 4 shows an example, where for various time steps, the time step when the most reliable measurement is available is the following: $\pi_s(k-1) = \pi_s(k) = \pi_s(k+1) = \pi_s(k+2) = k-1$, $\pi_s(k+3) = \pi_s(k+4) = k+3$, and $\pi_s(k+5) = \pi_s(k+6) = \pi_s(k+7) = k+5$.

For system s with missing or delayed evaluations for the state variables at time step k, the dynamics at this time step may be formulated as a function of the most recent evaluated state variable vector, i.e., $\boldsymbol{x}_s^{\text{eval}}\left(\pi_s(k)\right)$, and of all the control inputs that have affected the dynamics of the system from time step $\pi_s(k)$ until time step $k-1$.

Therefore, such systems possess an input-delayed dynamics formulation. A logical "If-then" rule for modeling the dynamics of system s for time step k is, in general, formulated by:

$$\begin{aligned}
\text{If}\quad & \boldsymbol{x}_s^{\text{eval}}\left(\pi_s(k)\right) \quad \text{is} \quad \left(\tilde{X}_1, \ldots, \tilde{X}_{|\boldsymbol{x}_s|}\right) \quad \text{and} \\
& \boldsymbol{u}_s\left(\pi_s(k)\right) \quad \text{is} \quad \left(\tilde{U}_{\pi_s(k),1}, \ldots, \tilde{U}_{\pi_s(k),|\boldsymbol{u}_s|}\right) \quad \text{and} \\
& \ldots \quad \text{and} \\
& \boldsymbol{u}_s(k-1) \quad \text{is} \quad \left(\tilde{U}_{k,1}, \ldots, \tilde{U}_{k,|\boldsymbol{u}_s|}\right), \\
\text{then}\quad & \boldsymbol{x}_s^{\text{est}}(k) = f\left(\boldsymbol{\theta}_s^{\text{con}}(k), \boldsymbol{x}_s^{\text{eval}}\left(\pi_s(k)\right), \boldsymbol{u}_s\left(\pi_s(k)\right), \ldots, \boldsymbol{u}_s(k-1)\right)
\end{aligned} \tag{1}$$

where the following definition holds in our mathematical notations:

$$\begin{aligned}
& \boldsymbol{x}_s^{\text{eval}}\left(\pi_s(k)\right) \quad \text{is} \quad \left(\tilde{X}_1, \ldots, \tilde{X}_{|\boldsymbol{x}_s|}\right) \Leftrightarrow \\
& x_{s,1}^{\text{eval}}\left(\pi_s(k)\right) \quad \text{is} \quad \tilde{X}_1 \quad \text{and} \quad \ldots \quad \text{and} \quad x_{s,|\boldsymbol{x}_s|}^{\text{eval}}\left(\pi_s(k)\right) \quad \text{is} \quad \tilde{X}_{|\boldsymbol{x}_s|}
\end{aligned} \tag{2}$$

and for $\kappa = \pi_s(k), \ldots, k-1$:

$$\begin{aligned}
& \boldsymbol{u}_s(\kappa) \quad \text{is} \quad \left(\tilde{U}_{\kappa,1}, \ldots, \tilde{U}_{\kappa,|\boldsymbol{u}_s|}\right) \Leftrightarrow \\
& u_{s,1}(\kappa) \quad \text{is} \quad \tilde{U}_{\kappa,1} \quad \text{and} \quad \ldots \quad \text{and} \quad u_{s,|\boldsymbol{u}_s|}(\kappa) \quad \text{is} \quad \tilde{U}_{\kappa,|\boldsymbol{u}_s|}
\end{aligned} \tag{3}$$

Moreover, $k \in \mathbb{K}$ and $\pi_s(k) \in \mathbb{K}_s^{\text{eval}}$, where $\mathbb{K}_s^{\text{eval}} \subseteq \mathbb{K}$, and $\tilde{X}_1, \ldots, \tilde{X}_{|\boldsymbol{x}_s|}$ and $\tilde{U}_{\kappa,1}, \ldots, \tilde{U}_{\kappa,|\boldsymbol{u}_s|}$ are type-2 sets (that may be described by a probabilistic-fuzzy or fuzzy-probabilistic membership function) that represent the uncertain elements of, respectively, $\boldsymbol{x}_s^{\text{eval}}$ and $\boldsymbol{u}_s$.

For instance, suppose that the state variable vector has one element, driving aggression. Human users and managers of the road, who are the main sources of evaluating this variable, usually consider no precise quantification for driving aggression, and instead, perceive this variable based on their heuristics. Then, the evaluation that is represented via "$x_{s,1}^{\text{eval}}\left(\pi_s(k)\right)$ is $\tilde{X}_1$" may correspond to "recent observations imply with a certainty following a **Gaussian probability density function** that the driving aggression on the roads has been **high**". Thus, $\tilde{X}_1$ in this case is a probabilistic-fuzzy type-2 set with primary fuzzy and secondary probabilistic uncertainties.

In (1) and (3), the past control inputs may also be represented by type-2 sets, due to the source (e.g., human perception) of reporting the information corresponding to these inputs, or due to limiting the storage requirements, where instead of recording the exact values, the data is stored under a limited number of linguistic categories (e.g., small, average, large). For instance, suppose that the control input vector has one element, the maximum preferred driving speed. The evaluation that is represented via "$u_{s,1}(\kappa)$ is $\tilde{U}_{\kappa,1}$" may correspond to "based on **most** of the received reports, the vehicles on the road followed a symmetric piece-wise linear probability density function, choosing a maximum preferred driving speed in the range of $[8, 13]$ meter per second". This implies that $\tilde{U}_{\kappa,1}$ is a fuzzy-probabilistic type-2 set with primary probabilistic and secondary fuzzy uncertainties.

Finally, in general $f(\cdot)$ in (1) is a nonlinear function, and $\boldsymbol{\theta}_s^{\text{con}}(k)$ is a vector containing the design parameters for function $f(\cdot)$ for time step k. This parameter vector may in the course of running the model be re-identified for improving the model precision.

Remark 1. A type-2 model of the system dynamics is composed of a rule base with several logical rules of the form (1). Each rule may generate a different value for an estimated state variable. The estimated value of that state variable can be obtained via a linear combination of all the values produced by the various rules.

Remark 2. In (1) in addition to the parameters that identify the outputs of the rules, adaptive parameters $\boldsymbol{\theta}_s^{\text{ant}}(k)$ may also be considered for the mathematical formulation of the type-2 sets in the antecedent of the rules. More specifically, the primary and secondary membership functions may involve tuning parameters that will be fine-tuned whenever needed.

Remark 3. In general, each system s is also prone to uncontrolled inputs (i.e., external disturbances) denoted by $\boldsymbol{v}_s(k)$. Disturbances in a traffic network could refer to the changes in the state variables due to accidents or unexpected inflow of vehicles to the network. In such cases in (1), the uncontrolled inputs and their qualifications should be included similarly to the controlled inputs.

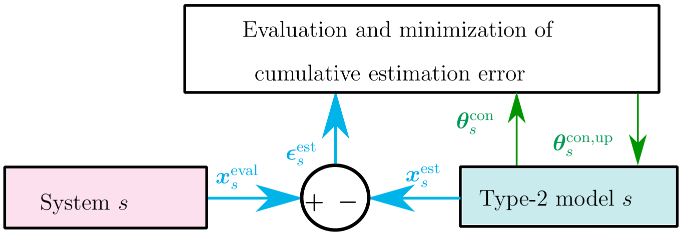

Fig. 5. Identification of the parameters of the type-2 model for system s via minimizing the estimation error of the model. The estimation error $\|\boldsymbol{x}_s^{\text{eval}} - \boldsymbol{x}_s^{\text{est}}\|$ of the type-2 model is denoted by ϵ_s^{est} and for the sake of simplicity time steps are not indicated. In fact the estimation error may be computed and accumulated within a time window and the accumulated error will then be minimized. Moreover, the superscript "up" is used for the updated consequent parameter vector.

4. Parameter identification for type-2 models

For the type-2 model of system s that involves a rule base composed of rules with formulation (1), the parameter vectors $\boldsymbol{\theta}_s^{\text{ant}}(k)$ and $\boldsymbol{\theta}_s^{\text{con}}(k)$ per rule in the rule base are identified or re-identified at the time steps in the set $\mathbb{K}_s^{\text{id}}$. The most recent element of $\mathbb{K}_s^{\text{id}}$ before the current identification time step ℓ is denoted by $\pi_s^{\text{id}}(\ell)$. Therefore, in (1) for $\pi_s^{\text{id}}(\ell) \le k < \ell$, we have $\boldsymbol{\theta}_s^{\text{con}}(k) = \boldsymbol{\theta}_s^{\text{ant}}\left(\pi_s^{\text{id}}(\ell)\right)$ and $\boldsymbol{\theta}_s^{\text{ant}}(k) = \boldsymbol{\theta}_s^{\text{con}}\left(\pi_s^{\text{id}}(\ell)\right)$.

4.1. Re-identification of the antecedent parameters

Re-identification of $\boldsymbol{\theta}_s^{\text{ant}}(\pi_s^{\text{id}}(\ell))$ at time step ℓ may be done through a mapping on the most recently identified vector $\boldsymbol{\theta}_s^{\text{ant}}\left(\pi_s^{\text{id}}(\ell)\right)$. Moreover, we consider the set $\mathbb{L}_s^{\text{ant}}(\ell)$, which includes time step ℓ and a number of earlier time steps. For a certain number of these time steps an evaluation of the state variable vector should exist, i.e., $\mathbb{K}_s^{\text{eval}} \cap \mathbb{L}_s^{\text{ant}}(\ell) \neq \emptyset$. Then, the set $\mathbb{X}_s^{\text{eval}}(\ell)$ including the evaluated state variable vectors for all the time steps in $\mathbb{K}_s^{\text{eval}} \cap \mathbb{L}_s^{\text{ant}}(\ell)$, and the set $\mathbb{U}_s(\ell)$ consisting of control input vectors for all these time steps are also included in the mapping. Note that if uncontrolled inputs also exist, they may also be included. We assume that all these inputs can be recovered (possibly not as precise quantities, but via type-2 sets, as was explained in an example in the third paragraph below (3)).

Then we have:

$$\boldsymbol{\theta}_s^{\text{ant}}(\ell) = \mathcal{O}\left(\boldsymbol{\theta}_s^{\text{ant}}\left(\pi_s^{\text{id}}(\ell)\right), \mathbb{X}_s^{\text{eval}}(\ell), \mathbb{U}_s(\ell)\right) \tag{4}$$

with $\mathcal{O}(\cdot)$ a generally nonlinear mapping (see, e.g., [47]). The definition and parameters corresponding to the antecedent terms, especially the fuzzy membership functions, are largely dependent on the perceptions of the humans who report this data. In practice, after a long enough interaction with the system and collecting and analyzing such data, the identified parameters will remain constant [48].

4.2. Re-identification of the consequent parameters

Re-identification of $\boldsymbol{\theta}_s^{\text{con}}(\pi_s^{\text{id}}(\ell))$ at time step ℓ is based on the information corresponding to the time steps in set $\mathbb{L}_s^{\text{con}}(\ell)$. This set includes time step ℓ and a number of earlier time steps, where for a certain number of these time steps an evaluation of the state variable vector exists. Therefore, we have $\mathbb{K}_s^{\text{eval}} \cap \mathbb{L}_s^{\text{con}}(\ell) \neq \emptyset$. The parameter vector $\boldsymbol{\theta}_s^{\text{con}}(\pi_s^{\text{id}}(\ell))$ of the consequent of the type-2 rules for system s may be updated at time step ℓ, by minimizing, within time window $\mathbb{L}_s^{\text{con}}(\ell)$, the cumulative error of the reported evaluations of the state variable vectors and these vectors when estimated by the type-2 model. For instance, the following optimization problem may be solved to determine the consequent parameter vector $\boldsymbol{\theta}_s^{\text{con}}(\ell)$:

$$\min_{\boldsymbol{\theta}_s^{\text{con}}(\ell)} \sum_{\kappa \in \left(\mathbb{K}_s^{\text{eval}} \cap \mathbb{L}_s^{\text{con}}(\ell)\right)} \left\| \boldsymbol{x}_s^{\text{eval}}(\kappa) - \boldsymbol{x}_s^{\text{est}}(\kappa) \right\| \tag{5}$$

where $\boldsymbol{x}_s^{\text{eval}}(\kappa)$ is retrieved from $\mathbb{X}_s^{\text{eval}}(\ell)$, and $\boldsymbol{x}_s^{\text{est}}(\kappa)$ is estimated in the loop of the optimization problem using (1) with $\boldsymbol{\theta}_s^{\text{ant}}\left(\pi_s^{\text{id}}(\ell)\right)$ for the antecedent parameters if (4) is not run (yet) or with the updated vector $\boldsymbol{\theta}_s^{\text{ant}}(\ell)$ otherwise. Moreover, the set $\mathbb{U}_s(\ell)$ that consists of control input vectors (recorded generally as type-2 sets) for all the time steps in the window of the optimization problem is assumed to be available.

In general, (5) is a nonlinear constrained optimization problem, in case the state variable vector should be bounded. To prevent the risk of infeasibility, one may make the optimization loop unconstrained by relaxing the implicit constraint that is defined on the optimization variable in (5), and by instead constraining $f(\cdot)$ in (1), e.g., by saturating the output of this function. Moreover, since (5) is in general non-convex, a multi-start optimization solver, e.g., based on a genetic algorithm or pattern search, may be considered.

The identification procedure explained above has been simplified and illustrated in Fig. 5: The difference between the state variable vector estimated via the type-2 model for system s and its evaluation (via qualified, quantified, or combined data) is shown

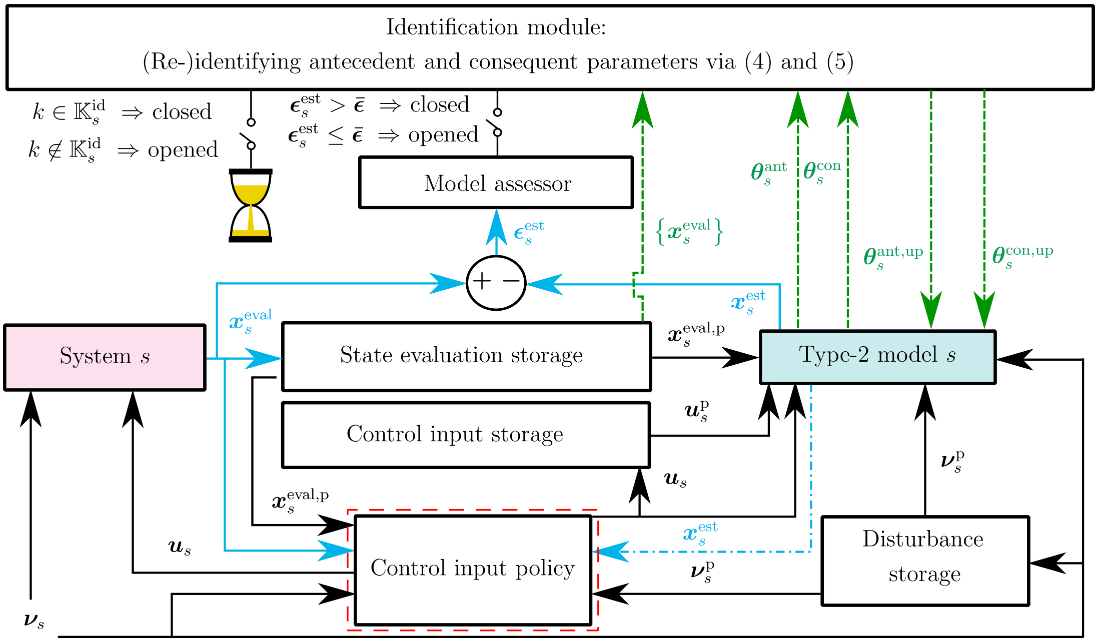

Fig. 6. Detailed illustration of the identification procedure for type-2 models of system s. The signals indicated by blue solid lines are activated when an evaluation (either qualified, quantified, or a combination of both) of the state variable vectors is received. The signals indicated by blue dash-dotted lines are activated when the ones indicated by the blue solid signals are not activated. The signals indicated by dashed green lines correspond to the model (re-)identification procedure and they are also activated at specific pre-set time steps $k \in \mathbb{K}_s^{\mathrm{id}}$ or at any time step at which the model evaluator notices, based on the estimation error $\epsilon_s^{\mathrm{est}}$, that an identification threshold $\bar{\epsilon}$ has been exceeded. In this figure, the superscripts "up" and "p" have been used for, respectively, "updated" and "past". The other notations follow the general notation rules of the paper, as given in Table 1, and also explained in detail in Section 1.

by ϵ_s^{e}. With more details, Fig. 6 shows the type-2 model of system s. Three storage modules for the past evaluated state variables and external disturbances, as well as the previously injected control inputs (which are all distinguished in the figure via a superscript "p" for the word "past") are considered. These are needed, according to (1) and (5), to re-identify the parameters of the type-2 model. Note that this data is in general recorded including the probabilistic and fuzzy uncertainties using type-2 sets introduced in Section 2. The control input policy that generates vector $\boldsymbol{u}_s$ is illustrated within a dashed red rectangle in Fig. 6. This controller receives the past evaluations of the state variable vector (indicated by $\boldsymbol{x}_s^{\mathrm{eval,p}}$) from the state evaluation storage, as well as the current evaluated state variable vector, indicated by $\boldsymbol{x}_s^{\mathrm{eval}}$, from the system. In case an evaluation is not available at the current time step, the current state variable vector estimated by the type-2 model, and indicated by $\boldsymbol{x}_s^{\mathrm{est}}$, as well as the corresponding current and previous corresponding external disturbances (indicated, respectively, by $\boldsymbol{\nu}_s$ and $\boldsymbol{\nu}_s^{\mathrm{p}}$) from the disturbance storage will be used. Next, the resulting control input vector $\boldsymbol{u}_s$ is injected into the system and into the control input storage.

In Fig. 6, the signals indicated by solid blue arrows correspond to the time steps for which an evaluation of the state variable vector of the system is available. For the other time steps, the signals marked by dash-dotted blue arrows will be activated. The signals indicated by dashed green arrows in Fig. 6 will be activated only at identification time steps. For pre-specified identification time steps $k \in \mathbb{K}_s^{\mathrm{id}}$ or whenever the model assessor identifies that the type-2 model should be updated (i.e., when $\epsilon_s^{\mathrm{est}}$ exceeds a pre-specified threshold $\bar{\epsilon}$), the identification module will be activated. This module will then receive the sequence (shown by $\{\boldsymbol{x}_s^{\mathrm{eval}}\}$ in Fig. 6) of all evaluated state variable vectors within time window $\mathbb{L}_s^{\mathrm{ant}}(\ell) \cup \mathbb{L}_s^{\mathrm{con}}(\ell)$ from the state evaluation storage, as well as the most recent parameter vectors $\boldsymbol{\theta}_s^{\mathrm{ant}}$ and $\boldsymbol{\theta}_s^{\mathrm{con}}$ from the type-2 model, and updates these vectors using, respectively, (4) and (5).

5. Case study: Traffic modeling

To illustrate and assess the proposed type-2 fuzzy modeling approaches for autonomous data analysis and modeling in smart cities, when additional uncertainties are introduced into the data via humans, we now present a case study that involves an urban traffic network. We estimate the accuracy of a type-1 fuzzy model that is used when only primary uncertainties exist in the traffic data. We then consider traffic data that includes a second level of uncertainty introduced by humans, and compare the estimations made by type-2 models that include fuzzy-probabilistic, probabilistic-fuzzy, and type-2 fuzzy sets with those estimated when data was uncertain only in one level. The goal is to obtain insights about how effectively these type-2 models can handle the additional level of uncertainty in estimation of traffic states.

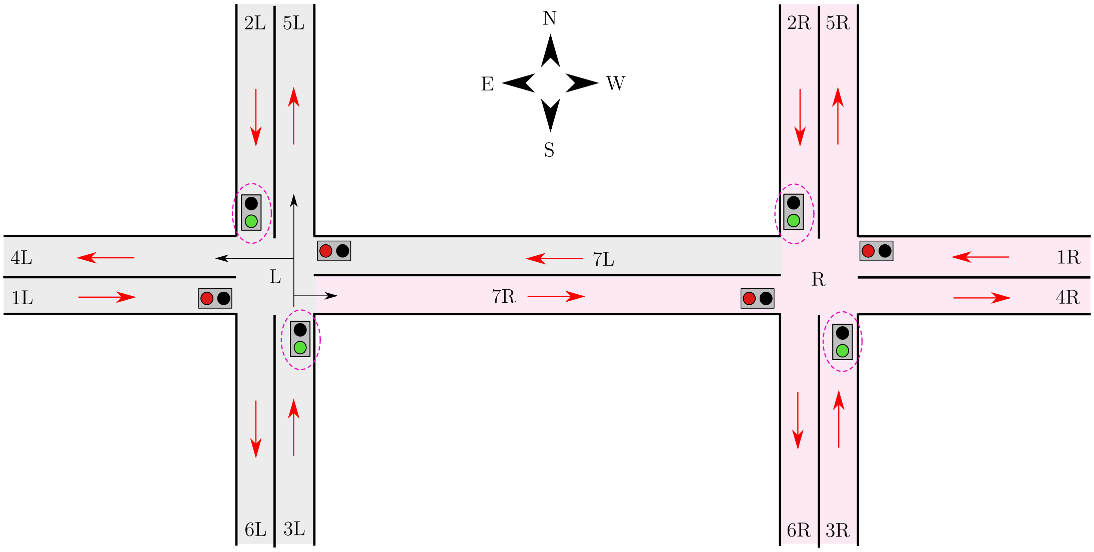

Fig. 7. Urban traffic network used for the case study.

5.1. Setup

The traffic network shown in Fig. 7 is considered. The network consists of two intersections (indicated by the labels "L" and "R") and seven links, each of which two lanes. The lanes are indicated by the labels 1L, …, 7L, and 1R, …, 7R. Lanes 1L, 2L, 3L, and 7L are the entrance lanes for intersection L, whereas lanes 4L, 5L, 6L, and 7R are the exit lanes of intersection L (in the figure the direction in which the vehicles drive in a given lane is indicated by a red arrow). Likewise, 1R, 2R, 3R, and 7R are the entrance lanes for intersection R, and 4R, 5R, 6R, and 7L are the exit lanes for intersection R.

In the remainder of the paper, lanes in links 1, …, 6 are referred to as "side lanes", and the two lanes in link 7 are called "connecting lanes". Turning, except for U-turns, is allowed at the intersections (as indicated by the black arrows in Fig. 7). Each intersection has four traffic signals, each of which controls all the rights-of-way of the entrance lane at which the traffic signal is located. The traffic signals at the opposite entrance lanes of an intersection are synchronized and the follow the same schedule (i.e., in Fig. 7 the green and red phases of the northern and southern traffic signals coincide, and a similar statement holds for those of the western and eastern ones). In the given traffic network, the length of the side lanes is 150 m, the length of the connecting lane is 300 m, the average vehicle length (including the safety distances from and to the preceding and following vehicles) is 7.5 m, and the cycle time of the traffic signals is 90 s.

5.2. Modeling

The given traffic network is divided into two sub-networks, called "sub-network 1" and "sub-network 2". In Fig. 7 these sub-networks are indicated in, respectively, gray and pink. More specifically, sub-network 1 contains intersection L and lanes 1L, …, 7L, while sub-network 2 consists of intersection R and lanes 1R, …, 7R. For each sub-network, one type-1 model and three different classes of type-2 models with a formulation following (1) that describe the traffic behavior are developed:

- "class 0" involves type-1 fuzzy membership functions and is considered to compare the efficacy of type-2 models versus type-1 models;
- "class 1" involves type-2 fuzzy membership functions;
- "class 2" includes probabilistic-fuzzy membership functions, and
- "class 3" includes fuzzy-probabilistic membership functions.

The models involve two state variables, i.e., the total number of vehicles per link, n, and the number of vehicles in the queue on a link, q. Note that defining the dynamics of urban traffic with 2 state variables is a common approach in literature (see, e.g., [45,49]). The control input and external disturbances are, respectively, the green time of the traffic signals and the external inflow of the vehicles.

For the sake of simplicity, we assume that the most recent evaluation of the state variables for each time step k, has been done at time step $k-1$. Each type-2 rule indexed by r is of the following from:

$$\begin{aligned}&\text{If } x(k-1) \text{ is } \tilde{X}_r \text{ and } u(k-1) \text{ is } \tilde{U}_r \text{ and } v(k-1) \text{ is } \tilde{N}_r,\\&\text{then } x_r(k) = a_{0,r} + a_{1,r}x(k-1) + a_{2,r}u(k-1) + a_{3,r}v(k-1),\end{aligned} \tag{6}$$

where x is a state variable of the traffic network (i.e., n or q for a particular lane) and the symbol x_r in the consequent refers to the estimated value of x via rule r. For the traffic scenarios considered in this case study, the range of variations of the parameters of the considered traffic network is limited. Therefore, the parameters of the type-2 sets $\tilde{X}_r$, $\tilde{U}_r$, and $\tilde{N}_r$ in the antecedents are assumed to be fixed, and only the parameter vectors $\left[a_{0,r}, a_{1,r}, a_{2,r}, a_{3,r}\right]^\top$ of the consequent are (re-)identified. The control input of each sub-network is the green time of the northern and southern traffic signals (marked by the red dashed ovals in Fig. 7). These traffic signals are synchronized. As a consequence, the green time of the other two traffic signals of each intersection (which are also synchronized) is the difference between the fixed cycle time of the intersection and the control input. In this case study the vehicle flows that enter the network via the source lanes (1L, 2L, 3L, 1R, 2R, 3R) are considered as the external disturbances.

When a fuzzy membership function is considered as the primary membership function of the type-2 sets $\tilde{X}_r$ and $\tilde{N}_r$, to which the state variables and the external disturbances belong, we use two qualitative terms "low" and "high" to describe the sets. Moreover, for the type-2 set $\tilde{U}_r$, to which the control inputs belong, two qualitative terms "short" and "long" are considered for the fuzzy primary membership function. For the secondary fuzzy membership functions, the qualitative terms "slightly likely", "moderately likely", and "highly likely" are used. Therefore, the statement $n(k-1)$ is $\tilde{X}_r$ in the antecedent, with $\tilde{X}_r$ represented via a type-1 fuzzy set, a type-2 fuzzy set, a type-2 probabilistic-fuzzy set, and a type-2 fuzzy-probabilistic set, may, for instance, be respectively given by:

- *The number of vehicles on the lane is* ***high***
- *It is* ***very likely*** *that the number of vehicles on the lane is* ***high***.
- *There is a chance according to a* ***Gaussian density function*** *that the number of vehicles on the lane is* ***high***
- *It is* ***likely*** *that the number of vehicles on the lane follows a* ***Gaussian density function*** *over the total range of the demand*

Since in the first case, only one level of uncertainty (which is fuzzy) exists, a natural expectation is that the most precise results are obtained based on such data, using the corresponding type-1 model, in comparison with all other cases where two levels of uncertainties exists.

However, one important aspect to consider in addition to the number of uncertainty levels, is the type of the uncertainties. In type-1 and class 1 type-2 models, the uncertainty is handled only after the linguistic terms have been translated into quantified uncertainties (via fuzzy membership functions) and after the corresponding parameters have been identified. Analyzing probabilistic data requires one step less, i.e., the quantification is not needed. Thus, it is again expected to experience less imprecision when uncertainties are already quantified (at least in one of the two uncertainty levels).

In combined fuzzy-probabilistic and probabilistic-fuzzy models, the fuzziness is accompanied by quantified statistical uncertainties. Thus, it is not unexpected to observe a competitive trend in the accuracy of the results of these combined models and the type-1 model.

In summary, we are in particular interested in seeking answers for the following questions, via our case studies and results:

Q1. Which model(s) will result in the most precise estimations of the traffic state variables in most case studies? Which model(s) will result in the least cases with the least precise estimations of traffic states?

Q2. How complex/demanding the computations corresponding to type-2 sets and hence type-2 models are in general? What the sources of these potential complexities are?

5.2.1. Type-1 trapezoidal membership functions

For the models in class 0, type-1 trapezoidal membership functions are considered because of two main reasons: next to the simplicity and low computation time, both empirical evidence and theory show that, compared to more complicated membership functions, trapezoidal membership functions are very efficient in various engineering applications [50].

Fig. 8 shows the type-1 trapezoidal membership functions used for the class 0 models. The maximal number of type-1 fuzzy rules with n^{in} inputs that can be constructed for a type-1 model within class 0 is $\prod_{i=1}^{n^{\text{in}}} \rho_i$, where ρ_i is the number of possible linguistic/fuzzy realizations for the ith input variable. Note that this is based on the assumption that an initial filtering has been implemented, such that the best output realization for each combinations of the n^{in} inputs has been selected.

Therefore, in this case the maximum number of type-1 fuzzy rules within class 0 is $2^3 = 8$, i.e., the antecedent statements concerning the state variable and the external disturbance can each adopt 2 descriptions from the set {low,high} and the antecedent statement for the control input can also adopt either of the 2 descriptions within the set {short,long}. Hence, the total number of parameters that should be identified for the type-1 fuzzy model is 4(8), i.e., 32 parameters, per state variable. Notice that rules of the class 0 model follow (6) with 4 identification parameters in the consequent for each of the two state variables, where the sets in the antecedent are type-1 fuzzy sets.

5.2.2. Type-2 fuzzy membership functions

In the linguistic description of the rules of a type-2 class 1 model, we consider *low* or *high* for the state variable and also for the external disturbance, *short* or *long* for the control input, and *slightly likely*, *moderately likely*, or *very likely* for the secondary uncertainties. The corresponding mathematical representations are illustrated in Fig. 9, where three different interpretations per term *low*, *high*, *short*, and *long* are considered.

The maximum number of type-2 fuzzy rules that can be constructed for a type-2 model within class 1 is $6^3 = 216$, i.e., the antecedent statements regarding the state variable and the external disturbance can adopt 6 descriptions within the set {slightly likely,moderately likely,very likely} × {low,high} and the antecedent statement for the control input takes on either of

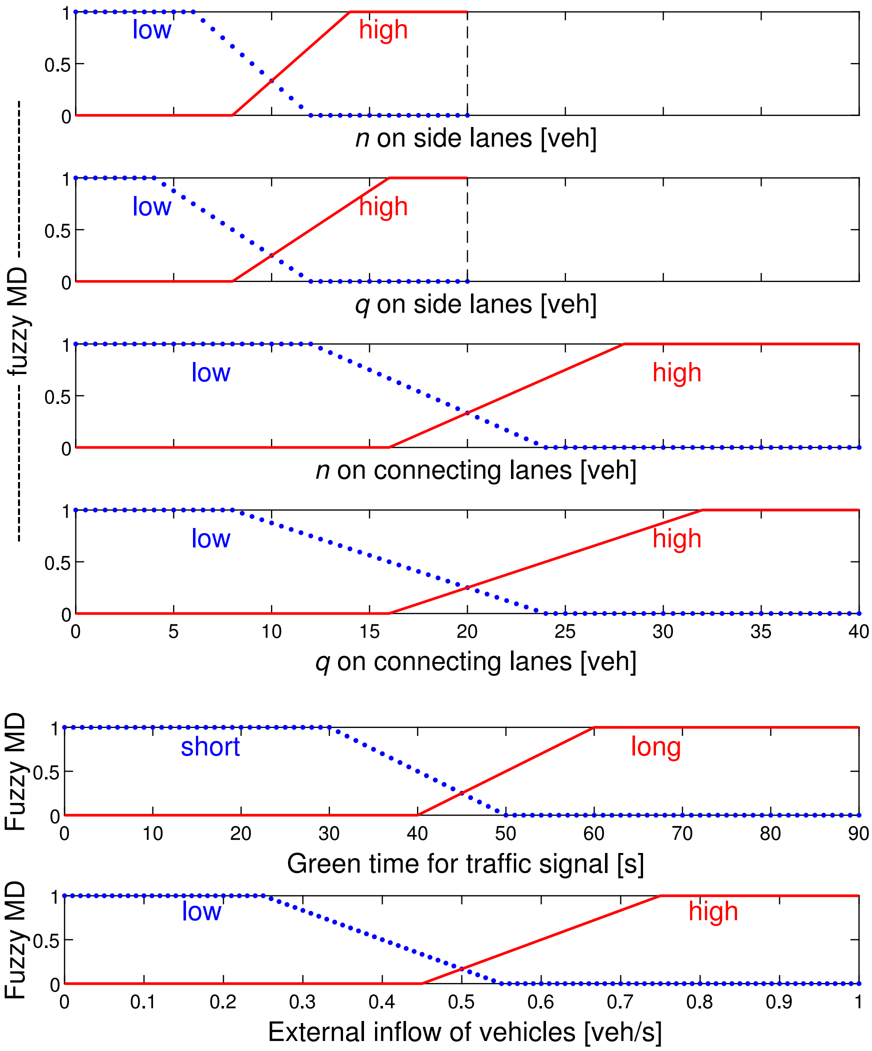

Fig. 8. *Class 0 type-1 model:* Type-1 triangular membership functions for state variables, control inputs, and external disturbances.

the 6 descriptions within the set {slightly likely,moderately likely,very likely} × {short,long}. This implies that the total number of parameters to be identified for the type-2 fuzzy model is $4 \cdot 216$, i.e., 864 parameters, per state variable.

Consider now a specific rule with the following antecedent: "if x is very likely high and u is moderately likely short and v is slightly likely low". Then the three type-2 fuzzy events involved are denoted by $e^{x:\text{VL-high}}$, $e^{u:\text{ML-short}}$, and $e^{v:\text{SL-low}}$, and their primary membership degrees are indicated by $\mu_{1,i}^{x:\text{high}}$, $\mu_{1,j}^{u:\text{short}}$, and $\mu_{1,k}^{v:\text{low}}$ for $i,j,k \in \{1,2,3\}$ corresponding to the three various interpretations given for the fuzzy terms *low*, *high*, *short*, *long* in the antecedent. Likewise, $\mu_2^{\text{VL}(x:\text{high})_i}$, $\mu_2^{\text{ML}(u:\text{short})_j}$, and $\mu_2^{\text{SL}(v:\text{low})_k}$ denote the secondary membership degrees corresponding to each of these primary membership degrees. Normally to find the membership degrees of the combination of these three type-2 fuzzy events in the antecedent, all possible combinations of the primary and secondary membership degrees should be considered. For the given antecedent, $3^3 = 27$ combinations are possible, i.e., each primary membership degree can adopt 3 values, as there are three definitions per fuzzy term *high*, *low*, *short*, *long*. Moreover, the secondary membership degree adopts 1 value depending on which fuzzy term, i.e., *slightly likely*, *moderately likely*, or *very likely* is used to formulate the rule. Hence, each variable in the antecedent adopts 3 possible combinations of the primary and secondary membership degrees, and for all the three variables (state variable, control input, external disturbance) in the antecedent the possible combinations are 3^3. For each combination $e^{x:\text{VL-high}} \wedge e^{u:\text{ML-short}} \wedge e^{v:\text{SL-low}}$, with $\wedge$ representing '**logical and**', the primary and secondary membership degrees of the combined fuzzy-fuzzy event can be determined as follows:

$$\mu_1\left(e^{x:\text{VL-high}} \wedge e^{u:\text{ML-short}} \wedge e^{v:\text{SL-low}}, i, j, k\right) = \min\left\{\mu_{1,i}^{x:\text{high}},\ \mu_{1,j}^{u:\text{short}},\ \mu_{1,k}^{v:\text{low}}\right\} \tag{7}$$

$$\mu_2\left(e^{x:\text{VL-high}} \wedge e^{u:\text{ML-short}} \wedge e^{v:\text{SL-low}}, i, j, k\right) = \tag{8}$$

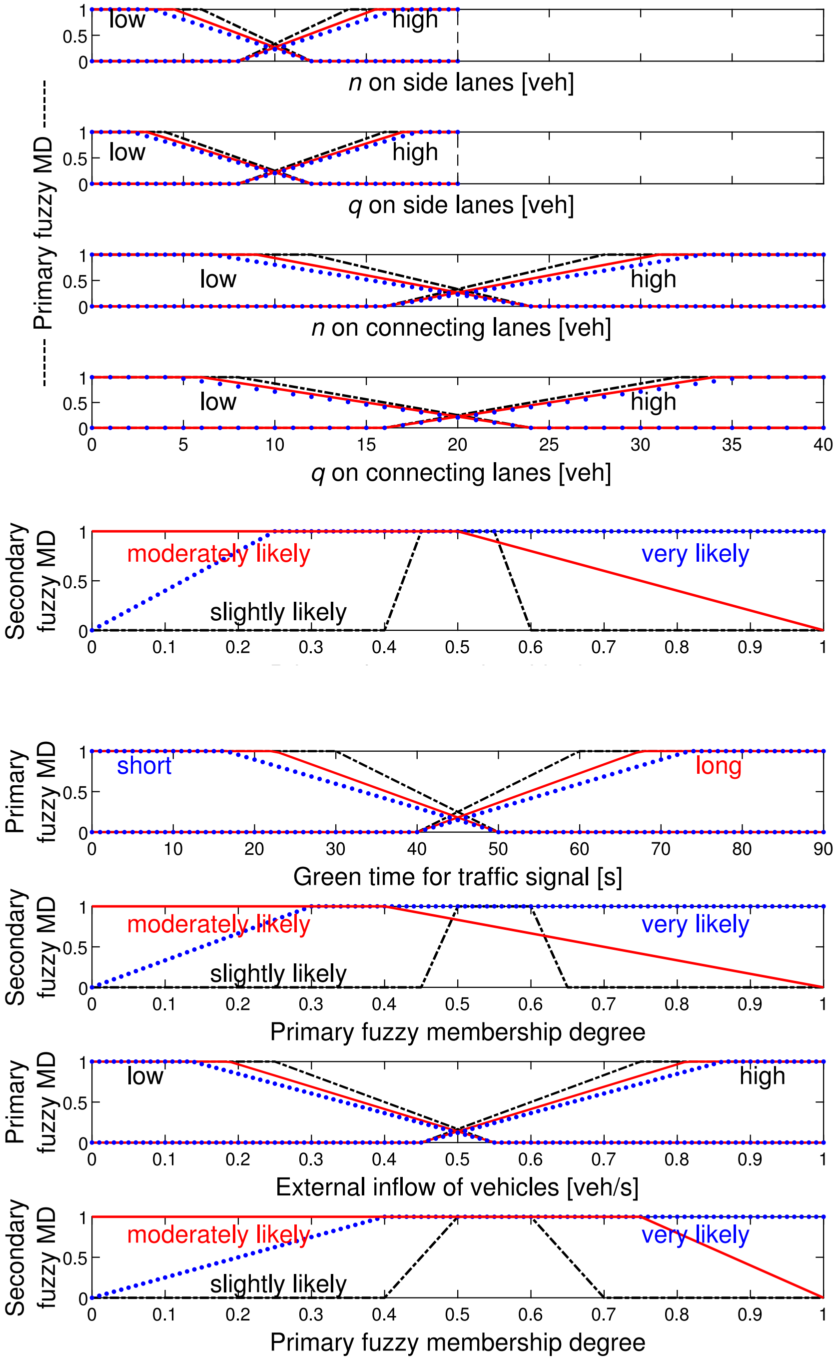

Fig. 9. *Class 1 type-2 model:* Type-2 fuzzy membership functions for state variables, control inputs, and external disturbances.

$$\min\left\{\mu_2^{\mathrm{VL}(x:\mathrm{high})_i},\ \mu_2^{\mathrm{ML}(u:\mathrm{short})_j},\ \mu_2^{\mathrm{SL}(v:\mathrm{low})_k}\right\}$$

If two combinations have the same primary membership degree, the combination with the largest secondary membership degree is kept, while the rest of the equal primary membership degrees and their corresponding secondary membership degrees are then excluded when computing the output of the fuzzy inference engine (the interested reader is referred [51] for more details). The approach of [52] is then used to compute the output of the inference engine of the type-2 fuzzy-fuzzy rule.

In order to reduce the number of possible combinations and hence, the computational burden and computation time of class 1 type-2 models, for both the identification procedure and the online computations, we have considered the following setup: as

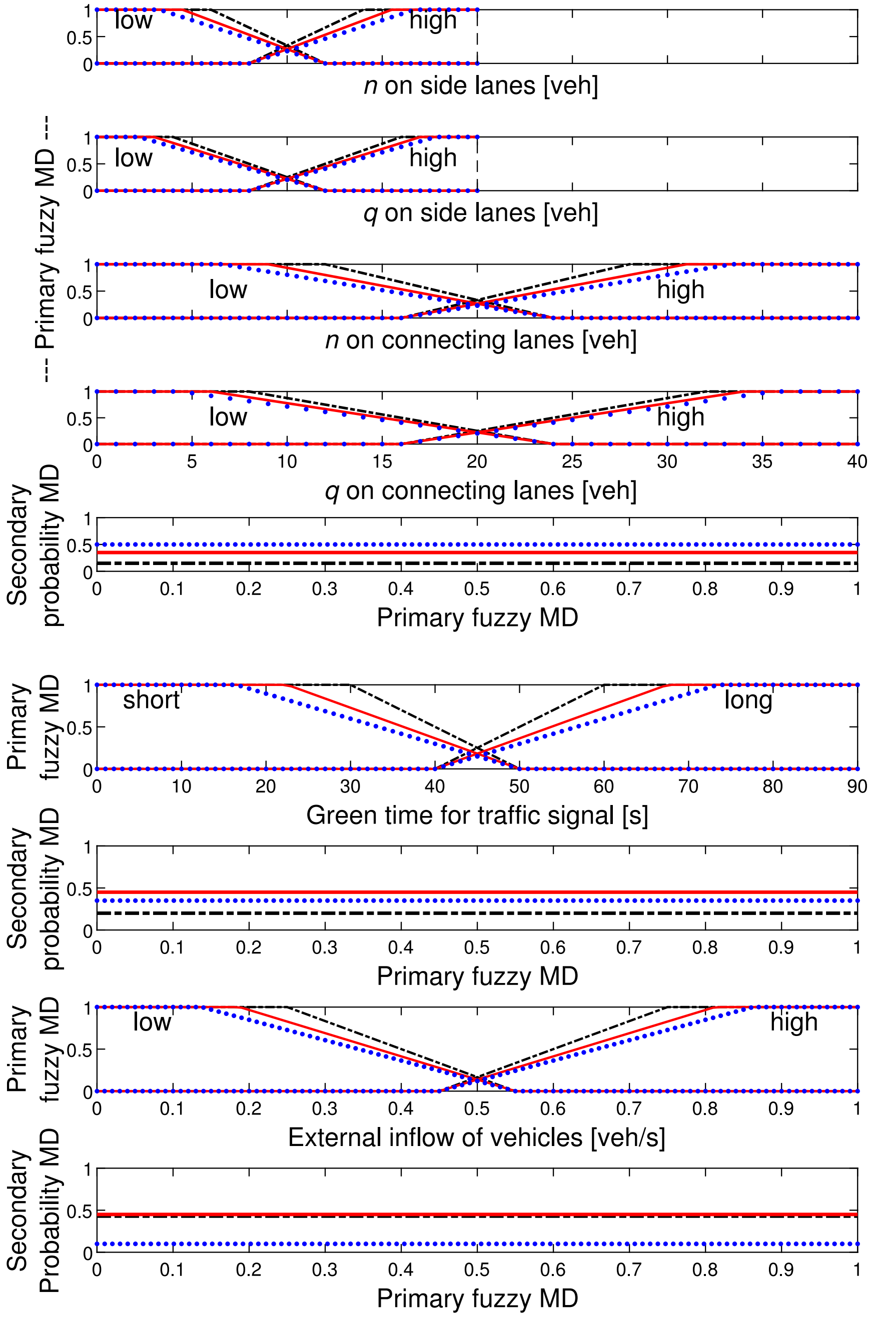

Fig. 10. *Class 2 type-2 model:* Probabilistic-fuzzy membership functions for state variables, control inputs, and external disturbances.

illustrated in Fig. 9, each fuzzy term used for the secondary membership degrees corresponds to only one of the three interpretations for the fuzzy terms used for the primary membership degrees. More specifically, in Fig. 9, the term *very likely* (illustrated by dotted blue curves) corresponds to those interpretations of the terms *low*, *high*, *short*, and *long* that are also represented by dotted blue curves. Similarly, the terms *moderately likely* (solid red curves) and *slightly likely* (dash-dotted black curves) correspond to those interpretations of *low*, *high*, *short*, and *long* that are represented by curves of a similar color and format.

5.2.3. Type-2 probabilistic-fuzzy membership functions

Fig. 10 illustrates the primary type-1 fuzzy membership functions and the secondary probability density functions of the type-2 probabilistic-fuzzy membership functions. The maximally possible number of type-2 probabilistic-fuzzy rules for a type-2 model within class 2 is $6^3 = 216$, i.e., the antecedent statements regarding the state variable, control input, and external disturbance can each adopt 6 options from, respectively, the sets {sPF1,sPF2,sPF3} × {low,high}, {cPF1,cPF2,cPF3} × {short,long}, {dPF1,dPF2,dPF3} × {low,high}, where sPF1, sPF2, sPF3 are the state probability density functions illustrated in the 5th plot of Fig. 10, cPF1, cPF2, cPF3 are the control probability density functions illustrated in the 7th plot of Fig. 10, and dPF1, dPF2, dPF3 are the disturbance probability density functions illustrated in the 9th plot of Fig. 10. As a result, total number of parameters to be identified for the type-2 probabilistic-fuzzy model is 4(216), i.e., 864 parameters, per state variable.

Consider the antecedent of a type-2 probabilistic-fuzzy rule that is described by "if x is $100 \cdot \pi_x\%$ (with $0 \le \pi_x \le 1$) low and u is $100 \cdot \pi_u\%$ (with $0 \le \pi_u \le 1$) long and v is $100 \cdot \pi_v\%$ (with $0 \le \pi_v \le 1$) high". The corresponding three events, denoted by $e^{x:\pi_x\text{-low}}$, $e^{u:\pi_u\text{-long}}$, and $e^{v:\pi_v\text{-high}}$, have to occur at the same time for this specific rule to be fired. The primary fuzzy membership degree of the combined event $e^{x:\pi_x\text{-low}} \wedge e^{u:\pi_u\text{-long}} \wedge e^{v:\pi_v\text{-high}}$ can be determined by aggregating the corresponding primary type-1 fuzzy membership functions using a t-norm (i.e., minimum or multiplication) of the primary fuzzy membership degrees of the three events, i.e., for $i, j, k \in \{1, 2, 3\}$:

$$\mu_1\left(e^{x:\pi_x\text{-low}} \wedge\ e^{u:\pi_u\text{-long}} \wedge e^{v:\pi_v\text{-high}}, i, j, k\right) = \min\left\{\mu_{1,i}^{x:\text{low}},\ \mu_{1,j}^{u:\text{long}},\ \mu_{1,k}^{v:\text{high}}\right\} \tag{9}$$

The probability of simultaneous occurrence of the three events and hence of activation of this type-2 probabilistic-fuzzy rule is

$$\begin{aligned} &p\left(e^{x:\pi_x\text{-low}} \wedge e^{u:\pi_u\text{-long}} \wedge e^{v:\pi_v\text{-high}}, i, j, k\right) = \\ &p\left(e^{x:\pi_x\text{-low}} | (x : \text{low})_i\right) \cdot p\left(e^{u:\pi_u\text{-long}} | (u : \text{long})_j\right) \cdot p\left(e^{v:\pi_v\text{-high}} | (v : \text{high})\right) \\ &= \pi_{x,i} \cdot \pi_{u,j} \cdot \pi_{v,k} \end{aligned} \tag{10}$$

where $p(\cdot)$ denotes the probability density function. For the sake of simplicity, we have assumed the three events to be independent. In case two different combinations have the same primary fuzzy membership degree, the combination with the highest secondary probability is considered.

Here the secondary membership functions are probability density functions, which have been considered to have a fixed value (see Fig. 10). Similarly to the class 1 model, to decrease the computational burden and the computation time, we assume that each probability density function corresponds to one of the interpretations given for the fuzzy terms *low*, *high*, *short*, and *long*. More specifically, the chances that the primary fuzzy membership degrees corresponding to the solid (red), dotted (blue), or dash-dotted (black) curves (see plots 1–4, 6, and 8 in Fig. 10) are realized, are the same for all primary fuzzy membership degrees that correspond to each of these plots, and these chances correspond to the fixed-value functions illustrated by, respectively, the solid (red), dotted (blue), and dash-dotted (black) curves.

5.2.4. Type-2 fuzzy-probabilistic membership functions

The primary membership functions in this case are probability density functions, meaning that the membership degrees corresponding to a certain state variable and all the primary membership functions sum up to 1. For class 2 and class 3 models, we considered two categories (i.e., *low* and *high*; *short* and *long*) for the state variables, control inputs, and external disturbances. For class 4 models we therefore also consider two categories/probability density functions per variable, supposing they form the world set together (see Fig. 11). These two functions can be considered as the information deduced from two different experiments for counting the number of vehicles on various lanes and measuring the green time length of the traffic signals, and the probability that each provided data may be valid in each experiment. In probability theory, the uncertainty does not arise due to the use of fuzzy terms (taken from human language), but the uncertainty is about the likeliness that a random event is realized or a realization is true. Therefore, in the class 4 model, there are not various interpretations per category, and the plots 1–4, 6, and 8 of Fig. 11 include two curves only. For the fuzzy secondary membership functions, we again consider the terms *very likely*, *moderately likely*, and *slightly likely*.

The maximal number of type-2 fuzzy-probabilistic rules one can construct for a type-2 model within class 3 is $6^3 = 216$, i.e., the antecedent statements regarding the state variable, control input, and external disturbance can each adopt 6 options from, respectively, the sets {slightly likely,moderately likely,very likely} × {sPF1,sPF2}, {slightly likely, moderately likely,very likely} × {cPF1,cPF2}, {slightly likely, moderately likely, very likely}×{dPF1,dPF2}, with sPF1 and sPF2 any of the state probability density functions shown in the first four plots of Fig. 11, cPF1 and cPF2 the control probability density functions shown in the 6th plot of Fig. 11, and dPF1 and dPF2 the disturbance probability density functions shown in the 8th plot of Fig. 11. Hence, the total number of parameters to be identified for the type-2 fuzzy-probabilistic model is 4(216), i.e., 864 parameters, per state variable.

Consider the antecedent of a type-2 fuzzy-probabilistic rule that is described by "if x is slightly likely 5 veh and u is very likely 65 s and v is moderately likely 0.35 veh/s". The corresponding three fuzzy-probabilistic events are $e^{x:\text{SL-5}}$, $e^{u:\text{VL-65}}$, and $e^{v:\text{ML-0.35}}$, with primary probability degrees $\pi_i^{x:5}$, $\pi_j^{u:65}$, $\pi_k^{v:0.35}$, where $i, j \in \{1, 2\}$, and with secondary fuzzy membership degrees $\mu_2^{\text{SL}(x:5)_i}$, $\mu_2^{\text{VL}(u:65)_j}$, $\mu_2^{\text{SL}(v:0.35)_k}$. There are 2^3, i.e., 8, possible combinations for a rule. To reduce the number of combinations, one may make a setup similar to that made for models in class 1 and class 2. For instance, *moderately likely* only corresponds to the probability density functions that are illustrated by the dotted blue curves and *very likely* and *slightly likely* only correspond to the solid red curves. This way, the number of combinations per rule reduces to one, as we had for the models in class 1 and class 2. In this particular case study, however, we did not impose these assumptions since the computation time for the class 3 model was still reasonable.

For a combined event $e^{x:\text{SL-5}} \wedge e^{u:\text{VL-65}} \wedge e^{v:\text{ML-0.35}}$ the primary probability degree and the secondary fuzzy membership degree are determined by:

$$p\left(e^{x:\text{SL-5}} \wedge e^{u:\text{VL-65}} \wedge e^{v:\text{ML-0.35}}, i, j, k\right) = \pi_i^{x:5} \cdot \pi_j^{u:65} \cdot \pi_k^{v:0.35}, \tag{11}$$

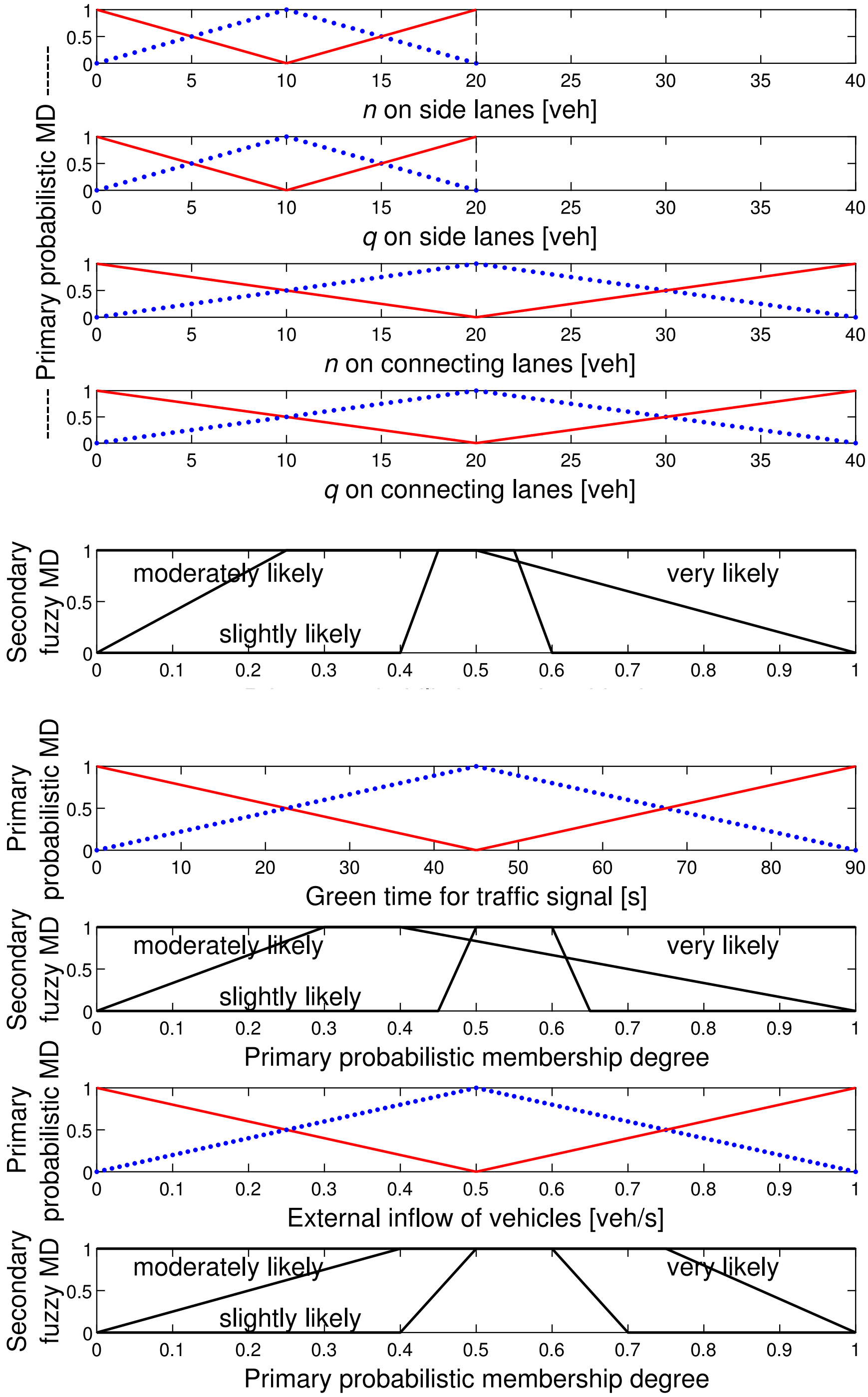

Fig. 11. *Class 3 type-2 model:* Fuzzy-probabilistic membership functions for state variables, control inputs, and external disturbances.

$$\mu_2\left(e^{x:\text{SL-5}} \wedge e^{u:\text{VL-65}} \wedge e^{v:\text{ML-0.35}}, i, j, k\right) = \min\left\{\mu_2^{\text{SL}(x:5)_i},\ \mu_2^{\text{VL}(u:65)_j},\ \mu_2^{\text{SL}(v:0.35)_k}\right\}. \tag{12}$$

5.2.5. Model identification

In this section, we explain the procedure of identifying the type-1 model in class 0 and the type-2 models within classes 1, 2, and 3.

First, an extensive dataset was collected for the urban traffic network shown in Fig. 7 using micro-simulation. NetLogo [53] was used to develop the micro-simulator and Gipps' car following model [54] was implemented into the overall model in order to ensure a more realistic simulation.

The resulting dataset included the realized values for the traffic state variables (i.e., the total number of vehicles on the lanes and the number of vehicles idling in the queues) for all the lanes of the traffic network for a long enough simulation, with varying demands at the source links and varying green lights for the traffic signals.

It was made sure that many variations for these inputs were considered, so that different modes of traffic were simulated and that the corresponding dataset was sufficiently comprehensive. Next, as is standard in the field [55], the dataset was split into 2 parts: one used for training, and one for validation, and these sub-datasets were used to identify and validate the models. More specifically, 80% of the collected data was used for the identification and the other 20% was used for the validation of the models.

We considered the shape and parameters of the type-1 and type-2 sets in the antecedent of the rules of the models to be fixed in our case studies (for the details and motivation of this choice see Section 4.1), whereas the consequent parameters in (6) were identified using the minimization approach explained in Section 4.2.

5.3. Results and discussions

We have assessed the accuracy of the different type-1 and type-2 models, based on their level of precision in estimating the two state variables (i.e., total number of vehicles and the number of vehicles idling in the queues per lane) for the two sub-networks of the urban traffic network that is shown in Fig. 7. The comparison is based on the relative validation errors of these models, after the parameters of each model have been identified. The relative error reflects how accurate the estimation of the state variables is, with respect to the ground truth values, that have been generated by a NetLogo micro-simulation model [56]. Moreover, we have run the Wilcoxon signed rank test for the identified models for various datasets, in order to rank the models based on their relative precision.

The average relative validation errors of the type-1 and type-2 models, after being trained, have been shown in Fig. 12. Note that since no restraining boundary conditions have been defined for the sink nodes of the urban traffic network, all the vehicles on the sink lanes (4L, 5L, 6L, 4R, 5R, 6R) can freely leave all the time. Therefore, there is no queue on these lanes, and hence the errors corresponding to the queue lengths of these lanes have not been shown. Overall, out of the 22 results shown in Fig. 12:

- In 12 (almost 55% of the) cases, the type-1 model results in the most accurate estimations, whereas in 5 (almost 23% of the) cases, it results in the least accurate estimations.
- In 5 (almost 23% of the) cases, the type-2 probabilistic-fuzzy model results in the most accurate estimations, while only in 1 case (i.e., less than 5% of the cases) it results in the least accurate estimation, jointly with the type-2 fuzzy model.
- In 3 (almost 14% of the) cases, and all jointly with the type-2 probabilistic-fuzzy model, the type-2 fuzzy membership function results in the most accurate estimations, whereas in 7 (almost 32% of the) cases, it results in the least accurate estimations.
- In 5 (almost 23% of the) cases, the type-2 fuzzy-probabilistic model results in the most accurate estimations, whereas in 10 (almost 45% of the) cases, it results in the least accurate estimations.

The main observations and deductions from these results are the following. The estimation error for none of these models exceeds 1%. More accurately, for the state variable n, for lanes 1L-7L the maximum error realized is almost 0.75% (which corresponds to the class 1 type-2 membership function) and for lanes 1R-7R it is 0.6% (which corresponds to the type-1 trapezoidal membership function). For the state variable q, for lanes 1L, 2L, 3L, and 7L the maximum estimation error is almost 0.93% (which corresponds to the class 1 type-2 membership function) and for lanes 1R, 2R, 3R, and 7R it is 0.9% (which corresponds to the fuzzy-probabilistic membership function). It is crucial to consider that these very low errors are likely due to the simplicity of the considered traffic network compared to a large-scale real-life traffic network that, e.g., covers a metropolitan. However, at this stage of the research, we are mainly interested in a comparative analysis for the newly introduced concepts and in finding answers for questions Q1 and Q2, rather than being interested in analysis of real-life traffic data.

In reply to question Q1, these results confirm our initial assumption that a model that runs based on data with one level of uncertainty (i.e., a type-1 model) is in general more accurate in more than half of the case studies (to be more precise, for almost 55% of the performed simulations), than the other models that perform on data with two levels of uncertainties. Next come the combined probabilistic-fuzzy and fuzzy-probabilistic models with outperforming in about a quarter (almost 23%) of the case studies. When we look at the worst performances, however, from the type-1 model and type-2 probabilistic-fuzzy and fuzzy-probabilistic models, the probabilistic-fuzzy model shows the worst performance for only less than 5% of the cases, whereas the type-1 model and the fuzzy-probabilistic model under-perform in their estimations in, respectively, 23% and 45% of the cases.

This, in the first place, may sound counter-intuitive, because probabilistic uncertainties, which are already quantified, are expected to be less prone to inaccuracies than fuzzy uncertainties, which are interpreted and quantified based on human perceptions. Accordingly, when the primary uncertainty is probabilistic (i.e., for a fuzzy-probabilistic model), since the fuzziness gets involved only in the secondary computations, the results are expected to be less erroneous than for a probabilistic-fuzzy model, where a primary fuzzy uncertainty is also involved in and affects the computations regarding the secondary uncertainties. The results of the case study, however, show the opposite, which may very likely be because of the flexibility of fuzzy membership functions, compared to probabilistic density functions, in being fine-tuned. In fact, fuzzy membership functions for various fuzzy events are not constrained to add up to 1 for the same input values, whereas the probabilities of various random events must satisfy that condition.

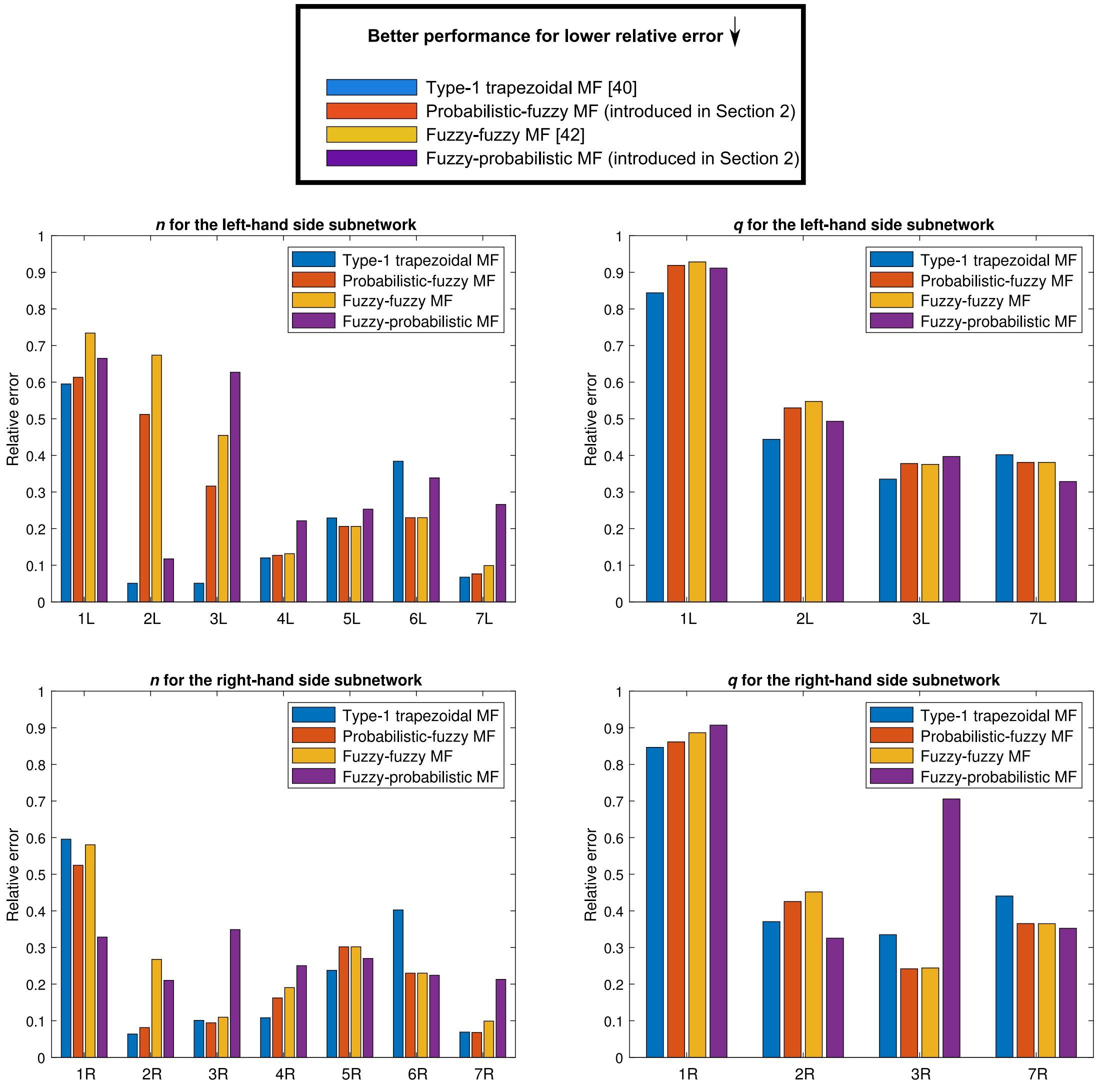

Fig. 12. Comparison of the relative validation errors (as percentages) for estimation of the state variables by the fuzzy models, for sub-network 1 (top row) and sub-network 2 (bottom row), where the ground truth values are taken from a microscopic urban traffic simulation model developed in NetLogo. The blue bars correspond to the type-1 model [40] and the yellow bars correspond to the type-2 fuzzy model [42], whereas the purple and the red bars correspond to type-2 models with a, respectively, fuzzy-probabilistic and probabilistic-fuzzy membership function, which have been introduced in Section 2 of the current paper. **Note that a shorter bar corresponds to a better performance for that particular estimation. In other words, from the 4 bars in each cluster of bars, the one with the least height corresponds to the best performing model and the one with the largest height corresponds to the worst performing model.**

These results, in summary, imply the effectiveness of incorporating fuzzy information and data into the analysis and modeling of traffic dynamics, as well as the significance of the identification procedure.

Finally, the worst-performing model, as was expected, is the class 1 type-2 model, which deals with fuzziness on both levels of uncertainties. Despite the positive aspects that were just mentioned regarding the flexibility of fuzzy membership functions, when such functions exist on two (or more) levels, the complexity in running the identification and later on model computations on the multiple fuzzy levels will rise significantly. Thus, simplifications (e.g., reducing the number of the rules) is inevitable for identifying the model in a reasonable time and for running it online. Therefore, the compensation of the performance for obtaining a computationally affordable model is inevitable for type-2 fuzzy models.

Note that when running the Wilcoxon signed rank test, the most accurate model for various datasets was the type-2 probabilistic-fuzzy model, followed by the type-2 fuzzy-probabilistic model.

In reply to question Q2, note that the computation time for identification and running a type-1 model was very reasonable for the performed case studies, because the total number of parameters that needed to be identified was only 32 and the number of rules that were involved in the online computations was at most 8.

The computational burden for the class 1 type-2 model, however, was generally very high and grew rapidly when adding extra rules. In particular, the iterative Karnik–Mendel algorithms [57] that are commonly used in type-reduction when interval class 1 type-2 fuzzy systems are considered, or the required discretization level (for achieving the desired accuracy) via α-planes [58] when general class 1 type-2 fuzzy systems are considered, are the main sources of this high computational cost. Due to these reasons, in our case studies, we have reduced the number of possible verbal interpretations for the fuzzy uncertainties and hence, reduced the computational burden and computation time accordingly. This, however, compensates the precision and the impact of (re-)identification of the model .

The same conditions hold for the type-2 probabilistic-fuzzy sets, where due to the high computation time experienced during the simulations, we had to reduce the number of linguistic interpretations for the fuzzy uncertainties. Although this compensates the precision of the model, our results showed that the resulting type-2 probabilistic-fuzzy model still outperformed, in terms of precision, the other type-2 models, and in some cases even the type-1 model. Therefore, based on our results, we foresee the highest potential for such type-2 models, although for a more general conclusion further case studies and designed scenarios will be needed.

In general, when two levels of uncertainties exist in the traffic data, type-2 models are preferred, in particular, those with a probabilistic-fuzzy membership function, when accuracy is crucial, and those with a fuzzy-probabilistic membership function, when a trade-off between accuracy, computational efficiency, and ease of implementation is desired. While the accuracy and computational efficiency of type-2 models were discussed earlier, regarding the ease of implementation, note that while statistical data may be collected and stored in advance for later use, the fuzzy uncertainty that is included to the data due to the perception and varying mental states of human users of the road is provided only in real time. An example for a rule that is formulated according to the historical observations for regular, off-peak traffic is "75% of the drivers avoid a detour in traffic area A". In case of heavy congestion in area A, due to road construction or an accident, the varying mental states of the road users will impact this rule, i.e., while in a normal situation there is certainty about 75% of the traffic flow not selecting a detour, this certainty is affected and should best be represented as a fuzzy term, which properly represents the direct feedback of humans. In other words, the rule will become "It is **quite likely** that 75% of the drivers avoid a detour in traffic area A". This natural way of adapting the rules when a secondary uncertainty is introduced to the dataset generates a type-2 model with a fuzzy-probabilistic membership function. Note that building a rule base with probabilistic-fuzzy membership functions, however, is not similarly straightforward (since the existing statistical information by nature corresponds to the primary uncertainties, not the secondary).

In addition to the above discussions, observing the variations of the accuracy for different models across different estimations in Fig. 12 of the results section, yields the following specific insights about the strong and weak points of each model.

First, all models are in general more accurate in estimating the first traffic state, i.e., the total number of vehicles per lane, compared to the second traffic state, i.e., the queue length per lane. This is not unexpected, since the dynamics of traffic queues is in general more complex and prone to variations at the microscopic level. Hence, when a macroscopic representation of the evolution of the queues is used, as is the case in (6), such microscopic dynamics are ignored and this will normally impact the estimations of the queue lengths more than the estimations of the total number of vehicles (also see [45,49]).

Second, in 4 out of the 8 cases, i.e., for lanes 1L, 2L, 3L, 1R, which are all source lanes (i.e., the traffic flow to the traffic network enters the network via these lanes) the type-1 model shows the best performance for estimation of the queue lengths. The best performing type-2 model for estimating the queue lengths on the source lanes is the one with a probabilistic-fuzzy membership function. A similar trend is observed for the estimation of the total number of vehicles on the source lanes, i.e., the type-1 model and the type-2 model with a probabilistic-fuzzy membership function show the highest accuracies.

Third, the most challenging lanes for a traffic model to estimate or predict the states of, include the sink lanes (i.e., lanes through which traffic leaves the traffic network), which in this case include lanes 4L, 5L, 6L, 4R, 5R, 6R in Fig. 7, and the connecting lanes (i.e., lanes that are neither a source nor a sink lane), which are lanes 7L and 7R in Fig. 7. In fact, while the inflow to the source lanes 1L, 2L, 3L, 1R, 2R, 3R is given or measured per simulation time step, for the sink lanes (numbered 4, 5, 6) and the connecting lane (numbered 7) the inflow and outflow are both estimated. Due to this, a larger cumulative error is expected for the states estimated by a model for these lanes. For 5 out of the 8 sink and connecting lanes, the type-1 model shows the highest accuracy in estimation of the total number of vehicles.

However, in case of lane 6, the type-1 model not only under-performs with respect to the other models in estimating the total number of vehicles, but also the estimation error of this model is relatively significant. The next best performing model in estimating the total number of vehicles on the sink and connecting lanes is the type-2 model with a probabilistic-fuzzy membership function, which shows a stable behavior for all the lanes. Thus, the best choice for estimation of the total number of vehicles for complex (i.e., sink and connecting) lanes, based on these results, is the type-2 model with a probabilistic-fuzzy membership function. Although, as discussed above, the type-1 model and the type-2 model with a probabilistic-fuzzy membership function show the highest accuracy for estimating the queue lengths of the source lanes, for neither of the complex lanes (i.e., 7L and 7R) does the type-1 model or the type-2 model with a probabilistic-fuzzy membership function perform the best. In fact, for both of these connecting lanes, the type-1 model under-performs compared to the other models, whereas the type-2 model with a fuzzy-probabilistic membership function shows the highest accuracy in estimating the queue lengths.

Generally speaking, type-1 models are the easiest to design [59,60]. However, they are not always the best choice considering the accuracy and efficiency of the computations, as confirmed by our results and by other comparisons run in the literature (see [61,62]). Our observations based on the results presented in this paper, in summary, imply that when larger cumulative errors are expected (e.g., because the lane is a sink or a connecting lane) modeling the traffic dynamics incorporating two levels of uncertainties is in general preferred over considering one level of uncertainty only. Moreover, when the primary and secondary uncertainties are represented via, respectively, probabilistic and fuzzy data, the type-2 model shows the best overall performance. Moreover, such a model is also preferred due to its ease of implementation in real-time, since the implementation simply requires including the newly received fuzzy information, as the secondary uncertainty, on top of the existing statistical information that exhibits the primary uncertainty about the traffic dynamics (this has been explained earlier in detail via an example above).

Note that in a realistic urban traffic network that is larger than that shown in Fig. 7, there are many more connecting lanes. In such cases, a wise choice for estimation of the queue lengths is, thus, again a type-2 model with a fuzzy-probabilistic membership function.

Finally, our case studies showed a reasonable time for identification and computations for the type-2 fuzzy-probabilistic model, which eliminated the need for shrinking the linguistic interpretation sets. However, as indicated before, by swapping the primary and secondary membership functions, i.e., for a probabilistic-fuzzy counter-part model, the precision of the estimations is still higher than the other type-2 models.

In summary, these results indicate that by using the proposed concepts of probabilistic-fuzzy and fuzzy-probabilistic membership functions (possibly depending on the type of the traffic scenarios and the data available), one can obtain estimations for traffic states with the introduced human-inspired models that are comparable to the estimations of a type-1 fuzzy model when data is prone to only one level of uncertainty. This implies that the type-2 sets and models introduced in this paper can properly handle the additional uncertainty that is introduced to traffic data via humans. In the design of such type-2 models, one may consider (to the level that the nature of the data allows) symmetrical type-2 membership functions. Such a symmetry, for instance, will translate into a trapezoidal primary and a triangular secondary membership function (more details can be deduced from [58]), and is expected to reduce the computational burden of these models.

Moreover, due to the large number of identification parameters for each type-2 model (i.e., 864 parameters per state variable) compared to the type-1 model (i.e., 32 parameters per state variable), and considering the fact that the identification procedure involves a non-convex optimization procedure, which is performed several times using multiple starting points, it is possible that the type-1 model is more accurately identified compared to the type-2 models.

6. Conclusions and future research

This paper has addressed an important challenge of autonomous driving in smart cities: Handling various levels and types of uncertainties in traffic, including both quantified and non-quantified data, in analysis and prediction of the traffic dynamics via mathematical modeling. Via the introduction of novel type-2 sets, called probabilistic-fuzzy and fuzzy-probabilistic sets, and by exploiting these sets through human-inspired rule-based models, the following major impact is expected for smart cities: Improved, in terms of performance and safety, autonomous control of traffic, incorporating both statistical and human factors within the decision making procedure.

In a case study, we used a type-1 model and the different type-2 models to mathematically represent the dynamics of an urban traffic network using extensive data collected from this traffic network, using NetLogo for micro-simulations. The case study was performed to show-case the potentials of these type-2 models. We noticed that for data-sets that already include uncertainties of fuzzy, probabilistic, or both types, the added value of using type-2 models is more prominent. In particular, type-2 probabilistic-fuzzy models provide a balanced trade-off between accuracy and efficiency of the computations. This is likely because of the presence of fuzzy membership functions in the primary level that handles the uncertainties and the higher flexibility of such functions in being tuned compared to probabilistic density functions.

In general, when data collected from the traffic network involves multiple levels of uncertainty, in particular both fuzzy and probabilistic, type-2 models are the most effective tools to capture and represent the dynamics of traffic. This is commonly the case when traffic dynamics is (significantly) impacted by the dynamics of the cognition and mental states of humans. For instance, in regular, ideal traffic conditions, such as during the off-peak hours when traffic flows freely, historical data that provides the pattern of traffic dynamics as a function of the time of day may be used to estimate and predict the upcoming traffic states. Such historical data, stored as probabilistic values or fuzzy terms, can properly be modeled via a type-1 model. However, when drivers and other road users are emotionally impacted (e.g., are angry, frustrated, stressed-out, or anxious due to a blocked road, slow traffic, or bad weather conditions), a second level of uncertainty will impact the traffic dynamics that is due to the dynamics of the mental states of the road users. This uncertainty, due to its nature, is interpreted and mathematically represented via fuzzy variables. For such examples, type-2 models (including type-2 fuzzy, fuzzy-probabilistic, and probabilistic-fuzzy models) outperform type-1 models in representing and predicting the traffic dynamics. Moreover, since road users send out such data about their mental states individually, and the data is thus perceived or received asynchronously and/or delayed, the general formulation of type-2 models provided by (1)–(3), which capture this asynchrony/delay, is particularly suited.

Future research should investigate how the choice of the probability density functions, when aligned with specific interpretations of fuzzy terms, may positively impact the computational burden of type-2 models. Moreover, further investigation about the impacts on the precision and computation time when the type of the primary and secondary membership functions are swapped for modeling the same phenomena for various traffic scenarios is required. Analysis of these aspects is relevant, specially for cases where the type and shape of these functions can be designed, noting that in the context of the current paper, we assume that data has already been collected in a combined way and we represent and model it according to the available dataset.

For future research, comparison with other validated traffic models that can handle uncertain data will be performed. Additionally, we propose to combine the proposed rule-based modeling methods with a supervisory optimization-based control system, in order to coordinate different traffic sub-networks, while optimizing global performance criteria, including total travel time and total emissions of the vehicles. Moreover, further research on collection, filtering, and calibration of heterogeneous data from human users and managers of the roads is a highly relevant and essential step for using the proposed models in real-life smart cities. These may require novel data fusion and/or soft sensing techniques based on machine learning or other relevant (combined) methods.

Various fields and applications that involve humans in the system will also benefit from the novel approaches of this paper. Accordingly, it is recommended to implement the proposed modeling approaches for the analysis and control of search-and-rescue robots (which should process data from dynamic environments and from firefighters and trapped victims), and for steering social robots that should understand and interact with humans. In such cases both statistical and fuzzy-logic-based models are relevant and thus such novel integration of probability theory and fuzzy logic, as proposed in this paper, will be of high relevance.

Finally, extensive analysis (based on traffic micro-simulation or real-life data) is needed to provide insights about traffic scenarios and datasets where type-2 models will exceptionally improve the accuracy of the estimations, thus the safety of traffic when a control system has been designed and is performing upon these estimations. Running statistical tests (e.g., the Wilcoxon signed-rank test) is a logical next step in validating the proposed models for larger scale, real-life traffic networks.

CRediT authorship contribution statement

Anahita Jamshidnejad: Conceptualization, Methodology, Validation, Writing – original draft, Review & editing, Project administration, Funding acquisition. **Bart De Schutter:** Conceptualization, Validation, Review & editing, Supervision, Project administration, Funding acquisition.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Anahita Jamshidnejad reports financial support was provided by Dutch Research Council. Bart De Schutter reports financial support was provided by European Research Council. If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The micro-simulator for the urban traffic network developed in NetLogo is available upon request. Moreover the identified values of the parameters for all the models used in the experiments are available on the personal website of the corresponding author https://sites.google.com/site/jamshidnejadanahita/publications/data.

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References

[1] Jain S, Ahuja NJ, Pulipeti S, Bhadane KV, Nagaiah B, Kumar A, Kumar A. Blockchain and autonomous vehicles: Recent advances and future directions. IEEE Access 2021;9:130264–328.

[2] Alawadhi M, Almazrouie J, Kamil M, Khalil KA. A systematic literature review of the factors influencing the adoption of autonomous driving. Int J Syst Assur Eng Manag 2020;11:1065–82.

[3] Kuderer M, Burgard SGW. Learning driving styles for autonomous vehicles from demonstration. In: International conference on robotics and automation. ICRA, IEEE; 2015, p. 2641–6.

[4] Lin JC, Liew S, Sabaliauskaite G, Zhou F. A review on safety failures, security attacks, and available countermeasures for autonomous vehicles. Ad Hoc Netw 2019;90:101823.

[5] Taeihagh A, Lim HSM. Governing autonomous vehicles: Emerging responses for safety, liability, privacy, cybersecurity, and industry risk. Transp Rev 2019;39:103–28.

[6] Shafiq M, Tian Z, Bashir AK, Jolfaei A, Yu X. Data mining and machine learning methods for sustainable smart cities traffic classification: A survey. Sustainable Cities Soc 2020;60:102177.

[7] Tian K, Markkula G, Wei C, Lee YM, Madigan R, Merat N, Romano R. Explaining unsafe pedestrian road crossing behaviours using a psychophysics-based gap acceptance model. Saf Sci 2022;154:105837.

[8] Fridman L, Brown DE, Glazer M, Angell W, Dodd S, Jenik B, Terwilliger J, Patsekin A, Kindelsberger J, Ding L, Seaman S, Mehler A, Sipperley A, Pettinato A, Seppelt BD, Angell L, Mehler B, Reimer B. Mit advanced vehicle technology study: Large-scale naturalistic driving study of driver behavior and interaction with automation. IEEE Access 2019;7:102021–38.

[9] McDonald AD, Alambeigi H, Engstrom J, Markkula G, Vogelpohl T, Dunne J, Yuma N. Toward computational simulations of behavior during automated driving takeovers: A review of the empirical and modeling literatures. Hum Factors J Hum Factors Ergon Soc 2019;61(4):642–88.

[10] Burger M, van den Berg M, Hegyi A, Schutter BD, Hellendoorn J. Considerations for model-based traffic control. Transp Res C 2013;35:1–19.

[11] Jamshidnejad A, Papamichail I, Papageorgiou M, De Schutter B. A mesoscopic integrated urban traffic flow-emission model. Transp Res C 2017;75:45–83.

[12] Jamshidnejad A, Papamichail I, Papageorgiou M, Schutter BD. Sustainable model-predictive control in urban traffic networks: Efficient solution based on general smoothening methods. IEEE Trans Control Syst Technol 2018;26(3):813–27.

[13] Koopman P, Wagner M. Autonomous vehicle safety: An interdisciplinary challenge. IEEE Intell Transp Syst Mag 2017;9(1):90–6.

[14] Pek C, Althoff M. Ensuring motion safety of autonomous vehicles through online fail-safe verification. In: Robotics: science and systems-pioneers workshop. 2019.

[15] Pek C, Zahn P, Althoff M. Verifying the safety of lane change maneuvers of self-driving vehicles based on formalized traffic rules. In: IEEE intelligent vehicles symposium. 2017, p. 1477–83.
[16] O'Kelly M, Abbas H, Mangharam R. Computer-aided design for safe autonomous vehicles. In: 2017 resilience week. RWS, IEEE; 2017, p. 90–6.
[17] Patrício MLM, Jamshidnejad A. Dynamic mathematical models of theory of mind for socially assistive robots. IEEE Access 2023;11:103956–75.
[18] Chen Z, Huang X. End-to-end learning for lane keeping of self-driving cars. In: IEEE intelligent vehicles symposium. IV, IEEE; 2017, p. 1856–60.
[19] Xu H, Gao Y, Yu F, Darrell T. End-to-end learning of driving models from large-scale video datasets. In: Proceedings of the IEEE conference on computer vision and pattern recognition. IEEE; 2017, p. 2174–82.
[20] Xiao Y, Codevilla F, Gurram A, Urfalioglu O, López AM. Multimodal end-to-end autonomous driving. IEEE Trans Intell Transp Syst 2022;23(1):537–47.
[21] Tampuu A, Matiisen T, Semikin M, Fishman D, Muhammad N. A survey of end-to-end driving: Architectures and training methods. IEEE Trans Neural Netw Learn Syst 2022;33(4):1364–84.
[22] Amini A, Rosman G, Karaman S, Rus D. Variational end-to-end navigation and localization. In: 2019 international conference on robotics and automation. ICRA, IEEE; 2019, p. 8958–64.
[23] Michelmore R, Wicker M, Laurenti L, Cardelli L, Gal Y, Kwiatkowska M. Uncertainty quantification with statistical guarantees in end-to-end autonomous driving control. In: 2020 IEEE international conference on robotics and automation. ICRA, IEEE; 2020, p. 7344–50.
[24] Feng D, Harakeh A, Waslander SL, Dietmayer K. A review and comparative study on probabilistic object detection in autonomous driving. IEEE Trans Intell Transp Syst 2022;23(8):9961–80.
[25] Franchi G, Bursuc A, Aldea E, Dubuisson S, Bloch I. Encoding the latent posterior of Bayesian neural networks for uncertainty quantification. IEEE Trans Pattern Anal Mach Intell 2023;1–13.
[26] Hubmann C, Becker M, Althoff D, Lenz D, Stiller C. Decision making for autonomous driving considering interaction and uncertain prediction of surrounding vehicles. In: 2017 IEEE intelligent vehicles symposium. IV, IEEE; 2017, p. 1671–8.
[27] Alghodhaifi H, Lakshmanan S. Autonomous vehicle evaluation: A comprehensive survey on modeling and simulation approaches. IEEE Access 2021;9:151531–66.
[28] Zheng P, McDonald M. Application of fuzzy systems in the car following behaviour analysis. In: Fuzzy systems and knowledge discovery. Lecture notes in computer science, Springer; 2005, p. 782–91.
[29] Gao Q, Hu S, Dong C. The modeling and simulation of the car-following behavior based on fuzzy inference. In: Workshop modeling, simulation and optimization. IEEE; 2008, p. 322–5.
[30] Shojaeefard MH, Mollajafari M, Ebrahimi-Nejad S, Tayebi S. Weather-aware fuzzy adaptive cruise control: Dynamic reference signal design. Comput Electr Eng 2023;110:108903.
[31] Rout RR, Vemireddy S, Raul SK, Somayajulu DVLN. Fuzzy logic-based emergency vehicle routing: An IoT system development for smart city applications. Comput Electr Eng 2020;88:106839.
[32] Sugeno M, Nishida M. Fuzzy control of a model car. Fuzzy Sets and Systems 1985;16:103–13.
[33] Kong S-G, Kosko B. Comparison of fuzzy and neural truck backer-upper control systems. In: International joint conference on neural networks and fuzzy systems. 1990, p. 349–58.
[34] Jou C-C, Wang N-C. Training a fuzzy controller to back up an autonomous vehicle. In: IEEE international conference on robotics and automation. 1993, p. 923–8.
[35] Miyata H, Ohki M, Ohkita M. Self-tuning of fuzzy reasoning by the steepest descent method and its application to a parallel parking. IEICE Trans Inf Syst 1996;E79D(5):561–9.
[36] Baturone I, Moreno-Velo FJ, Sanchez-Solano S, Ollero A. Automatic design of fuzzy controllers for car-like autonomous robots. IEEE Trans Fuzzy Syst 2004;12(4):447–65.
[37] Oketch TG. New modeling approach for mixed-traffic streams with nonmotorized vehicles. Transp Res Rec: J Transp Res Board 2000;1705(1):61–9.
[38] Hidas P. Modelling lane changing and merging in microscopic traffic simulation. Transp Res C 2002;10(5):351–71.
[39] Wei H, Lee J, Li Q, Li CJ. Observation-based lane-vehicle assignment hierarchy: Microscopic simulation on urban street network. Transp Res Rec: J Transp Res Board 2000;1710(1):96–103.
[40] Zadeh LA. Fuzzy sets. Inf Control 1965;8(3):338–53.
[41] Zadeh LA. Probability measures of fuzzy events. J Math Anal Appl 1968;23(2):421–7.
[42] Zadeh LA. The concept of a linguistic variable and its application to approximate reasoning. Inform Sci 1975;8(3):199–249.
[43] Eisen M. Introduction to mathematical probability theory. USA: Prentice-Hall; 1969.
[44] Portilla C, Espinosa J, De Schutter B. A multi-class urban traffic model considering heterogeneous vehicle composition: An extension of the S model. Transp Res C 2020;115.
[45] Lin S, De Schutter B, Hegyi A, Xi Y, Hellendoorn J. On a spatiotemporally discrete urban traffic model. IET Intell Transp Syst 2014;8:219–31.
[46] Papageorgiou M, Kotsialos A. Freeway ramp metering: An overview. IEEE Trans Intell Transp Syst 2002;3(4):271–81.
[47] Botta A, Lazzerini B, Marcelloni F, Stefanescu DC. Context adaptation of fuzzy systems through a multi-objective evolutionary approach based on a novel interpretability index. Soft Comput 2009;13(5):437–49.
[48] Ye C, Yung NHC, Wang D. A fuzzy controller with supervised learning assisted reinforcement learning algorithm for obstacle avoidance. IEEE Trans Syst Man Cybern 2003;33(1):17–27.
[49] Jamshidnejad A, Lin S, Xi Y, De Schutter B. Corrections to Integrated urban traffic control for the reduction of travel delays and emissions. IEEE Trans Intell Transp Syst 2019;20:1978–83.
[50] BaruaL A, Mudunuri S, Kosheleva O. Why trapezoidal and triangular membership functions work so well: Towards a theoretical explanation. J Uncertain Syst 2014;3(8):164–8.
[51] Zimmermann HJ. Fuzzy set theory – and its applications. USA: Kluwer Academic Publishers Norwell; 1996.
[52] Liang Q, Mendel JM. An introduction to type-2 TSK fuzzy logic systems. In: Proceedings of the IEEE international conference on fuzzy systems. South Korea; 1999, p. 1534–8.
[53] Wilensky U. Netlogo. Evanston, IL: Center for Connected Learning and Computer-Based Modeling, Northwestern University; 1999, https://ccl.northwestern.edu/netlogo/. [Accessed 28 March 2024].
[54] Gipps PG. A behavioural car-following model for computer simulation. Transp Res B 1981;15(2):105–11.
[55] Ljung L. System identification: theory for the user. 2nd ed.. Prentice-Hall; 1999.
[56] Jamshidnejad A, Mahjoob M. Traffic simulation of an urban network system using agent-based modeling. In: 2011 IEEE colloquium on humanities, science and engineering. Malaysia; 2011, p. 300–4.
[57] Wu D. Approaches for reducing the computational cost of interval type-2 fuzzy logic systems: Overview and comparisons. IEEE Trans Fuzzy Syst 2013;21(1):80–99.
[58] Ontiveros E, Melin P, Castillo O. High order α-planes integration: A new approach to computational cost reduction of general type-2 fuzzy systems. Eng Appl Artif Intell 2018;74:186–97.
[59] Sobrinho ASF, Junior FG. Type-1 fuzzy logic algorithm for low cost embedded systems. Comput Electr Eng 2020;88:106861.
[60] Prauzek M, Krömer P, Rodway J, Musilek P. Differential evolution of fuzzy controller for environmentally-powered wireless sensors. Appl Soft Comput 2016;48:193–206.
[61] Afrakhte H, Bayat P. A self-evolving type-2 fuzzy energy management strategy for multi-microgrid systems. Comput Electr Eng 2020;85:106702.
[62] Castillo O, Amador-Angulo L, Castro JR, Garcia-Valdez M. A comparative study of type-1 fuzzy logic systems, interval type-2 fuzzy logic systems and generalized type-2 fuzzy logic systems in control problems. Inform Sci 2016;354:257–74.