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Analysis of methods for determining ship speed during a sea trial

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Abstract

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Analysis of methods for determining ship speed during a sea trial

by Floris Buwalda

The aim of this study is to evaluate the performance of two methods for determining the speed of a ship relative to the water during a so called speed trial.

The first method is called the Mean of means method and has been traditionally. The second is called the Iterative method and has been recently developed and is already used for speed trial analysis.

The methods have been evaluated by simulation. A relationship between ship power and velocity was chosen based on a theoretical analysis, and this relationship is used to generate simulated speed trial data. Using the knowledge of the 'true' relationship the performance of the methods could then be evaluated after implementation.

The findings are that the Iterative method needs a minimum of 8 measurements obtained by sailing back and forth 4 times in order to find the true relationship with acceptable accuracy. The Mean of means method needs a minimum of 12 measurements, or 6 double runs. The conclusion is that the Iterative method is more time efficient for speed trial analysis. In a test case assuming realistic measurement noise, 99.3% of found speeds had an error smaller than the simulated error margin of 0.1 knots, out of a sample size of 1000.

When comparing the Iterative to the Mean of means method under the same conditions, the Mean of means method made smaller errors for all observed cases.

From the results of this study has been concluded that the Iterative method is usable method for determining ship speed during a sea trial that has both advantages and disadvantages over the traditional Mean of means method.

Advantages are that the Iterative method requires less measurements than the Mean of means method, and it remains accurate even if measurements have a large or varying spacing in time.

A disadvantage is that it is more sensitive to measurement noise than the Mean of means method, making it prone to making larger errors.

A small adjustment to the current function the method assumes and uses for calculating current effects has been proposed. The adjustment was to normalize the component of the current function that scales linearly with time. This has been found to have a positive impact on the performance of the method.

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List of Abbreviations

ISO International Organization for Standardization
ITTC International Towing Tank Conference

Physical Constants

knot	$kn = 0.51444 \text{ m s}^{-1}$
gravitational acceleration	$g = 9.81 \text{ m/s}^2$
period of tidal current	$T_C = 12.42 \text{ h}$

List of Symbols

P	power	W (J s^{-1})
V	velocity	m s^{-1}
ρ	density	kg/m^3
S	wet surface area	m^2
τ	shear stress	kg/ms^2
μ	dynamic viscosity	$\text{kg m}^{-1} \text{s}$
ν	kinematic viscosity	m^2/s
p	pressure	$\text{kg/s}^2\text{m}$

Introduction

Ships have been designed and built for a long time. More than often, people do not build their own ships, but have a shipyard build theirs for them.

When people use ships for commercial purposes, such as freight, the performance of the ship is important. Because of this, often the shipbuilder and the orderer agree on a certain speed the ship must be able to reach at a certain engine power. This is called the 'contract speed'.

After a ship is built, a sea trial is conducted to see if the new ship meets the contract criteria. The problem is that it is hard to measure the ship's speed. Its ground speed can be accurately measured using GPS, but the ship's speed relative to the water may be different because of current effects.

Traditionally a way to measure the ship's speed relative to the water was by using the 'Mean of means' method. The basic idea is to sail back and forth a number of times, and eliminate the current effects by repeatedly averaging the measurements.

Recently a new method has been developed called the Iterative method. It uses knowledge of the shape and period of the tidal current to eliminate current effects with less measurements, saving time and money. This method is already being used for sea trial analysis, but some issues are known to emerge, such as the method failing to provide an outcome, or one that is not physically possible. Both the Mean of means method and the new Iterative method will be explained more thoroughly in chapter 2.

The aim of this research is to examine the performance of the iterative method and investigate the probable causes of the issues appearing. The performance criteria of the mean of means method will also briefly be discussed in order to compare the two methods. The results are presented in chapter 4 and discussed in greater detail in chapter 5.

The Iterative method was tested by simulation. With knowledge of theoretical and empirically tested ship behavior, an artificial current and ship were created, and measurement data generated accordingly. In order to make this simulation of a ship, a literature study has been done on theoretical and historical work on ship modeling. The result of this can be found in chapter 1.

In order for the Iterative method to be tested it needed to be implemented. The implementation involved a non-linear least squares parameter estimation problem which required additional literature study in order to understand the methods required to solve it. The result of this study can be found in chapter 3. The resulting implementation of the method in MATLAB can be found in the appendix.

Chapter 1

Theoretical ship behaviour

1.1 Introduction

As described in the introduction, the aim of this research is to evaluate a method for determining the relative speed of a ship during a so called speed trial. The method is called the Iterative method and is evaluated by applying it to a simulated case. The method will be described in detail in chapter 2.

In order to simulate ship behavior, first a theoretical background is needed. The aim of this chapter is to provide insight into the workings of ship resistance and validation for the chosen ship model used in simulation.

1.2 Forces

The most basic model of a ship propagating through a body of water is to use Newton's first law, which states that if a ship propagates at a constant speed the net force on it must be zero.

After making approximations of the forces that work on the ship opposing its movement, the force the engine must produce can be derived, and from that its power expenditure can be found.

According to van Manen and van Oossanen, 1988 , the total 'calm water' resistance is made up of three main components:

- The frictional resistance due to the motion of the hull through a viscous fluid and the above-water part of the ship moving through the air.
- The wave-making resistance, caused by the transfer of the ship's kinetic energy into the waves it makes while moving through the water.
- The eddy resistance, which is caused by energy carried away by eddies shed from the hull or appendages. An eddy is the swirling of a fluid and the reverse current created when the fluid flows past an obstacle, in this case the hull or a ship appendage.

1.2.1 Frictional resistance

One of the most elementary forces acting on a body in a flowing medium is Bernoulli's force. It follows from Bernoulli's principle of conservation of energy in differential form:

$$\frac{1}{2}\Delta V^2 + g \Delta z + \frac{\Delta p}{\rho} = 0 \quad (1.1)$$

Where ΔV is the velocity difference of the fluid between two points in space, Δz the height difference, Δp the pressure difference, ρ is the fluid density, and g is the gravitational acceleration. Bernoulli's principle holds as long as the flow is incompressible and friction is negligible.

The net force a ship experiences equals the integral of the pressure along the hull area, or

$$R_{drag} = \int_{A_{hull}} \Delta p \cdot n \, dA \quad (1.2)$$

Where $\Delta p = p - p_0$ the pressure field on the hull resulting from the ship's velocity, A_{hull} the hull area, n the normal vector of the hull surface and p_0 the hydrostatic water pressure which depends on depth.

Assuming that at the interface between the hull and the water the water locally has the same velocity as the ship, and the pressure is constant along the hull area, it follows that the resistance force equals:

$$R_{drag} = S \frac{1}{2} \rho V^2 \quad (1.3)$$

With S the wet frontal area of the ship, ρ the water density and V the ship's velocity relative to the flow.

This force also acts on the part of the ship that is above water, with the density of air instead of water. Because the density of air is a thousand times less than that of water, the air friction component is assumed to be negligible when the air speed is comparable to the ship speed.

The drag force calculated using Bernoulli's principle is valid in this simplified model. In reality, the pressure on the hull is not constant along the hull area, friction is not negligible and at the interface between the hull and the water does not have exactly the same velocity as the ship. To account for these deviations, a so called drag coefficient was introduced, which will be explained in greater detail in section 1.5.

1.2.2 Skin friction

The skin friction, besides Bernoulli's force, is another frictional force. It is the result of the shear-stress between the hull of the ship and the water. There is currently no theoretical expression for this force, only empirical relations exist which will be analyzed in further sections. It is however known that it is a function of ρ , V , S , and μ , the kinematic viscosity.

It is also known to be dependent on whether the flow is laminar or turbulent.

Laminar flow is characterized by the fact that fluid flows in parallel layers with no currents perpendicular to the direction of flow.

Turbulent flow is the opposite, with rapid local changes in both pressure and flow velocity in space and time.

Whether flow is laminar or turbulent depends mostly on Reynold's number $Re = \frac{\rho V L}{\mu}$. Where ρ the fluid density, v the fluid velocity, L the characteristic linear dimension and μ the dynamic viscosity of the fluid.

Reynold's number is defined as the ratio of inertial forces and viscous forces in the fluid. Viscous forces are dominant in laminar flow and inertial forces in turbulent flow. Hence laminar flow occurs at low Reynold's numbers, and turbulent flow at higher ones.

For example, laminar flow in a pipe occurs for $Re < 2300$ and turbulent flow for $Re > 4000$. In between the flow is called transitional. These values of the Reynold's number are called critical Reynold's numbers and they are different for every geometry.

1.2.3 Wave-making resistance

A ship propagating through a body of water leaves behind a wavefront. These waves have a mass and a velocity and thus a corresponding amount of energy. This energy must be supplied by the ship, and thus a ship will experience a force called the wave-making resistance.

An important property of the wave-making resistance is that it is both a function of the length of the ship and its velocity. If the ship is moving at a certain speed called the hull speed, the waves created by the stern and the waves created by the bow will interfere constructively, increasing the size of the waves created and thus causing a sharp increase in the wave resistance.

This behavior is characterized by the Froude number of the ship. The Froude number is defined as $Fr = \frac{V}{\sqrt{gL}}$, with V the ship's velocity relative to the flow in m/s , g the gravitational acceleration of $9.81m/s^2$ and L the length of the ship's waterline in m . The Froude number characterizes the ratio of flow inertia to the external force field working on the flow, usually gravity. An important property is that objects with equal Froude numbers will generate equal wave patterns.

The wave-making resistance will start to significantly rise at a Froude number of about 0.35, reaching its maximum at 0.5, after which it starts to decrease again (van Manen and van Oossanen, 1988, p. 24). This sharp increase in resistance is what coined the term hull speed, as it limits the speed of a ship that is not able to overcome it.

In Misra, 2016 it is stated that for some of the larger ship types studied in this research, the wave-making resistance will be only 5 – 10% of the total resistance. For smaller ones this can be up to 70%.

1.2.4 Eddy resistance

An eddy is the swirling of a fluid and the reverse current created when the fluid flows past an obstacle, in this case the hull or a ship appendage. These both contain kinetic

energy which must be supplied by the object causing them, and thus resulting in a resistance force.

As with the skin friction resistance, there is currently no theoretical expression for the this force. It is however known that it is influenced by the hull shape.

Eddies are known to only occur in turbulent flow, making the Eddy resistance a function of Reynolds number.

Eddies are also known to be a function of the Euler number, which is used to characterize energy loss in flow, which will also be explained in section 1.4.

1.3 The Navier-Stokes equations

Flow type and properties play a large role in determining the frictional and eddy resistance of a ship. Flow behavior is described by the Navier-Stokes equations (Navier, 1822, Stokes, 1845). They can be derived from the conservation principles of mass, energy and momentum for an arbitrary control volume inside the flow.

1.3.1 Conservation principles

The conservation principles state that in a control volume, the change of any of these three quantities must be equal to the total in- or outflow through the boundaries of the volume plus the production within the volume.

The in- or outflow through the boundaries equal the integral of the flux of the quantity over the boundary surface. The flux is a vector field defined as the amount of a given quantity per unit time per unit area. This is equal to the quantity density times the flow velocity vector field or $J = \rho u$.

It follows then that the conservation law for a quantity ϕ in a control volume states:

$$\frac{\partial \phi}{\partial t} = \int_{\partial \Omega} \rho_{\phi} u \cdot dS + \int_{\Omega} Q_{\phi} dV \quad (1.4)$$

Where Ω is the control volume, $\partial \Omega$ the boundary of the volume, ϕ is the amount of the quantity contained in the control volume, ρ_{ϕ} the density of ϕ as a function of place, u the flow velocity field, and Q_{ϕ} the production of ϕ per unit volume.

Using Gauss' divergence theorem (Stolze, 1978), the integral of the flow over the boundary surface can be expressed as the volume integral of the divergence, leading to:

$$\frac{\partial \phi}{\partial t} = \int_{\Omega} \nabla \cdot (\rho_{\phi} u) + Q_{\phi} dV \quad (1.5)$$

The quantity ϕ is equal to the volume integral of its density or $\phi = \int_{\Omega} \rho_{\phi} dV$. Because ρ_{ϕ} is assumed to be continuously differentiable within the control volume, integration and differentiation may be interchanged in formula 1.5 leading to:

$$\int_{\Omega} \frac{\partial \rho_{\phi}}{\partial t} - \nabla \cdot (\rho_{\phi} u) - Q_{\phi} dV = 0 \quad (1.6)$$

This holds for any control volume, which means that the integral itself must be zero, and thus:

$$\frac{\partial \rho_\phi}{\partial t} - \nabla \cdot (\rho_\phi u) - Q_\phi = 0 \quad (1.7)$$

For the case of momentum, momentum density is defined as the product of mass density and the flow velocity field, or $\rho_p = \rho_m u$. Inserting this into equation 1.7 provides:

$$\frac{\partial}{\partial t} (\rho_m u) - \nabla \cdot (\rho_m u u) - Q_p = 0 \quad (1.8)$$

Expanding the derivative and the divergence terms provides:

$$\frac{\partial \rho_m}{\partial t} u + \frac{\partial u}{\partial t} \rho_m - u u \cdot \nabla \rho_m - \rho_m u \cdot \nabla u - \rho_m u \nabla \cdot u - Q_p = 0 \quad (1.9)$$

Since $u \cdot \nabla \rho_m + \rho_m \nabla \cdot u = \nabla \cdot (\rho_m u)$, equation 1.9 can be rewritten as:

$$u \left(\frac{\partial \rho_m}{\partial t} - \nabla \cdot (\rho_m u) \right) + \rho_m \left(\frac{\partial u}{\partial t} - u \cdot \nabla u \right) - Q_p = 0 \quad (1.10)$$

If equation 1.7 is examined for mass, which means $\rho_\phi = \rho_m$, and the fact that the source term Q_m is always zero since no sources or sinks of mass exist, it states that

$$u \left(\frac{\partial \rho_m}{\partial t} - \nabla \cdot (\rho_m u) \right) = 0 \quad (1.11)$$

And thus the principle of conservation of momentum states that

$$\rho_m \left(\frac{\partial u}{\partial t} - u \cdot \nabla u \right) - Q_p = 0 \quad (1.12)$$

From the Cauchy momentum equation (Acheson, 1990) follows that the source term is equal to:

$$Q_p = \nabla \cdot \tau - \nabla p + f \quad (1.13)$$

With τ the shear stress, p the pressure and f the body force density acting on the fluid. The force density can be expressed as acceleration times mass density, or $f = \rho_m a$. Inserting this into equation 1.12 gives:

$$\rho_m \left(\frac{\partial u}{\partial t} - u \cdot \nabla u \right) - \nabla \cdot \tau = -\nabla p + \rho_m a \quad (1.14)$$

1.3.2 The Navier-Stokes equations for Newtonian fluids in incompressible flow

Often the flow that is being described by the Navier-Stokes equations is that of water. Water has two properties that help with solving the Navier-stokes equations.

The first is that the flow of water can be assumed to be incompressible, which means that the density of the fluid is constant in an infinitesimal volume that moves with the flow velocity. This also implies that the divergence of the flow velocity field is zero, or $\nabla \cdot u = 0$.

Secondly water can be assumed to be a Newtonian fluid. A Newtonian fluid has a linear relation between the shear stress and the gradient of the local fluid velocity field. Or

$$\tau = \mu \nabla u \quad (1.15)$$

Where τ is the shear stress in the fluid, μ a constant of proportionality called the dynamic viscosity and u the flow velocity vector field.

Inserting these two properties into equation 1.14, noting that the acceleration a is equal to the gravitational acceleration g and dividing by the mass density produces the Navier-Stokes equations for Newtonian fluids in incompressible flow:

$$\frac{\partial u}{\partial t} + u \cdot \nabla u - \nu \nabla^2 u + \nabla \left(\frac{p}{\rho_m} \right) = g \quad (1.16)$$

Where ρ_m represents the uniform mass density of the fluid, u is defined as the three dimensional flow velocity vector field, ν is the kinematic viscosity defined as $\frac{\mu}{\rho_m}$ and p is the pressure in N/m . The gravitational acceleration g has a magnitude of $9.81m/s^2$.

The Navier-Stokes equations describe flow, and thus solving them can lead to an expression for the force a ship experiences as the result of that flow. However, the Navier-Stokes equations are too complex to solve analytically. Numerically solving the Navier-Stokes equations is a field of research that has undergone much development over the years, but is so computationally intensive and complex that it has not been included as a part of this study.

1.4 Dimensional analysis

One way to make a prediction of what kind of expression for the resistance to expect is by performing a dimensional analysis. If the physical quantities that play a role in the function to be determined are known, the constraint that the expression must be dimensionally consistent will provide a good guess of what the function will look like. From van Manen and van Oossanen, 1988:

$$R \propto \rho \cdot V^2 \cdot L^2 \cdot f \left[\left(\frac{\rho V L}{\mu} \right), \left(\frac{V}{\sqrt{gL}} \right), \left(\frac{\Delta p}{\rho V^2} \right) \right] = \rho \cdot V^2 \cdot L^2 \cdot f[(Re), (Fr), (Eu)] \quad (1.17)$$

With ρ the density of the water in kg/m^3 , L the ship's characteristic dimension, in this case the length, in m, μ the liquid's viscosity in kg/sm , Δp the pressure difference along the flow around the ship in kg/ms^2 , and g the acceleration due to gravity, $9.81m/s^2$

- Re is Reynold's number, $\frac{\rho V L}{\mu}$ the ratio of inertial forces and viscous forces.
- Fr is the Froude number, $\frac{V}{\sqrt{gL}}$ is the ratio of flow inertia to the external force field, usually gravity. It is used to compare the wave-making resistance of bodies.

- Eu is the Euler number, $\frac{\Delta p}{\rho V^2}$ the ratio of a pressure drop over the flows kinetic energy per volume. It is used to characterize energy losses in flow that is the result of a pressure difference. Perfect frictionless flow corresponds to an Euler number of 0.

Because the numbers inside the function's brackets are dimensionless, a dimension analysis cannot provide information about the function form.

Based on the dimensionless quantities, one would expect the form and frictional resistance to be a function of Reynold's number, the wave-making resistance to be a function of the Froude number, and the eddy resistance to be a function of the Euler number and Reynold's number.

1.5 Historical empirical relations

No exact theoretical expressions exist for any of the basic forces listed in section 1.2. This is due to the complexity of the problem of calculating flow around a body, as the Navier-Stokes equations have yet to be solved analytically.

Thus throughout the history of ship design, various mathematicians and physicists have conducted empirical research in order to predict ship resistance and behavior. Their work can be used in order to construct a model for simulated ship behavior.

1.5.1 The Froude model

William Froude was one of the pioneers in the field of frictional resistance of bodies moving through fluids. Around 1861, after measuring the resistance force on planks of various lengths in flowing water of various speeds, he came up with an empirical formula (Froude, 1961):

$$R_T = \alpha S V^\beta \quad (1.18)$$

With R_T the total resistance force in Newton, S the wetted surface of the body in square meters, and V the velocity of the body relative to the water in m/s.

He determined the constants α and β for various planks and found that they both depend on the roughness of the plank and its length.

Baker, 1915 came up with the idea of a friction coefficient defined as:

$$C_F = \frac{R_F}{0.5\rho S V^2} \quad (1.19)$$

With R_F the frictional resistance force, ρ the fluid density, S the wet surface area, and V the velocity of the body relative to the water.

Note that this is the ratio of the total resistance to the Bernoulli force of section 1.2, and that C_F is precisely the mystery function in equation 1.17 depending on Reynolds, Euler and Froude numbers found in the dimensional analysis of the resistance force.

The international Conference of Ship Tank Superintendents (ICST) initially adopted Froudes model when they were founded in 1935 . After multiple revisions the empirical relationship they agreed on at the conference of Madrid in 1957 was:

$$C_F = \frac{0.075}{(\log_{10}(Re) - 2)^2} \quad (1.20)$$

Granville, 1977 modeled ship resistance as a two dimensional flat plate in turbulent flow, and derived from this a general formula for the resistance coefficient. :

$$C_F = \frac{a}{(\log_{10}(Re) - b)^2} + \frac{c}{Re} \quad (1.21)$$

It should be clear that the ITTC formula given by equation 1.20 agrees with his general formula, with $a = 0.075$, $b = 2$ and $c = 0$.

Hughes, 1952 proposed that the total resistance coefficient consisted of the sum of a frictional resistance coefficient, a wave-making resistance coefficient and a residual resistance coefficient. He also proposed adding a form factor the expression in equation 1.20 to accommodate for different hull shapes.

The Froude model may seem crude, but over the years lots of empirical research has provided good methods for predicting resistance based on the Froude model.

1.5.2 Historical work on the wave-making resistance

The earliest theoretical work on the wave making resistance of ships is attributed to Lord Kelvin (1824-1907). He considered a single pressure point traveling in a straight line. This pressure point caused a system of transverse waves together with a series of divergent waves radiating from the point, with the combined wave front on a line with an angle of 19.5 degrees to the line of motion (Thompson, 1887).

An illustration of this phenomenon can be seen in figure 1.1.

The wave pattern as a whole moves together with the ship, this means the transverse waves move in the same direction with the same speed. Their length can thus be approximated as a free surface wave, of which the length is known to be:

$$L_w = 2\pi v^2/g \quad (1.22)$$

The divergent waves have a certain velocity in the direction tangent to their crests. However, in order to maintain the pattern the component of their velocity parallel to the line of the ship's motion must be equal to the ship's speed. If one defines the angle between the tangent and the parallel line as θ , it is easy to see that $V_{crest} = V_{ship} \cos(\theta)$ and thus the divergent wave length $L_{w_{div}} = \cos^2(\theta)L_w$

Havelock, 1910 derives a formula for the wave making resistance of the type:

$$R_{wm} = (A^2 + B^2 - 2AB \cos(Fr^{-2})) e^{-2 \left(\frac{Fr}{\sqrt{g}l} \right)^{-2}} \quad (1.23)$$

With A and B constants depending on the ship's hull form, Fr the Froude number, g the gravitational acceleration, v the velocity of the ship in knots, l the effective length

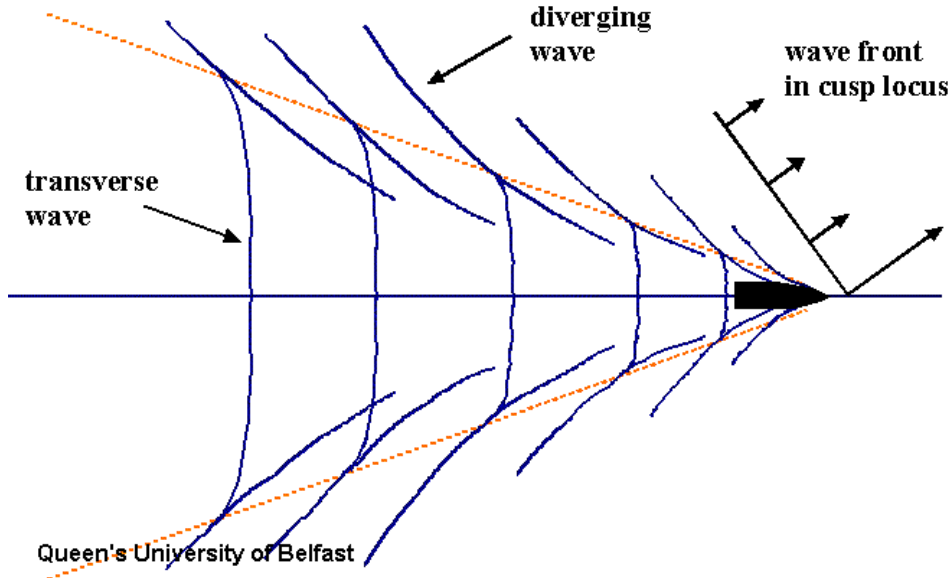


FIGURE 1.1: Wave pattern of divergent and transverse waves as modeled by the Kelvin model (source: Queen's University of Belfast)

of the ship and α the pressure system constant. The waves are in this case assumed to be caused by a one dimensional pressure system of the form:

$$p(x) = p_1(x) - p_2(x) = \frac{1}{\pi} \left[\frac{P_1 \alpha^2}{\alpha^2 + (x - 0.5l)^2} - \frac{P_2 \alpha^2}{\alpha^2 + (x + 0.5l)^2} \right] \quad (1.24)$$

Where P_1 and P_2 are the respective integral pressures such that $\int_{-\infty}^{\infty} p_n(x) = P_n$

Unfortunately, Havelock himself states that the parameter α has a significant impact on the wave-making resistance modeled, but he was unable to find a good approximation or guideline for what value of α to take, making his formula impractical.

Another pioneer in the field of wave making resistance was John Henry Michell (1863-1940). He found that if viscosity is neglected, a velocity potential for the flow due to the moving ship exist (Tuck, 1988).

This results in a non-linear Neumann-Stokes problem which he linearizes by assuming the ship is thin. The result is:

$$R_{wm} = \frac{4\rho g^2}{\pi v^2} \int_1^{\infty} (I^2 + J^2) \frac{\lambda^2}{\sqrt{\lambda^2 - 1}} d\lambda \quad (1.25)$$

The functions I and J are given by:

$$I = \int \int_H \eta_x(x, z) e^{\lambda^2 g z / v^2} \cos(\lambda g x / v^2) dx dz$$

$$J = \int \int_H \eta_x(x, z) e^{\lambda^2 g z / v^2} \sin(\lambda g x / v^2) dx dz \quad (1.26)$$

with η_x the Wigley hull equation, and H the hull surface.

Note the similarities between these two expressions and Havelock's equations 1.23 and 1.24, but instead of having to choose a value λ , it is an artificial variable that vanishes after integration, making the expression easier to implement provided it is possible to evaluate the integral.

Wigley, 1926 showed that the wave-making resistance could be constructed as the sum of 5 interference effects if one assumed a wedge shaped ship model. He proceeds to conclude that as there is one constant term plus four oscillating terms:

$$R_{wm} = kV^6 \quad (1.27)$$

He also noted up to a Froude number of 0.4 the transverse waves provide the main contribution to the wave making resistance, and above this the contribution from divergent waves starts to increase to significant levels.

1.6 Ship Power-Velocity relation

The goal of this chapter is to provide the theoretical background required to simulate a ship. This simulation is done by choosing realistic parameters of the relationship between Power and Velocity for the simulated ship. This relationship is assumed and used in the Iterative method which is explained in detail in chapter 2.

For the method the ship is assumed to have a Power-Velocity relation of the form:

$$P(V_s) = a + b \cdot V_s^q \quad (1.28)$$

With P the engine power in Watt and V_s the ship velocity in knots.

Since Power equals force times velocity, it follows that the resistance force is proportional to:

$$R_t \approx V_s^{q-1}$$

In this research the Iterative method has been applied on simulated test cases. This involved choosing a 'True' V-P relation for the simulated ship. From equation 1.27 it follows that the wave-making resistance component is of 6th order, but for large ships contributes only a fraction of the total resistance.

The skin friction can be approximated from equation 1.20. The base 10 logarithm of the Reynolds number squared can be approximated by $C_F \approx V^{-0.2}$.

Combining this with the definition of the friction coefficient in equation 1.19 yields the fact that the proportionality of the skin friction to the velocity of the form $R_{sf} \approx V_s^{1.8}$.

This agrees with the expression Prandtl and von Karman (1921) found for the friction coefficient in turbulent flow:

$$C_F = 0.072(Re^{-1/4}) \quad (1.29)$$

The simulated ship has a length of 300 metres. This means that the wave making component of the friction force will be around 10 – 20%, according to Misra, 2016. After combining this knowledge with equation 1.29 the parameter q of the simulated

Power-Velocity relationship 1.28 was chosen to be 3.2.

The parameter a in the relationship 1.28 equals the stationary power of the ship. It is the power the engine must supply in order for the ship to start moving. This represents the sum of the stationary resistance and the engine's internal resistance, which must both be overcome in order to have a net force on the ship.

This stationary power, and thus the value of the parameter a , has been estimated for the simulated ship to be 100 Kwatt.

The parameter b in the relationship 1.28 was chosen in such a way that the resulting ship velocity at contract speed was about 15 knots, which is a reasonable assumption for a ship of this size. The corresponding value of the parameter b is 5.

Chapter 2

Methods for determining current effect during speed trials

2.1 Introduction

Traditionally, the method used to determine the ships speed relative to the water by eliminating the effects of current is the mean of means method. The idea behind the method is that if you sail back and forth and then take the average, it should give you a good approximation of the ship's true speed. The method is very intuitive, if the current were constant, it would always provide you with the ships speed relative to the water.

A recently developed alternative method is the Iterative method. It uses non-linear least squares fitting of measured ship speed at different power settings to a function that relates engine power to ship velocity, followed by fitting of the derived current data to a general function with a known period. It is called the Iterative method because it uses its estimate of the Velocity-Power function to make a better estimate of the current, which in turn leads to a better estimate of the V-P function and so forth.

One of the main advantages of the method is that it requires less measurements than the Mean of means method, and that the measurements do not have to be evenly spaced.

2.2 The Iterative method

The Iterative method (Toki, 2016) is a recently developed method for calculating the current velocities of a sea trial. Its theoretical advantage over the mean of means method is that it utilizes knowledge of the tidal current in order to better approximate the effects of current at a sea trial. This results in more accurate results for longer sea trials.

A schematic overview of the iterative process is given in figure 2.1. The method will be described in detail in subsections 2.2.1 through 2.2.3.

As stated in the official sea trial guidelines described in ISO15016 (ISO and ITTC, 2015), the tidal current has a semidiurnal period $T_C = 12$ hours, 25 minutes and 12 seconds. It is assumed that the current over this period follows the relation:

$$V_c(t) = A \cos\left(\frac{2\pi}{T_c}t\right) + B \sin\left(\frac{2\pi}{T_c}t\right) + C t + D \quad (2.1)$$

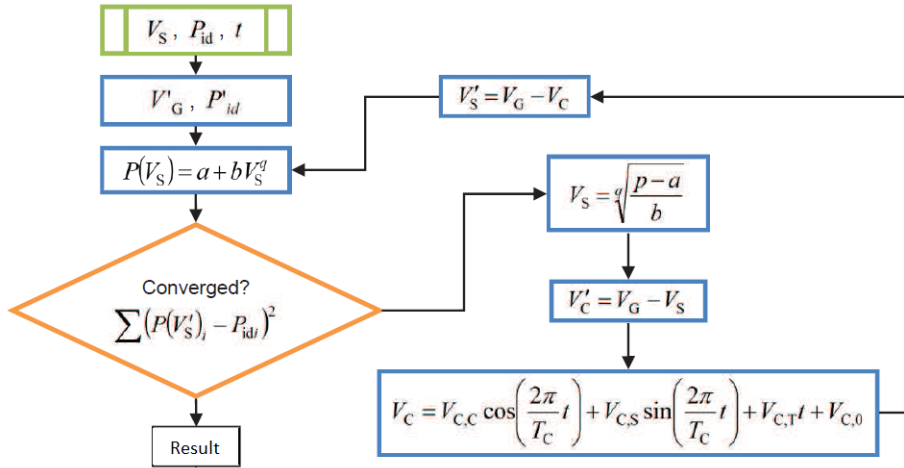


FIGURE 2.1: Flowchart of the Iterative method (source: ISO15016)

With V_c the current velocity, T_c the current period, t the time and A , B , C and D parameters to be determined by the Iterative method.

In ISO15016 it is also stated that the minimum measurements required to use the Iterative method are four double runs, of which two at contract speed. A double run is a back-and-forth run along the same trajectory. Contract power is the engine power setting with a corresponding contract speed, the minimum speed the ship must be able to reach at that power setting.

To minimize measurement errors, it helps to have an overdetermined system. Every measurement corresponds to an equation, and at least one equation is needed for every unknown variable. An overdetermined system has more equations than unknowns.

The ship's average speed over the ground is obtained by measuring the time between a start line and a finish line, determined by GPS, and then dividing distance by time.

The Iterative method consists of 3 main steps:

2.2.1 Step 1: initial guess of the speed-power relation

First an initial guess of the ship's speeds V_s is obtained by taking the average speed of the double runs. This produces (P, V_s) data pairs. Then, it is assumed that the relationship between engine power and ship velocity is of the form:

$$P(V_s) = a + b \cdot V_s^q \quad (2.2)$$

Inverting this produces:

$$V_s = \left(\frac{P(V_s) - a}{b} \right)^{\frac{1}{q}} \quad (2.3)$$

With unknown parameters a , b and q .

The parameters a , b and q by fitting the power-velocity data pairs to formula 2.3 using the Levenberg-Marquardt method described in chapter 3.

This step is represented starting with the green box containing V_s, P_{id} and t until the blue box containing $P(V_s) = a + bV_s^q$ in figure 2.1.

2.2.2 Step 2: initial guess of the current function

Since the measured speed is the ship's speed over the ground V_G is equal to the ship's own speed plus the current speed, we can approximate the current speeds V'_c by taking the difference between V_G and the initial approximation for V_s .

$$V'_c = V_G - V_s \quad (2.4)$$

Since the runs were timed, it is now possible to find the parameters A , B , C and D by fitting these values of V'_c to equation 2.1 using linear least squares fitting as described in chapter 2 with the following components:

$$X = \begin{bmatrix} \cos(2\pi \frac{t_1}{T}) & \sin(2\pi \frac{t_1}{T}) & \frac{t_1}{T} & 1 \\ \cos(2\pi \frac{t_2}{T}) & \sin(2\pi \frac{t_2}{T}) & \frac{t_2}{T} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \cos(2\pi \frac{t_n}{T}) & \sin(2\pi \frac{t_n}{T}) & \frac{t_n}{T} & 1 \end{bmatrix}$$

$$\beta = \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} \quad Y = \begin{bmatrix} V'_{c1} \\ V'_{c2} \\ \vdots \\ V'_{cn} \end{bmatrix}$$

Where V'_{c_n} is the n th component of the vector V'_c and t_n the time at which the corresponding measurement took place.

This step corresponds to the blue box containing equation 2.4 followed by the blue box containing equation 2.1 in figure 2.1.

2.2.3 Step 3: the iterative process

Now that the initial guesses for the parameters have been determined, the iterative process can start. It consists of the following steps:

- Find the updated calculated ship speed by subtracting the fitted current speeds from the measured ground speeds, as seen in equation 2.5

$$V'_s = V_G - V_c \quad (2.5)$$

- Fit the updated calculated ship speed V'_s and the measured engine power P to the V-P function given by 2.2 once again using least squares fitting.
- Calculate the new ship speed V_s by inserting the updated parameters from the fitting process into equation 2.3
- Calculate the updated current speed V'_c using equation 2.4
- Fit the updated calculated current speed V'_c to the current function given by 2.1

Step 3 corresponds to the entire loop beginning and ending with $P(V_s) = a + bV_s^q$ in figure 2.1.

Step 3 is repeated until the sum of squared differences between the calculated engine power and measured engine power is sufficiently small. This sum of squared errors is given by:

$$E = \sum_{i=1}^n (P(V'_s) - P_{id})^2 \quad (2.6)$$

Where the $P(V'_s)$ is the engine power as a function of ship speed as described in equation 2.2 using the parameters found in step 3, P_{id} is the measured engine power, and n is the number of double runs performed during the sea trial.

2.2.4 Measurements during a test run

As mentioned in the previous section, in order to find the parameters a , b , and q of the V-P and current functions an overdetermined system is desirable.

The Iterative method consists of two systems, that are linked. First there is the V-P system, with three unknown parameters thus requiring a minimum of three measurements. Next there is the current system, with four parameters and thus requiring a minimum of four measurements.

During a sea trial, data are gathered by performing runs back and forth. The ships ground speed at time t_m and power setting p_n is:

$$V_{G(m,n)} = V_s(p_n) \pm V_c(t_m) \quad (2.7)$$

Where V_s is the ship speed, V_G the ground speed and V_c is the current speed.

A P-V measurement then consists of taking the mean of a double run, so

$$\hat{V}_s = V_s(p_n) \pm \frac{V_c(t_m) + V_c(t_{m+1})}{2} \quad (2.8)$$

It is clear that one P-V measurement is obtained for every double run, so a minimum of three double runs are required to be able to determine the parameters of the P-V function.

A current measurement is obtained by taking the difference between the ground speed and \hat{V}_s , see equation 2.4. This means that every double run two current measurements are obtained, and thus a minimum of two double runs is required to have four measurements for the four parameters of the current function.

When examining equations 2.4 and 2.8 it is clear that the measurements have an error. This error is minimized by the rest of the iterative process.

2.3 The Mean of Means method

Traditionally the Mean of Means method has been used to determine the ship's speed without the influence of current. It originates from the principle that if you have a constant current, sailing back and forth and then taking the average will provide the desired result. Unfortunately, currents vary over time. However if it is possible to do all the measurements for a single power setting within a certain time frame the current function can be approximated as:

$$V_c(t) = a + b t + c t^2 + d t^3 \quad (2.9)$$

What is meant by taking the mean of means is the following process: If n measurements were obtained, the first means are the mean of the 1st and 2nd measurement, 2nd and 3rd, and so forth. This will result in $n - 1$ means. If the same process is applied on these means, this will result in $n - 2$ means of means. This can be repeated m times until $n - m = 1$ and a final $n - 1$ 'th order mean of means is obtained.

If the mean of means method is to give an accurate approximation of the ships speed, it is important that the current can be accurately approximated as a third order polynomial in time. However, especially for very large ships that require a long time to get up to the required speed and take a long time to turn around, this is not feasible and will result in an error in the estimation.

2.3.1 Pascal's triangle

If the process of mean taking is examined, it is evident that not every measurement is equally represented in the end result. It can be derived that the representation of the measurements follows Pascal's triangle. For example if four measurements are done, measurement 2 and 3 will both have three times the weight of measurements 1 and 4 when evaluating the final ship velocity.

An illustration of Pascal's triangle can be observed in figure 2.2.

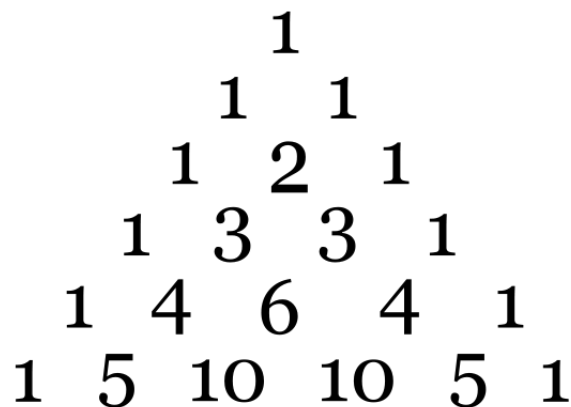


FIGURE 2.2: The first 6 rows of Pascal's triangle

2.4 Proof of convergence

In the preceding section was said that the mean of means method is accurate if the current can be approximated by a third degree polynomial. This is because for an additive function of this form the current effects will vanish after taking the fourth mean of mean.

If the current function is of the form:

$$V_c(t) = a + b t + c t^2 + d t^3 \quad (2.10)$$

And Measurements are done at regular intervals of time Δt , of the form:

$$V_G(nt) = V_s \pm V_c(n\Delta t) = V_s \pm a + b n t + c n^2 t^2 + d n^3 t^3 \quad (2.11)$$

Taking the first mean of measurement n and $n + 1$, assuming measurement 1 was taken with with the current, results in:

$$M_1 = V_s \pm 0.5 [b\Delta t + ((n + 1)^2 - n^2) c\Delta t^2 + ((n + 1)^3 - n^3) d\Delta t^3] \quad (2.12)$$

The sign of the current addition alternates between positive and negative for even and uneven n .

The second mean is then equal to:

$$M_2 = V_s \mp 0.25 [((n + 2)^2 - n^2) c\Delta t^2 + ((n + 2)^3 - n^3) d\Delta t^3] \quad (2.13)$$

and the third mean

$$M_3 = V_s \pm 0.25 [((n + 2)^2 - n^2 - (n + 3)^2 + (n + 1)^2) c\Delta t^2 + ((n + 2)^3 - n^3 - (n + 3)^3 + (n + 1)^3) d\Delta t^3]$$

Which equals

$$M_3 = V_s \pm 0.25 [4c\Delta t^2 + (12n + 18) d\Delta t^3] \quad (2.14)$$

Then the fourth mean:

$$M_4 = V_s \mp 0.125 [12d\Delta t^3] \quad (2.15)$$

and at the fifth mean V_s is obtained.

In the case of a second order polynomial the derivation is analogous but converges at the fourth mean.

An example of the mean of means method can be seen in table [2.3](#)

2.5 Accuracy of the polynomial approximations approximation

The preceding section shows that the Mean of means method is valid if the current can be approximated with a 2nd or 3rd order polynomial, depending on the amount of double runs performed.

Assessing the quality of this approximation is has been done in two ways. First various possible current functions of the form of equation [2.1](#) are compared to the Lagrange

t/T	V_s	mean 1	mean 2	mean 3	mean 4	mean 5	V_{true}
0	17.2978	15.1494	15.1644	15.2912	15.3092	15.3006	15.2978
0.1	13.001	15.1794	15.4181	15.3273	15.2919		
0.2	17.3578	15.6568	15.2364	15.2565			
0.3	13.9557	14.8161	15.2766				
0.4	15.6765	15.7371					
0.5	15.7978						

FIGURE 2.3: Example of the mean of means method

interpolating polynomial of this function. Second the same functions are compared to a linearly least squares fitted polynomial. The process of linear least squares fitting is explained in chapter 3.

2.5.1 Lagrange approximation

A good way to approximate a function with a polynomial is to use Lagrange interpolating polynomials.

A Lagrange polynomial is a unique polynomial of least degree that interpolates a set of function values $y_i = f(n_i)$. The Lagrange polynomial through k data points is given by:

$$L(x) = \sum_{j=0}^k f(n_j)l_j(x) \quad (2.16)$$

with

$$l_j(x) = \prod_{m=0, m \neq j}^k \frac{x - n_m}{n_j - n_m} \quad (2.17)$$

To minimize errors, n_i should be chosen to be the Chebychev nodes (Fink and Mathews, 1999) for an interval $[a, b]$ given by:

$$n_i = \frac{1}{2}(a + b) + \frac{1}{2}(b - a)\cos\left(\frac{(2i - 1)\pi}{2k}\right) \quad (2.18)$$

An example of a third order polynomial approximation can be seen in figure 2.4. One approximation is obtained by a linear least squares fit, the other by Lagrange approximation.

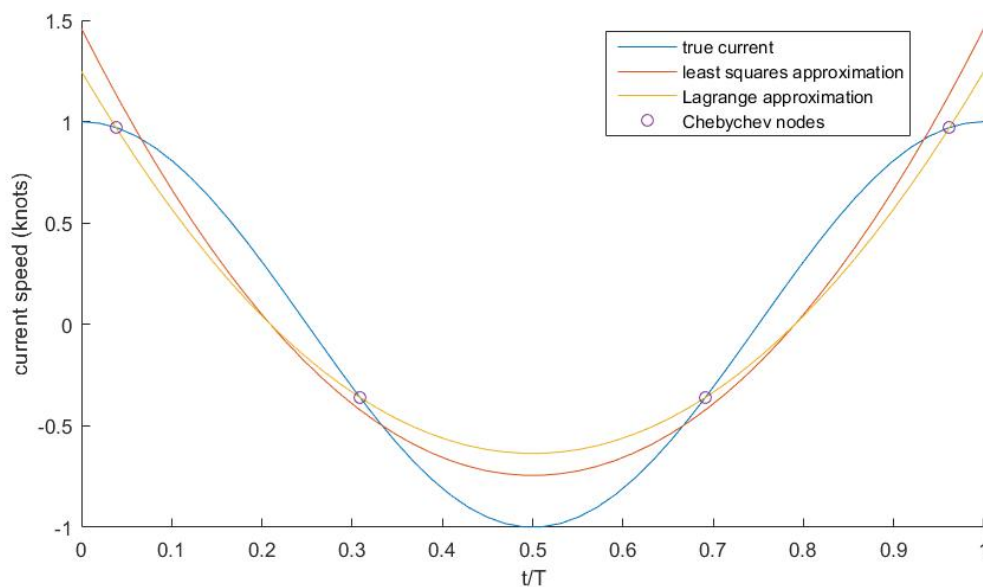


FIGURE 2.4: Example of a linear least squares and a Lagrange approximation of a current function over a full period.

2.5.2 The Lagrange remainder

If the Chebychev nodes are used, the interpolation error function for a polynomial of degree k can be shown to be bound by:

$$|E(x)| \leq \frac{(b-a)^{k+1}}{(k+1)!} \max(\zeta) |f^{k+1}(\zeta)| \quad (2.19)$$

With $\zeta \in [a, b]$

If the function that is being interpolated is of the form of equation 2.1, this upper bound can be calculated.

It is beneficial to transform t/T into a new variable τ , and use this for calculating the derivatives. If this is not done the value of the derivative depends on the units in which t is expressed.

The third derivative of this function with respect to τ is:

$$8\pi^3 (A \sin(2\pi\tau) - B \cos(2\pi\tau)) \quad (2.20)$$

While the fourth is:

$$16\pi^4 (A \sin(2\pi\tau) - B \cos(2\pi\tau)) \quad (2.21)$$

And

$$\max(\tau) [A \sin(2\pi\tau) - B \cos(2\pi\tau)] = \sqrt{A^2 + B^2} \quad (2.22)$$

Inserting this into equation 2.19 gives:

$$\begin{aligned} |E_2(x)| &\leq \frac{4\pi^3(b-a)^3}{3} \sqrt{(A^2 + B^2)} \\ |E_3(x)| &\leq \frac{2\pi^4(b-a)^4}{3} \sqrt{(A^2 + B^2)} \end{aligned} \quad (2.23)$$

Chapter 3

Theory of Numerical methods

3.1 Introduction

It is described in chapter 2 that during the execution of the Iterative method, fitting of data points takes place. Examples are fitting the current as a function of time or ship velocity as a function of engine power. In this chapter various numerical methods used for the fitting process are described.

3.1.1 Least squares fitting

In the description of the iterative method as given in ISO15016 (ISO and ITTC, 2015) it is said that the parameters are found using the least squares method. This consists of the following:

Given a set of data pairs x_n, y_n that is subject to a relation $y = f(x, \beta)$ with parameter vector β , we look for β belonging to the model function such that we minimize the sum of squares of the deviations.

$$E(\beta) = \sum_{n=1}^m [y_n - f(x_n, \beta)]^2 \quad (3.1)$$

With y_n, x_n the data pairs you want to fit a function to.

Generally, since the objective is to minimize these squares, a minimum can be found by setting the gradient of E equal to zero:

$$\frac{\partial E(\beta_i)}{\partial \beta_i} = -2 \cdot \sum_{n=1}^m [y_n - f(x_n, \beta)] \cdot \frac{\partial f(x_n, \beta_i)}{\partial \beta_i} \quad (3.2)$$

This results in as many equations as there are parameters β .

3.2 Levenberg Marquardt

The Levenberg-Marquardt algorithm (Levenberg, 1944, Marquardt, 1963, Gavin, 2015) is an algorithm that is used to solve non-linear least squares curve fitting problems. It interpolates between the Gauss-Newton Algorithm and the method of gradient descent, which will be discussed in the following sections. Generally the Levenberg Marquadt is a robust algorithm. Independently of an initial guess for the parameters it has to determine it will almost always find a solution.

3.2.1 Method of Gradient descent

The method of Gradient descent uses the gradient of the error function with respect to the parameter vector β , it is possible to perturb β in such way that the resulting step in parameter space nets the greatest negative change in the error function, which is the case when moving in the negative direction from the gradient. This results in a recursive relation for β :

$$\beta_{i+1} = \beta_i - h \cdot \nabla E(\beta_i) \quad (3.3)$$

With h a step size which depends on the 'area' where the gradient remains a good approximation of the true gradient.

The Error function defined in 3.1 can be written as a column – row vector product of the form:

$$E(\beta) = (Y - F(X, \beta))^T W (Y - F(X, \beta)) \quad (3.4)$$

With Y the vector of y_n 's and $F(X, \beta)$ the vector of individual $f(x_n, \beta_i)$'s, and W a diagonal matrix with the squared weights w_n .

The weight matrix W can be added if certain measurements have a greater importance in the sum than others, otherwise it is equal to the identity matrix.

Taking the gradient with respect to β :

$$\nabla E(\beta) = (Y - F(X, \beta))^T W \frac{\partial}{\partial \beta} (Y - F(X, \beta)) = -(Y - F(X, \beta))^T W \left[\frac{\partial F(X, \beta)}{\partial \beta} \right] \quad (3.5)$$

Inserting this expression into equation 3.3 to obtain:

$$\beta_{n+1} = \beta_n + h(Y - F(X, \beta_n))^T W \left[\frac{\partial F(X, \beta_n)}{\partial \beta} \right] \quad (3.6)$$

Equation 3.6 gives a recursive relation for the updates of the parameter vector β .

Finding the minimum of the sum of least squares by this method is called the method of Gradient descent, since it finds the solution by moving in the opposite direction of the gradient.

3.2.2 The Newton-Gauss method

The Newton-Gauss method (Björck, 1996) is very similar to the method of Gradient descent, but it makes use of the fact that the function $F(X, \beta)$ can be approximated using a Taylor polynomial. This results in extra information that is used for finding a better perturbation. Since $F(X, \beta)$ is a vector, the gradient of this vector with respect to β is actually the Jacobian of $f(x_m, \beta_n)$:

$$\frac{\partial F(X, \beta)}{\partial \beta} = \begin{bmatrix} \frac{\partial f(x_1, \beta)}{\partial \beta_1} & \frac{\partial f(x_1, \beta)}{\partial \beta_2} & \dots & \frac{\partial f(x_1, \beta)}{\partial \beta_n} \\ \frac{\partial f(x_2, \beta)}{\partial \beta_1} & \frac{\partial f(x_2, \beta)}{\partial \beta_2} & \dots & \frac{\partial f(x_2, \beta)}{\partial \beta_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f(x_n, \beta)}{\partial \beta_1} & \frac{\partial f(x_n, \beta)}{\partial \beta_2} & \dots & \frac{\partial f(x_n, \beta)}{\partial \beta_n} \end{bmatrix} = J$$

The approximation with a Taylor polynomial gives:

$$F(X, \beta + \delta) \approx F(X, \beta) + \left[\frac{\partial F(X, \beta)}{\partial \beta} \right] \beta = F(X, \beta) + J\beta \quad (3.7)$$

In where δ is a vector indicating the perturbation of the vector β .

Substitution into 3.4 gives:

$$E(\beta + \delta) \approx (Y - F(X, \beta) - J\delta)^T W (Y - F(X, \beta) - J\delta) \quad (3.8)$$

$$= Y^T W Y + F(X, \beta)^T W F(X, \beta) - 2Y^T W F(X, \beta) - 2(Y - F(X, \beta))^T W J\delta + \delta^T J^T W J\delta \quad (3.9)$$

Setting the derivative with respect to δ to 0 results in:

$$\frac{\partial}{\partial \delta} E(\beta + \delta) \approx -2(Y - F(X, \beta))^T W J + 2\delta^T J^T W J = 0 \quad (3.10)$$

Rearrangement of the terms provides an expression for the perturbation:

$$\delta = (Y - F(X, \beta))^T W J [J^T W J]^{-1} \quad (3.11)$$

Note that $[J^T W J]^{-1}$ is a square matrix so the inverse will almost always exist.

This results in the Newton-Gauss method, with update relationship:

$$\beta_{n+1} = \beta_n + (Y - F(X, \beta_n))^T W J [J^T W J]^{-1} \quad (3.12)$$

It should be noted that this parameter update is equal to the method of Gradient Descent if $[J^T W J] = I$.

3.2.3 Marquardt's modification

Marquardt noted that the Gauss-Newton method and the Gradient Descent method are similar, yet their behaviour is very different.

The method of Gradient descent is very reliable in the sense that it will almost always converge to a local minimum, however if the gradient is small it can do so very slowly. The Gauss-Newton algorithm is quicker, but more prone to diverging.

Marquardt proposed to take a linear combination of the two methods relating them by a so called dampening parameter λ , resulting in:

$$\delta = (Y - F(X, \beta))^T W \cdot J \cdot [J^T W J + \lambda \cdot I]^{-1} \quad (3.13)$$

For the algorithm to function as efficiently as possible, the choice of λ needs to be optimal. For large values of λ the behavior is similar to that of the method of Gradient descent and thus will converge quickly, but will be prone to error. For small values of λ the behavior will mimic G-N and thus be very accurate, albeit slow (Gavin, 2015).

From this it follows that for maximum efficiency λ must be chosen as large as possible without the algorithm making errors.

Various ways of determining the λ have been proposed. Marquardt suggested defining a factor λ_{up} and a factor λ_{down} . During the iteration process, if the taken step results in a lower value of the objective function, λ is multiplied by λ_{up} to speed up the algorithm.

However, if the taken step results in a higher value of the objective function, the step is aborted and λ is multiplied by λ_{down} for a more accurate retry.

3.3 Linear least squares fitting

The fitting of the current data to the current function is done with the method of Linear least squares. Since a measurement of a data pair (x_n, y_n) corresponds to an equation of the form:

$$\beta_1 f_1(x_n) + \beta_2 f_2(x_n) + \beta_3 f_3(x_n) + \beta_4 f_4(x_n) \cdots \beta_n f_n(x_n) = y_n + \epsilon_n \quad (3.14)$$

Where β_n 's are the parameters to be determined by the fit, f_n an arbitrary function of x_n and ϵ_n the measurement error in the n th measurement.

These equations can be put into matrix form:

$$X\beta = Y + \epsilon \quad (3.15)$$

Where

$$X = \begin{bmatrix} f_1(x_1) & f_2(x_1) & \cdots & f_n(x_1) \\ f_1(x_2) & f_2(x_2) & \cdots & f_n(x_2) \\ \vdots & \vdots & \ddots & \vdots \\ f_1(x_m) & f_2(x_m) & \cdots & f_n(x_m) \end{bmatrix}$$

In this case Y is the vector of measurement values and ϵ the vector of measurement errors.

To find the best fit for the parameters β_n the sum of squared errors must be minimized. Noting that

$$E(\beta) = \|Y - \beta X\|^2 = (Y - \beta X)^T (Y - \beta X) = Y^T Y - 2\beta^T X^T Y + \beta^T X^T X \beta \quad (3.16)$$

The minimum is obtained by setting the derivative with respect to $\bar{\beta}$ to zero:

$$-2X^T Y + 2(X^T X)\beta = 0 \quad (3.17)$$

and from this follows that the linear least squares fit for β is:

$$\beta = [X^T X]^{-1} X^T Y \quad (3.18)$$

Note that $X^T X$ is non singular and thus can be inverted if X has a full rank, which means that all its rows contain independent vectors. This is always the case if X is constructed from unique measurements.

Chapter 4

Results

A goal of this study is to investigate the Iterative method, a new method for determining a ship's speed relative to the water, which is described in chapter 2. First a literature study was done in order to accurately simulate a case to apply the method to, explained in chapter 1. After some research on non-linear least squares fitting problems, described in chapter 3, the method was ready to be implemented and tested.

During the implementation of the Iterative method, various problems arose. The solutions to these problems are part of the results of this research, as the method was implemented by the guidelines of ISO15016. After the method was functioning properly, first its behavior assuming measurements without noise was examined. This has been done in order to determine under what conditions the method properly converged.

Next an analysis without measurement noise was done to determine the effect of measurement noise on the iterative method's results.

Finally a comparison has been made with the Mean of means method, also described in chapter 2.

The Mean of means method makes the assumption that the current can be approximated by a polynomial, of which the validity has been evaluated in various cases. After that the method results obtained by using simulated measurements with artificial noise levels are presented in order to compare the performance of both methods.

4.1 The current function

One of the first problems encountered in the implementation of the iterative method is that a resulting current function that was encountered has a large non-physical increase. An illustration is given in figure 4.1.

The reason for this behavior is found to be the fact that in the current formula given by equation 4.1 the parameter C scales differently depending on the units in which the time is expressed.

$$V_c(t) = A \cos\left(\frac{2\pi}{T_c}t\right) + B \sin\left(\frac{2\pi}{T_c}t\right) + C t + D \quad (4.1)$$

In equation 4.1 the parameter C has dimension m/s^2 , while A , B and D have dimension m/s .

For example, if the tidal current are expressed in seconds its period would have a value of over 40000. This causes the parameter C to be 4 orders of magnitude smaller than the other three, while being fitted by the same linear least squares procedure. This

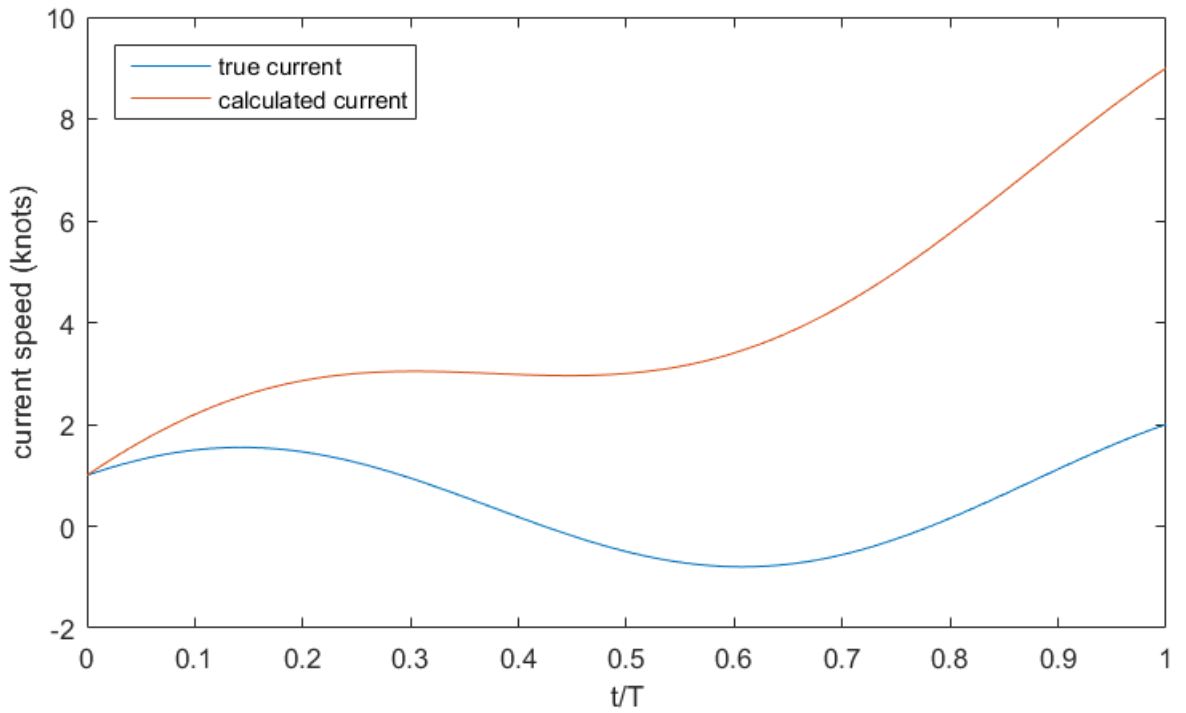


FIGURE 4.1: Example of gain in calculated current

disparity causes a lot of inaccuracy in the parameter C , causing the behavior seen in figure 4.1.

4.1.1 Solution

The proposed solution to the problem is to normalize the t component that belongs to the parameter C . The resulting formula can be seen in equation 4.2. This normalization is done by dividing by the current period, causing values of $\frac{t}{T_c}$ to lie between 0 and 1, which is of the same order of magnitude as the other three components for realistic tidal currents. Having all three parameters be of the same order of magnitude will result in more accurate fitting by the linear least squares method.

$$V_c(t) = A \cos\left(\frac{2\pi}{T_c}t\right) + B \sin\left(\frac{2\pi}{T_c}t\right) + C \frac{t}{T_c} + D \quad (4.2)$$

4.2 Failure to converge

Sometimes in usage of the Iterative method, it would simply fail to provide a solution. All parameter values became infinite, resulting in termination of the program. Upon closer inspection of this phenomenon, it was found that this occurred when the method tried to set the parameter b in the inverted V-P function $V_s = \left(\frac{P(V_s)-a}{b}\right)^{\frac{1}{q}}$ to 0, and compensated for this effect using the other two parameters.

During calculation the method uses the derivative of this function with respect to parameters a, b and q to calculate the next iterative step. However with b zero, the derivative approached infinity, which caused the . Limiting the value of the parameter b to 10^{-12} proved to solve this problem. The implications of this and whether this should be implemented or not will be discussed in chapter 5.

4.3 Parameter limiting

One of the problems with the iterative method was that the solution found by the method, the V-P function sometimes would 'not converge'. This means that the parameters of the function $P(V_s) = a + b \cdot V_s^q$ the method found were such that they could not be physically possible, and thus cannot be the true parameters of the ship. The proposed method is to limit the minimum and maximum values the parameters could attain during the iterative process. This proves to solve the problem, but the method fails to find the exact input parameters. A table with resulting parameters can be observed in table 4.1. Whether this is a viable addition to the iterative method or not will be discussed in chapter 5.

$$P(V_s) = a + b \cdot V_s^q$$

TABLE 4.1: Parameters of the V-P relation calculated by the iterative method for 4 double runs at 3 powersettings with and without parameter limiting.

Parameter:	a	b	q
Input:	100	5	3.2
Start:	0	3	3
Without limiting			
Guess:	1938.6231	4.6194	3.1821
Iterative:	100	5	3.2
With limiting			
Guess:	1973.3272	4.6768	3.1764
Iterative:	0	7.2997	3.0546
Lower limit:	0	0.001	2.8
Higher limit:	3000	1000	4.2

In table 4.1 the row after Input indicates the parameters used to generate the measurement data. The row after Start indicates the method's starting parameters. The row after Guess indicates the parameters guessed based on the average of the double run measurements. The row after Iterative indicates the parameters after the method converges. The row after Lower limit indicates the lower limit of the values the parameters are allowed to attain, the row after Higher limit is analogous.

4.4 Minimum number of runs required to use the Iterative method

One of the main goals of this study is to determine how many double runs and power-settings are required such that the iterative method can be used.

In order to do this a test case has been analyzed without measurement noise. In the first case the simulated measurement data consisted of three double runs at three different

power settings. In the second case it consisted of four double runs of which two runs performed at the simulated contract speed.

The results can be seen in figures 4.2 through 4.5, and the parameter values can be seen in table 4.2.

The Velocity-Power relationship that is assumed and calculated by the iterative method is given by equation 4.3.

$$P(V_s) = a + b \cdot V_s^q \quad (4.3)$$

TABLE 4.2: Parameters of the V-P relation calculated by the iterative method for 3 and 4 double runs at 3 powersettings.

Parameter:	a	b	q
Input:	100	5	3.2
Start:	0	3	3
3 double runs			
Guess:	2788	4.9	3.135
Iterative:	-7730	88.89	2.217
4 double runs			
Guess:	1575	5.458	3.124
Iterative:	100	5	3.2

In table 4.2 the row after Input indicates the chosen input parameters of the function given by 4.3. The row Start indicates the parameter values the method starts with. The rows Guess indicate the parameter values calculated after step 1, explained more thoroughly in chapter 2. The rows Iterative indicate the final parameter values after the iterative method converges.

From table 4.2 it can be concluded that for 3 double runs at zero noise, the method fails to find the correct parameters, and for 4 double runs it does. This leads to the conclusion that 3 double runs at 3 power settings are insufficient to be able to use the iterative method, while 4 double runs at 3 power settings are sufficient.

In the bottom plot of figure 4.2 the true V-P curve overlaps with the Iterative curve, as do the True velocity data points and the values calculated by the iterative method.

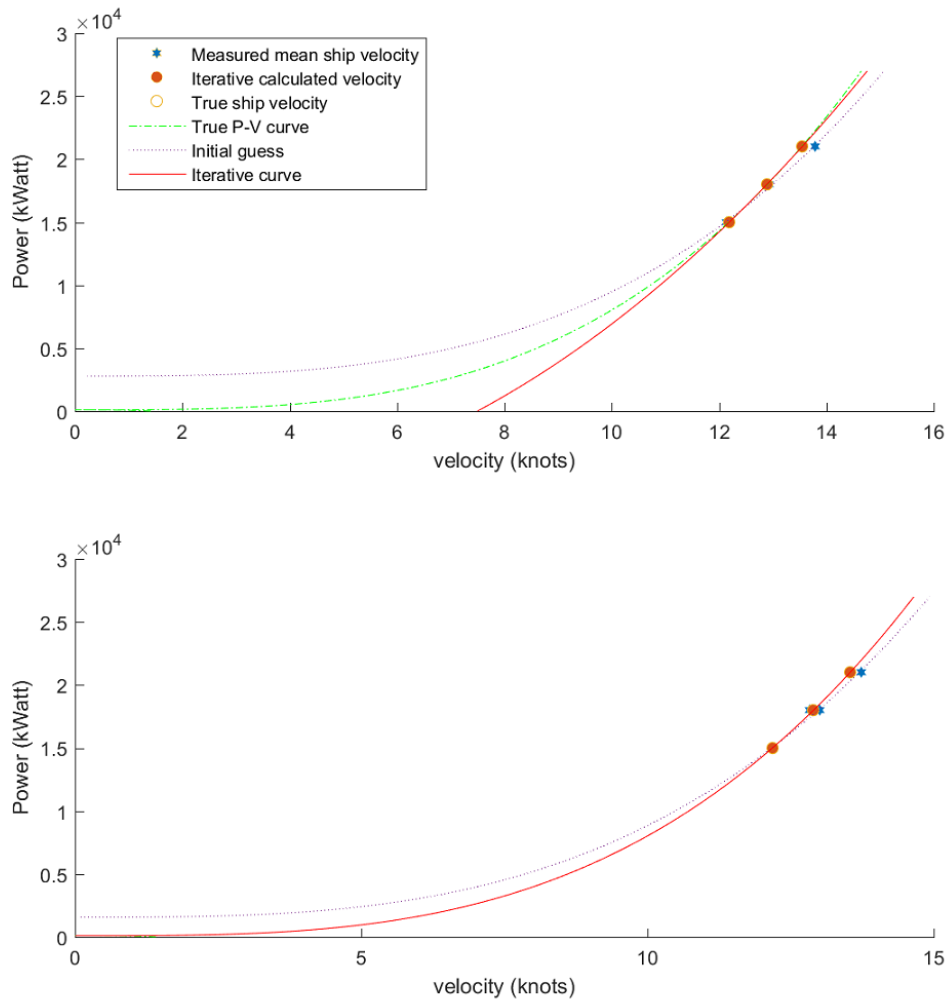


FIGURE 4.2: True V-P functions and the function found by the iterative method. Top are 3 double runs at 3 power settings, bottom are 4 double runs at 3 power settings.

The current function the iterative method assumes and calculates is given by equation 4.4.

$$V_c = A \cdot \cos\left(\frac{2\pi}{T_c}t\right) + B \cdot \sin\left(\frac{2\pi}{T_c}t\right) + C \cdot \frac{t}{T_c} + D \quad (4.4)$$

As in table 4.2 the rows after Start and Input indicate the parameter values of function 4.4 the method starts with and the true values, respectively.

The current parameters support the same conclusion as the V-P function parameter table 4.2. The method fails to exactly find the parameters at 3 double runs at 3 power settings, but it does find them for 4 double runs.

TABLE 4.3: Parameters of the current function calculated by the iterative method for three and four double runs at 3 powersettings.

Parameter:	A	B	C	D
Input:	0.25	0.25	1	-1
Start:	1	1	1	1
3 double runs				
Guess:	0.25791	0.065545	0.57754	-0.75356
Iterative	0.2617	0.22974	0.94735	-0.96929
4 double runs				
Guess:	0.24451	0.13825	0.751	-0.85994
Iterative	0.25	0.25	1	-1

Plots of the current functions with the parameters from 4.3 can be seen in figure 4.3. In figure 4.3 in the bottom graph the Iterative result coincides with the true current, as do the true values in the measurement points and the calculated iterative values.

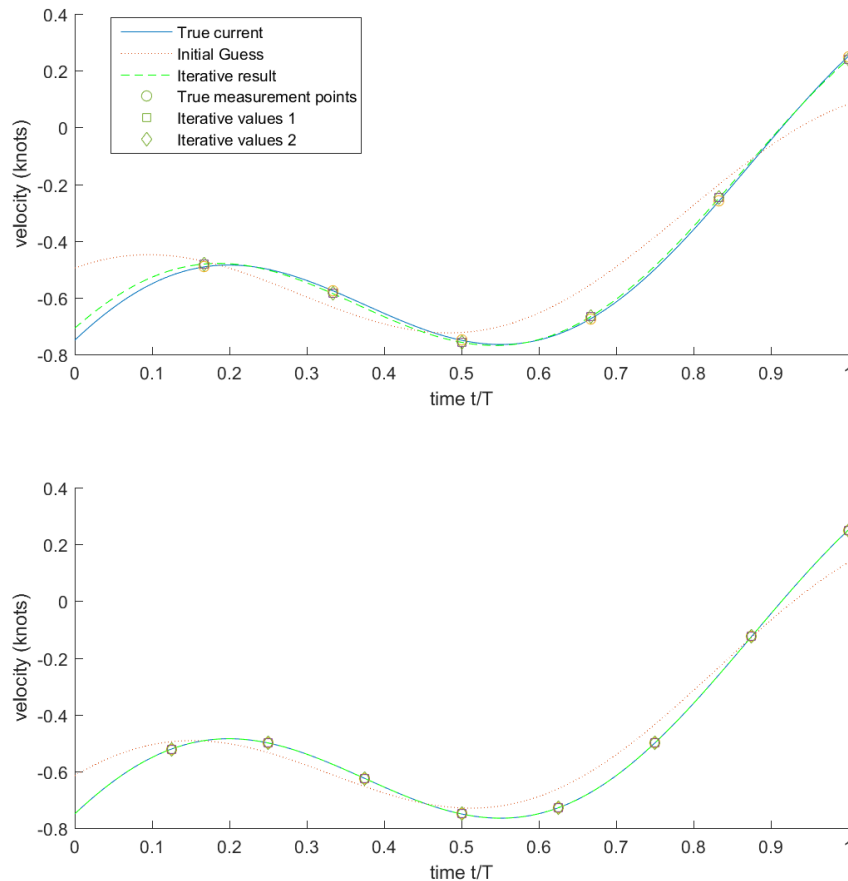


FIGURE 4.3: True current function and the function found by the iterative method. Top is the case of 3 double runs at 3 power settings, bottom is 4 double runs at 3 power settings.

In figure 4.4 and 4.5 the estimated and true standard error of the method is plotted against the Iteration number. The standard error is obtained with formula 4.5.

$$E(\hat{\beta}) = \sqrt{\frac{\sum_{n=1}^N (P(\hat{\beta}) - P(\beta))^2}{N}} \quad (4.5)$$

Where $\hat{\beta}$ the iterative method's parameter vector of the function given by equation 4.3 and 4.2, N the number of measurements performed and β the true parameter vector.

The estimated error is the estimate of the standard error given by equation 4.5 of the iterative method which it tries to minimize. The true error is the actual standard error it makes. They are obtained by taking the squared sum of the residuals in the data points.

The method estimates its error by assuming the current found in the previous iterative step is the true current, and using that to calculate the ship's true velocity. The opposite is true for estimating the current error. The disparity between the methods error estimate and the true error, which can be observed in the top graph of figure 4.4, is caused by the fact that the current found in the previous iterative step is not the true current.

From figures 4.4 and 4.5 it can be seen that the standard error becomes extremely small in the case of 4 double runs at 3 power settings, but for only 3 double runs it quickly stagnates at a much higher error. This is consistent with the fact that the method failed to correctly find the parameters of the functions.

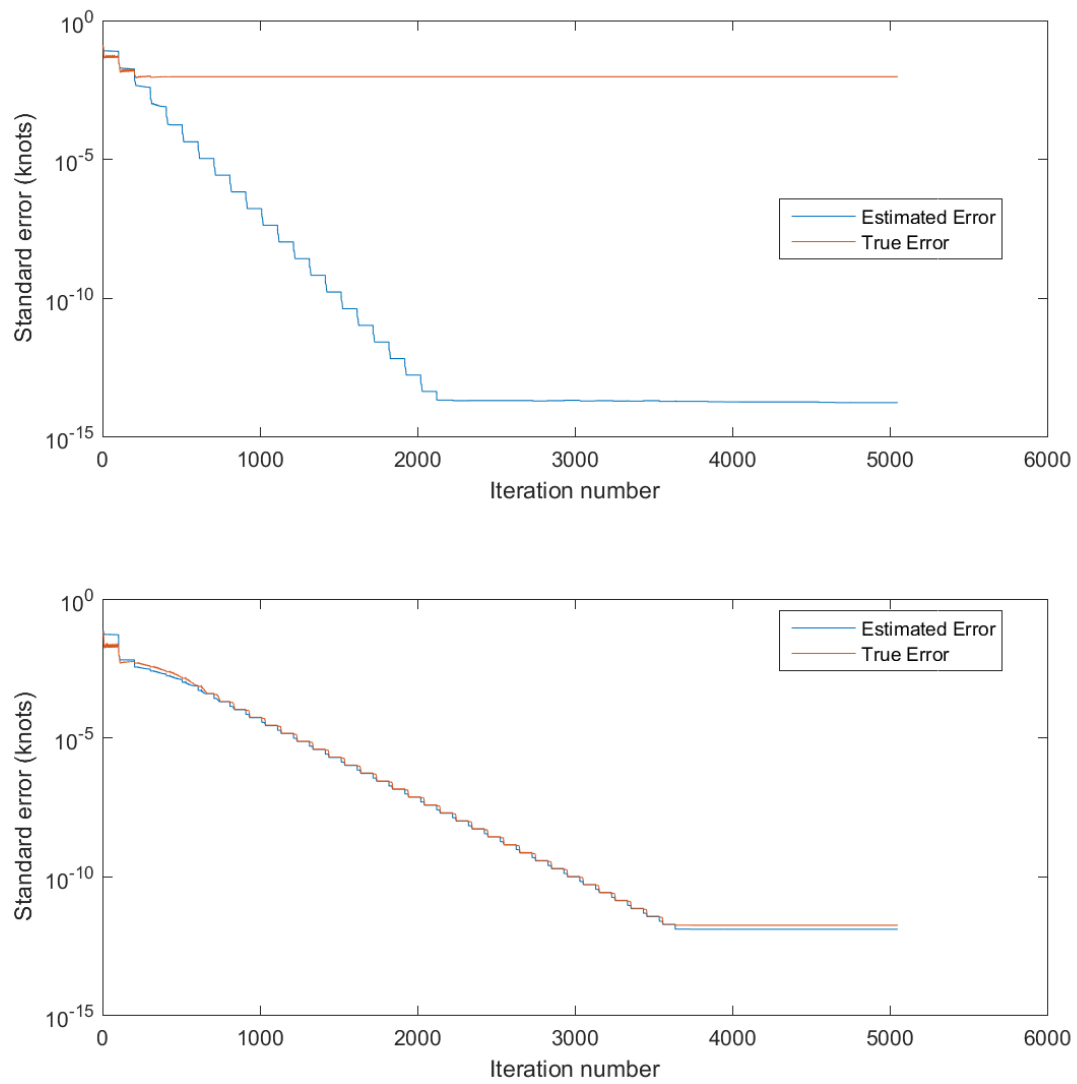


FIGURE 4.4: True standard error and estimated standard error of the ship speed in the measurement points. Top is the case of 3 double runs at 3 power settings, bottom is 4 double runs at 3 power settings.

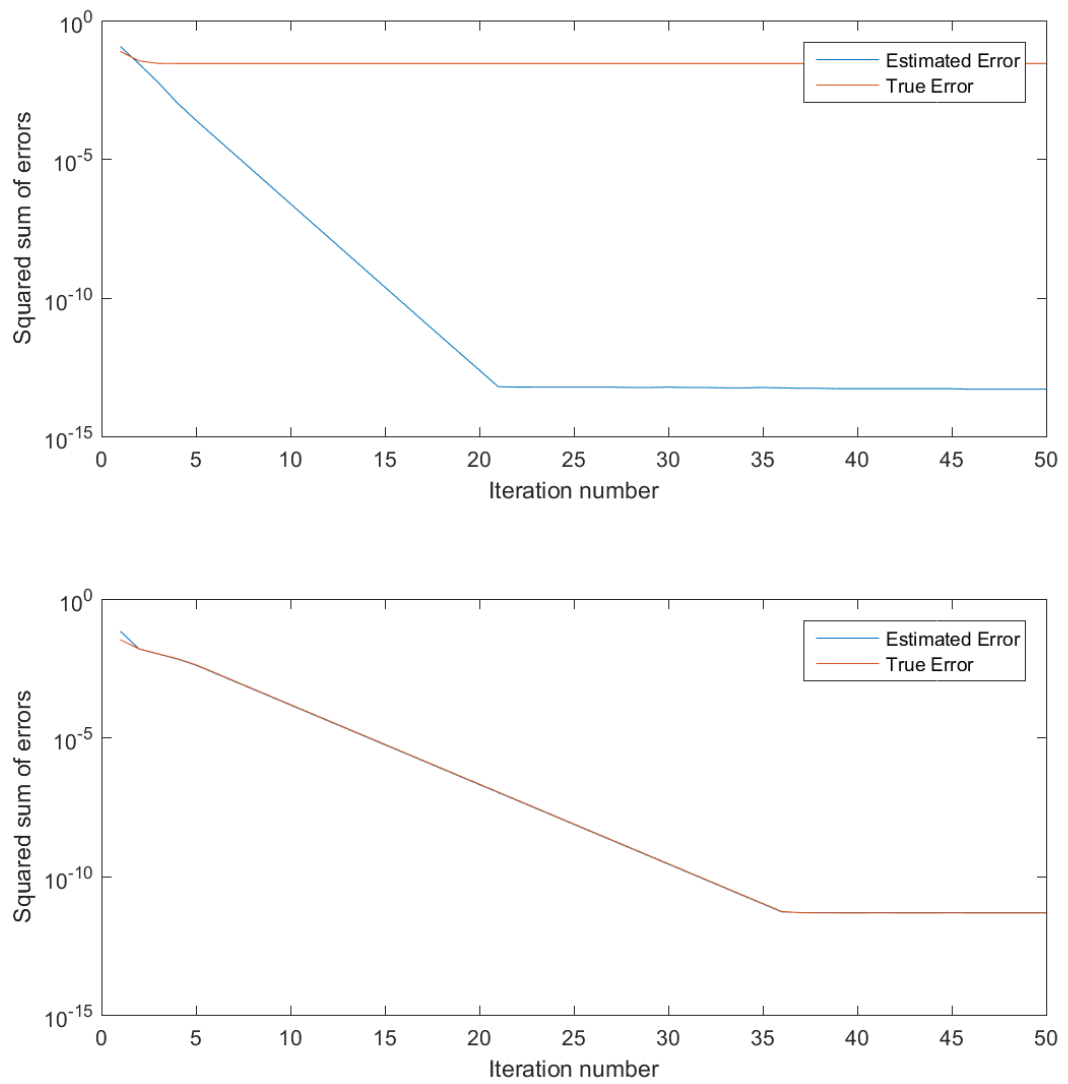


FIGURE 4.5: True standard error and estimated standard error of the current in the measurement points. Top is the case of 3 double runs at 3 power settings, bottom 4 double runs at 3 power settings.

4.5 Performance of the Iterative method with measurement noise

To evaluate the performance of the iterative method it is important to see how it is affected by measurement noise. In order to achieve this artificial noise is added to the three "measured" parameters of the Iterative method: The measured engine power, the run time and the ships ground velocity.

Noise was applied by the following procedure:

- As described in ISO15016 the engine power can be measured with a maximum error of 2%. The power settings were given 2% normal distributed noise levels.
- The ships ground speed is measured with a maximum error of 0.05 knots. This error is also assumed to be normally distributed.
- The measurement time is assumed to be normally distributed with an error of up to 0.66% of a measured run of 1.5 hours, or 36 seconds.

For generating the noise, the normal distribution was taken to have a mean of 0 and a standard deviation σ of one third of the maximum error. This ensures that 99.73% of generated errors lie within the maximum error range.

4.5.1 Powersettings comparison

From the simulation tests at without noise it was concluded that 3 double runs at 3 power settings was insufficient for the iterative method to find the parameters of the current and V-P function. 3 double runs corresponds to 6 current velocity measurements and 6 ship velocity measurements. The question is then whether an extra current measurement is needed or another power setting.

In order to test this, a comparison was made. The first tested case was 4 double runs at 4 different power settings. The second was 4 double runs at 3 different power settings. Of the 4 double runs at 3 different power settings, 2 double runs were at contract speed.

The noise levels are implemented as described in the preceding subsection, and the method calculated the ship speed 1000 times. The resulting histogram of the errors in the measurement points can be seen in figures 4.6 and 4.7.

The errors in the figures are defined as the difference between the true speed of the ship relative to the water and the speed of the ship calculated by the Iterative method during the double runs of the speed trial.

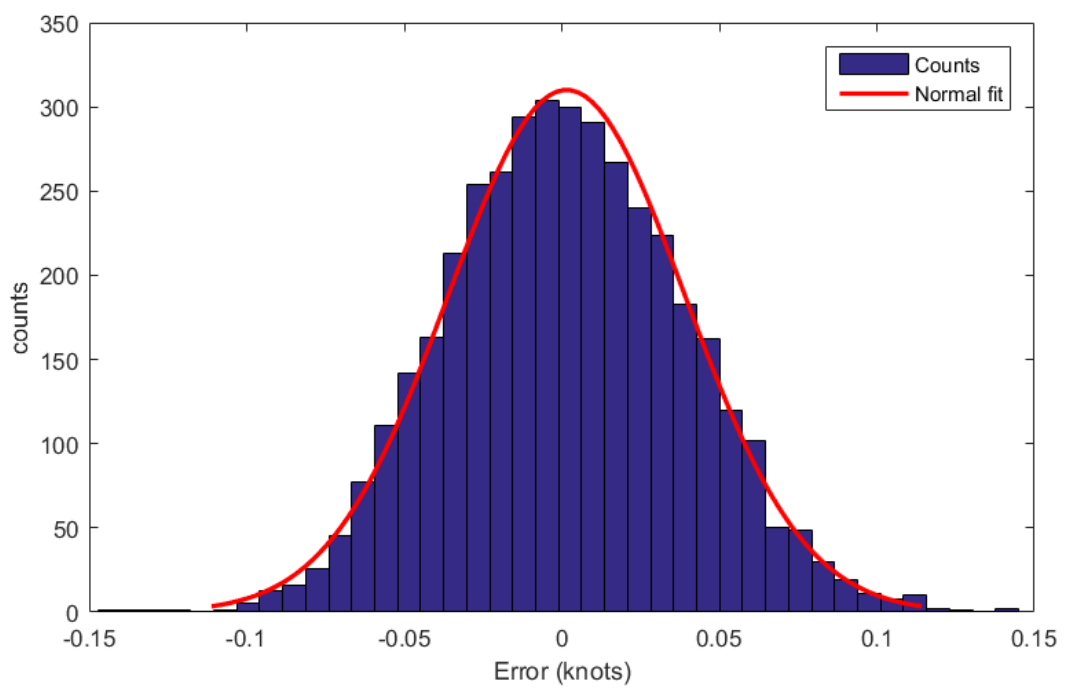


FIGURE 4.6: Histogram of errors in the calculated ship velocity of 1000 calculations with 4 double runs at 4 power settings. This provides a total of 4000 data points.

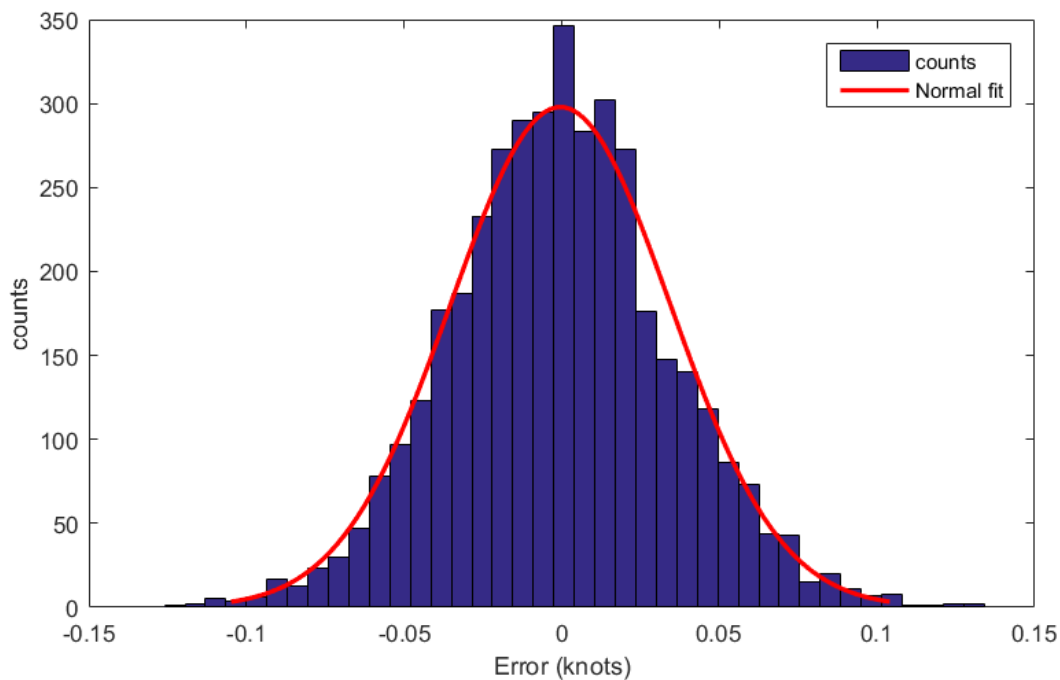


FIGURE 4.7: Histogram of errors in the calculated ship velocity of 1000 calculations with 4 double runs at 3 power settings.

In figures 4.6 and 4.7 the different histograms for the different power settings are presented. The blue bars labeled 'counts' indicate how many of the observed errors lie within the range of the bar. 1000 calculations of four double runs yields 4000 total counts. The red line labeled 'normal fit' indicates a fit of the normal distribution to the data of the histogram.

The distributions look fairly similar, the fitted mean μ and standard deviation σ can be observed in table 4.4.

In evaluating the performance of the Iterative method based on these histograms, it is important to look at the maximum error the method makes. The simulated ship is assumed to have a grace margin of 0.1 knot. This means that as long as the method is guaranteed to make an error smaller than this, there is no chance of making a wrong assessment of a ship that reaches its contract speed.

For 4 double runs at 3 power settings, the number of errors larger than the grace margin was 27/4000.

For 4 double runs at 4 power settings, the number of errors larger than the grace margin was 31/4000.

Since the parameters for the two distributions as seen in table 4.4 are comparable and the amount of errors greater than the grace is comparable, it can be concluded that there is no difference in performance for 4 double runs at either 3 or 4 power settings at these noise levels.

TABLE 4.4: Mean μ and standard deviation σ of the distribution fits of histograms 4.7 and 4.6

Power settings:	3	4
μ :	$-4.3e^{-4}$	$1.5e^{-3}$
σ :	0.0348	0.0375

4.6 Current approximations

One of the goals of this study was to evaluate the performance of the Iterative method. A part of this is to compare the Iterative method with the traditional Mean of Means method. In order to do this, first the current approximation the Mean of means method makes is examined, followed by an error analysis in section 4.7.

The criteria for perfect performance of the Mean of means method are that the current is a polynomial in time of degree equal to the amount of double runs performed. Thus, a good way of evaluating the performance of the method is to investigate the validity of the criteria.

As proven in chapter 2 the mean of means method is exact if the measurements follow a current function which is a 2nd or 3rd order polynomial in time, for 2 and 3 measured double runs, respectively.

Example current have been approximated with a 2nd and 3rd degree polynomial in time for a full and half period. Two different approximations were made, a Lagrange approximation and a least squares approximation. The results can be observed in figures 4.8 to 4.13.

The Lagrange approximation is explained in chapter 2 section 2.5.1.

The method for least squares fitting is explained in chapter 3 section 3.3.

The results can be seen in figures 4.8 to 4.13.

When comparing figures 4.8 and 4.9 the second and third order approximations seem to have comparable errors.

However, when comparing figures 4.10 and 4.11 it can be seen that the third order approximation makes a small error but the second order a large one, illustrating that the measured interval has a large influence on the accuracy of the approximation.

From the figures it is concluded that a third order polynomial works very well when approximating the current over half a period, but the errors can be large when a full period is used.

This means that if single power setting has three double runs they should be conducted in less than 6.2 hours to ensure optimal results.

The second order approximation was found to be accurate over a quarter period, but large errors start to occur if half a period is used. Since the current period is 12.42 hours, this means that if a single power setting has two double runs they should be conducted in less than 3.105 hours to ensure optimal results.

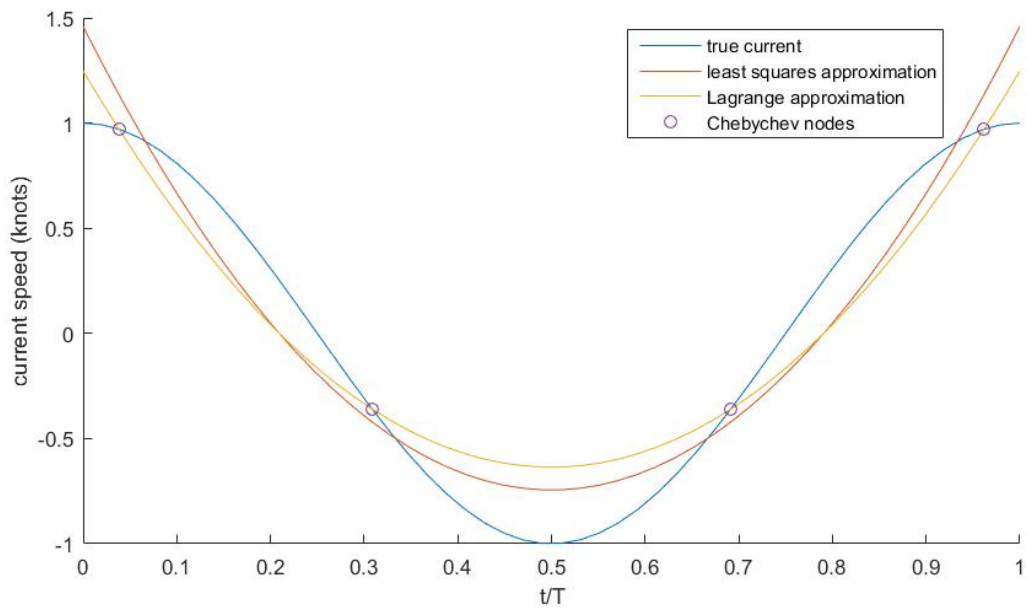


FIGURE 4.8: True current and 3rd order approximations for a full current period

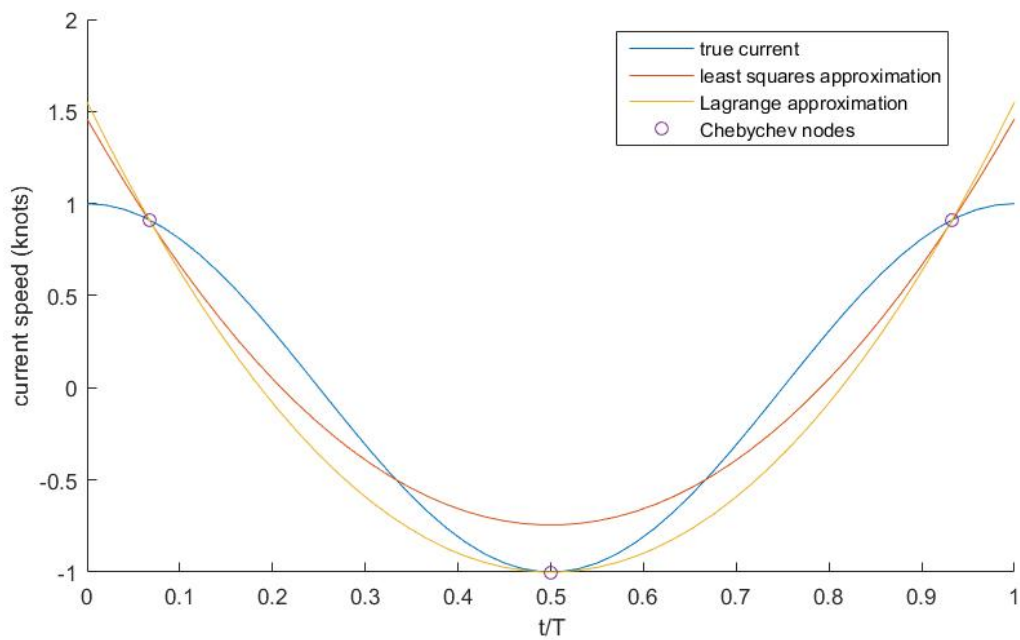


FIGURE 4.9: True current and 2nd order approximations for a full period

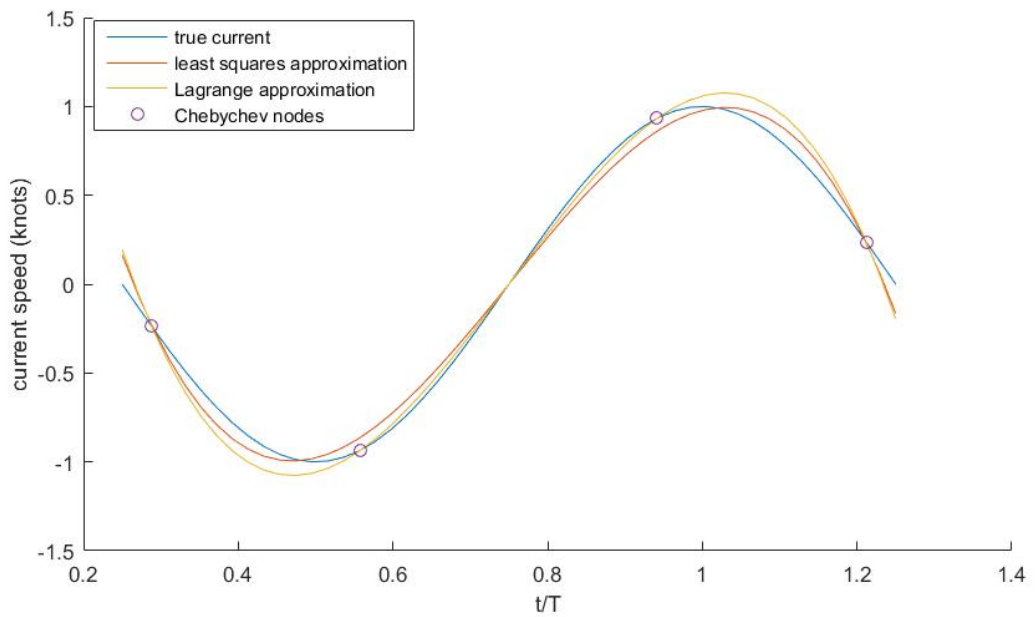


FIGURE 4.10: True current and 3rd order approximations for a full period

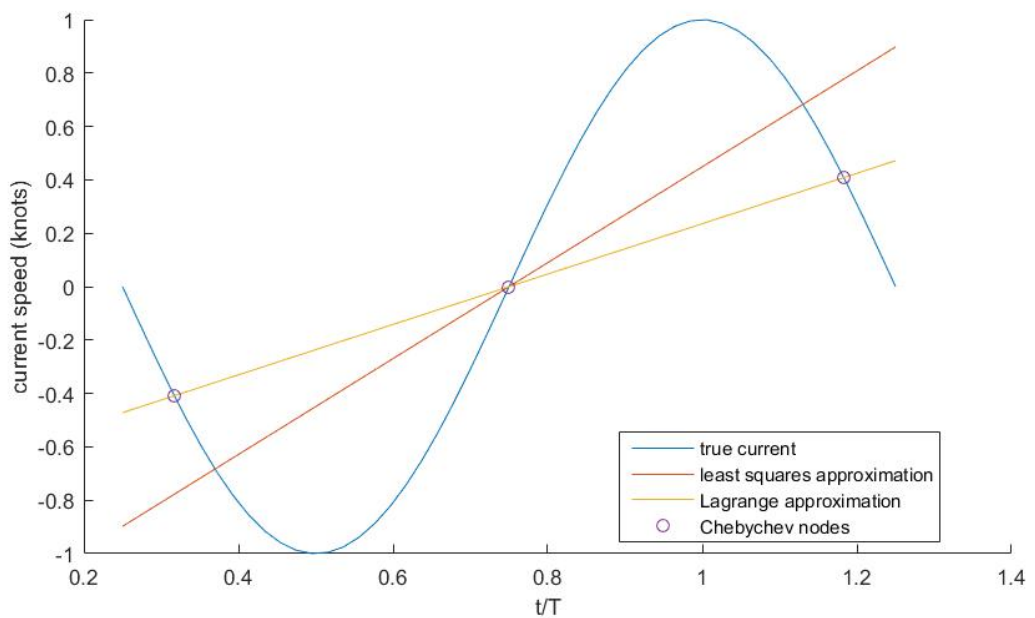


FIGURE 4.11: True current and 2nd order approximations for a full period

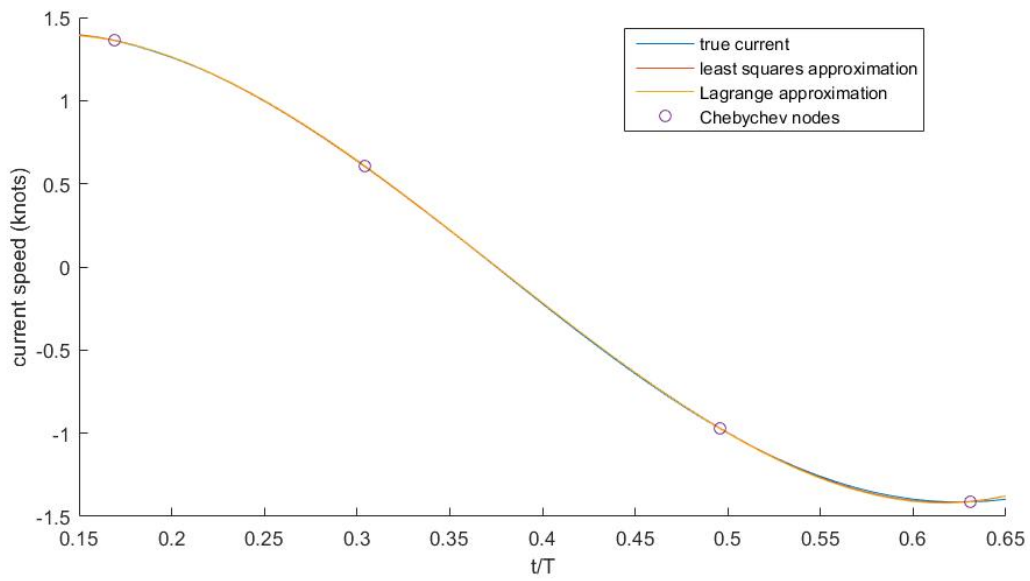


FIGURE 4.12: True current and 3rd order approximations for half a current period

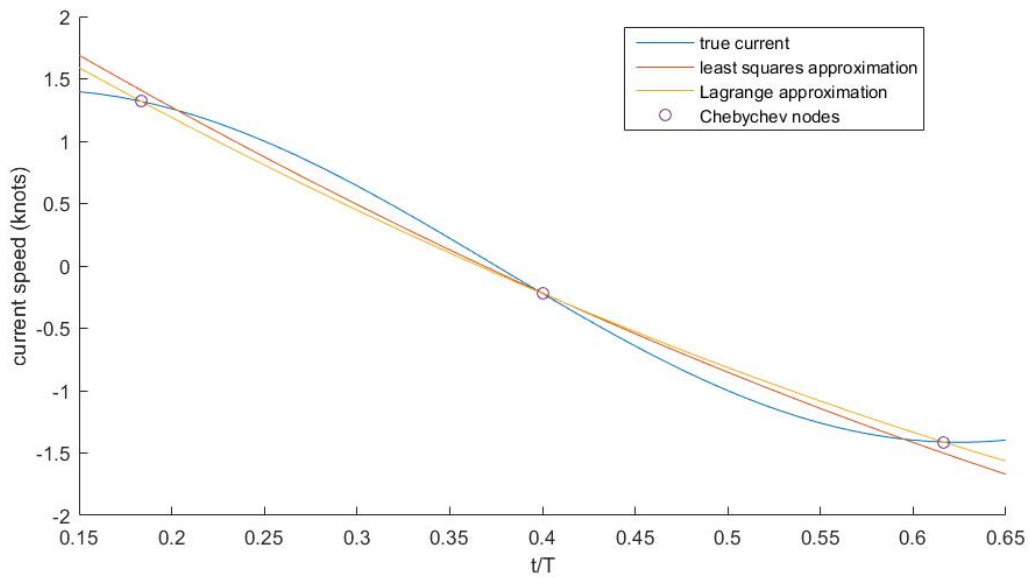


FIGURE 4.13: True current and 2nd order approximation for half a current period.

4.6.1 The Lagrange remainder

In chapter 2 section 2.5.2 the upper bounds for the Lagrange remainder were derived:

$$|E_2(x)| \leq \frac{4\pi^3(b-a)^3}{3T^3} \sqrt{(A^2 + B^2)}$$

$$|E_3(x)| \leq \frac{2\pi^4(b-a)^4}{3T^4} \sqrt{(A^2 + B^2)}$$

Inserting the parameters used for the approximations in figures 4.10 and 4.11 gives values between 6 and 30, which is a gross overestimation of the real error and thus not useful for assessing the approximations.

4.7 Performance of the Mean of means method with measurement noise

In order to compare the Iterative method with the Mean of means method, a noise analysis of the Mean of means method has been performed. The Mean of Means method and the Iterative method require different inputs however, so a direct input-output comparison was not possible. The noise levels, run times and current function were kept the same as in the analysis of the iterative method done in section 4.5.1.

In figures 4.15 to 4.17 error histograms can be seen of 1000 results of the Mean of means method. They are accompanied by plots of the current approximations made by the method to illustrate the relationship between this approximation and the errors. The error is defined as the difference in the true ship's speed and the ship's speed calculated by the Mean of means method during the double runs of the speed trial.

In the previous section it became clear that the accuracy of the current approximations used for the Mean of means method is influenced by the interval over which the measurements are done. In the case of the second order approximation, whose measurements consisted of two double runs, there was a lot of variation in the errors, of which the most striking cases are presented.

The worst case and best case scenarios of a two double run case are displayed in figures 4.15 and 4.14. The three double run case was very consistent over all intervals chosen, an average example can be seen in figure 4.16.

When examining figures 4.15 to 4.16 it is clear that in all cases the maximum error is smaller than the simulated grace margin of 0.1 knots. Another observation is that the maximum error made by the mean of means method in these cases is smaller than of the iterative method, seen in section 4.5.1. In some of the figures the error distribution is not centered around 0. The errors are the result of both noise and systematic errors. It is reasonable to believe that the spread is the result of the noise, while the offset is the result of the systematic error in the current approximation.

The duration of the runs has been found to have a large impact. For example when examining figure 4.17 where a double run was assumed to take 4.5 hours instead of the 3 used in figures 4.15 to 4.16, the error was significantly larger. The errors were centered around 0.12 knots with a spread up to 0.15. This is much higher than the

case of the iterative method in section 4.5.1.

From the results can be concluded that the mean of means method can have a lower error than the iterative method at these noise levels, but that it is dependent on the measured interval of the current function and the length of each run.

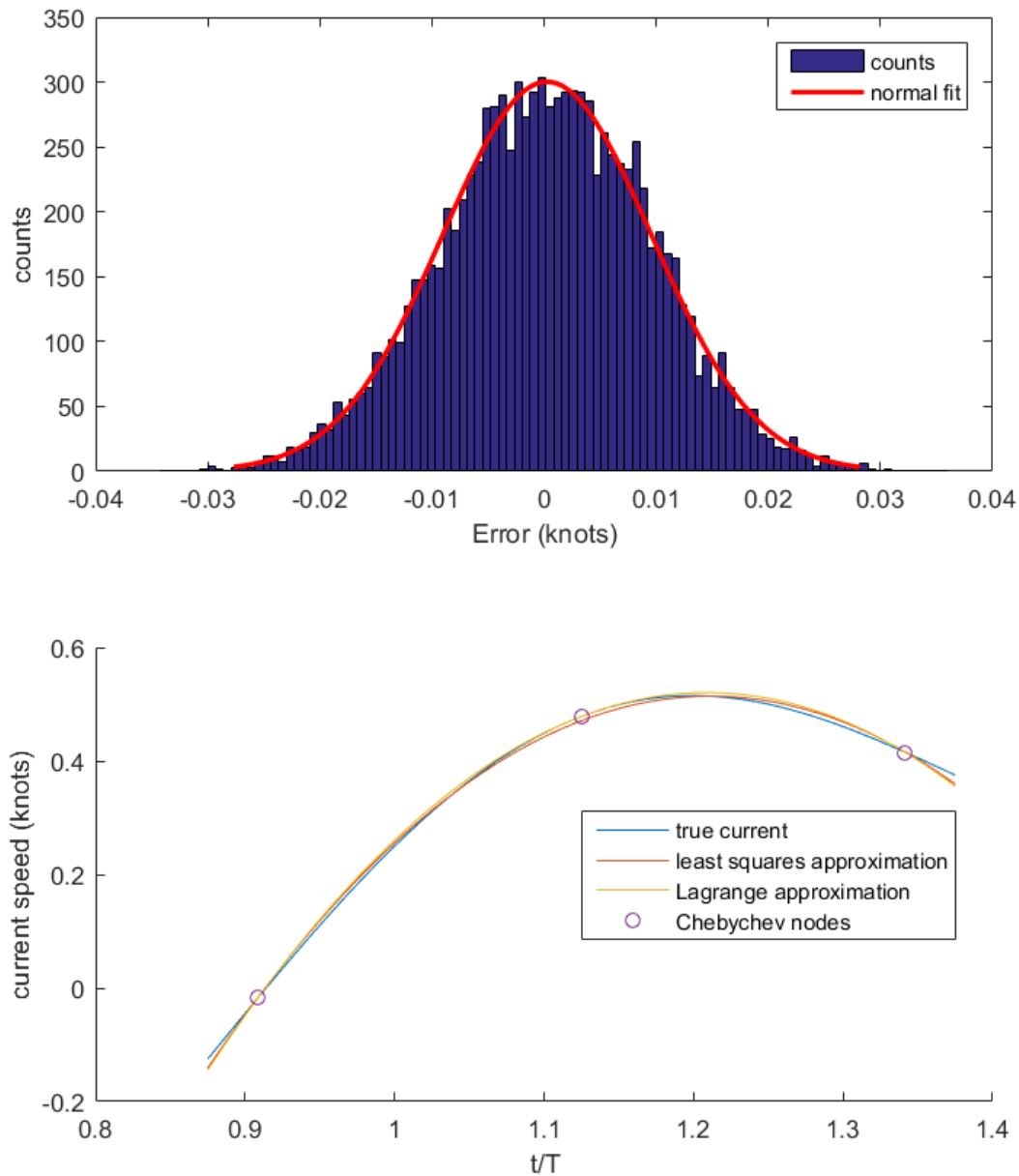


FIGURE 4.14: First error histogram and 2nd order current approximation for the Mean of Means method performed with 2 double runs of 3 hours per double run

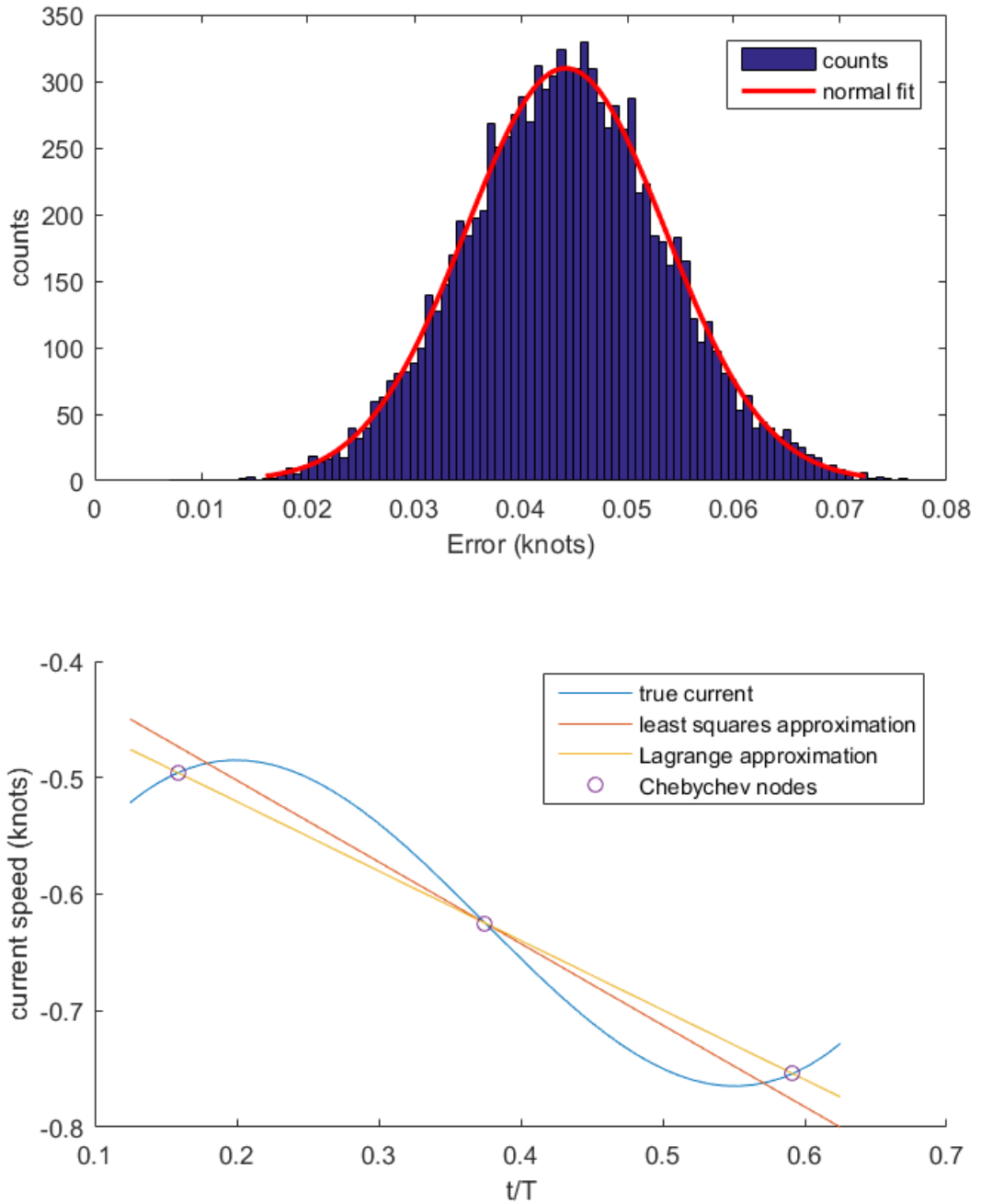


FIGURE 4.15: Second Error histogram and 2nd order current approximation for the Mean of Means method performed with 2 double runs of 3 hours per double run

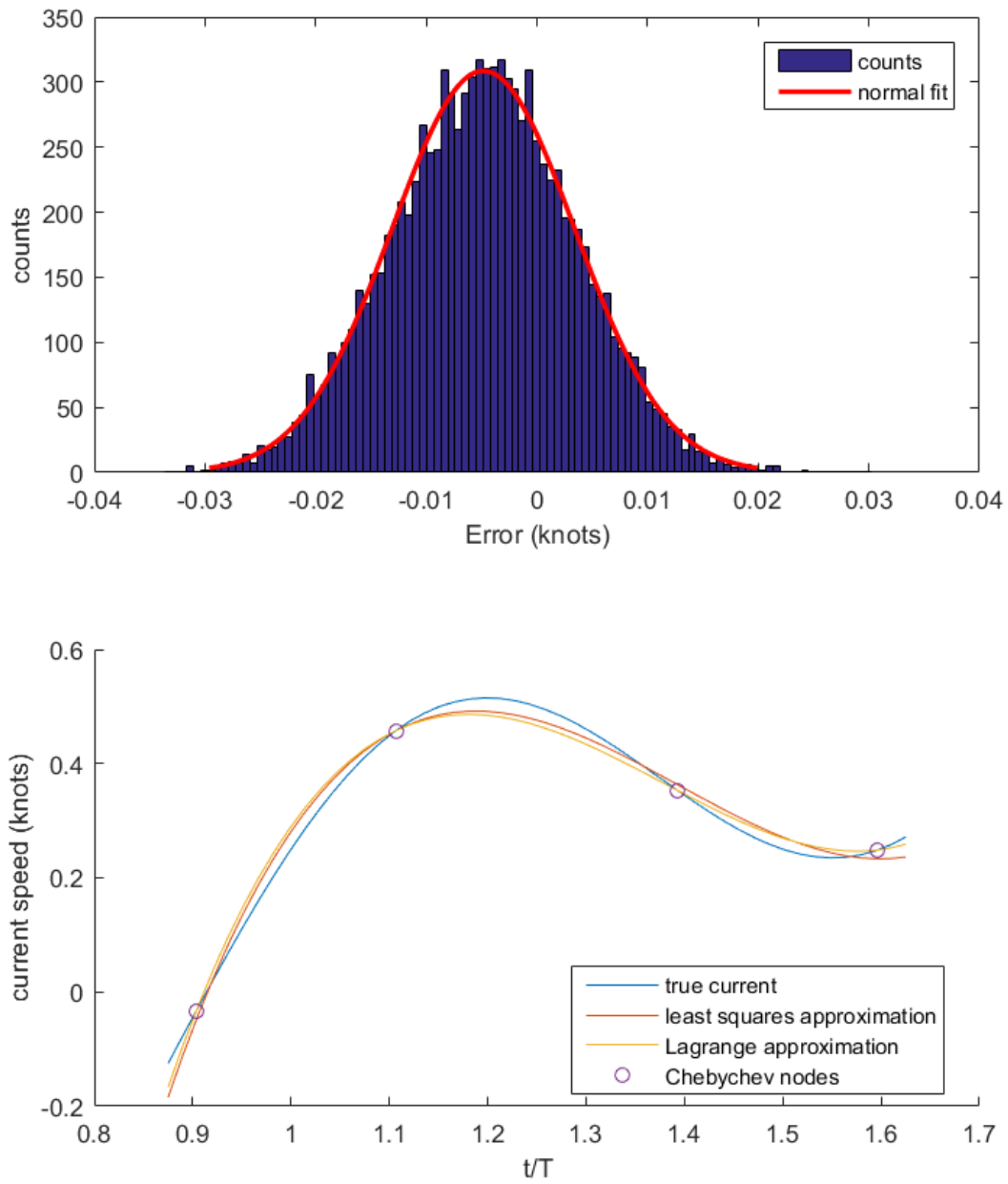


FIGURE 4.16: Error histogram and 3rd order current approximation for the Mean of Means method performed with 3 double runs of 3 hours per double run

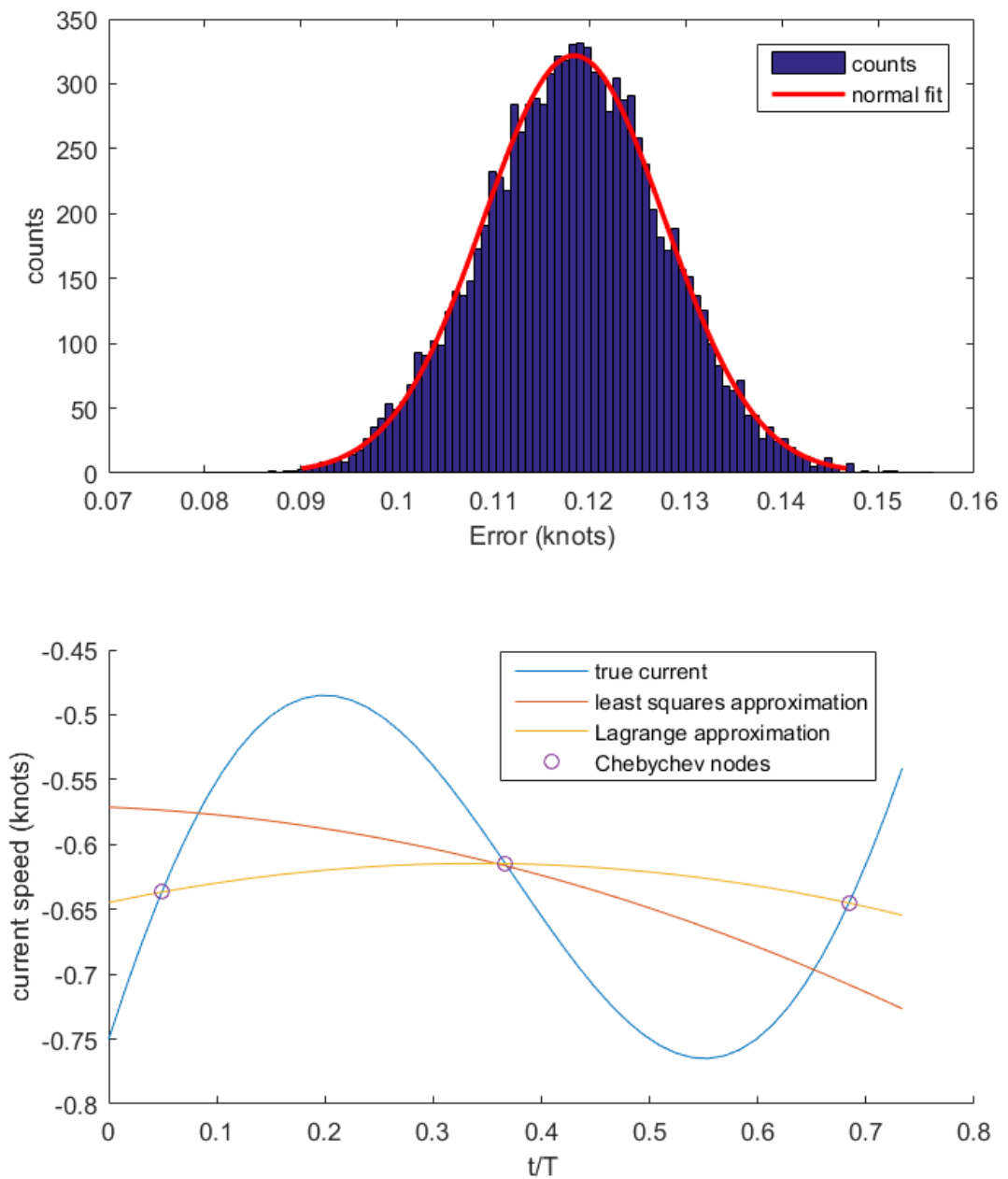


FIGURE 4.17: Error histogram and 2nd order current approximation for the Mean of Means method performed with 2 double runs of 4.5 hours per double run

Chapter 5

Conclusions and discussion

The main purpose of this research is to investigate the behavior of the Iterative method, which is used to eliminate the effects of current during a ship's speed trial. The main research questions consisted of:

1. What are the causes and possible solutions for the various problems users of the method have reported?
2. Is the Iterative method able to exactly find the input parameters using noiseless measurement data, and if so under what conditions?
3. How is the performance of the Iterative method affected by measurement noise?
4. How does the performance of the Iterative method measure up against that of the Mean of means method?

All these questions are important to be able to determine whether the Iterative method is fit to be used for sea trial analysis in its current form.

5.1 Causes and solutions for known problems.

5.1.1 The current function

One reported problem users of the iterative method have experienced was inaccuracy in the estimation of the parameter C of the current function by the iterative method.

$$V_c(t) = A \cos\left(\frac{2\pi}{T_c}t\right) + B \sin\left(\frac{2\pi}{T_c}t\right) + C t + D$$

The first observation was that in its current form not all parameters have the same units. A , B and D have unit m/s , and C has unit m . While this in itself not a problem, it causes the magnitude of the parameter C to be dependent on the chosen units in which time is expressed.

This causes the order of magnitude of the parameter C to be able to differ greatly from the other three, which are all more or less of equal order of magnitude. The conclusion is that the most likely cause for the inaccuracy in estimation of the parameter C is the fact that its order of magnitude can differ greatly from the other three parameters while being estimated by the same process.

The proposed and implemented solution for this is to normalize the values of t by replacing the term $C t$ of the current function with $C \frac{t}{T_C}$. This solved the inaccuracy problems as far as observed.

5.1.2 Convergence issues

A second reported problem was that the Iterative method sometimes failed to provide a solution. This same phenomenon was encountered during implementation as part of this research, and upon closer inspection this occurred whenever the value of the parameter b of the Power-Velocity function $V_s = \left(\frac{P(V_s)-a}{b}\right)^{\frac{1}{q}}$ becomes very small. It emerged that the value of the derivative of the function with respect to b becomes too large for a computer to work with, resulting in termination of the program. The proposed and implemented solution is to limit the values of b to 10^{-12} eliminated the issue without any side effects.

Finally another problem with the iterative method is that the the V-P function found by the method sometimes diverged. This means that the parameters of the function $P(V_s) = a + b \cdot V_s^q$ the method found were such that they could not be physically possible. The proposed solution is to limit the minimum and maximum values the parameters could attain during the iterative process. This proved to solve the problem, but the method failed to converge to the input parameters in the case of noiseless measurements.

The conclusion is that for both convergence issues, non-convergence or convergence to values outside of the physically acceptable range, limiting the parameters proved to solve the problem. In the case where parameters are limited to physically acceptable values, the method failed to find the input parameters exactly.

The question remains on whether this is something that should be implemented. One could argue that as long as the quality of the solution is not influenced by the limitations it is acceptable to implement them.

It has been observed that limiting the parameters does indeed have a negative influence on the accuracy of the method, even though the global minimum of the error function lies within the limitations.

The cause for this was theorized to be that the method needs to temporarily move its solution outside of the 'allowed' parameter space in order to reenter later from a better angle.

Based on these results it is recommended not to limit the parameters of the V-P function of the Iterative method to a degree where it influences the accuracy of the method. To accurately determine where exactly this occurs, further research is required.

5.1.3 Local minima in the Levenberg-Marquardt algorithm

The Levenberg-Marquardt algorithm that is part of the Iterative method finds a local minimum of a given function. In the case of the Iterative method, it finds a local minimum of the power error, the difference between measured engine power and calculated power.

The algorithm does not ensure convergence to a global minimum of this error function, as the Levenberg-Marquardt is not able to take steps that increase the error function. But in order to go from a local minimum to a global minimum, the error must momentarily increase.

A possibility for further research is to use a least squares problem solver that is guaranteed to converge to a global minimum for the iterative method. For example a method

called Simulated Annealing always finds a global optimum, but is very computationally intensive, which might make practical use infeasible.

5.2 Performance of the Iterative method

The investigation of the properties of the iterative method started with determining its behavior in an ideal test case, where the measurements have no measurement noise. In section 4.4 the results are presented, and it is concluded that the method requires at least 4 double runs during a speed trial in order to exactly find the ship's speed relative to the water.

Another property that was investigated was the performance of the method for 4 double runs at 3 or 4 power settings, respectively.

The results are presented in section 4.5.1, and show that in this case assuming a realistic current with realistic noise levels, the method is able to calculate the ship's speed relative to the water with an error that had a mean of almost zero in both cases and a standard deviation of between 0.035 and 0.0375 knots.

The amount of errors larger than the grace margin of 0.1 knot was approximately 0.75% in both cases.

Based on these results the conclusion is that there is no significant difference in performance of the iterative method between 3 and 4 power settings for 4 double runs. The errors made in both cases lie within the grace margin, ensuring that no ship which reaches its contract speed will be found to be outside of the grace margin during its speed trial.

5.2.1 Validity of the assumptions of the Iterative method

As described in chapter 2, the Iterative method assumes two things:

1. The relationship between Power and Velocity of a ship is of the form $P = a + b V^q$
2. The current experienced by the ship during a sea trial is of the form $V_c(t) = A \cos\left(\frac{2\pi}{T_c}t\right) + B \sin\left(\frac{2\pi}{T_c}t\right) + C \frac{t}{T_c} + D$

The method was simulated and tested under circumstances where both the ship's behavior and the current followed those relations exactly. However, from the literature study in chapter 1 it can already be concluded that this can not hold exactly. In section 1.6 was already derived that contribution of the skin friction is proportional to the ship's speed to the power of 2.8.

Also derived was that the wave making resistance could effectively be modeled as proportional to the velocity to the 6th power. The relationship assumed by the iterative method may be a good approximation of ship behavior, in this study is concluded that it is not exact.

The current is also known not to be of the form assumed in the iterative method. The function assumed is a good estimator of the macroscopic behavior of the current, but locally small irregularities have been observed that are not accounted for in the

current function assumed by the iterative method.

If the sea trial duration is longer than 12.5 hours, the assumed period of the current function, the Iterative method is no longer reliable. The assumed current function is only an approximation of the real current for a timeframe of 12.5 hours. For example the current velocity may steadily rise for one period and then proceed to steadily decrease over the next period. This behaviour is not accounted for by the current function, which assumes a constant gain of the current velocity.

In this study the accuracy of the current approximation compared to actual measured sea currents has not been evaluated. This is needed in order to assess whether the iterative method is viable when the measurements are taken over a period longer than 12.5 hours.

The results of this study are only valid under the assumption that in a sea trial the assumed V-P and current functions are an accurate representation of reality. Future research may be needed to determine the possible effects on the Iterative method when this is not the case.

5.3 Performance of the Mean of means method

In order to compare performance of the Mean of means method with the Iterative method, the performance of the Mean of means method has to be evaluated.

From literature is known that the Mean of means method is exact if the current function can be approximated by a polynomial whose degree is equal to the amount of double runs performed during the speed trial. Part of this study was to examine how good those approximations are when measurements are done over various time intervals. The results are presented in section 2.5.1.

The conclusion is that the validity of the approximation depends greatly on the length and phase of the approximation interval. It was found that a third order approximation is viable as long as there are no two maxima of the current function over the approximation interval. This means that as long as three double runs are measured over less than 12.5 hours, the quality of the approximation is acceptable.

It was found that a second order approximation was acceptable as long as the current is parabolic in shape. This was found to be true for up to half a period, approximately 6.3 hours.

A second part of the evaluation of the Mean of means method is to examine the errors made by the Mean of means method using the same circumstances as simulated during the testing of the Iterative method which is presented in section 4.5.1. The results of this are presented in section 4.7.

The Mean of means method made an error smaller than that of the iterative method in all recorded cases if the length between two consecutive runs was 1.5 hours. However if the length was increased to 2.2 hours, the Mean of means method made a larger error than the Iterative method.

From the results can be concluded that the performance of the method depends greatly on the length of the measuring interval and the phase of the current function during this interval. This is consistent with conclusion of section 5.2.1.

5.4 Summary of main conclusions

The most important conclusions that are drawn from this research are:

1. For both consistency and performance, the current function used for calculations by the Iterative method should be adjusted by replacing the term $C t$ with $C \frac{t}{T_C}$. This is discussed in greater detail in section 5.1.1.
2. The Iterative method requires at least 4 double runs to properly find the V-P function it calculates. This is discussed in greater detail in section 5.2.
3. When using the Iterative method with 4 double runs, there is no significant difference in performance between 4 double runs at 4 power settings, and 4 double runs at 3 power settings. This is discussed in greater detail in section 4.5.1.
4. Limiting the parameters of the V-P function during the iterative process ensures that the method finds a physically acceptable function, but negatively impacts the accuracy of the method. This is discussed in greater detail in section 5.1.2.
5. In the case of two double runs per power setting, if the time between two consecutive double runs is less than 1.5 hours, the Mean of means method has acceptable results. In the case of three double runs per power setting, the time between two double runs must be less than 2.1 hours for acceptable results. If the time between two consecutive double runs is higher than these values under these circumstances, the Iterative method will provide better results. This is discussed in greater detail in section 5.3.

Appendix A

Matlab implementation of the Iterative method

```
T = 1; %Current period
Pi0 = [15000,18000,18000,21000]; %Power settings of simulated ship

%% Functions
%Current functions
syms VCC VCS VCT VC0 t t2 VC %create symbolic variables
CP = [VCC VCS VCT VC0];
CP2 = [VCC VCS VCT];
dataL = length(Pi0);
VC = sym('VC', [1 2*dataL]);
t = sym('t', [1 2*dataL]);
Cur = VCC*cos(2*pi*t/T) + VCS*sin(2*pi*t/T) + VCT*t + VC0;
fc = matlabFunction(Cur, 'vars', {CP,t});

%Error function
E2 = VC - fc(CP,t);
ch2 = matlabFunction(E2, 'vars', {CP,VC,t});

%Jacobian
J2 = jacobian(fc(CP,t), CP);
ch3 = matlabFunction(J2, 'vars', {CP,t});

%Power-Velocity functions
X = sym('X', [1 dataL]);
Y = sym('Y', [1 dataL]);
B = sym('B', [1 3]);

g = ((Y-B(1))/B(2)).^(1/B(3));
fh = matlabFunction(g, 'vars', {B,Y});

g2 = B(1) + B(2).*Y.^B(3);
fh7 = matlabFunction(g2, 'vars', {B,Y});

%Error
E4 = X - fh7(B,Y);
fh6 = matlabFunction(E4, 'vars', {B,Y,X});

E = X - fh(B,Y);
fh2 = matlabFunction(E, 'vars', {B,X,Y});

%Jacobiaan
J = jacobian(fh(B,Y), B);
fh3 = matlabFunction(J, 'vars', {B,Y});
%% Parameters
```

```

Btrue = [100,5,3.2];%Parameters of the V-P relation of the ship

Bt = [0,3,3]; %Initial guess of the parameters

B_max = [3000 1000 4.2];%Maximum values of parameters

B_min = [0 0.001 2.8];%Minimum values of parameters

Vtrue =fh(Btrue,Pi0);%Generate ship speeds

Lp = 3; Lm = 2; L = 1;%Parameters for Lambda (used in Levenberg-Marquardt algorithm)

CPtrue = [0.25 0.25 1 -1];%Current parameters

CPt = [1 1 1 1];%Initial guess CP

timer = true;
noise = false; %enables or disables measurement noise
noiseV = 0; noiseP = 0; noiseT = 0;

if noise == true
noiseP = 0.02;
noiseT = 0.01;
noiseV = 0.05;
end

Paralim = true;%enables or disables parameter limiting

imax = 100;%maximum values per iterative step
kmax =50;
imax2 = 100;

autosave = false;

%% Data generation

for k = 1:2*dataL
tjkd(k) = k*3/24;
end

Vsh = Vtrue + calcC(CPtrue,tjkd(2:2:2*dataL)); %generate measurement values
Vsl = Vtrue - calcC(CPtrue,tjkd(1:2:(2*dataL-1)));

th= tjkd(2:2:2*dataL);
tl= tjkd(1:2:(2*dataL-1));

tcur = [th tl];

%add noise
for i = 1:length(Vsh)
Vsh(i) =Vsh(i)+ (0.5-rand)*noiseV ;
Vsl(i) =Vsl(i)+ (0.5-rand)*noiseV ;
end

for i = 1:dataL
Pim = Pi0 + (0.5-rand)*noiseP*Pi0(i) ;
end

for k = 1:length(tcur)

```



```

t2(k) = tcur(k) + (0.5-rand)*noiseT*tcur(k);
end

Vs = (Vsh + Vs1)./2;%Calculate mean velocity
Vctrue = fc(CPTrue,tcur);%Calculate true current velocity

%% Iterative process

%% Step 1:Levenberg-marquardt iteration process for finding an initial P-V relation
Bt2 = zeros(imax,3);
Bt2(1,:) = Bt;
Et = zeros(imax,dataL);
Er = zeros(imax,1);
if timer == true
tic
end
for i = 1:imax

Jt = fh3(Bt2(i,:),Pim);%Calculate Jacobian of the parameters
JtJ = Jt.'*Jt;

%Update Parameters according to Levenberg-Marquardt method
Bt2(i+1,:) = Bt2(i,:) + (fh2(Bt2(i,:),Vs,Pim)*Jt)/(JtJ+L*diag(diag(JtJ)));
if Paralim == true %Check if parameters are limited
Bt2(i+1,:) = min(max(B_min,Bt2(i+1,:)),B_max);
else
Bt2(i+1,2) = max(Bt2(i+1,2),10^(-12));
end

%Error vector
Et(i,:) = fh2(Bt2(i,:),Vs,Pim);

%Squared errors
Er(i) = sum((Et(i,:).^2));
if i > 1
if Er(i) < Er(i-1) %If the error becomes smaller, update the parameter L
L = L/Lm;
else
Bt2(i+1,:) = Bt2(i,:); %if the error is not smaller, discard the step and adjust L
L = L*lp;
end
end
end
if timer == true
toc
end

%% Step 2: initial calculation of current velocity

Vc = [(Vsh - Vs) (Vs-Vs1)];%Estimation current speeds

cp = zeros(imax,4);
cp(1,:) = CPt;
Et2 = zeros(imax,length(t2));
Er2 = zeros(imax,1);

Ac = ch3(CP,t2);
cp(end,:) = ((Ac.'*Ac)\Ac.'*Vc.').';%linear least squares solution of the current function

Vccalc0 = fc(cp(end,:),t2);

```

```

%% Redefine Jacobian
X1 = sym('X1', [1 2*dataL]);
Y1 = sym('Y1', [1 2*dataL]);
g2 = B(1) + B(2).*Y1.^B(3);
fh4 = matlabFunction(g2, 'vars', {B, Y1});
%Jacobian
J3 = jacobian(fh4(B, Y1), B);
fh5 = matlabFunction(J3, 'vars', {B, Y1});

%% Step 3: Start the iterative process
Pim2 = [Pim Pim];

%Ship V-P relation least squares fit
VsIt = zeros(kmax, 2*dataL); Vscal = zeros(kmax, dataL); VcIt = VsIt; Vccalc = zeros(kmax, 2*dataL);
%Starting values
Vccalc(1, :) = Vccalc0;
VcIt(1, :) = Vc;

%declare variables
Et2it = zeros(2*dataL, kmax); TrueEC = Et2it; Etit = zeros(imax2, 2*dataL, kmax); TrueE = zeros(imax2, 2*dataL, kmax);
Bt3 = zeros(imax2, 3, kmax); cp2 = zeros(4, kmax); Erit = zeros(kmax, imax2); TrueEsq = zeros(kmax, imax2);

%start with parameters from initial guess phase
cp2(:, 1) = cp(end, :);
Bt3(1, :, 1) = Bt2(end, :);

for k = 1:kmax

%resume process with parameters from last iterative round
if k > 1
Bt3(1, :, k) = Bt3(end, :, k-1);
cp2(:, k) = cp2(:, k-1);
end

%Reset convergence status
converged1 = false; converged2 = false;

%Reset counters
i = 1; L = 1;

%New shipspeeds (Vs')
VsIt(k, :) = [Vsh-Vccalc(k, 1:dataL) Vsl+Vccalc(k, dataL+1:2*dataL)];
if timer == true
tic
end
while converged1 == false

Jt2 = fh5(Bt3(i, :, k), VsIt(k, :));
JtJ2 = Jt2.'*Jt2;

%parameter update according to L-M
Bt3(i+1, :, k) = Bt3(i, :, k) + (EP(Bt3(i, :, k), VsIt(k, :), Pim2)*Jt2)/(JtJ2+L*diag(diag(JtJ2)));
if Paralim == true
Bt3(i+1, :, k) = min(max(B_min, Bt3(i+1, :, k)), B_max);
else
Bt3(i+1, 2, k) = max(Bt3(i+1, 2, k), 10^(-12));
if max(Bt3(i+1, 2, k), 10^(-12)) == 10^(-12)
disp('warning: b at limit') %display warning if parameter b reaches its limit
end
end
end

```

```

end

%Error vector
Etit(i, :, k) = EP(Bt3(i+1, :, k), VsIt(k, :), Pim2);
TrueE(i, :, k) = EP(Bt3(i+1, :, k), Vtrue, Pi0);

%Mean Squared relative errors
Erit(k, i) = sum((Etit(i, :, k).^2))/(2*dataL);
Erel(k, i) = sqrt(sum(((Etit(i, :, k)).^2))/(2*dataL));
TrueEsq(k, i) = sqrt(sum(((TrueE(i, :, k)).^2))/dataL);

%check if step was succesful
if i > 1
if Erit(k, i) < Erit(k, i-1)
L = L/Lm;
else
Bt3(i+1, :, k) = Bt3(i, :, k);
L = L*lp;
end
end

%Check if converged
if Erel(k, i) < 10^-14 || i > imax
converged1 = true;
iconv(k) = i;
Erit(k, i+1:end) = Erit(k, i);
TrueEsq(k, i+1:end) = TrueEsq(k, i);
Erit(k, i+1:end) = Erit(k, i);
TrueEsq(k, i+1:end) = TrueEsq(k, i);
Bt3(end, :, k) = Bt3(i+1, :, k);
TrueE(end, :, k) = EV(Bt3(end, :, k), Vtrue, Pi0);
end
if Erel(k, i) < 10^-14 || k == kmax
converged3 = true;
end
i = i+1;

end
if timer == true
toc
end
%reset counters
i = 1;
L = 1;

%New Vs after fitting (Vs)
Vscal(k+1, :) = calcV(Bt3(end, :, k), Pim);
%New current after calculating Vs (Vc')
VcIt(k+1, :) = [Vsh-Vscal(k+1, :) Vscal(k+1, :)-Vsl];

%Current least squares fit
Ac = ch3(CP, t2);
cp2(:, k) = ((Ac.'*Ac)\Ac.'*VcIt(k+1, :)).';

%Error vector
Et2it(:, k) = ch2(cp2(:, k).', VcIt(k+1, :), t2);
TrueEC(:, k) = ch2(cp2(:, k).', Vctrue, tcur);

%Squared errors
Er2it(k) = sum((Et2it(:, k).^2));
Ecrel(k) = sqrt(sum(((Et2it(:, k)).^2))./2*dataL);

```

```
TrueECsq(k) = sqrt(sum((TrueEC(:,k)).^2)/2*dataL);
```

```
% New current speed VC
```

```
Vccalc(k+1,:) = calcC(cp2(:,k).',t2);
```

```
end
```

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