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A real-time holding decision rule accounting for passenger travel cost

G. Laskaris, O. Cats, E. Jenelius, F. Viti

Abstract— **Holding has been extensively investigated as a strategy to mitigate the inherently stochastic nature of public transport operations. Holding focuses on either regulating vehicle headways using a rule-based approach or minimizing passenger travel cost by employing optimization models. This paper introduces a holding decision rule that explicitly addresses passenger travel cost. The decision to hold relies on the passenger demand distribution along the line. The passenger cost holding rule is tested using simulation for a high frequency bus line in Stockholm, Sweden and is compared with a no-control scheme and the currently used headway-based strategy. The results indicate that the new decision rule results in relatively minor reductions of passenger cost compared to the currently adopted strategy, and that it allocates the greatest share of holding time at the beginning of the route.**

I. INTRODUCTION

Public transport services are confronted with high variability in travel time and in passenger demand, which can yield long headways and undesired phenomena such as bus bunching. These cause longer waiting times for the commuters, discomfort and overcrowding at stops and increasing costs due to poor management of the available resources for the operators. In order to respond to the inherent stochastic nature of public transport operations, operators can utilize advanced public transport systems (APTS) to maintain regularity and minimize operational costs via control strategies that rely on real time information. Real time information availability from technologies such as automatic passenger counts (APC) and automatic vehicle location (AVL) give the capability of reacting dynamically to disturbances in travel time or passenger demand.

A control strategy thoroughly investigated in the literature and commonly used in practice is holding. A vehicle is held at a stop for a certain amount of time according to different criteria and characteristics of the line. For instance, for high frequency services with short headways, the goal is to maintain regularity, therefore headway-based holding strategies are recommended. On the other hand, for lines with high variability in passenger demand, holding strategies focus more on the minimization of passenger cost rather than headway variability [1].

Headway-based strategies are mostly based on rules which limit the maximum allowed headway, and their objective is to minimize headway variance between consecutive vehicles of the same line [1]. Some related studies are shown in TABLE 1. The first column presents the author and the year of publication; the second column presents the objective of the study, which is either minimizing travel cost (TC), or the variability of headway (HV) or schedule deviation (SD); the next column indicates the approach for modeling passenger demand and travel times, which can be either deterministic (Det) or stochastic (Stoch), and the last column indicates if a capacity constraint is applied.

TABLE 1 HEADWAY-BASED HOLDING STRATEGIES

Author	**Objective**	**Travel Time and Passenger Demand**	**Capacity Constraint**
Abkowitz and Lepofsky, 1990 [2]	HV	Det	No
Fu and Yang, 2002 [3]	TC	Det	No
Daganzo, 2009 [4]	HV	Stoch	No
Daganzo and Pilachowski, 2011 [5]	HV	Det	No
Xuan et al, 2011 [6]	SD	Det	No
Cats et al, 2011 [7]	HV	Stoch	Yes
Bartholdi and Eisenstein, 2012 [8]	HV	Stoch	Yes

In general, by regulating the headways, passengers benefit in terms of their travel cost [9] but only a few authors have considered the number of passengers that will experience these benefits. For instance, Ding and Chien [10] used the ratio of the arrival rate at a stop and the sum of the arrival rates along a route as a weight in their function to reduce headway variance and at the same time minimize waiting time. The number of passengers waiting at stops and on board is considered mostly in optimization models when minimizing travel cost, such as [11] [12] and [13].

This paper introduces a holding rule for minimizing the passenger travel times. The proposed holding rule calculates the recommended holding time according to the time needed to regulate headways between consecutive vehicles and adjusts it by accounting for the number of passengers that experience the extra time. The proposed strategy is tested for a high frequency bus line in Stockholm, Sweden and is compared with the currently used even headway strategy. The

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evaluation is performed using BusMezzo, a dynamic public transport operations simulation model.

The remainder of the paper is structured as follows: Section 2 describes the methodology, which is applied to the case study described in Section 3. Section 4 evaluates the new decision rule based on the results of the case study. In Section 5 conclusions are drawn.

II. METHODOLOGY

A. *Problem Description*

The problem considered in this study is how to design a holding strategy that instructs vehicles to be held at stops for a certain additional time to regulate the headway between arrivals of consecutive vehicles. The holding time is then determined subject to the passengers that will experience the additional travel time.

The variables used in the paper are denoted as follows

$\mathrm{AT_{jk}}$	arrival time of vehicle k at stop j
$\mathrm{DT_{jk}}$	dwell time of vehicle k at stop j
$\mathrm{w_{jk}}$	holding time assigned to vehicle k at stop j
$\mathrm{ET_{jk}}$	departure time of vehicle k from stop j
$\mathrm{L_{jk}}$	load of vehicle k at stop j after the completion of alighting and boarding
λ_j	passenger arrival rate at stop j
m	last stop visited by vehicle $\mathrm{k}+1$
N	number of stops along the line
$\mathrm{SRT_{mj}}$	scheduled riding time between stops m and j
$\mathrm{PH_{k-1,k}}$	planned headway between services $\mathrm{k}-1$ and k
α	threshold ratio parameter.

B. *Even Headway Holding Strategy*

The even headway strategy regulates departure times considering the proceeding and the succeeding vehicle on the same line and also limits the maximum holding time [7]. In order to be implemented, real time information on vehicle locations is needed. The decision rule for the departure (exit) time of vehicle *k* from stop *j* is:

$$\mathrm{ET_{jk}} = \max\left\{\min\left\{\frac{\mathrm{AT_{j,k-1}}+\mathrm{AT_{j,k+1}}}{2}, \mathrm{AT_{j,k-1}} + \alpha\mathrm{PH_{k-1,k}}\right\}, \mathrm{AT_{jk}} + \mathrm{DT_{jk}}\right\} \tag{1}$$

The arrival time $\mathrm{AT_{j,k+1}}$ of the succeeding bus needs to be predicted. The commonly applied delay preservation prediction scheme assumes $\mathrm{AT_{j,k+1}} = \mathrm{AT_{m,k+1}} + \mathrm{SRT_{mj}}$, where $\mathrm{AT_{m,k+1}}$ denotes the arrival time at the last visited stop m and $\mathrm{SRT_{mj}}$ is the scheduled riding time between m and j. Parameter α is a threshold ratio that determines the minimum allowed headway with values varying from 0.6 to 0.8 as found by previous studies [3, 7]. In previous simulation [7, 14] and field experiment [9] studies, the even headway strategy outperformed schedule-based strategies and demonstrated robust results.

C. *Passenger Cost Holding Strategy*

The proposed decision rule incorporates the effects on passenger travel cost due to holding in order to determine the optimal holding time. Real-time AVL data and historical data from APC should be available in order to implement this decision rule. Total passenger travel cost consists of passenger waiting time at stops (WT) and the in-vehicle delay of passengers on board (IVT). Since waiting at stops is considered a greater disturbance by passengers compared to in vehicle delay, a weight is assigned to the waiting time term. This is set to 2 in this study:

$$\mathrm{TT_k} = 2 * \mathrm{WT_k} + \mathrm{IVT_k} \tag{2}$$

Assuming that passengers arrive to stops at random, the expected waiting time per passenger is half the current headway at the stop. The number of passengers arriving at the downstream stops is the product of the sum of the arrival rates λ of passengers at each stop and the current headway, assumed to be preserved at downstream stops. Given that the current bus arrives at time $\mathrm{AT_k}$, the total passenger waiting time between the preceding and the succeeding vehicle is:

$$\mathrm{WT_k^0} = \frac{\sum_{i=j+1}^{N}\lambda_i(\mathrm{AT_{\kappa+1}}-\mathrm{AT_\kappa})^2}{2} + \frac{\sum_{i=j+1}^{N}\lambda_i(\mathrm{AT_\kappa}-\mathrm{AT_{\kappa-1}})^2}{2} \tag{3}$$

When a vehicle is instructed to hold, more passengers arrive at the downstream stops and will experience a longer waiting time. On the other hand, fewer passengers will arrive after the bus and have shorter average waiting time to the succeeding bus. Given that the current bus arrives at time $\mathrm{AT_k} + \mathrm{w_k}$, total waiting time is:

$$\mathrm{WT_k^H}(\mathrm{w_k}) = \frac{\sum_{i=j+1}^{N}\lambda_i\left(\mathrm{AT_{\kappa+1}}-(\mathrm{AT_\kappa}+\mathrm{w_k})\right)^2}{2} + \frac{\sum_{i=j+1}^{N}\lambda_i\left((\mathrm{AT_\kappa}+\mathrm{w_k})-\mathrm{AT_{\kappa-1}}\right)^2}{2} \tag{4}$$

Consequently, the change in total passenger waiting time due to holding is:

$$\mathrm{WT_k}(\mathrm{w_k}) = \mathrm{WT_k^H}(\mathrm{w_k}) - \mathrm{WT_k^0} = \sum_{i=j+1}^{N}\lambda_i\,\mathrm{w_\kappa}\left(\mathrm{w_\kappa} + (\mathrm{AT_\kappa} - \mathrm{AT_{\kappa-1}}) - (\mathrm{AT_{\kappa+1}} - \mathrm{AT_\kappa})\right) \tag{5}$$

The total delay that passengers on board experience due to holding at a stop is the product of the bus load and the holding time:

$$\mathrm{IVT_k}(\mathrm{w_k}) = \mathrm{L_k}\mathrm{w_k} \tag{6}$$

The optimal holding time is obtained by minimizing the travel time cost:

$$\min_{\mathrm{w_k}} \mathrm{TT_k}(\mathrm{w_k}) = 2\mathrm{WT_k}(\mathrm{w_k}) + \mathrm{IVT_k}(\mathrm{w_k}) \tag{7}$$

which gives

$$\mathrm{w_k} = \frac{(\mathrm{AT_{k+1}}-\mathrm{AT_k})-(\mathrm{AT_k}-\mathrm{AT_{k-1}})}{2} - \frac{\mathrm{L_k}}{4\sum_{i=j+1}^{N}\lambda_i} \tag{8}$$

In the first term of the formula, holding time is calculated based on the headway between consecutive vehicles. Then the time calculated is shortened according to the ratio of the number of on-board passengers at the stop and four times the sum of the expected passenger arrival rates at the downstream stops. The holding strategy implies that the holding time is shorter if the number of on-board passengers is high compared to the sum of the arrival rates at the downstream stops, and longer if the opposite relation holds.

In the limit $L_k \to 0$, the passenger cost holding strategy is equivalent to the even headway strategy. In other words, the even headway strategy minimizes waiting times but does not take in-vehicle delay into account. This can for example result in holding buses unnecessarily and delaying on-board passengers in cases where there are few or no passengers further downstream (i.e. towards the end of the line). Holding earlier at the terminal or at early stops of the route is considered to be beneficial in reducing potential holding costs at later stops [15, 16] and as the passenger ratio gradually increases along the route, the rule allows a higher holding tolerance at the beginning of the route, then restricts holding towards the end of the route where few or no passengers are waiting.

For consistency and comparability reasons, the passenger cost holding strategy is formulated as a departure time decision rule from the current stop j, including also a term to limit the minimum allowable headway between stops:

$$\mathrm{ET_{jk}} \\ = \max\left\{\min\left\{\frac{\mathrm{AT_{j,k-1}} + \mathrm{AT_{j,k+1}}}{2} - \frac{\mathrm{L_{jk}}}{4\sum_{j+1}^{N}\lambda_i}, \mathrm{AT_{jk-1}} \\ + \alpha \mathrm{PH_{k-1,k}}\right\}, \mathrm{AT_{jk}} + \mathrm{DT_{jk}}\right\}$$

III. CASE STUDY

A. *Line Description*

The proposed decision rule introduced in eq. (9) was tested for a high frequency bus line in Stockholm, Sweden. The bus system in the city center of Stockholm is mainly served by four trunk lines. The case study considers the southbound direction of Line 4, which serves 31 stops. Line 4 has the highest demand and operates between *Radiohuset* and *Gullmarsplan* (*Figure 1*).

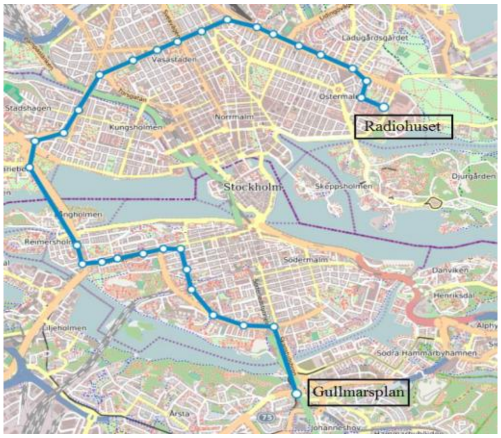

Figure 1 Route of line 4 in Stockholm, Sweden

Along the line there are connections with several subway, light rail and commuter train stations as well as bus terminals. During the peak hour approximately 500 passengers are boarding the line and the headway between successive departures is 5 minutes. In order to attain smooth operations and satisfy the demand, articulated buses are used and designated lanes and signal priority are provided along the route. Real time data concerning vehicle locations and aggregated passenger demand data were available for this study. *Figure 2* illustrates the demand profile of the southbound direction of bus line 4 for the afternoon period (15:00-18:00). The stops with the highest numbers of boarding and alighting passengers are those that allow transfers to other modes. These stops are also used as time point stops for relieving drivers and for service regulation.

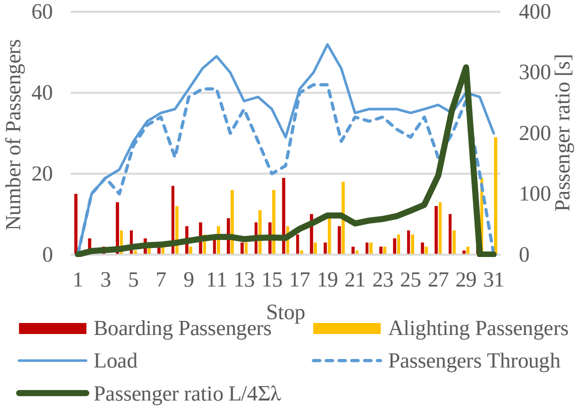

Figure 2 Demand Profile of Line 4

B. *Scenario Design*

The passenger cost strategy is compared with a do nothing scenario and the even headway strategy. The even headway strategy is currently used for the trunk lines in Stockholm. This strategy was implemented following a series of simulation and field experiment studies [7]. An empirical analysis of the performance of this strategy demonstrated that it resulted in passenger travel time savings when compared with the previous schedule-based holding control.

Passenger demand data were retrieved for three hours of afternoon operations of the line. Based on the data three different levels of demand were determined to test the two strategies: i) low passenger demand ii) normal (base) passenger demand and iii) high passenger demand. Base passenger demand corresponds to the observed demand level while low and high demand corresponds to 50% and 200% of the base level, respectively.

In the literature there are different approaches concerning the number and the allocation of the control points. Theoretically, all stops can serve as time control points. A common strategy is to allocate time control points prior to stops with high demand [17, 18, 19, 20]. As aforementioned, the new strategy is expected to be triggered more frequently and intensely at the beginning of the route while opting for even headway at the stops where headway variance will be high. In this study all stops can potentially be used for control. This allows identifying where and how frequently the new strategy assigns holding time considering the actual headway and the passenger cost and if it meets the initial assumptions. TABLE 2 summarizes the nine scenarios that were tested.

TABLE 2 SCENARIOS

	No Control (NC)	**Even Headway Strategy (EH)**	**Passenger Cost Strategy (PC)**
Low demand (50)	NC_50	EH_50	PC_50
Base demand (100)	NC_100	EH_100	PC_100
High demand (200)	NC_200	EH_200	PC_200

The scenarios were implemented and tested using the public transport simulation model BusMezzo, which is built on the mesoscopic traffic simulator Mezzo [21]. BusMezzo has a wide range of applications and has been previously used to analyze and evaluate real time control strategies [7, 14, 20]. Each scenario was analyzed based on the results of 20 simulation replications. Across the 20 replications the standard error of the headway standard deviation is 5%.

IV. RESULTS

The strategies were evaluated using key measurements of performance of the line, shown in TABLE 3. The coefficient of variation of headways represents the average variability of headways at all stops along the line. It is clear from the results that when a control strategy is applied, the headway variability decreases significantly. Both strategies result in considerable improvements for peak demand, but in general even headway outperforms passenger cost.

The second measure of performance is bunching. The share of buses that are bunched is the ratio of trips that arrive within a headway 50% lower or 50% greater than the planned headway and the total number of trips [22, 23]. Again, the even headway strategy yields the best results. However, the passenger cost strategy also reduces bunching significantly compared to when no control is applied. Although the results of the passenger cost strategy cannot be characterized as poor, the dominance of the even headway strategy for these two measures can be explained by the fact that they are consistent with the main objective of the strategy, which is to regulate headways between consecutive vehicles.

When a control strategy is applied, the average trip time is longer due to the additional holding time. Indeed, for all three demand levels the average trip time becomes longer when a control strategy is applied. The new strategy slightly decreases the average trip time due to the reduction of holding times caused by the passenger ratio.

Furthermore, the effects of the introduced passenger ratio can be observed by the reduced holding time by 20% to 30% when the passenger cost strategy is applied compared to the even headway strategy.

TABLE 3 KEY MEASUREMENTS OF PERFORMANCE OF THE LINE

	Average CV of line headway	**Bunching %**	**Average bus trip time (sec)**	**Average holding time per bus trip (sec)**	**Average waiting time per passenger (sec)**	**Average in vehicle time per passenger (sec)**	**Average weighted travel time per passenger (sec)**
NC_50	0.57	40%	3092	0	189.3	99.8	478.4
EH_50	0.32	8%	3291	6.78	156.7	106.2	420.4
PC_50	0.35	12%	3253	4.72	155.9	104.9	416.7
NC_100	0.76	54%	3473	0	230.5	112.4	573.4
EH_100	0.48	16%	3765	10.43	174.0	121.6	469.6
PC_100	0.53	27%	3717	8.15	179.8	119.9	479.5
NC_200	0.79	55%	4291	0	214.2	138.6	567
EH_200	0.57	24%	4484	7.28	177.8	145.8	501.4
PC_200	0.57	29%	4430	4.82	174.7	144.1	493.5

There are no significant differences between the control schemes in terms of waiting times. While both control strategies significantly reduce waiting times compared to the no control scenarios, there are marginal differences between them. At low and peak demand levels, the waiting time is slightly lower for the passenger cost strategy than the even headway strategy. However, the even headway strategy outperforms the passenger cost strategy for the base demand level.

When a control strategy is applied, passengers are experiencing longer in-vehicle delay because of holding time. Due to the reduction in holding time yielded by the passenger cost holding strategy, the in-vehicle delay is reduced. In all three demand scenarios the mitigation of in-vehicle delay is at a similar level, on average 1.2%.

The sum of waiting time and in-vehicle delay is the corresponding travel time of each passenger. For all three demand levels, the no control scheme is outperformed by the schemes with a control strategy and for low and peak demand, the passenger cost strategy is the most effective thanks to slightly shorter in-vehicle times as well as waiting times.

The results suggest that there is no significant gain from implementing the new strategy at system level since there are minor reductions in travel time cost while attaining less regular service compared with the even headway strategy. With the new strategy, more holding time is assigned at the beginning of the route and less after the middle of the route, because of the increasing magnitude of the passenger ratio as shown in *Figure 2*. This can be also seen in TABLE 4 where the travel time results are split into the first and second halves of the route. On the first part of the route the two control strategies have the same performance. Conversely, on the second half of the route, where the passenger cost strategy instructs vehicles to hold less frequently and for a shorter time, an increasing trend in waiting time is observed simultaneously with a decreasing trend in in-vehicle time, which are also reflected in total travel time.

TABLE 4 TRAVEL TIME FOR THE TWO HALVES OF THE ROUTE

	First half of the route			**Second half of the route**		
	Average waiting time per passenger (sec)	**Average in vehicle delay per passenger (sec)**	**Average weighted time per passenger (sec)**	**Average waiting time per passenger (sec)**	**Average in vehicle delay per passenger (sec)**	**Average weighted time per passenger (sec)**
NC_50	174.5	101.6	450.6	213.1	97.9	524.0
EH_50	155.1	107.2	417.5	159.8	105.0	424.7
PC_50	153.5	106.7	413.7	159.4	103.1	421.8
NC_100	189.6	115.6	494.8	296.6	108.9	702.1
EH_100	164.1	122.4	450.5	189.1	121.0	499.3
PC_100	167.4	122.1	456.8	198.7	117.6	514.9
NC_200	189.7	146.2	525.5	259.1	131.3	649.5
EH_200	174.2	150.9	499.3	184.6	139.5	508.6
PC_200	170.2	150.1	490.4	182.7	137.6	503.1

The overall performance of the passenger-based strategy is also shown by the variability of headways along the route (*Figure 3*). Both strategies are effective in improving regularity but the effect of the new strategy is more pronounced at the beginning of the route. Evidently, both strategies significantly improve headway variability compared to operation without control. Until the middle of the route, both strategies have the same performance. After the 16th stop, the sum of the arrival rates at the downstream stops is not sufficiently high and consequently the magnitude of the passenger ratio is higher. As a result, when the passenger cost strategy is applied, the final holding time assigned is lower. For peak demand the passenger ratio presents a slower increasing trend allowing the passenger cost strategy to perform identically to the even headway strategy.

The passenger ratio is a key determinant of the performance of the proposed strategy. By reducing holding time, it also creates a new pattern concerning the stops or route segments where the vehicle can be held. *Figure 4* shows the average holding time at each stop with even headway strategy and passenger cost strategy and the holding frequency of each strategy. It can be observed how holding time is mitigated due to the effect of the passenger ratio and the larger share of holding time is applied before the middle of the route, where the passenger ratio reduces holding time by less than a minute and then the new strategy allows holding time of several seconds as a vehicle approaches the end of the route.

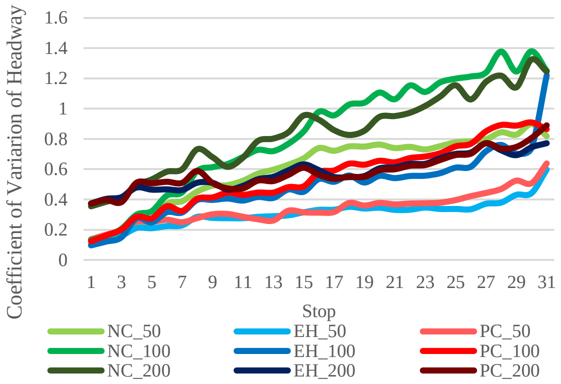

Figure 3 Coefficient of variation of headway at each stop

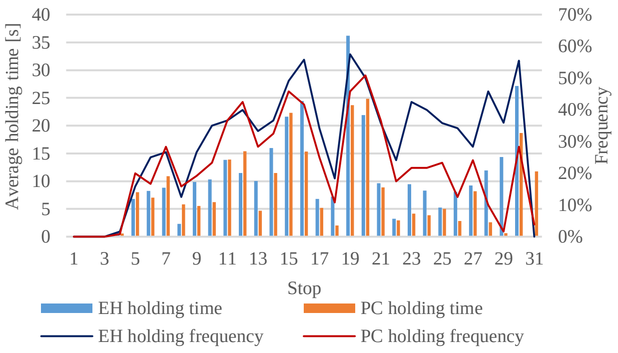

Figure 4 Average holding time and holding frequency per stop

At high demand stops, in particular those that provide connections to other modes, the variability in passenger demand is also high. At these stops holding rules should be more frequently applied. This can also be observed in the difference in the frequency with which the vehicles are instructed to hold between the different strategies. When approaching high demand stops, with the passenger cost strategy, vehicles are instructed to hold more frequently in contrast to the general decreasing trend due to the increase in the ratio's magnitude. Although there is a mitigation of holding time at stops, due to the high variability of travel time and demand at these stops more vehicles need to be held and vehicles arriving at these stops with lower occupancy are held longer, affecting the average holding time.

V. CONCLUSION

Rule-based holding control strategies aim at regulating the headway between consecutive vehicles and indirectly achieving reductions in passenger travel cost. In this paper a decision rule explicitly based on passenger travel cost is formulated in order to determine holding times at stops. Recommended holding times are calculated based on the headways of consecutive vehicles and the number of passengers that will be affected by the additional travel time. The new holding rule was tested for a high-demand high-frequency bus line in the city of Stockholm using a simulation model, and was compared with the even headway strategy, which is currently used.

The passenger cost strategy performs almost equally well compared with even headway strategy in terms of waiting time and travel time while it yields a minor reduction in in-vehicle time. The two control strategies provide satisfactory results in terms of vehicle-based reliability metrics which are the main objectives of the even headway strategy. For peak demand the benefits to the passengers from the reduction in in-vehicle delay is sufficient to be reflected in the travel time. Moreover, the passenger cost strategy gives similar results in terms of headway variability and similar or shorter waiting times compared to the even headway strategy while requiring shorter holding times.

With the new strategy holding is more prevalent at the first part of the route with similar performance to the even headway strategy, while the need to control diminishes at the second half of the route. The distribution and the frequency of holding along the line change due to the passenger distribution of the line and the occupancy of buses arriving at these stops.

Further research will focus on the passenger ratio term and testing the strategy's performance for different demand distributions along the line. Further, the strategy may be extended beyond single line operations by for example introducing passenger transfer costs in the context of service synchronization.

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