Surgery scheduling

Dealing with over-time

M. van der Tuin

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by

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TU Delft

Preface

As the final year of high school approached, I found myself pondering what university life would entail. It would be a mix of studying and enjoying time with friends beyond the confines of lecture halls. My primary concern at that point was choosing my field of study. From a young age, I always loved building, drawing, and designing, which led me to consider Architecture at the TU Delft. However, in the end, I made the decision to pursue Mathematics, which was also offered at the TU Delft, as I loved the subject in high school and since it provided a broad range of possibilities for my future. With that choice made, I believed the hardest part was behind me.

Little did I know that things wouldn't start off smoothly. COVID-19 abruptly brought my high school days to a halt and forced a virtual beginning to my university journey. It was only at the start of my second year that I finally experienced in-person lectures, joined the study association, and forged incredible friendships.

Having begun in isolation, I was unaware of what I had been missing, but I managed to adapt. Reflecting on those times, I now realize how swiftly one's social circle at university can change. This realization serves as motivation for me to delve deeper into the robustness of surgical planning. Although an event like COVID-19 might have an overwhelming impact that is beyond our control, having a solid foundation can provide hope during times of stress.

Before I proceed, I would like to express my gratitude to the bachelor end project coordinator and my supervisor at the TU Delft, Theresia van Essen, for granting me the opportunity to work in such an important field. Initially, the amount of feedback I received was intimidating, but looking back, I appreciated the time you dedicated to help me. I hope we were able to catch and correct most of the spelling errors.

I would also like to thank my friends and family for their support, especially during the past year. Lastly, I want to give a special thanks to my fellow RiCie committee members. While conducting this research, we had a demanding schedule preparing for our visit down the east. Your faith in me and patience while I completed my tasks are greatly valued.

M. van der Tuin
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Abstract

Surgical scheduling is a complex task that requires consideration of various factors, including the probability of overtime. In this study, we address the research problem of surgery scheduling while accounting for the likelihood of exceeding scheduled operating room (OR) time. To tackle this problem, we employ integer linear programming (ILP) models to determine the optimal number of surgeries per group, with the objective of maximizing OR utilization while incorporating the probability of overtime as a constraint.

To capture the probabilistic nature of surgery durations, we investigate suitable probability distributions. Existing literature suggests that surgery durations follow a lognormal distribution. However, since the sum of lognormally distributed random variables lacks a closed form solution, we initially assume a normal distribution for analytical convenience. Subsequently, we approximate the lognormal distribution using the Fenton-Wilkinson method to account for its realistic behavior. To incorporate the lognormalistic behavior and solve the ILP models efficiently, we employ a column based approach. This approach enables us to handle the complexities introduced by the lognormal distribution.

Our study utilizes data provided by a hospital in the Netherlands, including information on surgeries, specialties, groups, and the master surgery schedule (MSS). Given the consideration of both normal and lognormal distributions for surgery durations, we assess the goodness of fit using appropriate statistical tests.

Our results reveal that using averages or expected values yields the highest OR utilizations. However, there is a discussion regarding the validity of this method, as it does not explicitly incorporate the probabilistic overtime constraints. Nevertheless, we observe that all methods include cases which surpass the predetermined overtime threshold, suggesting that utilizing averages or expected values can be a valid alternative. However, utilizing averages or expected values gives rise to high percentage of cases surpassing our overtime threshold. So, we suggest to use a method which explicitly uses the probabilistic nature of the surgery durations.

During the examination of our methods, we had to take a minimum number of mandatory scheduled surgeries for each group into account. This means that another dataset, with different mandatory numbers, might lead to different results. Additionally, we noticed that our overtime definition might not be the most optimal, as we still have cases that surpass our overtime threshold. In future research, it would be valuable to include financial and staff factors, which can further enhance the scheduling process.

Overall, this study contributes to the field of surgery scheduling by addressing the probability of overtime and presenting insights into the trade-offs between OR utilization and the inclusion of probabilistic constraints. Further research can build upon these findings to refine the scheduling approaches and incorporate additional factors for a more comprehensive solution.

Contents

1
Introduction

Amidst the challenges posed by the COVID-19 pandemic, the field of surgery scheduling has gained even more significance. The disruption caused by the pandemic has highlighted even more the need for efficient and effective scheduling strategies in healthcare institutions. However, the motivation for optimizing surgery scheduling extends beyond the pandemic itself. It arises from the aim to enhance patient care, maximize resource utilization, and improve overall operational efficiency. By developing efficient scheduling strategies, we can minimize patient waiting times, ensure timely access to surgical interventions, and prevent unnecessary delays or cancellations. Moreover, optimal surgical scheduling plays a crucial role in maximizing the utilization of valuable resources such as operating rooms, equipment, and healthcare personnel. By efficiently allocating these resources, healthcare institutions can maximize the utilization of their available operating room capacity and enhance cost-effectiveness, since operating rooms are one of the most expensive resources in a hospital. So, efficiently scheduling surgical procedures is a fundamental aspect of healthcare management. However, considering limiting overtime complicates surgical scheduling. limiting overtime in surgical scheduling include restrictions intended for healthcare professionals regarding the number of hours they can work beyond their regular shifts. Including limiting overtime in surgical scheduling is crucial for ensuring a balanced workload and maintaining high-quality patient care. In the first place, these constraints are implemented to ensure the well-being and performance of healthcare staff as excessive work hours can lead to physical and mental health issues. Therefore, developing scheduling strategies that limit overtime is essential for achieving a sustainable and resilient healthcare workforce.

To determine these, we pose the following research questions:

1. Which methods have already been developed to incorporate limiting overtime?
2. How do these methods perform?
3. Which method performs best?

In order to address these questions, we undertake a thorough investigation of various methods for developing a tactical surgery scheduling system, which takes limiting overtime into account. In Chapter 2, we examine the relevant literature to gain insights into the challenges, implications, and potential solutions associated with the integration of overtime considerations in surgical scheduling. Next, in Chapter 3, the problem description and formulation are presented. This chapter includes a detailed examination of the fundamental constraints and objective function underlying the scheduling model. By explicitly mentioning the constraints and objectives, the framework for incorporating limiting overtime into the surgical scheduling process is established. In Chapter 4, the focus shifts towards the incorporation of limiting overtime within the formulated model, while taking previous research into account. Chapter 5 is dedicated to the analysis of the available data used to validate and test the proposed scheduling models. Following the data analysis, Chapter 6 presents the evaluation of the obtained results. This chapter critically assesses the performance of the developed models in terms of their ability to incorporate limiting overtime effectively. Finally, in Chapter 7, a conclusion is drawn based on the findings and outcomes of the research. The conclusion provides a concise summary of the key findings, an in-depth discussion of the study's implications, and valuable recommendations for future research directions.

2
Literature

Over the past twenty years, surgery scheduling has sparked more interest. To see an overview of the available literature dealing with this topic, we refer the reader to the articles published by Cardoen *et al.* [4], Guerriero and Guide [5] and Wang *et al.* [12] . Since this topic can take many factors into consideration, we define in this chapter the relevant definitions for our scope of research.

2.1. Elective patients

Elective surgeries are typically planned procedures that do not require immediate attention or emergency intervention. By scheduling elective patients, hospitals can allocate resources, including operating rooms, surgical teams, and equipment, in an organized and efficient manner. This helps to ensure that critical resources are available for both elective and emergency cases. Elective scheduling helps to reduce waiting times for patients who require non-emergency surgical procedures. By assigning specific time slots for elective surgeries, hospitals can provide patients with a clear schedule and minimize delays. In the next section, we explain how the master surgery scheduling can help with the allocation of the elective surgeries.

2.2. Master surgery scheduling

Master surgery scheduling is the strategic and organized planning of elective surgical procedures in a healthcare facility, typically spanning weeks or months. Its objective is to efficiently allocate operating rooms (ORs), surgical teams, and resources to meet the demand for surgical services while optimizing resource utilization and enhancing patient flow. The creation of a master surgery schedule (MSS) involves considering multiple factors, including surgeon availability, OR availability, patient priorities, surgical complexity, equipment requirements, and staff availability. Different definitions exist for an MSS. According to Van Oostrum *et al.* [11], it entails a cyclical arrangement of surgery types, while Beliën and Demeulemeester [1] define it as a cyclical schedule of time blocks within the ORs allocated to surgeons or specialized areas. For this research, we adopt the MSS definition proposed by Beliën and Demeulemeester [1]. However, the MSS has limited flexibility, primarily stemming from the assumption of uniform resource capacity across all periods. Periodically updating the MSS might be helpful, but this contradicts the cyclic nature of an MSS.

2.3. ILP

An Integer Linear Program (ILP) is a mathematical optimization model that incorporates discrete integer variables in a linear programming framework. Schneider [10] uses decision variables which determine the number of surgeries per group for each OR and each day. To maximize the utilization, we can use the approach proposed by Beliën and Demeulemeester [1], who studied the problems of building robust cyclic master surgery schedules. Their focus was on minimizing the shortage of beds, but even though we don't consider the bed shortage, their approach can still be applied effectively.

In their research, Santibanez *et al.* [9] explore the scheduling of surgical blocks for each specialty using a mixed integer programming model. Their objective is to find the optimal balance between the

availability of OR time, downstream resources, and patient waiting lists. By analyzing these factors, they aim to identify the best trade-off that ensures efficient utilization of resources while minimizing patient waiting times.
In another study, Van Oostrum *et al.* [11] focus on planning elective surgical procedures that are frequently performed within a cyclic schedule. The primary objective of their research is to minimize the utilization of OR-capacity while also leveling the requirement of hospital beds. The authors incorporate planned slack into their scheduling approach in order to include the stochastic nature of surgery durations. However, they assume that the length of stay (LoS) for patients is deterministic. The solution approach consists of two phases. In the first phase, they ignore the requirement of hospital beds and formulate an ILP. This ILP is then solved using an implicit column generation approach. By employing this method, they aim to generate an initial solution that meets the scheduling constraints while optimizing the OR capacity utilization. Moving to the second phase, the researchers formulate the problem as a mixed integer linear program (MILP). The objective in this phase is to minimize the maximum number of hospital beds needed. This MILP formulation takes into account the cyclic scheduling and the requirement of hospital beds, allowing for the identification of an optimal solution within reasonable time bounds. By utilizing this two-phase approach, Van Oostrum *et al.* [11] effectively address the challenges associated with elective surgical scheduling. Their research offers valuable insights into minimizing OR capacity utilization, leveling the requirement of hospital beds, and optimizing the scheduling of frequently performed surgical types within a cyclic schedule. For our research, only the first phase is relevant, since we do not take bed occupation into account.

2.4. Surgery duration

Normally, surgery duration is defined as the time span from the moment the patient enters the OR until the moment the patient leaves it. According to the findings presented in the review conducted by Wang *et al.* [12], the log-normal distribution emerged as the most frequently observed distribution for fitting surgery duration. Assuming a lognormal distribution, the calculation of the probability of overtime becomes more challenging as there is no exact expression for the sum of log-normal distributed random variables. Therefore, Van Oostrum *et al.* [11] and Hans *et al.* [6] assume that surgery durations follow a normal distribution. One advantage of assuming normal distributions for surgery durations is that the sum of these durations is also normally distributed. This allows for relatively straightforward calculation of the probability of overtime. On the other hand, Nguyen [8] incorporated the lognormal distribution with a column generation based approach. Before her model was executed, she made a set with all feasible group combinations satisfying the probabilistic overtime constraint. Based on this set, the model chose a combination for each OR and each day.

2.5. Overtime

To account for the probability of overtime, Hans *et al.* [6] use a scheduling approach for surgeries that integrates the average durations of surgeries. They also allocate supplementary slack time, based on managerial assessment of overtime tolerance, to accommodate the potential occurrence of overtime. Similarly, Schneider [10] incorporates the probability of overtime by explicitly incorporating the probability distribution of the normal distribution as a constraint. This constraint incorporates the expected duration and variance of the duration of surgeries, enabling a more comprehensive consideration of overtime probabilities. Alternatively, Kauwenbergh [7] determines the end time of a surgery based on the starting time and the expected surgery duration. By obtaining the estimated end time, we can minimize the probability of overtime.

3

Model

This chapter describes the model that has been constructed for our research. In the first section, we give the problem description. In Section 3.2, we formulate the corresponding mathematical model, including the constraints and objective function.

3.1. Problem description

In our model, we let J be the set of operating rooms (ORs), T the set of days and S be the set of specialties. In our scheduling framework, we utilize a Master Surgery Schedule (MSS), which specifies the allowable scheduling of specialties in each OR $j \in J$ on day $t \in T$, referred to as OR-day (j, t). Our primary objective in this problem is to maximize OR utilization while minimizing overtime occurrence. To achieve this, we define our objective function as the maximization of OR utilization, with the constraint of minimizing the probability of overtime. In order to simplify this problem, we divide each specialty into groups of surgeries, based on surgery duration. In total, each specialty has at most three groups. With $|S|$ specialties, this means we have to consider at most $3 \cdot |S|$ groups of surgeries. For each specialty $s \in S$, we define I_s as the set of surgery groups belonging to specialty s. Consequently, the set of all groups $\bigcup_{s \in S} I_s$, is denoted by I. Since the groups belong to one specialty, we have that $\bigcap_{s \in S} I_s = \emptyset$.

Based on this simplification, we establish the following constraints. Firstly, surgeries belonging to specialty $s \in S$ can only be scheduled on specific OR-days assigned by the MSS. Secondly, we impose a minimum number of scheduled surgeries for each group to ensure that all groups are included in the schedule. These constraints are our basis constraints, since they can be easily implemented in our (M)ILP-model. Now, let D_{jt} be the random variable representing the total surgery duration in OR-day (j, t). Each OR-day (j, t) has a specific capacity, denoted by c_{jt}. Consequently, overtime will occur if $D_{jt} \geq c_{jt}$ for some j, t. Besides our basis constraints, we aim to incorporate the non-linear constraint of overtime probability. This research explores multiple approaches to address this non-linearity and compares their effectiveness.

3.2. Problem formulation

Now that we have given a description of our problem, the next crucial step in the process of mathematical modeling is formulating the mathematical model itself. In the following subsections, we delve into the details of formulating the mathematical model. Therefore, we identify the decision variables, define the objective function and express the constraints mathematically.

3.2.1. Constraints

For this model, we do not consider every surgery separately. Instead, we look at each group, and decide how many surgeries from this group we should schedule. This gives rise to the following non-negative integer decision variables:

$$N_{ijt} = \text{Number of surgeries of group } i \in I \text{ scheduled in OR } j \in J \text{ on day } t \in T.$$

Besides the decision variables, we need some input parameters. We need to schedule the surgeries according to the MSS, which is given by input parameters a_{sjt}.

$$a_{sjt} = \begin{cases} 0 & \text{surgeries of specialty } s \in S \text{ are not allowed to be scheduled in OR } j \in J \text{ on day } t \in T. \\ 1 & \text{surgeries of specialty } s \in S \text{ are allowed to be scheduled in OR } j \in J \text{ on day } t \in T. \end{cases}$$

Note, that the MSS allows at most one specialty for each OR-day (j, t).

As our first constraints, for each specialty $s \in S$, we need to take into account whether or not those surgeries are allowed to be planned in OR-day (j, t). The corresponding constraints are given by

$$\sum_{i \in I_s} N_{ijt} \leq M_s \cdot a_{sjt}, \ \forall s \in S, j \in J, t \in T, \tag{3.1}$$

where M_s is the maximum number of surgeries for specialty $s \in S$ which can be scheduled in one OR-day. Although Constraints (3.1) are sufficient for the scheduling of our surgeries, they do not guarantee that surgeries of every group are considered in the schedule. In order to provide more equal healthcare, management also provides us with a minimum required number of planned surgeries per group, denoted by β_i.

$$\sum_{t \in T} \sum_{j \in J} N_{ijt} \geq \beta_i, \ \forall i \in I. \tag{3.2}$$

Lastly, we need to incorporate the overtime constraints. Since the occurrence of overtime cannot be determined beforehand with certainty, we look at the probabilistic constraints, given as

$$\mathbb{P}(D_{jt} \geq c_{jt}) \leq \alpha \ \forall j, t, \tag{3.3}$$

where $\alpha \in [0, 1]$. In most hospitals, α is chosen by management. Since Constraints (3.3) are not linear, we have to find a way to linearly incorporate it in our model. In the next chapter, we investigate several ways to deal with this issue, as discussed in Chapter 2.

3.2.2. Objective function

Now that we have defined our decision variables, we can mathematically formulate our objective function. We want to maximize the utilization of ORs, i.e. we want to maximize the total surgery duration per OR-day. Let D_i be stochastic variables representing the duration of surgeries from group $i \in I$. In order to estimate D_i, we use $\mathbb{E}[D_i]$, the expected surgery duration of group $i \in I$. This results in the following objective function:

$$\max \sum_{t \in T} \sum_{j \in J} \sum_{i \in I} N_{ijt} \cdot \mathbb{E}[D_i]. \tag{3.4}$$

Since we incorporate the probabilistic overtime constraints in multiple ways, we use different values for $\mathbb{E}[D_i]$. Therefore, we specify the value of $\mathbb{E}[D_i]$ for each method.

4

Solution methods

Relying solely on the expected duration of elective surgeries for scheduling tends to lead to high OR-utilization. However, as the variability of surgery duration increases, there is a higher probability of overtime. Balancing this variability leads to a trade-off between maximizing OR utilization and minimizing the probability of OR overtime. In this chapter, we incorporate the probability overtime constraint into our model. In Section 4.1, we reduce the available capacity in order to take the possible overtime into account. Next, in Section 4.2, we approximate the duration with a normal distribution. Lastly, in Section 4.3, we approximate the surgery duration with a lognormal distribution.

4.1. Average duration

Our first approach for dealing with the probabilistic constraints, is to reduce the available capacity c_{jt} for each OR-day (j, t) by a factor, which we refer to as our overtime factor q. Let e_i be the average surgery duration for group $i \in I$. Since every group $i \in I$ contains different surgeries, with different durations, we use the average group duration e_i in our model to approximate $\mathbb{E}[D_i]$. Using the reduced capacity, we determine the allowed surgery duration for each OR-day (j, t). These constraints are given by

$$\sum_{i \in I} N_{ijt} \cdot e_i \leq (1 - q) \cdot c_{jt}, \ \forall j, t. \tag{4.1}$$

Since the duration of mandatory surgeries might exceed the reduced capacity of some OR-days, we only consider valid input minimum numbers β_i for each group $i \in I$.

This model incorporates the probability of overtime by reducing the available capacity using a factor q. Consequently, the choice of q is crucial for our model. Our goal is to select a value for q that ensures the overtime fraction, given by

$$F_q = \frac{\text{Number of OR-days having overtime}}{\text{Total number of OR-days}},$$

is less than or equal to a specified threshold α, which lies between 0 and 1. In order to determine this factor q, we utilize binary search. Initially, we create a set containing values from 0 to 1 in increments of 0.01: $[0, 0.01, 0.02, ..., 0.98, 0.99, 1]$. In each iteration, we calculate the overtime fraction for the middle value, which is 0.50 at the start. If $F_{0.50}$ is greater than α, we focus on the right half of the interval; otherwise, we consider the left half. The chosen half becomes the set for the next iteration. We repeat this process until we find a value of q that satisfies the conditions $F_q \leq \alpha$ and $F_{q-0.01} \geq \alpha$.

4.2. Normal distribution

Schneider [10] uses the normal distribution to incorporate the probabilistic overtime constraints. This means we have to fit the corresponding normal distribution for each surgery group duration D_i for all $i \in I$ with parameters μ_i and σ_i.

4.2.1. Expected duration

At first, we use the expected value of the random variables D_i assuming they have the normal distribution for each group $i \in I$. Consequently, it becomes necessary to estimate the distribution parameters for each individual group. The expected value of a normally distributed random variable X with parameters μ_X and σ_X is given by μ_X. We use the same model as outlined in Section 4.1. Assuming the normal distribution, we know that $\mathbb{E}[D_i] = \mu_i$. Consequently, in our objective function (3.4) $e_i = \mu_i$.

4.2.2. Closed form

In the previous subsection, we incorporated the probability of overtime by reducing the available capacity with a factor q. However, this approach does not consider the probability distribution of the random variable D_{jt}, the total duration of surgeries of OR-day (j, t). In this subsection, we use the normal distribution to approximate D_{jt}. Therefore, we assume that D_i has a normal distribution with parameters (μ_i, σ_i). It is known that, the sum of normally distributed random variables also has the normal distribution. Consequently, the total duration of surgeries in OR-day (j, t) is approximated by a normal distribution with parameters $\mu_{jt} = \sum_{i \in I} N_{ijt} \cdot \mu_i$ and $\sigma_{jt}^2 = \sum_{i \in I} N_{ijt} \cdot \sigma_i^2$. To incorporate the probabilistic overtime constraint, we proceed as follows:

$$\mathbb{P}(D_{jt} \geq c_{jt}) \leq \alpha \iff \mathbb{P}(D_{jt} \leq c_{jt}) \geq 1 - \alpha \iff \mathbb{P}\left(\frac{D_{jt} - \mu_{jt}}{\sigma_{jt}} \leq \frac{c_{jt} - \mu_{jt}}{\sigma_{jt}}\right) \geq 1 - \alpha.$$

Using the properties of the normal distribution, we get that

$$\Phi\left(\frac{c_{jt} - \mu_{jt}}{\sigma_{jt}}\right) \geq 1 - \alpha \iff \mu_{jt} + \Phi^{-1}(1 - \alpha)\sigma_{jt} \leq c_{jt}, \ \forall j \in J, t \in T. \tag{4.2}$$

Furthermore, we can use $\mu_{jt} = \sum_{i \in I} N_{ijt} \cdot \mu_i$ and $\sigma_{jt}^2 = \sum_{i \in I} N_{ijt} \cdot \sigma_i^2$ to rewrite Constraints (4.2) as

$$\sum_{i \in I} N_{ijt} \cdot \mu_i + \Phi^{-1}(1 - \alpha) \sqrt{\sum_{i \in I} N_{ijt} \cdot \sigma_i^2} \leq c_{jt}, \ \forall j \in J, t \in T. \tag{4.3}$$

Since we are working with an ILP, we need to linearize Constraints (4.3), i.e. we need to linearly approximate the square root function. To achieve this, we use the approach described by Schneider *et al.* [10] to approximate the square root function by piecewise linear functions. Following this approach, we first determine the interval $[x_{min}, x_{max}]$ in which we approximate the square root function. We choose $x_{min} = 0$, since this means there are no surgeries scheduled in some OR-day. To determine x_{max}, we calculate the largest possible variance in an OR-day. Subsequently, we split the interval into smaller sub-intervals, such that each linear function approximates the square root function in some sub-interval. Let $x_0, \ldots, x_N$ be the $N + 1$ breakpoints, where each consecutive pair of breakpoints are the starting and ending points of a respective sub-interval. These breakpoints are the values on the x-axis in our interval. Note that $x_0 = x_{min}$ and $x_N = x_{max}$.

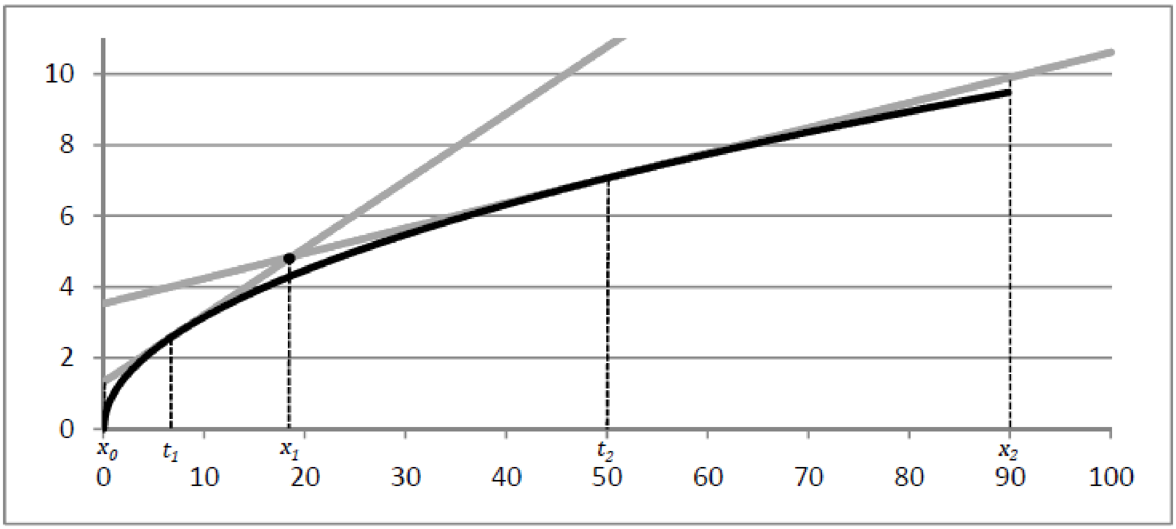


Figure 4.1: Approximating the square root function in the interval [0:90] by two piecewise linear functions (Source: Bosch [3])

In Figure 4.1, an example is demonstrated, in which the square root function is approximated in the interval $[0, 90]$ by using two piecewise linear funcions. It can be observed that the linear approximations (gray) and square root function (black) are equal in the tangent points t_1 and t_2. The breakpoints x_0 and x_2 correspond with the minimum and maximum values of the interval, while breakpoint x_1 is the intersection point of the two piecewise linear functions.

In general, each linear function is a tangent line of the square root function in the point t_n for $n \in \{1, 2, \dots, N\}$. The linear functions are described by $h_n(x) = a_n + b_n x$ for $n \in \{1, 2, \dots, N\}$. Here, b_n is the derivative of the square root function in the point t_n. Since our approximation functions are equal to the square root function in the point t_n, we can calculate the value of a_n.

$$\sqrt{t_n} = a_n + b_n t_n,\ b_n = (\sqrt{t_n})' = \frac{1}{2\sqrt{t_n}} \implies a_n = \frac{1}{2}\sqrt{t_n}. \tag{4.4}$$

Using the values of a_n and b_n, we get

$$h_n(x) = \frac{1}{2}\sqrt{t_n} + \frac{1}{2\sqrt{t_n}}x. \tag{4.5}$$

Let $y_n = h_n(x_n)$, i.e. y_n is the function value of the linear approximation in the breakpoint x_n. The λ-formulation, as described in AIMMS Optimization Modelling Manual [2], can be used to model the piecewise linear functions. According to this formulation, the function value of any point between two breakpoints is the weighted sum of the function values of these two breakpoints. Let λ_n be nonnegative weights for $n \in \{1, 2, \dots, N\}$ such that their sum equals one.

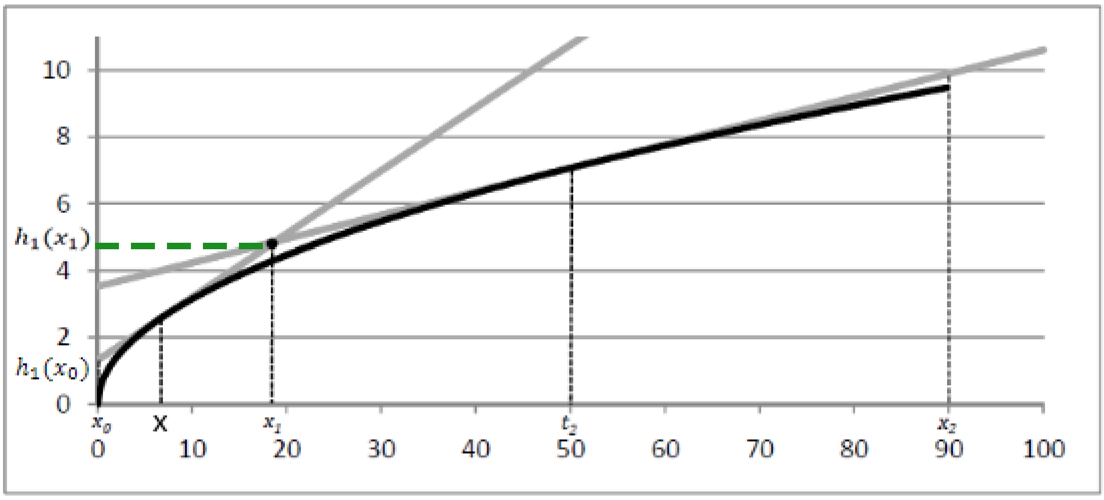


Figure 4.2: Approximating the square root function in the point x.

Say, we need to determine the square root of x, as shown in Figure 4.2. Then we have the following situation:

$$\lambda_0 x_0 + \lambda_1 x_1 = x \tag{4.6}$$
$$\lambda_0 + \lambda_1 = 1 \tag{4.7}$$
$$\lambda_2 = 0 \tag{4.8}$$
$$\lambda_0 h_1(x_0) + \lambda_1 h_1(x_1) = h_1(x) \tag{4.9}$$

Firstly, Constraint (4.6) makes sure x is the weighted sum of two consecutive breakpoints x_0 and x_1. Next, Constraint (4.7) makes sure that the weighted sum of these two breakpoints sum up to one. This means that the other λ's are zero, as given by Constraint (4.8). At last, Constraint (4.9) uses the determined weights to determine the linear approximation of x. In general, the following constraints are incorporated to model the use of piecewise linear functions.

$$\lambda_0 h_1(x_0) + \sum_{n=1}^{N} \lambda_n h_n(x_n) = h_{\tilde{n}}(x), \tag{4.10}$$
$$\sum_{n=0}^{N} \lambda_n x_n = x, \tag{4.11}$$
$$\sum_{n=0}^{N} \lambda_n = 1. \tag{4.12}$$

Note that in Constraint (4.10), the right-hand side incorporates a linear approximation function $h_{\tilde{n}}$. This particular function is chosen in such a way that the positive consecutive breakpoints align with the start and end points in which this function is defined. Now that we have described how we can approximate the square root function with piecewise linear functions, we need to determine the values of the breakpoints and the tangent points. For a detailed procedure, we refer the reader to Schneider *et al.* [10]. Once the tangent points, breakpoints and corresponding approximation function values have been determined, we can use the λ-formulation for each OR-day (j,t), consequently, giving rise to the introduction of decision variables λ_{jtn}. By combining the λ-formulation for each OR-day (j,t) with Constraints (4.3), we obtain the following set of constraints:

$$\sum_{i\in I} N_{ijt} \cdot \mu_i + \Phi^{-1}(1-\alpha) \sum_{n=0}^{N} \lambda_{jtn} y_n \leq c_{jt},\ \forall j \in J, t \in T, \tag{4.13}$$

$$\sum_{n=0}^{N} \lambda_{jtn} x_n = \sum_{i\in I} N_{ijt} \sigma_i^2,\ \forall j \in J, t \in T, \tag{4.14}$$

$$\sum_{n=0}^{N} \lambda_{jtn} = 1,\ \forall j \in J, t \in T. \tag{4.15}$$

There are some necessities in order to make sure these constraints are satisfied for all rooms on every day. Firstly, we must make sure that for every closed room, we have that:

$$\sum_{i\in I} N_{ijt}\sigma_i^2 = \sum_{n=0}^{N} \lambda_{jtn} x_n = 0 \implies \sum_{n=0}^{N} \lambda_{jtn} x_n = x_0 = 0 \implies \sum_{n=0}^{N} \lambda_{jtn} y_n = 0. \tag{4.16}$$

In Schneider's [10] approximation, we use a linear approximation tangent to t_1 to determine y_0. So, even though $x_0 = 0$, we have that $y_0 \neq 0$, since it is an over approximation with maximum error Δ^{max}. Consequently, the constraint is not satisfied for empty rooms. To address this issue, we consider Schneider's approach again. However, this time it is applied in the interval $[\epsilon, x_{max}]$ for some sufficiently small ϵ to determine breakpoints $x_1, x_2, ..., x_{N+1}$ with their corresponding y_n-values. Afterwards, we add $x_0 = y_0 = 0$ and the parameter $m = N + 1$ is introduced for convenience. Since no approximations are made between x_0 and x_1, the shift does not affect the other approximations. Secondly, we must make sure that at most two consecutive λ's attain positive values, in order to make sure our approximation errors are less than or equal to Δ^{max}. In order to do this, we introduce binary decision variables δ_{jtn} and the following constraints:

$$\sum_{n=0}^{m-1} \delta_{jtn} = 1, \forall j \in J, t \in T, \tag{4.17}$$

$$\lambda_{jt0} \leq \delta_{jt0}, \forall j \in J, t \in T, \tag{4.18}$$

$$\lambda_{jtm} \leq \delta_{jtm-1}, \forall j \in J, t \in T, \tag{4.19}$$

$$\lambda_{jtn} \leq \delta_{jtn-1} + \delta_{jtn}, \forall j \in J, t \in T, n \in \{1, \dots, m-1\}. \tag{4.20}$$

The Constraints (4.15) combined with (4.17) - (4.20) correspond to special ordered set type two (SOS2) constraints. Most software packages are already taking these SOS2 constraints into account, without explicitly including Constraints (4.17) - (4.20).

4.2.3. Column based approach

This method uses a column generation based approach to incorporate the probabilistic overtime constraint assuming the normal distribution into our model. We define K_s as the set of feasible combinations containing only groups of specialty $s \in S$, such that $\mathbb{P}(D_k \geq c_{jt}) \leq \alpha$, for some $\alpha \in [0, 1]$. Here, D_k is the duration of surgeries in combination $k \in K$. For each specialty $s \in S$, we consider all possible combinations of number of surgeries within each group associated to this specialty. Given our assumption that the durations of surgery groups follow a normal distribution, we approximate the sum of these durations with a normal distribution. This enables us to determine the parameters and evaluate whether our probabilistic constraint is met for every OR-day (j, t), which in turn determines if this combination is added to K_s. Moreover, we define K as the set of all feasible combinations, i.e. $K = \bigcup_{s\in S} K_s$. Again, since groups only belong to one specialty, we have that $\bigcap_{s\in S} K_s = \emptyset$. The set K can be found in Appendix B. In this model, we choose at most one combination $k \in K$ for each OR-day (j, t). This leads to the following decision variables:

$$U_{kjt} = \begin{cases} 0 & \text{combination } k \in K \text{ is not scheduled in OR } j \in J \text{ on day } t \in T. \\ 1 & \text{combination } k \in K \text{ is scheduled in OR } j \in J \text{ on day } t \in T. \end{cases}$$

As input, we know for all combinations $k \in K$ the number of surgeries from each group $i \in I$.

$$v_{ki} = \text{Number of surgeries of group } i \in I \text{ in combination } k \in K.$$

Note, that N_{ijt} can be replaced by $\sum_{k \in K} U_{kjt} \cdot v_{ki}$.

Since OR-days can have different available capacities, we need to indicate which combinations can be used for each OR-day. Therefore, we introduce the following input parameters:

$$p_{kjt} = \begin{cases} 0 & \text{combination } k \in K \text{ cannot be scheduled in OR } j \in J \text{ on day } t \in T. \\ 1 & \text{combination } k \in K \text{ can be scheduled in OR } j \in J \text{ on day } t \in T. \end{cases}$$

For each OR-day, we can choose at most one combination $k \in K$ and we must make sure that the MSS is preserved:

$$\sum_{k \in K} U_{kjt} \leq 1, \; \forall j \in J, t \in T. \tag{4.21}$$

$$U_{kjt} \leq p_{kjt}, \; \forall k \in K, j \in J, t \in T. \tag{4.22}$$

We also need to make sure that the minimum required number β_i is met for each group $i \in I$:

$$\sum_{k \in K} \sum_{j \in J} \sum_{t \in T} U_{kjt} \cdot v_{ki} \geq \beta_i, \; \forall i \in I. \tag{4.23}$$

Combining $N_{ijt} = \sum_{k \in K} U_{kjt} \cdot v_{ki}$ and using the expected value μ_i in (3.4), we get

$$\max \sum_{k \in K} \sum_{i \in I} \sum_{j \in J} \sum_{t \in T} U_{kjt} \cdot v_{ki} \cdot \mu_i. \tag{4.24}$$

4.3. Lognormal distribution

In the previous section, we considered the normal distribution to approximate the probability of overtime. However, it is important to note that the surgery duration is lognormally distributed, as mentioned by Wang *et al.* [12]. Hence, the total surgery duration is approximated by the sum of lognormal distributions. Yet, there has been no closed form to calculate this sum.

4.3.1. Expected duration

At first, we again use the expected value of the random variables D_{jt}. However, this time, we approximate them using the lognormal distribution for each group $i \in I$. Consequently, it becomes necessary to estimate the distribution parameters for each individual group. The expected value of a lognormally distributed random variable X with parameters μ_X and σ_X is given by $\exp\left(\mu_X + \frac{\sigma_X}{2}\right)$. We use the same model as outlined in Section 4.1, where $e_i = \exp\left(\mu_{D_i} + \frac{\sigma_{D_i}}{2}\right)$.

4.3.2. Column based approach

Again, we use a column generation based approach to incorporate our probabilistic overtime constraints. However, this time we assume the surgery durations are lognormally distributed. Secondly, we assume that the sum of surgery durations also has the lognormal distribution. To determine the corresponding parameters we use the Fenton-Wilkinson approximation.

Consider $X_1, \dots, X_H$ as H independent two-parameter log-normal random variables. Let X be the log-normal random variable which approximates $\sum_{i=1}^{H} X_i$. The Fenton-Wilkinson method is used to approximate the scale and shape parameters of X by matching the first and second moments of X about the origin to those of $\sum_{i=1}^{H} X_i$. For each $i \in I$, we determine parameters of D_i and use those to calculate the corresponding expected value and variance. Consequently, we get that $\mathbb{E}[X] = \sum_{i=1}^{H} \mathbb{E}[X_i]$ and $\text{Var}(X) = \sum_{i=1}^{H} \text{Var}(X_i)$. Once we calculated $\mathbb{E}[X]$ and $\text{Var}(X)$, we can use the explicit formula for expected value and variance of a lognormally distributed random variable, given by

$$\mathbb{E}[X] = \exp\left(\mu_X + \frac{\sigma_X^2}{2}\right),$$
$$\text{Var}(X) = (\exp(\sigma_X^2) - 1)(\mathbb{E}[X])^2,$$

to calculate the parameters of X. Consequently,

$$\sigma_X^2 = \log\left(\frac{\text{Var}(x)}{\mathbb{E}[X]^2} + 1\right),$$
$$\mu_x = \log(\mathbb{E}[X]) - \frac{\sigma_X^2}{2}.$$

Once these parameters are known, we use the same model as described in Subsection 4.2.3.

5
Data

In this chapter, we examine the data which is used to test our models. In Section 5.1, we elaborate on the surgeries, specialties and MSS we used. In the next section, we execute goodness of fit tests to evaluate which distribution is most suitable for the surgery duration.

5.1. Context

This data has been retrieved from an academic hospital in the Netherlands. It consists of 6426 surgeries, distributed among 9 specialties, which are listed in Table 5.1.

Table 5.1: Specialties

Abbreviation	Specialty
ENT	Ear, Nose and Throat
EYE	Ophthalmology
GYN	Gynaecology
NS	Neurological surgery
OB	Obstetrics
OMS	Oral and maxillofacial surgery
ORT	Orthopaedic urgery
PLA	Plastic surgery
URO	Urology

Given that each specialty has at most 3 groups, in total we have at most 27 groups. In fact, it turns out there are 25 groups. In Table 5.2, the number of groups per specialty is listed.

Table 5.2: Group division per specialty

Specialty	ENT	EYE	GYN	NS	OB	OMS	ORT	PLA	URO
Number of groups	3	3	3	3	2	3	2	2	3

We also use a MSS of 14 days (2 weeks, with 5 working days per week) to schedule the surgeries, as shown in Table 5.3.

Table 5.3: Master surgery scheduling for first and second week

OR	Monday	Tuesday	Wednesday	Thursday	Friday
OR1	ORT	ORT	ORT	ORT	
OR2	ORT	ORT / NS	ORT	ORT	ORT
OR3	URO	URO	URO	URO	URO
OR4			OB		
OR5	NS	NS		NS	NS
OR6	NS	NS	NS	NS	NS
OR7	NS	GYN	NS	NS	NS
OR8	NS	NS	NS	NS	NS
OR9	GYN	PLA / GYN	GYN	GYN	GYN
OR10	OMS	OMS	NS	OMS	
OR11	EYE	EYE	EYE	EYE	EYE
OR12	ENT	ENT / —	ENT	ENT	ENT
OR13	ENT	ENT	ENT	ENT	ENT
OR14	PLA		PLA		PLA

This MSS also determines if OR-day (j, t) is open. If it is open, we assume it can be open for 240, 480, 780 or 900 minutes. The opening times are specified in Table 5.4.

Table 5.4: Opening times for first and second week in minutes

OR	Monday	Tuesday	Wednesday	Thursday	Friday
OR1	480	480	480	480	
OR2	480	480 / 480	480	480	480
OR3	480	480	480	480	480
OR4			480		
OR5	480	480		480	480
OR6	480	480	480	480	480
OR7	480	480	480	480	480
OR8	480	480	480	480	480
OR9	480	900 / 480	480	480	480
OR10	240	480	240	480	
OR11	480	480	480	480	480
OR12	480	480 / 0	780	480	480
OR13	480	480	480	480	480
OR14	240		240		240

5.2. Surgery duration fitting

As described by Wang *et al.* [12], we saw that surgery durations have the lognormal distribution. Since we also work with the normal distribution, we compare the fit of both distributions per group using the two-sided Kolmogorov-Smirnov test (KS-test), Cramer-Von Mises test (CvM-test) and Anderson-Darling test (AD-test).

Let $X_1, \ldots, X_n$ be a sample having an unknown cumulative probability distribution F. The KS-test uses the empirical distribution function F_n of our sample, which is given by

$$F_n(x) = \frac{1}{n}\sum_{i=1}^{n} \mathbb{1}_{X_i \leq x} = \frac{\text{Number of observations} \leq x}{\text{Total number of observations}}.$$

The KS-test tests the null hypothesis H_0 that $F = F_0$, which tests if our distribution F equals the tested distribution F_0. Therefore, it uses the statistic

$$T = \sup_{x \in \mathbb{R}} |F_n(x) - F_0(x)|.$$

For large values, the KS-test will reject the null-hypothesis. Another test used in statistics, is the CvM-test. This test uses another statistic to test the null hypothesis, given by

$$T = n \int_{-\infty}^{\infty} \left(F_n(x) - F_0(x)\right)^2 \, dx.$$

Lastly, we use the AD-test, which uses

$$T = n \int_{-\infty}^{\infty} \frac{\left(F_n(x) - F_0(x)\right)^2}{F_0(x)(1 - F_0(x))} \, dx$$

as its test statistic. The AD-test uses another weight function compared to the CvM-test, to take the observations in the tail more into account.

Now, we evaluate the null hypothesis for each group. This involves determining whether we can reject the null hypothesis based on the available evidence. To make this determination, we calculate the p-value and compare it with a predetermined significance level, denoted as ξ. A commonly used significance level is $\xi = 0.05$. The p-values are computed using the following formula:

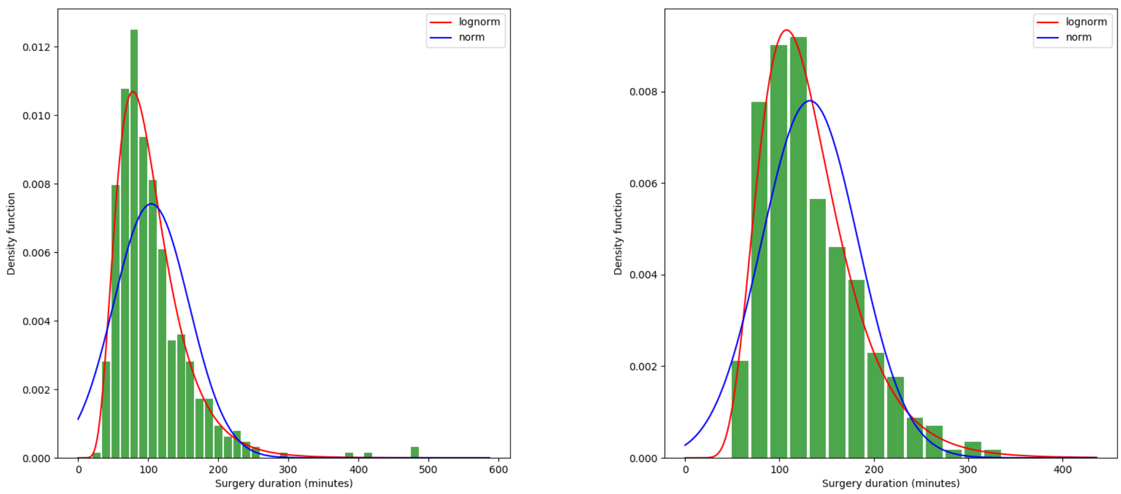
$$2 \min\left(\mathbb{P}(T \leq t), \mathbb{P}(T \geq t)\right),$$

where T represents the observed statistic. If the p-value is below or equal to ξ, we have strong evidence to reject the null hypothesis. Conversely, if the p-value exceeds ξ, we lack sufficient evidence to reject the null hypothesis.

Table 5.5: Goodness of fit p-values for the distribution per group.

	KS-test		CvM-test		AD-test	
group	normal	lognormal	normal	lognormal	normal	lognormal
NS1	0.006	**0.489**	0.061	**0.620**	0.058	**0.681**
NS2	0.001	**0.684**	0.001	**0.703**	0.000	**0.756**
NS3	0.000	**0.176**	0.000	**0.189**	0.000	**0.171**
GYN1	**0.320**	0.214	**0.346**	0.227	**0.239**	0.221
GYN2	0.002	**0.636**	0.005	**0.476**	0.002	**0.554**
GYN3	0.321	**0.857**	0.380	**0.857**	0.338	**0.934**
OMS1	0.450	**0.801**	0.534	**0.743**	0.422	**0.785**
OMS2	0.272	**0.858**	0.256	**0.856**	0.243	**0.903**
OMS3	0.026	**0.295**	0.018	**0.254**	0.012	**0.223**
ENT1	**0.020**	0.010	**0.041**	0.028	**0.026**	0.023
ENT2	0.000	**0.315**	0.000	**0.368**	0.000	**0.368**
ENT3	0.000	**0.033**	0.000	**0.032**	0.000	**0.020**
EYE1	0.001	**0.160**	0.015	**0.325**	0.013	**0.257**
EYE2	0.030	**0.046**	0.039	**0.081**	0.014	**0.076**
EYE3	0.000	**0.032**	0.000	**0.144**	0.000	**0.147**
ORT1	0.469	**0.514**	**0.616**	0.475	**0.591**	0.517
ORT2	0.035	**0.457**	0.070	**0.219**	0.021	**0.203**
ORT3	0.000	**0.045**	0.000	**0.031**	0.000	**0.024**
PLA1	0.795	**0.825**	0.775	**0.884**	0.794	**0.920**
PLA2	0.100	**0.680**	0.093	**0.778**	0.084	**0.852**
URO1	0.011	**0.600**	0.020	**0.571**	0.013	**0.486**
URO2	**0.628**	0.427	**0.856**	0.612	**0.928**	0.689
URO3	0.508	**0.537**	**0.858**	0.663	**0.853**	0.728
OB1	0.459	**0.835**	0.454	**0.969**	0.445	**0.974**
OB2	0.023	**0.371**	0.030	**0.236**	0.019	**0.202**

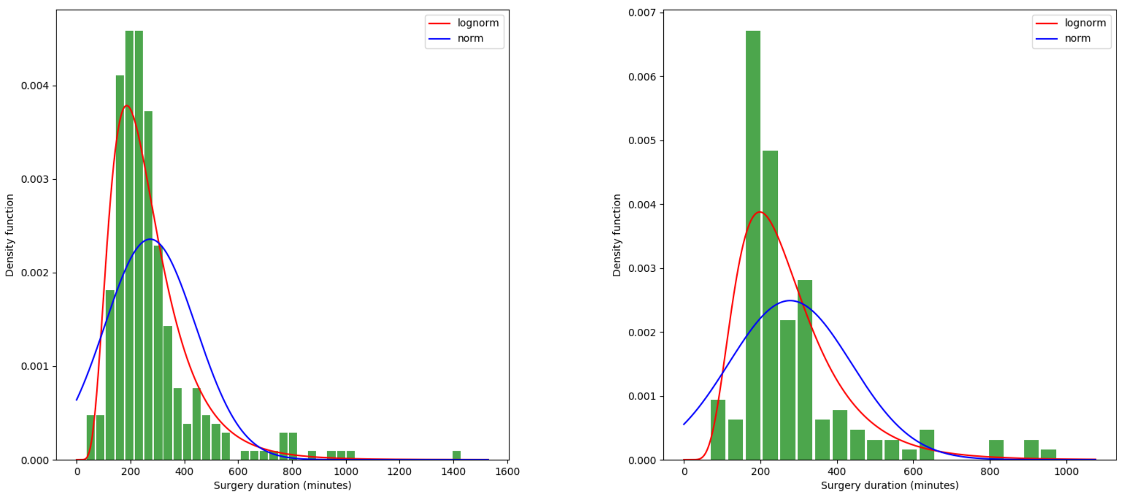
Table 5.5 shows that most groups are more likely to have the lognormal distribution, compared to the normal distribution. For example, in Figure 5.1 we see that groups ENT2 and GYN2 are more likely to have the lognormal distribution.



(a) Group ENT2 (b) Group GYN2

Figure 5.1: Two groups which are more likely to have the lognormal distribution.

However, we cannot ignore that some groups are not very likely to have the lognormal distribution. One reason could be, because these groups have some values which cannot be included in the tail of their approximated lognormal distribution, as shown in Figure 5.2.

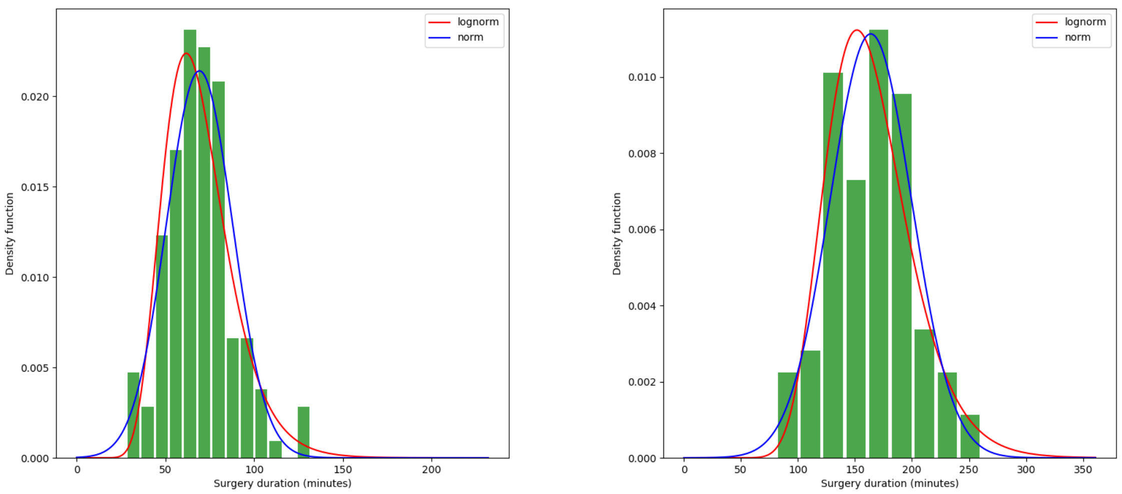


(a) Group ENT3

(b) Group ORT3

Figure 5.2: Two groups which have problems fitting the distribution in the tail.

Another reason could be, that some groups have the tendency to follow a normal distribution, because they are more symmetrical around the mean, as shown in Figure 5.3.



(a) Group ORT1

(b) Group URO2

Figure 5.3: 2 groups which are more symmetrical.

Although, we see that not all groups have the same distribution, we consider in each method at most one and the same distribution for all groups.

6
Results

In this chapter, the models are implemented using the PuLP module version 2.4 in Python version 3.7.5. In PuLP, multiple solvers can be used. For our test, we use CPLEX version 20.1.0. Furthermore, to test our models, we use a PC with an Intel Core i5-10210U 1.60 GHz with 8.00 GB RAM. We execute our different models, simulate and compare the results. In order to compare the different models, we consider different aspects, such as utilization, overtime and computing times. In our overtime constraints, we take $\alpha = 0.05$.

Firstly, we determine OR-utilization (percentage) obtained by running the corresponding ILP. Our model determines for each group and each OR-day the number of surgeries that are scheduled. To simulate how the resulting schedule would perform in practice, we randomly pick surgeries per group from our data set. We perform both 1000 and 10,000 simulations to compare the simulated OR-utilizations with our model. Moreover, we use the simulations to calculate the overtime percentage, which is defined as

$$\text{Overtime percentage} = \frac{\text{Number of OR-days with overtime}}{\text{Total number of OR-days}} \cdot 100\%.$$

6.1. Binary search

In this section, we test our binary search models. Since we have three models which use binary search, we show the results individually first. For the results of each method, we look at the objective function value of our models. Afterwards we look at the average simulated utilization and overtime. Lastly, we examine the computing time, which has been divided into two components: model and simulation. Consequently, their sum represents the total computing time. After executing the three models, we compare the results.

Firstly, we test the binary search method which uses the average group duration, as described in Section 4.1. The results obtained from this method are presented in Tables 6.1 and 6.2. Interestingly, in both simulations, it is found that an overtime factor of $q = 0.25$ is the optimal choice for this method.

Table 6.1: Simulating by reducing the capacity with factor q using the average, 1000 times.

q	F_q	comp. time model	comp. time simulation	total comp. time
0.5	infeasible	1.5 s	0 s	1.5 s
0.25	0.043	8.2 s	11.3 s	19.5 s
0.12	0.144	8.9 s	13.9 s	22.8 s
0.18	0.094	8.7 s	11.5 s	20.2 s
0.21	0.077	4.7 s	11.3 s	15.9 s
0.23	0.054	5.3 s	11.5 s	16.8 s
0.24	0.053	7.3 s	11.0 s	18.3 s

Table 6.2: Simulating by reducing the capacity with factor q using the average, 10000 times.

q	F_q	comp. time model	comp. time simulation	total comp. time
0.5	infeasible	1.2 s	0 s	1.2 s
0.25	0.042	9.4 s	102.2 s	111.6 s
0.12	0.143	9.7 s	103.6 s	113.3 s
0.18	0.094	8.2 s	104.3 s	112.5 s
0.21	0.076	4.8 s	103.7 s	108.5 s
0.23	0.054	5.3 s	106.0 s	111.3 s
0.24	0.053	7.5 s	105.4 s	112.9 s

Secondly, we test the binary search method using the normal distribution, as described in Section 4.2.1. The obtained results are presented in Tables 6.3 and 6.4. Again, in both simulations, it is found that an overtime factor of $q = 0.25$ is the optimal choice for this method.

Table 6.3: Simulating by reducing the capacity with factor q using the expected value of a normal distribution, 1000 times.

q	F_q	comp. time model	comp. time simulation	total comp. time
0.5	infeasible	1.5 s	0 s	1.5 s
0.25	0.042	8.4 s	11.1 s	19.5 s
0.12	0.143	7.9 s	11.6 s	19.5 s
0.18	0.094	8.5 s	11.7 s	20.2 s
0.21	0.077	5.5 s	11.3 s	16.8 s
0.23	0.056	5.4 s	12.0 s	17.4 s
0.24	0.051	7.2 s	11.7 s	18.9 s

Table 6.4: Simulating by reducing the capacity with factor q using the expected value of a normal distribution, 10000 times.

q	F_q	comp. time model	comp. time simulation	total comp. time
0.5	infeasible	1.4 s	0 s	1.4 s
0.25	0.043	10.9 s	104.0 s	114.9 s
0.12	0.143	7.6 s	103.9 s	111.6 s
0.18	0.094	8.2 s	108.2 s	116.4 s
0.21	0.076	5.4 s	106.3 s	111.8 s
0.23	0.054	5.0 s	105.7 s	110.8 s
0.24	0.052	7.3 s	104.9 s	112.2 s

Finally, we show the results from the binary search method using the lognormal distribution, as descibed in Section 4.3.1. These results are presented in Tables 6.5 and 6.6. Interestingly, in both simulations, it is found that an overtime factor of $q = 0.23$ is the optimal choice for this method.

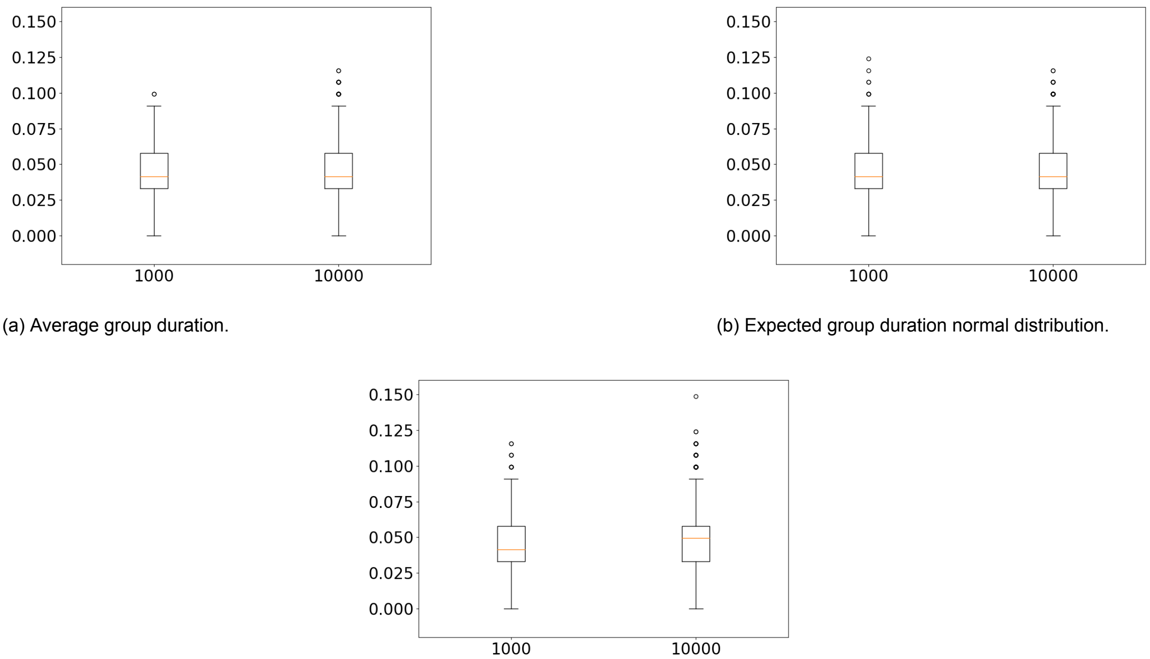
Table 6.5: Simulating by reducing the capacity with factor q using the expected value of a lognormal distribution, 1000 times.

q	F_q	comp. time model	comp. time simulation	total comp. time
0.5	infeasible	1.3 s	0 s	1.3 s
0.25	infeasible	7.1 s	0 s	7.1 s
0.12	0.131	5.9 s	12.3 s	18.1 s
0.18	0.077	7.9 s	11.1 s	19.0 s
0.21	0.062	5.7 s	11.3 s	16.9 s
0.23	0.046	4.9 s	11.1 s	16.0 s
0.22	0.057	6.2 s	12.6 s	18.8 s

Table 6.6: Simulating by reducing the capacity with factor q using the expected value of a lognormal distribution, 10000 times.

q	F_q	comp. time model	comp. time simulation	total comp. time
0.5	infeasible	1.5 s	0 s	1.5 s
0.25	infeasible	9.0 s	0 s	9.0 s
0.12	0.131	8.0 s	111.6 s	119.6 s
0.18	0.077	8.1 s	104.6 s	112.7 s
0.21	0.060	5.5 s	109.1 s	114.6 s
0.23	0.047	4.8 s	104.9 s	109.8 s
0.22	0.057	5.5 s	109.1 s	114.6 s

Now that we have looked at the results of our model, let us have a look at the results of the corresponding simulations. In Figure 6.1, the simulated overtime factors for the binary search methods are displayed. Followed by the results from our model and simulation in Table 6.7.



(a) Average group duration.

(b) Expected group duration normal distribution.

(c) Expected group duration lognormal distribution.

Figure 6.1: Boxplot showing the simulated overtime factors for binary search methods.

Table 6.7: Comparing the results.

Binary search method	Average		Normal distribution		Lognormal distribution	
Simulated scenarios	1000	10000	1000	10000	1000	10000
Model OR-utilization	38640	38640	38643	38643	40863	40863
Model OR-utilization percentage	67.9%	67.9 %	67.9%	67.9%	71.8%	71.8%
Average simulation OR-utilization	38655	38649	38643	38651	40856	40864
Average simulation OR-utilization percentage	67.9%	67.9 %	67.9%	67.9%	71.8%	71.8%
Average simulation overtime percentage	4.3%	4.2%	4.2%	4.3 %	4.7%	4.7%
Percentage of cases exceeding the threshold	25.4%	25.6%	26.0%	25.5 %	32.3%	34.3%
Computing time	115.1 s	671.6 s	114.0 s	679.2 s	97.5 s	582.2 s

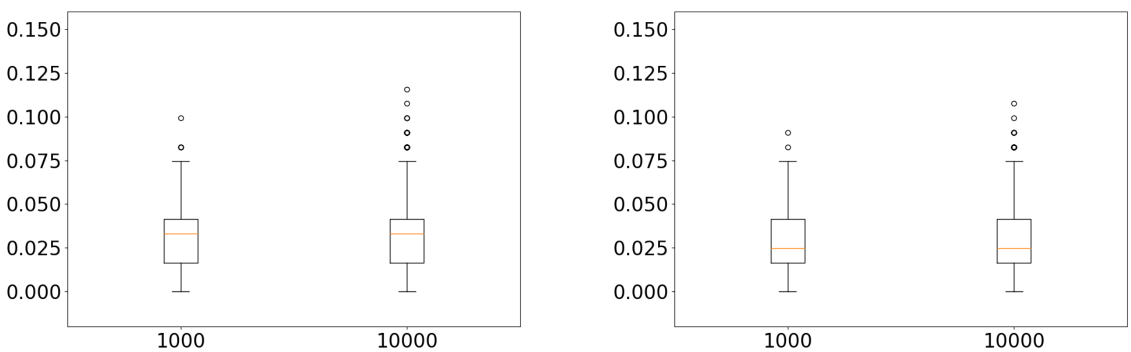
The results obtained from conducting the three binary search methods are presented in Table 6.7.

Upon examining the table, we observe that the simulated OR-utilization closely resembles the model's expected values. Additionally, the average percentages of overtime derived from the simulations are slightly lower than our predefined 5% overtime threshold. This can be explained by the nature of the binary search method, which selects the largest value that is still below the specified threshold.

However, looking at Figure 6.1, we see the range of simulated overtime factors. Notice, that there are values surpassing our threshold. Defining the overtime factor as an average, might be the cause of this phenomenon. Our binary search method stops when the average simulated overtime is below our threshold of 0.05. However, as Figure 6.1 suggests, there are still some values above this threshold. In fact, it turns out that the percentage of simulated cases exceeding this threshold is way higher for all three methods. Consequently, employing a 95% confidence interval could potentially yield better results. This interval implies that 95% of the calculated overtime factors should be equal to or below our predetermined threshold. Upon analysis, it becomes apparent that increasing the simulation sample size does not yield substantial improvements in the results. The marginal benefits obtained from larger samples are overshadowed by the substantial increase in computational time. Considering the trade-off between computational resources and the marginal gains achieved, it is advisable to refrain from employing larger sample sizes.

6.2. Column based approach

In this section, we test our column based approach, considering both the normal and lognormal distribution. In order to use this method, we first have to create the set K, containing combinations of feasible group numbers for the different used capacities, as described in Section 4.3.2. This set can be found in Appendix B. When considering OR-days with capacities of 240, 480, or 900 minutes, there is minimal difference observed between the sets generated assuming normal and lognormal distributions. However, when we examine the set of combinations for OR-days with a capacity of 780 minutes, a noticeable difference becomes apparent. Specifically, assuming the lognormal distribution results in a higher number of feasible group combinations compared to the normal distribution. This discrepancy suggests that the lognormal distribution assumption allows for a wider range of group combinations, enhancing the flexibility of our column generating based approach.



(a) Normal distribution. (b) Lognormal distribution.

Figure 6.2: Boxplot showing the simulated overtime factors for column based approach.

Table 6.8: Comparing the results.

Column based approach	Normal distribution		Lognormal distribution	
Simulated scenarios	1000	10000	1000	10000
Model OR-utilization	39036	39036	38908	38908
Model OR-utilization percentage	68.6%	68.6 %	68.3%	68.3%
Model computing time	8.7 s	8.7 s	8.9 s	8.9 s
Average simulation OR-utilization	39053	39030	38869	38879
Average simulation OR-utilization percentage	68.6%	68.5 %	68.3%	68.3%
Average simulation overtime percentage	3.2%	3.2 %	2.9%	2.9%
Percentage of cases exceeding the threshold	7.5%	8.6 %	6.7%	6.4%
Simulation computing time	25.0 s	251.5 s	24.7 s	247.1 s

Looking at the results in Table 6.8, we again see that the model's expected values are similar to the average simulated values. However, in contrast to the binary methods, this time the average simulated overtime percentage is significantly lower, as well as the percentage of exceeding cases in the simulations. This observation can be attributed to the incorporation of a probabilistic constraint utilized in the creation of a predefined set. While it is true that the average simulated overtime percentage is below our predefined threshold, it is important to note, that there are still instances where the values exceed this threshold, as shown in Figure 6.2. Though, in this case, the percentage of simulated cases exceeding the threshold is not as high as in the binary search methods.

6.3. Closed form

Laslty, we use a closed form model to incorporate the probabilistic overtime constraints assuming the normal distribution, as described in Section 4.2.2. Firstly, we have to determine x_{max}. With the retrieved data, we have that $x_{max} = 45367$ minutes. We choose the maximum approximation error $\Delta^{max} = 1$ minute. This means we need 11 piecewise linear functions, such that our approximation error is always less than or equal to 1.

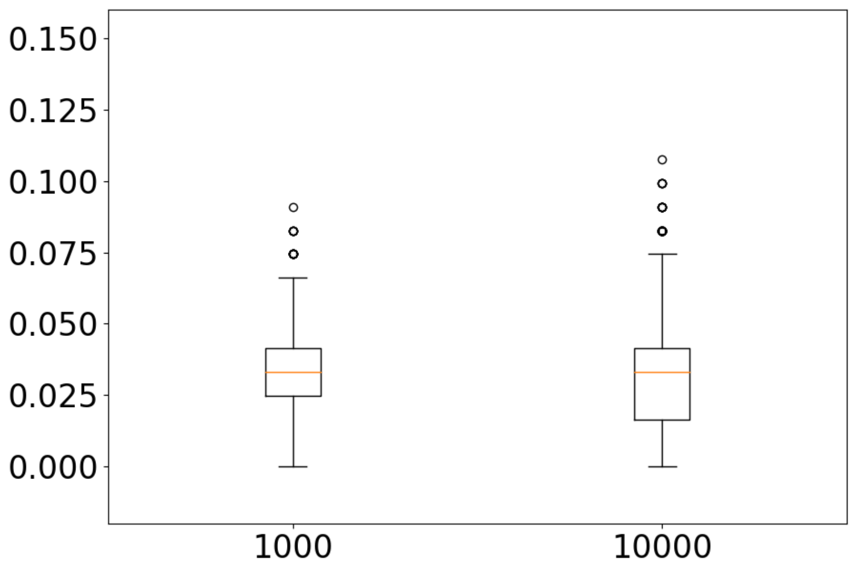


Figure 6.3: Boxplot using closed form.

Table 6.9: Comparing the results.

Simulated scenarios	1000	10000
Model OR-utilization	39036	39036
Model OR-utilization percentage	68.6%	68.6 %
Model computing time	55.4 s	55.4 s
Average simulation OR-utilization	39036	39044
Average simulation OR-utilization percentage	68.6%	68.6 %
Average simulation overtime percentage	3.1%	3.1%
Percentage of cases exceeding the threshold	9.3%	8.9%
Simulation computing time	20.8 s	211.8 s

Table 6.9 displays the results for reference. Additionally, Figure 6.3 visually represents the range of average overtime factors obtained from the simulations. Examining the average values in Table 6.9, we notice minimal differences between the two simulation scenarios. However, there is a notable variation in the spread of the data. This suggests that when conducting a higher number of simulations, a greater number of outliers can be observed. It is interesting to note that although the percentage of cases for simulating 10,000 instances is lower than simulating 1000 cases, the presence of outliers is more pronounced in the former.

Considering the nature of the simulations, where random surgeries fill the Master Surgery Schedule (MSS), the variation in the spread of data could potentially be attributed to chance occurrences. As each simulation creates a set of random surgeries, the presence of outliers might be coincidental.

6.4. Distribution time OR-day

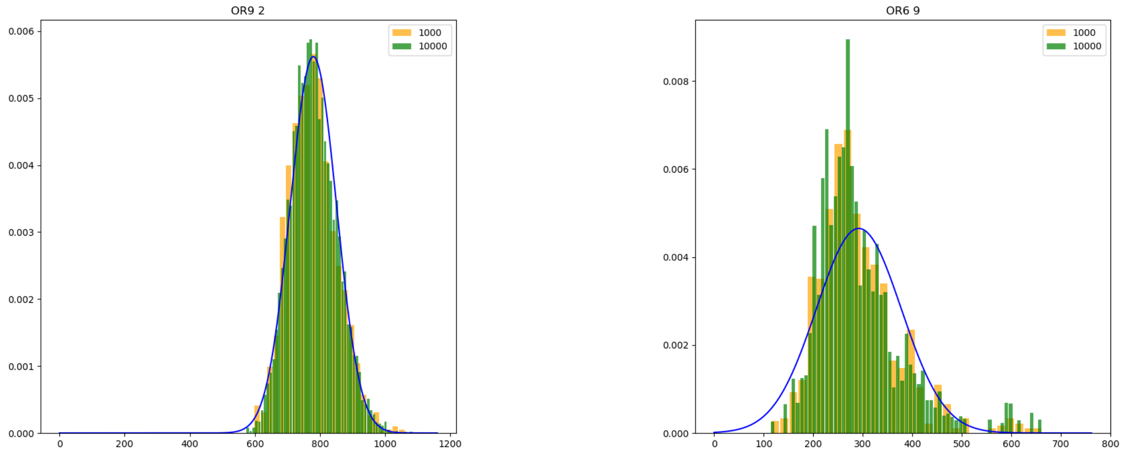
In this section, we examine the fit of OR-days by considering the assumption that the total surgery durations D_{jt} for OR-day (j,t) either have a normal or lognormal distribution. Therefore, we again use the KS-test, CvM-test and AD-test, as described in Section 5.2. As displayed in Figure 6.4, the p-values of the lognormal distribution have higher values, for both number of simulations. This implies that the duration of OR-days are more likely to have the lognormal distribution.

(a) 1000 simulations.

(b) 10000 simulations.

Figure 6.4: Boxplot with p-values testing normal and lognormal distribution for OR-days.

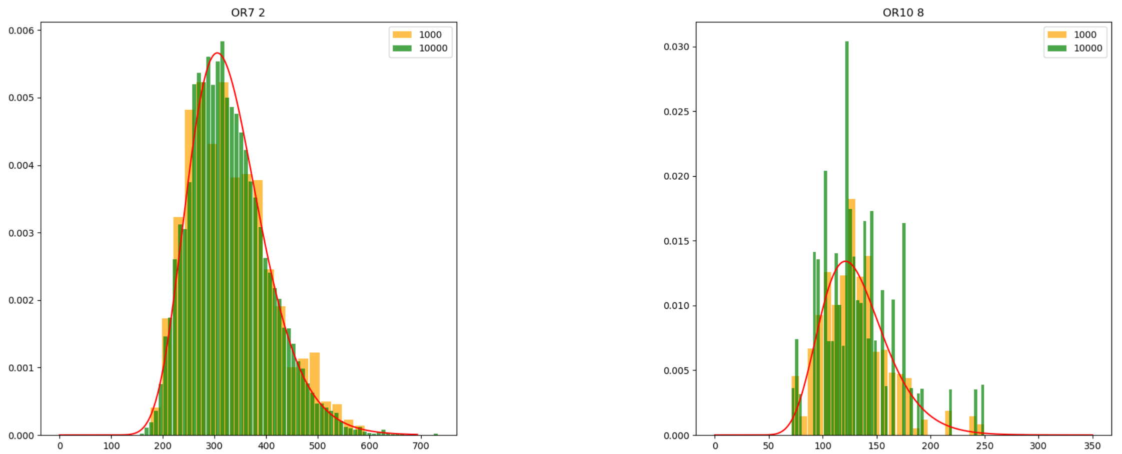
Figures 6.5 and 6.6 present the graphical representations of some fitted OR-day distributions.



(a) null hypothesis is not rejected.

(b) null hypothesis is rejected.

Figure 6.5: Fitted normal distribution.



(a) null hypothesis is not rejected.

(b) null hypothesis is rejected.

Figure 6.6: Fitted lognormal distribution.

Observing some results in Figures 6.5 and 6.6, it becomes apparent that not all OR-days adhere to either a normal or lognormal distribution. In Chapter 7, we provide an in-depth analysis and discussion of this phenomenon. It is important to note that comparing the same OR-day under different distributions is not feasible, as our column based models may yield different combinations for each OR-day.

6.5. Overview

In this section, we give an overview of the results, given in the previous sections. Once this overview has been shown, we compare the results and draw a conclusion. For convenience, we use the following abbreviations in our overviews:

- $M1$ = Average duration;
- $M2$ = Expected duration normal;
- $M3$ = Closed form normal;
- $M4$ = Column based approach normal;

- $M5$ = Expected duration lognormal;
- $M6$ = Column based approach lognormal.

Table 6.10: Comparing the results for models.

	$M1$	$M2$	$M3$	$M4$	$M5$	$M6$
OR-utilization	38640	38643	39036	39053	40863	38908
OR-utilization percentage	67.9%	67.9%	68.6%	68.6%	71.8%	68.3%
Computing time	115.1 s	114.0 s	55.4 s	8.7 s	97.5 s	8.9 s

Table 6.11: Comparing the results for 1000 simulations.

	$M1$	$M2$	$M3$	$M4$	$M5$	$M6$
Average OR-utilization	38655	38643	39036	39053	40856	38869
Average OR-utilization percentage	67.9%	67.9%	68.6%	68.6%	71.8%	68.3%
Average overtime percentage	4.3%	4.2%	3.1%	3.2%	4.7%	2.9%
Percentage of cases exceeding the threshold	25.4%	26.0%	9.3%	7.5%	32.3%	6.7%
Computing time	115.1 s	114.0 s	20.8 s	25.0 s	97.5 s	24.7 s

Table 6.12: Comparing the results for 10000 simulations.

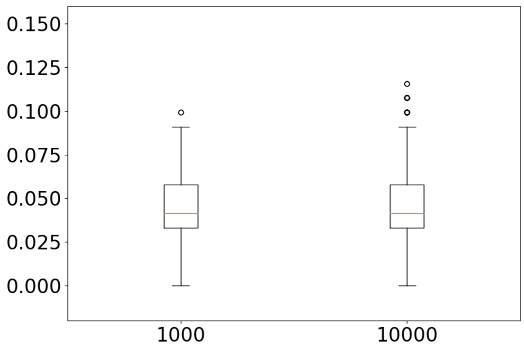
	$M1$	$M2$	$M3$	$M4$	$M5$	$M6$
Average OR-utilization	38649	38651	39044	39030	40864	38879
Average OR-utilization percentage	67.9%	67.9%	68.6%	68.5%	71.8%	68.3%
Average overtime percentage	4.2%	4.3%	3.1%	3.2%	4.7%	2.9%
Percentage of cases exceeding the threshold	25.6%	25.5%	8.9%	8.6%	34.3%	6.4%
Computing time	671.6 s	679.2 s	211.8 s	251.5 s	582.2 s	247.1 s

Upon analyzing the outcomes presented in Table 6.10, we can draw several observations. Firstly, method $M5$ demonstrates superior performance in terms of OR-utilization compared to the other methods. However, it is important to note that this improvement comes at the expense of increased computing time. Furthermore, the reduced capacity methods, namely $M1, M2,$ and $M5$, exhibit significantly higher average overtime percentages compared to the closed form method ($M3$) and the column based approach methods ($M4$ and $M6$). This discrepancy arises due to the nature of the binary search method, which aims to identify an overtime factor q that remains slightly below the predefined threshold of 0.05. However, it is worth noting, that despite this optimization strategy, there are still cases where the threshold is exceeded, as indicated by the range of overtime values in Figure 6.7.

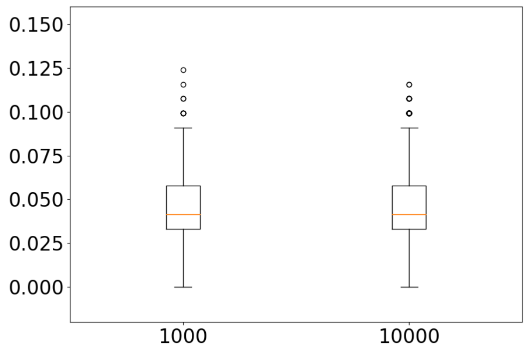
While method $M5$ stands out in terms of OR-utilization, it may not be the most suitable approach for our problem, since it does not explicitly consider the probabilistic nature of our constraints. Instead, it relies on reducing the capacity to ensure that the average overtime percentage of the simulation remains below the threshold. However, the presence of cases exceeding the threshold indicates the limitations of this approach. In contrast, methods $M3, M4,$ and $M6$ explicitly incorporate probability distributions into the model. Among these methods, the normal distribution slightly outperforms the lognormal distribution, although the difference is not as significant as observed in method $M5$.

Examining the results of the simulations in Tables 6.11 and 6.12, we find minimal differences between the simulations for each model in terms of average values. However, there is a slight variation in the average overtime percentage and the percentage of cases exceeding the threshold.

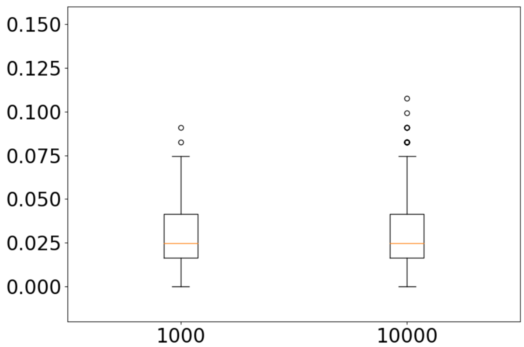
Considering these observations, it is important to strike a balance between OR-utilization, overtime percentage, and computing time when selecting an appropriate method for our problem. While method $M5$ demonstrates impressive utilization, the presence of cases exceeding the threshold and the longer computing time raise concerns. On the other hand, methods $M3, M4,$ and $M6$ offer a more probabilistic approach, resulting in lower overtime percentages. Moreover, when considering the percentage of cases exceeding the threshold, an important factor in evaluating the performance of the methods, we observe that methods $M1, M2,$ and $M5$ raise concerns. These methods exhibit a relatively high percentage of cases (at least 25%) that surpass our predefined threshold. This finding highlights the need to carefully assess the suitability of these methods for our problem. In contrast, methods $M3, M4,$ and $M6$ demonstrate better performance in terms of the percentage of cases exceeding the threshold, as well as computing time, compared to the aforementioned methods. Although these methods slightly underperform in terms of OR-utilization, the significant improvement in the percentage of cases below the desired threshold and the reduced computing time make them viable alternatives to consider. Overall, when making a decision, we should weigh the trade-offs between OR-utilization, the percentage of cases exceeding the threshold, and computing time. While methods $M1, M2,$ and $M5$ excel in OR-utilization, their higher percentage of cases exceeding the threshold raises concerns. On the other hand, methods $M3, M4,$ and $M6$ demonstrate better performance in terms of controlling overtime and computing time, albeit with a slight sacrifice in OR-utilization.



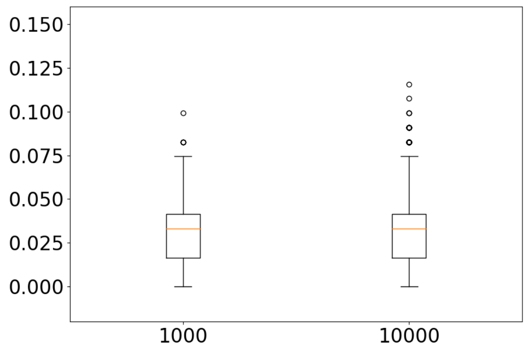
(a) Average.



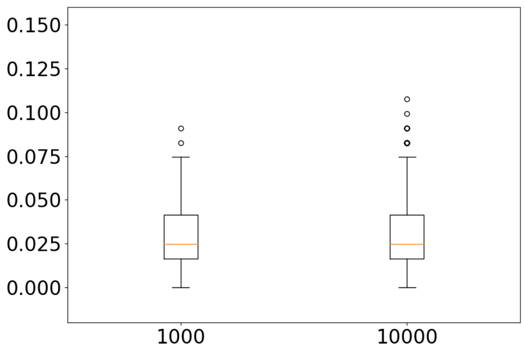
(b) Normal expected.



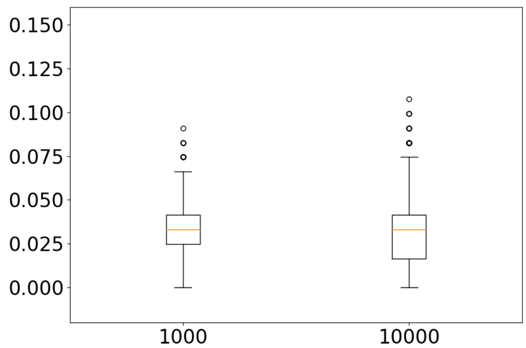
(c) Lognormal expected.



(d) Normal column.



(e) Lognormal column.



(f) Norm closed.

Figure 6.7: Boxplot showing the simulated overtime factors.

7 Conclusions and recommendations

In Chapter 1, we have stated our research goals:

- maximizing OR utilization
- minimizing overtime

Section 7.1 discusses the conclusions from this research. In Section 7.2, we give recommendations for further research.

7.1. Conclusions

In Chapter 2, we defined the relevant topics for our research. We discussed the elective patient, master surgery scheduling (MSS), integer linear programming (ILP), surgery duration and overtime. In Chapter 3, we gave a formal problem description and formulated the corresponding mathematical model. In Chapter 4, we incorporated the overtime constraints. Firstly, we used the average surgery group duration combined with an overtime factor to reduce the available capacity. This method was developed by us. Secondly, we used a normal distribution for the total OR-day durations. Three methods were applied assuming the normal distribution, namely overtime factor, direct form and column based approach. Lastly, we incorporated the probabilistic constraints using the lognormal distribution with an overtime factor and column based approach. In Chapter 5, we elaborated the data which was used to test our models. By fitting the data, we examined if the lognormal and normal distribution were an appropriate distribution to base our research on. In Chapter 6, we tested our models. Next, we evaluated the resulting utilization, overtime and computation time obtained from our simulation. Upon examination, it appears that using the expected value assuming the lognormal distribution yields the highest level of OR-utilization. However, it is important to note that this approach does not explicitly consider the probability of overtime. Moreover, looking at the simulated cases exceeding the threshold, we should use a column based or closed form method.

7.2. Recommendations

Our findings are derived from a specific dataset that includes a minimum number of mandatory surgeries for each group. It is important to acknowledge that results can significantly vary when using different datasets. Conducting experiments with diverse datasets can provide valuable insights into the robustness and generalizability of our model and findings. It is worth noting, that the current definition of overtime may not be the best approach, as the range of our simulations still allows numerous values surpassing the predefined threshold. Considering the use of a (95%-)confidence interval could be more appropriate in order to provide a more comprehensive assessment of overtime probabilities. In the future, it might be possible that a more accurate approximation method may be developed to effectively incorporate the sum of lognormally distributed random variables, providing a more efficient and accurate modeling framework. Moreover, it would be valuable to consider the financial aspects of surgery scheduling, as well as taking into account the specific needs and preferences of the hospital staff. Incorporating these factors into the scheduling process can contribute to a more comprehensive, realistic and well-rounded decision-making framework. Although we used the normal and lognormal distributions to approximate the surgery duration, we suggest for further research to use the empirical distribution. The advantage is that you do not need to fit a distribution to your data. Lastly, it is important to acknowledge that despite our efforts to stay below the overtime threshold, the inherent variability in surgery duration introduces the possibility of forced overtime. This variability is a natural occurrence in surgical settings and must be taken into account when designing our model. In other words, it is crucial to recognize that there will be instances where the actual surgery duration exceeds the planned duration, leading to unavoidable overtime. This is a reality that needs to be accepted and appropriately incorporated into a scheduling model.

A

Mathematical model

This appendix is written to give a general overview of the different models we used in our research.

A.1. Average duration & expected duration

$$\max \sum_{t\in T}\sum_{j\in J}\sum_{i\in I} N_{ijt}\cdot e_i. \tag{A.1}$$

s.t.

$$\sum_{i\in I} N_{ijt}\cdot e_i \leq (1-q)\cdot c_{jt},\ \forall j\in J, t\in T, \tag{A.2}$$

$$\sum_{t\in T}\sum_{j\in J} N_{ijt} \geq \beta_i,\ \forall i\in I, \tag{A.3}$$

$$\sum_{i\in I_s} N_{ijt} \leq M_s\cdot a_{sjt},\ \forall s\in S, j\in J, t\in T, \tag{A.4}$$

$$N_{ijt}\in \mathbb{Z}_{\geq 0},\ \forall i\in I, j\in J, t\in T. \tag{A.5}$$

A.2. Column based approach

$$\max \sum_{k\in K}\sum_{i\in I}\sum_{j\in J}\sum_{t\in T} U_{kjt}\cdot v_{ki}\cdot e_i. \tag{A.6}$$

s.t.

$$\sum_{k\in K} U_{kjt} \leq 1,\ \forall j\in J, t\in T, \tag{A.7}$$

$$U_{kjt} \leq p_{kjt},\ \forall k\in K, j\in J, t\in T, \tag{A.8}$$

$$\sum_{k\in K}\sum_{j\in J}\sum_{t\in T} U_{kjt}\cdot v_{ki} \geq \beta_i,\ \forall i\in I. \tag{A.9}$$

A.3. Normal distribution closed form

$$\max \sum_{t\in T}\sum_{j\in J}\sum_{i\in I} N_{ijt}\cdot e_i. \tag{A.10}$$

s.t.

$$\sum_{i\in I} N_{ijt}\cdot \mu_i + \Phi^{-1}(1-\alpha)\sum_{n=0}^{N}\lambda_{jtn}y_n \le c_{jt},\ \forall j\in J, t\in T, \tag{A.11}$$

$$\sum_{n=0}^{N}\lambda_{jtn}x_n = \sum_{i\in I} N_{ijt}\sigma_i^2,\ \forall j\in J, t\in T, \tag{A.12}$$

$$\sum_{n=0}^{N}\lambda_{jtn} = 1,\ \forall j\in J, t\in T, \tag{A.13}$$

$$\sum_{n=0}^{m-1}\delta_{jtn} = 1,\ \forall j\in J, t\in T, \tag{A.14}$$

$$\lambda_{jt0} \le \delta_{jt0},\ \forall j\in J, t\in T, \tag{A.15}$$

$$\lambda_{jtm} \le \delta_{jtm-1},\ \forall j\in J, t\in T, \tag{A.16}$$

$$\lambda_{jtn} \le \delta_{jtn-1} + \delta_{jtn},\ \forall j\in J, t\in T, n\in\{1,\dots,m-1\}, \tag{A.17}$$

$$\sum_{t\in T}\sum_{j\in J} N_{ijt} \ge \beta_i,\ \forall i\in I, \tag{A.18}$$

$$\sum_{i\in I_s} N_{ijt} \le M_s\cdot a_{sjt},\ \forall s\in S, j\in J, t\in T, \tag{A.19}$$

$$N_{ijt}\in\mathbb{Z}_{\ge 0},\ \forall i\in I, j\in J, t\in T. \tag{A.20}$$

$$\lambda_{jtn}\in[0,1],\ \forall j\in J, t\in T, n\in\{0,\dots,N\}. \tag{A.21}$$

$$\delta_{jtn}\in[0,1],\ \forall j\in J, t\in T, n\in\{0,\dots,m-1\}. \tag{A.22}$$

B Combinations

In this appendix, the combinations sets for both normal and lognormal column generating methods are displayed, divided into the different capacity categories.

B.1. 240-capacity

Table B.1: Feasible combination set for 240-capacity OR-days.

index	specialty	combination	normal distribution	lognormal distribution
0	NS	(0, 1, 0)	1	0
1	NS	(1, 0, 0)	1	1
2	NS	(2, 0, 0)	1	1
3	OMS	(0, 1, 0)	1	1
4	OMS	(1, 0, 0)	1	1
5	PLA	(0, 1)	1	1
6	PLA	(1, 0)	1	1
7	PLA	(1, 1)	1	1
8	PLA	(2, 0)	1	1

B.2. 480-capacity

Table B.2: Feasible combination set for 480-capacity OR-days.

index	specialty	combination	normal distribution	lognormal distribution
9	NS	(0, 0, 1)	1	1
10	NS	(0, 1, 0)	1	1
11	NS	(0, 2, 0)	1	1
12	NS	(1, 0, 0)	1	1
13	NS	(1, 1, 0)	1	1
14	NS	(2, 0, 0)	1	1
15	NS	(2, 1, 0)	1	1
16	NS	(3, 0, 0)	1	1
17	NS	(3, 1, 0)	1	1
18	NS	(4, 0, 0)	1	1
19	NS	(5, 0, 0)	1	1
20	GYN	(0, 0, 1)	1	1
21	GYN	(0, 1, 0)	1	1
22	GYN	(0, 2, 0)	1	1
23	GYN	(1, 0, 0)	1	1

24	GYN	(1, 0, 1)	1	1
25	GYN	(1, 1, 0)	1	1
26	GYN	(1, 2, 0)	1	1
27	GYN	(2, 0, 0)	1	1
28	GYN	(2, 1, 0)	1	1
29	GYN	(3, 0, 0)	1	1
30	GYN	(3, 1, 0)	1	1
31	GYN	(4, 0, 0)	1	1
32	GYN	(5, 0, 0)	1	1
33	GYN	(6, 0, 0)	1	0
34	OMS	(0, 0, 1)	1	1
35	OMS	(0, 1, 0)	1	1
36	OMS	(0, 1, 1)	0	1
37	OMS	(0, 2, 0)	1	1
38	OMS	(1, 0, 0)	1	1
39	OMS	(1, 0, 1)	1	1
40	OMS	(1, 1, 0)	1	1
41	OMS	(1, 2, 0)	1	1
42	OMS	(2, 0, 0)	1	1
43	OMS	(2, 1, 0)	1	1
44	OMS	(3, 0, 0)	1	1
45	ENT	(0, 1, 0)	1	1
46	ENT	(0, 2, 0)	1	1
47	ENT	(0, 3, 0)	1	1
48	ENT	(1, 0, 0)	1	1
49	ENT	(1, 1, 0)	1	1
50	ENT	(1, 2, 0)	1	1
51	ENT	(2, 0, 0)	1	1
52	ENT	(2, 1, 0)	1	1
53	ENT	(2, 2, 0)	1	1
54	ENT	(3, 0, 0)	1	1
55	ENT	(3, 1, 0)	1	1
56	ENT	(4, 0, 0)	1	1
57	ENT	(4, 1, 0)	1	1
58	ENT	(5, 0, 0)	1	1
59	ENT	(6, 0, 0)	1	1
60	EYE	(0, 0, 1)	1	1
61	EYE	(0, 0, 2)	1	1
62	EYE	(0, 0, 3)	1	1
63	EYE	(0, 1, 0)	1	1
64	EYE	(0, 1, 1)	1	1
65	EYE	(0, 1, 2)	1	1
66	EYE	(0, 1, 3)	1	1
67	EYE	(0, 2, 0)	1	1
68	EYE	(0, 2, 1)	1	1
69	EYE	(0, 2, 2)	1	1
70	EYE	(0, 3, 0)	1	1
71	EYE	(0, 3, 1)	1	1
72	EYE	(0, 4, 0)	1	1
73	EYE	(0, 4, 1)	1	1
74	EYE	(0, 5, 0)	1	1
75	EYE	(1, 0, 0)	1	1
76	EYE	(1, 0, 1)	1	1

77	EYE	(1, 0, 2)	1	1
78	EYE	(1, 0, 3)	1	1
79	EYE	(1, 1, 0)	1	1
80	EYE	(1, 1, 1)	1	1
81	EYE	(1, 1, 2)	1	1
82	EYE	(1, 2, 0)	1	1
83	EYE	(1, 2, 1)	1	1
84	EYE	(1, 2, 2)	1	1
85	EYE	(1, 3, 0)	1	1
86	EYE	(1, 3, 1)	1	1
87	EYE	(1, 4, 0)	1	1
88	EYE	(1, 5, 0)	1	0
89	EYE	(2, 0, 0)	1	1
90	EYE	(2, 0, 1)	1	1
91	EYE	(2, 0, 2)	1	1
92	EYE	(2, 0, 3)	1	1
93	EYE	(2, 1, 0)	1	1
94	EYE	(2, 1, 1)	1	1
95	EYE	(2, 1, 2)	1	1
96	EYE	(2, 2, 0)	1	1
97	EYE	(2, 2, 1)	1	1
98	EYE	(2, 3, 0)	1	1
99	EYE	(2, 3, 1)	1	0
100	EYE	(2, 4, 0)	1	1
101	EYE	(3, 0, 0)	1	1
102	EYE	(3, 0, 1)	1	1
103	EYE	(3, 0, 2)	1	1
104	EYE	(3, 1, 0)	1	1
105	EYE	(3, 1, 1)	1	1
106	EYE	(3, 1, 2)	1	0
107	EYE	(3, 2, 0)	1	1
108	EYE	(3, 2, 1)	1	1
109	EYE	(3, 3, 0)	1	1
110	EYE	(3, 4, 0)	1	0
111	EYE	(4, 0, 0)	1	1
112	EYE	(4, 0, 1)	1	1
113	EYE	(4, 0, 2)	1	1
114	EYE	(4, 1, 0)	1	1
115	EYE	(4, 1, 1)	1	1
116	EYE	(4, 2, 0)	1	1
117	EYE	(4, 2, 1)	1	0
118	EYE	(4, 3, 0)	1	1
119	EYE	(5, 0, 0)	1	1
120	EYE	(5, 0, 1)	1	1
121	EYE	(5, 1, 0)	1	1
122	EYE	(5, 1, 1)	1	1
123	EYE	(5, 2, 0)	1	1
124	EYE	(6, 0, 0)	1	1
125	EYE	(6, 0, 1)	1	1
126	EYE	(6, 1, 0)	1	1
127	EYE	(6, 2, 0)	1	1
128	EYE	(7, 0, 0)	1	1
129	EYE	(7, 0, 1)	1	1

130	EYE	(7, 1, 0)	1	1
131	EYE	(8, 0, 0)	1	1
132	EYE	(8, 1, 0)	1	0
133	EYE	(9, 0, 0)	1	1
134	ORT	(0, 1, 0)	1	1
135	ORT	(0, 2, 0)	1	1
136	ORT	(1, 0, 0)	1	1
137	ORT	(1, 1, 0)	1	1
138	ORT	(1, 2, 0)	1	1
139	ORT	(2, 0, 0)	1	1
140	ORT	(2, 1, 0)	1	1
141	ORT	(3, 0, 0)	1	1
142	ORT	(3, 1, 0)	1	1
143	ORT	(4, 0, 0)	1	1
144	ORT	(5, 0, 0)	1	1
145	PLA	(0, 1)	1	1
146	PLA	(0, 2)	1	1
147	PLA	(0, 3)	1	1
148	PLA	(1, 0)	1	1
149	PLA	(1, 1)	1	1
150	PLA	(1, 2)	1	1
151	PLA	(1, 3)	1	1
152	PLA	(2, 0)	1	1
153	PLA	(2, 1)	1	1
154	PLA	(2, 2)	1	1
155	PLA	(3, 0)	1	1
156	PLA	(3, 1)	1	1
157	PLA	(3, 2)	1	1
158	PLA	(4, 0)	1	1
159	PLA	(4, 1)	1	1
160	PLA	(5, 0)	1	1
161	PLA	(6, 0)	1	1
162	URO	(0, 0, 1)	1	1
163	URO	(0, 1, 0)	1	1
164	URO	(0, 2, 0)	1	1
165	URO	(1, 0, 0)	1	1
166	URO	(1, 0, 1)	1	1
167	URO	(1, 1, 0)	1	1
168	URO	(2, 0, 0)	1	1
169	URO	(2, 1, 0)	1	1
170	URO	(3, 0, 0)	1	1
171	URO	(3, 1, 0)	1	1
172	URO	(4, 0, 0)	1	1
173	URO	(5, 0, 0)	1	1
174	OB	(0, 1)	1	1
175	OB	(0, 2)	1	1
176	OB	(0, 3)	1	1
177	OB	(0, 4)	1	1
178	OB	(1, 0)	1	1
179	OB	(1, 1)	1	1
180	OB	(1, 2)	1	1
181	OB	(1, 3)	1	1
182	OB	(1, 4)	1	0

183	OB	(2, 0)	1	1
184	OB	(2, 1)	1	1
185	OB	(2, 2)	1	1
186	OB	(2, 3)	1	1
187	OB	(3, 0)	1	1
188	OB	(3, 1)	1	1
189	OB	(3, 2)	1	1
190	OB	(4, 0)	1	1
191	OB	(4, 1)	1	1
192	OB	(4, 2)	1	1
193	OB	(5, 0)	1	1
194	OB	(5, 1)	1	1
195	OB	(6, 0)	1	1
196	OB	(6, 1)	1	1
197	OB	(7, 0)	1	1
198	OB	(8, 0)	1	1

B.3. 780-capacity

Table B.3: Feasible combination set for 780-capacity OR-days.

index	specialty	combination	normal distribution	lognormal distribution
199	ENT	(0, 0, 1)	1	1
200	ENT	(0, 1, 0)	1	1
201	ENT	(0, 1, 1)	1	1
202	ENT	(0, 2, 0)	1	1
203	ENT	(0, 2, 1)	0	1
204	ENT	(0, 3, 0)	1	1
205	ENT	(0, 4, 0)	0	1
206	ENT	(0, 5, 0)	0	1
207	ENT	(1, 0, 0)	1	1
208	ENT	(1, 0, 1)	1	1
209	ENT	(1, 1, 0)	1	1
210	ENT	(1, 1, 1)	1	1
211	ENT	(1, 2, 0)	1	1
212	ENT	(1, 3, 0)	0	1
213	ENT	(1, 4, 0)	0	1
214	ENT	(1, 5, 0)	0	1
215	ENT	(2, 0, 0)	1	1
216	ENT	(2, 0, 1)	1	1
217	ENT	(2, 1, 0)	1	1
218	ENT	(2, 2, 0)	1	1
219	ENT	(2, 3, 0)	0	1
220	ENT	(2, 4, 0)	0	1
221	ENT	(3, 0, 0)	1	1
222	ENT	(3, 0, 1)	1	1
223	ENT	(3, 1, 0)	1	1
224	ENT	(3, 2, 0)	0	1
225	ENT	(3, 3, 0)	0	1
226	ENT	(3, 4, 0)	0	1
227	ENT	(4, 0, 0)	1	1
228	ENT	(4, 1, 0)	1	1
229	ENT	(4, 2, 0)	0	1

230	ENT	(4, 3, 0)	0	1
231	ENT	(5, 0, 0)	1	1
232	ENT	(5, 1, 0)	1	1
233	ENT	(5, 2, 0)	0	1
234	ENT	(5, 3, 0)	0	1
235	ENT	(6, 0, 0)	1	1
236	ENT	(6, 1, 0)	0	1
237	ENT	(6, 2, 0)	0	1
238	ENT	(7, 0, 0)	1	1
239	ENT	(7, 1, 0)	0	1
240	ENT	(7, 2, 0)	0	1
241	ENT	(8, 0, 0)	0	1
242	ENT	(8, 1, 0)	0	1
243	ENT	(9, 0, 0)	0	1
244	ENT	(9, 1, 0)	0	1
245	ENT	(10, 0, 0)	0	1
246	ENT	(11, 0, 0)	0	1

B.4. 900-capacity

Table B.4: Feasible combination set for 900-capacity OR-days.

index	specialty	combination	normal distribution	lognormal distribution
247	PLA	(0, 1)	1	1
248	PLA	(0, 2)	1	1
249	PLA	(0, 3)	1	1
250	PLA	(0, 4)	1	1
251	PLA	(0, 5)	1	1
252	PLA	(0, 6)	1	1
253	PLA	(0, 7)	1	1
254	PLA	(1, 0)	1	1
255	PLA	(1, 1)	1	1
256	PLA	(1, 2)	1	1
257	PLA	(1, 3)	1	1
258	PLA	(1, 4)	1	1
259	PLA	(1, 5)	1	1
260	PLA	(1, 6)	1	1
261	PLA	(1, 7)	1	1
262	PLA	(2, 0)	1	1
263	PLA	(2, 1)	1	1
264	PLA	(2, 2)	1	1
265	PLA	(2, 3)	1	1
266	PLA	(2, 4)	1	1
267	PLA	(2, 5)	1	1
268	PLA	(2, 6)	1	1
269	PLA	(3, 0)	1	1
270	PLA	(3, 1)	1	1
271	PLA	(3, 2)	1	1
272	PLA	(3, 3)	1	1
273	PLA	(3, 4)	1	1
274	PLA	(3, 5)	1	1
275	PLA	(4, 0)	1	1
276	PLA	(4, 1)	1	1

277	PLA	(4, 2)	1	1
278	PLA	(4, 3)	1	1
279	PLA	(4, 4)	1	1
280	PLA	(4, 5)	1	1
281	PLA	(5, 0)	1	1
282	PLA	(5, 1)	1	1
283	PLA	(5, 2)	1	1
284	PLA	(5, 3)	1	1
285	PLA	(5, 4)	1	1
286	PLA	(6, 0)	1	1
287	PLA	(6, 1)	1	1
288	PLA	(6, 2)	1	1
289	PLA	(6, 3)	1	1
290	PLA	(6, 4)	1	0
291	PLA	(7, 0)	1	1
292	PLA	(7, 1)	1	1
293	PLA	(7, 2)	1	1
294	PLA	(7, 3)	1	1
295	PLA	(8, 0)	1	1
296	PLA	(8, 1)	1	1
297	PLA	(8, 2)	1	1
298	PLA	(9, 0)	1	1
299	PLA	(9, 1)	1	1
300	PLA	(9, 2)	1	1
301	PLA	(10, 0)	1	1
302	PLA	(10, 1)	1	1
303	PLA	(11, 0)	1	1
304	PLA	(12, 0)	1	1

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