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Empirical analysis and modeling of the allometric scaling of urban freight systems

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Abstract – Heavy trucks which undertake the majority of freight volume play an important role in urban freight systems. By analyzing heavy truck trip data, we find a superlinear scaling relationship for heavy truck trips and a sublinear scaling relationship for heavy truck numbers relative to urban population size. Although these allometric scaling relationships that widely appear in nature and social systems have been explained by many models, a simple model that can cover a wide range of scaling exponents in these systems is still lacking. Here, we develop a partially mixing city operation model by quantifying the mixability of the urban population to explain why the superlinear and sublinear scaling exponents are in the range of 1 and $1 \pm 1/3$. This simple model not only helps us understand the mechanism of allometric scaling of urban freight systems, but also provides a new framework for other superlinear and sublinear scaling relationships in cities.

Introduction. – Cities are places of socioeconomic interaction dependent on massive goods delivery through urban freight systems [1,2]. Thus, it is of great significance to study urban freight systems from the perspective of complex system. Previous studies on the complex systems of urban freight mainly include the analysis of the overall structure of city logistics networks [3], the prediction of highway freight transportation networks using the gravity model and radiation model [4,5] and the quantification of the robustness and resilience of freight transportation networks [6,7]. However, studies of the allometric scaling of urban freight systems are still lacking.

Although whether allometric scaling exists in urban freight systems remains unknown, allometric scaling appears universally in nature and social systems [8–11]. For example, researchers have found many properties of cities that have superlinear allometric scaling relationships with population, such as income [12], GDP [13], patents [14], crimes [15] and total communication activity [16]. These properties of cities related to wealth and innovation are called socioeconomic outputs (the scaling exponent is between 1 and 4/3) [17]. Many properties of cities have sublinear scaling relationships with population, such as road area [18], urban area [19], gas station numbers [20] and power grid length [8]. These properties of cities associated with infrastructure are called material quantities (the scaling exponent is between 2/3 and 1) [8]. The allometric scaling relationship was first observed by evolutionary biologists [21] and designates the power function relationship between changes in certain parts of an organism and changes in overall size. Metabolic rate follows a 3/4 power relationship with the mass of an organism [22–27]. Moreover, in river systems, Pelletier found a similar sublinear allometric scaling relationship between the average river discharge and the drainage area [28]. In addition, there is a sublinear allometric scaling relationship between the corpus size and the vocabulary size of books [29]. These studies show that allometric scaling relationships widely appear in nature and social systems and inspire us to study the allometric scaling of urban freight systems.

In recent years, with the improvement of massive intracity freight trip data [30,31], it has become possible for us to study the allometric scaling relationships between many properties of urban freight systems and urban population size. In urban freight systems, heavy trucks mainly undertake logistics to ensure the supply of goods and raw material for people and firms, accounting for most

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of the urban freight volume and playing an important role in cities. Therefore, in this letter, we take heavy trucks as our research object. We find a superlinear allometric scaling relationship between heavy truck trips and urban population size and a sublinear allometric scaling relationship between heavy truck numbers and urban population size. At present, researchers have proposed many models to explain the allometric scaling relationships between properties of cities and urban population size. For example, Batty used a diffusion-limited aggregation model to simulate the growth process of city which shows that the scaling relationship of urban area and population is determined by the fractal dimension of a city's geometry [32]. However, the scaling exponent in Batty's model is not consistent with the empirical data. Um *et al.* established a microdynamics model to explain the scaling relationship of commercial and public facility density relative to urban population density [33], but it remains unknown how to relate it with the urban scaling based on urban population size [34]. Ribeiro and Rybski summarized the majority of the theoretical models that explain urban scaling laws from different premises [35]. Jusup *et al.* concluded the most representative model of urban scaling laws [34]. At the individual level, Pan *et al.* established a social density model by assuming that the probability of the interaction of each two agents in a city is inversely proportional to distance to produce the log-form (but not power-form) superlinear relationship between socioeconomic outputs and urban population [36]. At the macro level, Bettencourt proposed the simplest city operation model from the perspective of the balance of socioeconomic outputs and interaction costs by assuming that the urban population is fully mixing [17]. In his model, the superliner scaling exponents of socioeconomic outputs is 4/3. However, the empirical superliner scaling exponents are generally between 1 and 4/3. Although Bettencourt introduced the road fractal dimension and infrastructure network to modify the initial model [17] such that the theoretical scaling exponent could be consistent with the empirical data, his model considered too many other factors instead of interactions, which made the modified model more complex. In addition, Arbesman *et al.* used a hierarchical network to quantify the social distance (network distance) of each social interaction whereby the contribution of each interaction to the output that reflects the amount of innovation is affected by their social distance [37]. This model can reflect the superliner scaling relationships between socioeconomic outputs and urban population size if parameters are set to specific values. However, their models focus too much on the microstructure of social interaction networks, which makes the model slightly more complicated. Therefore, we aim to develop a simple model that quantifies the total amount of social interaction at the macro level without considering other detailed factors such as network structure and road fractal dimension to explain why the superlinear and sublinear scaling exponents are in the range of 1 and $1 \pm 1/3$. In this regard, Samaniego and Moses studied the allometric scaling relationship between the rescaled travel distance and urban population size and used its scaling exponent to quantify the centrality of human mobility pattern within a city [18]. Barthelemy considered the effect of distance on the amount of social interaction and established a link between the centrality of human mobility pattern and destination choice behavior [38]. This link tells us that the centrality of human mobility pattern is positively correlated with population mixability, which provides us with a means for expanding Bettencourt's simplest city operation model and proposing the partially mixing population city operation model to derive the theoretical scaling exponents of socioeconomic outputs and material quantities. In this letter, we first find a superlinear scaling relationship for heavy truck trips and a sublinear scaling relationship for heavy truck numbers relative to urban population size through an empirical analysis of heavy truck trip data. Then, we introduce a method for quantifying the mixability of urban population and develop a model based on the simplest city operation model to derive a superlinear scaling exponent (from 1 to 4/3) and sublinear scaling exponent (from 1 to 2/3). Finally, we discuss the potential application value of our model.

Empirical analysis. – The data we use in this work are intracity heavy truck trip [30,31] (origin and destination, OD) data from 335 cities in China for over 2 weeks in 2018, urban population data for China for 2018 (downloaded from `http://www.stats.gov.cn/`) and urban area data for China for 2018 (downloaded from `https://www.mohurd.gov.cn/`). The heavy truck trip data include heavy truck trips, heavy truck numbers and the longitudes and latitudes of origins and destinations.

We study three statistical relationships between quantities (heavy truck trips, heavy truck numbers and rescaled travel distances of heavy trucks) and urban population size. The results show a superlinear allometric scaling relationship between heavy truck trip numbers and urban populations size with scaling exponent $\beta = 1.08 \pm 0.078$ as shown in fig. 1. This scaling exponent is similar to the scaling exponents of socioeconomic outputs $\beta \in [1.07, 1.34]$, such as GDP, patents and crimes [8]. We believe this is because the number of heavy truck trips is also a socioeconomic output that reflects the amount of interaction in a city [17]. With an increase in urban population size, the *per capita* socioeconomic outputs in cities increase, which further increases the *per capita* freight demand, resulting in a corresponding increase in the number of *per capita* heavy truck trips.

Figure 2 shows a sublinear allometric scaling relationship between heavy truck numbers and urban population size with scaling exponent $\beta = 0.95 \pm 0.078$, echoing the scaling exponents of material quantities $\beta \in [0.77, 0.83]$, such as road area, urban area, gas station numbers and power grid length [8]. We believe this is the case because heavy truck numbers are material quantities reflecting the

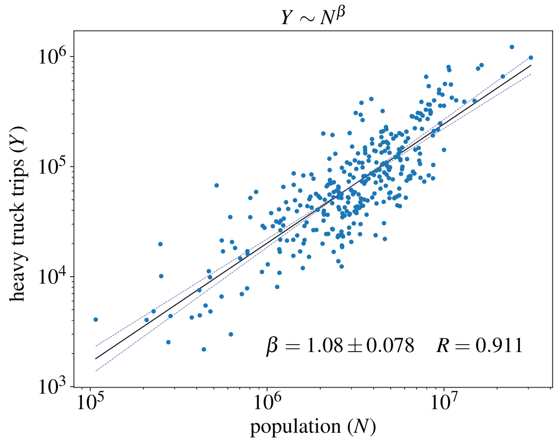

Fig. 1: The superlinear allometric scaling relationship between heavy truck trips Y and urban population size N. Each point is a single city. The solid line represents regression to the points and the scaling exponent $\beta = 1.08$. The dashed lines indicate the 95% confidence interval of scaling exponent β.

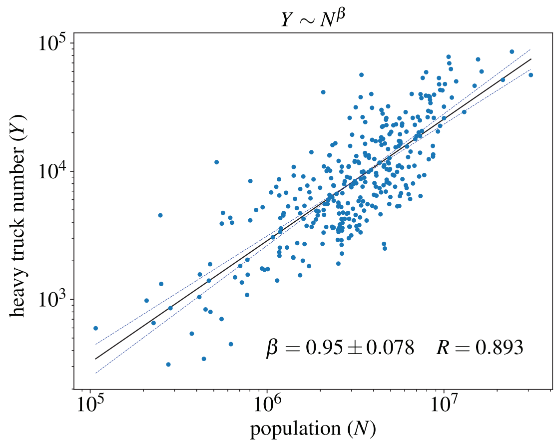

Fig. 2: The sublinear allometric scaling relationship between heavy truck numbers Y and urban population size N. The solid line represents regression to the points and the scaling exponent $\beta = 0.95$. The points and dashed lines have the meaning defined in fig. 1.

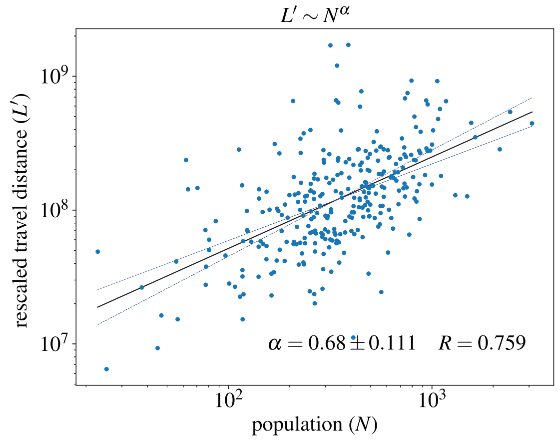

Fig. 3: The allometric scaling relationship between rescaled travel distance L' and urban population size N. The solid line represents regression to the points and the scaling exponent $\alpha = 0.68$. The points and dashed lines have the meaning defined in fig. 1.

infrastructure scale of cities. With an increase in urban population size, the *per capita* material quantities decrease, resulting an increase in the number of people served by each heavy truck.

Furthermore, we calculate the rescaled travel distance of heavy trucks in each city. We first calculate the spherical distance of each heavy truck trip using the coordinates of the origin and destination and obtain the total travel distance L of each city. Then, we can obtain the rescaled travel distance of heavy trucks which can be expressed as the total travel distance divided by the square root of urban area $L/A^{1/2}$. Figure 3 shows a sublinear scaling relationship between the rescaled travel distance and urban population size ($\alpha = 0.68 \pm 0.111$), which is similar to the sublinear scaling relationship between the rescaled travel distance of urban motor vehicles and population size ($\alpha \approx 0.66$) found in previous studies [18].

Partially mixing population city operation model. – We carry out our work based on the simplest city operation model established by Bettencourt [17]. The hypothesis of the model reflecting how a city operates includes the following: 1) *per capita* interaction is proportional to urban population density; 2) urban socioeconomic outputs are proportional to the total amount of social interaction; 3) the interaction costs are proportional to the total travel distance; 4) the urban population is fully mixing, that is, the *per capita* travel distance is proportional to the radius of the city; and 5) urban socioeconomic outputs are proportional to the interaction cost. Bettencourt points out that the total amount of social interaction is the key factor that determines the total amount of socioeconomic output in cities [17]. Thus, according to hypotheses 1) and 2), the socioeconomic output of a city can be expressed as

$$Y \sim I \sim Ni \sim N\rho \sim N\frac{N}{A} \sim \frac{N^2}{A}, \tag{1}$$

where Y is the socioeconomic output, I is the total amount of interaction, i is *per capita* interaction, and ρ is urban population density. Bettencourt assumes that the urban population is fully mixing; that is, each person can randomly interact with anyone else in the city. Therefore, according to hypothesis 4), the *per capita* travel distance can be expressed as

$$l \sim A^{1/2}. \tag{2}$$

According to hypothesis 3), the interaction cost of a city can be expressed as

$$T \sim L \sim N \cdot l \sim NA^{1/2}, \tag{3}$$

where T is the total cost of interaction, L is the total travel distance, and l is the *per capita* travel distance. By combining hypothesis 5), eq. (1), eq. (2) and eq. (3), one can obtain

$$\frac{N^2}{A} \sim NA^{1/2}, \tag{4}$$

and further obtain the sublinear scaling relationship between urban area and urban population size

$$A \sim N^{2/3}. \tag{5}$$

In combination with eq. (1) and eq. (5), the superliner scaling relationship between socioeconomic output and urban population size can be derived as

$$Y \sim N^{4/3}. \tag{6}$$

However, the theoretical scaling exponent of 4/3 is obviously too large relative to the actual scaling exponents [8] of socioeconomic outputs. Moreover, the assumption of a fully mixing population in this model is also inconsistent with the actual interaction behavior. In an actual city, some people are very active, and interact with anyone in the city, while some people only interact with people within the area around their homes. This means that the urban population is not fully mixing.

To make our model more realistic, we consider the mixability of urban population by assuming that the urban population mixability ranges from fully mixing to not mixing. In this respect, the centrality of human mobility proposed by Samaniego and Moses [18] and further extended by Barthelemy [38] provides important precedent for us. The work of Samaniego and Moses establishes a link between human mobility patten and rescaled travel distance in the city. The authors argue that there are two extreme cases of human mobility patten in cities, namely, centralized and decentralized. When human mobility pattern is centralized, everyone interact with others randomly in the city. Therefore, according to eq. (2), the *per capita* travel distance is proportional to the square root of urban area. This is similar to the assumption of fully mixing population in the simplest city operation model [17]. In this case, the rescaled travel distance L' can be expressed as

$$L' = \frac{L}{A^{1/2}} = \frac{N \cdot l}{A^{1/2}} \sim N. \tag{7}$$

However, when human mobility pattern is decentralized, the interaction area of each person is reduced to the *per capita* area in the city, which means that each person only interacts with his nearest neighbor. Therefore, the *per capita* travel distance is proportional to the square root of the *per capita* urban area, which means that the urban population is not mixing. In this case, the rescaled travel distance can be expressed as

$$L' = \frac{L}{A^{1/2}} = \frac{N \cdot l}{A^{1/2}} = \frac{N\rho^{-1/2}}{A^{1/2}} \sim N^{1/2}. \tag{8}$$

In an actual city, the human mobility pattern is between these two extremes. Samaniego and Moses [18] assumed an allometric scaling relationship between the rescaled travel distance and urban population size that can be expressed as

$$L' = \frac{L}{A^{1/2}} \sim N^{\alpha}, \tag{9}$$

where α is the scaling exponent ranging from 1/2 to 1. Then, the authours conducted a statistical analysis of the relationship between the rescaled travel distance of motor vehicles and the urban population size of 425 cities in the United States within a day and found a sublinear scaling exponent of 0.66 [18]. This result indicates that the human mobility pattern in a city is between centralized and decentralized and that the scaling exponent α in eq. (9) can be used to quantify the centrality of human mobility pattern in cities at a macro level. When scaling exponent α is closer to 1, the human mobility pattern in the city is more centralized. When scaling exponent α is closer to 1/2, the human mobility pattern in the city is more decentralized.

Based on Samaniego and Moses's work [18], Barthelemy considered the effect of distance on the probability of social interaction between two individuals and established a link between human mobility centrality and destination choice behavior [38]. Barthelemy assumed that the probability of the social interaction between two individuals can be expressed as

$$f(x) \sim x^{-\tau}, \tag{10}$$

where x is the distance between two individuals and τ is a decay exponent that reflects the interaction range. In his work, when τ is very small, each individual interacts with others randomly, which means that the urban population is fully mixing. In this case, the human mobility pattern in city is centralized and α approaches 1. In contrast, when τ is very large, each individual only interacts with his nearest neighbor, which means that the urban population is not mixing. In this case, the human mobility pattern in city is decentralized and α approaches 1/2. This result shows that the centrality of human mobility is positively correlated with the mixability of urban population.

To further verify the positive correlation between human mobility centrality and urban population mixability, we conduct a numerical simulation to show how scaling exponent α in eq. (9) changes with parameter τ. The steps of our simulation include the following: 1) build m cities with different areas $n \times n$; 2) distribute the same number of individuals per unit area; 3) set $\tau = 0$; 4) each individual interacts with others with a probability proportional to $x^{-\tau}$, see eq. (10); 5) calculate the rescaled travel distance in eq. (9) for each city; 6) estimate scaling exponent α of eq. (9) from m cities; and 7) set $\tau = \tau + s$, where s is the step size, and return to step 4).

According to these steps, we set the city number as $m = 10$, the n range to 10 to 20 and the step size to

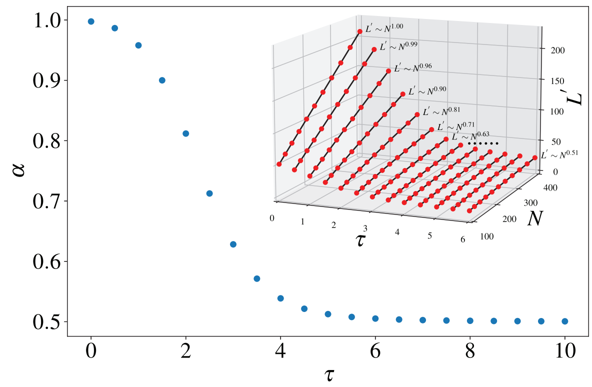

Fig. 4: The relationship between decay exponent τ and scaling exponent α. The blue points are the simulation results, which show that the scaling exponent α decreases form 1 to 1/2 with the increase of τ. This result reflects the positive correlation between human mobility centrality and urban population mixability. The insert diagram shows the scaling relationship between rescaled travel distance L' and urban population size N when τ takes specific values, which verifies the assumption in eq. (9).

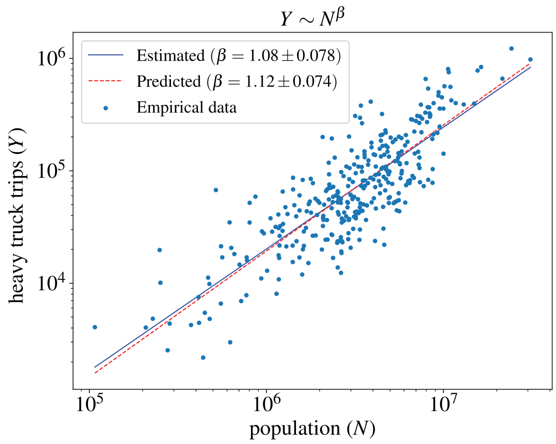

Fig. 5: Comparison between the scaling exponent of heavy truck trip number Y derived from empirical data and that derived from the PMPCO model. Each point represents the empirical data of a single city. The solid line denotes the scaling relationship estimated from empirical data and the dashed line is the scaling relationship predicted by the PMPCO model.

$s = 0.5$ to conduct the simulation. The result of our simulation is shown in fig. 4. We can see that as parameter τ grows, the scaling exponent α of eq. (9) decreases from 1 to 1/2. This indicates that urban population mixability is positively correlated with the centrality of the human mobility pattern. We also find that regardless of how τ changes, the allometric scaling relationship between the rescaled travel distance and urban population size exists. This result verifies the allometric scaling relationship between rescaled travel distance and urban population size given in eq. (9).

Therefore, we can generalize eq. (3) and extend the simplest city operation model by using the method for quantifying urban population mixability. We call this the partially mixing population city operation model (PMPCO), *i.e.*,

$$T \sim L \sim N^{\alpha} A^{1/2}. \tag{11}$$

Since socioeconomic output in hypothesis 5) is proportional to the total cost of social interaction, according to eq. (1) and eq. (11), we can obtain

$$\frac{N^2}{A} \sim N^{\alpha} A^{1/2}. \tag{12}$$

Consequently, the sublinear scaling relationship between urban area and urban population size is

$$A \sim N^{\frac{4-2\alpha}{3}}. \tag{13}$$

By combining eq. (13) and eq. (1), we derive the superliner scaling relationship between heavy truck trips and urban population size, *i.e.*,

$$Y \sim N^{\frac{2+2\alpha}{3}}, \tag{14}$$

which means that its scaling exponent is

$$\beta = \frac{2+2\alpha}{3}. \tag{15}$$

This indicates that the higher the mixability of an urban population is, the stronger the superlinear scaling relationships between the socioeconomic outputs and urban population size are. When $\alpha = 1/2$, the mixability of the urban population is the lowest, and the scaling exponent is $\beta = 1$. When $\alpha = 1$, the mixability of urban population is the highest and the scaling exponent is $\beta = 4/3$, which is consistent with the simplest city operation model [17]. When $\alpha \in (1/2, 1)$, the scaling exponent $\beta \in (1, 4/3)$ can cover the range of scaling exponent of the majority of socioeconomic outputs including the number of heavy truck trips.

The PMPCO model can reproduce not only the superlinear scaling relationships between socioeconomic outputs and urban population size, but also the sublinear relationship between urban area and urban population, the scaling exponent of which is

$$\beta = \frac{4-2\alpha}{3}. \tag{16}$$

This shows that the higher the mixability of the urban population is, the stronger the sublinear scaling relationships between material quantities and urban population size are. When $\alpha = 1/2$, the mixability of an urban population is the lowest, and the scaling exponent is $\beta = 1$. When $\alpha = 1$, the mixability of the urban population is the highest, and the scaling exponent is $\beta = 2/3$. When $\alpha \in (1/2, 1)$, the scaling exponent $\beta \in (2/3, 1)$ can cover the range of scaling exponent of the majority of material quantities including heavy truck numbers.

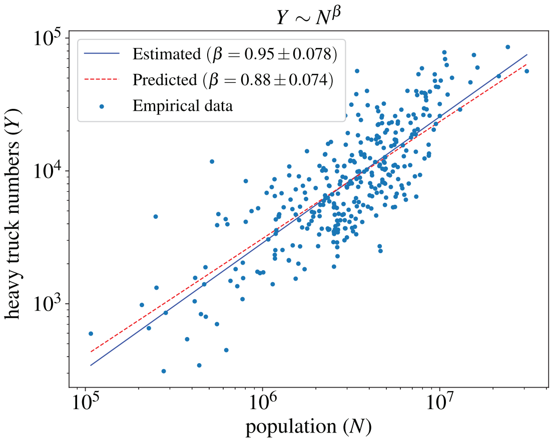

Fig. 6: Comparison between the scaling exponent of heavy truck number Y derived from empirical data and that derived from the PMPCO model. The points, solid lines and dashed lines have the meaning defined in fig. 5.

Model validation. – Here, we use the scaling relationship between rescaled travel distance and urban population size to explain the allometric scaling relationships of socioeconomic outputs and material quantities relative to urban population size. The scaling exponent α of the rescaled travel distance relative to urban population is 0.68 ± 0.111 as shown in fig. 3, which indicates that the mixability of an urban population is between fully mixing and not mixing. We set the scaling exponent in eq. (15) as $\alpha = 0.68 \pm 0.111$ and use the PMPCO model to derive the theoretical scaling exponent of heavy truck trip $\beta = 1.12 \pm 0.074$, which is close to the empirical scaling exponent $\beta = 1.08 \pm 0.078$, as shown in fig. 5. The result shows that the PMPCO model can effectively reproduce the superlinear allometric scaling of socioeconomic outputs. Moreover, we can also obtain the theoretical sublinear scaling exponent of material quantities $\beta = 0.88 \pm 0.074$ by letting the scaling exponent be $\alpha = 0.68 \pm 0.111$ in eq. (16). The scaling exponent derived from the PMPCO model is very close to empirical value $\beta = 0.95 \pm 0.078$ in fig. 6. This result indicates that our model is also helpful for understanding the sublinear scaling relationship between heavy truck number and urban population size.

Conclusion. – In this letter, we use intracity heavy truck trip data for China to obtain heavy truck trips, heavy truck numbers and rescaled travel distances of heavy trucks for 335 cities. We in turn find a superlinear scaling relationship between the number of heavy truck trips and urban population size and a sublinear scaling relationship between the heavy trucks number and urban population size through statistical analysis. To explain the cause of this allometric scaling in urban freight systems, we propose urban population mixability as a key determinant to establish the PMPCO model. Our simple model can effectively reproduce the observed allometric scaling relationships we find.

In the PMPCO model, the change in urban population mixability can greatly affect the allometric scaling relationships between urban quantities and population size, which can explain why the superlinear scaling exponents of socioeconomic outputs are in the range of $[1, 4/3]$ and the sublinear scaling exponents of material quantities are in the range of $[2/3, 1]$. The PMPCO model can cover almost all the scaling exponents found in nature and social systems, including socioeconomic outputs such as income levels [12], GDP [13], patents [14], crimes [15] and material quantities such as road area [18], city area [19], the number of gas stations [20], and power grid length [8]. This simple model provides a new perspective for the study of urban scaling. Not only that, but our model may also help to explain the allometric scaling laws in other systems including living organisms [21], rivers [28] and books [29].

Our work can also serve as a reference for practical applications. For example, the PMPCO model reveals that the mixability of the urban population is an important factor affecting the scaling relationship between urban quantities and population size. We can measure the deviations of urban quantities from average value in each city [11] and adjust the urban population mixability to make the urban quantities more closely reflect the empirical allometric scaling relationships. This also provides more reference for policy makers seeking to take measures to adjust urban quantities to improve the performance of urban freight systems or even the entire city.

* * *

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Data availability statement: The data that support the findings of this study are available upon reasonable request from the authors.

REFERENCES

[1] Behrends S., *Transp. Res. Proc.*, **12** (2016) 278.
[2] Demir E., Bektaş T. and Laporte G., *Eur. J. Oper. Res.*, **237** (2014) 775.
[3] Ma J. C., Wang L., Jiang Z. Q., Yan W. and Zhou W. X., *Physica A*, **583** (2021) 126333.
[4] Wang L., Ma J. C., Jiang Z. Q., Yan W. and Zhou W. X., *EPJ Data Sci.*, **8** (219) 1.
[5] Zhou W. X., Wang L., Xie W. J. and Yan W., *Phys. Rev. E*, **102** (2020) 052314.
[6] Madar G., Maoh H. and Anderson W., *J. Geogr. Syst.*, **22** (2020) 309.
[7] Jansuwan S., Chen A. and Xu X., *Transp. Res. Part A*, **151** (2021) 154.
[8] Bettencourt L. M., Lobo J., Helbing D., Kühnert C. and West G. B., *Proc. Natl. Acad. Sci. U.S.A*, **104** (2007) 7301.
[9] West G. B. and Brown J. H., *J. Exp. Biol.*, **208** (2005) 1575.

[10] Batty M., *Science*, **319** (2008) 769.
[11] Li R., Dong L., Zhang J., Wang X., Wang W. X., Di Z. and Stanley H. E., *Nat. Commun.*, **8** (2017) 1.
[12] Lobo J., Bettencourt L. M., Strumsky D. and West G. B., *PLoS ONE*, **8** (2013) e58407.
[13] Ribeiro H. V., Oehlers M., Moreno-Monroy A. I., Kropp J. P. and Rybski D., *PLoS ONE*, **16** (2021) e0245771.
[14] Bettencourt L. M., Lobo J. and Strumsky D., *Res. Policy*, **36** (2007) 107.
[15] Glaeser E. L. and Sacerdote B., *J. Polit. Econ.*, **107** (1999) S225.
[16] Schläpfer M., Bettencourt L. M., Grauwin S., Raschke M., Claxton R., Smoreda Z., West G. B. and Ratti C., *J. R. Soc. Interface*, **11** (2014) 20130789.
[17] Bettencourt L. M., *Science*, **340** (2013) 1438.
[18] Samaniego H. and Moses M. E., *J. Transp. Land Use*, **1** (2018) 21.
[19] Lee Y., *Environ. Plan A*, **21** (1989) 463.
[20] Kühnert C., Helbing D. and West G. B., *Physica A*, **363** (2006) 96.
[21] Gayon J., *Am. Zool.*, **40** (200) 748.
[22] Kleiber M., *Hilgardia*, **6** (1932) 315.
[23] West G. B., Brown J. H. and Enquist B. J., *Science*, **284** (1999) 1677.
[24] Enquist B. J., West G. B., Charnov E. L. and Brown J. H., *Nature*, **401** (1999) 907.
[25] Ahluwalia A., *Sci. Rep.*, **7** (2017) 1.
[26] West G. B., Brown J. H. and Enquist B. J., *Science*, **279** (1997) 122.
[27] Banavar J. R., Maritan A. and Rinaldo A., *Nature*, **399** (1999) 130.
[28] Pelletier J. D., *J. Geophys. Res. Solid Earth*, **104** (1999) 7359.
[29] Petersen A. M., Tenenbaum J. N., Havlin S., Stanley H. E. and Perc M., *Sci. Rep.*, **2** (2012) 943.
[30] Yang Y., Jia B., Yan X. Y., Jiang R., Ji H. and Gao Z., *Transp. Res. Part C*, **136** (2022) 103564.
[31] Yang Y., Jia B., Yan X. Y., Li J., Yang Z. and Gao Z., *Transp. Res. Part E*, **157** (2022) 102590.
[32] Batty M. and Longley P. A., *Fractal Cities: A Geometry of Form and Function* (Academic Press, London) 1994.
[33] Um J., Son S. W., Lee S. I., Jeong H. and Kim B. J., *Proc. Natl. Acad. Sci. U.S.A*, **106** (2009) 14236.
[34] Jusup M., Holme P., Kanazawa K., Takayasu M., Romić I., Wang Z., Geček S., Lipić T., Podobnik B., Wang L., Luo W., Klanjšček T., Fan J., Boccaletti S. and Perc M., *Phys. Rep.*, **948** (2022) 1.
[35] Ribeiro F. L. and Rybski D., *Phys. Rep.*, **1012** (2023) 1.
[36] Pan W., Ghoshal G., Krumme C., Cebrian M. and Pentland A., *Nat. Commun.*, **4** (2013) 1.
[37] Arbesman S., Kleinberg J. M. and Strogatz S. H., *Phys. Rev. E*, **79** (2009) 016115.
[38] Barthelemy M., *Phys. Rep.*, **499** (2011) 1.