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Simultaneously Identifying the System Dynamics and Fault Isolation for Air Data Sensor Failures: A Convex Approach

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Abstract: This paper addresses the key question that when faults occur either the aircraft system dynamics changes due to the fault or these dynamics are unknown (precisely). This question is addressed for the important case of Air Data Sensor failures, due to e.g. icing, for fixed wing aircraft operating in a nominal flight condition. The solution to this question uses basic ideas from subspace identification to cast this problem in linear least squares problem with convex constraints (nuclear norm and 1-norm constraints). The latter are relaxations of a rank and cardinality constraint.The presented solution is validated using real-life flight test data.

Keywords: Signal and identification-based methods; Flight control, fault detection; FDI for linear systems

1. INTRODUCTION

Fault Detection and Isolation (FDI) has been a key attention topic in control of high performance aircraft. See for its importance and its role in state-of-the-art in terms of fault tolerant control applicable to civil aircraft to the overview in Edwards et al. (2010). In that book but also in many additional papers, different approaches have been presented for sensor or actuator FDI for Aerospace Systems, see e.g. Marzat et al. (2012) and the many references in it.

One crucial subset of faults in aircraft operation are faults with Air Data Sensors (ADS). These sensors make use of pitot tubes and wind vanes, mounted on the exterior of the airplane. From these measurements quantities like airspeed, angle of attack or sideslip angle are derived. These quantities provide essential information to the pilot on the state of the aircraft to safely conduct a flight (Houck and Atlas, 1998). Its exterior mounting make these sensors vulnerable to icing or water accumulation. These environmental effects may result in fault such as blocked pitot tubes (Freeman et al., 2013). The consequence of these faults may severely influence the information provided to the pilot, possibly even leading to catastrophic accidents. Examples are the faults in ADS in Austral Lineas Aeroeas Flight 2553 where an improper referenced airspeed led to structure failure due to exceeding the safe airspeed limits (Eubank et al., 2010). More recently the Air France 447 accident was due erroneous airspeed measurements by improper operation of the pitot probes (Balzano et al., 2018). In the period between 2003 and 2010, commercial aircraft have suffered more than 35 recorded incidents of multiple ADS faults (Eubank et al., 2010).

This high relevance of FDI for ADS faults has triggered a lot of research in this area (Freeman et al., 2013; Ellsworth and Whitmore, 2007). Solutions have been sought in developing alternative hardware modification, such as (regular) flushing of the sensing system (Ellsworth and Whitmore, 2007) or using redundant air data systems and majority sensor voting. Possible software extensions aim at developing virtual sensor capabilities derived from navigation sensors (Looye and Joos, 2001). These virtual methods use analytical redundancy provided by mathematical models of the aircraft dynamics. In general a bottleneck in these analytical approaches is the reliance on model information of the aerodynamic forces and moments acting on the aircraft, which have to be estimated prior to the virtual sensor design methodology. That model information might be time consuming to obtain and/or may be inaccurate as a consequence of storing only a limited number of models for selected operation conditions. To overcome this shortcoming, alternative kinematic models have been proposed, such as in Lu et al. (2016). These kinematic models rely on the use of Inertial Measurement Units (IMU), to reconstruct the aircraft state. However that as well may suffer sensor limitations that introduce noise and biases (Van Den Hoek et al., 2018).

In this paper we take a radically new approach that aims at *simultaneous* identification of the aircraft system dynamics (in a particular operation (or trim) point) and the diagnosis of the ADS faults. The novelty stems from the fact that we either do not assume the operating point, usually defined by the ADS, to be known, nor that we restrict our contribution to a 'classical' sensor configuration, not relying on IMU data (and its inherent bias and noise disturbances). The first generalization excludes the use of many model based approaches that rely on

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gain-scheduling, while the second generalization makes the method a competitor of new approaches relying on IMU data, such as in Lu et al. (2016).

For the system dynamics we assume that the aircraft dynamics in an operating point can be well described by an LTI (state space) model and for the fault diagnosis we assume the availability of a dictionary of different possible scenarios as represented by the "basis" signals in the dictionary. As in Zhang (2021) such a dictionary is allowed to be too "rich" to model the (additive) fault scenario as well as does not require the magnitude of the faults to be known. Also the case of linear combinations of the "basis" signals should be allowed.

The paper is outlined in the following way. We start in section 2 with a brief recap on the essential step of formulating a state space identification problem in the subspace identification framework (Verhaegen and Verdult, 2007). Then we outline briefly in section 3 the modeling of ADS sensor faults as additive output failures with particular signatures. After that we are ready to formulate in the next section 4 the joint identification of the system dynamics and the diagnosis of the fault as a rank and cardinality constrained least squares problem. That problem is relaxed (convexified) by replacing these constraints resp. by a nuclear norm and a 1-norm constraint. This formulation is based on our recent contribution for "general" faults in Noom et al. (2023a). However we now specialize this method to the isolation of ADS faults. In this formulation we are able to (automatically) deal with identifying the aircraft dynamics when flying through turbulence and we do not require the determination of the order of a state space model first, as that model is *never* explicitly identified. The goal in this paper, for the sake of brevity, is to completely focus on the isolation of ADS faults. The validation of the new methodology in section 5, is demonstrated using flight test obtained models of the Cessna II Delft-NLR test aircraft operating in the longitudinal mode. On this real-life flight test data, we synthetically introduce certain fault scenarios inspired by the work in Lu et al. (2016). The paper is concluded with some final remarks looking towards some future potentials.

2. REVIEW OF THE "ESSENCE" OF THE SUBSPACE PERSPECTIVE

We consider the Aircraft operating in single operation condition, possibly experiencing turbulence, to be modelled by the following Linear Time-Invariant (LTI) system:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + w(k) \\ y(k) &= Cx(k) + v(k) \end{aligned} \tag{1}$$

where $x(k) \in \mathbb{R}^{n_x}, u(k) \in \mathbb{R}^{n_u}, y(k) \in \mathbb{R}^{n_y}$ are resp. the state, input and output; A, B, C are the state space matrices; $w(k)$ and $v(k)$ are the process- and measurement noise. This model can therefore also accommodate the aircraft flying through turbulent media. This is an advantage over (kinematic) model based methods (such as the DMAE method Lu et al. (2021) that was selected as benchmark reference to compare the new methodology later on in Section 5), as the latter requires (selective) reinitialization that does not assume process noise. The

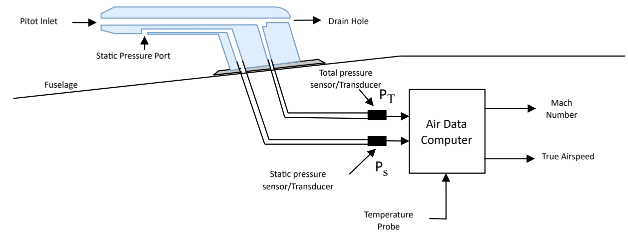

Fig. 1. Schematic diagram of a Pitot Tube Air Data System (based on Fig. 7.1 in Collinson (2011))

assumption on considering the Aircraft to be described by an LTI model when operating in a single operating point is generally made in gain-scheduling. Bett (2005).

In Subspace identification we consider the *observer* form representation of the above LTI system, based on the assumption that the conditions hold for the existence of the following observer (Verhaegen and Verdult, 2007):

$$\begin{aligned} \hat{x}(k+1) &= \underbrace{(A - KC)}_{\Phi}\hat{x}(k) + Bu(k) + Ky(k) \\ \hat{y}(k) &= C\hat{x}(k) \end{aligned} \tag{2}$$

with $\hat{x}(k), \hat{y}(k)$ resp. the estimated state and output vectors. Using this observer form allows to write that output as:

$$\hat{y}(k) = C\Phi^s\hat{x}(k-s) + \sum_{i=1}^{s} C\Phi^{i-1}\Big(Bu(k-i) + Ky(k-i)\Big) \tag{3}$$

If K is assumed to make Φ asymptotically stable, the effect of the (initial) state vector $\hat{x}(k-s)$ in (3) fades away as s increases. This is referred to as the *Subspace Trick*. This leads to the following approximate Vector Auto-Regressive model with eXogenous input (VARX):

$$\hat{y}(k) \approx \sum_{i=1}^{s} B_i u(k-i) + K_i y(k-i) \tag{4}$$

with matrices B_i, K_i of compatible dimensions approximating the observer Markov parameters.

3. MODELING ADS FAULTS

For the sake of brevity we restrict in this paper to the modeling faults on the measurement of the aircraft's airspeed. This measurement is derived from a pitot-tube device illustrated in Figure 1. This device measures the total pressure at the inlet p_t and the static pressure p_s. Let p_d be the difference $p_t - p_s$, then using Bernoulli's equation (for compressible media), the True Airspeed (TAS) is given as (Hu et al., 2022):

$$V_{TAS} = \sqrt{\frac{2\gamma}{\gamma-1}R_g T_s\left(\frac{p_d}{p_s}+1\right)^{\frac{\gamma-1}{\gamma}} - 1}, \tag{5}$$

where, γ is the specific heat ratio of air, R_g is the ideal gas constant and T_s is the static temperature.

When treating anomalies (due to e.g. icing or clogging) as small deviations from the nominal values of p_t and p_s, these anomalies an be modelled as additive *faults* to the measured V_{TAS} (Freeman et al., 2013). A similar reasoning

Fault	Mathematical Representation
Abrupt blockage	Bias
Gradual blockage	Drift
Partial water blockage	Sinusoidal

Table 1. Typical Patterns of the additive fault $f(k)$ in (6) to the measurement of V_{TAS} and α (Freeman et al., 2013).

holds for the other air data sensor signals like the angle of attack α. Therefore when denoting $\begin{bmatrix} V_{TAS} \\ \alpha \end{bmatrix}$ as the output $y \in \mathbb{R}^2$, the measured (faulty) output is given as:

$$y_m(k) = y(k) + f(k) \tag{6}$$

Where these additive faults behave according to certain typical patterns as illustrated in Table 1 (Freeman et al., 2013).

4. SIMULTANEOUS SYSTEM DYNAMICS IDENTIFICATION AND FAULT DIAGNOSIS

4.1 The Problem Formulation

When substituting the (faulty) measurement into the VARX model (4) we get:

$$\hat{y}_m(k) \approx \sum_{i=1}^{s} \Big(B_i u(k-i) + K_i y_m(k-i) - K_i f(k-i) \Big) + f(k). \tag{7}$$

Making the key assumption as e.g. in Zhang (2021); Noom et al. (2023a), and based on known patterns of the time evolution of the air data sensor faults, as indicated in Table 1, the sensor fault $f(k)$ can be modelled as:

$$f(k) = \omega(k) z, \tag{8}$$

where $\omega(k)$ are known signal patterns in the signal dictionary of faults. The actual fault (or fault combination) is determined by the unknown vector $z \in \mathbb{R}^{n_z}$. Based on this assumption and the VARX model representation of the output, allows to model the measured output $y_m(k)$ as:

$$\begin{aligned} \hat{y}_m(k) &\approx \sum_{i=1}^{s} \Big(B_i u(k-i) + K_i y(k-i) - K_i f(k-i) \Big) + f(k) \\ &= \sum_{i=1}^{s} \Big(B_i u(k-i) + K_i y(k-i) - K_i \omega(k-i) z \Big) + \omega(k) z \\ &= \sum_{i=1}^{s} \Big(B_i u(k-i) + K_i y(k-i) - K_i (I_2 \otimes z^T) \mathrm{vec}(\omega^T(k-i)) \Big) \\ &\quad + \omega(k) z \\ &= \sum_{i=1}^{s} \Big(B_i u(k-i) + K_i y(k-i) - M_i \mathrm{vec}(\omega^T(k-i)) \Big) + \omega(k) z \end{aligned} \tag{9}$$

Now we are ready to define the rank, cardinality constrained least squares problem that is at the heart of this paper. For that purpose, define the following quantities (assuming that we have the following input-output data $\{u(j), y_m(j)\}_{j=k}^{k+N}$ available):

$$Y_m = \begin{bmatrix} y_m(k+s) \\ y_m(k+s+1) \\ \vdots \\ y_m(k+N) \end{bmatrix}; T_u = \begin{bmatrix} u(k+s-1) & \cdots & u(k) \\ u(k+s) & \cdots & u(k+1) \\ \vdots & \ddots & \vdots \\ u(k+N-1) & \cdots & u(k+N-s) \end{bmatrix}$$

Like the (block-)Toeplitz matrix T_u we can define the matrices T_{y_m} and T_ω from the signals $y_m(k)$ and $\mathrm{vec}(\omega^T(k))$. Let the products $K_i(I_2 \otimes z^T)$ be denoted as M_i and let the unknowns B_i be stored as follows:

$$\mathbf{B} = \begin{bmatrix} B_1^T \\ B_2^T \\ \vdots \\ B_s^T \end{bmatrix}$$

and similarly we define the matrices $\mathbf{K}, \mathbf{M}$ from the matrices K_i, M_i. Finally let the matrix Ω be defined as Y_m but now from the signal $\mathrm{vec}(\omega^T(k))$, then we can define based on (9) the following constrained (linear) Least Squares problem:

$$\min_{\mathbf{B},\mathbf{K},\mathbf{M},z} \| Y_m - [T_u \ T_{y_m} \ -T_\omega \ \Omega] \begin{bmatrix} \mathbf{B} \\ \mathbf{K} \\ \mathbf{M} \\ (I_2 \otimes z) \end{bmatrix} \|_F^2 \tag{10}$$

subject to a cardinality (ℓ_0) constraint on the vector z and the following rank constraint:

$$\mathrm{rank} \begin{bmatrix} M_1 & K_1 \\ \vdots & \vdots \\ M_s & K_s \\ (I_2 \otimes z^T) & I_2 \end{bmatrix} = \mathrm{rank}(P) = 2$$

Using the nuclear norm (denoted as $\|.\|_\star$) and the 1-norm as convex relaxations of the above constraint, the simultaneous identification of the model dynamics and the isolation of the faults is formulated via the following convex optimization problem:

$$\min_{\mathbf{B},\mathbf{K},\mathbf{M},z} \| Y_m - [T_u \ T_y \ -T_\omega \ \Omega] \begin{bmatrix} \mathbf{B} \\ \mathbf{K} \\ \mathbf{M} \\ (I_2 \otimes z) \end{bmatrix} \|_F^2 + \tau \|P\|_* + \lambda \|z\|_1 \tag{11}$$

where τ, λ are hypertuning parameters. In this brief paper we simply assume that the compound matrix $[T_u \ T_{y_m} \ -T_\omega \ \Omega]$ has full column rank. This in essence means that the joint input $\begin{bmatrix} u(k) \\ \mathrm{vec}(\omega^T(k)) \end{bmatrix}$ is persistently exciting of at least order s, see Verhaegen and Verdult (2007).

4.2 A Solution to (11)

The convex problem (11) can be solved in a large number of different ways. Standard tools like `cvx`[1] can be readily applied. However more efficient implementations are available, like those based on proximal algorithms. For example for the optimization problem (11) that has three terms of which two are non-differentiable, can be handled by multiple-operator splitting schemes, such as the Parallel ProXimal Algorithm (`PPXA`) (Combettes and Pesquet, 2008), generalized forward-backward splitting (Raguet et al., 2013) or the Davis-Yin algorithm. In this paper we use for prototyping the new idea the `cvx` toolbox. The new algorithmic approach based on solving (11) is indicated in this paper as `MF2D` ("Model-Free Fault Diagnosis")

[1] `http://cvxr.com/cvx`

5. VALIDATION STUDY

5.1 Organization of the Experiment

The newly presented data driven approach is benchmarked against the state of the art Double-Model Adaptive Estimation (`DMAE`) Approach for Air Data Sensor Fault detection and diagnosis presented in Lu et al. (2021). In this approach two Kalman filters are run in parallel: one using a fault-free model and the other a combination of the fault free model with its state augmented with the faults. The faults are modelled as random walk models and in Lu et al. (2021) an extension is formulated to update the covariance matrices needed in the Kalman filter design.

When using (classical) sensor data for the longitudinal aircraft mode, the input (that is used by the `MF2D`) method) is the elevator angle (δ_e) and the output is the airdata sensor vector y_m. Use is made of real-life recorded flight test data with the Cessna II laboratory aircraft of the TU Delft and NLR. The recordings for a single flight condition that we used in this validation study are displayed in Figure 2.

When using this “standard” (limited) sensor data in a model based approach, like the multiple model based approach in Hallouzi et al. (2009), such model needs knowledge of the aerodynamic derivatives. This would make many of such model based approaches very ineffective as this would require dedicated flight testing and flight test data analysis methods for capturing these derivatives. And even then the models might never describe the actual operating conditions accurately. To overcome this major drawback the use of kinematic models was proposed in Lu et al. (2021). This one hand, frees the approach from requiring access to the aerodynamic derivatives, but on the other hand requires the aircraft to be equipped with (very accurate) Inertial Measurement units (IMUs). Such equipment is often present in navigation (and higher) grade IMUs where there is a redundancy in terms of triple or even quadruple sets of duplicates. A part from the fact that even then special precaution is still needed to deal with operational bias and noise effects, the kinematic approach requires the aircraft to be *rigid*. The measurements used with kinematic models are listed in Table 2 (`DMAE`).

In the conducted experiments the data related to the flight condition from which the recorded data as in Figure 2 is derived, is used to simulate air-data sensors faults. As introducing such sensor faults in real-life might lead to endangering the operators and aircraft, we opted for introducing these errors synthetically afterwards by adding errors to the measured data. Additive faults are introduced on the measured V_{TAS} and α as depicted in Figure 3 (under the label ”True”). In this paper the raw data (part of which shown in this figure is used). This raw data was made to be recorded as a sample rate of 100 seconds. We note hereby that the air data sensor was recorded at 10 Hz, but upsampled (ZOH) to 100Hz by the flight test instrumentation system.

5.2 Setting of the Algorithms

The `DMAE` kinematic model used is the one reported in Lu et al. (2021) This model has state dimension 6 containing $V_{\text{TAS}}, \alpha, \beta, \phi, \theta, \psi$, the input vector containing 3 accelerations A_x, A_y, A_z and angular velocities p, q, r in describing the nonlinear kinematic equations. The output vector in this case is the full state vector (plus the added faults and sensor noise).

The tuning parameters used for the `DMAE` algorithm are as follows:

- The process noise covariance matrix $Q_k = \begin{bmatrix} 10^{-4} I_3 & 0 \\ 0 & 3 \times 10^{-8} I_3 \end{bmatrix}$
- The measurement noise covariance matrix $R_k = \begin{bmatrix} 10^{-2} & 0 & 0 \\ 0 & 3 \times 10^{-6} I_2 & 0 \\ 0 & 0 & 3 \times 10^{-8} I_3 \end{bmatrix}$
- The initial state covariance matrix of the nominal (fault-free = ff) model $P_{x_0,ff} = I_6$
- The initial state covariance matrix of the augmented fault (=af) model $P_{x_0,af} = I_8$
- $N_Q = 40$ (width of moving window)

For the new algorithm `MF2D` the following settings have been used:

- $s = 4$
- $\tau = \lambda = 0.1$
- $\omega(k) \in \mathbb{R}^{2 \times 80}$ consisting of unit steps with 40 possible starting times for f_V, plus 40 possible starting times for f_α (evenly distributed).
- I/O data is detrended by subtracting the sample means.
- The two sensor outputs in y are normalized such that both outputs have a variance of 1.

5.3 Results

For the sake of brevity we focus only on the estimation of the additive faults. These are for both methods displayed in Figure 3. From this figure it is clearly observed that the new method `MF2D` outperforms the `DMAE` approach. The latter furthermore makes use of a much more elaborate sensors infrastructure (which are themselves prone to errors that need continuous calibration) and needs careful adaptation of the Kalman filter parameters, such as a careful reinitialization Lu et al. (2021). This especially for flying through turbulence might not be trivial. The performance of the `DMAE` approach is tested using the code available on `github`[2], with no efforts to finetune the defaults given. Figure 3 shows that this implementation is indeed able to *detect* that there is a fault in the ADS, but is not able to precisely *diagnose* the magnitude and the particular part of the ADS sensor failing.

On the other hand the new `MF2D` captured both faults very accurately in onset and shape. The magnitude is however not captured fully accurately due to the use of 1-norm as a convex relaxation of the 0-norm. This can however easily be improved by a re-estimation of that magnitude as outlined in Noom et al. (2023b). In that re-estimating the support and shape of the fault is then used as pictured in Figure 3. One challenging element for the new `MF2D` is the design of the dictionary ω. However prior testing might provide useful information here, and the consideration of

[2] https:github.com/lplp8899/ADS_FDD_Turbulence

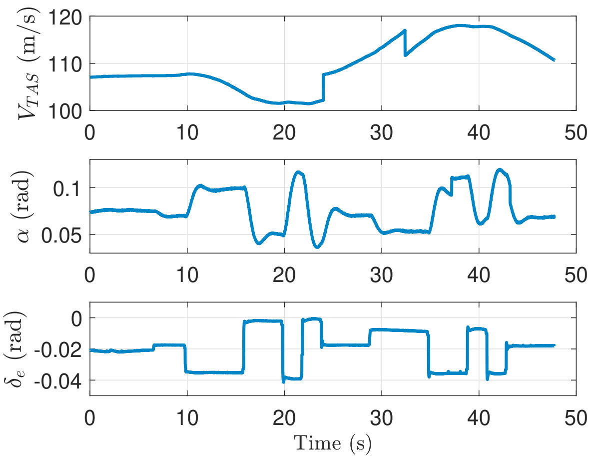

Fig. 2. The flight data utilized in the MF2D approach: true airspeed V_{TAS} (top), angle of attack α (middle) and elevator deflection δ_e (bottom).

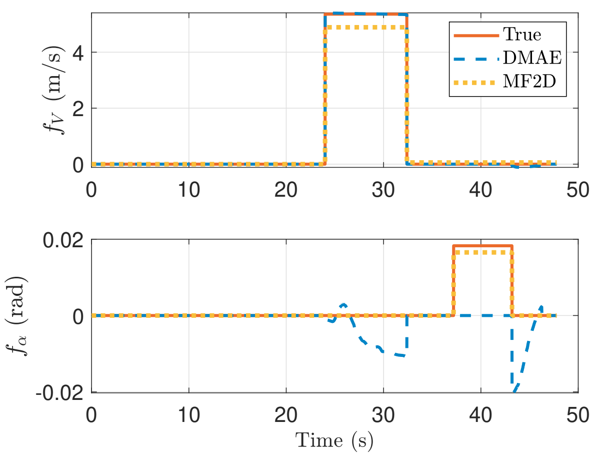

Fig. 3. The faults f_V and f_α introduced in V_{TAS} and α, respectively: true faults (red, solid), estimation using the extended DMAE-approach (blue, dashed), and estimation using the MF2D approach (yellow, dotted).

the cardinality (or 1-norm) constraint as in the current experiment enabled to correctly and accurate estimate faults that were not in the dictionary!

Table 2. Required sensors for the two approaches

	DMAE	MF2D	Description
V_{TAS}	✓	✓	true airspeed, m/s
α	✓	✓	angle of attack, rad
δ_e		✓	elevator deflection, rad
β	✓		sideslip angle, rad
p, q, r	✓		roll, pitch & yaw rates, rad/s
ϕ, θ, ψ	✓		roll, pitch & yaw angles, rad
A_x, A_y, A_z	✓		linear accelerations, m/s^2

6. CONCLUSION

This paper has presented a new method to *simultaneously* identify the (LTI) system dynamics and additive faults on the Air Data Sensors for civil aircraft. The new methods, indicated as the `MF2D` (Model-Free Fault Diagnosis) extends basic ideas of subspace identification and models the fault via basis (time-)functions in a dictionary. The `MF2D` is presented here via convex relaxation as convex optimization problem. This can efficiently and reliably be solved. Its comparison with the state-of-the-art solution based on multiple Kalman filters that avoids accurate knowledge of model information through the aerodynamic derivatives, as presented in Lu et al. (2021) demonstrated the superiority of the newly developed methodology. This comparison makes use of real-life flight test data with the Cessna Citation II aircraft.

Though this comparison is preliminary, it shows the great potential of the new `MF2D` approach. It makes use of a minimal set of sensor devices that are standardly available on aircrafts (and drones), it is able to deal with these flying objects flying through turbulence as well as laminar flow and can also deal with non-rigid flying devices. The difference in sensor configuration is highlighted for the current study in Table 2.

Through these encouraging results there is plenty of room for future extensions and validations of this approach. One is the design of the dictionary and making that dictionary adaptive by letting the basis functions move in a moving window. This extension would make the presented offline version in the current paper applicable in a recursive manner. This in combination with making the whole approach adaptive to different flight conditions, would turn the method into a fully online applicable procedure.

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