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Optimizing the Bridge Maintenance Schedule of Transportation Networks under Uncertainty: A Simheuristic Approach Considering Reliability

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ABSTRACT: This paper presents a two-stage simheuristics-based framework for optimizing bridge maintenance scheduling strategies in a highway transportation network under uncertainty considering bridge life-cycle reliability and the effects of different maintenance interventions. The design variables of the optimal bridge maintenance scheduling problem are the preventive maintenance schedules for all bridges in the network, with the conflicting objectives of minimizing total maintenance cost and total travel time over the life cycle of bridges. The outcome of the first stage of the framework is a Pareto front of deterministic optimal solutions, which are then tested in the second stage to measure their performance under uncertainty.

1. INTRODUCTION

The importance of bridges as critical infrastructure assets in transportation networks cannot be overlooked, as their breakdown and hence transportation disruptions have a catastrophic impact on facilitating a country's economy and societal development. Unfortunately, the extension of the service life of bridges coupled with other factors such as increasingly heavier truck traffic, more harsh environmental conditions, and natural hazards can lead to deterioration and thus their partial or complete failures over time.

To mitigate deterioration, it is crucial to establish effective asset maintenance and management policies for bridges within transportation networks. However, these networks are complex systems and are subject to a wide range of uncertainties, such as bridge degradation patterns, traffic behaviours, and the extent of reliability improvement after maintenance interventions. These stochastic factors can significantly influence structures' performance and the decision-making process for maintenance policies. Therefore, uncertainties must be taken into account to provide a more robust and realistic ap-

proach to bridge maintenance scheduling.

In recent years, the field of bridge maintenance scheduling has seen a growing interest in methods for addressing the inherent uncertainty involved in these activities. One notable approach is a probability-based framework proposed by Bocchini and Frangopol (2011), which utilizes a combination of genetic algorithms, Monte Carlo simulation, and various probability models in order to effectively deal with uncertainty. The authors incorporate previously established bridge life-cycle reliability models, as presented by Frangopol et al. (2001), into their framework. They propose an optimization-based approach to identify the optimal schedule associated with the minimum total cost of maintenance activities while guaranteeing the overall network performance based on total travel time and total distance travelled. An alternative framework was presented by Zhang et al. (2015), which focused on optimizing the maintenance scheduling policies based on travel-time reliability as the key metric. These studies highlight the need for comprehensive frameworks to account for uncertainty and multiple performance indicators in optimizing bridge maintenance schedules. Given that metaheuristics (e.g., genetic algorithms) are often utilized to tackle real-world optimization problems, it makes sense to combine them with simulation techniques to handle the uncertain variants of the problems. Simheuristics algorithms, first introduced by Juan et al. (2014), are a type of simulation-optimization method that is effective in addressing combinatorial optimization problems including elements of uncertainty. Simheuristics can be used to replicate real-world systems with a high level of accuracy. Additionally, they provide valuable insights into the system's behaviour by offering direct methods to interpret results through visual and statistical means (Chica et al., 2020).

In this paper, we propose a simulation-based optimization methodology for bridge maintenance scheduling policies considering total maintenance costs and network performance in terms of total travel time. The proposed method optimizes bridge maintenance scheduling strategies under monetary constraints using the NSGA-II algorithm (Deb

et al., 2002) and then tests the performance of deterministic solutions under uncertainty employing a simheuristic approach. The proposed framework presents a novel simheuristic approach for optimal bridge maintenance scheduling under stochastic conditions. Additionally, this study makes contributions not only to optimal bridge maintenance scheduling but also to the state of the art on the simheuristics concept since it provides a deeper understanding of the overall benefits and limitations of using simheuristics by introducing the concept into a new field.

The rest of the document is organized as follows. Section 2 provides an introduction to bridge maintenance components and insights into uncertainty, as well as a detailed explanation of the life-cycle bridge reliability model. In Section 3, the proposed simheuristic framework for optimal bridge maintenance scheduling is explained. In Section 4, the framework is applied to a case study, and some conclusions are drawn in Section 5.

2. BRIDGE RELIABILITY AND MAINTENANCE INTERVENTIONS

2.1. Life-cycle Reliability of Bridges

Determining the reliability of existing bridges can be challenging because it requires a comprehensive evaluation of the bridge's structural integrity and ability to withstand various loads and environmental factors. Therefore, the reliability of bridges in a network is not always certain and may only be studied in detail for critical bridges. To account for this uncertainty, analytical models can be used to assess the time-dependent reliability of individual bridges. Different models have been suggested in the literature for assessing the reliability of bridges, such as the bi-linear, quadratic, square-root and exponential models (Frangopol et al., 2001; Bocchini and Frangopol, 2011). In this study, the bi-linear model is used to reflect bridge life-cycle reliability;

$$\beta_b(t) = \begin{cases} \beta_b^0 & \text{for } 0 \leq t \leq T'_b \\ \beta_b^0 - (t - T'_b)r_b & \text{for } t > T'_b \end{cases} \quad (1)$$

where $\beta_b(t)$ is the reliability index of bridge b at any time instant t , β_b^0 is the initial value of the reliability index, r_b is the degradation rate of bridge

b , and T'_b is the time instant where degradation of bridge b starts at. Uncertainty plays a critical role in the reliability index of bridges since it is highly dependent on the construction process, the quality of materials used as well as external factors such as heavy traffic and aggressive environmental conditions. To reflect the uncertainty in this case, variables of the bridge life-cycle reliability model β_0 , r_b , and T'_b are modelled as random variables, whose parameters are shown in Table 1.

2.2. Maintenance Interventions

Bridge maintenance interventions are crucial in maintaining the safety and functionality of bridges for public use. The necessary interventions vary based on the condition and intended use of each bridge. Bridge maintenance interventions can be classified into preventive and corrective maintenance. Preventive Maintenance (PM) interventions are planned in advance to maintain the good condition of bridges and prevent collapse, whereas Corrective Maintenance (CM) interventions are not scheduled ahead of time and are implemented when a bridge is out of service due to failure. Interventions within the PM can be: (i) predetermined; (ii) based on the observed condition of the bridge; and (iii) predicted based on degradation models. For the sake of simplicity and without loss of generality, the predetermined PM and CM are taken into consideration in the proposed framework.

The effect of PM interventions on the bridge's reliability can be modelled by assuming an increase in the reliability index following different trends or a delay in the degradation process (Bocchini and Frangopol, 2011). A sudden increase in the reliability index, λ_{PM} , and a delay in the degradation process, δ_{PM} , are considered in this paper. Since these components are subjected to uncertainty, their effects on the bridge reliability index are modelled as random variables whose parameters are shown in Table 1. CM interventions have similar effects on bridges. Regarding its impact on reliability, it is assumed that CM interventions are not perfect and hence, cannot fully recover the reliability index of bridges. Therefore, the obtained reliability after a CM intervention will be $\beta_b^0 - \Delta$. Additionally, no degradation occurs for T'_b years after a CM inter-

vention is applied. In regard to the degradation of bridges, since CM interventions are also subjected to uncertainty, Δ is modelled as a random variable as shown in Table 1. The effects of PM and CM on the reliability and service life are illustrated in Figure 1.

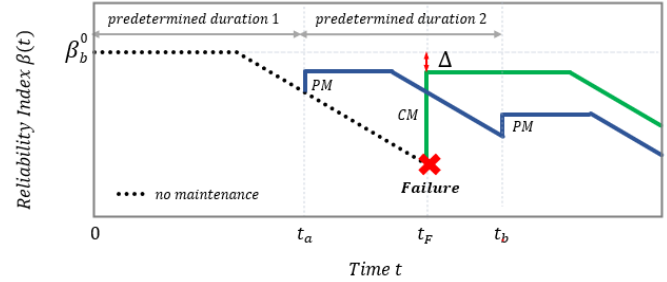


Figure 1: Behaviour of Reliability Index of a bridge over time without maintenance and with PM and CM interventions.

3. SIMHEURISTICS FRAMEWORK

Simheuristics enables decision-makers to test deterministic optimal or near-optimal solutions in a stochastic environment. As a part of the proposed framework, simheuristics, therefore, entail a deterministic optimal bridge maintenance scheduling problem as well as a stochastic simulation as illustrated in Figure 2. The main structure of Stage 1 involving the deterministic problem is explained first, and then, the implementation of the stochastic part, Stage 2 is presented.

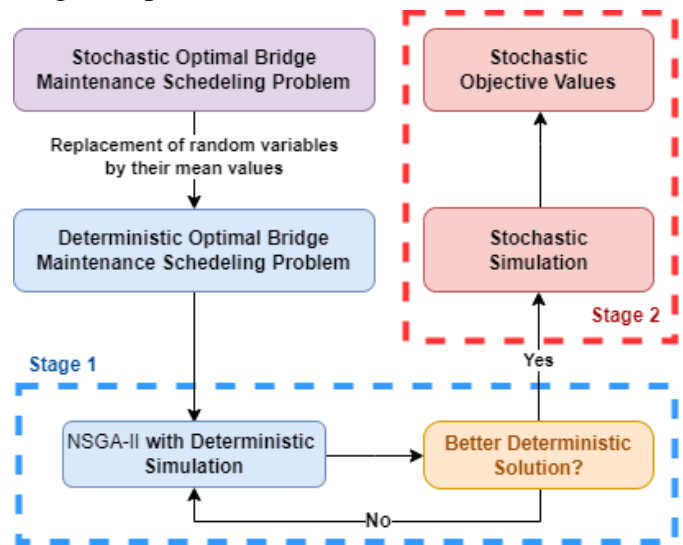


Figure 2: The logic behind Simheuristics

Table 1: Probability distributions of the variables of the bridge life-cycle reliability model.

Variables	Units	Distribution	Mean	Standard deviation	Minimum	Maximum
β_b^0	-	Lognormal	4	1.5	0	∞
T'	years	Lognormal	15	5	0	∞
r_b	years ⁻¹	Uniform	0.102	0.056329	0.005	0.2
λ_{PM}	-	Uniform	0.055	0.14434	0.3	0.8
δ_{PM}	years	Triangular	7.667	1.0274	5	10
Δ	-	Triangular	0.233	0.10276	0	0.5

3.1. Stage 1: Deterministic Optimal Bridge Maintenance Scheduling

The presented approach considers a road traffic network consisting of A interconnected roads involving B bridges. The transport network is modelled by a set of nodes and edges. The nodes represent the cities within the network, while the edges, $a = 1 \dots A$, represent the highway segments connecting these cities. Subsequently, bridges are assigned to the edges of the transportation network, considering their physical location within the network. The road traffic network is studied over a discretized period of time $t = \{0, d, 2d, \dots, T\}$. Based on the given traffic demand/supply associated with each node, the traffic flow has been distributed between all origin-destination (OD) pairs using the gravity model. Then, the traffic flow between each OD pair is assigned to each edge located on the shortest path connecting the corresponding OD pair following the all-or-nothing approach. This approach is based on the assumption that all users travelling between an OD pair will use only the shortest path available, and that other routes will be ignored. Additionally, it assumes that there will be no congestion, as free travel times for all vehicles will remain the same regardless of traffic flow on each link. The travel time on each edge at time t is calculated as follows:

$$\tau_{a,t} = f_a v_{a,t} \quad (2)$$

where f_a is the free-flow travel time of edge a and $v_{a,t}$ is the traffic flow assigned to the corresponding edge. The life-cycle bridge reliability model is implemented by assigning the initial values of β_b^0 , r_b , and T'_b . Despite the stochastic nature of these variables, for the deterministic version of the optimization problem, they are replaced by their mean val-

ues. The same principle applies to the parameters of PM and CM interventions outlined in Table 1.

The deterministic optimal bridge maintenance scheduling problem is defined as follows; The design variables are the timings of the PM interventions on each bridge in the network, $x_{b,i}$, $i = 1 \dots I$, with I being the maximum number of possible PM interventions per bridge during the studied time. The main goal is to find the optimal schedules that balance trade-offs between conflicting objectives, namely, minimizing the total cost of maintenance, $Cost$, while adhering to a budget constraint, C , and minimizing the total travel time, TT , over the studied time period. This is mathematically formalized as follows:

$$\min_{\mathbf{x}} \{Cost(\mathbf{x}); TT(\mathbf{x})\} \quad (3)$$

$$Cost = \sum_{b=1}^B \sum_{i=1}^I \frac{C_{PM}}{(1+r)^{x_{b,i}}} + \sum_{b=1}^B \sum_{j=1}^J \frac{C_{CM}}{(1+r)^{y_{b,j}}} \quad (4)$$

$$TT = \sum_{t=0}^T \sum_{a=1}^A \tau_{a,t} \quad (5)$$

subject to

$$\mathcal{D} : \mathbf{x} \rightarrow \{\tau, \mathbf{y}\} \quad (6)$$

$$Cost \leq C \quad (7)$$

$$0 \leq x_{b,i} \leq T \quad b = 1 \dots B, i = 1 \dots I \quad (8)$$

where $y_{b,j}$ is the timing of the CM interventions on each bridge in the network, with $j = 1 \dots J$, r is the interest rate, and C_{PM} and C_{CM} are the constant cost of a PM and CM intervention, respectively. Note that there is no restriction regarding the number of possible interventions (i.e., I and J) and time resolution considered beyond the available computational budget.

The function \mathcal{D} in Eq. (6) refers to the deterministic simulation, which takes \mathbf{x} as input to calculate the corresponding road network performance and CM interventions over time. The pseudo-code of the deterministic simulation algorithm is shown in Figure 3.

Algorithm for Deterministic Simulation

```

for Every Candidate Solution (PM Schedules) do
- Take PM Schedule as Input for Simulation
- Create Bridges and Transportation Network
- Assign Reliability and Maintenance Parameters to Bridges
- For every time instant t=T
    • Update the reliability index of bridges due to degradation
    • Apply PM Interventions
    • Simulate the bridge in-out of service status
    • For every failed bridge, close the corresponding highway segment
    • Do Traffic Assignment
    • Calculate travel time for t=T
    • Apply CM Interventions for every out-of-service bridge.
- Go to next time instant t = T + 1
end for

```

Figure 3: Pseudo-code for Deterministic Simulation.

At every time instant t , the reliability index of each bridge, $\beta_b(t)$, is updated using the bi-linear reliability index model, Eq. (1). Additionally, the scheduled PM interventions are applied to the corresponding bridges. Then, the service state of every bridge in the network is simulated considering the current $\beta_b(t)$. This is achieved through the use of the service state variable, $s_b(t)$, which is defined for the deterministic version of the optimization problem as follows:

$$s_b(t) = \begin{cases} 0 & \text{for } \beta_b(t) \leq 1 \\ 1 & \text{for } \textit{otherwise} \end{cases} \quad (9)$$

Accordingly, reliability values less than or equal to 1 imply that the corresponding bridge becomes out-of-service ($s_b = 0$). For any other case, the bridge is in service. Upon simulating the service state of all bridges, the subsequent step is to close the edges on which out-of-service bridges are located. This is achieved by assigning a significantly high value to the free-flow travel time, f_a , of the corresponding edge for each out-of-service bridge within the network. As a result, these edges will not be considered part of the shortest path for any origin-destination pair in the network. Additionally, the algorithm applies a penalty to $\tau_{a,t}$ of the affected edge if there is any PM intervention scheduled at the current time instant, as follows:

$$\tau_{a,t}^{updated} = \tau_{a,t} (1 + \gamma n_{a,t}^{PM}) \quad (10)$$

where γ is a penalization parameter and $n_{a,t}^{PM}$ represents the number of PM intervention on edge a at the current time step, t . For instance, if there are two PM interventions scheduled for two bridges located on the same edge, $n_{a,t}^{PM} = 2$. The main purpose of this penalty is to account for disruptions in traffic caused by PM interventions and therefore, the increase in travel time. After the traffic assignment process, CM interventions are applied to the bridges that are out-of-service. At this point, the deterministic simulation proceeds to the next time instant, and the entire process is repeated until the end of the time horizon T is reached. To solve the multi-objective optimal bridge maintenance scheduling problem, Eqs. (3)–(8), the NSGA-II is employed. For every generated solution, NSGA-II runs the deterministic simulation that takes the deterministic candidate solutions as input.

3.2. Stage 2: Evaluating uncertainty via stochastic simulation

In Stage 2, the main purpose is to test the optimal deterministic solutions under uncertainty. This is done by introducing a stochastic simulation, \mathcal{S} that involves Monte Carlo Simulations. For every deterministic solution obtained in Stage 1, the stochastic simulation is run n times. Therefore,

$$\mathcal{S} : \mathbf{x}_{det} \rightarrow \{Cost_k, TT_k\} \quad k = 1 \dots n \quad (11)$$

where \mathbf{x}_{det} is a deterministic solution obtained in Stage 1. Probabilistic analysis can be performed to evaluate the quality of each solution when introducing uncertainty.

The main difference between the deterministic, \mathcal{D} and the stochastic simulation, \mathcal{S} is that the life-cycle bridge reliability model parameters and the parameters of both PM and CM components are modelled as random variables as shown in Table 1.

The approach for simulating the service state of the bridges is also different. In contrast to the deterministic version, the service state of the bridges is simulated through the use of Bernoulli's distribution, $s_b(t) \sim Bernoulli(m_b(t))$, with parameter

$$m_b(t) = \Phi[\beta_b(t)] \quad (12)$$

where Φ is the standard Gaussian cumulative distribution function. The analysis allows decision-makers to make the final decision among the deterministic solutions on the basis of engineering judgment considering their stochastic performance under uncertainty.

4. NUMERICAL EXPERIMENTS

4.1. Case description

The benefits of using the presented approach are exemplified by the toy infrastructure network shown in Figure 4, consisting of 6 nodes, 16 edges (two-direction roads) and 13 bridges. The network and traffic data are based on Bocchini and Frangopol (2011).

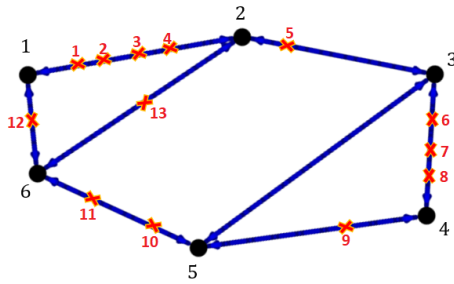


Figure 4: Toy infrastructure network. Cross signs refer to bridges located on the edges.

For the bridges in the network, the bi-linear life-cycle reliability model has been assigned with the parameters given in Table 1. The mean values of these parameters are used for the deterministic simulation \mathcal{D} in Stage 1. For the stochastic simulation \mathcal{S} , all of the parameters are generated randomly following the corresponding probability distributions in Stage 2. The life cycle of bridges and the time-step are assumed $T = 75$ and $d = 5$ years, respectively. The maximum number of PM interventions per bridge, I , is assumed to be 2. The penalization parameter γ is assumed to be 0.3 for the network analysis. The cost of PM and CM interventions, C_{PM} and C_{CM} , are assumed to be 500 and 1000 monetary units, respectively, and the budget is set equal to 3300 units for an interest rate $r = 5\%$.

4.2. Deterministic Optimization

Following the procedure explained in Section 3.1, the deterministic Pareto optimal solutions for

Table 2: Summary of scheduling for Pareto solutions

Pareto solution	Average Timing 1 st PM (years)	Average Timing 2 st PM (years)
X1	36.45	53.85
X12	49.95	63

the bridge maintenance scheduling problem (i.e., the PM schedules) are found as shown in Figure 5. The Pareto front displays the trade-off between conflicting objectives $Cost$ and TT by presenting a set of non-dominated solutions, where no other solution is superior in both objectives. With no other information about the relative importance of each of these objectives, all the points on the Pareto front are equally good. Table 2 provides a summary of the scheduling obtained for the Pareto solutions X1 and X12, in which travel time and cost are respectively prioritized.

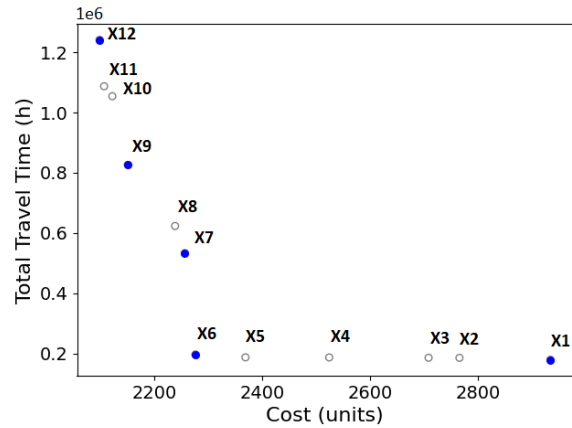


Figure 5: The deterministic Pareto optimal solutions for the bridge maintenance scheduling problem.

4.3. Stochastic Simulation

In Stage 2, the stochastic simulation has been run $n = 100$ times for each of the deterministic solutions of the Pareto front. The stochastic performances of 5 selected deterministic solutions, X1, X6, X7, X9 and X12, highlighted in Figure 5, are further analyzed. Figure 6 provides the corresponding cost and total travel time associated with each of these points under the 100 stochastic scenarios. The stochastic performance of the selected deterministic solutions for each objective, $Cost$ and TT , is also represented by their marginal probability distribu-

tions in Figures 7 and 8, respectively. The deterministic values of the objective functions $Cost$ and total travel time, TT , and the corresponding mean values under stochastic conditions, \overline{Cost} and \overline{TT} , for the selected deterministic solutions are shown in Table 3.

Table 3: $Cost$ (monetary units) and total travel time (10^5 hours) for selected deterministic solutions (PM Schedules) and the corresponding mean values under stochastic conditions.

Pareto Point	$Cost$	TT	\overline{Cost}	\overline{TT}
X1	2934	1.80	3727	3.81
X6	2275	1.96	3180	4.03
X7	2256	5.33	3218	3.83
X9	2150	8.25	2965	3.85
X12	2098	12.41	3005	4.63

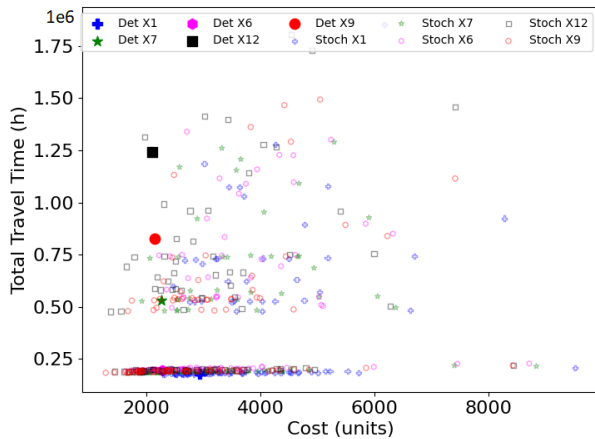


Figure 6: Stochastic performance of the deterministic optimal solutions X1, X6, X7, X9 and X12.

4.4. Discussion

Several deterministic solutions of the Pareto front (i.e., X1, X6, X7, X9 and X12) are investigated due to their significance. Solutions X1 and X12 represent the extreme cases, with X12 showcasing the highest total travel time and the lowest cost, while X1 presents the opposite scenario. Additionally, X6 demonstrates a considerable improvement in TT while incurring only a slight increase in cost, as compared to X7. Finally, X9 is also included based on its stochastic performance presented in Table 3.

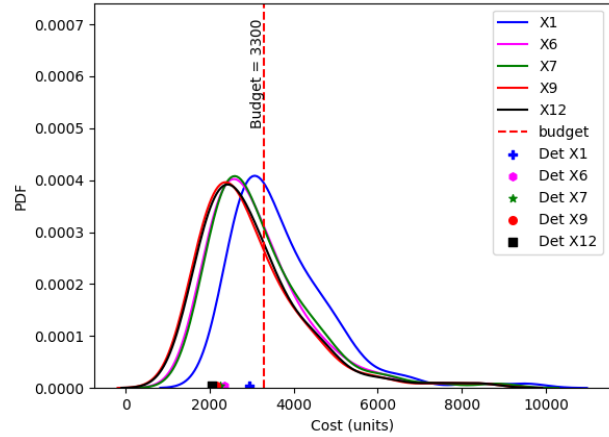


Figure 7: Marginal PDF of the cost for solution X1, X6, X7, X9 and X12.

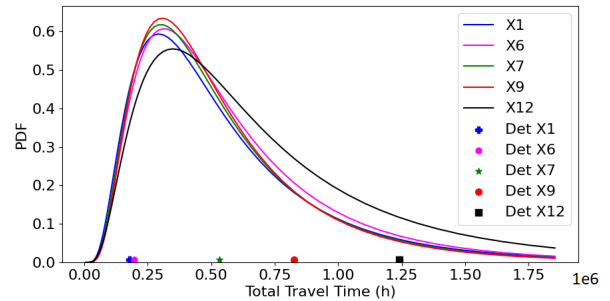


Figure 8: Marginal PDF of the total travel time for solution X1, X6, X9 and X12.

The findings shown in Figure 8 and Table 3 indicate that the expected value of the total travel time under uncertainty for the schedules of solutions X7, X9, and X12 exhibit better performance when compared to the deterministic environment. The probability of experiencing less travel time than the ones considered with the deterministic approach is 0.71, 0.92 and 0.92, respectively. Given that all bridges were assigned the same initial reliability index in the deterministic simulation and that out-of-service conditions were assumed to occur below a specific threshold, it is reasonable to anticipate a higher TT in the deterministic environment as multiple bridges may become unrealistically out-of-service simultaneously based on the PM schedule. In contrast, solutions X1 and X6 were found to be only at the 12th and 17th percentiles, respectively. This implies that the schedules of X1 and X6 are likely to experience more total travel time under uncertainty. Although these solutions are chosen when

travel time is the most important objective, the assumed risk of experiencing larger travel times is the largest.

Regarding the cost, none of the deterministic solutions is likely to exhibit good performance under uncertainty (see Figure 7). X6, X7, and X12 are at the 23rd, 20th, and 22nd percentile, respectively, while X1 and X9 are at a slightly higher 26th percentile. The main reason behind the poor cost performance of the schedules under uncertainty is that the bridge life-cycle reliability is stochastic in Stage 2. This results in some bridges having a lower initial reliability index, leading to a higher need for CM interventions over the life-cycle of bridges based on the simulation of their service state. In addition, as shown in Figure 7, there is a high probability of cost overruns for the five schedules associated with the deterministic solutions. The probability that the schedule of solution X1 is larger than the maximum budget is 53%, followed by X6 and X7, with 34% and 36%, respectively. X9 and X12 have a higher likelihood of staying within budget, with a probability of exceeding the budget of 29% and 30%, respectively. These probabilities are inadmissible in a real situation.

5. CONCLUSIONS

The proposed framework is based on the concept of simheuristics and offers decision-makers the chance to examine the results of using deterministic solutions under uncertainty. The numerical study shows that considering a deterministic approach for the optimal bridge maintenance scheduling problem can result in extra total travel times and costs that were not foreseen, making a solution that was believed to be optimal non-suitable to fulfilling requirements such as a maximum budget.

The different objectives of the optimal bridge maintenance scheduling problem, i.e., the total cost of maintenance and the total travel time, are impacted differently by the presence of uncertainty. While the schedules can experience an increase or decrease in the total travel time with respect to the value considered in the deterministic approach, they tend to perform poorly financially due to the noticeable rise in the total cost of maintenance when uncertainty is present.

Finally, the proposed framework provides a valuable understanding of the behaviour of deterministic solutions under uncertainty, however, it also has some limitations. The stochastic simulation is not incorporated into the optimization process, it only serves as an evaluator in Stage 2. This could result in a possibility that non-Pareto optimal solutions perform better than the Pareto optimal solutions under uncertain conditions. To address this, integrating stochastic simulation into the optimization process will be a focus of future work. In addition, future efforts will include incorporating additional bridge reliability models and implementing the user equilibrium method as a traffic assignment algorithm to achieve more precise results in uncertain conditions.

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