Benefits of space-time diversity for radar - Les bienfaits de la diversité spatio-temporelle en radar

le Chevalier, Francois; Petrov, Nikita

DOI
10.1016/j.crhy.2019.03.002

Publication date
2019

Document Version
Final published version

Published in
Comptes Rendus Physique

Citation (APA)
le Chevalier, F., & Petrov, N. (2019). Benefits of space-time diversity for radar - Les bienfaits de la diversité spatio-temporelle en radar. *Comptes Rendus Physique*, *20*(3), 183-191.
https://doi.org/10.1016/j.crhy.2019.03.002

C. R. Physique 20 (2019) 183–191

Contents lists available at ScienceDirect

Comptes Rendus Physique

www.sciencedirect.com

URSI-France 2018 Workshop: Geolocation and navigation / *Journées URSI-France 2018 : Géolocalisation et navigation*

Benefits of space-time diversity for radar

Les bienfaits de la diversité spatio-temporelle en radar

François Le Chevalier [*], Nikita Petrov

Delft University of Technology, The Netherlands

ARTICLE INFO

Article history:
Available online 26 March 2019

Keywords:
Radar
Diversity
Wideband
MIMO

Mots-clés :
Radar
Diversité
Large bande
MIMO

ABSTRACT

When designing a new radar system, standard resolution trade-offs play a major role, providing the basic parameters of the radar, such as size, update rate, and range. Besides, diversity has long been used for mitigating fading effects due to the fluctuation of targets and clutter.

However, with the arrival of more flexible systems, using multiple parallel channels on transmit and receive, and wider instantaneous bandwidths, these standard trade-offs are becoming less simple—and more flexible. In this communication, we will analyse the benefits of diversity and its relations with range, Doppler, and angle, for detection and location of moving targets with wideband/wide-beam radar systems. The idea is to contribute to a better understanding of the real benefits of agile transmissions for detection/localization of moving targets, focusing on range, velocity, and angular measurement improvements, as well as on the benefits for detection of moving targets.

Special attention will be given to the quality of the different wideband wide-beam sensor modes for long-range surveillance, and new results on detection of moving targets in clutter will be provided to demonstrate the effectiveness of these new architectures for small target detection at long range, in difficult environments.

RÉSUMÉ

Lors de la conception d'un nouveau système radar, les compromis classiques de résolution jouent un rôle majeur, pilotant les paramètres de base du radar, tels que la taille, la cadence de renouvellement et la portée. Par ailleurs, la diversité a longtemps été utilisée pour atténuer les effets de *fading* dus à la fluctuation des cibles et du fouillis.

Cependant, avec l'arrivée de systèmes plus souples, l'utilisation de plusieurs canaux parallèles en transmission et en réception, et des largeurs de bande instantanées plus larges, ces compromis standard deviennent moins simples – et plus flexibles. Dans cette communication, nous analyserons les apports de la diversité et ses relations avec la distance, le doppler et l'angle, pour la détection et la localisation de cibles en mouvement avec des systèmes radar à large bande/large faisceau. L'idée est de contribuer à une meilleure compréhension des avantages réels des transmissions agiles pour la détection/localisation de cibles mobiles, en se concentrant sur les améliorations de portée,

---

[*] Corresponding author.
E-mail addresses: F.LeChevalier@TUDelft.nl (F. Le Chevalier), N.Petrov@TUDelft.nl (N. Petrov).

https://doi.org/10.1016/j.crhy.2019.03.002

de vitesse et de mesure angulaire, ainsi que sur les avantages pour la détection des cibles en mouvement.

Une attention particulière sera accordée à la qualité des différents modes de détection pour la surveillance à longue portée, et de nouveaux résultats sur la détection des cibles mobiles en présence de fouillis seront présentés pour démontrer l'efficacité de ces nouvelles architectures pour la détection de petites cibles à longue portée, dans des environnements difficiles.

1. Introduction

1.1. Objectives

In order to design a surveillance radar,[1] a critical point is the "illumination time", also called the "time on target": this time duration should be long enough to allow Doppler analysis, and to gain a sufficient signal-to-noise ratio (SNR), but also sufficiently small to allow a fast update rate, required by the user. This well-known trade-off between update rate and velocity resolution also involves the antenna beamwidth (the wider the beam, the better the velocity resolution, for a given update rate), and the clutter rejection capability (the wider the beam, the higher the clutter level), and has also direct consequences on the power budget (the wider the beam, the lower the antenna gain, but also the higher the coherent integration gain, for a given update rate).

These intricate relations between beamwidths, velocity resolution, and power budget (hence, range) are getting even more complex when taking into account the fluctuation characteristics of the targets and clutter, since performances can be improved through an increased averaging of clutter and target echoes—averaging which may itself be eased through widening of the beam, or longer illumination time. Such improvements are often more difficult to analyse, because they arise through modifications of the clutter and targets distribution functions, more complex than mere mean or standard deviation modifications: clutter and targets being generally not Gaussian, averaging several samples generally changes the shape of the resulting distribution functions.

Moreover, modern radar systems generally operate over significant relative bandwidths—typically 10%—which can be exploited either coherently, with a wide instantaneous bandwidth (providing a fine range resolution), or non-coherently, with a collection of measurements in different sub-bands. Again, these different operating modes have consequences on the power budget, but also on the distribution functions of targets and clutter.

The purpose of this paper is to try and clarify, with intuitive reasoning rather than precise equations, the order of magnitudes of these competing effects, so as to provide the designer with some basic insight necessary for building new radar architectures, involving multiple transmitting/receiving channels and arbitrary waveform agility.

1.2. Canonical problem

Detection being a 2-hypothesis problem (H0: no target, H1: a target), it basically comes down to comparing a certain quantity X, function of the received signals and of the expected situations (e.g., energy of the output of a matched filter), to a threshold depending on the required probability of detection P_d and probability of false alarm P_fa. This situation is shown in Fig. 1, where the position of the threshold T defines the probability of detection Pd (area with oblique lines) and the probability of false alarm P_fa (area with horizontal lines).

Obviously, the shape of the probability density functions $p_{x/\mathrm{H}i}$ (probability of the received signal, under hypothesis H_i) is critical here. Using *diversity* is a means to improve the separation: generally speaking, averaging quantities is a way to reduce the spread (and change the shape) of each probability density functions, and to bring it closer to a Gaussian (central limit theorem); using *coherent integration*, or more generally matched filtering, is a way to increase the mean value of X under hypothesis H_1. Both techniques thus improve the separation, in different ways: narrowing each distribution function, or shifting them along the horizontal axis. Our objective here is to clarify these effects, and their consequences, for typical situations.

2. Standard detection

Statistical detection of radar fluctuating targets in the presence of noise is limited by the presence of noise and by the fact that the target may provide only very small signals for certain presentation angles or frequencies of illumination (a phenomenon also known as target fading in the literature). In order to mitigate target fading, most radars use frequency agility:

---

[1] This paper focuses on the basic issue of detection of moving targets with a ground surveillance radar, taking into account ground clutter.

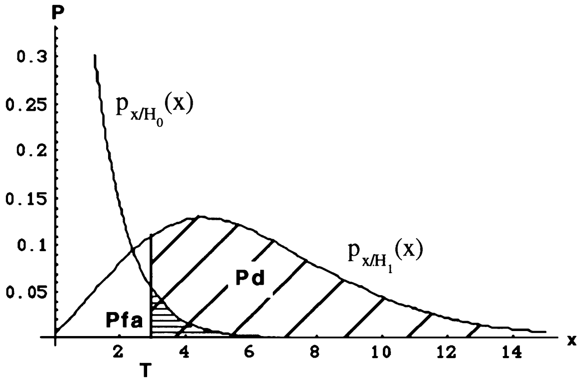


Fig. 1. Detection and false alarm, after thresholding of the quantity X with threshold T (from [1]).

(1) they transmit successive bursts at different carrier frequencies;
(2) when received, each burst is coherently processed as usual (Doppler filtering in each range cell);
(3) the outputs of these coherent summations are non-coherently summed (sum of the modulus, or the squared modulus), before final detection thresholding is applied.

This way improves the signal-to-noise ratio through each coherent burst processing. The resulting non-coherent summation allows consideration of observations at different frequencies, involving different target radar cross sections (RCS). In practice, these different bursts also generally use different repetition frequencies, allowing removal of the ambiguities in range and Doppler [1].

For a high required probability of detection, “some” non-coherent integration is preferable, in order to avoid getting trapped in a low RCS zone, especially for highly fluctuating targets (e.g., Swerling 1 targets, in the standard classification of targets fluctuations [1,2]). “Some” means that coherent integration must first be used to get a sufficient signal-to-noise ratio (SNR) which should typically be larger than 0 dB after coherent integration, so that it is not too much degraded by the modulus operation, which, as every nonlinear operation, severely reduces the detection capability if it is done at low SNR. This explains the often used “golden rule”: first improve SNR through coherent integration, and then mitigate the low-RCS zones by sending a few bursts with frequency agility from burst to burst. The price to pay for that noncoherent integration (and the associated diversity gain) when the available time-on-target is limited, is a lower Doppler resolution because of shorter coherent bursts.

The next question is: how many bursts are required during the time on target? Fig. 2 gives a generic answer by showing the required SNR per burst as a function of the number of bursts on the target: $N = 6$, or 10, for Swerling targets [1–3]. The diversity gain can be defined as the ratio of the required SNRs per sample, between the coherent summation case and the noncoherent summation case. It clearly appears that the diversity gain is maximum for $N = 6$: between 2.6 and 7 dB, depending on the required probability of detection, for this case of Swerling cases 1 and 2. There is still a gain for higher numbers of bursts, but it is much smaller.

Similar analyses with Swerling case 3 and 4 targets show that the gain is lower—as expected, since the fluctuations of Swerling 3 targets are smaller than those of Swerling 1—but still exists, at least for detection probabilities larger than 0.8 (i.e. gain between 2.7 and 1 dB). The diversity gain would then become a loss for very high resolution of Swerling 3 targets (1 dB loss for a target analysed in 30 cells and a required $P_\mathrm{D} = 0.8$, not shown in the figure).

This basic analysis of diversity leads to the following conclusions:

- the more the target fluctuates, the higher the diversity gain;
- higher requirements in the probability of detection P_d lead to a higher diversity gain;
- the number of bursts should be between 5 and 10, not more.

A very similar reasoning could apply in the angular/spatial domain, as just shown here in the range/frequency domain. For a multistatic system with a few radar sites, some non-coherent integration will nicely complement coherent integration of the signals received by each site. Depending on the exact signature of the target, spatial diversity, or frequency diversity, could be preferable: frequency diversity when the scatterers are distributed in range, spatial diversity when the scatterers are distributed in angle. A good solution, if possible, consists in combining both, for example with two or three frequencies per site, and two or three transmitting and/or receiving sites. However, it should be emphasized that frequency diversity is very generally an existing feature on most medium/long-range monostatic radars (because most of them use multi-bursts operation, for ambiguity/eclipses removal), whereas spatial diversity, requiring multisite implementations, is only applicable for specific situations, such as the passive radars as discussed by Cherniakov in [4] and Chernyak in [5]).

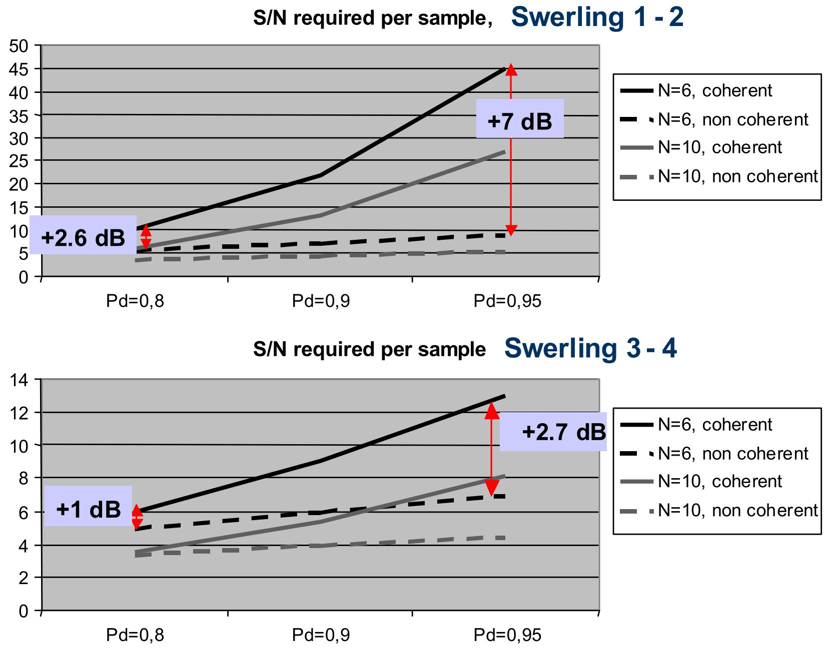


Fig. 2. The diversity gain for fluctuating target detection. The traces show noncoherent versus coherent integration for a $P_{\mathrm{fa}} = 10^{-6}$. The vertical scale is linear (not in dB).

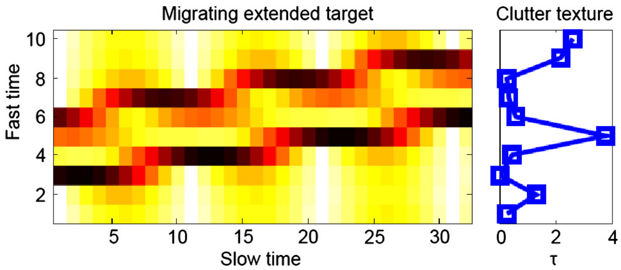


Fig. 3. Range migrating extended target in spiky clutter.

3. Wideband radar detection of moving targets

An essential limitation for standard narrowband radars using bursts of periodic pulses comes from the well-known pulsed radar range–Doppler ambiguity relation, which states that the ambiguous velocity V_{a} and the ambiguous range D_{a} are related by $D_{\mathrm{a}} \times V_{\mathrm{a}} = \left(\frac{c}{2}\right) F_{\mathrm{r}} \times \left(\frac{\lambda}{2F_{\mathrm{r}}}\right) = \lambda \times c/4$. This relation means that many ambiguities, either in range or velocity (or both), need consideration. This in turn implies the transmission of successive pulse trains with different repetition frequencies, requiring more time to be spent on target for ambiguity and blind speeds removal (without a corresponding gain in Doppler resolution, since the successive coherent pulse trains are then processed incoherently).

An alternative solution [1] consists in improving the range resolution (through increasing the instantaneous bandwidth) so that the moving target range variation (range walk or range migration, as shown on Fig. 3) during the pulse train becomes non-negligible compared with the range resolution. Such radars may use bursts with a low pulse repetition frequency (no range ambiguities) and wideband pulses such that the range walk phenomena during the whole burst is significant enough to remove the velocity ambiguity (the range walk being a non-ambiguous measurement of the radial velocity). It then becomes possible to detect the target and measure range and velocity with only one long coherent (and wideband) pulse burst (Fig. 3).

With such wideband radars, using Parseval's theorem, we first observe that the energy in the squared modulus of the impulse response (range profile) is the same as the energy in the squared modulus of the frequency response. So, summing the energy of the impulse response along the length of the target is equivalent, from a detection point of view, to summing the energy of the corresponding frequency response.

Integration along the range profile of the target for a pre-assumed length of the targets of interest (e.g., 15 m for air targets) is a way to combine coherent integration, used to obtain the range profile with its associated Doppler spectra in each range cell, with noncoherent integration. In other words, for wideband radars, coherent integration time—and the

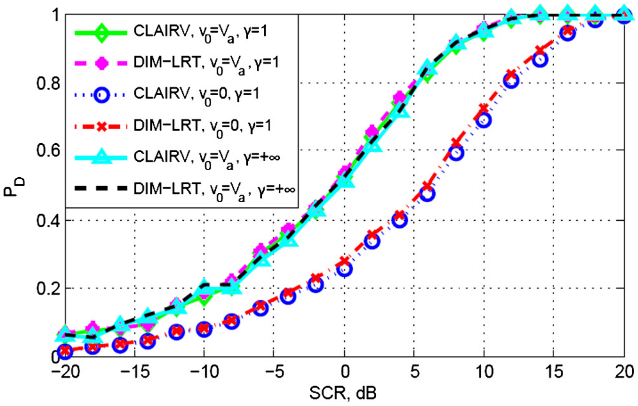


Fig. 4. Detection probability of a range-migrating target in CG clutter with $v = 0$ m/s and $v = 15$ m/s, SCR is the power of clutter after whitening, exponential correlation in range with $\gamma = 1$ and $\gamma = +\infty$; PFA $= 10^{-5}$. The parameters of the radar are: $f_c = 10$ GHz, $B = 1$ GHz, $\delta_R = 0.15$ m, $T_r = 1$ ms, M (number of pulses) $= 32$.

clutter separation in Doppler that it provides—needs not be reduced to take benefit from the diversity gain. Thus, summing the bursts in each range cell of a narrowband agile radar is equivalent to summing the samples of the range profile of a high-range resolution radar.

An appropriate detector for such wideband radars has been designed [6], ensuring CFAR performance with respect to the clutter texture, speckle correlation matrix, and target velocity. Comparison in Fig. 4 includes the CFAR detector DIM-LRT (for Dependent Interference Model, Likelihood Ratio Test), and the clairvoyant detector (assuming known the correlation matrix of clutter). The loss of the proposed detector in comparison to the clairvoyant one is about 1 dB in each scenario. The analysis shows that target detection performance does not depend on the clutter's spatial correlation γ, but the detection performance depends on the target's velocity. Thus, the detection gain for the target with velocity $v_0 = 15$ m/s, which obeys a range-walk of about 3 range cells during the CPI, with respect to the stationary one, is about 7 dB in K-distributed clutter with shape parameter $\nu = 0.5$.

This phenomenon—improved detection of fast-moving targets—can be well explained by the diversity of clutter, obtained by coherent integration of the target response during its migration over a few range cells. The faster the target, the more it migrates during the whole burst, the lower the probability to miss the target due to a possible clutter spike in one range cell, so the higher the probability of detection: the target's range-walk (migration) along non-Gaussian clutter thus provides a new way to exploit clutter diversity. This behaviour is similar to the detection of range-extended targets in CG clutter, where the detection performance depends on the target's extent (see, e.g., [7]). The observed diversity gain is not linear and saturates as the number of the range cells increases: we have observed that the major improvement is obtained by the first three-range-cell migration, and fully saturates when the range-walk exceeds five range cells.

Generally speaking, the radar range resolution should be selected such that the target of interest (given its expected dimensions and radial velocity) is spread over 5–10 range cells as a result of its range extent and its range migration. So meter resolution of the surveillance radar is sufficient for the detection of typical air targets with spatial diversity.

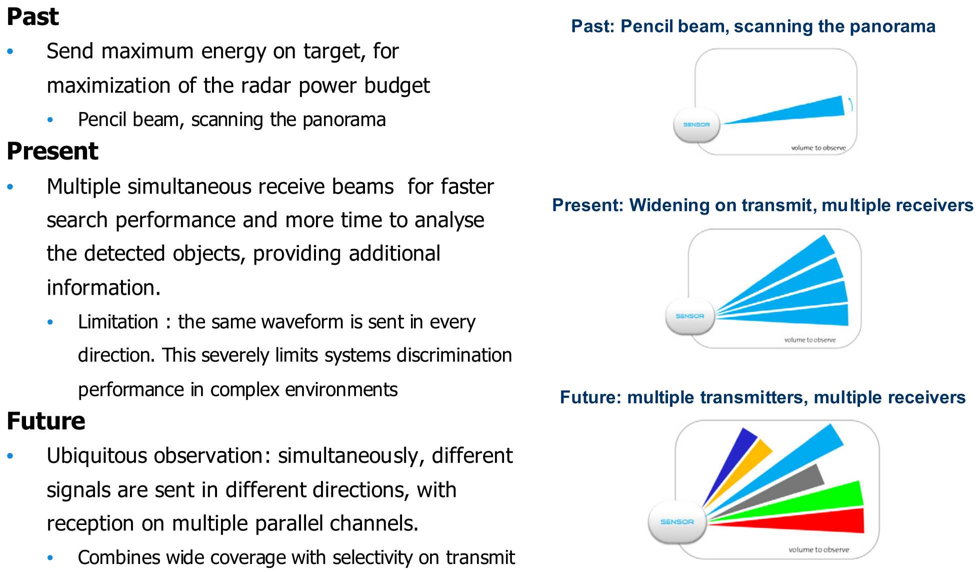
4. Wide-beam surveillance

Standard digital beamforming provides wide angular sector instantaneous coverage with a wide-beam illumination on transmit by transmitting through one relatively small subarray, or through multiple sub-arrays with appropriate phase coefficients for widening the beam. In this technique, also known as "beam spoiling", the multiple directive beams are simultaneously formed on receive through coherent summations of signals received on different subarrays, in parallel for each aiming direction.

Digital beamforming generally does not essentially change the power budget, compared to standard focused exploration, since the lower gain on transmit (due to wider illumination) is traded against a longer coherent integration time (made possible by the simultaneous observation of different directions). In fact, the main benefit provided by digital beamforming is an improved velocity resolution obtained through this longer integration time, especially useful for target identification purposes, or for detection of slow targets in clutter.

However, the improved velocity resolution of this wide beam exploration comes at a cost: i.e. the non-directive beam on transmit, which induces a poorer rejection of echoes coming from adjacent directions. For ground or surface applications, this means that detection of small targets in the presence of strong clutter will become more difficult: clutter echoes from different directions, which were cancelled not only through the Doppler rejection, but also through the angular separation on transmit and on receive, are now less easily rejected.

Moreover, the use of a wide beam on transmit implies that the angular resolution—and accuracy—is only obtained on receive; the angular resolution is thus poorer—approximately by a factor $\sqrt{2}$—compared to the standard pencil beam solution.



Past

- Send maximum energy on target, for maximization of the radar power budget
  - Pencil beam, scanning the panorama

Present

- Multiple simultaneous receive beams for faster search performance and more time to analyse the detected objects, providing additional information.
  - Limitation : the same waveform is sent in every direction. This severely limits systems discrimination performance in complex environments

Future

- Ubiquitous observation: simultaneously, different signals are sent in different directions, with reception on multiple parallel channels.
  - Combines wide coverage with selectivity on transmit

Past: Pencil beam, scanning the panorama

Present: Widening on transmit, multiple receivers

Future: multiple transmitters, multiple receivers

Fig. 5. Pencil beam, widebeam digital beamforming, space-time coding (from [4]).

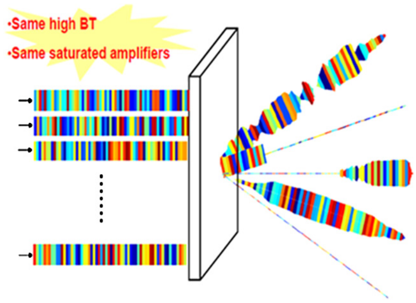


Fig. 6. Space-time coding, generic case (from [3], courtesy J.-P. Guyvarch).

In order to recover this angular separation on transmit (which was basic to standard focused beam techniques), it is necessary to code the transmitted signals (space-time coding), such that the signals transmitted in the different directions are different—and then become separable on receive as shown in Fig. 5.

This method uses different signals simultaneously transmitted in different directions—thus jointly coding space and time—and then coherently processed in parallel on receive. Such concepts, first proposed and demonstrated by Drabowitch and Dorey [8,9], should now be considered as mature techniques to be implemented in operational systems. Basically, the main advantages to be gained are a better extraction of targets—especially slow targets—from clutter, multipath, and noise, and a better identification of targets obtained through longer observation times, and possibly wider bandwidths.

The generic configuration is shown in Fig. 6: different codes (preferably with constant amplitude, for better efficiency of the amplifiers) are transmitted through the different antenna elements, or sub-arrays (e.g., lines, or columns of a 2D flat antenna array); the resulting signal is transmitted by the antenna, resulting in different modulated signal in the different directions.

The main properties of space-time coding—as far as we are concerned here—are summarized in their range-angle ambiguity function. Indeed, due to the fact that different signals are transmitted through the different antenna elements, the result is a coupling between the range and angle information, so the angular diagrams cannot be analysed without looking simultaneously at the range domain (the Doppler domain is not affected, since these coding are inside each pulse, and repetitive from pulse to pulse: the Doppler selectivity and Doppler sidelobes are then just as usual).

Let us consider this ambiguity function for “circulating codes”, which are a good example of such space-time coding, with nice properties as detailed in [3] and [10]. Circulating codes are generated, as shown in Fig. 7, by the same waveform (e.g., a chirp) “circulating” with a relative time shift δt through N transmitter channels. The relative time shift δt between adjacent circulating signals is equal to a one-time sample, $\Delta t = 1/\Delta F$, where ΔF is the signal bandwidth. As detailed in [3] and [11], it is possible to get different properties with space-time codes based on those circulating codes (or analogously on frequency diverse arrays, which are very similar when the circulating signal is a chirp, since a time shift is then equivalent to a frequency shift).

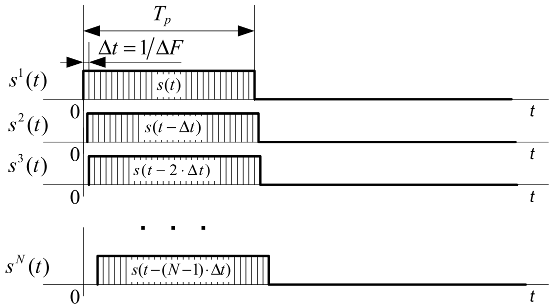


Fig. 7. Circulating codes for space-time coding (from [10]). $s^i(t) = s[t-(i-1)\Delta t]$ is the signal sent through the ith transmitting element.

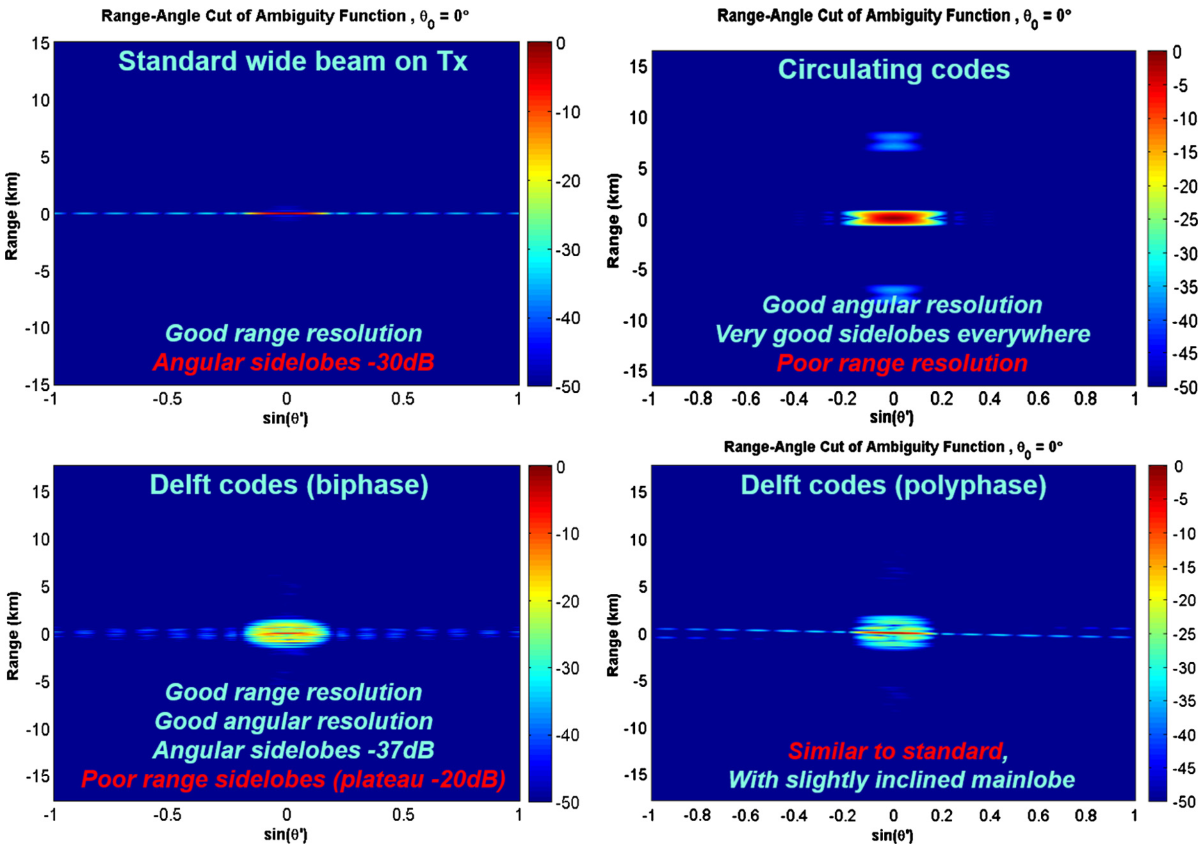


Fig. 8. Ambiguity functions for space-time coding with circulating codes and their variations (from [4]; Delft codes are circulating codes combined with a fixed phase code applied along the antenna array).

For this analysis, the following characteristics are chosen:

- Tx/Rx Array: linear array, 15 elements, spacing $\lambda/2$;
- Tx: Wide beam, or space-time coding; Rx: Taylor weighting, 30 dB;
- Signal: circulating chirps, pulse duration $t = 100$ µs, BT = 256, time delay between adjacent chirps: $\Delta t = 1/B = 0.34$ µs; time weighting (Hamming) on receive;
- Space-time processing: mismatched “optimal” filtering for Delft codes [12].

The main results can be outlined by looking at the range-angle ambiguity functions (output of the processing for every range and angle, when one target is present at range 0—arbitrary value—and angle 0), shown in Fig. 8:

- the angular resolution is improved with space-time coding, by a factor $\sqrt{2}$, (or 2 for a bidimensional coding of a rectangular array); this is not obvious on the figure, but careful analysis indeed gives this result (a natural result,

actually, since there is now directivity both on transmit and receive, compared with the directivity on receive only for the standard digital beam forming with wide beam on transmit);
- the position and level of the sidelobes are different with the different codes: in angle only for standard beam forming, almost no sidelobes but a degradation in range resolution for pure circulating codes, at different levels and positions for Delft codes [3].

As a result, the increased degrees of freedom provided by space-time coding on transmit open the way to adaptive systems where range and angle resolutions can be traded, depending on the mission and the actual environment (knowledge-aided system). This trade-off can be operated differently for each burst, allowing some diversity on clutter to be obtained when several bursts are used for ambiguity removal, for instance. And of course, space-time coding also provides an improvement (mentioned above) in both accuracy and resolution higher than 2, for 2-dimensional antennas, compared with modern wide beam DBF Systems.

5. Discussion

A question then arises: how to combine diversity effects when using agile waveforms? Let us take a baseline example, with a typical modern radar using digital beamforming in elevation only, and a chirp waveform with pulse length 100 μs, pulse repetition frequency 1 kHz. Any designer would like to benefit from:

1) high Doppler resolution, for visibility of slow and weak targets;
2) high angular resolution, in elevation (for altitude measurement) and azimuth (for tracking);
3) diversity on the target, for improved detection in noise;
4) diversity on clutter, for improved detection in clutter.

The first point requires long coherent integration times—but anyway this coherent integration time is limited by the fluctuations of the aspect angle of the target, typically to less than 100 ms.[2]

The second point requires accurate angular measurements (monopulse) with narrow beams on transmit and receive.

The third point requires observations at different carrier frequencies, or from different aspect angles (multistatic system), or integration along a high-resolution range profile.

The fourth point requires the target to be superposed to different patches of clutter, either through range migration or range extent of the target, or through multi-bursts with different range ambiguities (so that the target folds over different clutter patches).

These requirements tend to eliminate standard solutions, such as pencil beam with low range resolution (because of a limited velocity resolution due to a short time on target), or standard digital beam forming with no ambiguity in range (because of a limited angular resolution due to the wide beam on transmit, and limited diversity on clutter).

Facing those trade-offs, several baseline solutions can be sketched:

a) pencil beam, high range resolution, unambiguous in range (low prf): this satisfies all requirements if the coherent integration time is sufficient—and also provides valuable target analysis capabilities, made possible through the exploitation of the high-resolution range-doppler signatures;
b) space-time coding, low range resolution, ambiguous in range (high/medium prf): this also satisfies all requirements; pure circulating codes could be a preferred solution in strong clutter environments, providing very low sidelobes everywhere;
c) space-time coding with high range resolution, unambiguous in range (low prf): low sidelobes, high diversity, combined with valuable target analysis capabilities, exploiting the high-resolution range-Doppler signatures.

Remark on countermeasures and interferences. A detailed analysis of the specific properties of these systems with respect to countermeasures is beyond the scope of this article (see e.g. [13]); however, some significant properties should be considered—particularly when dealing with repeater jammers:

- space-time coding on transmit allows radiating nulls in different directions, thus limiting the efficiency of jammers in those directions;
- with space-time coding on transmit, any repeating jammer will repeat the code it receives, thus providing its direction to the radar: this will help labelling the false echoes as such, and then cancelling them in data processing;
- wideband coherent system obliges the jammer to use similar wideband signals, and to simulate range migration for generating credible false echoes—an increased complexity for the jammer;

---

[2] There may also exist rapid fluctuations of the target due to moving parts (jet engines, rotating blades, wheels); however, for the detection of moving targets, these parts have lower radar cross-sections than the cell of the target, so they can be ignored in this general assessment.

- wideband coherent systems provide a classification capability, which can be used to discriminate between real and false targets;
- wideband systems are of course susceptible to mutual interferences; whereas specific measures can be taken against such degradations (e.g., stepped frequency agile coherent waveforms), it must be noted that this limitation it not more severe than with SAR systems, currently operating with wider bandwidths from airborne platforms.

These baseline descriptions should not, of course, be considered as definitive solutions to the very complex task of defining a multifunction radar (for instance, multistatic solutions could also make sense, possibly combined with space-time coding for solving the “rendezvous” issue, cf. [5]). The objective was rather, as outlined in introduction, to highlight and clarify some specificities of diversity effects that have to be considered when designing future systems. Many other aspects, from complexity and cost to multifunction requirements, have also to be taken into consideration—and should also bring out different advantages of high-resolution and space-time coding for surveillance radars.

References

[1] F. Le Chevalier, Principles of Radar and Sonar Signal Processing, Artech House, Norwood, MA, USA, 2002.
[2] M.A. Richards, J.A. Scheer, W.A. Holm, W.L. Melvin, Principles of Modern Radar, SciTech Pub., Raleigh, NC, USA, 2010.
[3] F. Le Chevalier, Wideband wide beam motion sensing, in: J. Taylor (Ed.), Advanced Ultrawideband Radar: Targets, Signals and Applications, CRC Press, 2016 (Chapter 12).
[4] M. Cherniakov (Ed.), Bistatic Radar: Principles and Practice, Wiley, Chichester, UK, 2007.
[5] V. Chernyak, Fundamentals of Multisite Radar Systems, CRC Press, 1998.
[6] F. Le Chevalier, N. Petrov, Diversity considerations in wideband radar detection of migrating targets in clutter, Sci. China Inf. Sci. 62 (4) (2019) 040302, https://doi.org/10.1007/s11432-018-9735-6.
[7] K. Gerlach, Spatially distributed target detection in non-Gaussian clutter, IEEE Trans. Aerosp. Electron. Syst. 35 (3) (1999) 926–934.
[8] S. Drabowitch, C. Aubry, Pattern compression by space-time binary coding of an array antenna, in: Proceedings of the AGARD CP 66, Advanced Radar Systems, 1969.
[9] J. Dorey, Y. Blanchard, F. Christophe, G. Garnier, Le projet RIAS, une approche nouvelle du radar de surveillance aérienne, L'Onde électrique 64 (4) (1978).
[10] G. Babur, P. Aubry, F. Le Chevalier, Space-time radar waveforms: circulating codes, J. Electr. Comput. Eng. 2013 (2013) 809691.
[11] G. Babur, P. Aubry, F. Le Chevalier, Research Disclosure: Delft Codes: Space-Time Circulating Codes Combined with Pure Spatial Coding for High Purity Active Antenna Radar Systems, Research Disclosure No. 589037, May 2013.
[12] T. Faucon, G. Pinaud, F. Le Chevalier, Mismatched filtering for space-time circulating codes, in: Proceedings of IET International Radar Conference, Hangzhou, PR China, October 2015.
[13] A. De Maio, M. Greco, Modern Radar Detection Theory, SciTech Pub., Raleigh, NC, USA, 2016.