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Azzollini, Ilario Antonio; Yu, Wenwu; Yuan, Shuai; Baldi, Simone

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Adaptive leader-follower synchronization over heterogeneous and uncertain networks of linear systems without distributed observer

Ilario A. Azzollini, Wenwu Yu, Shuai Yuan, and Simone Baldi

***Abstract*— A challenging task in network synchronization is steering the network towards a coherent solution, when the dynamics of the constituent systems are heterogeneous and uncertain. In this situation, synchronization can be achieved via adaptive protocols (with adaptive feedback gains, or adaptive coupling gains, or both). However, as state-of-the-art synchronization methods adopt a distributed observer architecture, they require to communicate extra observer variables among neighbors, in addition to the neighbors' states (or outputs). The distinguishing feature of this technical note is to show that, for heterogeneous and uncertain networks of some classes of linear systems, synchronization is possible without the need for any distributed observer. Such classes are in line with those in model reference adaptive control literature. Lyapunov analysis is used to derive a new adaptive synchronization protocol with the simplest communication architecture, in which both feedback and coupling gains are adapted without any extra communication other than neighbors' states (in the full-state information case) or neighbors' outputs (in the partial-state information case).**

***Index Terms*— Adaptive control, synchronization, heterogeneous uncertain networks.**

I. INTRODUCTION

In recent years, coordination of multi-agent systems has been studied by different scientific communities, motivated by its applicability to biology [1], energy systems [2], autonomous vehicles [3], and many other fields. A common objective in multi-agent systems is to achieve a desired collective behavior through local actions, i.e. by updating the behavior of each system (agent) using only its own information and the information of its neighbors: typical examples are synchronization or the closely-related topic of consensus [4]. An established way to solve the synchronization problem is to formulate it in a cooperative output regulation framework, where synchronized tracking and disturbance rejection can be treated in a unified way, even for multi-input multi-output systems. In [5], it was shown that an internal model requirement is necessary and sufficient for synchronizability of a network to an autonomous exogenous system, denoted as exosystem. This means that the well-known internal model principle [6] can be used to solve synchronization problems. Motivated by this result, synchronization protocols were designed for both linear [7], [8] and nonlinear networks [9]. It has to be noticed that synchronization via cooperative output regulation always requires the communication of extra auxiliary variables, i.e. the variables of the distributed observer to reconstruct the exosystem information.

Initial research on synchronization has focused on systems sharing the same (homogeneous) dynamics, possibly uncertain. Synchronization of these homogeneous networks has been achieved by adopting either adaptive coupling [10]–[12], or adaptive feedback [13]–[15], respectively. In the first case, one increases the coupling strength according to the synchronization error, exploiting the fact that synchronization in stable homogeneous networks can be achieved if the coupling strength is large enough [16], [17]. In the second case, static couplings have been used, while a stabilizing feedback gain has been determined in an adaptive way for special classes of homogeneous uncertain systems [13]–[15], [18].

A more challenging task is that of achieving synchronization when the systems of the network differ from each other, and also their dynamics lie in a possibly large uncertainty set (heterogeneous and uncertain networks). Adaptive feedback strategies have been mostly explored, namely for unknown linear systems [19], chaotic systems [20], systems with unknown identical control directions [21], passifiable systems [22], and systems in the Euler-Lagrange form [23]–[26]: a notable exception is [27], where a discontinuous protocol with both adaptive feedback and adaptive couplings is implemented. Differently from homogeneous approaches that might not require a distributed observer [13], [14], all heterogeneous approaches share the need for implementing some form of distributed observer, thus requiring communication of extra variables to reconstruct the leader information. Therefore, relevant questions arise: what is the simplest distributed adaptive architecture for synchronization of heterogeneous uncertain networks? In which cases is it possible to get rid of any distributed observer, and reach synchronization by adapting both the feedback and the coupling gains with no further local communication than the neighbors' states (or outputs)?

The main contribution of this work is to show that, for certain classes of linear systems, we can get rid of the distributed observer architecture. This results in a direct adaptive control approach having the simplest communication architecture, without any extra local communication than neighbors' states (in the full-state information case) or neighbors' outputs (in the partial-state information case). These classes of systems are in line with those for which Model Reference Adaptive Control (MRAC) can be adopted [28], [29], i.e. systems with matched uncertainties. Such systems broadly appear in literature on networks of cooperative vehicles [3], oscillators [30], fully-actuated Euler-Lagrange systems [26], [27], etc., making the proposed approach applicable in all these settings.

The rest of the paper is organized as follows: the problem formulation is given in Sect. II, while the full-state and partial-state designs are given in Sect. III and Sect. IV, respectively. Numerical examples are provided in Sect. V, with conclusions in Sect. VI.

Notation: The notation in this paper is standard. The transpose of a matrix or of a vector is indicated with X^T and x^T respectively. A vector signal $x \in \mathbb{R}^n$ is said to belong to $\mathcal{L}_2$ class ($x \in \mathcal{L}_2$), if $\int_0^t \|x(\tau)\|^2 \, \mathrm{d}\tau < \infty$, $\forall t \geq 0$. A vector signal $x \in \mathbb{R}^n$ is said to belong to $\mathcal{L}_\infty$ class ($x \in \mathcal{L}_\infty$), if $\max_{t\geq 0} \|x(t)\| < \infty$, $\forall t \geq 0$. An undirected graph of order N is completely defined by the pair $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, \ldots, N\}$ is a finite nonempty set of nodes, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of corresponding non-ordered pair of nodes, called edges. Let $\mathcal{N}_i$ denote the subset of $\mathcal{V}$ which consists of all the neighbors of node i. The adjacency matrix $\mathcal{A} = [a_{ij}]$ of an unweighted undirected graph is defined as $a_{ii} = 0$ and $a_{ij} = a_{ji} = 1$

I. A. Azzollini is with CASY - DEI "Guglielmo Marconi", University of Bologna, Italy. W. Yu is with School of Mathematics, Southeast University, Nanjing 210096, China. S. Yuan is with School of Astronautics, Harbin Institute of Technology, China. S. Baldi (corresponding author) is with School of Mathematics, Southeast University, Nanjing 210096, China, and with Delft Center for Systems and Control, TU Delft, 2628 CD, Delft. (emails: ilario.azzollini@unibo.it, wwyu@seu.edu.cn, shuaiyuan@hit.edu.cn, and s.baldi@tudelft.nl).
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if $(i,j)\in\mathcal{E}$, where $i\neq j$. The Laplacian matrix of the unweighted graph is defined as $\mathcal{L}=[l_{ij}]$, where $l_{ii}=\sum_j a_{ij}$ and $l_{ij}=-a_{ij}$, if $i\neq j$. An undirected graph $\mathcal{G}$ is said to be connected if, taken any arbitrary pair of nodes (i,j) where $i,j\in\mathcal{V}$, there is a path that leads from i to j. Let $\mathcal{T}\subseteq\mathcal{V}$ be the set of those nodes, called target nodes, which receive information from a leader. The target nodes can access the leader state through the diagonal target matrix $\mathcal{M}\in\mathbb{D}^N_{\geq 0}$, which is defined as follows: $\mathcal{M}=[m_{ij}]$, where $m_{ii}=1$ if $i\in\mathcal{T}$ and $m_{ii}=0$ otherwise. Let the leader-follower topology matrix be defined as $\mathcal{B}=\mathcal{L}+\mathcal{M}$, which is positive definite by construction [19].

II. Problem formulation

A network of linear heterogeneous systems with unknown dynamics is considered in this work

$$\begin{aligned}\dot{x}_i &= A_i x_i + b_i u_i\\ y_i &= c_i^T x_i, \qquad i\in\mathcal{V}\end{aligned} \tag{1}$$

where $x_i\in\mathbb{R}^n$ is the state, $u_i\in\mathbb{R}$ is the input, and $y_i\in\mathbb{R}$ is the output. Time index t is omitted when obvious. The triple (A_i,b_i,c_i) is unknown with matrices of appropriate dimensions, and possibly $A_i\neq A_j$, $b_i\neq b_j$ and $c_i\neq c_j$, $i\neq j$, $i,j\in\mathcal{V}$ (uncertain heterogeneous systems). As common in adaptive literature [28], let us focus on the single-input single-output case. The equivalent transfer function form of (1) is

$$y_i = k_i\frac{Z_i(s)}{R_i(s)}u_i, \quad i\in\mathcal{V}. \tag{2}$$

Analogously, the triple (k_i,Z_i,R_i) is unknown with $R_i(s)$ being monic polynomials of order n, $Z_i(s)$ being monic polynomials of order $q<n$, k_i being constants referred to as the high-frequency gains. In addition to the N systems in (1) (or (2)), a special role is played by system 0 (leader system), with state x_0 and output y_0, whose dynamics can be completely determined by the designer as clarified later in Assumptions 2 and 3.
The following connectivity assumption is made.

Assumption 1: The graph $\mathcal{G}$ of the network is undirected and connected, and the leader interacts with at least one system ($\mathcal{T}\neq\emptyset$).

The following problem is considered:

Problem 1 (Adaptive synchronization): Let Assumption 1 hold for a network of uncertain heterogeneous systems (1) plus a leader. Find a state-feedback (resp. output-feedback) adaptive distributed strategy (i.e. exploiting only measurements from neighbors) for the control input u_i such that synchronization to the leader state (resp. output) is achieved, i.e. $x_i-x_0\to 0$ (resp. $y_i-y_0\to 0$), $\forall i\in\mathcal{V}$.

III. Full-state measurement adaptive synchronization

Consider the following assumption.

Assumption 2: There exist a family of vectors $k_i^*\in\mathbb{R}^n$ and a family of scalars $l_i^*\in\mathbb{R}$ (with $sgn(l_i^*)$ known) such that the following matching conditions are satisfied for some desired (A_0,b_0)

$$\begin{cases} A_i + b_i k_i^{*T} = A_0\\ l_i^* b_i = b_0\end{cases}, \qquad i\in\mathcal{V}. \tag{3}$$

Remark 1: [The structural issue] *The equations* (3) *remind the well-known matching conditions of standard MRAC [28, Sect. 6.2.3]. Analogously to MRAC, satisfying conditions* (3) *requires* (A_i,b_i) *and* (A_0,b_0) *to share some common structure. The knowledge of* $sgn(l_i^*)$ *is typically assumed in MRAC [28, Chapt. 6], which amounts to having knowledge of the systems control direction. Systems with such matched uncertainties broadly appear in literature, and examples include networks of cooperative vehicles [3], oscillators [30], and fully-actuated Euler-Lagrange systems [26], [27], among others.*

Motivated by Assumption 2, let us choose the leader dynamics as

$$\dot{x}_0 = A_0 x_0 \qquad x_0(0)=x_{00} \tag{4}$$

where $x_0\in\mathbb{R}^n$ is the leader state, accessible to the target nodes only, as per Assumption 1.

Two results are now given which are instrumental to solving Problem 1.

Proposition 1: [*Ideal state-feedback homogenization*] Under Assumptions 1 and 2, there exists an *ideal* controller

$$u_i^* = k_i^{*T}x_i + l_i^* f^T\left(\sum_{j=1}^N a_{ij}(x_i-x_j) + m_{ii}(x_i-x_0)\right) \tag{5}$$

with $f\in\mathbb{R}^n$ to be designed, giving the closed-loop dynamics

$$\dot{x}_i = A_0 x_i + b_0 f^T\left(\sum_{j=1}^N a_{ij}(x_i-x_j) + m_{ii}(x_i-x_0)\right). \tag{6}$$

Proof: The proof directly follows from applying the control input (5) to system (1), and using (3).

The following result allows us to design f to achieve synchronization for the homogeneous dynamics in (6).

Proposition 2: [*Homogeneous network state synchronization*] The homogeneous network (6) synchronizes to the reference state x_0 if

$$\lambda_i A_0 + b_0 f^T \text{ is Hurwitz}, \quad \forall i\in\mathcal{V} \tag{7}$$

with λ_i the eigenvalues of the inverse of the leader-follower topology matrix $\mathcal{B}^{-1}$.

Proof: Similar results as Proposition 2 have appeared in literature, but let us nevertheless sketch the proof, because it will be useful to understand the upcoming adaptive design. Define $x=[x_1^T,x_2^T,\dots,x_N^T]^T\in\mathbb{R}^{Nn}$ and $x_m=[x_0^T,x_0^T,\dots,x_0^T]^T\in\mathbb{R}^{Nn}$, and the local synchronization error

$$e_i = \left(\sum_{j=1}^N a_{ij}(x_i-x_j)\right) + m_{ii}(x_i-x_0) \tag{8}$$

where $e=[e_1^T,e_2^T,\dots,e_N^T]^T$ can be written as [19]

$$e = (\mathcal{B}\otimes I_n)(x-x_m). \tag{9}$$

Moreover, the overall homogeneous network dynamics (6) can be written in the compact form

$$\begin{aligned}\dot{x} &= (I_N\otimes A_0)x + (\mathcal{B}\otimes b_0 f^T)(x-x_m)\\ &= (I_N\otimes A_0)x + (I_N\otimes b_0 f^T)e.\end{aligned} \tag{10}$$

Positive-definiteness of $\mathcal{B}$ leads to the existence of a unitary matrix $\mathcal{U}\in\mathbb{R}^{N\times N}$ such that $\mathcal{U}^T\mathcal{B}^{-1}\mathcal{U}=\mathrm{diag}(\lambda_1,\lambda_2,\dots,\lambda_N)\triangleq\Lambda$. This can be used to define the transformation $e=(\mathcal{U}\otimes I_n)\bar{e}$ with $\bar{e}=[\bar{e}_1^T,\bar{e}_2^T,\dots,\bar{e}_N^T]^T$ [4].

We can now write the overall error dynamics, using (9) and (10)

$$\begin{aligned}\dot{e} =&(\mathcal{B}\otimes I_n)(I_N\otimes A_0)x + (\mathcal{B}\otimes I_n)(I_N\otimes b_0 f^T)e\\ &-(\mathcal{B}\otimes I_n)(I_N\otimes A_0)x_m\\ =&[(I_N\otimes A_0)+(\mathcal{B}\otimes b_0 f^T)]e\ .\end{aligned} \tag{11}$$

Consider the Lyapunov candidate

$$V_1 = e^T(\mathcal{B}^{-1}\otimes P)e \tag{12}$$

where $P \in \mathbb{R}^{n\times n}$ is a symmetric positive definite matrix. We have

$$\begin{aligned}\dot{V}_1 &= 2e^T(\mathcal{B}^{-1}\otimes P)[(I_N\otimes A_0)+(\mathcal{B}\otimes b_0 f^T)]e\\ &= 2\bar{e}^T(\Lambda\otimes PA_0 + I_N\otimes Pb_0 f^T)\bar{e}\\ &= \sum_{i=1}^N \bar{e}_i^T\left[P\left(\lambda_i A_0 + b_0 f^T\right)+\left(\lambda_i A_0+b_0 f^T\right)^T P\right]\bar{e}_i\end{aligned} \tag{13}$$

which is negative definite if

$$\left[P\left(\lambda_i A_0 + b_0 f^T\right)+\left(\lambda_i A_0+b_0 f^T\right)^T P\right] < \mathbf{0},\ \forall i\in\mathcal{V}\ . \tag{14}$$

This completes the proof.

Remark 2: [Need for adaptation] *Since A_i and b_i in* (3) *are unknown, the ideal control* (5) *cannot be implemented to solve Problem 1. Therefore, some distributed adaptation mechanisms must be devised to estimate the unknown ideal gains in Proposition 1.*

The following edge-based state synchronizing protocol is proposed

$$u_i = k_i^T x_i + f^T\left(\sum_{j=1}^N l_{ij}a_{ij}(x_i-x_j)+l_{im}m_{ii}(x_i-x_0)\right) \tag{15}$$

where k_i is the estimate of k_i^*, while, l_{ij} and l_{im}, are the edge-based estimates of l_i^*. All the estimates are time-dependent, driven by distributed adaptive laws to be designed. In the next Theorem 1 we present the adaptive laws.

Theorem 1 (Heterogeneous network state synchronization): Under Assumptions 1 and 2, the heterogeneous uncertain network (1), controlled using the protocol (15) and the adaptive laws

$$\begin{aligned}\dot{k}_i^T &= -sgn(l_i^*)\gamma e_i^T P b_0 x_i^T\\ \dot{l}_{ij} &= -sgn(l_i^*)\gamma e_i^T P b_0 f^T(x_i-x_j)\\ \dot{l}_{im} &= -sgn(l_i^*)\gamma e_i^T P b_0 f^T(x_i-x_0)\end{aligned} \tag{16}$$

with adaptive gain $\gamma > 0$, reaches synchronization to the reference state x_0, provided that the matrix P and the vector f are chosen such that condition (14) holds.

Proof: The closed-loop formed by (1) and (15) can be rewritten as a function of the estimation errors

$$\begin{aligned}\dot{x}_i =& A_0 x_i + b_i\tilde{k}_i^T(t)x_i\\ &+ b_0 f^T\sum_{j=1}^N a_{ij}(x_i-x_j) + b_i f^T\sum_{j=1}^N \tilde{l}_{ij}(t)a_{ij}(x_i-x_j)\\ &+ b_0 f^T m_{ii}(x_i-x_0) + b_i f^T\tilde{l}_{im}(t)m_{ii}(x_i-x_0)\end{aligned}$$

where $\tilde{k}_i(t)=k_i(t)-k_i^*$, $\tilde{l}_{ij}(t)=l_{ij}(t)-l_i^*$ and $\tilde{l}_{im}(t)=l_{im}(t)-l_i^*$. By defining for compactness

$$\begin{aligned}B_k(t) =& \mathrm{diag}(b_1\tilde{k}_1^T(t),\dots,b_N\tilde{k}_N^T(t))\\ B_l(t) =& \mathrm{diag}\Big(b_1 f^T\sum_{j=1}^N \tilde{l}_{1j}a_{1j}(x_1-x_j),\dots\\ &\dots, b_N f^T\sum_{j=1}^N \tilde{l}_{Nj}a_{Nj}(x_N-x_j)\Big)\\ B_m(t) =& \mathrm{diag}\Big(b_1 f^T\tilde{l}_{1m}m_{11}(x_1-x_0),\dots\\ &\dots, b_N f^T\tilde{l}_{Nm}m_{NN}(x_N-x_0)\Big)\end{aligned} \tag{17}$$

the closed-loop for the overall network can be written as

$$\dot{x} = (I_N\otimes A_0 + B_k(t))x + (I_N\otimes b_0 f^T)e + B_l(t) + B_m(t)\ .$$

From the synchronization error (9), we obtain the error dynamics

$$\begin{aligned}\dot{e} =& [(I_N\otimes A_0)+(\mathcal{B}\otimes b_0 f^T)]e +\\ &+ (\mathcal{B}\otimes I_n)(B_k(t)x + B_l(t) + B_m(t)).\end{aligned} \tag{18}$$

The adaptive laws (16) arise from the Lyapunov candidate $V = V_1 + V_2 + V_3 + V_4$, where V_1 is (12), and

$$\begin{aligned}V_2 &= \sum_{i=1}^N \frac{\tilde{k}_i^T(t)\gamma^{-1}\tilde{k}_i(t)}{|l_i^*|}\ ,\qquad V_3 = \sum_{i=1}^N \frac{\tilde{l}_{ij}(t)\gamma^{-1}\tilde{l}_{ij}^T(t)}{|l_i^*|}\ ,\\ V_4 &= \sum_{i=1}^N \frac{\tilde{l}_{im}(t)\gamma^{-1}\tilde{l}_{im}^T(t)}{|l_i^*|}.\end{aligned} \tag{19}$$

In fact, following the same procedure as in (13), we have

$$\begin{aligned}\dot{V}_1 =& 2e^T(\mathcal{B}^{-1}\otimes P)[(I_N\otimes A_0)+(\mathcal{B}\otimes b_0 f^T)]e\\ &+ 2e^T(\mathcal{B}^{-1}\otimes P)[(\mathcal{B}\otimes I_n)(B_k(t)x+B_l(t)+B_m(t))]\\ =& \sum_{i=1}^N \bar{e}_i^T\left[P\left(\lambda_i A_0 + b_0 f^T\right)+\left(\lambda_i A_0+b_0 f^T\right)^T P\right]\bar{e}_i\\ &+ 2\sum_{i=1}^N \tilde{k}_i^T(t)x_i b_i^T P e_i + 2\sum_{i=1}^N(\tilde{l}_{im}m_{ii}(x_i-x_0))^T f b_i^T P e_i\\ &+ 2\sum_{i=1}^N\left(\sum_{j=1}^N \tilde{l}_{ij}(t)a_{ij}(x_i-x_j)\right)^T f b_i^T P e_i.\end{aligned} \tag{20}$$

Moreover, by using (16) we have

$$\begin{aligned}\dot{V}_2 &= -2\sum_{i=1}^N \frac{sgn(l_i^*)}{|l_i^*|}\tilde{k}_i^T(t)x_i b_0^T P e_i\\ \dot{V}_3 &= -2\sum_{i=1}^N \frac{sgn(l_i^*)}{|l_i^*|}\left(\sum_{j=1}^N \tilde{l}_{ij}(t)a_{ij}(x_i-x_j)\right)^T f b_0^T P e_i\\ \dot{V}_4 &= -2\sum_{i=1}^N \frac{sgn(l_i^*)}{|l_i^*|}(\tilde{l}_{im}m_{ii}(x_i-x_0))^T f b_0^T P e_i\end{aligned}$$

leading to

$$\dot{V} = \sum_{i=1}^N \bar{e}_i^T\left[P\left(\lambda_i A_0 + b_0 f^T\right)+\left(\lambda_i A_0+b_0 f^T\right)^T P\right]\bar{e}_i$$

which is negative semi-definite provided that condition (14) holds. Using standard Lyapunov arguments we can prove boundedness of all closed-loop signals and convergence of e to 0. In fact, since $V > 0$ and $\dot{V}\le 0$, it follows that $V(t)$ has a limit, i.e.,

$$\lim_{t\to\infty} V(e(t),\tilde{\Omega}(t)) = V_\infty < \infty \tag{21}$$

where $\tilde{\Omega}$ collects all parametric errors. The finite limit implies V, e, $\tilde{\Omega}\in\mathcal{L}_\infty$. In addition, by integrating $\dot{V}$ it follows that

$$\int_0^\infty e^T(\tau)Qe(\tau)\,\mathrm{d}\tau \le V(e(0),\tilde{\Omega}(0)) - V_\infty$$

for some $Q > 0$, from which we establish that $e\in\mathcal{L}_2$. Finally, since $\dot{V}$ is uniformly continuous in time (being $\ddot{V}$ finite), the Barbalat's lemma implies $\dot{V}\to 0$ as $t\to\infty$ and hence $e\to 0$, from which we derive $x_i\to x_0$, $\forall i\in\mathcal{V}$. This concludes the proof.

Remark 3: [Non-convergence to ideal gains] *The proof shows that $e\to 0$, but cannot guarantee that $k_i\to k_i^*$, $l_{ij}\to l_i^*$, $l_{im}\to l_i^*$, $\forall i$. This is a typical result in direct adaptive control approaches, where unless the closed-loop signals are persistently exciting, convergence of the tracking error to zero does not necessarily come with convergence of the estimates to the ideal gains* *defined by the matching conditions [28, Sect. 6.2.3]. This implies that, while the ideal protocol* (5) *imposes all systems to homogenize to* (A_0, b_0)*, the adaptive protocol* (15),(16)*, although being thought as an estimate of* (5)*, achieves synchronization without necessarily leading to homogenization.*

Remark 4: [Advances wrt the state of the art] *In contrast with [17] (with only adaptive coupling gains) or [19] (with only adaptive feedback gains), here we manage to adapt both sets of gains. In contrast with [27], where both feedback and coupling gains are adapted, we have removed any need for a distributed observer for the leader velocity.*

Remark 5: [Synchronization error and Laplacian eigenvalues] *Two errors have been considered in synchronization problems: the tracking error with the leader/exosystem [8], [27], or the disagreement error with neighbors [19], [21]. Since the former error is not locally computable, a distributed observer is mandatory in all heterogeneous networks designs we are aware of. Therefore, we resorted to the latter error, which is locally computable, to remove the need for a distributed observer. Although the proposed simpler architecture requires some information of the Laplacian eigenvalues, c.f.* (14)*, it has to be remarked that, to the best of the authors' knowledge, there exists no adaptive or non-adaptive protocol for heterogeneous networks based on the latter error that can get rid of any information of the Laplacian eigenvalues.*

IV. Output measurement adaptive synchronization

In place of Assumption 2, in this section we consider the following assumption.

Assumption 3: Consider some desired homogeneus dynamics defined by (A_0, b_0, c_0) or, equivalently, by the transfer function (k_0, Z_0, R_0), with n_0 and q_0 representing the order of R_0 and Z_0, respectively. There exist a family of vectors $h_i^* \in \mathbb{R}^{n-1}$, $g_i^* \in \mathbb{R}^{n-1}$ and a family of scalars $c_i^*, l_i^* \in \mathbb{R}$ (with $sgn(l_i^*)$ known) such that the following matching conditions are satisfied

$$\begin{cases} (\Lambda(s) - h_i^{*T}\alpha(s))R_i - k_i Z_i(s)(g_i^{*T}\alpha(s) + c_i^*\Lambda(s)) \\ \qquad\qquad\qquad = Z_i(s)\Lambda_0(s)R_0(s) \\ l_i^* = k_0/k_i \end{cases} \tag{22}$$

with

$$\begin{cases} \alpha(s) \triangleq [s^{n-2}, s^{n-3}, \dots, s, 1] & \text{for } n \geq 2 \\ \alpha(s) \triangleq 0 & \text{for } n = 1 \end{cases} \tag{23}$$

and with $\Lambda(s)$ being a monic Hurwitz polynomial of degree $n-1$ that contains Z_0 as a factor

$$\begin{aligned} \Lambda(s) &= \Lambda_0(s)Z_0(s) \\ &= s^{n-1} + \mu_{n-2}s^{n-2} + \mu_{n-3}s^{n-3} + \ldots + \mu_0 \end{aligned} \tag{24}$$

where $\Lambda_0(s)$ is an arbitrary monic Hurwitz polynomial of degree $n-1-q_0$.

Remark 6: [Structural requirements] *Analogously to the full-state measurement case,* (22) *remind the matching conditions of output-feedback MRAC: [28, Lemma 6.3.1] shows that the matching conditions* (22) *always have a solution when (i)* Z_0*,* R_0 *are monic polynomials with* $n_0 \leq n$*; (ii)* Z_i *are monic Hurwitz polynomials (leading to minimum-phase); (iii) the relative degree of* (k_0, Z_0, R_0) *is the same as that of* (k_i, Z_i, R_i)*, i.e.* $n_0 - q_0 = n - q$*.*

For simplicity, and in line with [13], [14], we consider unitary relative degree for both the desired homogeneous dynamics and the systems, i.e. $n_0 - q_0 = n - q = 1$.

Motivated by Assumption 3, let us choose the leader dynamics as

$$\begin{aligned} \dot{x}_0 &= A_0 x_0 \\ y_0 &= c_0^T x_0 \end{aligned} \qquad x_0(0) = x_{00} \tag{25}$$

where $x_0 \in \mathbb{R}^n$ is the state, $y_0 \in \mathbb{R}$ is the output, and the matrix A_0 and the vector c_0 have appropriate dimensions.

Two results are now given which are instrumental to solving Problem 1.

Proposition 3: [*Ideal output-feedback homogenization*] Under Assumptions 1 and 3, there exists an *ideal* controller

$$\begin{aligned} u_i^* &= h_i^{*T}\frac{\alpha(s)}{\Lambda(s)}u_i + g_i^{*T}\frac{\alpha(s)}{\Lambda(s)}y_i + c_i^* y_i \\ &\quad + l_i^*\phi\left(\sum_{j=1}^N a_{ij}(y_i - y_j) + m_{ii}(y_i - y_0)\right) \end{aligned} \tag{26}$$

with $\phi \in \mathbb{R}$ to be designed, giving the closed-loop dynamics

$$\begin{aligned} \dot{x}_i &= A_0 x_i + b_0\phi\left(\sum_{j=1}^N a_{ij}(y_i - y_j) + m_{ii}(y_i - y_0)\right) \\ y_i &= c_0^T x_i, \qquad i \in \mathcal{V}. \end{aligned} \tag{27}$$

Proof: The proof follows from [28, Sect. 6.3]. Details are not given for lack of space.

The following result allows us to design ϕ to achieve synchronization for the homogeneous dynamics in (27).

Proposition 4: [*Homogeneous network output synchronization*] The homogeneous network (27) synchronizes if

$$\left(\lambda_i A_0 + b_0 f c_0^T, b_0, c_0^T\right) \quad \text{is SPR}, \qquad \forall i \in \mathcal{V} \tag{28}$$

where λ_i's, $i \in \mathcal{V}$, are the eigenvalues of the $\mathcal{B}^{-1}$ matrix.

Proof: The overall homogeneous network (27) can be written in the more compact form

$$\begin{aligned} \dot{x} &= (I_N \otimes A_0 + \mathcal{B} \otimes b_0\phi c_0^T)(x - x_m) \\ y &= (I_N \otimes c_0^T)x \end{aligned} \tag{29}$$

where $y = [y_1, y_2, \dots, y_N]^T \in \mathbb{R}^N$. Let us now define the state and output synchronization errors as

$$\begin{aligned} e_i &= \left(\sum_{j=1}^N a_{ij}(x_i - x_j)\right) + m_{ii}(x_i - x_0) \\ \epsilon_i &= \left(\sum_{j=1}^N a_{ij}(y_i - y_j)\right) + m_{ii}(y_i - y_0) \end{aligned} \tag{30}$$

with $e = [e_1^T, e_2^T, \dots, e_N^T]^T$ and $\epsilon = [\epsilon_1, \epsilon_2, \dots, \epsilon_N]^T$. The overall homogeneous network can be now written as

$$\begin{aligned} \dot{x} &= (I_N \otimes A_0)x + (I_n \otimes b_0\phi c_0^T)e \\ y &= (I_N \otimes c_0^T)x. \end{aligned} \tag{31}$$

Recalling that $e = (\mathcal{B} \otimes I_n)(x - x_m)$, the error dynamics result in

$$\begin{aligned} \dot{e} &= (\mathcal{B} \otimes I_n)[(I_N \otimes A_0)x + (I_N \otimes b_0\phi c_0^T)e - (I_N \otimes A_0)x_m] \\ &= [(I_N \otimes A_0) + (\mathcal{B} \otimes b_0\phi c_0^T)]e. \end{aligned}$$

Now, let us use a similar decomposition as in Proposition 1 and consider the Lyapunov candidate

$$\Upsilon_1 = e^T(\mathcal{B}^{-1} \otimes P)e \tag{32}$$

where $P \in \mathbb{R}^{n\times n}$ is a symmetric positive definite matrix satisfying the Kalman-Yakubovich lemma [28, Sect. 3]

$$\begin{aligned} &P\left(\lambda_i A_0 + b_0\phi c_0^T\right) + \left(\lambda_i A_0 + b_0\phi c_0^T\right)^T P < -Q \\ &Pb_0 = c_0, \qquad \forall i \in \mathcal{V}. \end{aligned} \tag{33}$$

Then, we have

$$\dot{\Upsilon}_1 = \sum_{i=1}^{N} \bar{e}_i^T \left[P\left(\lambda_i A_0 + b_0\phi c_0^T\right) + \left(\lambda_i A_0 + b_0\phi c_0^T\right)^T P\right]\bar{e}_i$$

which is negative definite if

$$\left[P\left(\lambda_i A_0 + b_0\phi c_0^T\right) + \left(\lambda_i A_0 + b_0\phi c_0^T\right)^T P\right] < \mathbf{0},\ \forall i \in \mathcal{V} \tag{34}$$

implied by the first in (33). This completes the proof.

The following edge-based output synchronizing protocol is proposed

$$\begin{aligned} u_i(t) = {}& h_i^T(t)\frac{\alpha(s)}{\Lambda(s)}u_i + g_i^T(t)\frac{\alpha(s)}{\Lambda(s)}y_i + c_i(t)y_i \\ &+ \phi\left(\sum_{j=1}^{N} l_{ij}(t)a_{ij}(y_i - y_j) + l_{im}(t)m_{ii}(y_i - y_0)\right) \end{aligned} \tag{35}$$

where h_i, g_i, and c_i are the estimates of h_i^*, g_i^* and c_i^*, respectively, while l_{ij} and l_{im} are the edge-based estimates of l_i^*. The following synchronization result holds.

Theorem 2 (Heterogeneous network output synchronization): Under Assumptions 1 and 3, the heterogeneous uncertain network (1), controlled using the following distributed adaptive controller

$$\begin{aligned} &u_i(t) = \theta_i^T(t)\omega_i, \qquad \dot{\theta}_i = -sgn(l_i^*)\gamma\epsilon_i\omega_i \\ &\dot{\omega}_{i_1} = F\omega_{i_1} + du_i, \qquad \dot{\omega}_{i_2} = F\omega_{i_2} + dy_i \\ &\theta_i = \begin{cases} \left[h_i^T\ g_i^T\ c_i\ [l_{ij}]_{j\in\mathcal{N}_i}\ l_{im}\right]^T & \text{if } i \in \mathcal{T} \\ \left[h_i^T\ g_i^T\ c_i\ [l_{ij}]_{j\in\mathcal{N}_i}\right]^T & \text{otherwise} \end{cases} \\ &\omega_i = \begin{cases} \left[\omega_{i_1}^T\ \omega_{i_2}^T\ y_i\ [\phi(y_i - y_j)]_{j\in\mathcal{N}_i}\ \phi(y_i - y_0)\right]^T & \text{if } i \in \mathcal{T} \\ \left[\omega_{i_1}^T\ \omega_{i_2}^T\ y_i\ [\phi(y_i - y_j)]_{j\in\mathcal{N}_i}\right]^T & \text{otherwise} \end{cases} \\ &F = \begin{bmatrix} -\mu_{n-2} & -\mu_{n-3} & \cdots & -\mu_0 \\ & I_{n-2} & & 0_{(n-2)\times 1} \end{bmatrix},\ d = \begin{bmatrix} 1 \\ 0_{(n-2)\times 1} \end{bmatrix} \end{aligned} \tag{36}$$

with adaptive gain $\gamma > 0$, reaches synchronization to the reference output y_0, provided that the scalar ϕ is chosen such that condition (28) holds. The notation $[v]_{j\in\mathcal{N}_i}$ is used to indicate row vectors that collect all the components associated to the neighbors of system i. Please notice that u_i in (36) is equivalent to (35), as (F, d) is a state-space realization of $\alpha(s)/\Lambda(s)$. Also notice that, analogously to what emphasized in Remark 3, convergence of θ_i to the ideal gains cannot be guaranteed.

Proof: The proof follows very similar steps as the one of Theorem 1. The Lyapunov candidate Υ_1 in (32) should be used together with

$$\Upsilon_2 = \sum_{i=1}^{N} \frac{\tilde{\theta}_i^T(t)\gamma^{-1}\tilde{\theta}_i(t)}{|l_i^*|}. \tag{37}$$

Then, similar with (20), we have

$$\begin{aligned} \dot{\Upsilon}_1 = {}& 2e^T(\mathcal{B}^{-1}\otimes P)[(I_N \otimes A_0) + (\mathcal{B}\otimes b_0\phi c_0^T)]e \\ &+ 2e^T(\mathcal{B}^{-1}\otimes P)[(\mathcal{B}\otimes I_n)(B_\theta(t)\omega)] \end{aligned} \tag{38}$$

where

$$\begin{aligned} B_\theta(t) &= \mathrm{diag}(b_1\tilde{\theta}_1^T(t), \dots, b_N\tilde{\theta}_N^T(t)) \\ \omega &= [\omega_1^T, \omega_2^T, \dots, \omega_N^T]^T \end{aligned} \tag{39}$$

and, following a similar procedure as in (20), we obtain

$$\begin{aligned} \dot{\Upsilon}_1 = {}& \sum_{i=1}^{N} \bar{e}_i^T \left[P\left(\lambda_i A_0 + b_0\phi c_0^T\right) + \left(\lambda_i A_0 + b_0\phi c_0^T\right)^T P\right]\bar{e}_i \\ &+ 2\sum_{i=1}^{N} e_i^T P b_i \tilde{\theta}_i^T(t)\omega_i \\ = {}& \sum_{i=1}^{N} \bar{e}_i^T \left[P\left(\lambda_i A_0 + b_0\phi c_0^T\right) + \left(\lambda_i A_0 + b_0\phi c_0^T\right)^T P\right]\bar{e}_i \\ &+ 2\sum_{i=1}^{N} \frac{sgn(l_i^*)}{|l_i^*|}\tilde{\theta}_i^T(t)\omega_i\epsilon_i \end{aligned}$$

where we have used the second equation in (33). Moreover, from (36) we have

$$\dot{\Upsilon}_2 = -2\sum_{i=1}^{N} \frac{sgn(l_i^*)}{|l_i^*|}\tilde{\theta}_i^T(t)\omega_i\epsilon_i$$

leading to

$$\dot{\Upsilon} = \sum_{i=1}^{N} \bar{e}_i^T \left[P\left(\lambda_i A_0 + b_0\phi c_0^T\right) + \left(\lambda_i A_0 + b_0\phi c_0^T\right)^T P\right]\bar{e}_i$$

which is negative semi-definite provided that (34) holds. Using standard Lyapunov arguments as in Theorem 1 we can prove boundedness of all closed-loop signals and convergence of e to 0, from which we derive $\epsilon \to 0$, i.e. $y_i \to y_0$, $\forall i \in \mathcal{V}$.

V. Numerical Examples

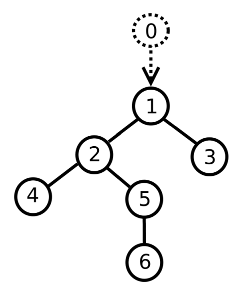

Fig. 1. The undirected communication graph.

Simulations using controllers (15)-(16) and (36) are carried out on the graph of Figure 1, where system 0 is the leader node and system 1 is the only target node. The heterogeneous systems (1) are taken as second-order linear systems with relative degree equal to one

$$\begin{aligned} \dot{x}_i &= \underbrace{\begin{bmatrix} 0 & 1 \\ -d_{2_i} & -d_{1_i} \end{bmatrix}}_{A_i} x_i + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{b_i} u_i \\ y_i &= \underbrace{\begin{bmatrix} n_{2_i} & n_{1_i} \end{bmatrix}}_{c_i^T} x_i \end{aligned} \tag{40}$$

where the second equation is used only in the output-feedback case. The parameters and initial conditions for each system (unknown to the designer and used only for simulation) are reported in Table I.

TABLE I
PARAMETERS AND INITIAL CONDITIONS FOR THE SYSTEMS

	d_{1_i}	d_{2_i}	n_{1_i}	n_{2_i}	$x_i(0)$
system #1	0.75	2.5	0.5	1	$[-0.25 \quad 1]^T$
system #2	1	2	1	1.5	$[0.25 \quad -1]^T$
system #3	0.5	1	0.75	0.75	$[-0.5 \quad 0.5]^T$
system #4	1.25	2	1.25	1	$[0.5 \quad -0.5]^T$
system #5	1.5	1.5	1	1.25	$[-1 \quad 0.25]^T$
system #6	0.75	1	1.5	2	$[1 \quad -0.25]^T$

For the *state synchronization* case, the desired homogeneous dynamics in Assumption 2 and the initial conditions for the leader (4) are chosen as

$$A_0 = \begin{bmatrix} 0 & 1 \\ -(0.8^2) & 0 \end{bmatrix}, \; b_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \; x_0(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

The vector f and matrix P that satisfy condition (14) are

$$P = \begin{bmatrix} 0.4774 & 0.0641 \\ 0.0641 & 0.5681 \end{bmatrix}, \qquad f^T = \begin{bmatrix} -1 & -10 \end{bmatrix}.$$

Finally, the adaptive gain is $\gamma = 50$ and all estimated gains k_i, l_{ij} and l_{im} are initialized to 0. The resulting adaptive state synchronization is shown in Figure 2, with adaptive gains shown in Figure 3.

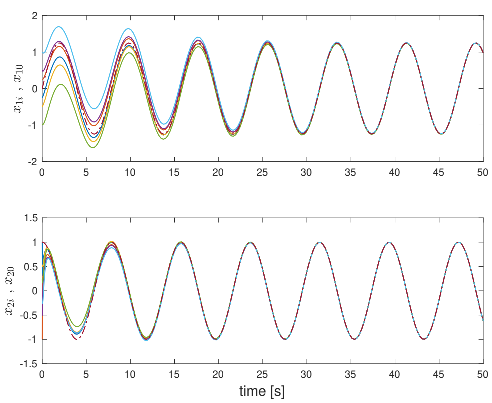

Fig. 2. Synchronization of the states of each system to the leader reference state using (15) and (16).

For the *output synchronization* case, the same parameters and initial conditions as in Table I are taken. The desired homogeneous dynamics in Assumption 3 and the initial conditions for the leader (25) are chosen as

$$A_0 = \begin{bmatrix} 0 & 1 \\ -(0.8^2) & 0 \end{bmatrix}, \; b_0 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \; c_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \; x_0(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{41}$$

that in transfer function form is $(s+0.64)/(s^2+0.64)$. Therefore we have $F = -0.64$ and $d = 1$. The scalar ϕ that satisfies condition (28) is $\phi = -1$. The adaptive gain is taken $\gamma = 50$ and all estimated gains θ_i are initialized to 0. The resulting adaptive output synchronization is shown in Figure 4 together with the adaptive gains.

VI. CONCLUSIONS

The contribution of this work was to show that, for heterogeneous and uncertain networks of certain classes of linear systems, synchronization is possible without the need for any distributed observer. Such classes are in line with those proposed in model reference adaptive control literature. As a result, any local communication except from neighbors' states (or outputs) has been removed.

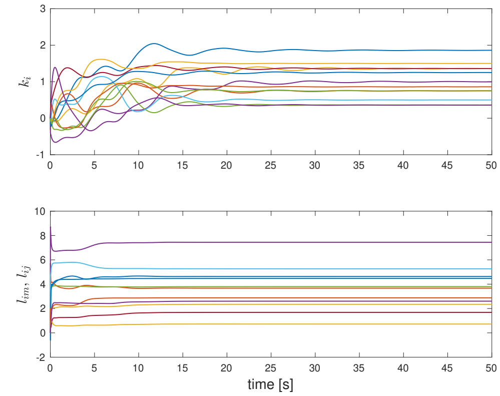

Fig. 3. Adaptive gains resulting from (16).

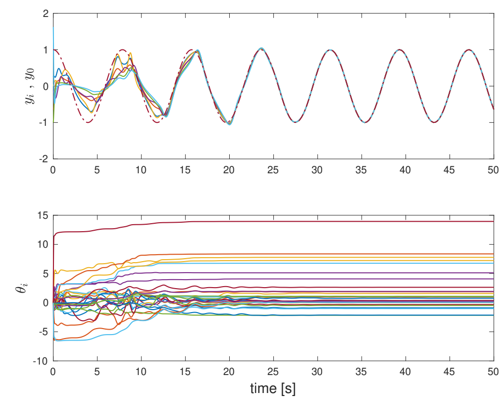

Fig. 4. Synchronization of the outputs of each system to the leader reference output using (36), and corresponding adaptive gains.

Future work could involve studying the effects of delays in the computation of the protocols [31] or extending the results in the switching topology scenario, e.g. using adaptive switching tools [32].

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