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DOI
10.1109/icassp.2016.7472389

Publication date
2016

Document Version
Accepted author manuscript

Published in
2016 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)

DIRECTIONAL MAXIMUM LIKELIHOOD SELF-ESTIMATION OF THE PATH-LOSS EXPONENT

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ABSTRACT

The path-loss exponent (PLE) is a key parameter in wireless propagation channels. Therefore, obtaining the knowledge of the PLE is rather significant for assisting wireless communications and networking to achieve a better performance. Most existing methods for estimating the PLE not only require nodes with known locations but also assume an omni-directional PLE. However, the location information might be unavailable or unreliable and, in practice, the PLE might change with the direction.

In this paper, we are the first to introduce two directional maximum likelihood (ML) self-estimators for the PLE in wireless networks. They can individually estimate the PLE in any direction merely by locally collecting the related received signal strength (RSS) measurements. The corresponding Cramér-Rao lower bound (CRLB) is also obtained. Simulation results show that the performance of the proposed estimators is very close to the CRLB. Additionally, also for the first time, the RSSs based on only a geometric path loss are found to follow a truncated *Pareto* distribution in wireless random networks. This might be of great help in the analysis of wireless communications and networking.

***Index Terms*—** Random networks, received signal strength (RSS), path-loss exponent (PLE), maximum likelihood estimation, Cramér-Rao lower bound (CRLB), Pareto distribution

1. INTRODUCTION

The path-loss exponent (PLE) is very crucial for efficiently designing wireless communications and networking systems. For instance, the information-theoretic capacity of large ad hoc networks highly depends on the PLE, which might lead to different routing strategies [1]. Source localization based on the received signal strength (RSS) measurements requires the knowledge of the PLE to estimate the target location [2]. The interference in wireless ad hoc networks is greatly affected by the PLE [3], which has a strong impact on the quality of the transmission link. Therefore, the PLE needs to be accurately estimated.

Current methods for estimating the PLE can mostly be found in the field of RSS-based localization [4–6], where some nodes with known locations, i.e., anchors, are required. However, the anchors are sometimes not available and the location information might also be unreliable, especially in military scenarios where adversaries can maliciously sabotage wireless networks by spoofing some specific information. Some other estimators of the PLE are presented in [7], which however require the network density or the receiver sensitivity, and even require changing them. Besides, in practice, the PLE might change with direction, as depicted in Fig.1. Yet, all existing estimators assume the PLE to be omni-directionally the same.

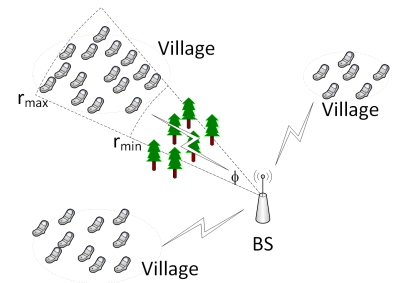


Fig. 1: Example in $\mathbb{R}^2$ based on a spherical coordinate system: mobile users cluster in different separate villages, but remotely connect to the BS; the PLEs to different villages are often different (e.g., some forests might result in a large PLE).

Therefore, a directional self-estimator, which can solely and individually estimate the PLE merely by locally collecting the RSSs, is urgently required. Driven by this motivation, two (weighted) total least squares self-estimators of this kind have already been proposed in our previous work [8]. However, obtaining the maximum likelihood (ML) solution and the Cramér-Rao lower bound (CRLB) still remains a problem. Additionally, our previous work assumes a homogeneously random deployment of the surrounding nodes, i.e., a homogeneous random network (HRN), and such an assumption can easily be violated, when nodes are clustered. For example, considering cellular networks, the mobile users tend to cluster in different villages with different PLEs, as shown in Fig. 1. In this case, all the aforementioned estimators become unfeasible and certainly cannot estimate the PLE to every village. A possible solution is to consider the mobile users in each village to be locally randomly deployed, i.e., those villages are viewed as several locally random networks (LRNs), which remotely connect to the base station (BS). Then, this issue can be well-resolved if we propose a self-estimator for the PLE based on the RSS measurements from those LRNs. In fact, if considering a spherical coordinate system, the LRN is more general (and includes the HRN), thus leading to more general solutions.

The contributions of this paper can be listed as follows:

1. The RSSs based only a geometric path-loss are first found to follow a truncated *Pareto* distribution. This is derived using properties of LRNs. This finding might be of great help in the analysis of wireless communications and networking.
2. Based on the RSS distribution, two ML self-estimators of the PLE are derived, which meet the mentioned requirements. Further, the CRLB for this kind of estimator is computed.
3. The two proposed ML self-estimators are both close to the CRLB. For comparison, we especially consider the case of a

This work is supported by the China Scholarship Council (CSC) and the Circuits and Systems (CAS) group, Delft University of Technology, the Netherlands.

HRN and both our estimators outperform two existing ones: one weighted total least squares estimator from our previous work (WTLS-PLE) and another estimator based on the cardinality of the transmitting set (C-PLE).

2. RSS DISTRIBUTION IN WIRELESS RANDOM NETWORKS

Since the self-estimation of the PLE only relies on the RSS measurements, it is obviously significant to obtain the RSS distribution in wireless random networks. However, this has never been studied before, to the best of our knowledge. If this distribution can be found, it might not only help to obtain the CRLB as well as the ML solution for the self-estimation of the PLE, but also lead to other insightful properties of wireless networks.

To begin, we first have to study the distribution of the nodal distance r for a random node deployment. Two distributions for ordered nodal distances were already given in [9,10]. However, they were limited to (infinite) HRNs. Therefore, for the remote LRNs depicted in Fig. 1, we actually need a more general distribution.

A random deployment of nodes implies that every node holds an equal chance ρ to reside in a considered area. Therefore, if all the nodes are bounded by an LRN in $\mathbb{R}^m$, we can obtain $\rho = 1/(c_{m,\phi}\ r_{max}^m - c_{m,\phi}\ r_{min}^m)$, where ϕ is the angular window, r_{min} and r_{max} are considered the smallest and the largest nodal distances, and for $m = 1, 2, 3$ we have $c_{1,\phi} = 1$, $c_{2,\phi} = \phi/2$ and $c_{3,\phi} = \frac{2\pi}{3}(1 - cos\phi)$. Therefore, the cumulative density function (CDF) of r is given by

$$\mathbb{F}(r) = \rho c_{m,\phi}(r^m - r_{min}^m) = \frac{r^m - r_{min}^m}{r_{max}^m - r_{min}^m}, \quad \text{for } r \in [r_{min}, r_{max}). \tag{1}$$

and hence the probability density function (PDF) of r can be obtained as

$$\mathbb{P}(r) = \frac{\partial\ \mathbb{F}(r)}{\partial r} = \frac{m\ r^{m-1}}{r_{max}^m - r_{min}^m}, \text{ for } r \in [r_{min}, r_{max}). \tag{2}$$

Then, for the wireless propagation channel, we currently only consider the geometric path loss [11], i.e., the RSS can be presented (in *Watt*) by

$$P_r = Cr^{-\gamma}, \tag{3}$$

where γ is the PLE and $C \triangleq G_t G_r P_t$ with G_t the transmitter antenna gain, G_r the receiver antenna gain and P_t the transmit power. Admittedly, the *shadowing* effect is very important, yet considering it will complicate the following derivations. Besides, the proposed ML solutions are also very resilient to the *shadowing* effect if considered, which will be discussed later on.

One may also consider the *small-scale* fading, which mainly decides the instantaneous received power. In fact, the instantaneous received signal envelope follows a *Nakagami* distribution [12] and accordingly the distribution of the instantaneous received power p follows a *Gamma* distribution, which is given by

$$\mathbb{P}(p) = \frac{(\frac{d}{E(p)})^d p^{d-1} e^{-\frac{dp}{E(p)}}}{\Gamma(d)}, \tag{4}$$

where d is the fading parameter and a small value of d indicates a stronger fading. Precisely speaking, the *small-scale fading* just causes the instantaneous power p to rapidly fluctuate within a very small scale around the expectation that is determined by the RSS, i.e., $E(p) = P_r$. Therefore, compared with the geometric path-loss, the impact of *small-scale fading* is relatively small. In practice, the RSS P_r is obtained by taking the average over K consecutive time slots of instantaneous received powers p_k, i.e. $P_r = \frac{1}{K}\sum_{i=1}^{K} p_k$. From (4), we have $Var(P_r) = \frac{E(p_k)^2}{Kd}$, which implies that, when K is large enough, the impact of the small-scale fading almost vanishes. Therefore, the term "received signal strength (RSS)" does not consider the *small-scale fading*, i.e., the RSS in this paper refers to P_r.

Obviously, the geometric path-loss in (3) follows the Zipf's law, which enlightens us that, in this case, the RSS in wireless random networks might be subject to one of the power-law distributions [13], e.g., the *Pareto* distribution, but this has never been observed before. Note that this kind of distribution has rather wide applications in research on the city population [14], the sizes of earthquakes [15], etc., yet so far not in the field of wireless networks.

Based on (1) and (3), the CDF of the RSS can be obtained after a simple transformation of variables as

$$\mathbb{F}(P_r|m, \gamma, P_{r,min}, P_{r,max}) = \begin{cases} \frac{1-(P_{r,min}/P_r)^{m/\gamma}}{1-(P_{r,min}/P_{r,max})^{m/\gamma}}, & \text{for } P_{r,min} \le P_r \le P_{r,max}, \\ 0, & \text{otherwise}, \end{cases} \tag{5}$$

where $P_{r,min} \triangleq Cr_{max}^{-\gamma}$, and $P_{r,max} \triangleq Cr_{min}^{-\gamma}$ in the LRN ($r_{min} \gg 0$), or $P_{r,max} \triangleq P_t$ in the HRN to avoid the singularity issue in (3). And, the PDF can finally be obtained as

$$\mathbb{P}(P_r|m, \gamma, P_{r,min}, P_{r,max}) = \frac{\partial \mathbb{F}(P_r|m, \gamma, P_{r,min}, P_{r,max})}{\partial P_r} = \begin{cases} \frac{m}{\gamma} \frac{P_{r,min}^{m/\gamma} P_r^{-m/\gamma-1}}{1-(P_{r,min}/P_{r,max})^{m/\gamma}}, & \text{for } P_{r,min} \le P_r \le P_{r,max}, \\ 0, & \text{otherwise}, \end{cases} \tag{6}$$

which apparently follows a truncated *Pareto* distribution Type I [16].

3. DIRECTIONAL MAXIMUM LIKELIHOOD SELF-ESTIMATION OF THE PLE

After obtaining the distribution for the RSS measurements, we can introduce the CRLB for the self-estimation of the PLE and our proposed ML solutions.

3.1. CRLB

If n RSS samples are locally collected from an LRN, where the i-th sample is denoted by P_i, the truncated *Pareto* distribution (6) directly leads to the CRLB for the self-estimation of the PLE, which can be given by $CRLB(\gamma) = \frac{1}{\mathcal{I}(\gamma)}$, where $\mathcal{I}(\gamma) = -E\left[\sum_{i=1}^{n} \frac{\partial^2 ln(\mathbb{P}(P_i|m,\gamma,P_{r,min},P_{r,max}))}{\partial \gamma^2}\right]$ is the Fisher information shown in (7) on the top of page. 3. As shown in Fig. 3a, the CRLB decreases with a large sample size or a small PLE. We also notice that, the farther the LRN is located from the considered node, the larger the CRLB becomes.

3.2. Two ML Self-Estimators for the PLE

Now, let us focus on the ML solution to the self-estimation of the PLE. Based on the truncated *Pareto* distribution in (6), the log-likelihood function can be expressed as

$$\begin{aligned}\mathcal{L}(\gamma) &= \sum_{i=1}^{n} ln(\mathbb{P}(P_i|m, \gamma, P_{r,min}, P_{r,max})) \\ &= nln(\frac{m}{\gamma}) + \frac{nm}{\gamma} ln(P_{r,min}) - (\frac{m}{\gamma} + 1)\sum_{i=1}^{n} ln(P_i) \\ &- nln(1 - (P_{r,min}/P_{r,max})^{m/\gamma}),\end{aligned} \tag{8}$$

$$\mathcal{I}(\gamma) = -\frac{n}{\gamma^2} - \frac{2mnln(P_{r,min})}{\gamma^3} + \frac{2n}{\gamma^3}\frac{[(\gamma + mln(P_{r,max}))(\frac{P_{r,min}}{P_{r,max}})^{\frac{m}{\gamma}} - (\gamma + mln(P_{r,min}))]}{(\frac{P_{r,min}}{P_{r,max}})^{\frac{m}{\gamma}} - 1} + \frac{nm(\frac{P_{r,min}}{P_{r,max}})^{\frac{m}{\gamma}} ln(\frac{P_{r,min}}{P_{r,max}})[2\gamma(\frac{P_{r,min}}{P_{r,max}})^{\frac{m}{\gamma}} - 2\gamma - mln(\frac{P_{r,min}}{P_{r,max}})]}{(1-(\frac{P_{r,min}}{P_{r,max}})^{\frac{m}{\gamma}})^2\gamma^4} \quad (7)$$

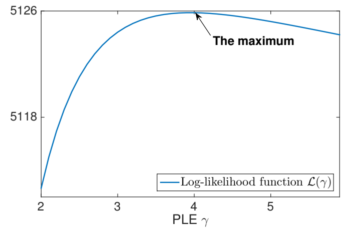


Fig. 2: Demonstration of the convexity of the log-likelihood function $\mathcal{L}(\gamma)$,when the PLE is set to 4.

which is required to be convex to facilitate the proposed ML estimators. To prove that, the derivative of $\mathcal{L}(\gamma)$ should be a strictly decreasing function. For convenience, it would also be sufficient to prove the monotonicity of $f(\gamma) \triangleq \gamma^2\frac{\partial\mathcal{L}(\gamma)}{\partial\gamma}, \forall\gamma : \gamma > 0$, which is what we will do next. Considering any two values of γ that satisfy $\forall\gamma_1, \gamma_2 : \gamma_1 > \gamma_2 > 0$, we have

$$f(\gamma_1) - f(\gamma_2) = -n\left(\gamma_1 - \gamma_2 + \frac{\gamma_1 ln(t^{\frac{m}{\gamma_1}})t^{\frac{m}{\gamma_1}}}{1-t^{\frac{m}{\gamma_1}}} - \frac{\gamma_2 ln(t^{\frac{m}{\gamma_2}})t^{\frac{m}{\gamma_2}}}{1-t^{\frac{m}{\gamma_2}}}\right), \quad (9)$$

where $t \triangleq \frac{P_{r,min}}{P_{r,max}}$. Finally, noticing that $t^{\frac{m}{\gamma}} \in (0, 1)$, we complete the proof of convexity by using some bounds on the natural logarithm, i.e., $1-1/t^{\frac{m}{\gamma}} \leq ln(t^{\frac{m}{\gamma}}) \leq t^{\frac{m}{\gamma}} - 1$, and observing that (9) is always negative as $f(\gamma_1) - f(\gamma_2) \leq -n\left(2\gamma_1 - \gamma_2 + \gamma_2 t^{\frac{m}{\gamma}}\right) < 0$. The convexity of $\mathcal{L}(\gamma)$ is also demonstrated in Fig. 2.

When $P_{r,min}$ and $P_{r,max}$ can be calculated based on some prior knowledge, the ML self-estimate of the PLE can be obtained by forcing the derivative of $\mathcal{L}(\gamma)$ to 0, i.e., the ML solution solves

$$\frac{n\gamma}{m} - \sum_{i=1}^{n}(ln\frac{P_i}{P_{r,min}}) + \frac{n(\frac{P_{r,min}}{P_{r,max}})^{m/\gamma} ln(\frac{P_{r,min}}{P_{r,max}})}{1-(\frac{P_{r,min}}{P_{r,max}})^{m/\gamma}} = 0. \quad (10)$$

When $P_{r,min}$ and $P_{r,max}$ are unknown, we can rank the RSSs, leading to the following set of ordered RSSs: $P_{(1)} < \cdots < P_{(n)}$. We further notice that the log-likelihood function in (8) is an increasing function of $P_{r,min}$ for $P_{r,min} \leq P_{(1)}$ and a decreasing function of $P_{r,max}$ for $P_{r,max} \geq P_{(n)}$. Therefore, for a fixed γ, this log-likelihood function is maximized when $P_{r,min} = P_{(1)}$ and $P_{r,max} = P_{(n)}$.

By respectively using the weakest RSS $P_{(1)}$ and the strongest RSS $P_{(n)}$ to replace the unknown $P_{r,min}$ and $P_{r,max}$, this ML self-estimate of the PLE can be obtained by solving

$$\frac{n\gamma}{m} - \sum_{i=1}^{n}(ln\frac{P_i}{P_{(1)}}) + \frac{n(\frac{P_{(1)}}{P_{(n)}})^{m/\gamma} ln(\frac{P_{(1)}}{P_{(n)}})}{1-(\frac{P_{(1)}}{P_{(n)}})^{m/\gamma}} = 0. \quad (11)$$

Both (10) and (11) can be efficiently solved by a simple bisection method. In our Matlab simulations, the function *fzero* helps us to calculate the solution.

Finally, it is worth noting that, even if the *shadowing* effect is considered, the term $\sum_{i=1}^{n} ln(P_i)$, which is the only sample-related part in our proposed ML solutions, becomes $\sum_{i=1}^{n} ln(P_i) + \sum_{i=1}^{n}\xi_i$, where ξ_i is a zero-mean *Gaussian* variable, i.e., the *shadowing* effect by definition. Obviously, compared to $\sum_{i=1}^{n} ln(P_i)$, the impact of $\sum_{i=1}^{n}\xi_i$ is relatively small, when the sample size n increases. Therefore, due to the limited space, we will not consider the case of the *shadowing* effect in the following simulations.

4. NUMERICAL RESULTS

We have conducted two simulations to evaluate the performance of our two proposed ML estimators. The first simulation assumes an LRN and our two ML estimators are compared with the CRLB. Since no existing method is capable to estimate the PLE in an LRN, we decide to conduct the second simulation for an HRN, where our two ML estimators can be compared with two existing methods: our previously proposed weighted total least squares estimator (WTLS-PLE) of [8] and the estimator based on the cardinality of the transmitting set (C-PLE) of [7] (see also the Appendix). The two node deployments are shown in Fig. 3d. The mean square error (MSE) is adopted to determine the accuracy of the estimators.

4.1. First Simulation

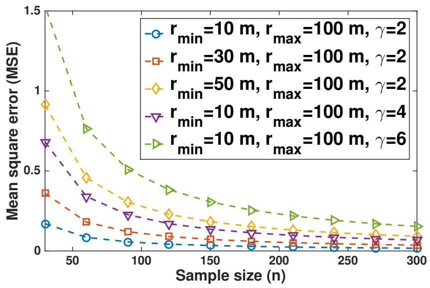
The numerical results are shown in Fig.3b and Fig.3c, from which we can observe that both proposed ML self-estimators yield a very good performance that is very close to the CRLB. Even without the exact knowledge of $P_{r,min}$ and $P_{r,max}$ and using $P_{(1)}$ and $P_{(n)}$ instead, our ML estimator barely suffers any notable decrease in accuracy. Additionally, the performance of our two proposed estimators becomes better with a high node density and a small PLE.

4.2. Second Simulation

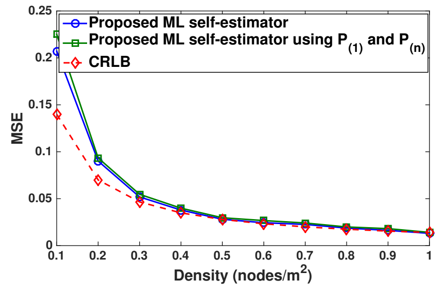
For comparison, the HRN, a special case of the LRN, is considered in this simulation to allow the use of existing estimators. In this case, $P_{r,max}$ is set to the transmit power P_t for our proposed ML self-estimators. As shown in Fig. 3e and Fig. 3f, our ML self-estimators remarkably outperform the WTLS-PLE and the C-PLE. This can be explained by the fact that the WTLS-PLE requires ranking the RSSs, which adopts the rank numbers as a new set of observations. This incurs an extra impact on the estimation quality. The C-PLE, on the other hand, requires changing the receiver sensitivity. However, it simply depends on only two observations, i.e., the neighborhood size before and after the receiver sensitivity change, which makes this estimator very inaccurate and vulnerable.

5. APPLICATIONS AND FUTURE WORKS

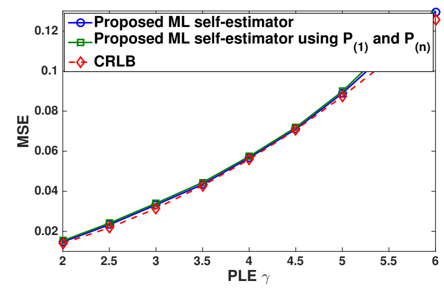
Due to their simplicity, the proposed ML self-estimators can be incorporated into any kind of wireless network. Hence, adapting existing wireless networking and communication designs to this change in PLE might lead to a better performance. We have already elaborated on many applications in [8]. Also note that the proposed self-estimators in this paper can also deal with the case when there exist node clusters, which might lead to broader applications. For example, as shown in Fig. 1, the BS can directionally adjust the transmit



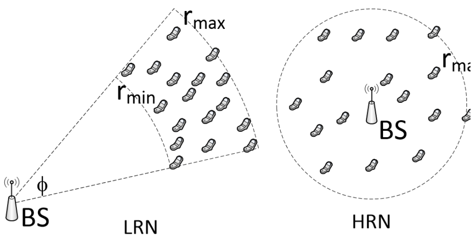
(a) CRLB for the self-estimation of the PLE in $\mathbb{R}^2$.



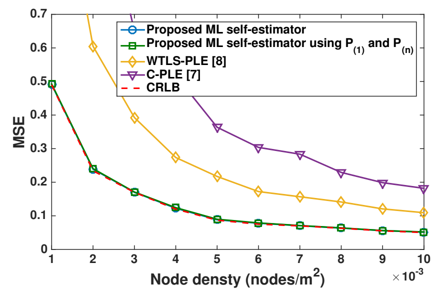
(b) First simulation: impact of the node density when the PLE is set to 2.



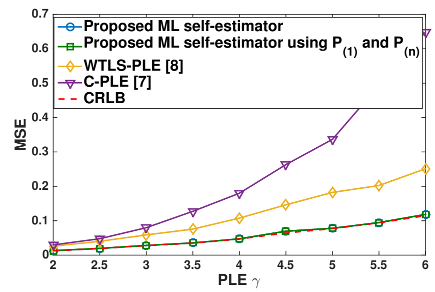
(c) First simulation: impact of the PLE when the node density is set to $1\ nodes/m^2$.



(d) The first simulation considers in an LRN, which is shown on the left side. The second simulation considers in an HRN, which is shown on the right side. Note that the HRN is a special case of the LRN.



(e) Second simulation: impact of the node density when the PLE is set to 4.



(f) Second simulation: impact of the PLE when the node density is set to $0.01\ nodes/m^2$.

Fig. 3: The simulations assume a 2-dimensional space, where nodes are randomly deployed. The carrier frequency is $2.4\ GHz$. The transmit power is $1\ Watt$. The antenna gains G_t and G_r are both 1. For the first simulation, $r_{min} = 50\ m$, $r_{max} = 100\ m$ and $\phi = \pi/6$. For the second simulation, $r_{max} = 100\ m$.

power to different remote villages according to the estimated PLEs such that the coverage of the signal or the energy efficiency can be guaranteed.

In this paper, the *shadowing* effect is ignored for convenience. To be more realistic, if it is considered, then the RSSs in wireless random networks are *log-normally* distributed with the expectation subject to the truncated *Pareto* distribution of (6). Therefore, if we intend to propose ML self-estimators for the PLE over *log-normal shadowing* fading channels, first a new distribution of the RSSs has to be obtained by blending the truncated *Pareto* distribution of (6) with the *log-normal* distribution, which might be mathematically very difficult and complicated.

6. CONCLUSION

Two directional ML self-estimators for the PLE are proposed: one with known $P_{r,min}$ and $P_{r,max}$ and another one using $P_{(1)}$ and $P_{(n)}$ instead. The CRLB is also obtained. Only by locally collecting the RSSs, this kind of estimator can solely and individually estimate the PLE without any external information. Superior to all existing estimators, our two proposed ML self-estimators not only have a very good performance but are also feasible when nodes appear in clusters (all the existing methods assume a homogeneously random node deployment). Two simulations have been conducted: the first one shows that the performance of our two proposed ML self-estimators is very close to the CRLB; the second one shows that they outperform two existing methods, i.e., our previously proposed WTLS-PLE and the C-PLE.

Most importantly, it is the first time that the RSSs based only a geometric path-loss in wireless random networks are found to follow a truncated *Pareto* distribution, which might be of great help for the analysis of future wireless networking and communication systems.

7. APPENDIX

The PLE estimator based on the cardinality of the transmitting set (C-PLE) is proposed in [7], and requires changing the receiver sensitivity for a successful communication. More specifically, when the SINR of a nodal link at the considered receiver exceeds a certain threshold Θ, i.e., $\Theta \leq \frac{P_r}{I+N_0}$ where I is the interference and N_0 is the background noise, this communication link can be determined successful. The cardinality of the transmitting set is simply the number of successful communication links, which is also called the neighborhood size.

By changing the receiver sensitivity from Θ_1 to Θ_2, the transmission range of the considered receiver changes and hence the cardinality of the transmitting set also varies from $N_{T,1}$ to $N_{T,2}$. Then, in $\mathbb{R}^2$, the PLE can be estimated by

$$\hat{\gamma}_{\text{C-PLE}} = \frac{2ln(\Theta_2/\Theta_1)}{ln(N_{T,1}/N_{T,2})}. \quad (12)$$

The C-PLE is only feasible for the HRN, where Θ_1 and Θ_2 are respectively calculated when the transmission ranges are $50\ m$ and $100\ m$.

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