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Van Der Wijk, Volkert

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Abstract— This paper proposes a design of a reconfigurable force balanced planar 4R four-bar linkage that can be applied for a variety of three-position motion tasks. Based on given and fixed link lengths of the four-bar linkage, from three-position synthesis the locations of the coupler pivots and base pivots can be derived for a desired set of three poses. With respect to the end-effector at the coupler link the two coupler pivots can be reconfigured in a balanced way with balanced five-bar parallelogram linkages such that force balance of the complete linkage is maintained at any time. Similarly the two base pivots can be reconfigured in a balanced way with balanced five-bar parallelogram linkages such that force balance of the complete mechanism is maintained for motion of the end-effector through any chosen set of three poses. The balance conditions for the complete mechanism are derived step by step with mass equivalent models.

I. INTRODUCTION

Planar 4R four-bar linkages belong to one of the most applied mechanisms in industry, if not the most applied. In one category of applications the motion of the coupler link is used to execute specific tasks [1]. For the design of such mechanisms the theory of finite-position synthesis has been developed in which the designer, depending on the desired task, can choose the poses - one, two, three, four, five or more - through which the end-effector at the coupler link has to move. While for one, two, and three desired poses a variety of solutions can be found, for four or five desired task poses solutions are obtained only for specific cases by Burmester's theorem [2], [3]. Although four-bar mechanisms can move through more than five poses, it is in general not possible to design a four-bar that can move through more than five chosen desired task poses.

In one- two- and three-position synthesis there is quite some freedom in choosing the free variables. In three-position synthesis generally two cases are considered: (1) to derive the link lengths with the positions of the coupler pivots when the fixed pivots are given and (2) to derive the link lengths with the positions of the fixed pivots when the coupler pivots are given. However, there is also a third possibility, which is to derive the positions of the coupler pivots and the positions of the fixed pivots when the link lengths are given. This possibility was first considered just recently for the purpose of Kinetic-Geometric synthesis enabling the link lengths to be derived from dynamic requirements independently [4].

Volkert van der Wijk, Faculty of Mechanical Engineering, Delft University of Technology, the Netherlands v.vanderwijk@tudelft.nl

An advantage of this third possibility is that it becomes possible to use the same developed linkage for different desired tasks. For a new set of three desired poses the new positions of the coupler pivots and base pivots can be derived and then solely by remounting the end-effector on the coupler and repositioning the base pivots in the base the mechanism can move through the three new desired poses without the need to change the design of the links. Then a single four-bar linkage can be applied for multiple desired tasks in sequence.

The goal of this paper is to propose a mechanism solution for the reconfiguration of the base pivots and the coupler pivots and to present a design of a 4R four-bar mechanism that can be reconfigured for multiple three-position tasks and is force balanced for any motion during reconfiguration and during task motion. First the force balanced four-bar linkage is presented and the reconfiguration is explained. Subsequently the reconfiguration mechanism is presented and the force balance conditions are derived by mass equivalent modeling.

II. Balanced 4R four-bar linkage and reconfiguration for different tasks

Figure 1 shows a 4R four-bar linkage $A_0A_1A_2A_3$ with link lengths l_i and degree of freedom θ_1. Link A_1A_2 is the coupler link and A_0A_3 is the base link. On the coupler link an end-effector is mounted which passes through the three illustrated task poses when the four-bar linkage is moved.

From three-position synthesis with the link lengths given, the base pivots and the coupler pivots can be obtained graphically from the pole triangle [4]. This means that the base pivot A_0 must be on the center-point curve $c_{e,1}$ and the base pivot A_3 must be on the center-point curve $c_{e,3}$ with the distance between them equal to l_4. Figure 1 shows one of the six solutions that can be obtained for this condition in this situation.

A common way to force balance a four-bar linkage is shown in Fig. 2 [5]. Here coupler link A_1A_2 has a mass m_2 of which the center of mass (CoM) is located along the link at a distance e_2 from A_1, link A_0A_1 has a mass m_1 with its CoM located at a distance e_1 from A_0, as illustrated, and link A_2A_3 has a mass m_3 with its CoM located at a distance e_3 from A_3, as illustrated. In this solution the masses m_1 and m_3 act as countermasses of which the parameter values can be obtained from the

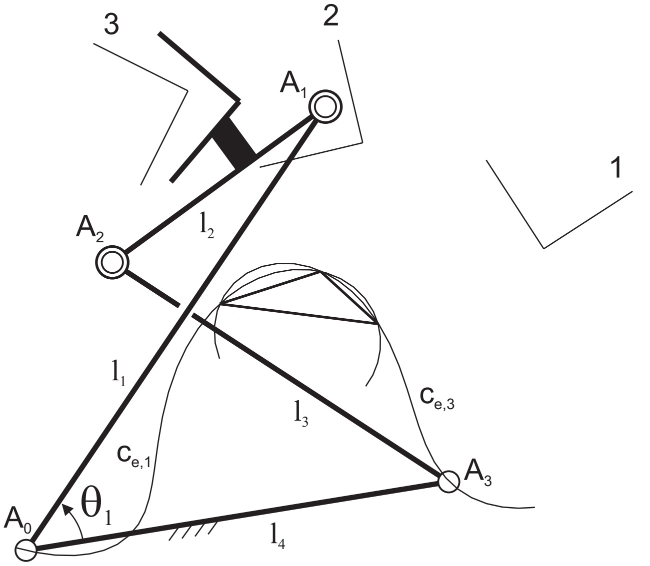

Fig. 1. 4R four-bar mechanism with base link A_0A_3 obtained from three-position synthesis when the end-effector on coupler link A_1A_2 passes through the three defined poses with the link lengths given [4].

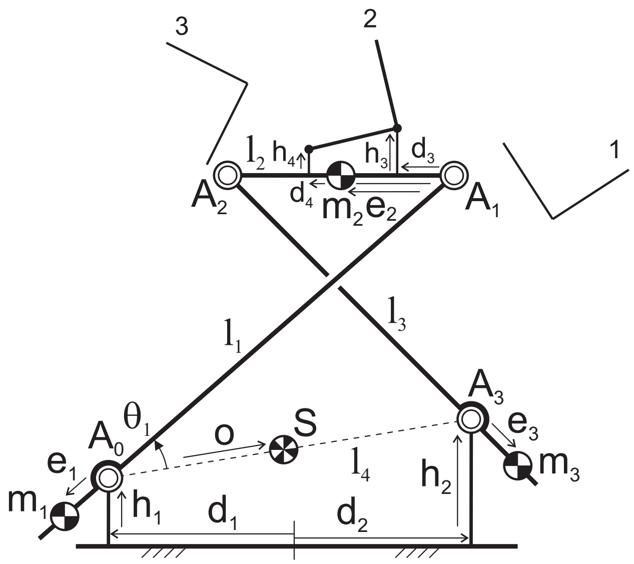

Fig. 2. Design parameters and link masses m_i for force balance with the common CoM in S for any motion of the mechanism.

force balance conditions [4]:

$$m_1 e_1 = m_2 (1 - \frac{e_2}{l_2}) l_1 \tag{1}$$

$$m_3 e_3 = m_2 \frac{e_2}{l_2} l_3 \tag{2}$$

For these conditions the common CoM of the three links is stationary in point S, located at a distance o from A_0 along the line through A_0 and A_1, as illustrated. Point S can be obtained from:

$$m_{tot} o = (m_2 \frac{e_2}{l_2} + m_3) l_4 \tag{3}$$

with $m_{tot} = m_1 + m_2 + m_3$.

The locations of the base pivots A_0 and A_3 relative to a fixed reference frame are defined with parameters d_1, h_1, d_2, and h_2, as illustrated, and the coupler pivots A_1 and A_2 relative to the end-effector are defined with parameters d_3, h_3, d_4, and h_4, as illustrated.

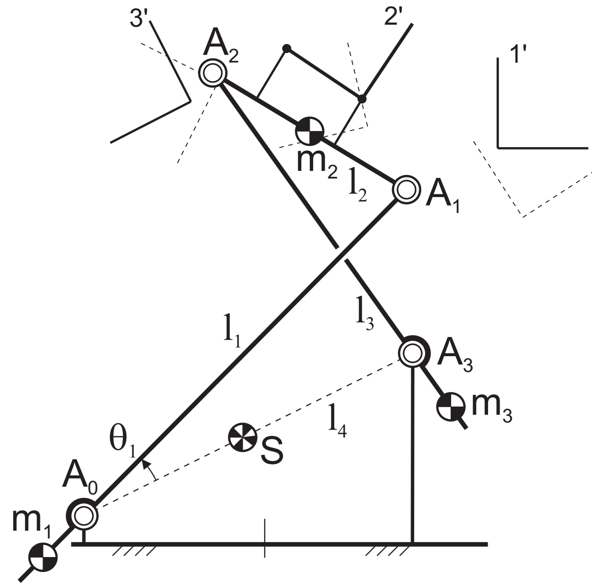

Fig. 3. Reconfigured force balanced four-bar linkage for a different set of three desired task poses with the same link lengths and mass distributions and with different locations of base pivots and of coupler pivots.

The linkage in Fig. 2 can be applied for force balanced motion through infinitely many sets of three desired task poses by solely adapting the parameters d_1, h_1, d_2, and h_2 of the base pivots and the parameters d_3, h_3, d_4, and h_4 of the coupler pivots. Figure 3 shows as an example the configuration that is obtained for the illustrated new set of three task poses. From three-position synthesis the new positions of the base pivots and the new positions of the coupler pivots have been derived for the same link lengths. Relocation of the four pivots does not affect the force balance and the force balance conditions (1-3), which remain equal since the link lengths and the mass distributions of the four-bar mechanism do not change.

III. Reconfiguration mechanism

Reconfiguration of the four pivots for a new set of task poses can be done manually with numerous devices and techniques to release the joints from their old location and fix them one by one at their new location on either the base or the coupler. However, the possible designs to enable this with the required adaptability in all directions may become rather complex and bulky and it may be cumbersome to obtain the new positions accurately. Therefore it is interesting to automate the reconfiguration process with a robotic solution.

In Fig. 4 a design is proposed in which the relocation of the pivots is obtained by four 2-DoF parallel manipulators being five-bar parallelogram mechanisms in specific.

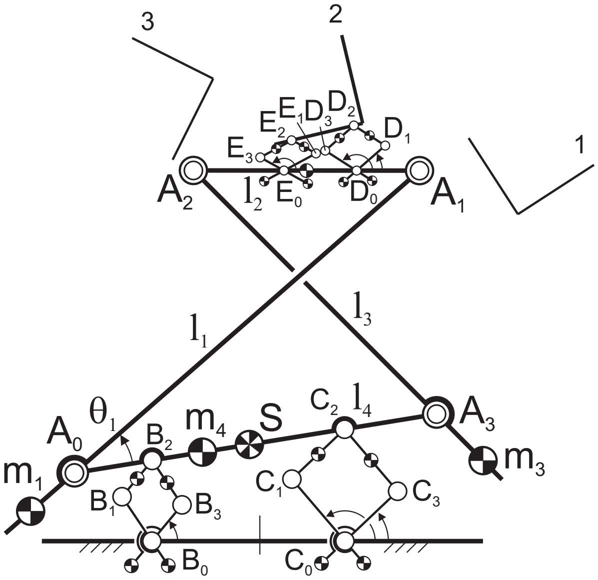

Fig. 4. Design of a force balanced reconfigurable four-bar linkage with four balanced 2-DoF parallel manipulators to relocate the base pivots and to relocate the coupler pivots for an end-effector motion through a different set of three desired poses.

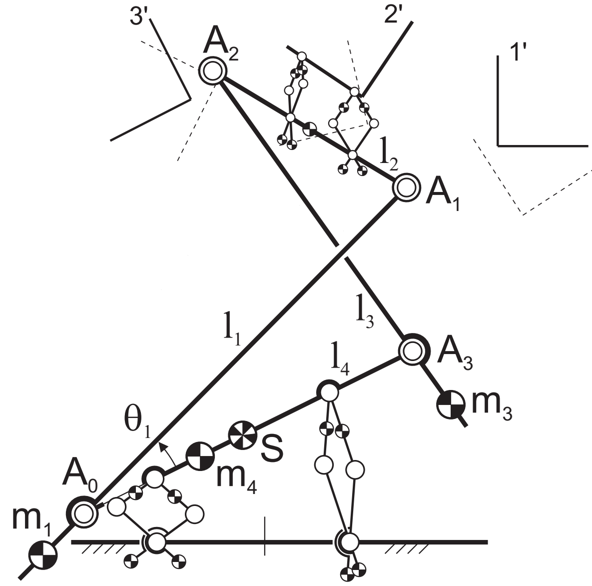

Fig. 5. Example of the relocation of the base pivots and the coupler pivots by the four reconfiguration mechanisms for the new set of three task poses.

With the two reconfiguration mechanisms $B_0B_1B_2B_3$ and $C_0C_1C_2C_3$ the base pivots can be relocated as their endpoints B_2 and C_2 determine the location and orientation of the base link A_0A_3. Similarly with the two reconfiguration mechanisms $D_0D_1D_2D_3$ and $E_0E_1E_2E_3$ the coupler pivots can be relocated as their joints D_0 and E_0 determine the location and orientation of the coupler link A_1A_2 with respect to the end effector. Figure 5 shows the linkage of Fig. 4 when reconfigured for the new set of three task poses.

One rather simple way to reconfigure the base link and the coupler link is with manual locks on at least three joints of both reconfiguration mechanisms of each link, on two joints in one and on one joint in the other. Then by unlocking the three joints the coupler link or the base link can be moved by hand to its new position where the joints are locked again. If also at least three encoders to measure and display the joint angles are installed in a similar way, then an accurate reconfiguration by hand becomes possible.

For an automated reconfiguration process the reconfiguration mechanisms need to be controlled with actuators. For relocating the base link three actuators are needed, either two actuators to drive $B_0B_1B_2B_3$ and one actuator to drive $C_0C_1C_2C_3$ or vice versa and also for relocating the coupler link three actuators are needed, either two actuators to drive $D_0D_1D_2D_3$ and one actuator to drive $E_0E_1E_2E_3$ or vice versa. Since these six actuators are only needed for reconfiguration, they may be small and lightweight and preferably include brakes for locking their motion without the need of energy. It is also possible to apply four actuators for reconfiguration of either the base pivots or the coupler pivots, two for each reconfiguration mechanism, which results in actuation redundancy and is helpful for increased accuracy while each actuator might be smaller.

By means of the actuators the reconfiguration can be obtained rapidly and realtime during the process. The complete design then functions with six small actuators for reconfiguration and one large actuator for moving the four-bar linkage for the intended task. While 1-DoF mechanisms are typically designed for an effective high-speed execution of a single specific task and multi-dof manipulators are typically designed for a wide variety of tasks and therefore are slower and demand dedicated control, the proposed multitask mechanism can be regarded as combining the best of both worlds: high-speed task execution with the flexibility for a wide variety of tasks.

IV. Force balance conditions

The masses of the reconfiguration mechanisms need to be included for force balance in order to have the motions of the four-bar linkage and the motions for reconfiguration not cause any base vibrations due to the produced shaking forces. Each reconfiguration mechanism could be, in principle, a general planar 5R five-bar linkage. However, the choice for the specific design where the planar 5R five-bar linkage is a parallelogram linkage or pantograph linkage is advantageous for a lightweight balanced design [6].

The force balance conditions for which the complete 4R four-bar multitask mechanism in Fig. 4 and 5 is balanced can be derived conveniently by mass equivalent modeling. Starting from the end-effector, Fig. 6 shows how the coupler link design can be modeled in a mass equivalent way.

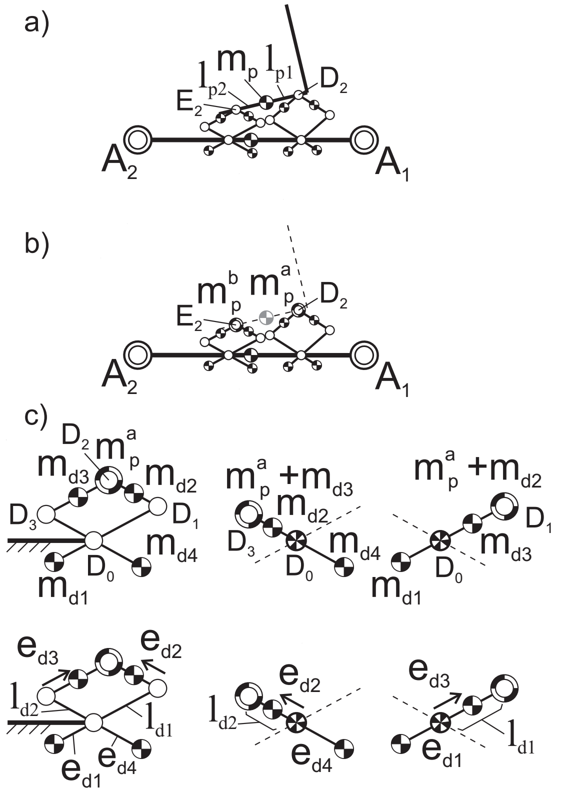

Fig. 6. Mass equivalent models of the coupler link design with its two reconfiguration mechanisms for deriving the force balance conditions.

In the coupler link design in Figure 6a the mass m_p is the mass of the end-effector which has its CoM on the line through the endpoints D_2 and E_2 of the two reconfiguration manipulators at a distance l_{p1} from D_2 and at a distance l_{p2} from E_2 as illustrated. This mass can be modeled with equivalent masses $m_p^a = m_p \frac{l_{p2}}{l_{p1}+l_{p2}}$ in D_2 and $m_p^b = m_p \frac{l_{p1}}{l_{p1}+l_{p2}}$ in E_2 as illustrated in Fig. 6b. With this model each reconfiguration manipulator can be investigated for balance individually.

Figure 6c shows how the reconfiguration mechanism $D_0D_1D_2D_3$ can be modeled with two mass equivalent links that each need to have their common CoM in the central pivot D_0. One model is represented by link D_0D_1 with a mass m_{d1} onto which a mass $m_p^a + m_{d2}$ is projected in D_1 and a mass m_{d3} is projected at a distance e_{d3} from D_0, as illustrated. The other model is represented by link D_0D_3 with a mass m_{d4} onto which a mass $m_p^a + m_{d3}$ is projected in D_3 and a mass m_{d2} is projected at a distance e_{d2} from D_0, as illustrated. From these two equivalent models the force balance conditions for which they have their common CoM in D_0 are derived as:

$$(m_p^a + m_{d2})l_{d1} + m_{d3}e_{d3} = m_{d1}e_{d1} \tag{4}$$
$$(m_p^a + m_{d3})l_{d2} + m_{d2}e_{d2} = m_{d4}e_{d4} \tag{5}$$

The derivation of the force balance conditions for the reconfiguration mechanism $E_0E_1E_2E_3$ is exactly the same and results in the conditions:

$$(m_p^b + m_{e2})l_{e1} + m_{e3}e_{e3} = m_{e1}e_{e1} \tag{6}$$
$$(m_p^b + m_{e3})l_{e2} + m_{e2}e_{e2} = m_{e4}e_{e4} \tag{7}$$

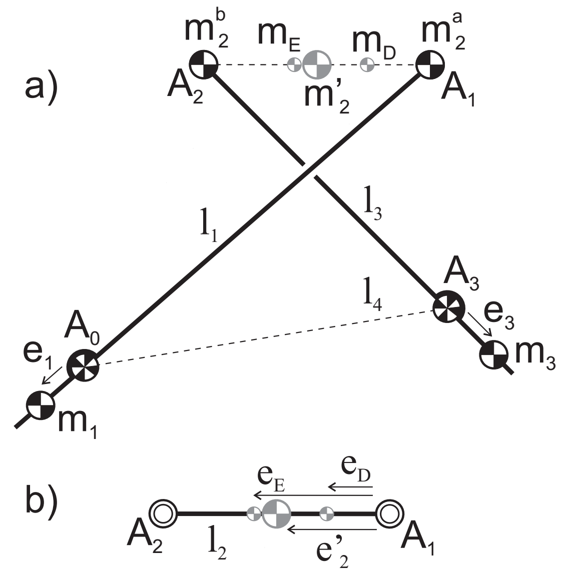

Fig. 7. Mass equivalent model of the four-bar linkage design for deriving the force balance conditions.

After the coupler link design, the design of the four-bar linkage is evaluated for force balance, of which the mass equivalent model is shown in Fig. 7. On the coupler link there are the three masses $m_D = m_p^a + m_{d1} + m_{d2} + m_{d3} + m_{d4}$ and $m_E = m_p^b + m_{e1} + m_{e2} + m_{e3} + m_{e4}$ of both reconfiguration mechanisms and mass m_2' of the link A_1A_2. These masses can be modeled mass equivalently with an equivalent mass m_2^a in joint A_1 and an equivalent mass m_2^b in joint A_2, as illustrated, for the conditions:

$$m_2^a l_2 = m_2'(l_2 - e_2') + m_D(l_2 - e_D) + m_E(l_2 - e_E)$$
$$m_2^b l_2 = m_2' e_2' + m_D e_D + m_E e_E \tag{8}$$

The force balance conditions then are obtained from both equivalent mass models as:

$$m_1 e_1 = m_2^a l_1 \tag{9}$$
$$m_3 e_3 = m_2^b l_3 \tag{10}$$

Another way is to calculate the new parameters for m_2 and e_2 with:

$$m_2 = m_2' + m_D + m_E \tag{11}$$
$$e_2 = \frac{m_2' e_2' + m_D e_D + m_E e_E}{m_2} \tag{12}$$

for which the balance conditions remain as in (1) and (2).

To calculate the location of the common CoM in S of the four-bar linkage with the two reconfiguration mechanisms of the coupler link, also the mass of the reconfigurable base link A_0A_3 must be included as illustrated in Fig. 8. Figure 8b shows the equivalent

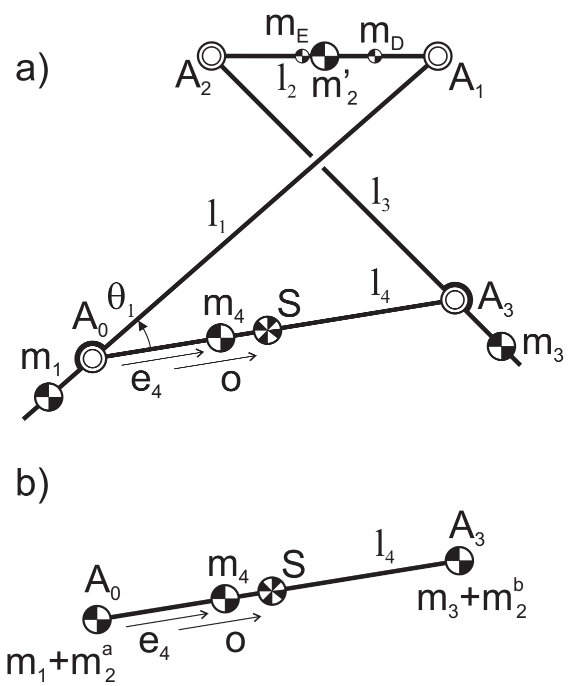

Fig. 8. Calculation of the common CoM in S with the mass m_4 of the link A_0A_3 included.

mass model of the complete four-bar linkage with a mass $m_1 + m_2^a$ projected in A_0 and a mass $m_3 + m_2^b$ projected in A_3. Point S is obtained as the common CoM of this model by:

$$m_{sum}o = (m_3 + m_2^b)l_4 + m_4e_4 \tag{13}$$

with $m_{sum} = m_1 + m_2 + m_3 + m_4$.

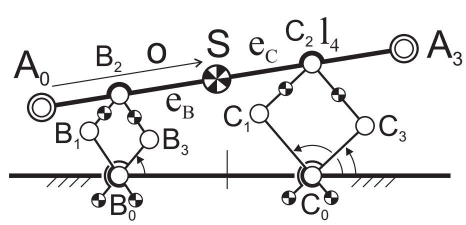

Fig. 9. Design of the base link with its two reconfiguration mechanisms of which the balance conditions can be derived in the same way as illustrated in Fig. 6.

Since the base link has become movable for reconfiguration, point S is no longer a stationary point and for force balance of the complete design also the two reconfiguration mechanisms of the base pivots need to be included. The procedure for this is equal to the coupler link design in Fig. 6 with the mass m_{sum} modeled mass equivalently with an equivalent mass $m_{sum}^a = m_{sum}\frac{e_C}{e_B+e_C}$ in B_2 and an equivalent mass $m_{sum}^b = m_{sum}\frac{e_B}{e_B+e_C}$ in C_2. The force balance conditions of the two reconfiguration mechanisms including these equivalent masses are obtained as illustrated in Fig. 6b and c resulting for mechanism $B_0B_1B_2B_3$ into:

$$\begin{aligned} (m_{sum}^a + m_{b2})l_{b1} + m_{b3}e_{b3} &= m_{b1}e_{b1} && (14)\\ (m_{sum}^a + m_{b3})l_{b2} + m_{b2}e_{b2} &= m_{b4}e_{b4} && (15) \end{aligned}$$

and for mechanism $C_0C_1C_2C_3$ into:

$$\begin{aligned} (m_{sum}^b + m_{c2})l_{c1} + m_{c3}e_{c3} &= m_{c1}e_{c1} && (16)\\ (m_{sum}^b + m_{c3})l_{c2} + m_{c2}e_{c2} &= m_{c4}e_{c4} && (17) \end{aligned}$$

V. CONCLUSIONS

This paper proposed a design of a reconfigurable force balanced multitask mechanism that can be applied for a variety of three-position motion tasks. The mechanism is based on a planar 4R four-bar linkage and has four reconfiguration mechanisms, being 2-DoF five-bar parallelogram linkages, for reconfiguration of the base pivots and the coupler pivots such that an end-effector on the coupler link can move through different sets of three desired poses with the same balanced mechanism. It was explained how the reconfiguration can be done by hand and by automated control of the five-bar linkages. The force balance conditions of the complete mechanism have been derived step by step from mass equivalent models. While the design includes six actuators that are just for reconfiguration and one actuator for the functional motion, if all seven actuators would be used continuously to generate motions, the complete design would be a 7-DoF force balanced parallel manipulator with superb level of mobility.

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