A cellular automaton for modelling territories

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Abstract

A cellular automaton for simulating territories is presented. In it, cells have a certain amount of markings of two different groups. The amount of markings for each group gets higher based on the amounts of that group in neighboring cells and the amount of markings of the opposite group in the current cell to simulate an avoidance tendency. The markings also decay at a certain rate. Depending on the parameters, the simulation can end up in a mixed state, where there are no clear territories, or a segregated state, where both groups have a large amount of connected cells where they are dominant. Small changes in these parameters can change the outcome significantly. Unless an unconsidered combination of parameters changes it, this model is not the most realistic. It could however have uses in more influence-based processes, such as the spread and boundaries of languages or religion.

1. Introduction

The formation of territories between groups is a complex process. It can of course be found between humans in gangs or tribes, but also in nature struggles for space are common, with ants [HL80] and bacteria [GMLF19] for example. Creating models of these phenomena can result in a better understanding of them and allows the possibility to act on them.

A model for simulating the evolution of gang territories already has been made previously by utilizing a random-walk approach [AB18]. This means that in a two-dimensional lattice, after each time interval, agents (i.e. gang members) take a step in a random direction, of which the probabilities are weighted to prefer avoidance of rival gang territory. During this, they leave behind markings to signify their territory.

Simulating these agents, whose numbers can be into the tens of thousands, is computationally expensive, however. Using another approach would allow for faster and more expansive simulations. One possibility for such an approach would be a cellular automaton. Here, each cell on the lattice has a state, and with each time interval it updates its state depending on the state of the cells around it. This can be used in simulations about competition for space, as shown by Young [You84] in the context of skin pattern formation. An adaptation of the aforementioned random-walk model still needs to be made though.

The question to be answered in this paper is therefore: *"Does a cellular automaton for simulating territories, using only territorial markings, get similar outcomes as a random-walk algorithm?"* Answering this would mean the construction of a cellular automaton that provides comparable results to a random-walk model modelling the same gang territory problem. Specifically, only the state of the territories will be used for this, without any agents, to see whether a model of this kind would be sufficient. As stated before, this could be a much more scalable model due to the simplified dynamics.

The paper is be structured as follows. section 2 explains related concepts and literature. In section 3, the model and algorithm are discussed. After this, section 4 describes the experiments performed and outlines the results of them. section 5 then follows with a discussion of the results. section 6 contains the conclusion and recommendations for possible future research. Lastly, section 7 talks about the ethical aspects of the research.

2. Related Literature

In this section, all the prerequisite knowledge and outcomes from previous research is listed.

2.1. Cellular automata

Cellular automata consist of a lattice with discrete cells, and each of these cells has a discrete state, which updates every time step to a potentially new state depending on the states of its surrounding cells, also known as the 'neighborhood' [Wol83]. The way in which the neighborhood affects the cell is dependant on predefined update rules. This can be done in any possible number of dimensions. Depending on the possible states and update rules, they can generate interesting patterns. The best-known example of a cellular automaton is "Conway's Game of Life" or simply "Life" [Gar70]. Here, the number of dead or alive cells is counted in the considered cell and the eight cells directly around it. Then, depending on these amounts, it is determined whether the cell should be dead or alive in the next iteration. With these rules, a myriad of complex behaviours can occur.

For a regular cellular automaton, the set of states a cell can take on is finite. However, it is also possible to use a cellular automaton with continuous states. This is called a "continuous cellular automaton" [Wol02, p. 155-160], and is what will be used for the model described in this paper.

2.2. Modelling skin patterns

A cellular automaton for situations where the occupancy of space is the defining characteristic is used by Young [You84] to simulate skin patterns. Here, there are colored and uncolored cells. The colored cells produce activators and inhibitors. The activators try to convert nearby cells to colored ones, while the inhibitors make far away cells uncolored.

This can be converted to a cellular automaton by making cells get a positive effect from cells inside a certain radius, but a negative effect from ones between that radius and a larger one. If the sum of all these values is then positive the cell becomes colored, and if it negative it becomes uncolored. A sum of 0 means the cell will keep its state. Eventually, it converges to have a stable state where none of the cells change anymore.

With this model, skin patterns found on certain animals are present in the final state. Depending on the parameters set, this can be spotted, but also striped.

2.3. Random-walk model

The model for simulating gang territories by Alsenafi and Barbaro (from here on out called the random-walk model) [AB18] is what will be adapted into a cellular automaton in this paper. While this model is made in the context of simulating gangs, this paper generalizes this to groups in the light of ethical concerns (see subsection 7.1). Therefore, from here on out "gangs" will be referred to as "groups" and "graffiti" as "markings".

In the random-walk model, N agents divided over two groups (A and B) walk around in biased random directions on a two-dimensional lattice S of size $L \times L$ and leave their markings behind. This lattice has periodic boundary conditions, meaning that for cells on the edges the cell on the opposite edge is a neighbor as well. This can be imagined as a torus shape – the grid wraps around and does not have edges. The probability of an agent of group i moving from one cell (x_1, y_1) to a neighbouring cell (x_2, y_2) is:

$$M_i(x_1 \to x_2,\ y_1 \to y_2,\ t) = \frac{e^{-\beta\xi_j(x_2,y_2,t)}}{\sum_{(\tilde{x},\tilde{y})\sim(x_1,y_1)} e^{-\beta\xi_j(\tilde{x},\tilde{y},t)}} \quad (1)$$

with $i \in \{A, B\}$ and $j \neq i$. Several variables and parameters can be found: $\xi_i(x,y,t)$ is the density of markings left behind by agents of group i, and $\beta \geq 0$ is the parameter that indicates the avoidance of the other group's markings. $(\tilde{x},\tilde{y}) \sim (x_1,y_1)$ represents all neighbours of (x_1,y_1).

As can be seen from Equation 1, the probability of an agent going to a specific neighboring cell is influenced by two things: the avoidance and the opposite group's markings. If the target cell has

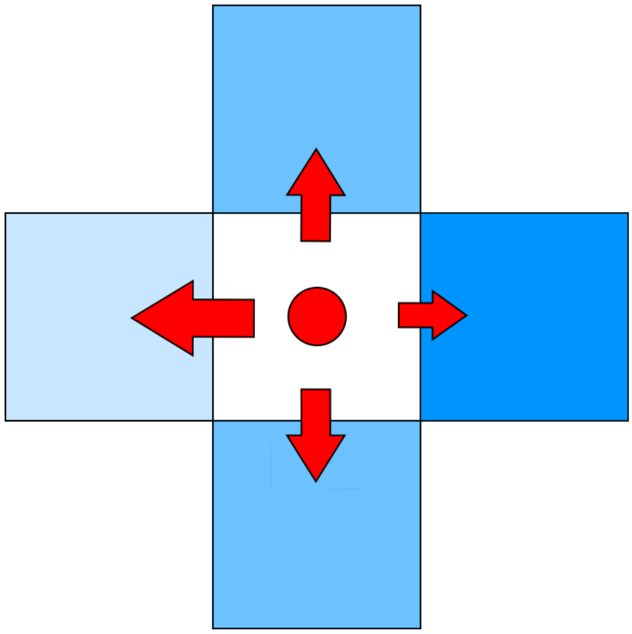

Figure 1: *A visualization of a red agent moving to a different cell in the random-walk model [AB18]. A darker shade of blue indicates more markings of the opposite group. The size of the arrows denote the probability of moving to the neighboring cell. If a cell is a darker shade of blue, the arrow is smaller to denote a smaller probability of going there for the agent.*

a relatively high amount of markings from the opposite group, the agent is more likely to avoid that cell. Additionally, if the avoidance parameter β is set to a higher value this effect will be amplified. A lower β would result in the agents caring less about the opposite group's markings during their decision making. An abstract visualization of this process can be seen in Figure 1.

The expected agent density $\rho_i(x,y,t+\delta t)$ of gang i for each cell (x,y) with a time step δt can then be calculated as follows:

$$\begin{aligned}\rho_i(x,y,t+\delta t) = \rho_i(x,y,t) + \delta t \sum_{(\tilde{x},\tilde{y})\sim(x,y)} \rho_i(\tilde{x},\tilde{y},t) M_i(\tilde{x} \to x,\ \tilde{y} \to y,\ t) \\ - \delta t\ \rho_i(x,y,t) \sum_{(\tilde{x},\tilde{y})\sim(x,y)} M_i(x \to \tilde{x},\ y \to \tilde{y},\ t) \quad (2)\end{aligned}$$

If the model is considered with discrete time steps (i.e. $\delta t = 1$), the first and last terms cancel out and the new agent density is only dependant on the agents coming in from neighboring cells.

The expected updated graffiti density after every time step is then:

$$\xi_i(x,y,t+\delta t) = \xi_i(x,y,t) - (\lambda \cdot \delta t)\xi_i(x,y,t) + (\gamma \cdot \delta t)\rho_i(x,y,t) \quad (3)$$

where $\lambda \in [0,1]$ is the decay rate of graffiti, and $\gamma \in [0,1]$ is the probability for an agent to put down graffiti.

Running this model with a well-mixed starting state on a $100 \times$

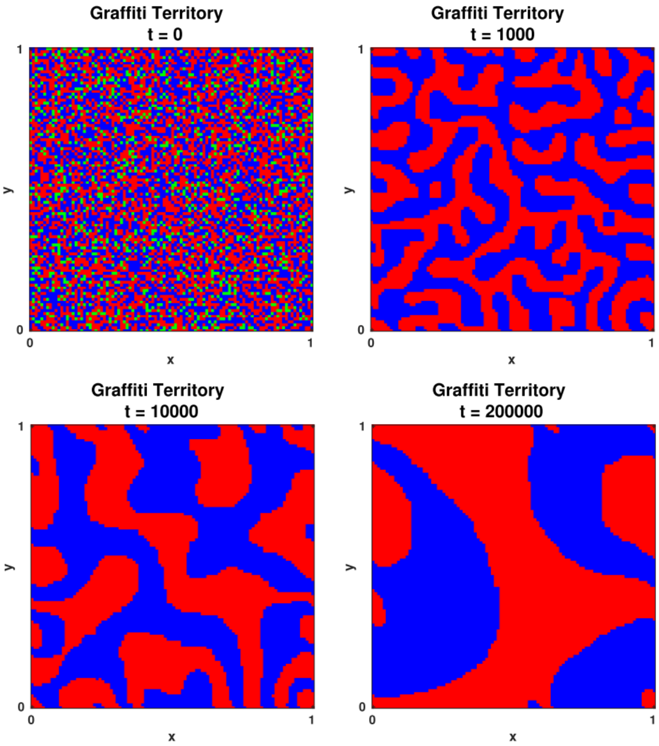

Figure 2: *Evolution of a well-segregated state from the random walk model. A red or blue color means the respective group has a majority in graffiti there, and a green color means the amount is equal.* $L = 100$, $N_A = N_B = 100\ 000$, $\lambda = \gamma = 0.5$, $\delta t = 1$ *and* $\beta = 2 \cdot 10^{-5}$. *Increasingly large connected territories from both groups can be seen. Adapted from [AB18, p. 771].*

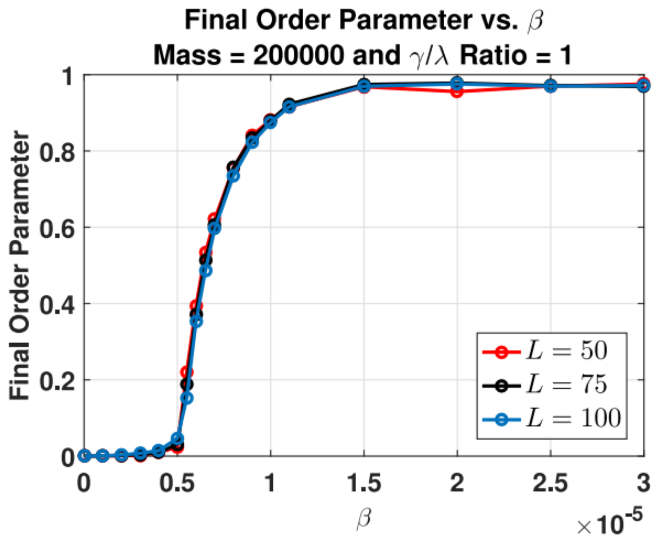

Figure 3: *Influence of* β *on the final order parameter for different lattice sizes.* $N_A = N_B = 100\ 000$, $\lambda = \gamma = 0.5$ *and* $\delta t = 1$. *From* $\beta = 0.5$, *the final value of the order parameter becomes higher with an increase of* β, *up to around* 1.5. *This range is where the* phase transition *occurs. From [AB18, p. 775].*

100 lattice with 100 000 agents for each group, $\lambda = \gamma = 0.5$, $\delta t = 1$, and a low avoidance of $\beta = 1 \cdot 10^{-6}$ keeps the state well-mixed, meaning no clear large connected territories have formed. Increasing β to $2 \cdot 10^{-5}$ gives a well-segregated state, where those *do* form, as seen in Figure 2.

2.3.1. Order parameter

In the random-walk model, an order parameter is also used. This is a value that signifies the order in a certain state, i.e. how mixed or segregated it is. A value of 0 indicates a completely well-mixed state, while a value of 1 occurs when one group controls the entire lattice. Thus, the order parameter is always in between those two for realistic states.

The order parameter is calculated as follows:

$$\varepsilon(t) = \left(\frac{1}{2LN}\right)^2 \sum_{(x,y)\in S} \sum_{(\tilde{x},\tilde{y})\sim(x,y)} (\rho_A(x,y,t) - \rho_B(x,y,t)) \cdot (\rho_A(\tilde{x},\tilde{y},t) - \rho_B(\tilde{x},\tilde{y},t)) \quad (4)$$

If the left and right part of the multiplication inside the sum have the same sign they contribute positively, but if they are different it will be a subtraction from the final value. The magnitude of the contribution to the final result is determined by how large the difference is. The total is then normalized by dividing by 4, the total number of cells, and the total amount of agents. This is intuitive, as the first sum always gives 4 values between -1 and 1, the second sum sums over every cell, and the values of the agent densities depend on the total amount of agents available.

Overall, this value gets larger if neighboring cells both have the same group's agents as majority, and smaller if they have different groups as majority. This means that for mixed states, the order will be small, as there are a lot of neighboring cells with differing groups. Conversely, for a state where each group has large connected territories most neighboring cells will be controlled by the same group, resulting in a large value.

One way in which the order parameter is used is to show the evolution of the order over time. Additionally, the influence of parameters on the final outcome can be made visible. Plotting it against β shows a *phase transition*, meaning that between certain β values it changes from a well-mixed state at low β values to a well-segregated state at high values (Figure 3).

3. Methodology

In this section, the way in which the experiments were carried out is described. First, in subsection 3.1 the designed cellular automaton is described. Then, in subsection 3.2, an order parameter used for analyzing states is defined.

3.1. Model

The expected markings density in the random walk model (Equation 3) depends partially on the previous amount of the markings in the current cell and neighboring cells, and partially on the agent density. The agent density itself depends on surrounding states and the markings therein. This dependence on a previous state of neighbors is exactly what a cellular automaton relies on. Therefore, a cellular automaton makes for a logical model as well here.

To investigate a different way of territory evolution, this cellular automaton will work solely based on markings. Each of its cells has two positive continuous values as its state: one for each group, representing the markings density of that group. It has a predefined size of $L \times L$, and just like in the random-walk model the lattice uses periodic boundary conditions. Part of the update rule is based on Equation 3: The first two terms, regarding the influence of the previous state, are similar: the only difference is the removal of the δt factor, as we are using discrete time steps. However, the third term uses agent density, which is not present in this model. Therefore, a different way of accommodating this interaction between neighboring cells is used.

Similar to how the movement probability for agents is determined in the random-walk model, the new value of the markings of each group is based on the markings of the four cells directly neighboring it. The average of these values is taken, and multiplied by an exponential factor and a positive real number α. The exponential factor uses the negative of an avoidance parameter β and the markings of the opposite group in the current cell. This signifies the tendency of avoiding the other group's territory: both a larger avoidance value or more markings of the opposite color in the current cell will lessen the effect of neighboring markings. The factor α is then used to scale the influence of these neighboring markings.

The final update rule is therefore:

$$\xi_i(x,y,t+1) = \xi_i(x,y,t) - \lambda\xi_i(x,y,t) + \alpha e^{-\beta\xi_j(x,y,t)} \cdot \frac{\sum_{(\tilde{x},\tilde{y})\sim(x,y)} \xi_i(\tilde{x},\tilde{y},t)}{4} \tag{5}$$

with $i \in \{A,B\}$ and $j \neq i$. $\lambda \in [0,1]$ represents the decay of markings, $\beta \geq 0$ the avoidance of the opposite group's markings, and $\alpha \geq 0$ scales the influence of neighboring markings.

This update rule makes the model work in a way that can best be conceptualized using influence or pressure. Instead of actively avoiding certain cells or not, cells can be seen as receiving a certain amount of pressure from surrounding cells. More markings in the neighboring cells results in more pressure, but this can be lessened by the current cell having a large amount of markings from the opposite group. This is visualized in Figure 4.

In section 4 experiments are performed with this model to see if behaviour and results similar to that of the random-walk model can be recreated.

3.2. Order parameter

As can be seen in subsection 4.1 and 4.2, for some values a well-mixed state is reached, and for others a well-segregated state. In order to distinguish these in an exact – non-visual – way, and to find the phase transition between them, an order parameter is used.

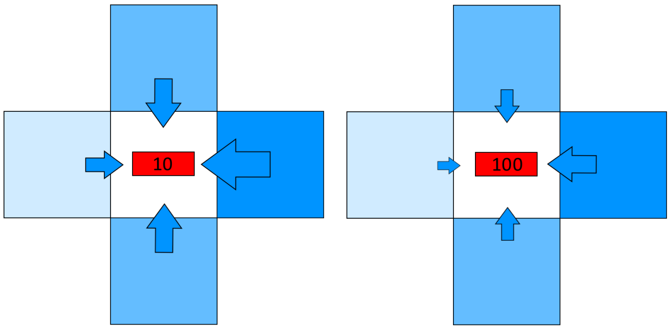

Figure 4: *A visualization of how neighboring cells affect the current one in the cellular automaton. A darker shade of blue means more blue markings in the cell. The red in the center cell is an indication of how much red markings are there. The arrows denote the influence of the blue markings on the center cell, which gets larger if more blue markings are present in the neighboring cell. In the left picture, a relatively lower value of red markings is present, meaning the blue markings from surrounding cells have a lot of influence. In the right picture, the red value is higher, meaning the blue from the surrounding cells has less influence, resulting in smaller arrows. This process happens for both groups in every cell.*

Simply replacing agent density with the amount of markings in Equation 4 would not work. This is because in the random-walk model, the number of agents is constant and can therefore be used to normalize the order parameter. In the cellular automaton model, the total amount of markings will vary per iteration. Thus, there is nothing to scale with to ensure a normalized value.

To solve this, ratios between the markings of each group in each cell are used instead. As outlined in subsubsection 2.3.1, the value of this order parameter should be 0 for a perfectly-mixed state, and 1 for a perfectly-ordered state. The order parameter for this model becomes:

$$\varepsilon(t) = \left(\frac{1}{2L}\right)^2 \left| \sum_{(x,y)\in S} \sum_{(\tilde{x},\tilde{y})\sim(x,y)} \frac{\xi_A(x,y,t) - \xi_B(x,y,t)}{\xi_A(x,y,t) + \xi_B(x,y,t)} \cdot \frac{\xi_A(\tilde{x},\tilde{y},t) - \xi_B(\tilde{x},\tilde{y},t)}{\xi_A(\tilde{x},\tilde{y},t) + \xi_B(\tilde{x},\tilde{y},t)} \right| \tag{6}$$

Using this formula, if in a cell the amount of markings for both groups is similar, the numerator and thus the fraction will be close to 0. If the difference is very large, the numerator will approximately be equal to the larger group, as will the denominator, resulting in a value close to 1 or -1, depending on which group was the larger one. Thus, after multiplication, the value will always be between -1 and 1. As each cell has 4 neighbours and there are $L \times L$ cells, dividing by $4L^2$ (or $(2L)^2$) after summation gives a normalized value for the order parameter.

This order parameter will be used in subsection 4.3. There, the order parameter over time for the examples given in subsection 4.1 and 4.2 can be found. With this, the evolution of the order over time

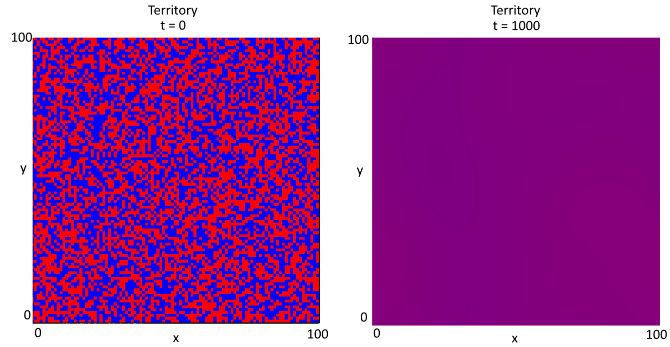

Figure 5: *A well-mixed state from a* 100×100 *cellular automaton at* $t = 0$ *and* $t = 1000$*. The used values for the parameters are* $\lambda = \alpha = 0.5$ *and* $\beta = 1 \cdot 10^{-5}$*. The color of each cell depends on the ratio of red and blue markings in it: if the red group has more markings the color will be redder, and the opposite for blue. A purple color means the cell has roughly the same amount for both.*

can be seen. After this, the final order parameter is plotted against β directly, to show the trend of how β affects the final result.

4. Results

In this chapter, the outcomes of the cellular automaton will be shown. In subsection 4.1 and 4.2 examples of the model are shown: one with a low avoidance ($\beta = 1 \cdot 10^{-5}$), and one with a high avoidance($\beta = 2$) respectively. For both instances a 100×100 cellular automaton is used, with $\lambda = 0.5$ and $\alpha = 0.5$. These values are chosen as a starting point, as they roughly balance the influence of the previous state of a cell and that of its neighbours. At $t = 0$, every cell is randomly assigned a markings value of 1 of one of the groups and 0 of the other. It should be noted here that if a different starting value is used for each cell, β must change by the same factor but inverted to keep results the same. This is because if done so, the exponential part in Equation 5 stays at the same value.

After this, in subsection 4.3, an analysis of the order parameter is performed using the same parameters. Then it is shown how changing a parameter, specifically λ, can change the outcome significantly in subsection 4.4.

4.1. Well-mixed state

Using the parameters described above, with $\beta = 1 \cdot 10^{-5}$, a well-mixed state is reached. From Figure 5 it can be seen that every cell becomes a shade of purple. This is due to the graffiti of both groups being roughly equal, thus resulting in an approximately even mix between red and blue. β is thus indeed too small to create segregation, and the markings spread around evenly.

4.2. Segregated state

Again using the parameters from the introduction of this section, but with $\beta = 2$ this time, a segregated state forms. In Figure 6 it can be seen that the territories cluster together into increasingly large ones. At $t = 1000$, both groups have a connected territory. The boundaries between them are blurry, as there the state is still somewhat mixed. These territories stay mostly unchanged for a large amount of iterations, usually hundreds or even a couple thousand. However, eventually one of the groups takes over and consumes the entirety of the other. This can be seen at $t = 5000$, where the blue group has almost entirely eradicated the red group. The part in which the state is still semi-stable does have segregated territories, and thus will be considered as such. The value of β is high enough to let separated territories form.

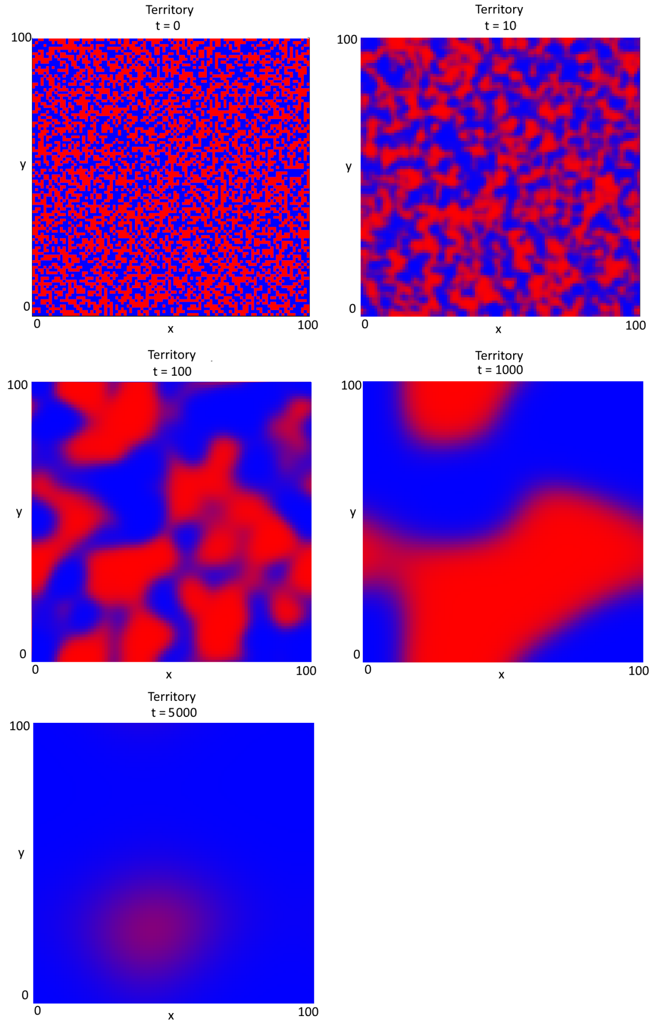

Figure 6: *A well-segregated state from a* 100×100 *cellular automaton at* $t = 0$*,* $t = 10$*,* $t = 100$*,* $t = 1000$ *and* $t = 5000$*. The used values for the parameters are* $\lambda = \alpha = 0.5$ *and* $\beta = 2$*. The state seen at* $t = 1000$ *is a semi-stable state, which means it stays roughly like that for hundreds of iterations before the blue group takes over.*

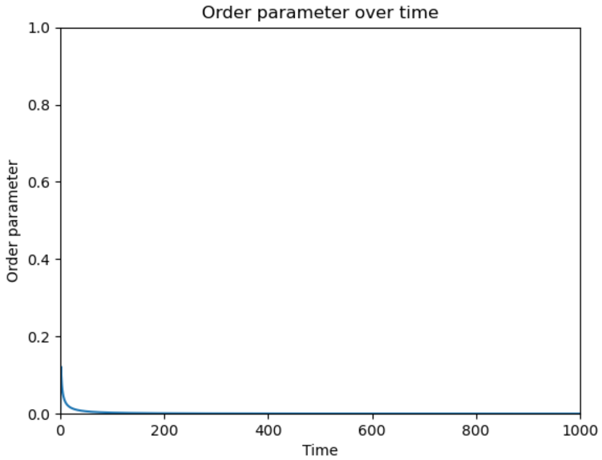

Figure 7: *Order parameter for the well-mixed state in Figure 5. $\lambda = \alpha = 0.5$ and $\beta = 1 \cdot 10^{-5}$. Due to the mixed state it stays close to* 0*.*

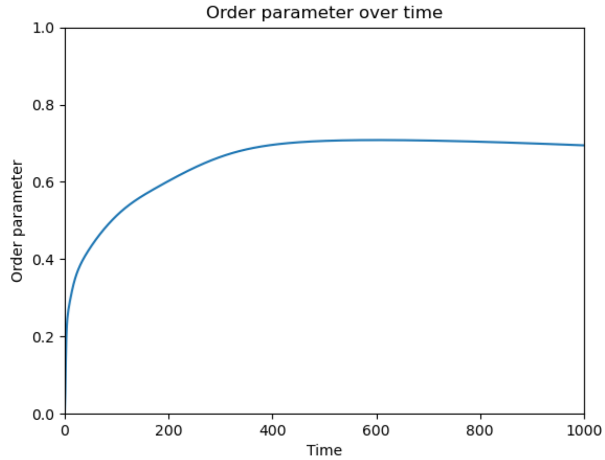

Figure 8: *Order parameter for the segregated state in Figure 6 up to $t = 1000$. $\lambda = \alpha = 0.5$ and $\beta = 2$. It stays relatively steady from approximately $t = 400$, indicating the semi-stable segregated state has been reached.*

4.3. Order parameter

The patterns shown in the evolution of the mixed and segregated states can also be made visible using the order parameter. By plotting the order parameter at each step of the cellular automaton, the coming together of the territories into larger ones can be identified.

Figure 7 shows that for the well-mixed state the order parameter quickly goes towards 0 and stays there. The spike at the start is caused by the territories being more well-defined temporarily at the start due to the way the cellular automaton is initialized.

In Figure 8 the evolution of the order parameter of the segregated state shown in subsection 4.2 is plotted for a 1000 iterations. It can be seen that the order parameter starts low but rises towards a higher value, indicating that the territories get more coherent. At around 0.7, the semi-stable state is reached. If we lengthen the time axis to go up to $t = 6000$, as is shown in Figure 9, it can be seen that the order parameter slowly declines at first, which likely happens due to increased mixing at the boundaries of the territories. At approximately $t = 4000$ the value of the order parameter suddenly starts to go up rapidly. This is the point at which the blue territory starts to take over.

To show the effect of the avoidance on the order parameter, in Figure 10 the order parameter after 1000 iterations is plotted against β. A 1000 iterations is enough for the semi-stable segregated states to form, while not yet reaching the time when one of the groups takes over. For each of the β values sampled, the average of five different tries is taken to lessen the effect of outliers (although some are still slightly visible). An upwards trend is visible, where low β values stay mixed and high values get to their segregated states.

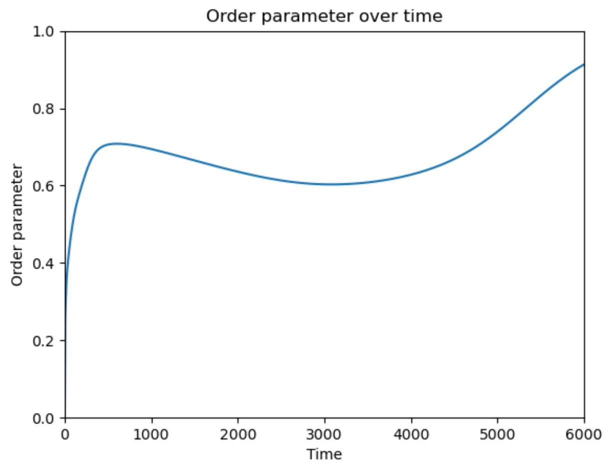

Figure 9: *Order parameter for the segregated state in Figure 6 up to $t = 6000$. $\lambda = \alpha = 0.5$ and $\beta = 2$. After a long period of slow decline, a sudden increase can be spotted when the blue group takes over.*

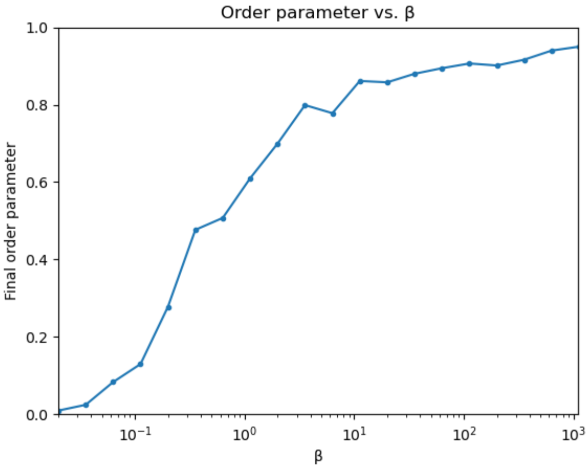

Figure 10: *Order parameter after 1000 iterations plotted against the avoidance parameter* β. $\lambda = 0.5$ *and* $\alpha = 0.5$. *For each point measured the value shown is the average of 5 different simulations to eliminate outliers.*

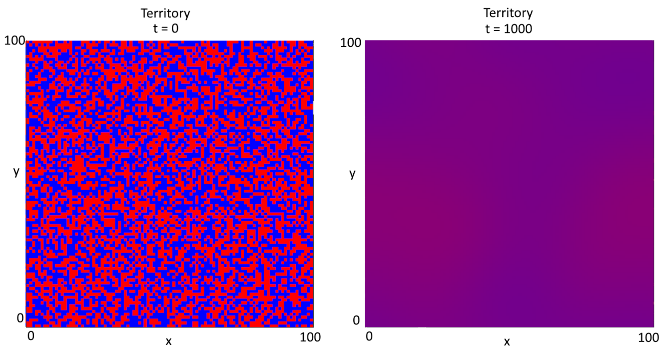

Figure 11: *A segregated state from the cellular automaton at* $t = 0$ *and* $t = 1000$, $\lambda = 0.6$, $\alpha = 0.5$ *and* $\beta = 2$. *The higher* λ *results in less well-defined borders, but does form large connected territories. Slightly redder and bluer patches can bee seen.*

4.4. Influence of λ

By adjusting the decay rate λ, but keeping $\alpha = 0.5$ and $\beta = 2$, the clearness between territory boundaries changes. With an increase of λ, which means graffiti decays more, the boundaries between the (semi)-stable states begin to fade, but connected territories can still (vaguely) be seen, as shown in Figure 11. Doing the opposite, meaning graffiti decays less, gives more defined boundaries between territories, but this comes at the expense of the grouping of territories, as can be seen in Figure 12.

If we change λ to 0.48, both groups still have one big segregated territory, as visible in Figure 13, but boundaries are now well-defined. This difference in state is also visible in the order parameter, which now is even closer to 1 (Figure 14). Moreover, even

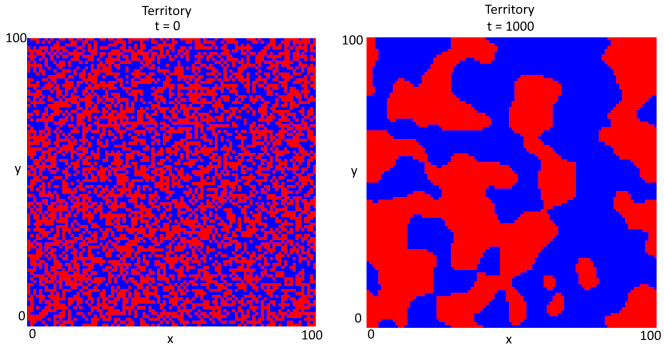

Figure 12: *A segregated state from the cellular automaton at* $t = 0$ *and* $t = 1000$, $\lambda = 0.4$, $\alpha = 0.5$ *and* $\beta = 2$. *The lower* λ *results in more well-defined borders and a stable, converged state, but does make the territories form less like large patches.*

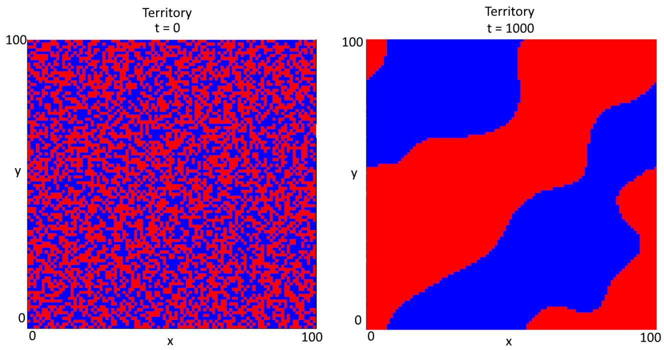

Figure 13: *A well-segregated state from the cellular automaton at* $t = 0$ *and* $t = 1000$, $\lambda = 0.48$, $\alpha = 0.5$ *and* $\beta = 2$. *Territories are large and connected, but the vague borders of Figure 6 are not present anymore.*

after 10000 steps the territories remain stable, indicating a stable, converged state is reached.

However, by changing λ by this light amount, the well-mixed state can't be reached anymore. Instead, it looks approximately mixed for a while, but then suddenly separates into stable large territories.

5. Discussion

The cellular automaton described in this paper does produce well-mixed and segregated states for the parameters used in subsection 4.1 and 4.2. However, in particular for the segregated states, the structures of the territories that can be seen while it gets to the semi-stable state (Figure 6) look different from those in the random-walk model (Figure 2). Additionally, the eventual dominance of one of the groups is a noticeable difference from what happens in the random-walk model.

Just like in the random-walk model, the change from mixed to segregated can be seen when looking at the order parameter plotted

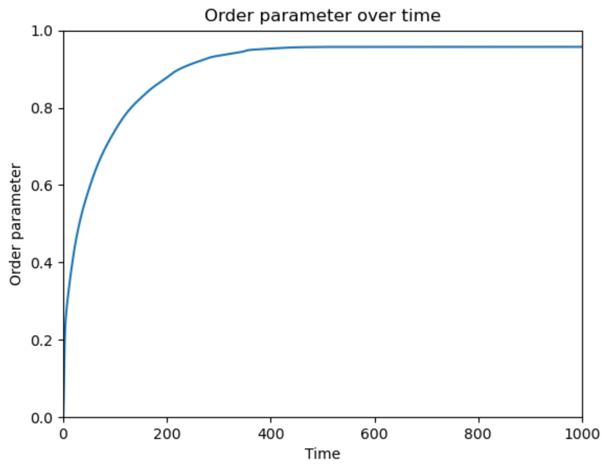

Figure 14: *The order parameter plot over time corresponding to Figure 13. The order parameter is closer to* 1 *compared to the one in Figure 8.* $\lambda = 0.48$, $\alpha = 0.5$ *and* $\beta = 2$.

against β in Figure 10. Compared to the random-walk model (Figure 3) it rises way slower however, and as such does not have the abrupt sharp phase transition that occurs with that one.

Another difference from the random-walk model is that the parameters do not work in the same way, as seen in subsection 4.4. A slight change in λ changes the way the result looks and behaves significantly. By looking at how the order parameter behaves when comparing different combinations of the parameters, this behaviour could be further investigated in future research.

The cellular automaton does have the advantage that it takes less iterations for the final states to be reached. In the random-walk model, t goes up into the hundreds of thousands before the final state is achieved. For the model from this paper, the (semi-)stable states are reached before $t = 1000$. Thus, computation times are shorter.

Overall, with the parameters used for this paper, the cellular automaton is not a replacement for the random-walk model. The parameters do not change the outcome in quite the same way, and the results come close but do not mimic those of the random-walk model.

6. Conclusion

The question to be answered was whether it is possible to construct a cellular automaton which has comparable outcomes to the random-walk model, by only using markings. While the random-walk model uses agents walking around and leaving behind their markings, the developed model uses the influence of the markings of cells around it to determine how it should update its own amount of markings.

With the model used it is possible to get end results which are similar to those of the random-walk model, but the way in which they evolve differs. The influence of the parameters which are present in both – λ and β – is also not equal. The model is thus not a substitution for the random-walk model.

However, there is a possibility that a certain combination of parameters that was not found in this research does generate more comparable results. As seen in this paper, a small change of a parameter can make outcomes vary quite drastically. Thus, a more in-depth analysis of parameters could be performed in the future by comparing the values of the order parameter generated by their combinations.

While it may be less realistic for territorial evolution simulations, it can potentially be useful for other processes. The cellular automaton provides a way of simulating processes based on influence or pressure instead of moving agents. Examples of this could occur in more sociological areas, such as how languages, religions, or cultures spread and mix at their borders. Thus, in future research it could be interesting to apply the model to these scenarios and see how it compares to the real world.

7. Responsible research

This section concerns ethical concerns; both in the content of the research and the reproducibility.

7.1. Ethics

The observant reader might have noticed the paper about the random-walk model talks about gangs. Gangs are usually affiliated with crime. This means that a model that models their territories and evolution of them can be used by law enforcement to act against this, before it actually happens. This is called predictive policing. This is a major ethical concern, as this often results in a self-reinforcing effect [EFN*18]. This problem only gets worsened by the fact that gangs are often more present in less well-off communities, and thus this self-reinforcement will only lead to them becoming even more marginalized [KHA*99].

Because of this, in this paper I have chosen to use the word "group" instead of "gang" like in the paper of Alsenafi and Barbaro to signify that it is a simulation and not necessarily a direct conversion from real-world dynamics. Additionally, a model like this has relevance outside of gang territories, such as in nature, where territorial processes happen too, like with ants [HL80] and bacteria [GMLF19].

As the term "graffiti" does not make sense outside of the context of gangs, the broader term of "markings" is used to signify how groups claim territory.

7.2. Reproducibility

To ensure the research performed in this paper can be verified, the model is clearly presented (in subsection 3.1). Together with the mentioning of what parameters are used, the results are easily reproduced, or even expanded upon.

References

[AB18] ALSENAFI A., BARBARO A.: A convection-diffusion model for gang territoriality. *Physica A: Statistical Mechanics and its Applications*

510 (2018), 765–786. doi:10.1016/j.physa.2018.07.004. 1, 2, 3

[EFN*18] Ensign D., Friedler S. A., Neville S., Scheidegger C., Venkatasubramanian S.: Runaway feedback loops in predictive policing. In *Proceedings of the 1st Conference on Fairness, Accountability and Transparency* (23–24 Feb 2018), Friedler S. A., Wilson C., (Eds.), vol. 81 of *Proceedings of Machine Learning Research*, PMLR, pp. 160–171. URL: https://proceedings.mlr.press/v81/ensign18a.html. 8

[Gar70] Gardner M.: Mathematical games. *Scientific American 223*, 4 (1970), 120–123. URL: http://www.jstor.org/stable/24927642. 1

[GMLF19] Granato E. T., Meiller-Legrand T. A., Foster K. R.: The evolution and ecology of bacterial warfare. *Current Biology 29*, 11 (2019), R521–R537. URL: https://www.sciencedirect.com/science/article/pii/S0960982219304221, doi:https://doi.org/10.1016/j.cub.2019.04.024. 1, 8

[HL80] Hölldobler B., Lumsden C. J.: Territorial strategies in ants. *Science 210*, 4471 (1980), 732–739. URL: https://www.science.org/doi/abs/10.1126/science.210.4471.732, arXiv:https://www.science.org/doi/pdf/10.1126/science.210.4471.732, doi:10.1126/science.210.4471.732. 1, 8

[KHA*99] Kyriacou D. N., Hutson H. R., Anglin D., Peek-Asa C., Kraus J. F.: The relationship between socioeconomic factors and gang violence in the city of los angeles. *The Journal of trauma 46*, 2 (1999), 334–339. 8

[Wol83] Wolfram S.: Statistical mechanics of cellular automata. *Reviews of Modern Physics 55*, 3 (1983), 601–644. 1

[Wol02] Wolfram S.: *A New Kind of Science*. Wolfram Media, 2002. URL: https://www.wolframscience.com. 2

[You84] Young D. A.: A local activator-inhibitor model of vertebrate skin patterns. *Mathematical Biosciences 72*, 1 (1984), 51–58. URL: https://www.sciencedirect.com/science/article/pii/0025556484900609, doi:https://doi.org/10.1016/0025-5564(84)90060-9. 1, 2