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Full-Wave Solver for Radiation from Thermal Sources

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***Abstract*—The thermal emission from finite-size bodies is directly investigated without resorting to reciprocity. Specifically, an integral equation representing the electromagnetic field distribution on a conductive body under investigation is proposed. The source of the electromagnetic field is classic as it is an extension of Johnson noise sources to volumetric problems. The solution of the integral equation allows one to study the radiometric properties for geometries that are smaller than the investigated wavelengths, and in observation points, both in the near and in the far-field. The limits of validity of the formulation are clarified.**

I. Introduction

The thermal emission has been widely investigated since the middle of the twentieth century. Extending on the original works of Rytov [1], many authors have been studying the brightness of conductive bodies resorting to reciprocity, which allows replacing the emissivity of a body with its absorptivity under plane wave incidence. The direct emission of a thermally excited body has been only lightly discussed in the literature. For instance, [2] and [3] address the generation of electromagnetic energy from thermally excited bodies but fail to provide results from numerical simulations in the near fields. To our knowledge, there is a complete lack of either measurements or simulations that provide information about the near-field.

In this work, the radiometric problem is formulated resorting to an integral equation representing the homogenized constitutive relations $\vec{J} = \sigma\vec{E}$ for a conductive body. This becomes an integral equation by representing the electric field as the superposition of the incident field and the field scattered by the unknown conducting currents, $\vec{E} = \vec{E}_i + \vec{E}_s$. In turn, the conduction currents are expanded into sub-domain basis functions, and the integral equation is transformed into a linear system that can be solved numerically. The innovative part of the procedure, otherwise standard, is the introduction of the thermal sources. These are defined by generalizing to scattering problems the noise sources that were proposed in [4]. The dependence on the geometry and the specific material properties can then be investigated in a rigorous full-wave manner. Moreover, the numerical full-wave solution is compared with asymptotic equivalent circuits valid in the high and low-frequency regimes.

II. Thermal Emission Models

A voxel of size δ of a dielectric having permittivity ε_r and conductivity σ is kept at temperature T while embedded in an unbounded homogeneous and lossless dielectric of relative permittivity ε_r. If the body is electrically small, the current and the corresponding electric field can be approximated as uniform. Correspondingly, resorting to the constitutive relation, $\vec{J}/\sigma = \vec{E}_i + \vec{E}_s$ and integrating the volumetric distributions over the voxel gives rise to voltage and currents that are related by

$$v = (Z_\Omega + Z_{\text{rad}})i. \quad (1)$$

The voltage v can be taken as a standard thermal source v=$\sqrt{4k_B T R_\Omega}$ with $R_\Omega = \text{Re}\{Z_\Omega\}$, $Z_\Omega = 1/\sigma\delta$, and k_B is the Boltzmann constant. Z_{rad} represents the scattering impedance of an infinitesimal cube, which can be approximated as

$$Z_{\text{rad}} \approx \frac{80\pi^2}{\sqrt{\varepsilon_r}}\frac{\delta^2}{\lambda_0^2} - j\frac{\zeta_0}{3k_0\varepsilon_r}\frac{1}{\delta}, \quad (2)$$

where λ_0 , k_0 and ζ_0 are the free-space wavelength, propagation constant, and wave impedance, respectively. A body composed of many material voxels will contribute to thermal radiation with the superposition of the energies generated by its different parts. Accordingly, one can set up an integral equation for the overall finite-size body and then solve the discretized volumetric equation finding the current distribution on all the voxels.

A Volumetric Method of Moments based on [5] has been formulated to study the thermal radiation from an arbitrary body. The linear system governing the problem is

$$\boldsymbol{v} = \left(\frac{1}{\sigma\delta}\boldsymbol{I} + \boldsymbol{Z}\right) \cdot \boldsymbol{i}, \quad (3)$$

with $\boldsymbol{Z}$ the radiation matrix calculated in the unbounded lossless dielectric medium, $\boldsymbol{I}$ the identity matrix, δ the size of the basis function, and $\boldsymbol{v}$ the voltage forcing term. This treatise allows modeling the radiative interaction between the different parts of the body, even if the sources are assumed to be independent.

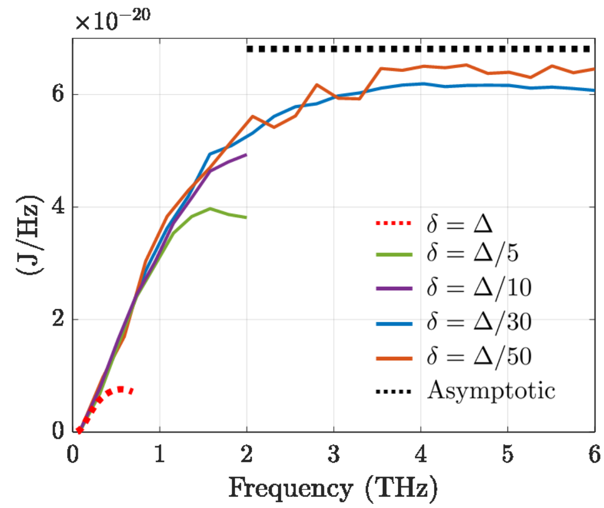

Fig. 1. Comparison between the spectral energy density calculated using different basis function size and its asymptotic value.

III. Results Discussion

The thermal radiation models of Sec. II are applied to a cube of edge $\Delta = 75\,\mu$m, permittivity $\varepsilon_r = 11.7$, and kept at temperature $T = 300$ K. Its conductivity $\sigma = 1/\rho$ is described by Drude's model, according to which the resistivity has the following frequency dependence

$$\rho(\omega) = \rho_{\mathrm{qs}}(1 + j\omega\tau) \tag{4}$$

with $\rho_{\mathrm{qs}} = m_{\mathrm{eff}}/ne^2\tau$, $m_{\mathrm{eff}} = 0.29\, m_{\mathrm{el}}$ for silicon, m_{el} is the free-electron mass, e is the electron's charge, $\tau = 2 \cdot 10^{-13}$ s is the scattering time, and the electron density is $n = 8.8 \cdot 10^{21}\ \mathrm{m}^{-3}$. Fig. 1 compares the radiated energy density calculated with the Method of Moments for different basis function sizes. The case corresponding to one basis function only, $\delta = \Delta = 75\ \mu$m, is representative of the low-frequency limit discussed at the beginning of Sec. II. It is apparent that it provides results equivalent to those using more basis functions as long as the frequency is such that $\Delta < \lambda_d/20$. When the cube's dimensions become larger, more and more basis functions must be adopted. The overlapping of the solutions indicates the range of validity of the less well-sampled ones. Finally, for the high-frequency regime, the power density tends to the asymptotic value

$$\lim_{f\to\infty} P_{rad}^{ave}(f) = 3\Delta^3 \frac{k_B T}{\rho_{\mathrm{qs}}\tau^2} 80 \frac{\sqrt{\varepsilon_r}}{c^2} \tag{5}$$

Taking the limit for $f \to \infty$ one is only meaningful for frequencies at which the homogenization process that leads to the definition of the conductivity is valid.

REFERENCES

[1] S. M. Rytov, Y. A. Kravtsov, and V. I. Tatarskii, *Principles of Statistical Radiophysics Vol. 3*, Springer-Verlag, Berlin, 1989.
[2] D. Polder and M. Van Hove, "Theory of Radiative Heat Transfer between Closely Spaced Bodies" *Phys. Rev. B*, vol. 4, no. 10, pp. 3303-3314, Nov. 1971.
[3] J.J. Greffet, P. Bouchon, G. Brucoli, F. Marquier, "Light Emission by Nonequilibrium Bodies: Local Kirchhoff Law," Phys. Rev. X, vol. 8, no. 2, pp. 0210008 -0210020, Apr. 2018.
[4] J.B. Johnson, "Thermal Agitation of Electricity in Conductors," *Phys. Rev.*, vol. 32, no. 1, pp. 97-109, Jul. 1928.
[5] H. Gan and W.C. Chew, "A discrete BCG-FFT algorithm for solving 3D inhomogeneous scatterer problems," *J. Electromagn. Waves Appl.*, vol. 9, no. 10, pp. 1339-1357, 1995.