Form-finding of gridshells generated from hanging-chain models by using the Dynamic Relaxation method and the NURBS technique

Li, Qingpeng; Wu, Yue; Borgart, Andrew

Publication date
2015

Document Version
Final published version

Published in
Proceedings of the International Association for Shell and Spatial Structures (IASS)

Proceedings of the International Association for Shell and Spatial Structures (IASS)
Symposium 2015, Amsterdam
Future Visions
17 - 20 August 2015, Amsterdam, The Netherlands

Form-finding of gridshells generated from hanging-chain models by using the Dynamic Relaxation method and the NURBS technique

Qingpeng LI [*], Yue WU [a], Andrew BORGART [b]

[*] Key Lab of Structures Dynamic Behavior and Control of Ministry of Education, Harbin Institute of Technology, Harbin, 150090, China

Faculty of Architecture and Built Environment, Delft University of Technology, Julianalaan 134, 2628 BL Delft, The Netherlands

hitssrclqp@gmail.com

[a] Key Lab of Structures Dynamic Behavior and Control of Ministry of Education, Harbin Institute of Technology

[b] Faculty of Architecture and Built Environment, Delft University of Technology

Abstract

Hanging models play an important role in shaping a structure since a very early age, and were favored by A. Gaudi, H. Isler, F. Otto and other architects or engineers. Nowadays, with the development of numerical analysis theory and computer technique, it is more accurate and convenient to simulate these physical models via numerical means. Based on the background, this paper presents a numerical form-finding method of gridshell structures generated from hanging-chain models by using Dynamic Relaxation method and the NURBS technique, which aims to obtain more complex structural forms with multiple control points.

This method uses global NURBS surface interpolation to describe the initial cable-net model passing through the given target points, which serve as the fitting points of the NURBS surface. The cable elements of the cable-net are not allowed to elongate after form-finding, and clearly, this kind of cable-nets belongs to geometrically unstable system, whose form-finding process of it has a very strong nonlinearity. To solve this problem, it uses the Dynamic Relaxation method, which can complete the form-finding of geometrically unstable systems but with some special sets, to get the equilibrium form of the hanging cable-net under the gravity. However, this structural form may no longer pass through the given target points, and then it introduces the inverse iteration method to adjust the coordinates of the fitting points of the NURBS, which actually means to find the initial structural form which after form-finding can just right meet the target requirements. At last, some numerical examples are presented to demonstrate the validity of the proposed method in this paper.

Keywords: form-finding, gridshells, hanging-chain models, Dynamic Relaxation method, NURBS, inverse iteration method

1. Introduction

The hanging model experiment is an effective form generation method for shell or gridshell structures, whose structural behavior strongly depends on their shape. This kind of experiment is based on the principle of mechanical equilibrium and represents structural behavior independent of scale. Because of its appealing characteristics, such as its clear mechanics concept and direct visual image, it was appreciated by A. Gaudi[1][4][5][6], H. Isler[2][4][5][6], F. Otto[3][4][5][6], and other architects or engineers, and they designed many projects with attractive shapes and sensible structures using this method. Among them, F. Otto used hanging-chain models to design the shape of grid shells, for instance, the Model for the Lath Dome at the German Building Exhibition (1962) shown in Figure 1, and the roof for the Multihalle in Mannheim (1970-1975) shown in Figure 2.

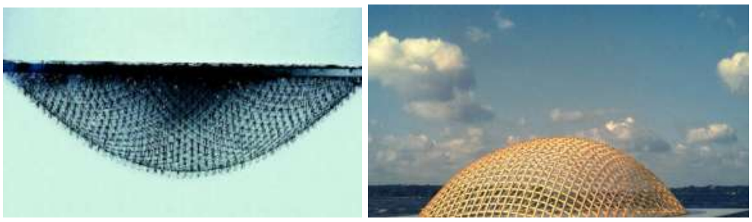


Figure 1: Model for the Lath Dome at the German Building Exhibition, 1962, Essen, Germany

Figure 2: Roof for Multihalle (multi-purpose hall), 1970-1975, Mannheim, Germany

Nowadays, with the development of numerical analysis theory and computer technique, it is more accurate and convenient to simulate these physical models for form-finding of grid shell structures via numerical means, and many numerical form-finding methods were built and are building by scholars. The dynamic relaxation method, force density method, thrust network analysis method and particle-spring systems can be used to generate the optimal shape of gridshells[6].

In this paper, it firstly introduces the basic theory or method used in the proposed method, which contains the Dynamic Relaxation method, the NURBS surface interpolation method and the inverse iteration method, and a simple example is shown respectively to demonstrate its efficiency. And then the form-finding method for gridshells generated from hanging-chain models is proposed. This method uses the NURBS surface interpolation method to describe the initial cable-nets model, in which the cable elements are not allowed to elongate after form-finding. And it uses the Dynamic Relaxation method to get the equilibrium form of the hanging cable-net. In order to generate a gridshell form which should pass through some target points, it introduces the inverse iteration method to adjust the coordinates of the fitting points of the NURBS surface.

2. Basic theory and method

In this part, it introduces the basic theories or methods used in this paper, and some simple examples are shown to explain them clearly.

2.1. Form-finding of hanging cable-nets by the Dynamic Relaxation method

The dynamic relaxation method was first proposed by British engineer A.S.Day[9], after the development by British scholar M.R.Barnes [10] and other scholars, it was widely used in the form-finding of tension structures.

The basis of the method is to trace step-by-step for small time increments, dt, the motion of each node of a structure until, due to artificial damping (kinetic damping is widely used nowadays), the structure comes to rest in static equilibrium. In this procedure "kinetic damping", the undamped motion of the structure is traced and when a local peak in the total kinetic energy of the system is detected, all velocity components are set to zero. The process is then restarted from the current geometry and repeated through further (generally decreasing) peaks until the energy of all modes of vibration has been dissipated and static equilibrium is achieved. By using the DR method, it can get the equilibrium structural form from any unbalanced state with arbitrary and inaccurate specification of geometry.

In this paper, form-finding of the hanging cable-nets composed of unelongated cable elements is solved by the Dynamic Relaxation method. There are some basic assumptions as follows:

1) The initial system is composed of cable elements and nodes. The weight of cable-net is focused on nodes, it ignores the influence of elements' weight. The external loads are also applied just on nodes.The cable element is space straight line with a given length, which means the cable cannot be longer than this certain value.

2) It just considers the axial rigidity of the cable element, and in order to ensure the unelongation of the cable, the cable elements are applied with a very large elastic modulus. For one cable, when the distance between the two nodes is smallar than the given value of length the axial force of the cable is set to zero, and when it reaches to the certain value the cable behaves according to the Hooke's law.

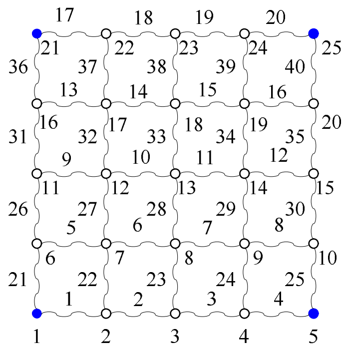
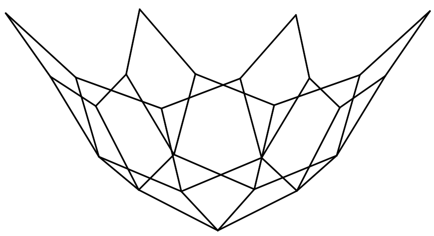


Figure 3: The initial structural system of EX.1

An example is shown here to illustrate the effectiveness of the method. Shown in Figure 4, all the nodes are in one plane and applies with a vertical force of 1N, the distance of two adjacent points is

1.00m. All the cable elements has a same cross-sectional area of 1.0E-4 m^2, a same very big elastic modulus of 1.0E10N/m^2, and a same given length of 1.20m. Point 1, 5, 21, 25 are set as the supports.

Using the DR method with the above basic assumptions, it obtains the equilibrium structural shape of the hanging cable-net and the 'kinetic energy - time' curve shown in Figure 5. And it should be noticed that all the cable elements of the equilibrium structure have a length of 1.20m with a very small error.



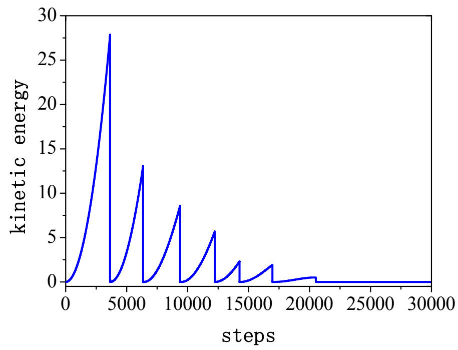


Figure 4: The equilibrium structural model and the 'kinetic energy - time' curve of EX.1

Finally, by inversing the equilibrium structural model, it obtains an inverted gridshell structure with an optimal mechanical behavior based on the principle of inverse hanging method.

2.2. Global NURBS surface interpolation

Non-uniform rational basis spline (NURBS) is a mathematical model commonly used in computer graphics for generating and representing curves and surfaces. It offers great flexibility and precision for handling both analytic (surfaces defined by common mathematical formulae) and modeled shapes. A NURBS surface is defined as a tensor product form by:

$$S(u,v)=\frac{\sum_{i=0}^{m}\sum_{j=0}^{n}w_{i,j}P_{i,j}N_{i,p}(u)N_{j,q}(v)}{\sum_{i=0}^{m}\sum_{j=0}^{n}w_{i,j}N_{i,p}(u)N_{j,q}(v)} \tag{11}$$

where $\{w_{i,j}\}$ are the weights, the $\{P_{i,j}\}$ are the control points, and the $\{N_{i,p}(u)\}$ and $\{N_{j,q}(v)\}$ are the normalised B-spline basis functions defined on the non-periodic knot vectors $U=\{0,0, . . ., 0,u_{p+1}, . . ., u_m,1,1, . . .,1\}$ and $V=\{0,0, . . ., 0,v_{q+1}, . . .,v_n, 1,1, . . . 1\}$, respectively. Here p and q are the degrees in the u and v directions. And it should be noted that the weigh vector $\{w_{i,j}\}$ of the NURBS surface is set to 1, it is actually a B-spline surface.

In this paper, an initial shape of cable-net, the surface of which should pass through some appointed fitting points, is needed. And the global NURBS surface interpolation method is introduced here.

The problem can be described as follows: firstly given a grid of $(m+1)\times(n+1)$ data points $\{D_{ij}\}$ ($0 <= i <= m$ and $0 <= j <= n$) and a degree (p, q), find a NURBS surface of degree (p,q) defined by $(m+1)\times(n+1)$ control points that passes all data points in the given order. And then an example is shown here to illustrate the procedure of the method.

A grid of 3×3 data points is given, shown in Figure 6, the coordinates are {0,0,0}, {10,0,0}, {20,0,0}, {0,10,0}, {10,10,5}, {20,10,0}, {0,20,0}, {10,20,0}, {20,20,0}. And a degree (2,2) is given. Use the method as follows:

1) Compute the knot vector U in the u-direction: {0, 0, 0, 1, 1, 1}.

2) Compute the knot vector V in the v-direction: {0, 0, 0, 1, 1, 1}.

3) Compute the control points $\{P_{i,j}\}$, the coordinates are {0,0,0}, {10,0,0}, {20,0,0}, {0,10,0}, {10,10,20}, {20,10,0}, {0,20,0}, {10,20,0}, {20,20,0}.

4) Use the control points $\{P_{ij}\}$, the knot vector U&V, the degree (2,2) and the weight vector of 1, to get the NURBS surface.

5) Get a grid with 20×20 points in the surface, whose shadow in the X, Y plane is uniform, shown in Figure 6.

This grid will be used as the initial shape of the cable-net before its form-finding process by the DR method, and the length of each cable element which equals to the distance between the two connected points will not be elongated.

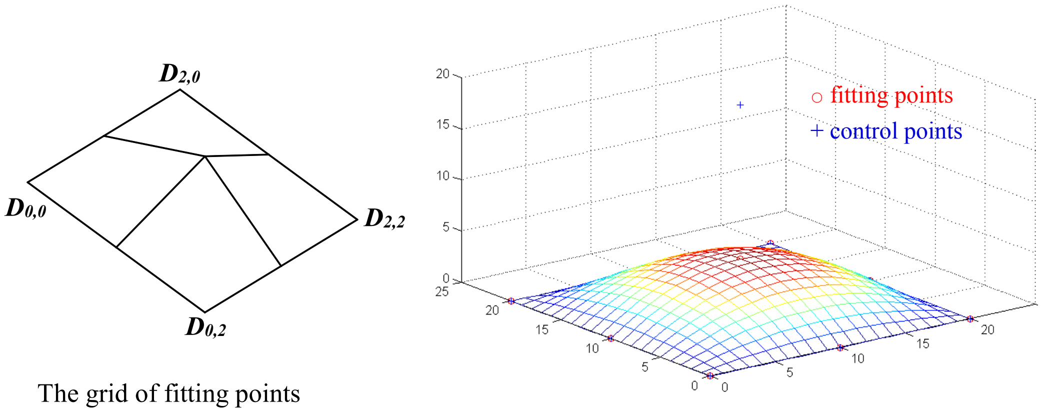


Figure 5: The surface of EX.2

2.3. The inverse iteration method

By using the DR method, it can get the equilibrium structural form from any unbalanced state, while what we want to get is an equilibrium structural from which should meet some given target points. In this case, the problem is to find a specific initial structural system, which after form-finding the equilibrium system can just pass through the target points, by adjusting the parameters of it.

In this paper, the inverse iteration method is introduced to adjust the shape of the initial structural system. And a simple example is shown here to illustrate that how it works.

Shown in Figure 7, the problem is to get the equilibrium shape of a cable, whose support points are A(0.00,0.00) and B(1.00,0.00), and target point is C(0.50,0.35). Shown in the Table 1, it gives the first two steps of the adjusting process.

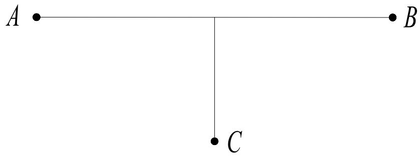


Figure 6: The problem of EX.3

1) Firstly, establish the initial structural model along line *AC* and line *BC* with five elements in each of them. The length of each element is the distance between the two nodes of it, which will not change after form-finding. By using the DR method explained in 1.1, it get the equilibrium structural form which has an error of ($\triangle_1$) with the target point.

2) Secondly, establish the initial structural model along line AC_1 and line BC_1 with same sets of Step.1, in which point C_1 is generated from point *C* with a displacement of (-$\triangle_1$). After form-finding by DR method, it get the equilibrium structural form which has an error of ($\triangle_2$) with the target point, which is much smaller than ($\triangle_1$).

3) In the following steps, it also adjusts the initial structural model as Step.2, and after few steps it gets the structural form across the target point with a very small error.

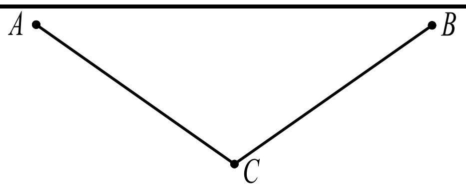
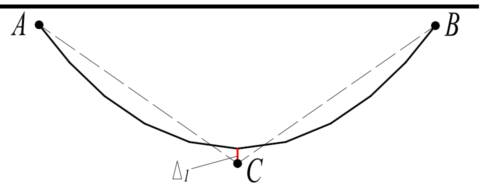
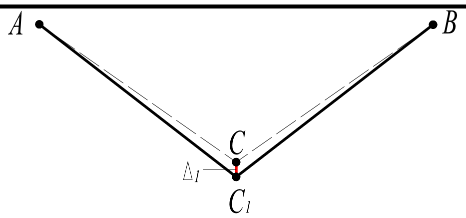
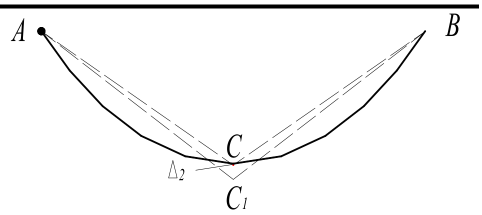
Steps	**Initial Structural Model**	**Equilibrium Structural Form**
Step. 1		
Step. 2		
•••	•••	•••

Table 1: The adjusting process of EX.3

In this section, it introduces basic theories or method used in this paper, and some simple examples are shown to illustrate the validity of each method. It can be seen clearly that the DR method can effectively solve the form-finding problem of hanging cable-net composed of unelongated cable elements, and can find the equilibrium structural form from any unbalanced state. And the global NURBS surface interpolation method is very convenient to describe the initial shape of the cable-net with several given fitting points. Moreover, the inverse iteration method is effective and efficient to get the target structural form by adusting the parameters of the initial structural system.

3. Numerical form-finding of edge-supported gridshells

In this section, numerical form-finding problem of edge-supported gridshells is discussed. Firstly, it uses global NURBS surface interpolation method to determine the initial cable-net which passes through several target points, and uses the DR method to get the equilibrium structural form of the cable-net. After then, it compares the equlibrium shape with the initial one, and when there is a large error (greater than the error required) between them, the inverse iteration method is introduced to adjust the coordinates of the fitting points, in order to find the specific initial shape of the cable-net which after form-finding can just right pass through the required target points.

Three examples are shown here to illustrate the presented method.

3.1. An edge-supported gridshell with 3×5 fitting points

Shown in Figure 8, it is the grid of fitting points, whose coordinates are shown in Table 2. It can be seen clearly the points in the boundary lines are in the same plane, and it takes points *A*, *B* and *C* as target points of the final equilibrium structural form.

$D_{i,0}$	$D_{i,1}$	$D_{i,2}$	$D_{i,3}$	$D_{i,4}$
(0,0,0)	(10,0,0)	(20,0,0)	(30,0,0)	(40,0,0)
(0,10,0)	***A* (10,10,8)**	***B* (20,10,5)**	***C* (30,10,8)**	(40,10,0)
(0,20,0)	(10,20,0)	(20,20,0)	(30,20,0)	(40,20,0)

Table 2: The coordinates of fitting points of EX.4

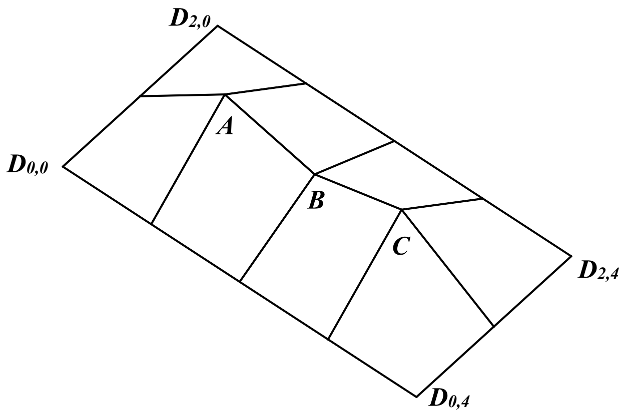


Figure 7: The grid of fitting points of EX.4

Using the global NURBS surface interpolation method, and giving the degree of (2,2) in the *u* and *v* direction, it can get a NURBS surface passing through all the fitting points and the initial cable-net with a grid of 20×40 cable elements, shown in Figure 9. The length of each element equals to the distance between the two nodes of it, which will not be elaongated after form-finding.

And then, with support points in all the four boundary lines, it uses DR method to find the equilibrium structural form of the initial cable-net, shown in Figure 10.

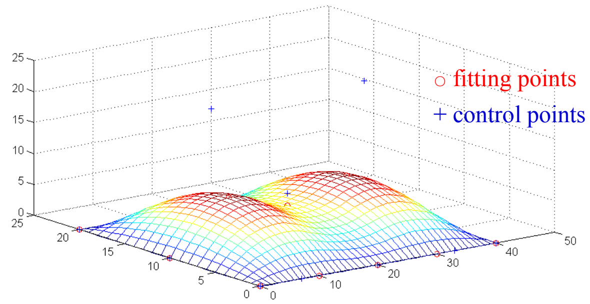


Figure 8: The initial structural model of EX.4

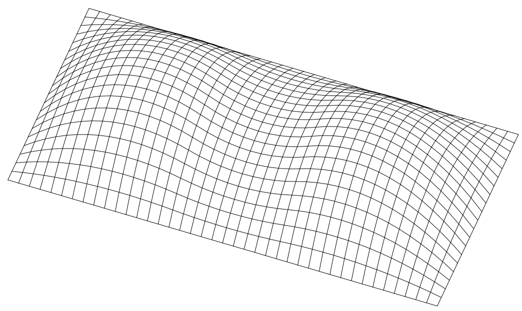


Figure 9: The equilibrium structural model of EX.4

It can easily get the vertical coordinates of the points *A*, *B*, *C* are 7.9999, 5.0002, 7.9999 respectively, which are very similar with the target points. And then it compares the equilibrium structural model (Figure 10) with the initial cable-net (Figure 9), and the result is shown in Figure 11. It can be seen that the two shapes are almost exactly the same (errors in all points are less than 0.3%).

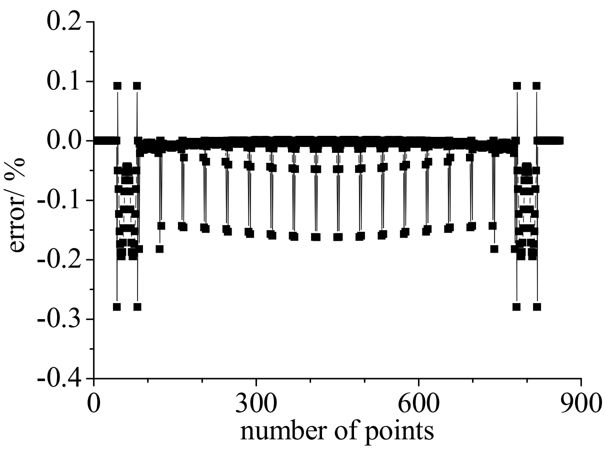


Figure 10: The comparison result of EX.4

3.2. An edge-supported gridshell with 4×5 fitting points

Shown in Figure 12, it is the grid of fitting points, whose coordinates are shown in Table 3. It can be seen clearly the points in the boundary lines are in the same plane, and it takes points *A*, *A′*, *B*, *B′*, *C*, *C′* as target points of the equilibrium structural form.

$D_{i,0}$	$D_{i,1}$	$D_{i,2}$	$D_{i,3}$	$D_{i,4}$
(0,0,0)	(10,0,0)	(20,0,0)	(30,0,0)	(40,0,0)
(0,7,0)	***A* (10,7,6)**	***B* (20,7,4)**	***C* (30,7,6)**	(40,7,0)
(0,13,0)	***A′* (10,13,6)**	***B′* (20,13,4)**	***C′* (30,13,6)**	(40,13,0)
(0,20,0)	(10,20,0)	(20,20,0)	(30,20,0)	(40,20,0)

Table 3: The coordinates of fitting points of EX.5

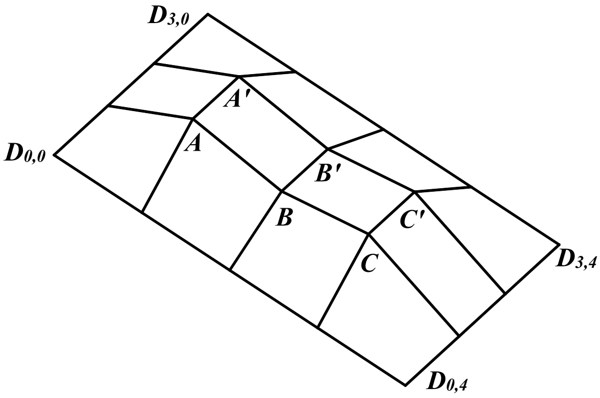


Figure 11: The grid of fitting points of EX.5

Using the global NURBS surface interpolation method, and giving the degree of (2,2) in the *u* and *v* direction, it can get a NURBS surface passing through all the fitting points and the initial cable-net with a grid of 20×40 cable elements, shown in Figure 13.

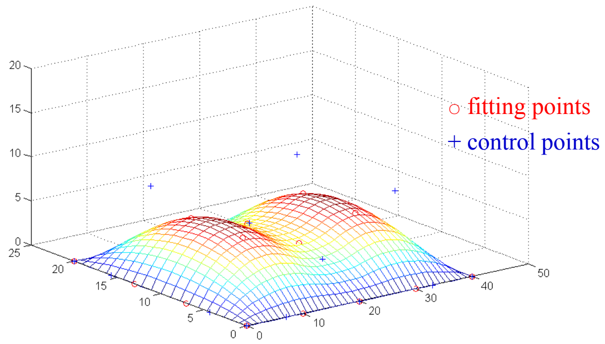


Figure 12: The initial structural model of EX.5

And then, with support points in all the four boundary lines, it uses DR method to find the equilibrium structural form of the initial cable-net, shown in Figure 14.

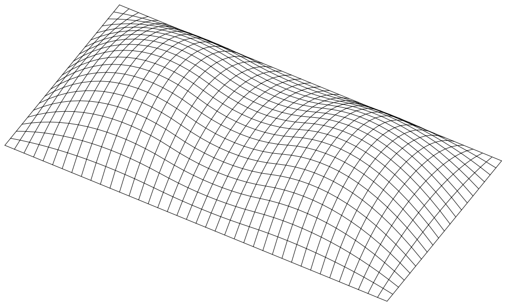


Figure 13: The equilibrium structural model of EX.5

It can easily get the vertical coordinates of points *A(A')*, *B(B')*, *C(C')* are 6.0257, 4.0607, 6.0257 respectively, which has much bigger errors with the target points. And it compares the equilibrium structural model (Figure 14) with the initial cable-net (Figure 13), and the result is shown in Figure 15. It can be seen that the two shapes have a very large difference.

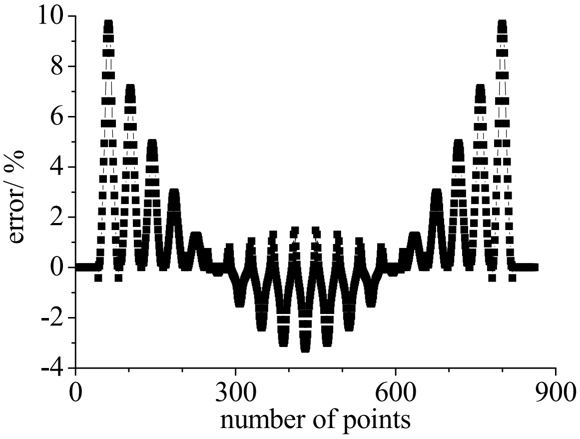


Figure 14: The comparison result of EX.5

And then it uses inverse iteration method to adjust the vertical coordinates of points *A(A')*, *B(B')*, *C(C')* to find the specific initial cable-net, which after form-finding can just right pass through the target points. Shown in Table 4, with a 3-step-adjustment when the coordinates of fitting points *A*, *A'*, *B*, *B'*, *C*, *C'* are (10,7,5.9743), (10,13,5.9743), (20,7,3.9405), (20,13,3.9405) , (30,7,5.9743), (30,13,5.9743), it can get the required structural model after a global NURBS surface interpolation and form-finding process by DR method. Shown in Figure 16, it is the finial equilibrium structural model which can pass through the target points with coordinates of (10,7,6), (10,13,6), (20,7,4), (20,13,4), (30,7,6), (30,13,6). And because of the edge-supports, also shown in Figure 16, there is a very small difference between the two equilibrium shapes.

Steps	Z-coordinate of point A(A')	Z-coordinate of point B(B')	Z-coordinate of point C(C')
Step 1	6.0000	4.0000	6.0000
Step 2	5.9743	3.9393	5.9743
Step 3	5.9746	3.9405	5.9746

Table 4: The adjusting process of EX.5

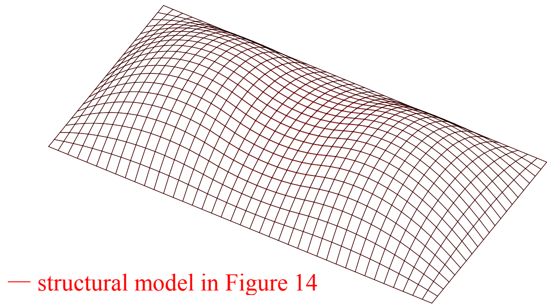


Figure 15: The final equilibrium structural model of EX.5

4. Numerical form-finding of point-supported gridshells

In this section, numerical form-finding problem of point-supported gridshells is discussed. It has the same procedure with Section 3, and just uses an example is shown here to illustrate.

Shown in Figure 17, it is the grid of fitting points, whose coordinates are shown in Table 5. It takes point $D_{0,0}$, $D_{2,0}$, $D_{0,2}$, $D_{2,2}$, $D_{0,4}$, $D_{2,4}$ as the support points, and takes points *A*, *B*, *C* as target points of the equilibrium structural form.

$D_{i,0}$	$D_{i,1}$	$D_{i,2}$	$D_{i,3}$	$D_{i,4}$
(0,0,0)	(10,0,2)	(20,0,0)	(30,0,2)	(40,0,0)
(0,10,5)	***A* (10,10,10)**	***B* (20,10,7)**	***C* (30,10,10)**	(40,10,0)
(0,20,0)	(10,20,2)	(20,20,0)	(30,20,2)	(40,20,0)

Table 5: The coordinates of fitting points of EX.6

Using the global NURBS surface interpolation method, and giving the degree of (2,2) in the *u* and *v* direction, it can get a NURBS surface passing through all the fitting points and the initial cable-net with a grid of 20×40 cable elements, shown in Figure 18. The length of each element also equals to the distance between the two nodes of it, which will not be elaongated after form-finding.

And then, with the six support points, it uses DR method to find the equilibrium structural form of the initial cable-net, shown in Figure 19.

It can easily get the vertical coordinates of points *A*, *B*, *C* are 10.7547, 7.1286, 10.7547 respectively, which have very large errors with the target points. And then it uses inverse iteration method to adjust the vertical coordinates of points *A*, *B*, *C* (and other fitting points are not changed) to find the specific initial cable-net, which after form-finding can just right pass through the target points.

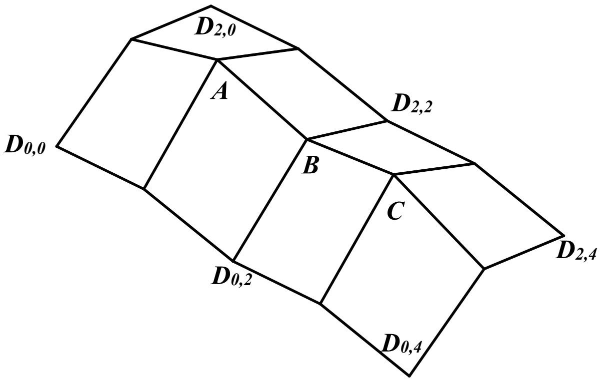


Figure 16: The grid of fitting points of EX.6

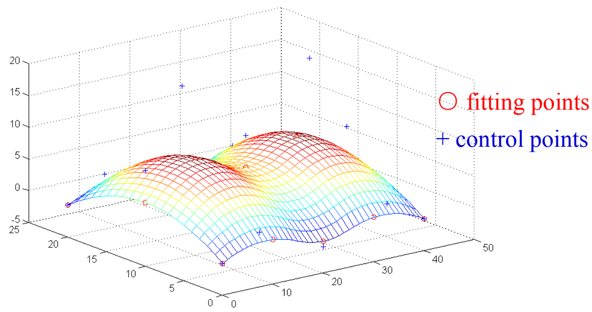


Figure 17: The initial structural model of EX.6

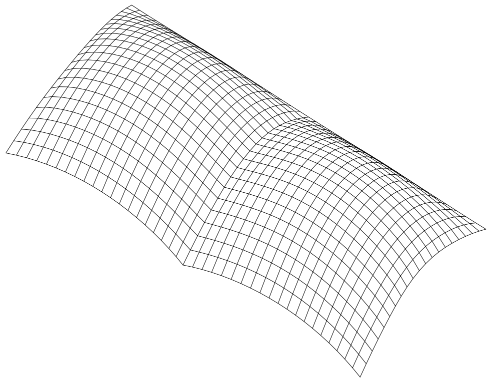


Figure 18: The equilibrium structural model of EX.6

Shown in Table 4, with a 3-step-adjustment when the coordinates of fitting points A, B, C are (10,10,9.1407), (20,10,6.8782), (30,10,9.1407), it can get the required structural model after a global NURBS surface interpolation and form-finding process by the DR method.

Steps	Z-coordinate of point A(A')	Z-coordinate of point B(B')	Z-coordinate of point C(C')
Step 1	10.0000	7.0000	10.0000
Step 2	9.2453	6.8732	9.2453
Step 3	9.1535	6.8772	9.1535
Step 4	9.1407	6.8782	9.1407

Table 6: The adjusting process of EX.6

Shown in Figure 20, it is the finial equilibrium structural model which can pass through the target points with coordinates of (10,10,10), (20,10,7), (30,10,10). And it can be seen clearly that there is a very large difference between the two equilibrium shapes.

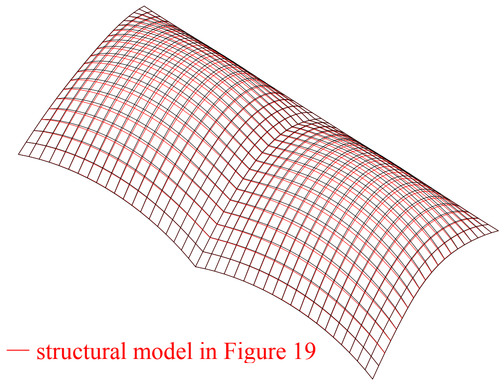


Figure 19: The final equilibrium structural model of EX.6

5. Conclusions and prospects

By using Dynamic Relaxation method and the NURBS technique, this paper discussed the numerical form-finding problems of gridshell structures which are generated from hanging-chain models. And it also introduces the inverse iteration method to find the specific initial structural system, which after form-finding can just right pass through the target points. The main conclusions are:

1) It presents an conceptual scheme of numerical form-finding method, and on this basis, a practical methodology of numerical form-finding problems is proposed.

2) It presents a form-finding method for edge-supported gridshells and point-supported gridshells. It uses global NURBS surface interpolation method to describe the initial cable-net, uses the Dynamic Relaxation method to find the equilibrium structural form, and introduces the inverse iteration method to adjust the fitting points of NURBS surface in order to generate a equilibrium structural form with several target points.

3) For a B-spline surface (a NURBS surface with weights of 1) with $(m+1)\times(n+1)$ fitting points and a degree of (2,2) in the u and v direction respectively, when i or j equals to 2 and all the middle fitting points are in the same orientation, it is almost exactly the same as the hanging equilibrium surface which contains the same fitting points .

By using the method proposed in this paper, it can describe an initial structural form with several fitting points by global NURBS surface interpolation, can do form-finding of geometrically unstable systems by the Dynamic Relaxation method, and can adjusting the coordinates of the fitting points of the NURBS surface to generate an equilibrium structural form with several target points. And it can be foresaw clearly that much more complex structural forms with multiple control points can be generated by this method, which takes advantages of both the hanging models and modern computer technique.

References

[1] Tomlow J. Gaudí’s reluctant attitude towards the inverted catenary. Engineering History and Heritage. 2011; 164(4): 219 - 233.

[2] Chilton J. The Engineer's contribution to contemporary architecture: Heinz Isler. London: Thomas Telford Publishing; 2000.

[3] Nerdinger W., With I.C., Meissner I., Möller E., Grdanjski M., editors. Frei Otto, Complete Works: Lightweight Construction, Natural Design. Birkhäuser Verlag für Architektur, Basel, Boston, Berlin, Architekturmuseum der Technischen Universität München; 2005.

[4] Saitoh M. Story of space and structure structural design’s future. Translated by Ji X, Xu H. Beijing: China Architecture ＆ Building Press; 2003. (in Chinese)

[5] Mungan I, Abel JF, editors. Fifty years of progress for shell and spatial structures. Essex: Multi-Science Publishing Co., Ltd; 2009.

[6] Adriaenssens, S., Block, P., Veenendaal, D., Williams, C., editors. Shell structures for architecture: form-finding and optimization. London: Routledge Taylor and Francis; 2014.

[7] http://en.wikipedia.org/wiki/Finite_element_method

[8] Schek H.J. The force density method for form-finding and computations of general networks. Computer Methods in Applied Mechanics and Engineering. 1974; 3; 115-134.

[9] Day A.S. An introduction to dynamic relaxation. The Engineer. 1965; 29(2); 218-221.

[10] Barnes M.R. Form-finding and Analysis of Tension Structures by Dynamic Relaxation[J]. International Journal of Space Structures. 1999; 14(2); 89-104.

[11] Ting E. C., Shih C., Wang Y. K. Fundamentals of a vector form intrinsic finite element: Part I. Basic procedure and a plane frame element [J]. Journal of Mechanics. 2004; 20(2); 113-122.