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Performance assessment of interpolation methods for orbits of altimetry satellites

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Abstract

Global and regional sea level variations are important indicators of climate change and are derived from accurate sea surface height measurements and precisely determined orbits of altimetry satellites. To validate and improve the quality of these orbits, comparisons with external solutions are important. Since orbit solutions of different institutions are not necessarily provided at the same time instants, interpolation is required for comparison. In this study, we investigate the appropriate interpolation method and its degree to reduce interpolation errors to sub-millimetre levels. We also assess the magnitude of errors occurring at transformations when expressing orbit differences not only in the terrestrial reference frame (Cartesian coordinates), but also in local orbital and ellipsoidal coordinates. The analyses conducted in this study provide good results for Hermite interpolation of degrees 7–11 and Newton interpolation of at least degree 9 with a three-dimensional interpolation error of 0.6 mm and a scattering of 0.2 mm on average for satellite coordinates given with an accuracy of 1 mm in the SP3 format. These interpolation settings limit transformation errors between coordinate systems to ±0.01 mm and incorrect mapping of interpolation errors into certain components in the target system to ±0.02 mm. The spectral analysis of orbit differences is affected up to 0.1 mm in magnitude with appropriate interpolation settings. Extending the number of decimal digits of the satellite position and velocity in SP3 files by one digit benefits the orbit comparisons and reduces the interpolation error by 90% from 0.6 to 0.06 mm. The results are obtained using piece-wise interpolation and a validity interval inside the interpolation interval to minimise the effects of the Runge phenomenon.

Keywords Orbit interpolation, Altimetry satellite orbits, Jason-2, Hermite, Lagrange, Newton, Cubic spline, Interpolation methods

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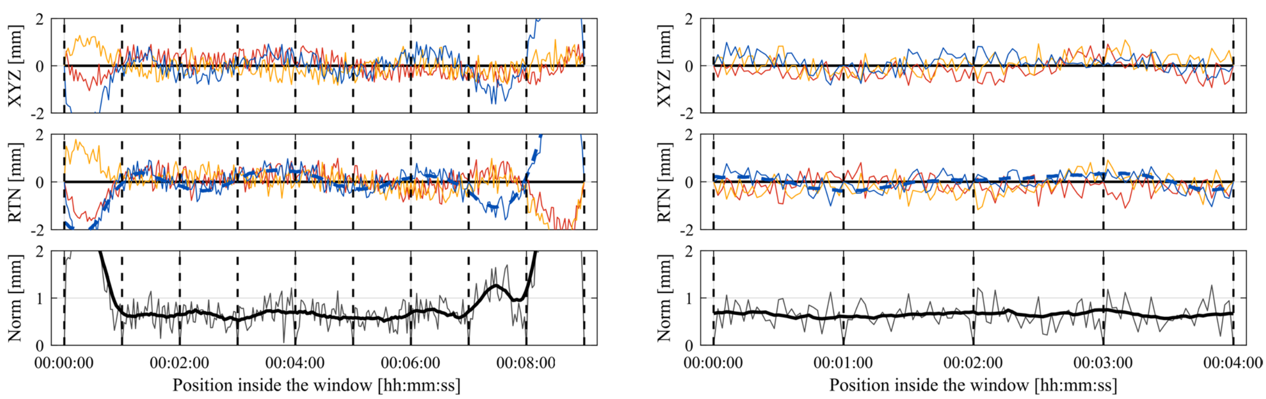
Graphical Abstract



1 Introduction

Precise orbits of altimetry satellites are the basis for investigations related to global, regional, and coastal sea level heights and support the determination of sea surface variations to reliably monitor and interpret climate change phenomena. The precise orbit determination (POD) of altimetry satellites at the sub-centimetre level is a prerequisite for these investigations, since it provides accurate positions of an altimetry satellite radar in space from which the distance to the water surface is measured. The analysis of satellite altimetry measurements and the POD of these satellites underwent significant progress in the last 30 years (International Altimetry Team 2021). Based on an analysis of altimetry observations between January 1993 and December 2021, the global mean sea level rise is quantified to be 3.3 ± 0.3 mm/year, and its acceleration is 0.12 ± 0.05 mm/year^2 (Guérou et al. 2023). Orbit errors, however, still remain the main contributors to regional sea level errors (Prandi et al. 2021). Contemporary requirements to sea level investigations require centimetre and even sub-centimetre accuracy of satellite positions (Meyssignac et al. 2023). The accuracy of state-of-the-art altimetry satellite orbits based on observations of the space-geodetic techniques GNSS (Global Navigation Satellite System) and DORIS (Doppler Orbitography and Radiopositioning Integrated by Satellite) is at the level of 0.4–1.0 cm for Jason and Sentinel-3 satellites (Rudenko et al. 2023).

For further improvement of the satellite orbit quality and many applications, such as a careful quantification of the radial orbit error (Rudenko et al. 2023), analyses of different spacecraft attitude approaches (Bloßfeld et al. 2020; Zeitlhöfler et al. 2024), and the development of correction models and their respective influences on the orbits (Zeitlhöfler et al. 2023), a comparison and validation of internal orbit solutions with those of other institutions is important. One possibility to exchange satellite orbits is the well-established Standard Product 3 (SP3) format (Hilla 2010), which provides, amongst other optional information, the satellite position and velocity with millimetre accuracy. For orbit comparisons between different institutions, one has to consider specific orbit parameters like the time scale, reference frame, or the orbit step size (usually 30 or 60 s for altimetry satellite orbits). In most cases—especially when orbits are given in different time scales like Coordinated Universal Time (UTC), International Atomic Time, or Terrestrial Time—the orbits to be compared are not provided at the same epochs. This requires interpolating the respective satellite positions (and velocities) with sufficient accuracy. In order to obtain profound orbit differences originating in the use of various up-to-date background POD models and to properly quantify the impact of orbit errors on the global and regional mean sea level at the millimetre level, one needs to limit the orbit interpolation error to well below one millimetre. Furthermore, when comparing orbits in coordinate systems other than those provided in the SP3 file, transformations are used which introduce uncertainties in the form of transformation errors.

Comparative studies within the International GNSS Service focus on appropriate methods for interpolating GNSS ephemeris data, which is mainly provided at a temporal resolution of 15 min. Neta et al. (1996), Feng and Zheng (2005), and Song et al. (2021) discuss polynomial methods as well as cubic splines and trigonometric functions and obtain an interpolation accuracy at the 1 cm level. Schenewerk (2003) reviews polynomial and trigonometric interpolators for GNSS ephemeris with respect to their strengths and weaknesses. The mitigation of discontinuities at the day boundary is discussed in Song et al. (2021).

The orbits of altimetry satellites, however, are generally given with a smaller step size and require higher ((sub-)millimetre) interpolation accuracy than those investigated in the studies mentioned before. Therefore, a separate investigation of interpolation methods for altimetry satellite orbits is important. Realistically quantifying the interpolation errors allows the differentiation between orbit differences caused by interpolation errors, transformation errors, and geophysical signals. In the POD of altimetry satellites, the user is primarily interested in the latter.

In this paper, we investigate the suitability of common interpolation approaches and analyse which methods fulfil the high demands of (sub-)millimetre accuracy for the interpolation of altimetry satellite orbits. Furthermore, we relate the magnitude of interpolation and transformation errors to orbit differences caused by different POD approaches. We focus on the transformations between the terrestrial reference frame (TRF), in which most orbits are given, and the orbital system and ellipsoidal coordinates. The transformation between the terrestrial and celestial reference frames is not analysed.

This paper is structured as follows: Sect. 2 provides an overview of investigated interpolation methods and Sect. 3 describes strategies for an effective and accurate interpolation result. Section 4 describes the interpolation experiments carried out in the study. The results on interpolation errors and the spectral analysis of orbit differences are presented in Sect. 5. Finally, conclusions and recommendations of our study are given in Sect. 6.

2 Overview of investigated interpolation methods

The choice of the optimal interpolation method depends on several factors, such as the required level of accuracy, the orbit step size, and the computational complexity. An advantage of the cubic spline interpolation method is its computational efficiency (de Boor 1978). A major drawback is its limited interpolation accuracy, which is analysed in the case of GNSS orbits in Neta et al. (1996), Yousif and El-Rabbany (2007), and Song et al. (2021). As discussed in Alkahal (2023), using the spline interpolation results in an interpolation error at a centimetre level in certain components. A possibility to resolve this issue is using the so-called middle-point approach (Alkahal 2023), in which both orbit solutions to be compared must have the same step size (resulting in the same magnitude of interpolation error) and both solutions are interpolated to time instants in between the orbit epochs. Despite the reduced interpolation errors by applying this approach in the case of spline interpolation, it is still of insufficient accuracy. Thus, the cubic spline method is not an option for the high demands required at the interpolation of altimetry orbits and is not further discussed within this study. Since orbit ephemeris data are close to periodic, Neta et al. (1996) applied trigonometric polynomial interpolation but concluded that it is too computationally expensive for practical use. The Chebyshev fitting method, as demonstrated by Song et al. (2021), provides good results. The limitations of this method are the larger computation time compared to other polynomial methods and a slightly degraded accuracy in the Up component (Wang et al. 2018), which is of particular importance for altimetry satellites.

Due to these reasons, we focus on Lagrange, Newton, and Hermite polynomial interpolation methods. This section briefly introduces and gives an overview of the methods. Detailed formulae are provided in Burden and Faires (2010).

2.1 Lagrange interpolation method

The Lagrange interpolation polynomial of degree n is formulated by Burden and Faires (2010) as follows. For the n + 1 distinct points $x_0, x_1, \ldots, x_n$ and their corresponding values, which are expressed by function f, a unique polynomial function $P(x)$ with degree n exists:

$$f(x_k) = P(x_k), \quad \text{with } k = 0, 1, \ldots, n. \tag{1}$$

The polynomial is written as

$$P(x) = \sum_{k=0}^{n} f(x_k) L_{n,k}(x), \tag{2}$$

where $L_{n,k}(x)$ is the basis function of degree n formulated as

$$L_{n,k}(x) = \prod_{i=0, i\neq k}^{n} \frac{x - x_i}{x_k - x_i}, \tag{3}$$

which satisfies

$$L_{n,k}(x_j) = \prod_{i=0, i\neq k}^{n} \frac{x_j - x_i}{x_k - x_i} = \delta_{ij} = \begin{cases} 1 & \text{if } i = j, \\ 0 & \text{if } i \neq j. \end{cases} \tag{4}$$

δ_{ij} is the Kronecker delta.

2.2 Newton interpolation method

In the case of equally spaced points $x_0, x_1, \ldots, x_n$, i.e. $x_i = x_0 + sh$ with $s = 0, 1, \ldots, n$ and $h = x_{i+1} - x_i$, which is given in our case for orbit position and velocity, Newton's forward divided difference formula is suitable for interpolation. According to Burden and Faires (2010), the polynomial in this case is

$$P(x) = P(x_0 + sh) = f(x_0) + \sum_{k=1}^{n} \binom{s}{k} \Delta^k f(x_0), \quad (5)$$

using the binomial-coefficient

$$\binom{s}{k} = \frac{s(s-1)\cdots(s-k+1)}{k!}, \quad (6)$$

and the forward difference notation Δ according to Aitken's Δ^2 method (Aitken 1927).

The Newton interpolation method is more efficient than the Lagrange method, since it does not perform recurrent calculations of the polynomial when new points are considered. One can only add a new term for each new point (Neta et al. 1996). However, since only one interpolation polynomial exists for a chosen set of points, the Lagrange and Newton methods yield the same results.

2.3 Hermite interpolation method

Since SP3 files contain the orbit velocity, i.e. the first derivative of the orbit position, this information can also be used for interpolation. The Hermite interpolation is known for its capability to include derivatives of the original function for the interpolation (Burden and Faires 2010).

The Hermite polynomial considering values of the function f and its first derivative f' is of up to degree $2n+1$ and given as

$$H_{2n+1}(x) = \sum_{k=0}^{n} f(x_k) H_{n,k}(x) + \sum_{k=0}^{n} f'(x_k) \hat{H}_{n,k}(x). \quad (7)$$

The functions $H_{n,k}(x)$ and $\hat{H}_{n,k}(x)$ use the Lagrange basis functions and are defined by

$$H_{n,k}(x) = \left[1 - 2(x - x_k) L'_{n,k}(x_k)\right] L^2_{n,k}(x) \quad (8)$$

and

$$\hat{H}_{n,k}(x) = (x - x_k) L^2_{n,k}(x). \quad (9)$$

$L_{n,k}(x)$ is the Lagrange basis function (Eq. 3) and $L'_{n,k}(x)$ is its first derivative:

$$L'_{n,k}(x) = L_{n,k}(x) \sum_{m=0, m \neq k}^{n} \frac{1}{x - x_m}. \quad (10)$$

Compared to the previously explained methods, an advantage of the Hermite interpolation is the smaller number of points required to compute the function of the same degree (Zheng and Zhang 2020). For interpolation of degree 5, Newton/Lagrange interpolation require six points, whereas Hermite interpolation only requires three, since one data point contributes both the function and first derivative values. In addition, Hermite interpolation experiences less impact on the interpolated results due to effects related to the Runge phenomenon (for details, see the next section).

3 Strategies for effective and accurate interpolation results

One disadvantage of polynomial interpolation methods is the high computational effort, especially for polynomials of high degrees. Another disadvantage—and this is more distinct in the case of equidistant spacing of supporting points—are the large differences between the interpolation polynomial and the original function (de Boor 1978). These interpolation errors scale with increasing polynomial degrees and are largest at both ends of the interpolation interval. This effect is known as the Runge phenomenon (Dahlquist and Björck 2008). In contrast to large values at the interval edges, the interpolation errors remain small in the central part. The application of piece-wise polynomial interpolation resolves both limitations. Another measure, the choice of Chebyshev points (de Boor 1978), is not possible in our case, since we have an equidistant, preset spacing of supporting points.

Figure 1 illustrates the piece-wise interpolation approach and how it is used in this study. The orbit information in the form of position and velocity is provided at equidistant supporting points and is temporally separated by a particular number of seconds (step size). For the interpolation of a certain point, a subset of supporting points is selected in that way to place the interpolation point in the central part of the interval and to satisfy the demanded number of points the interpolation degree requires. This selection of points is called an interpolation interval or window. The window's central part is the so-called validity interval (coloured area of the respective window in Fig. 1). The validity intervals of adjacent windows adjoin each other but do not overlap. All interpolation points located within one validity interval are interpolated using the same interpolation polynomial expressed by the supporting points which form the window. After processing the interpolation points, the window is moved forward, and the next interpolation points are processed. This approach is called moving window (or "Walk-along interpolation" in other literature; Feng and Zheng 2005). As discussed in Horemuž and Andersson (2006), the size of the validity interval can be extended depending on the interpolation method and degree.

4 Experiments on interpolation accuracy

This section briefly introduces each of the six experiments carried out to determine suitable methods and settings for highly accurate interpolation, assess interpolation and transformation errors, and analyse the consequences of such errors on the interpretation of orbit differences.

The basis for most experiments described hereafter is a 3-day covering orbital arc of Jason-2 with a temporal resolution of one second. This orbit is the reference orbit. The corresponding SP3 file contains the orbit data (position and velocity) in the TRF in Cartesian coordinates (XYZ) and in the time scale UTC. The orbit is computed using DGFI-TUM's POD software DOGS-OC (DGFI-TUM Orbit and Geodetic Parameter Estimation Software – Orbit Computation; Bloßfeld 2015) using satellite laser ranging observations and similar options as listed in Zeitlhöfler et al. (2023). Based on this reference orbit, three non-reference orbits with temporal resolutions of 30, 60, and 120 s are created by choosing the respective data.

Figure 2 illustrates the processing steps, concomitant data, and experiments related to the following investigations. The top box represents the reference (RO) and non-reference (NO) orbits, which are given in the TRF in Cartesian coordinates. First, orbit positions are interpolated to the epochs being in the middle of each non-reference solution using the orbit information of the respective non-reference solution, i.e. the time instants of the first non-reference orbit are 00:00:00, 00:00:30, 00:01:00, ..., and the interpolated epochs are 00:00:15, 00:00:45, 00:01:15, ... in the format hh:mm:ss. The interpolation methods used are Lagrange, Newton, and Hermite of different degrees. The other non-reference solutions are treated in the same theoretical manner. The interpolated orbit position is then compared to the (error-free) position of the reference orbit resulting in three-dimensional (3D) orbit differences expressed in XYZ coordinates. Analysing these orbit differences implies which interpolation method and polynomial degree are required to obtain sufficient interpolation accuracy at the sub-millimetre level (Experiment 1).

Since the orbital system, which is composed of the radial, transverse, and normal (RTN) directions, is the usual frame to express and compare orbit differences of altimetry satellites, we transform the component-wise XYZ orbit differences ($\boldsymbol{\Delta}_{\mathrm{XYZ}}$) into component-wise RTN orbit differences ($\boldsymbol{\Delta}_{\mathrm{RTN}}$) using the formulae

$$\boldsymbol{\Delta}_{\mathrm{RTN}} = \boldsymbol{R}_{\mathrm{XYZ}}^{\mathrm{RTN}} \cdot \boldsymbol{\Delta}_{\mathrm{XYZ}}, \tag{11}$$

where $\boldsymbol{R}_{\mathrm{XYZ}}^{\mathrm{RTN}}$ is the rotation matrix from the Cartesian to the local orbital coordinate system

$$\boldsymbol{R}_{\mathrm{XYZ}}^{\mathrm{RTN}} = \begin{bmatrix} \boldsymbol{e}_1^{\mathrm{T}} \\ \boldsymbol{e}_2^{\mathrm{T}} \\ \boldsymbol{e}_3^{\mathrm{T}}, \end{bmatrix}, \tag{12}$$

which is according to Montenbruck and Gill (2000) composed of the unit vectors

$$\boldsymbol{e}_1 = \frac{\boldsymbol{r}}{|\boldsymbol{r}|}, \quad \boldsymbol{e}_2 = \boldsymbol{e}_3 \times \boldsymbol{e}_1, \quad \boldsymbol{e}_3 = \frac{\boldsymbol{r} \times \boldsymbol{v}}{|\boldsymbol{r} \times \boldsymbol{v}|}. \tag{13}$$

The vectors $\boldsymbol{r}$ and $\boldsymbol{v}$ contain the orbit position and velocity of the reference solution in the TRF, respectively. Thus, the orbit differences in RTN are expressed in the orbital system of the reference solution.

A third type of coordinates for the interpretation of orbit differences are ellipsoidal coordinates (latitude φ, longitude λ, and height h). While we transformed the orbit differences themselves in the previous case, this approach is based on the transformation of orbit positions from Cartesian to ellipsoidal coordinates (https://gssc.esa.int/navipedia/index.php/Ellipsoidal_and_Cartesian_Coordinates_Conversion; latest access: 11 October 2024) and subsequent computation of orbit differences in ellipsoidal coordinates ($\boldsymbol{\Delta}_{\varphi\lambda h}$).

To detect possible errors or systematics in both transformation approaches, we determine the norm and statistical values of orbit differences expressed in the three coordinate systems and compare the values before and after the transformations (Experiment 2).

In Experiments 1 and 2, the interpolation epoch is located exactly in the middle of its corresponding validity interval. In reality, this situation is not necessarily given, but the position can vary within the validity interval. In order to determine, on the one hand, whether the validity interval can be extended and according to which general rule, and on the other hand, whether there arise certain systematics inside the interpolation interval for different interpolation methods and degrees, we interpolate an orbit position at each second within the interpolation window in Experiment 3. Besides analysing interpolation errors in Cartesian coordinates, we transform the orbit differences into the orbital system to evaluate possible patterns in that frame.

Apart from analysing the solutions' orbit differences themselves, the spectral analysis of the differences is also of particular interest, since it reveals systematic long-term and short-term signals in the data. In Experiment 4, we use the Lomb–Scargle periodogram (Lomb 1976; Scargle 1982) to determine any impact of the interpolation and transformation on the results of the spectral analysis. This analysis is based on the three non-reference orbits derived from the 1-s orbit and focuses on the orbit differences in the Cartesian and orbital coordinate

Table 1 Average (avg) and standard deviation (std) values of three-dimensional orbit differences in Cartesian coordinates (XYZ)

	Degree	Temporal step size of the reference and non-reference solutions					
		≤ 30 s		60 s		120 s	
		avg	std	avg	std	avg	std
Newton	5	0.61	0.23	**1.18**	**0.36**	**66.62**	**3.66**
	6	0.62	0.24	0.62	0.23	**3.88**	**0.96**
	7	0.62	0.24	0.61	0.23	**0.83**	0.40
	8	0.63	0.24	0.62	0.23	0.76	0.35
	9+	0.62	0.24	0.62	0.23	0.72	0.31
Hermite	5	0.57	0.21	0.57	0.21	**2.72**	**0.39**
	7+	0.56	0.21	0.56	0.20	0.57	0.21
	7+[a]	0.06	0.02	0.06	0.02	0.07	0.03
	7+[b]	0.01	0.01	0.03	0.01	0.05	0.02

The values are based on using the Newton and Hermite interpolation methods of different degrees and different temporal step sizes of the orbits. The plus sign indicates similar results for next-higher degrees. The unit is millimetre

[a]Seven instead of six (default) decimal digits in the SP3 file; [b]Eight instead of six (default) decimal digits in the SP3 file

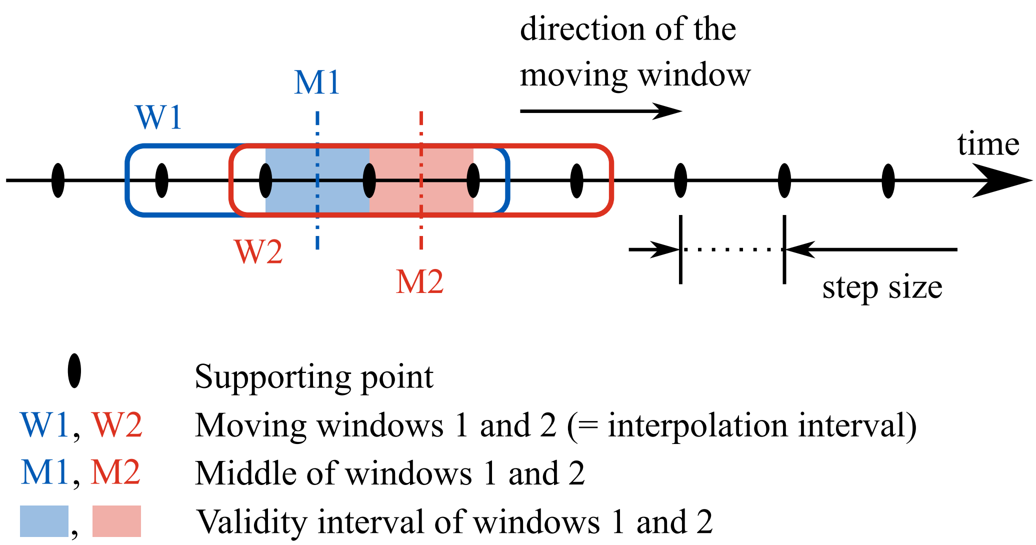


Fig. 1 Illustration of the moving window approach. Interpolation points are interpolated in that window, whose validity interval they are located in. This example refers to Hermite interpolation of degree 7 and/or Newton interpolation of degree 3

systems. This enables to examine whether specific interpolation setups create artificial oscillations in the orbit differences. As in Experiments 1 and 2, the moving window approach is used.

Since the orbit information is given at the 1-mm level in SP3 files (six decimal digits), the interpolation accuracy might stagnate at a certain level and cannot be improved by choosing other interpolation settings. In Experiment 5, we increase the number of decimal digits from six to seven and eight and examine whether these changes benefit the interpolation results. Therefore, we use the non-reference solutions with temporal resolutions of 60 and 120 s.

After the analysis of artificial solutions based on the 1-s orbit, we relate the interpolation and transformation errors to the magnitude of orbit differences based on real data in Experiment 6. Therefore, we compare two orbits of Jason-2 computed at different institutions. The first solution is provided by the French space agency Centre National d'Études Spatiales (CNES) and is based on the Precise Orbit Ephemeris-F (POE-F) orbit standards (ftp://ftp.ids-doris.org/pub/ids/data/POD_configuration_POEF.pdf; latest access: 11

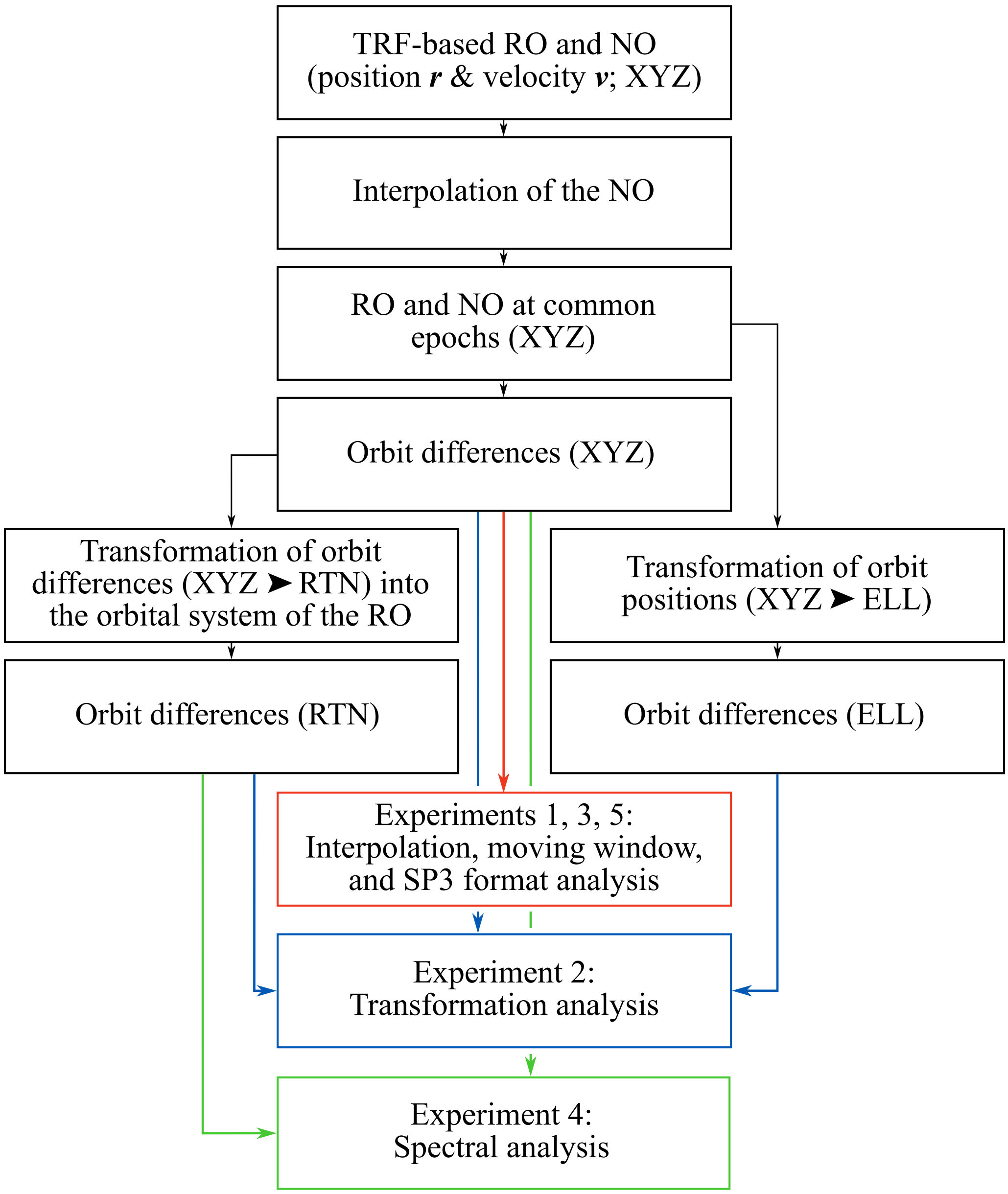


Fig. 2 Diagram of processing steps and experiments to investigate the interpolation accuracy of altimetry satellite orbits. RO and NO refer to the reference and non-reference orbits, respectively. XYZ are Cartesian coordinates, RTN the orbital system, and ELL ellipsoidal coordinates

October 2024). The second solution is computed at DGFI-TUM using DOGS-OC with a similar setup of physical background models. However, some models differ, e.g. POE-F uses the Finite Element Solution 2014 (FES2014) ocean tide model (Lyard et al. 2021), while DGFI-TUM uses the empirical ocean tide model (EOT11a; Savcenko and Bosch 2012). The resulting orbit differences are a combination of all computation setups realised differently for both solutions, e.g. different background models, orbit integration approaches, treatment and choice of observations, and other aspects. Both orbits cover the manoeuvre-free period between 21 December 2008 and 3 May 2009 and have a step size of 60 s. Based on the findings of the preceding experiments, we use Hermite interpolation of degree 7 applying piece-wise interpolation.

5 Results

This section summarises the results obtained in the Experiments 1–5 and in the comparison of the DGFI-TUM and CNES orbits (Experiment 6).

5.1 Experiment 1: Interpolation accuracy

Experiment 1 aims to assess the interpolation method, in combination with its polynomial degree, necessary to obtain precisely interpolated orbits. Table 1 shows the results within this experiment for the Newton and Hermite interpolation methods using degrees 5 to 9+ and 5 and 7+, respectively. The plus sign indicates similar results for next-higher degrees. The Lagrange interpolation method is not listed, since it produces, due to the uniqueness of the interpolation polygon inside an interpolation interval, results identical to the Newton method, which is computationally more efficient than Lagrange.

The orbit differences between the reference orbit and the interpolated non-reference orbit depend on the following factors: the step size of the interpolating orbit, the choice of the interpolation method, and the choice of its degree. The analysis of the 3D orbit differences using the standard deviation and average values allows to express the amplitude of scattering and possible offsets, respectively.

We first focus on all rows except the last two in Table 1. For an orbit step size of up to 30 s, Newton and Hermite interpolation provide similar results for all degrees with slight advantage for the Hermite method. The scattering is about 0.2 mm and the average 3D interpolation error is about 0.6 mm. The results are in the same range for an orbit step size of 60 s with an exception for the Newton interpolation of degree 5. This degree is insufficient for orbit ephemeris data separated by 60 s, since it introduces larger interpolation errors and increased scattering (bold values). This effect even intensifies for a step size of 120 s, where more than 3.6 mm scattering and a 3D orbit error of more than 66 mm on average arise. Degree 5 is also insufficient for Hermite interpolation and results in 0.4 mm scattering and 2.7 mm interpolation error on average. With degrees 7+ for Hermite and 8+ for Newton, the results are in similar ranges as for the other step sizes.

The investigations indicate that degree 8 for the Newton interpolation and degree 7 for Hermite interpolation are suitable for reaching sub-millimetre accuracy when comparing satellite orbits of up to 120 s step size.

5.2 Experiment 2: Transformation accuracy

After investigating the choice of interpolation settings, we highlight the uncertainties of expressing orbit differences in other coordinate systems via transformations in Experiment 2. Whereas the orbit differences were analysed in the 3D space in the previous experiment, we now focus on the individual components. These are the directions in the TRF (XYZ), the components in the orbital system (RTN), and ellipsoidal coordinates ($h\lambda\varphi$).

Despite the transformation of either the orbit differences from Cartesian into local orbital coordinates or the transformation of Cartesian orbit positions into ellipsoidal coordinates with subsequent computation of orbit differences, the norm of the orbit differences has to be identical and preserved in all three systems. This is independent of the interpolation method and degree. To verify this necessity, we compare the norm of orbit differences after applying Hermite interpolation of degree 7 to orbits with a temporal resolution of 60 s. Since the orbits used are still based on the 1-s solution, the resulting interpolation error is zero in an ideal case.

Figure 3 displays in the top row the norm of the orbit differences, i.e. the interpolation error, in Cartesian coordinates and its corresponding histogram for a period of three days. Most values are between 0.25 and 0.85 mm with an average of about 0.6 mm (this magnitude of error has already been determined in Experiment 1, cf. Table 1). The blue- and red-coloured dots in the bottom plot express the deviations of the norm in the local orbital and ellipsoidal coordinates from the black-coloured values, respectively. In other words, it is the transformation error. The histograms of both approaches are very similar and show a maximum transformation error of about 0.01 mm, which is two orders smaller than the interpolation error and certifies a negligible transformation error.

With verified preservation of the 3D orbit differences during transformations, we analyse possible redistributions of interpolation errors to individual components due to transformations. To show the consequences of inappropriate and appropriate interpolation settings on individual components, we use Newton interpolation of degree 5 with temporal resolutions of 120 and 30 s. This is of special interest for the radial direction, since it is the most important for satellite altimetry and is directly related to altimeter measurements and the derived products. Figure 4a shows the effects of using Newton interpolation with insufficient degree 5 and a step size of 120 s. The interpolation error in XYZ acts as an oscillation around the average value 0 with an amplitude of approximately 30–50 mm in each component. The 3D error is 66 mm (cf. Table 1). After the transformation into the orbital system and to ellipsoidal coordinates, the magnitudes of the oscillations reduce to approximately 5–20 mm, but a large offset of over 60 mm in absolute value arises in the radial and height directions. This means that an inappropriate interpolation degree introduces an error constantly acting in one direction

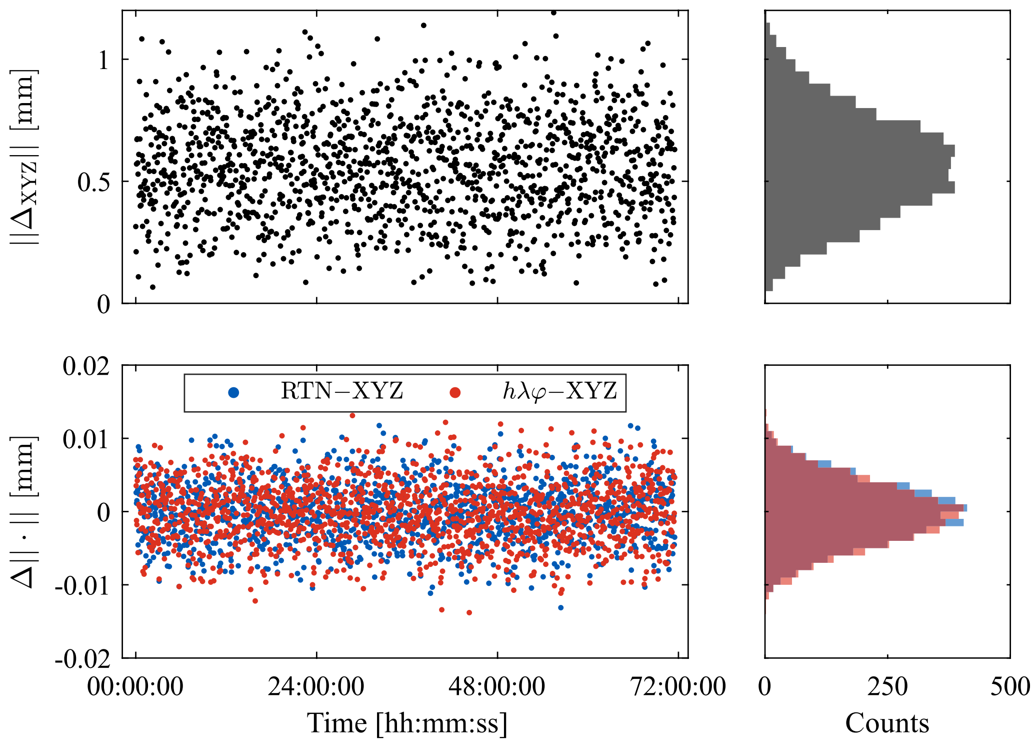


Fig. 3 Transformation results for Hermite interpolation of degree 7 applied to orbits of 60 s step size. Top: norm of orbit differences in Cartesian coordinates. In this case, the norms of differences are simultaneously the interpolation errors, since the values are zero for error-free interpolation. Bottom: deviation of the norm of orbit differences obtained after applying transformations into the local orbital coordinate system (RTN) and ellipsoidal coordinates ($h\lambda\varphi$). The plots on the right side are the histograms of the values on the left

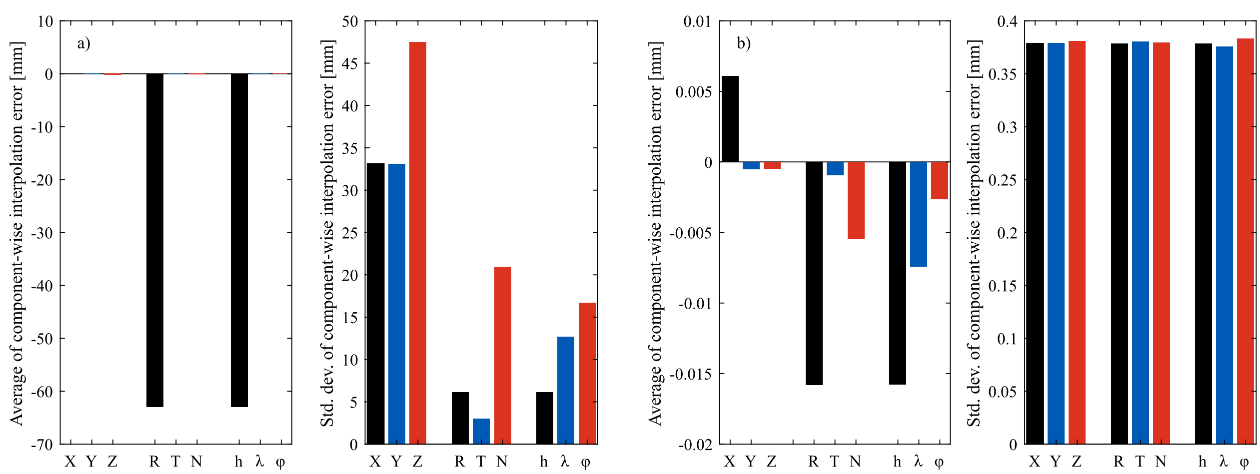


Fig. 4 Component-wise standard deviation and average values of the orbit differences in Cartesian coordinates (XYZ), the orbital system (RTN), and ellipsoidal coordinates (h, λ, φ). The interpolation method is Newton's divided differences, the polynomial degree is 5, and the orbit step size is **a** 120 s and **b** 30 s

and suggests an offset between two orbits which is actually not present. We want to stress that, depending on the interpolation method and its degree as well as the choice of supporting points, the component affected differs and is not always the radial direction. Using Newton interpolation of degree 5 with a step size of 30 s leads to clearly smaller values for both the component-wise averages and the standard deviations (cf. Fig. 4b). Also in this case, interpolation errors affect the radial and height components most, but now with just an offset of about −0.015 mm. This is accurate enough for plausible orbit comparisons. The similar values for the standard deviations of approximately 0.38 mm imply a homogeneously distributed interpolation error on each component.

5.3 Experiment 3: Runge phenomenon

In both previous experiments, the interpolation epoch is located in the middle of the interpolation interval. However, this is not the usual situation, but the requested epoch can be anywhere in the window in real-data comparisons. To assess the interpolation error depending on the position inside the interval, we interpolate an orbit position at each second for different interpolation methods, degrees, and step sizes. The left and right columns in Fig. 5 are based on step sizes of 120 and 60 s, respectively. Rows 1 and 3 contain results using Newton and rows 2 and 4 using Hermite interpolation. Figure 5a–d use degree 5 and e–h degree 9. The three panels in each subfigure show from top to bottom the component-wise orbit differences in Cartesian coordinates (XYZ), in the orbital system (RTN), and the 3D interpolation error with a smoothing moving mean. A major difference between the interpolation methods Newton and Hermite and common to all panels is the smaller number of supporting points required when using Hermite interpolation (rows 2 and 4).

We first discuss the left column of Fig. 5. As already shown in Experiment 1, Newton interpolation of degree 5 applied to an orbit of a temporal resolution of 120 s causes large interpolation errors (approximately 66 mm on average). The 3D interpolation error is—apart from supporting points, where the interpolation error is zero by nature—smallest at the epochs close to the middle of the interpolation interval (between 00:04:00 and 00:06:00 in Fig. 5a). Due to the Runge phenomenon, the errors increase the closer the interpolation epoch is to the limits of the window. As demonstrated in Experiment 2, the large disadvantage of an insufficient interpolation degree is the mapping of interpolation errors into a single component. This can affect the radial direction (blue curve in Fig. 5c), which is of utmost importance for altimetry missions. Both other directions are affected to a lesser extent. Using Hermite interpolation of degree 5 (Fig. 5c) clearly reduces the 3D interpolation error throughout the window from up to 350 to 3 mm. While in this case the largest errors occur in the Y component, the error distribution, in general, strongly depends on the choice of supporting points and shows no clear systematic pattern. In other windows, the X and Z components are most affected. However, the radial direction shows again the largest interpolation errors. As shown in Experiment 1, increasing the interpolation degree for both methods from 5 to 9 provides suitable results (Fig. 5e, g). Using degree 9 limits the interpolation error in all components and 3D to less than 1 mm in the middle of the interpolation interval. However, the Newton method again experiences the effects of the Runge phenomenon. Reducing the step size of supporting points from 120 to 60 s (right column of Fig. 5) significantly reduces the interpolation errors for degree 5. The average errors in the central part of the interval are for Newton and Hermite interpolation of degree 5 about 1 mm and 0.6 mm, respectively, and of degree 9 about 0.6 mm for both methods. These values coincide with those in Table 1.

This experiment shows that the interpolation error is relatively constant inside the interpolation window for Hermite interpolation of low degrees, whereas the Newton method is highly affected by the Runge phenomenon in all cases. However, the Hermite method also experiences the Runge phenomenon with increasing interpolation degrees. For temporal resolutions of 30, 60, and 120 s, the 3D interpolation errors increase to more than 1 mm at the window edges when using Hermite interpolation of degree 13. Thus, we recommend degrees up to 11. The experiment also indicates that the validity interval can theoretically be extended for both methods. However, since the behaviour of the interpolation error cannot be generalised and the error is smallest in the centre of the interval, we recommend interpolations in the middle of the window despite a slightly increased computation time due to a higher number of interpolation polynomial determinations.

5.4 Experiment 4: Spectral analysis

A further important aspect is whether the interpolation of orbit positions introduces artificial signals of specific periods into the orbits. This should be avoided to prevent misinterpretation in the orbit comparisons. To determine possible consequences due to interpolation, we spectrally analyse the orbit differences using the Lomb–Scargle periodogram (Lomb 1976; Scargle 1982). We choose degrees 5, 7, and 9 of Hermite and Newton interpolation methods, as well as temporal resolutions of 60 and 120 s. As in Experiment 3, we analyse the components individually for periodic behaviour and use the non-reference solutions and the moving window approach.

The periodograms in Fig. 6 show the results for 60 and 120 s. The first two and last two panels refer to Hermite and Newton interpolation, respectively. Columns 1 and 3 present the values for the X (circle), Y (square), and Z (triangle) coordinates. Columns 2 and 4 use the same symbols for the radial, transverse, and normal components in the orbital system. The symbol colours black,

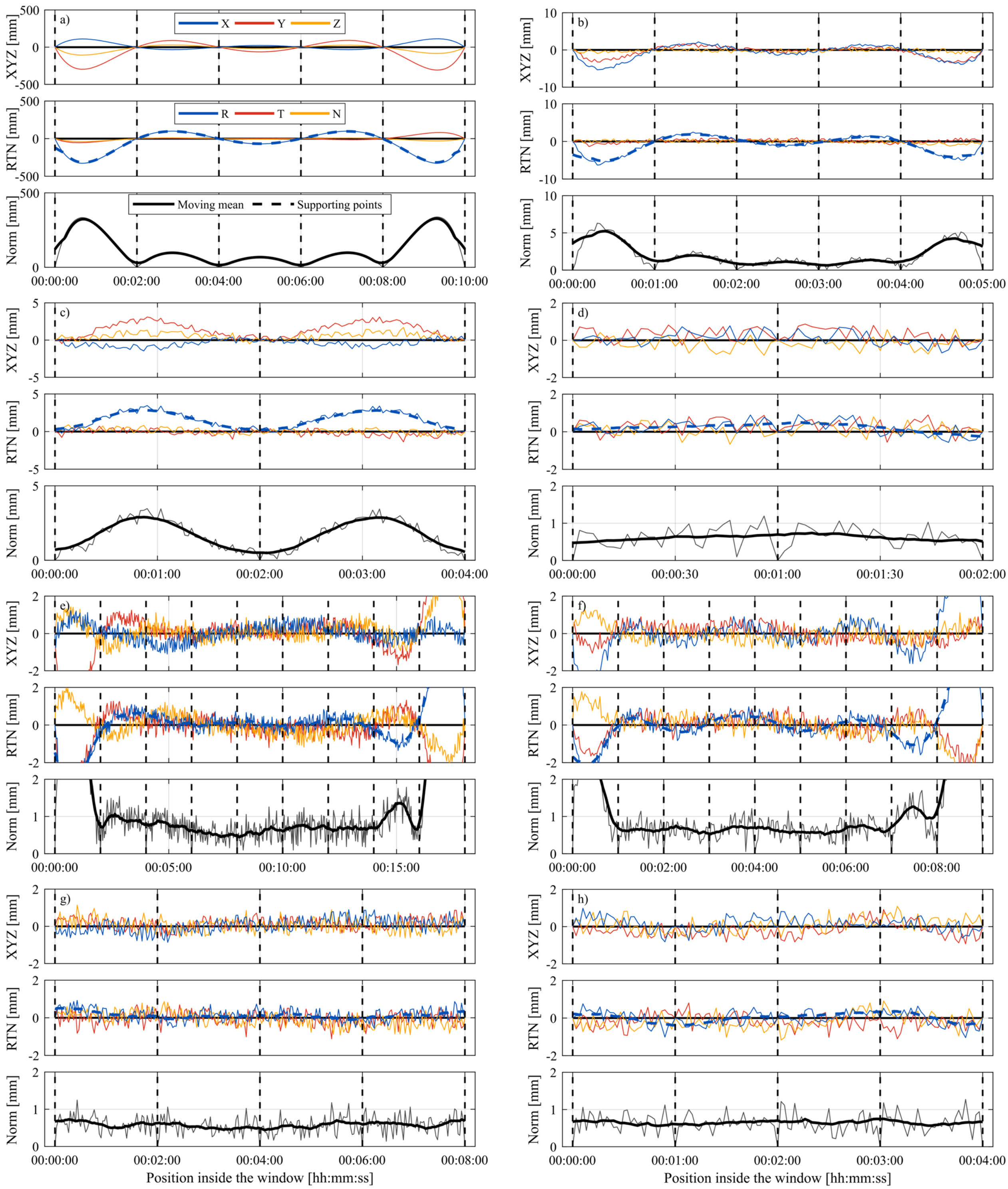


Fig. 5 Component-wise and 3D interpolation errors in the Cartesian (XYZ) and orbital (RTN) coordinate systems for different degrees and step sizes. Each subfigure covers one interpolation interval. An orbit position is interpolated at each second within the interval. Note the different scales of the y-axis in (**a–c**). **a** Newton, degree 5, 120 s; **b** Newton, degree 5, 60 s; **c** Hermite, degree 5, 120 s; **d** Hermite, degree 5, 60 s; **e** Newton, degree 9, 120 s; **f** Newton, degree 9, 60 s; **g** Hermite, degree 9, 120 s; **h** Hermite, degree 9, 60 s

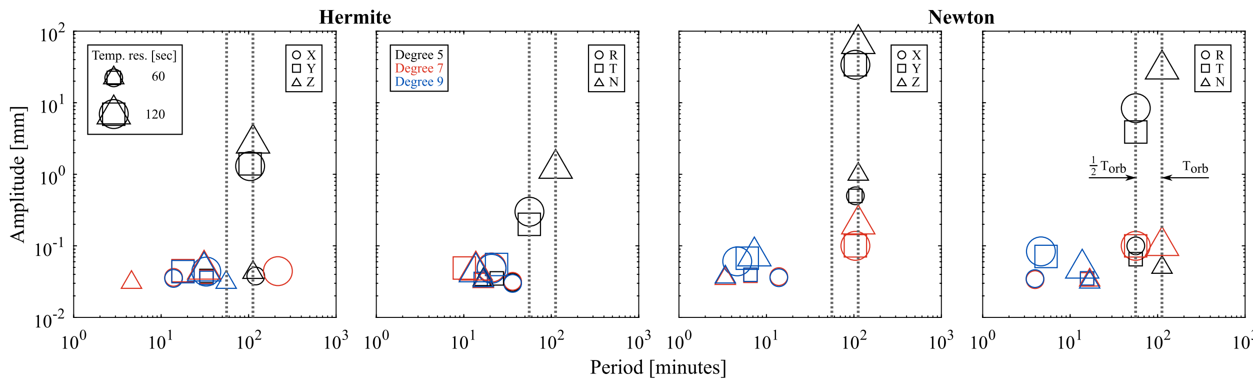


Fig. 6 Periodograms of the spectral analysis of orbit differences based on interpolated orbit information. The first two panels refer to the Hermite interpolation and both last panels to the Newton method. Columns 1 and 3 show the components XYZ (TRF) and columns 2 and 4 the direction in the orbital system (radial, transverse, normal). The symbol size and colour denote further settings like the step size and interpolation degree, respectively. The vertical lines refer to the orbital period T_{orb} (and its half) of Jason-2, which is approximately 112 min (56 min)

red, and blue indicate the interpolation degrees 5, 7, and 9, respectively, and the symbol size refers to the orbit step size. The two vertical lines mark Jason-2's full and half orbital periods (T_{orb}) of approximately 112 and 56 min, respectively. Note the logarithmic scale of both axes.

Interpolation degree 5 (black symbols) introduces in the case of step size 120 s significant periodic signals in all components and for both methods. The maximum values in each system are 2.7 mm (Z) and 1.2 mm (N) for Hermite and 66.4 mm (Z) and 29.3 mm (N) for Newton interpolation. The periods are always the full or half orbital period. Increasing the interpolation degree reduces the maximum amplitudes to the sub-millimetre level. While the orbital period does not appear for Hermite of degree 7, Newton requires degree 9 to avoid these periods. With degree 9 (blue symbols), the periods are randomly below 30 min with amplitudes between 0.04 and 0.09 mm.

For a temporal resolution of 60 s, the artificial interpolation signals and orbital periods only occur in the case of Newton interpolation of degree 5 (small black symbols on both panels on the right). For Hermite interpolation, the X and Z components are affected (black circle and triangle on the left). Degree 7 is sufficient for both methods to resolve this issue and to obtain interpolation results with unbiased spectral analysis.

This analysis demonstrates the consequences of insufficient choice of interpolation settings on the spectral analysis of orbit differences. Inappropriate interpolation results weaken the plausibility and lead to wrong conclusions in orbit difference analysis.

5.5 Experiment 5: Number of digits in the SP3 format

As demonstrated in the previous experiments, the 3D interpolation error is, despite higher interpolation degrees, about 0.5 to 0.6 mm on average. One limitation of achieving better interpolation results, and thus more reliable orbit comparisons, is the accuracy of the basis information, i.e. the number of digits provided in the SP3 files. We identify the orbit position as the current main limiting factor, which is given in the unit kilometre on the millimetre level with default six decimal digits.

To investigate the effects of increasing the number of digits for the position and velocity information in SP3 files, we use Hermite interpolation of degree 7 for step sizes of 30, 60, and 120 s. Table 1 shows the 3D interpolation error for the default SP3 settings in line 7 and those for extended seven and eight decimal digits in the last two lines. For all three temporal resolutions, the average and standard deviation values of the interpolation error decrease by about 90% to approximately 0.06 and 0.02 mm, respectively, when using seven instead of six decimal digits, i.e. by providing satellite coordinates and velocities with an accuracy of 0.1 mm and 10^{-5} mm/s, respectively. The extension to eight digits is further beneficial and yields values between 0.01 and 0.05 mm, but this is a minor improvement compared to the previous case. This clearly demonstrates the advantages and benefits for orbit comparison when applying small adjustments to the current SP3 format.

5.6 Experiment 6: Orbit comparison

After quantifying interpolation and transformation errors when comparing precisely determined orbits, we relate these errors to the differences between two orbits with a temporal resolution of 60 s using Hermite interpolation of degree 7. The minimum number of seconds between both orbits is 27 s (close to the possible maximum of 30 s), e.g. the orbital epochs of one orbit are 00:00:00, 00:01:00, ... and of the other orbit 00:00:27,

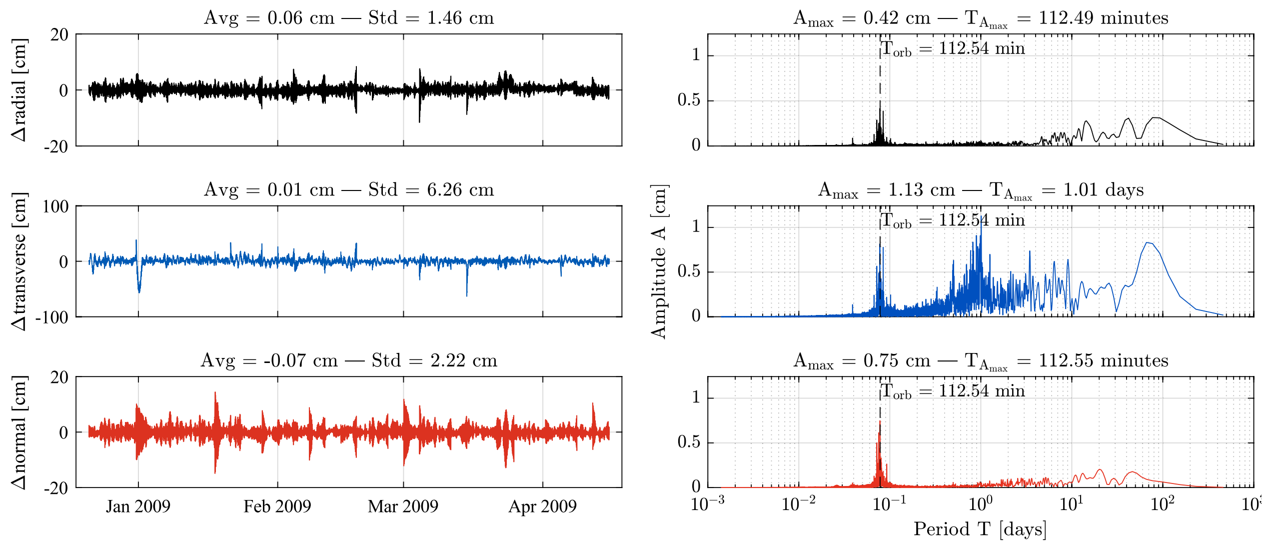


Fig. 7 Comparison of an orbit solution computed at the DGFI-TUM and a solution provided by CNES based on POE-F orbit standards. The panels on the left side show the orbit differences and graphs on the right the respective periodograms

00:01:27, ... (hh:mm:ss). The panels on the left of Fig. 7 show the orbit differences between the DGFI-TUM and POE-F Jason-2 orbits, and the panels on the right the respective periodograms. Most orbit differences are up to 5 cm in the radial and normal directions and up to 10 cm with some extreme values of several decimetres in the transverse direction. The standard deviations reflect the scattering of the time series, and the small average values indicate a good overall mean accordance of both orbits. The periodograms show a prominent peak in all components at Jason-2's orbital period of about 112 min. It is the period with the largest amplitude in the radial and normal directions with 0.42 and 0.75 cm, respectively, whereas a diurnal period with the amplitude of 1.13 cm dominates in the transverse direction. The component's periodograms and the amplitude values of up to about 1 cm emphasise the importance of appropriate interpolation settings since erroneous settings cause amplitudes of up to several millimetres (cf. Fig. 6). A major aspect for the comparably large difference between the orbits in this comparison are the observations used. The DGFI-TUM orbit is based on SLR observations, the CNES orbit on DORIS and GNSS. We do not want to further examine the potential sources of the depicted orbit differences at this point but rather relate the magnitude of interpolation and transformation errors to the differences.

According to the previous experiments, the orbit differences of up to several centimetres contain an average 3D interpolation error of about 0.6 mm at each epoch (cf. Table 1 and top of Fig. 3). Since the differences are shown in the orbital coordinate system, they also contain a transformation error of up to 0.01 mm in 3D. The distribution of the total error (interpolation and transformation) to each of the three components varies and depends on the choice of supporting points and orbit geometry. The latter aspect refers to the highly dynamic character of an altimetry satellite orbit. It is influenced by changes in the Earth's time-variable gravity field, the atmospheric drag, Earth's albedo and infrared radiation and solar radiation pressure, and experiences at entries into and exits from the Earth's shadow accelerations in certain directions. The effects cause subtle variations in the determined orbit, and as mentioned in Schenewerk (2003), a too small interpolation degree is not capable of considering these variations, whereas a too high degree exaggerates these variations and increases the interpolation error.

6 Conclusions

The comparison of orbits with external solutions is important for quality checks and validation. Since internal and external orbits are not necessarily given at the same time instants, one orbit has to be interpolated to the epochs of the other. To obtain reliable orbit differences, purely induced by differences in measurements, models, and computation approaches used in POD and not affected by interpolation errors, the expected interpolation error should be preferably zero. In this article, we assess, based on several experiments, the interpolation method and degree required to obtain accurate interpolation results at the sub-millimetre level for altimetry satellite orbits. We use the Hermite and Newton interpolation methods with varying degrees applied to orbits of different temporal resolutions. The expression of orbit differences in different coordinate systems requires transformations of either the orbit differences or satellite positions, which is accompanied by transformation errors. We quantify the interpolation and transformation errors and determine their influence on individual components and the spectral analysis of orbit differences.

Orbits of altimetry satellites are usually provided with a temporal resolution of 30 or 60 s and are distributed using the SP3 format. We conduct experiments for step sizes of 30, 60, and 120 s to also cover lower temporal resolutions. The experiments show that suitable interpolation results, i.e. minimised interpolation errors, are obtained using degrees 7−11 for Hermite and at least degree 8 for Newton interpolation. Too low degrees cause large interpolation errors, and too high degrees are computationally expensive and very much affected by the Runge phenomenon (extreme differences between the original function and the interpolation polynomial towards the edges of the interpolation interval). The recommended degrees induce an average 3D interpolation error of 0.6 mm with an error scattering of about 0.2 mm. The Hermite interpolation slightly outperforms the Newton interpolation in accuracy and computational efficiency.

The transformation process of orbit differences into other coordinates induces 3D transformation errors of up to ±0.01 mm. This indicates a sufficient retention of the orbit differences norm when applying transformations. A rather uncontrollable aspect is the mapping of orbit differences into certain components at transformations. This depends on the interpolation settings (method and degree) and the orbit geometry. The choice of appropriate degrees minimises these effects to about ±0.02 mm.

The Hermite and Newton interpolations are differently affected by the Runge phenomenon. Since the Hermite method includes the function derivatives (in our case the orbit velocity), the effects are limited with sufficient interpolation settings to about ±1 mm inside the interpolation interval. In contrast, for the Newton method, one must use piece-wise interpolation, i.e. the moving window approach, and only interpolate points in the central region of the window to avoid large interpolation errors.

The spectral analysis of orbit differences also emphasises the necessity to choose appropriate interpolation settings. Too low degrees (like degrees below 8 for Newton and below 7 for Hermite interpolation) introduce systematics at the satellite's orbital period of several millimetres. This might cause misleading interpretations of orbit differences. Sufficient interpolation degrees reduce the amplitude of erroneously introduced periods to values up to 0.1 mm for random periods up to 30 min.

The choice of suitable interpolation settings prevents misinterpretation of real-data comparisons. Both the direct comparison of orbit differences and extended analyses (e.g. spectrally) benefit, since the investigated parameters are often at the millimetre level.

Increasing the accuracy of position and velocity data in SP3 files to 0.1 mm and 10^{-5} mm/s, respectively, reduces the interpolation error by 90% from 0.6 to 0.06 mm and the scattering of interpolation errors decreases from 0.2 to 0.02 mm. This significant improvement clearly benefits the comparison of altimetry orbits, for which a highly accurate radial component is of the largest interest. Thus, we recommend revising the SP3 file format and extending the number of decimal digits of the position and velocity values at least by one digit to seven (0.1 mm for the position and 10^{-5} mm/s for the velocity).

The investigations in this article prove good interpolation results when applying orbit interpolation and transformation of orbit differences with appropriate settings. Consideration of the recommendations made within this study limits the uncertainties to the sub-millimetre level.

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Author contributions
SR and JZ initialised this study. JZ and RA developed the software for orbit comparison used for this study. JZ and RA carried out the experiments. JZ wrote the manuscript and created the figures. All authors helped to improve the final version of the manuscript.

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Availability of data and materials
The POE-F orbit is available in the CDDIS repository, https://cddis.nasa.gov/archive/doris/products/orbits/ssa/ja2/, and the DGFI-TUM orbit is available from the corresponding author on reasonable request.

Declarations

Competing interests
The authors declare that they have no conflict of interest, nor conflict of interest.

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