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Multi-resolution modeling based on quotient space and DEVS

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ABSTRACT

As simulation systems get more and more complex, the study of multi-resolution modeling (MRM) remains an exciting and fertile area of research. Contrasting its abundant successful use cases, a rigorous mathematical foundation is still lacking in MRM. In this paper, we propose a quotient space based multi-resolution modeling (QMRM) theory based on granular computing in artificial intelligence and on discrete-event system specification (DEVS) in modeling and simulation. Based on quotient sets, resolution, multi-resolution modeling and other related concepts are defined and a general concept framework is constructed. Based on the concepts of quotient set and natural projection, several MRM principles are derived. The internal consistency principle guarantees consistency among different perspectives of an atomic model, whereas the external consistency principle guarantees that different components in a coupled model are consistent. The false-preserving principle indicates that if a construction relation or state transformation relation of a component does not exist in a low resolution model, then the corresponding relations should not exist in its high resolution model. The true-preserving principle tells us that a high resolution model can be simplified by choosing the proper low resolution model. QMRM is not only a formal specification, but also a fundamental framework to understand MRM concepts, a guiding ideology to design specific MRM methods, and a modeling methodology to develop MRM systems. QMRM is created from a general simulation perspective, not limited by any specific application or problem domain aspects. The results of this paper can serve as a starting point for further study of multi-resolution problems in different domains.

1. Introduction

Resolution is an intrinsic notion in modeling and simulation, so every simulation study has to deal with resolution issues one way or other. The format of models, the validation of simulations, the reuse of components, and the cost of a simulation are all resolution related. But when talking about the resolution of a model, we often refer it intuitively. Many authors use it without definition and just within their specific contexts. In most of these contexts, model resolution refers to the level of detail, scale, or abstraction used by the model [1]. Furthermore, simulation models have time resolution, space resolution, state resolution, input and output resolution, and behavior resolution.

Though the label of multi-resolution modeling is relatively recent in modeling and simulation [2], the idea of recognition and thinking in different levels of detail already exists for a long time and has been widely applied in different domains.

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Conceptualizing the world at different resolutions and switching among them as a natural problem-solving methodology is deeply rooted in human thinking [3], and modeling and simulation is no exception. Generally, multi-resolution modeling means incorporating multiple models from different resolution levels to address the problem [1,4,5]. The most widely cited definition of MRM was given by Davis [6]: (1) building a single model with alternative user modes involving different levels of resolution for the same phenomena; (2) building an integrated family of two or more mutually consistent models of the same phenomena at different levels of resolution; or (3) both. We will use this definition as our start point to development a more formal definition of MRM.

Though many excellent research results have been achieved on MRM, most of them are application-oriented with a focus on a specific field or a specific problem. Even the terminology is not rigorously defined and similar terms are used without distinction. Resolution is often used interchangeably with detail, level, hierarchy, layer, granular, and so on, which makes it more difficult to differentiate these related terms. Multi-resolution modeling is often called variable-resolution modeling, multi-level modeling, multi-scale modeling depending the idiomatic usage of different domains. Multi-perspective modeling (MPM) often appears together with MRM in literature. Multi-perspective modeling refers to building models from different perspectives of the problem being modeled [7].

Actually, the concept of resolution and the idea of multi-resolution thinking have been widely discussed in different domains. Most of these domains have their own theories of multi-resolution. Granular computing is the theoretical basis for multi-granular problems in AI, wavelet analysis is the theoretical basis for multi-scale problems in mathematics and physics, and topology is the theoretical basis for multi-resolution problem in GIS. But what is the theoretical basis for MRM in modeling and simulation? The study of MRM still lacks a fundamental and commonly accepted theory to direct our modeling activity. We argue that a theory of multi-resolution modeling as a whole is especially needed.

From the viewpoint of information theory, a simulation system is an information processing system. The process of modeling and simulation is essentially the process of information processing. One of the purposes of simulation is to obtain information about the system. In the area of information processing, granular computing is the main theory on multi-level information processing. The theoretical foundation of granular computing includes fuzzy sets, rough sets and the comparatively new quotient space theory [8–11]. Based on this similarity, we argue the applicability of the research findings from granular computing in the area of multi-level information process, especially quotient space theory, to the theory of MRM in simulation.

Like in any science, the development of a theory for multi-resolution modeling must be based on a set of solid definitions, foundational observations and key principles, which are considered within a single context and form an integrated whole. The aim of this paper is to set up such a unified concept framework for multi-resolution modeling based on the quotient space theory and discrete-event system specification (DEVS). Quotient space provides a mathematical foundation for the common concept of resolution and multi-resolution modeling. From quotient space based granular computing, we can derive useful insights for multi-resolution modeling. We also built our MRM concept framework based on DEVS, which is a system theory based model description specification [12]. DEVS has the potential to describe system structure and behavior at various level of abstraction. Combining quotient space and DEVS, we build a general theory framework named Quotient space based Multi-Resolution Modeling(QMRM) for MRM.

QMRM can help to form an agreement on terms and definitions in MRM and to gain deeper insight into MRM for simulation researchers. QMRM provides guidelines on the fundamental beliefs for simulation practice echelons. QMRM also finds a formal way to express the well-known MRM disciplines practiced by most MRM developers. Many modeling terminologies especially multimodel, multi-paradigm model, hybrid model, multifacetted model, multi-formalism model, coupled model, multi-method model and composite model share the characteristic of resolution [1]. A sound MRM theory is also beneficial to general M&S community, since resolution is a basic concept in M&S. This work will encourage the M&S community to think the M&S concepts in a more rigorous manner.

The aim of this paper is to develop a theory of multi-resolution modeling approach. We will discuss the basic philosophy, core concepts, and universal principles of resolution and multi-resolution modeling. In Section 2, the foundation for this research, including background information on MRM, quotient sets, granular computing and DEVS is introduced. Based on the quotient space theory and DEVS, the concepts of resolution and multi-resolution are given, and some fundamental observations on MRM are presented in Section 3. In Section 4, the fundamental principles are induced from the concept framework. These principles are key for the design and implementation of MRM systems. In order to highlight the benefit of QMRM, a use case study is given in Section 5. In Section 6, the MRM concept framework is discussed with its key notion of quotient space. Quotient space based MRM and existing MRM theories are compared, and an indication is given how quotient based MRM can be applied to simulation projects. Some additional comments and discuss quotient-based MRM theory are also given in Section 6. In the final section, we give our conclusion and propose some further research suggestions.

2. Foundations

In this section, We will first give some background knowledge for those who do not familiar with MRM, then introduce related set theory, the concept of quotient space in granular computing and DEVS briefly.

2.1. Background

Though the concept of MRM was proposed in 1990s, the idea of multi-level thinking in the simulation domain has existed for many years. For example, Gardner [13] proposed a multi-level modeling method for system architecting. Fishwick [14] studied the problem of multi-level model abstraction, which is similar to the idea of multi-resolution modeling. Hill [15] described the SABLE tool (structure and behavior linking environment) for structured, multi-level simulation generation. At that time, different terms for MRM were used such as variable-resolution, cross-resolution or selectable-resolution modeling.

There have been quite some remarkable research results and many excellent projects on this subject, for an overview of the theory and application of multi-resolution modeling see [16]. Initially, the application of MRM was mainly focused on military simulation, where some typical studies were reported in [17–22] and [23]. Petty [24] provided a comprehensive list of the implemented systems in multi-resolution combat modeling. Recently, MRM has been applied to areas outside the military domain, such as traffic simulation [25,26], logistics system simulation [27], and biological simulation [28]. In the area of simulation optimization, the nested partitions method [29] also reflects the thought of multi-level modeling. Other potential applications of MRM are for instance multi-resolution modeling based VV&A [30], multi-resolution modeling for agent-based simulation [31], and service-oriented simulation with MRM [32]. Recently, modeling and simulation is becoming an important tool for the study of system of systems (SoS) [30,33]. Since an SoS spans multiple levels of hierarchy, the requirement for MRM is deeply rooted in the characteristics of SoS. The models of an SoS usually have different formalisms and resolutions. How to integrate these different resolution models is an essential problem for MRM.

Some literature also discusses the theoretical aspect of MRM. Liu [16] proposes a DEVS-based multi-resolution model system specification (MRMS). MRMS is a DEVS variant of multi-resolution model system specification, with the concept of a multi-resolution family (MF) and dynamic structure DEVS (DSDEVS). Kara [34] presents a methodology for resolution mapping for cross-resolution simulation using Event-B. It is actually a software engineering solution for implementing simulations via composition of models at different resolution levels with the help of Event-B as the formal specification language and DEVS as the model composition framework. Uhrmacher et al. [35] propose an ML–DEVS (Multi-Level-DEVS) specification to meet the requirement of describing the micro and macro level and the downward and upward causation in computational biology. Hong and Kim [36] propose a specification of multi-resolution modeling space (MRMS) for multi-resolution system simulation, which is based on concepts of decoupling MRM and multi-resolution simulation and is applicable to all object-oriented models. All of these methodologies except the last one extend the existed modeling specifications to provide the MRM ability. They are more model specifications than modeling methodologies. These specifications lack the ability to interpret widely used common rules in MRM practice, let alone to provide general guidelines for model development. A common mathematical foundation for multi-resolution modeling is still needed.

2.2. Quotient set

Some fundamental concepts related to quotient set are given below. Further information on classic set theory can be found in [37].

Definition 1 (Equivalence relation)**.** Let X be a set, and R a binary relation on X. Suppose x, y and $z \in X$. If R is

1. reflexive, i.e., xRx
2. symmetric, i.e., if xRy, then yRx
3. transitive, i.e., if xRy and yRz, then xRz

then R is an equivalence relation. xRy can be written as $x \sim y$.

Definition 2 (Equivalence class)**.** For any $x \in X$, we define $[x] = \{y|xRy\}$ as the equivalence class of x.

Definition 3 (Quotient set)**.** Let $[X] = \{[x]|x \in X, [x] = \{y|xRy\}$. $[X]$ is called the quotient set of X by R. It can be written as $[X] = X/R$ or $[X]_R$.

Definition 4 (Natural projection)**.** Let p: $X \to [X]$, p is called the natural projection from X to $[X]$. $p(x) = [x]$, $p^{-1}(u) = \{x|p(x) \in u\}$.

2.3. Basic knowledge on quotient space based granular computing

In artificial intelligence, a problem space is described by a triplet (X, T, f), where X denotes the problem universe, T is the structure of universe X, and f indicates the attributes (or features) of universe X [38]. Suppose we have a problem (X, T, f), and an equivalence relation R on X. When we derive a quotient set $[X]$ from R, then we can study the corresponding problem ($[X]$, $[T]$, $[f]$), where $[f]$ is the quotient attribute function on $[X]$, $[T]$ is the quotient structure of $[X]$, and ($[X]$, $[T]$, $[f]$) is the quotient space of (X, T, f). This transformation studies a problem (X, T, f) at a different granularity.

The "false-preserving" and "true-preserving" principles are two main properties in quotient space based granular computing [38,39].

Theorem 1 (False-preserving principle)**.** *If the coarse-grained space* ([*X*], [*T*], [*f*]) *has no solution then its original space* (*X, T, f*) *must have no solution.*

Theorem 2 (True-preserving principle)**.** *If a problem has a solution in the coarse-grained space* ([*X*], [*T*], [*f*]), *and for any* [*x*], $p^{-1}([x])$ *is a connected set in X, then the problem has a solution on the original space* (*X, T, f*), *where p is the natural projection from X to* [*X*] *according to Definition 4.*

Zhang [38] proved these properties mathematically. The properties are essential for quotient space based multi-level problem solving. These principles can be used for the study of multi-resolution modeling in simulation with some modification. We need more elements to specify a simulation system than to specify a problem space in artificial intelligence, so the property preserving is more complex than in artificial intelligence.

2.4. A brief introduction to DEVS

Here we only introduce the basic concepts of DEVS for our specification of multi-resolution modeling. A detailed description of DEVS can be found in [12]. As a theoretical framework, QMRM is independent of the sequential or parallel implementation of DEVS, so only classic DEVS is introduced here. In classic DEVS, a basic discrete event system specification is a structure

$$M = (X, Y, S, \delta_{int}, \delta_{ext}, \lambda, ta) \tag{1}$$

Where
X is the set of inputs
Y is the set of outputs
S is the set of states
δ_{int}: $S \rightarrow S$ is the internal transition function
δ_{ext}: $Q \times X \rightarrow S$ is the external transition function, where
$Q = \{(s, e) | s \in S, 0 \leq e \leq ta(s)\}$ is the total state set
e is the time elapsed since the last transition
λ: $S \rightarrow Y$ is the output function
$ta : S \rightarrow R_0^+ \bigcup \infty$ is the time advance function.
The classic DEVS coupled model can be described as:

$$N = (X, Y, Z, \{M_z | z \in Z\}, EIC, EOC, IC, select) \tag{2}$$

Where
X is the input set of the coupled model
Y is the output set of the coupled model
Z is a set of component references, usually component names
M_z is a basic DEVS structure
EIC is the external input coupling set which connects external inputs to component inputs
EOC is the external output coupling set which connects component outputs to external outputs
IC is the internal coupling set which connects component outputs to component inputs
$select : 2^Z - \emptyset \rightarrow Z$, is the tie-breaking function.

DEVS is a general specification for modeling and simulation with a rigorous mathematical foundation. So it provides a good starting point to discuss multi-resolution problems in modeling and simulation.

3. Conceptual framework

3.1. What is resolution

Literature reveals a wide variety of definitions for the term of resolution. Most of the definitions are based on a context or a domain. Some typical definitions of resolution include:

- "Resolution refers to the number of variables in the model and their precision or granularity" [12].
- "The degree of detail used to represent aspects of the real world or a specified standard or referent by a model or simulation" [40].
- "Resolution in simulation is a question of detail and is closely related to fidelity"... The major component of resolution is "the level of detail that the simulation can input and output" [41].
- "The level of detail at which system components and their behaviors are depicted" [6].
- "The resolution of a model or a simulation is the degree of detail and precision used in the representation of real-world aspects in a model or simulation" [5].

On the whole, all authors agree that resolution is closely related to level of detail, but some focus on input and output, some focus on behavior and some focus on components. These differences primarily originate from the even more different application perspectives and contexts in which these resolution approaches have been developed and tailored to suite specific needs. Further more, none of these definitions are based on rigorous mathematical theory.

In order to give a unified and general definition of resolution, we define resolution in modeling and simulation through the concept of quotient set. To a real or imagined system $\mathbb{S}$, $\mathbb{U}$ is its base model [12]. The base model is the idealized ground truth model with highest level of detail. It is a hypothetical, abstract representation of the system's properties and behavior, which is valid in all possible contexts, and it describes all the system's facets [42].

Definition 5 (Resolution)**.** Suppose M is a model of system $\mathbb{S}$, and the resolution of M is defined by an equivalence relation R on $\mathbb{U}$. For each element $x \in M$, $x = [u], u \in \mathbb{U}$, i.e., there exists a natural projection p from base model $\mathbb{U}$ to model M, $p : \mathbb{U} \to M,\ p(u) = [u]$ ($u \in \mathbb{U}$ and $[u] \in M$). The process of getting a low resolution model from $\mathbb{U}$ based on the equivalence relation R is called the quotient operation on $\mathbb{U}$.

The base model is the referent for the definition of resolution and R is the basis for determining model resolution. The definition of R is closely related to the notion of the experimental frame (EF) [12]. Different EFs define different simulation requirements, including requirements for the resolution. The equivalence relation R can be defined by the EF.

Let M be a model of $\mathbb{S}$, define an equivalence relation R on M, then

$$[M]_R = M/R \tag{3}$$

M is called the original model, and $[M]_R$ is the quotient model of M under R. The subscript R is usually omitted when there is no confusion. Equivalence relation is a mechanism for producing models at different resolutions. Zeigler et al. [12] use morphism to find equivalence relation between two atomic models. The natural projection p in our definition is also a morphism from M to $[M]$. Unlike [12], we use the concept of equivalence relation to generate models at different levels explicitly and it can also be used in DEVS coupled models.

People often differentiate between models with a high-resolution and a low-resolution, and between an aggregated-model and a disaggregated-model. An aggregated-model represents objects that are composed of other objects. On the other hand, a disaggregate-model represents objects that are not normally further decomposable [43]. In military simulation, simulation systems are classified into engineering level, engagement level, mission level and theater/campaign level according to their resolution [5]. Gross [44,45] gives four theorems for the measure of fidelity. Similarly, we can define the measure of resolution using similar theorems. But it is often difficult and usually unnecessary to measure model's resolution quantitatively. So we can define the comparison of resolution as following.

Definition 6 (Comparison of resolution)**.** Let $\mathcal{R}$ be all equivalence relations on $\mathbb{U}$, $R_1, R_2 \in \mathcal{R}$, if $\forall x, y \in \mathbb{U}$, $xR_1y \Rightarrow xR_2y$, then we can say that the resolution of R_1 is higher than R_2, denoted as $R_2 < R_1$.

The resolution comparison can be carried out from different perspectives. The relation "$<$" is a partial ordering on the set $\mathcal{R}$, since the resolutions of some models may be incomparable [37]. It is also important to note that the higher or lower of model resolution is relative.

The concept of resolution is closely related to the concepts of abstraction, scale, level, fidelity and granularity [46]. Table 1 gives the definition of these concepts and their relationship with resolution.

3.2. Multi-perspective nature of resolution

Definition 5 is an abstract definition of resolution. This section focuses on the multidimensional and multifaceted character of model resolution. We will apply the abstract definition of resolution to DEVS. According to the elements of atomic and coupled DEVS, we know that the model's resolution can be described from the viewpoint of input and output, states, structure, the behavior of state transformation, etc. Given different equivalence relations, model resolution can be defined from these different perspectives.

Different perspectives in a model can be described by different elements in DEVS. Based on the definition of DEVS (Eqs. (1) and (2)) and resolution (Definition 5), we can further define resolution for different perspectives. Let $\mathbb{U} = (I, O, S, \delta_{int}, \delta_{ext}, \lambda, T, ta)$, where T is the structure of the model, usually described by the coupled DEVS formalism. Define R_e as an equivalence relation on e, where $e \in \{I, O, S, \delta_{int}, \delta_{ext}, \lambda, T, ta\}$. R_e defines the resolution of different model elements. We can now define the following types of resolution:

- *Input resolution:* I is the input set of the base model. R_I is an equivalence relation on I. The input resolution is determined by R_I, $[I]_{R_I}$ is a lower resolution description of the ideal input I.
- *Output resolution:* O is the output set of the base model. R_O is an equivalence relation on O. The output resolution is determined by R_O, $[O]_{R_O}$ is a lower resolution description of the ideal output O. Output resolution is closely related to input resolution. Together they are called IO resolution.
- *State resolution:* S is the state set in the base model. R_S is an equivalence relation on S. The state resolution is determined by R_S, $[S]_{R_S}$ is a lower resolution description of the ideal state S. The relationship from S to $[S]_{R_S}$ is a homomorphism.

Table 1
Relationship among resolution-related concepts.

Concept	Definition	Relationship with resolution	Notes and references
abstraction	Suppressing details and dealing instead with a generalized, idealized model of a system	Different resolution models have different abstraction levels. Abstraction is the fundamental technique for MRM	Much work on model abstraction has been done [14,47]
scale	A dimension of analysis on which the phenomenon of interest can be measured. This dimension can be spatial or temporal, but also quantitative	Closely related to the concept of resolution and often used interchangeably in some disciplines	The concept of scale is frequently characterized by an adjective that relates it to space or time [46]
level	A position on a scale of intensity or amount or quality	The word "level" can refer to resolution level, abstraction level, fidelity level, etc. Usually used as a term for qualifier not for quantifier, i.e. a higher level of abstraction, a lower level of resolution, etc.	Hierarchy and layer can be exchanged with level [48]
fidelity	The degree of correspondence between the simulated situation and the reference situation	Resolution is closely related with fidelity. But high resolution does not imply high fidelity	This definition is given in [49]. There is also no authoritative definition of fidelity. Roza [50] gives a comprehensive analysis on the definition of fidelity
granularity	The ability to represent and operate on different levels of detail in data, information, and knowledge	In simulation especially combat simulation, granularity usually refers to entity resolution	Some authors think that granularity is equivalent to resolution [5,51], but others treat them as different concepts [3,52]

- *Behavior resolution*: We can also define equivalence relations on δ_{int}, δ_{ext}, and λ to get internal transition resolution, external transition resolution, and output transition resolution respectively. They are collectively called behavior resolution, noted as R_B.
- *Structure resolution: T* is the component structure in the base model, R_T is an equivalence relation on *T*. Structure resolution is determined by R_T, $[T]_{R_T}$ is a lower resolution structure model of the ideal structure *T*.
- *Time rsolution: ta* is the time advance function in the base model, R_{ta} is an equivalence relation on *ta*. Time resolution is determined by R_{ta}, $[ta]_{R_{ta}}$ is the lower resolution time advance function. Different time advance functions in a time-step based simulation means a different density of time steps. In discrete-event simulation, it is related to state resolution.

Using the definition of resolution from different perspectives, we can further define the comparison of resolution from different perspectives based on Definition 6. The concepts of granularity, aggregation and disaggregation are further defined by structure resolution.

Definition 7 (Granularity)**.** *M* is a model of system $\mathbb{S}$, *E* is the set of entities in base model $\mathbb{U}$, *R* is an equivalence relation on *E*, the granularity of *M* can be defined as $[E]_R = \{[e]_R | e \in E\}$. $|[E]|$ represents the number of elements in $[E]$, and the granularity can be measured by $Gr = |[E]|/|E|$. The greater *Gr*, the larger the granularity and the coarser the model. R_1 and R_2 are two equivalence relations on *E*, for any e_i, $e_j \in E$, if $e_i R_1 e_j$, then $e_i R_2 e_j$, then we say the granularity of $[E]_{R_2}$ is larger than or equal to that of $[E]_{R_1}$.

Definition 8 (Aggregation and disaggregation)**.** M_1 and M_2 are two models of system $\mathbb{S}$, *E* is the entity set of base model $\mathbb{U}$, E_1 and E_2 are respectively the entity set of M_1 and M_2 defined by equivalence relation R_1 and R_2 on set *E*. The granularity of E_1 is larger than that of E_2, i.e., $R_2 > R_1$. the aggregation operation is defined by natural projection g: $E_2 \to E_1$, and disaggregation relation is defined by $g^{-1} : E_1 \to E_2$.

The aggregation operation maps the attributes and structure of entities in E_2 to that of entities in E_1. Similarly, the disaggregation operation maps the attributes and structure of entities in E_1 to that of entities in E_2. But for an aggregated model, different disaggregation scheme exists. So relation g^{-1} is not a proper mathematical mapping. If we can find and save extra information ω on disaggregated model when aggregating, we can get a unique disaggregated model when disaggregating. We note this operation as $g^{-1}(E_1; \omega)$.

In modeling and simulation, we usually deal with model resolution in some perspective. For example, entity resolution is often addressed in combat simulation [24], dynamic behavior resolution often gets attention in multi-level model abstraction [47], input and output resolution is emphasized in decision making simulation [7]. When talking about model resolution, we usually refer to some perspective of model resolution that is our most concern. But we often do not point out it explicitly. Actually, the different perspectives of model resolution are closely related. When you change input resolution from a lower resolution to a higher resolution, the state resolution and behavior resolution are likely to be changed as well. We will further discuss this in Section 4.

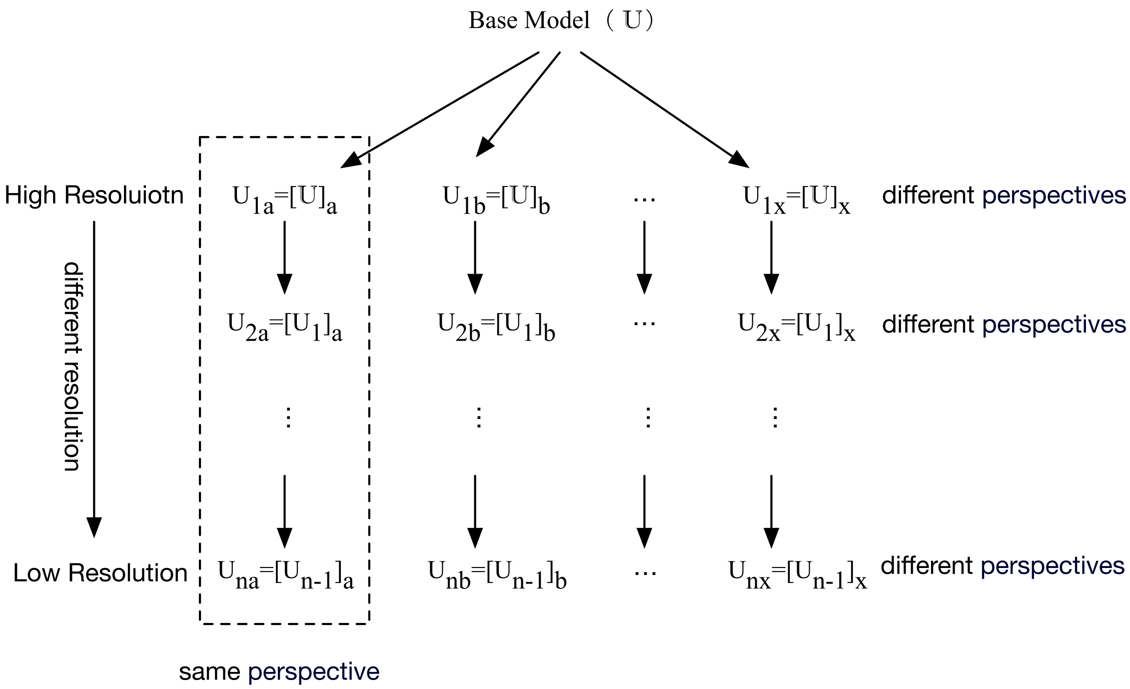

Fig. 1. Illustration of MRM and MPM.

3.3. Multi-resolution modeling

Davis's definition of MRM is straightforward and easy to understand. But it is not rigorous. The key elements of this definition are “different levels of resolution” and “the same phenomena”. A single model or a family of models is only related through the implementation pattern, which does not reflect the core of MRM. Let's restate this definition in a more formal way.

Definition 9 (Multi-resolution modeling)**.** For a system $\mathbb{S}$, given a subset of $\mathcal{R}$, i.e., $\{R_1, R_2, R_3, \ldots, R_n\} \in \mathcal{R}$, where $\mathcal{R}$ is all equivalence relations on base model $\mathbb{U}$ of system $\mathbb{S}$. For any $i, j \in n$, $R_i < R_j$ or $R_j > R_i$ exists. Let $\mathbb{M} = \{M_{R_i} | i \in n\}$, where $M_{R_i} = \mathbb{U}/R_i$ is the resultant quotient model under R_i. Let $\mathbb{P} = \{p^*_{ij} | i, j \in n\}$, if $R_i > R_j$, define p^*_{ij} as natural projection from high resolution model M_{R_i} to low resolution model M_{R_j}, i.e., $p : M_{R_i} \rightarrow M_{R_j}$, else if $R_j > R_i$, define p^*_{ij} as $p^{-1} : M_{R_i} \rightarrow M_{R_j}$. $(\mathbb{M}, \mathbb{P})$ is a multi-resolution model of system $\mathbb{S}$.

In theory, we can build a simulation system with arbitrary resolution levels. the selection of subset of $\mathcal{R}$ depends on the experiment frame in practice. In combat simulation, for instance, there are usually two resolution levels from the perspective of structure resolution: unit level which represents combat in terms of military units and entity level which represents individual battlefield objects or platforms.

Liu [16] presents the concept of “Multi-resolution model Family” (MF). An MF can be described as $MF = (\gamma, \{M_r\}, \{R_{i,j}\})$, where: γ is the set of model resolutions, which can be regarded as the index of models with different resolutions of the same entity. For $r \in \gamma$, M_r represents the model with resolution r, $\{M_r\}$ means the set of all models of the same entity and M_r can be specified by DEVS.

Theorem 3. *Under Definition 9, we can get an MF.*

Proof. In MF, the set γ can be defined by $\{R_i, R_2, \ldots, R_n\}$, $\{M_r\}$ can be defined by the set $\mathbb{M}$, and the set $\{R_{i,j}\}$ can be defined by the set $\mathbb{P}$.

Thus we proved that all element in MF can be derived from our MRM definition. □

We have proved that an MF can be formalized by coupled DEVS in [16]. So our MRM definition can also be rewritten as DEVS coupled model. Still we expect that other MRM formalizations can be expressed by our MRM definition. Like MF, Hong and Kim [36] use multi-resolution model family (MRMF) to represent the set of different resolution models of the same real-world object. With similar proof procedure given above, we can also prove that Hong and Kim's MRMF is equivalent to Definition 9.

Multi-perspective models may be at different resolutions or at the same resolution. Fig. 1 illustrates the relationship between MRM and MPM. From the viewpoint of quotient space, we can define multi-perspective modeling as following:

Definition 10 (Multi-perspective modeling)**.** For a system $\mathbb{S}$, given a subset of $\mathcal{R}$, i.e., $\{R_1, R_2, R_3, \dots, R_n\} \in \mathcal{R}$, where $\mathcal{R}$ is all equivalence relations on base model $\mathbb{U}$ of system $\mathbb{S}$. For some *i, j* ∈ *n*, R_i and R_j are equivalence relations defined from different perspectives of $\mathbb{U}$. $M_{R_i} = \mathbb{U}/R_i (i \in n)$ is the resultant quotient model under R_i. Then $\{M_{R_1}, M_{R_2}, \dots, M_{R_n}\}$ is a multi-perspective model of system $\mathbb{S}$.

There are five levels in DEVS specification, i.e., I/O frame, I/O behavior, I/O function, I/O structure and coupled systems [12]. It should be noted that the levels of system specification in DEVS are closely related with, but not the same as, multi-resolution modeling. Usually, high resolution models give more information which need a detailed system specification. But models with the same system specification can have different resolutions. For example, both a unit model and an entity model can be specified at the level of a coupled systems in DEVS, but they have different resolutions.

3.4. Basic observations for MRM

Many common sense rules are widely applied in the practice of multi-resolution modeling. These rules are based on our understanding on modeling and simulation. They are accepted without proof. We summarize these common sense rules as basic observations for MRM. These observations are neither exhaustive nor dogmatic.

- Observation 1: The resolution of a model is mainly determined by the experimental frame, i.e., the purpose or the intended use of simulation [12].
- Observation 2: Choosing a resolution is a tradeoff among many factors, such as the question to be answered, the required accuracy, the allocated budget, the time restrictions, the existing reusable models, the technique capabilities of the team, and other resources [7].
- Observation 3: Resolution is closely related to fidelity, but high resolution does not necessarily mean high fidelity and vice versa [6,7].
- Observation 4: A low resolution aggregated model can correspond to several high resolution disaggregated models. The mapping from an aggregated model to a disaggregated model is not unique without additional knowledge about the system [16].
- Observation 5: Even though computing power and storage capacity are increasing, a higher resolution is not always the better [7].
- Observation 6: Different modeling methodologies are often exercised for different resolution models. For example, a high resolution model can be an agent-based model, while a low resolution model may be a statistical meta-model [2,7].
- Observation 7: Models with different resolutions can be calibrated with each other. Often a lower resolution model can provide background information for higher resolution model, while a higher resolution model can provide inputs for lower resolution models [30].
- Observation 8: Different resolution models in a model family should be consistent within tolerable errors and under a certain experimental frame [53,54].

Some of these observations can be derived from the principles given in the following section, some are derived from common sense of simulation, some others come from simulation experience.

4. Basic principles for MRM

Based on the concepts and specifications proposed in the last section, we will discuss two fundamental sets of questions in multi-resolution modeling. The first one is: what is the relationship among different elements in a DEVS atomic model, and what is the relationship among different components in a DEVS coupled model? The other one is: what is the relationship between different resolution models of a system. Are the conclusions derived from low resolution simulation still valid in high resolution ones? Will the high resolution simulation results be reflected in a low resolution simulation? Based on the concepts of quotient set and natural projection, we can derive some principles for multi-resolution modeling to answer the above questions.

4.1. Internal consistency principle

We can describe a system from different perspectives, such as input and output, state, behavior and structure [12]. A basic observation about resolution is that the resolution of different perspectives in a model should be consistent [5,12,53,55,56]. This observation can help us to avoid over-modeling some part of a system while under-modeling another part of the system.

Lemma 1 (Resolution correlation)**.** *In a DEVS atomic model, each operation can only process information with a specific resolution. If the resolution is not matching, a transformation should be made before the operation can be executed.*

A model can only receive input with some resolution. An internal transition function can only process state with specified resolution. In practice, a resolution processing function or a special component can be used to deal with these resolution transformations. Based on Lemma 1, we give the following internal consistency principle.

Theorem 4 (Internal consistency principle)**.** *Given a model $M = (X, Y, S, \delta_{int}, \delta_{ext}, \lambda, ta)$, if we transform any one element e ($e \in \{X, Y, S, \delta_{int}, \delta_{ext}, \lambda, ta\}$) to another resolution, then the other elements should also be changed accordingly, in order to be compatible with the resolution of e.*

Proof. For $M = (X, Y, S, \delta_{int}, \delta_{ext}, \lambda, ta)$, let $L = \{\diamond X, \diamond Y, \diamond S, \diamond \delta_{int}, \diamond \delta_{ext}, \diamond \lambda, \diamond ta\}$, where L is the set of symbols of DEVS elements. For example, $\diamond X$ means the symbol of X in DEVS. We define a relation R on L, $\forall x, y \in L$, if the resolution of x is related with that of y, then xRy. We can easily understand that in this case:

1. xRx
2. if xRy, then yRx
3. $\forall x, y, z \in L$, if xRy and yRz, then xRz

so R is an equivalence relation.

Let's analyze the relationships among the elements in *DEVS* based on Lemma 1 and Eq. (1).

- From the definition of δ_{int}, $(\diamond \delta_{int}, \diamond S) \in R$
- From the definition of δ_{ext}, $(\diamond \delta_{ext}, \diamond S) \in R$, $(\diamond \delta_{ext}, \diamond X) \in R$, $(\diamond X, \diamond S) \in R$ and $(\diamond S, \diamond ta) \in R$. Then because R is an equivalence relation, $(\diamond X, \diamond ta) \in R$, $(\diamond \delta_{int}, \diamond \delta_{ext}) \in R$.
- From the definition of λ, we get $(\diamond S, \diamond Y) \in R$, $(\diamond \lambda, \diamond S) \in R$, and $(\diamond \lambda, \diamond Y) \in R$, Then because R is an equivalence relation, $(\diamond X, \diamond Y) \in R$

Through the above analysis of all elements in *DEVS*, it is clear that the resolution of all elements in *DEVS* are related. So when we change the resolution of one element in a model, the resolution of other elements should be changed accordingly. □

Understanding this theorem is helpful for designing a simulation model. From this theorem, we conclude that although there are many perspectives in a model, their resolutions should be kept consistent. When the resolution of one perspective is changed, that of other perspectives should be changed accordingly. When the resolution of model elements is not compatible, a transformation should be made. From this principle, we also know that when we try to get a refined model, improving the resolution of a single perspective is useless. We should not hope to get a fine-grained model by improving the input resolution without changing the internal state set of the model. Conversely, without high resolution inputs, a high resolution state model offers little help in improving the quality of models.

The internal consistency focuses on the DEVS atomic model. It belongs to the first part ("building a single model with alternative user modes involving different levels of resolution for the same phenomena") of Davis' definition on MRM [6].

A typical application of the internal consistency principle is the data engineering in the NATO code of best practice (COBP), where a data transformation process is specified to resolves resolution issues [57,58]. After transformation, the data will meet the resolution requirements of the model.

4.2. External consistency principle

There are different opinions on whether (sub)models of different resolutions can interact. Some advocate that cross-resolution interaction should be forbidden completely [53,54], some others propose methods for cross-resolution interaction [59,60]. Li [59] implements resolution transformations by separate resolution transformation components, while Powell [60] implements the same transformations by a built-in resolution management function. In the end, when (sub)models with different resolutions interact, resolution consistency between the (sub)models should always be maintained.

Theorem 5 (External consistency principle)**.** *In a coupled DEVS model, the resolution of each component should be compatible, otherwise a resolution transformation should be made in order to guarantee that all interactions can be conducted at the same resolution.*

Proof. A system composed of component models with different resolutions can be described by a *DEVS* coupled model $N = (X, Y, Z, \{M_z | z \in Z\}, EIC, EOC, IC)$. Let us discuss the case when the resolution of different components $d (d \in D)$ is different. We assume $\gamma(M_i)$ and $\gamma(M_j) (i, j \in Z)$ are not equal, and let $\gamma(X) = \gamma(Y) = r$.

Suppose that $\gamma(X) \neq \gamma(M_i)$ and $(X, i) \in EIC$. From the "Internal Consistency Principle", the elements in M_i should be transformed in order to be compatible with X.

When $\gamma(Y) \neq \gamma(M_j)$ and $(j, Y) \in EOC$. From the "Internal Consistency Principle", the resolution of M_j's output should be transformed in order to be compatible with Y.

Assuming, $(i, j) \in IC$, again from the "Internal Consistency Principle", the state, behavior and output of M_i or M_j should be transformed in order to maintain the compatibility among resolutions.

So, our conclusion is that different parts of a coupled model should have compatible resolutions. When models with different resolutions have to interact, a resolution transformation should be made. □

This theorem indicates that in a simulation system (centralized or distributed) with multiple components, the resolution consistency should always be maintained, either explicitly or implicitly. The overall resolution of the system is not determined by individual components, but by the resolution of all components. So improving the resolution of individual component alone can not get a better simulation result. When we intend to reuse a model, its resolution should be compatible with the resolution of the other models in the intended simulation system. If this is not the case, a transformation should be made. When we integrate models with different resolutions, the direction of transformation, i.e., whether to aggregate some components or disaggregate others should be considered. The function and performance requirements, the time and budget limitations, the acquirable input data and other factors should be taken into consideration when choosing the direction of the resolution transformation.

A formal specification named Multi-Resolution Model system Specification (MRMS) is given to describe a multi-resolution simulation system in [16]. In MRMS, a multi-resolution simulation system is described by $MRMS = (X, Y, \kappa, M_k, \chi, M_\chi)$, where κ is the set of entities, $M_k \in MF_k$ is the subset of multi-resolution model families of entity $k(k \in \kappa)$. χ is the model resolution controller, M_χ is the model of χ. M_χ can be described with a DEVS atomic model. In MRMS, the function of resolution maintenance is implemented by M_χ. Readers can find a complete description of MRMS in [16].

Multi-resolution entity (MRE) [54] and the subsequent UNIFY framework [53] fully embody the external consistency principle. UNIFY is a framework for maintaining consistency among representations of jointly executing models. MRE is a conceptual entity that can interact at multiple levels of resolution concurrently by maintaining consistency among corresponding attributes at different levels of resolution. The main aim of UNIFY and MRE is to maintain consistency among different resolution models. Seck and Honing [61] offer an interesting implementation in DEVS of the external consistency principle.

External consistency focuses on the DEVS coupled model. It belongs to the second part("building an integrated family of two or more mutually consistent models of the same phenomena at different levels of resolution") of Davis' definition on MRM [6].

What happens when internal consistency meets external consistency in a real multi-resolution simulation system? Often quotient operations on a coupled DEVS model can be unrelated to quotient operations on its atomic DEVS models. For example, we can use high resolution entity-level simulation in multi-agent simulation from the perspective of construction resolution, while each entity has only few state variables which means its state resolution is relatively low.

4.3. False-preserving principle

In [38] and [62], the authors have proven the true-preserving and false-preserving in artificial intelligence. This subsection and the next one will discuss similar properties in multi-resolution modeling.

System theory distinguishes between system structure and system behavior. So a simulation system specification is composed of two major parts [63]: the static part and the dynamic part. The static part is a structural description which specifies the inner structural constitution of a system, which can be formalized by a DEVS coupled model. The dynamic part is a behavioral description which specifies how the system's characteristics change over time, which can be formalized by a DEVS atomic model. We will study the problem of properties-preserving from both static and dynamic perspectives respectively.

The proof of the following theorems is based on a basic theorem in topology [38,64]:

Theorem 6. *If p: $(X, T) \to ([X], [T])$ is a natural projection, then p is continuous. If $A \subset X$ and A is a connected set in X, then $p(A)$ in also a connected set in $[X]$.*

Firstly, we present the static false-preserving principle.

Theorem 7 (Static false-preserving principle)**.** *If there is no connection from component $[A]$ to $[B]$ in a low-resolution model, there will no connection from A to B in its corresponding high-resolution model, where A and B are high resolution component models.*

The converse-negative proposition of this theorem is that if there is a connection from component A to B in a high resolution model, then there is a connection from $[A]$ to $[B]$ in its corresponding low resolution model.

Proof. From a static point of view, a DEVS coupled model can be written as (D, T, f), where D is the set of components, T is the set of relations among components which describes the static topology of a model, and f is the set of attributes of the components. Comparing with DEVS, D is the set of components $\{M_z | z \in Z\}$ described by the DEVS model, T is the total set of coupling relations, expressed by $\{EIC, EOC, IC\}$, f is expressed by the union of the states of all components. Here we are only interested in the connections between components, so the attributes of the component can be ignored. Actually, now (D, T, f) is equivalence with (X, T, f) in Theorem 1, so that theorem holds.

Now, let us prove this theorem based on Theorem 6. Because a connection $A \to B$ exists, A and B fall within the same connected set C in (D, T, f). Let p: $D \to [D]$ be a natural projection, p is continuous, so $p(C)$ is also connected in $([D], [T], [f])$. So we get $p(A) = [A],\ p(B) = [B]$, fall within the same connected set in $([D], [T], [f])$, consequently a connection from $[A]$ to $[B]$ exists. □

The transformation from high resolution components to low resolution components is implemented by the aggregation operation. Through aggregation, connections among components in set D will be omitted or described by aggregated attributes in $[D]$. The connections between different quotient sets on D should be modeled explicitly. It actually provide a principle for entity aggregation.

Theorem 8 (Dynamic false-preserving principle). *Assuming S_1 and S_2 are high resolution simulation state, If there is no state transformation from state $[S_1]$ to $[S_2]$ in a low-resolution model, there will no state transformation from S_1 to S_2 in its corresponding high-resolution model.*

Proof. From the dynamic point of view, a simulation model can be written as (S, K, g), S is the total states of system in additional with input set and output set, which are specifically external states. K is the structure of the state transformation, g is the dynamic attributes of states, which are the function of external event and time.

Comparing with DEVS, S is the set of states, input and output in DEVS, $K \subseteq S \times S$ is the transition relationship among states, $g = \{\delta_{int}, \delta_{ext}, \lambda\}$. This formalization is equivalent with the problem space description (X, T, f). From Theorem 2, we can derive that this theorem holds.

It can also be proven using Theorem 6. □

In simulation practice, if we cannot draw some result for a specific problem from a low resolution simulation, from Theorem 8 we know it is unnecessary to build a high resolution equivalent. So when studying a problem through simulation, we can first build a low resolution model to verify the feasibility of our idea.

4.4. True-preserving principle

The true-preserving principle answers the question whether we can generalize the properties of a low resolution model to its high resolution one. The proofs are similar to the ones for the false-preserving principle, so we omit the proofs here.

Theorem 9 (Static true-preserving principle). *If a connection from component $[d_1]$ to $[d_2]$ exists in the low resolution structure model (D, T, f) and $\forall[d] \in [D]$, $p^{-1}([d])$ is a connected set on D, then there should be a connection from d_1 to d_2 on (D, T, f).*

As we know, the mapping from an aggregated model to a disaggregated model is not unique. If $p^{-1}([d])$ is a connected set, and all connection relations between the set $[d_1]$ and $[d_2]$ are considered, then the relation between d_1 and d_2 will be restored in the disaggregated model.

Theorem 10 (Dynamic true-preserving principle). *If a state transformation exists from $[s_1]$ to $[s_2]$ in a low resolution state model $([S], [K], [g])$, and $\forall[s] \in [S]$, $p^{-1}([s])$ is a connected set on S, then there should be a state transformation from s_1 to s_2 on (S, K, g).*

The precondition for this theorem is that $p^{-1}([s])$ is a connected set on S. A complete disaggregation operation needs additional information. If some key information is not considered, the precondition for Theorem 10 may be false, then the high resolution state transformation may not exist.

5. Use case study

MRM is especially important in military simulation. So we will illustrate our QMRM framework with a simple tank combat use case. In our scenario, a red tank company and a blue one approach some specific target from different different directions. When encountering at some area called engagement area, they fight against each other. Meanwhile, a blue flight formation is ordered to bomb any red tanks entering the engagement area. Petty et al. [24] provide a detailed description of this scenario.

Defining equivalence relation R_t as belonging to the same tank in tank model, we get the entity level model TK_t. By defining equivalence relation R_c as belonging to the same company, we get the unit level model TK_c. If we define R_b as belonging to the same brigade, we get an even low resolution model TK_b. We can even define equivalence relation R_s as belonging to the same side, combined with a proper temporal resolution, we get the lowest resolution model usually expressed by Lanchester equation.

Specific to our experiment frame, we define the resolution set as $\{R_t, R_c\}$, the MRM model is described as $(\{TK_t, TK_c\}, \{p_{tc}, p_{tc}^{-1}\})$, where p_{tc}: $TK_t \to TK_c$. From the viewpoint of structure resolution, the tank models are divided into entity level model and unit level model. The structure resolution is main resolution, the resolutions of other perspectives are derived resolutions.

The terrain information is the input of tank models. According to Theorem 4, The low resolution maneuver model is a unit level troop model and a grid-based terrain model that partitions the terrain into squares or hexagons and abstracts the terrain characteristics of each grid element into one or more attributes that apply to the entire element. In its corresponding higher resolution models, the unit is disaggregated into entity level models, and the terrain is represented in greater detail, with individual roads, buildings, trees, bridges and rivers.

The tank model usually runs in unit resolution when maneuvering out of the engagement region. Once into the engagement region, the aggregated model should be disaggregated. But there are many disaggregation schemes. So extra knowledge such as combat doctrine and tank operation rules should be used in order to get reasonable entity level model. In the engagement area, the tank companies are disaggregated into tank entities. According to Theorem 5, If a formation of aircrafts attack the tanks in the engagement area, the formation model should be disaggregated into aircraft models or the interaction sent to the tank model be transformed into high resolution one.

From Theorem 7, we can deduce that if there is no engagement relationship between tank company A and tank company B, then there is no such relationship between each tank in A and each tank in B by static false-preserving principle.

From Theorem 8, we know that if there is no route for a tank company to maneuver from position pos_i to position pos_j, it is impossible that each tank in that company to maneuver from position pos_i to position pos_j. We can further study why this maneuver can not be achieved using high resolution model.

Applying the converse-negative proposition of Theorem 8, if we have a validated entity-level engagement simulation such as an agent-based tank combat simulation, with the information provided by this high-resolution simulation, we can build a meta-model using the equivalence class according to the identification of friend or foe (IFF). We can further use the equivalence class to build generalized Lanchester equations [65] supporting joint arms fighting.

From Theorem 9, we can deduce that if there is an engagement relationship between tank company A and tank company B, then such relationship between tanks in company A and tanks in company B should also exist.

Let's explain Theorem 10 with tank maneuver simulation. If the unit level simulation model shows that the troop can march from location A to B in some time duration, we can get the approximate same state transformation only when the unit is properly disaggregated (i.e., $p_{tc}^{-1}([s])$ is a connected set). The troops can pass a cell with a bridge in it with a lower speed in the low resolution simulation, which may not be true in high resolution simulation when the unit is not properly disaggregated. For example, without additional military knowledge such as combat regulations, the entities may pass the bridge using a completely different total time from that of the unit model. In some special situations, some entities may even march in the river at some time, which makes them unable passing the river. Only when the unit is properly disaggregated, we can get the same location state transformations at both resolution levels.

6. Discussion

We have proposed a conceptual framework for MRM based on quotient spaces and DEVS, and derived some important conclusions from this conceptual framework. We also further developed the related theorems in granular computing which is based on quotient space. In this section we will give some additional comments and discuss our quotient-based MRM theory.

6.1. QMRM is a general theory for multi-resolution modeling

QMRM is based on general systems theory, general simulation specification theory, and general quotient set theory. As long as a model can be expressed by set theory, Our quotient-set-based resolution concepts can be used to understand MRM problems in these models. since DEVS is general enough to cover discrete event system, differentialequation system and discrete time system, QMRM is applicable to most simulation methods. QMRM is not restricted by specific application domains of MRM. Modelers can develop MRM methods for any given domain based on QMRM. By applying the concept of a quotient set, we can get a unified understanding of resolution and its related concepts. Fig. 2 summarizes the concepts of resolution, MRM, and their relationship with other concepts.

This figure gives an overall picture of QMRM theory. The base model is the starting point for multi-resolution modeling and multi-perspective modeling. Base model is an idealized referent of the modeled system. In practice, we usually choose some high resolution model as base model, for example, entity-level model is usually regarded as base model in combat modeling. A subset of the base model is extracted from the base model within the context of a specific experimental frame. Models in QMRM are formalized with DEVS. Different kinds of equivalence relations define multi-perspective models and different levels of equivalence relations define multi-resolution models. Different elements in DEVS correspond to different perspectives of model resolution, such as IO resolution, state resolution, behavior resolution and structure resolution. Especially, Z and $\{M_z\}$ correspond to the concept of granularity. Consistency is the fundamental requirement for MRM. Internal consistency requires that the resolution of different elements in an atomic DEVS model should be compatible. External consistency requires that different components in a coupled DEVS model should be compatible. The consistency problem also exists in different perspective models. Many efforts have been made to maintain the consistency of multi-resolution models, such as [17,53,54,66,67]. The false-preserving principle maintains the observations when transforming from a higher resolution model to its lower resolution one. The true-preserving principle maintains the observations when transforming from a lower resolution model to its higher resolution one.

As pointed out in [7], a visualized modeling environment can be a key enabler of multi-resolution modeling. Automated or semi-automated resolution methods for transformation and consistency checking are also essential for the success of MRM. A general mathematical theory like QMRM can provide fundamental support for constructing such environments. An MRM modeling environment can apply the model structure provided by QMRM and enforce the principles given in QMRM.

It should be pointed out that model resolution should be considered at the semantic, syntactic and pragmatic dimensions. This paper focuses on the semantic and syntactic aspects of model resolution. When dealing with resolution problems for a specific domain context, pragmatics should also be considered. Consistency among different resolution models should also be viewed from the semantic, syntactic and pragmatic perspectives.

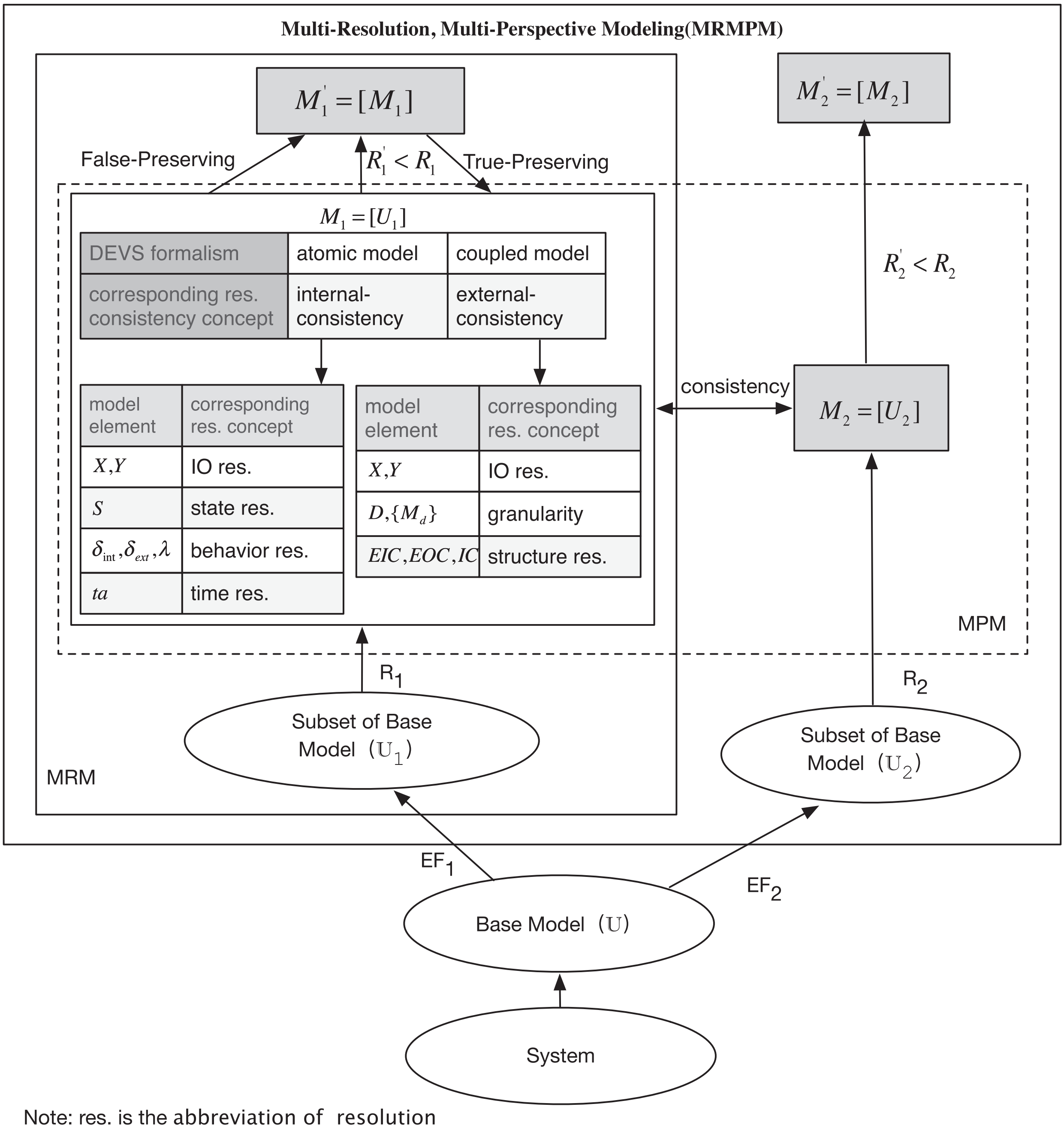

Fig. 2. Conceptual QMRM framework.

6.2. When QMRM theory meets the complex reality of multi-resolution modeling

Many aspects in MRM are difficult to quantify. What kind of state set is consistent with behavior resolution or IO resolution? The state resolution itself is difficult to measure, not to mention the measurement of consistency among different resolution perspectives in an atomic model. In practice, it is neither possible nor necessary that lower resolution states are strictly consistent with its higher resolution ones. There is also no quantitative criterion to specify the resolution consistence between different components in a simulation. Mostly, it depends on the modelers' experience or conventional rules. For example, in combat simulation, entity level models usually correlate with terrain component in great detail with individual roads, buildings, trees and obstacles, while unit level models usually correlate with grid-based low resolution terrain components where the impact of the terrain on the unit is represented with some parameters [5].

Undoubtedly, MRM is so complex that no single theory can address all MRM problems. On the one hand, a model family in QMRM is a clean hierarchical tree. This kind of structure is beneficial for modularization, model calibration, system development in general, and easy for resolution transformation and consistency maintenance, especially for MRM. On the other hand, the perfect tree structure rarely exists in real systems [7], so approximations must be made before using QMRM. When doing natural projection, usually not all elements in the base model will be considered — the base model of a complex system is too large to handle, and might even be infinitely large. Therefore, we often omit elements that are unrelated to our area of interest, and do the natural projection on the remaining elements. A perfect natural projection also almost never exists in complex systems, so the true-preserving and false-preserving principles would be true only within a certain error range. Based on QMRM proposed in this paper, we can further study fuzzy quotient spaces or rough quotient spaces based on fuzzy sets or rough sets in order to deal with the uncertainty in real systems.

6.3. QMRM and existing MRM methods

QMRM is based on DEVS and is equivalent with the MRMS presented by Liu [16]. So it would be interesting to map the abundant related research results onto QMRM. For example, the system entity structure (SES) in DEVS is a structural knowledge representation scheme that contains knowledge of decomposition, taxonomy, and coupling of a system [12]. The SES is a labeled tree with attached variable types that satisfy certain axioms. SES emphasizes the concept of hierarchy. The fact that SES describes model structure in a formal and structured manner, offers opportunities to link SES to QMRM.

Many existing MRM methods can be further formalized using the notion of quotient space. The choice of model resolution is the process of choosing an equivalence relation at a specific level from some perspective. The process of adjusting model resolution is the process of combining the corresponding quotient sets into one quotient set or splitting one quotient set into several quotient sets. Aggregation and disaggregation is a widely practiced MRM method. Aggregation is the process of quotient computing from the viewpoint of structure resolution, i.e., natural projection from a high resolution entity set to a low resolution set. Disaggregation is the inverse operation of natural projection. Selective viewing [68] is another traditional MRM approach, in which the most detailed model is simulated at all times. In selective viewing, only one model at a high level of resolution exists. At run time, a natural projection from the viewpoint of output is made dynamically when needed. IHVR (Integrated Hierarchical Variable Resolution Modeling) is an MRM method developed by Davis [69]. It is a procedure-oriented method using a hierarchical variable tree. In IHVR, natural projection is made from the perspective of behavior resolution and IO resolution.

7. Conclusions and future work

Although there have been a lot of applications using some form of resolution, multi-resolution modeling has not been fully explored in its own right. It is time to extract the common aspects from the studies from diverse fields, and to systematically and formally develop domain independent principles of multi-resolution modeling in a unified and well-formulated framework. This paper can be viewed as a step toward this goal.

Based on general set theory and DEVS model specification, we presented a mathematical framework named QMRM for multi-resolution modeling, including concepts, classification, measurement, basic observations, and principles. QMRM has the following characteristics:

(1) *Unification*. The concepts of resolution, multi-resolution modeling, and other related concepts are all based on the same mathematical theory. These concepts are defined rigorously. The modeling principles are also derived from the same mathematical theory, which gives the fact-based and widely-practiced MRM ideas a sound theoretical basis. The unification guarantees that our QMRM theory is self-consistent and integrated. QMRM lays a solid foundation for multi-resolution modeling. Such a unified conceptual framework is also beneficial for automated resolution transformation, developing new MRM methods, and model verification based on different resolution models.

(2) *Genericity*. QMRM is based on quotient space theory and DEVS. Quotient space theory is a generic mathematical theory based on the even more generic set theory. DEVS is a widely used common modeling specification based on general systems theory. The QMRM framework is developed from a generic simulation perspective, not limited to any specific application or problem domain. QMRM provides a common understanding and a theoretical foundation for multi-resolution problems in different domains. When dealing with specific application areas of simulation, these theorems can be refined. For example, the false-preserving principle and true-preserving principle could be given specific meaning in different contexts.

(3) *Extensibility*. QMRM is based on classic quotient space theory, and it can only deal with multi-resolution problems in ideal conditions. Fuzzy sets, rough sets and other extended set theories can be used to deal with more specific multi-resolution problems in the real world. The achievements in problem-solving based on quotient space theory such as hierarchical solving, synthesis techniques and inference modeling [38,62] can also be borrowed to enrich QMRM. Many DEVS-based extended formalisms have been developed, such as dynamic structure DEVS (DSDEVS), symbolic DEVS, real time DEVS (RT–DEVS) and fuzzy DEVS [12]. These extensions of DEVS can be used to extend QMRM. For example, classic DEVS can be replaced with RT–DEVS in QMRM to deal with multi-resolution problems in real-time simulation.

Building and verifying multi-resolution models is still a challenging work, especially maintaining consistency among models and validating and verifying of individual model in a MRM system. A theory-based general framework for validation and verification(V&V) of simulation models is beneficial [30], QMRM can serve as one of the theory foundations for V&V itself and the V&V of multi-resolution models. QMRM would potentially ease the burden of the modeler during the V&V phases assuming that they produced a MRM model that adhered to all of the definitions and principles contained in the QMRM.

QMRM theory is not the final result, but serves as a starting point for further theory study and a road map for simulation system development. Further research will focus on extending the quotient-base MRM theory as mentioned above, to develop an applicable version of QMRM, and study its application for different domains.

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