Smart Curtailment of Renewable Energy Resources for Increasing Capacity of Distribution Grids

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Master of Science Thesis

Delft University of Technology

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Abstract

Nowadays renewable energy sources (RES) are growing at a rapid pace, particularly the wind energy. The high amount of wind power penetration into the power grids poses a major challenge to Transmission System Operators (TSO) in terms of the operational management, as wind power is highly uncertain. The highly uncertain nature of the wind power leads to the scenarios where the power flow exceeds the grid limits leading to the capacity problem of the grid. There are multiple solutions to prevent this grid capacity problem. The first solution is the physical extension of the grid, but this requires considerable capital investments. Moreover, the frequency of the worst-case scenarios (maximum generation coinciding with minimum load) is very low and grid expansion is a much slower process, so this solution is not optimal. The second solution is to store the excessive power using batteries. The batteries cannot store power efficiently because of the storage losses and they also degrade with time. The initial setup of the batteries and their replacement (in the case of degradation) would require considerable capital investments. The third solution is reserve regulation of the generation units to deal with the uncertainty of wind power. The final solutions is to curtail excessive wind power in the grid. The last two solutions are feasible from an economic point of view when compared to the initial two solutions. However, both these approaches are expensive. But, an optimal combination of these approaches might result in an enhanced solution in terms of total energy procurement costs (a cheaper solution).

The objective of this thesis is to formulate a chance-constrained multivariate stochastic optimization problems which would perform the stochastic unit commitment and simultaneously would create an optimal combination of wind power curtailment and reserve scheduling to reduce overall cost of the system. As an initial step, the combination of the reserves and wind power curtailment (the convex combination approach) was modeled using the convex combination approach. The optimization problem corresponding to the convex combination model was formulated to find an optimal combination of reserve dispatch and wind power curtailment. Later on, the combination of reserve scheduling and the wind power curtailment was modeled using the mixed logic dynamical systems framework (MLD approach). The optimization problem corresponding to the MLD approach was formulated to find an optimal combination of reserve dispatch and wind power curtailment.

A randomization technique was used to generate various scenarios of the uncertain wind power. Based on a prior violation levels of the grid limits, we perform scenario-based stochastic optimization to obtain an optimal combination of reserve scheduling and wind power curtailment in both the approaches for each and every scenario to lower the overall costs of the system. The theoretical developments proposed were evaluated on an IEEE-30 bus network. The static and the dynamic demand cases were simulated. In both cases, the proposed approaches outperformed the reserve scheduling method.

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Mitta Sreekar Reddy

Chapter 1

Introduction

The main motive of this thesis is to develop optimal and efficient stochastic optimization strategies in order to deal with uncertain wind power problem in the smart grids. Especially in cases where the actual wind power is greater than the estimated wind power. This chapter presents a brief motivation of this thesis regarding the problem of the uncertain wind power penetration in the smart grid and how to deal with this problem. Further, we describe the problem statement in a detailed manner. Later on, a short description of the approaches solving this problem and the contributions of this thesis are presented. Finally, the chapter is concluded by etching a road map of this thesis and describing the organization of this thesis.

1-1 Motivation

With ever increasing in population, the need for the electricity (demand or load) has been increasing. The consistent increase in the fuel prices, depletion of fossil fuels and the environmental concerns paved a path for the integration of the renewable energy resources into the power grids. The pollution levels and the energy generation costs have reduced in a drastic manner with integration of the renewable energy resources. But, integration of the renewable energy resources poses a lot of operational challenges to the Transmission systems operators (TSO). The major challenges are, the renewable energy sources (RES) are highly uncertain, chaotic in nature [6] and the energy from these RES sources cannot be estimated with 100% accuracy.

In the power systems, a day ahead planning of the generation units takes place to minimize the generation costs of the system, this is called unit commitment [2]. When the RES sources are integrated into the grid, then the renewable energy should be estimated in a day ahead manner to plan the power from the generation units. This type of a day ahead estimation of the energy from the RES units is called short-term estimation [7]. Over a last few years, there has been enriched literature produced on a day ahead wind power estimation. But, chaotic nature of wind power does not allow any of these methods to predict the wind power with very high accuracy. As a result, the actual power from the RES sources would not be the same as the estimated power. This would lead to load shedding when the power from the RES sources is less than the estimated power. The frequency deviations and grid limits violation will be induced when the actual wind power is greater than the estimated wind power. Here, we focus on the problem of grid limits violations in the presence of the excessive wind power because it would lead to damage of the power systems. There are multiple solutions to deal with this problem of the grid limit violations. They are:

1. Physical extension of the grid with new cables which have higher limits.
2. Store the excessive power in the batteries.
3. Provide the down-spinning reserves from the generation units.
4. Curtail the excessive wind power in the grid.

The first solution to extend the grid is not viable since the physical extension of the grid needs a lot of money as an investment. Moreover, the frequency of the worst case scenarios (the maximum wind power with minimum load conditions) is too low. However, the physical extension of the grid doesn't solve the power congestion problem. So there is no need to extend the grid to deal with these very low frequent worst case scenarios. The second solution is to store excess wind power in the batteries. The batteries do not store power efficiently as they have storage losses. Moreover, batteries require maintenance and they must be replaced after their lifetime. Both of these operations require considerable capital investments. So this wouldn't be an efficient solution.

The third solution is reserve scheduling. In reserve scheduling, the down spinning reserves from the generation units are provided to reduce the power imbalance created by uncertain wind power in the smart grids. However, employing these down-spinning reserves in the system is also a costly approach, but it is cheaper when compared to the first two solutions. The fourth solution is to curtail the excessive wind power. By curtailing only 5% of the excessive wind power will enhance the capacity of the smart grids by 20%. But, the system operators will be penalized by the government for the curtailment of the wind power [8]. So this also turns out to be an expensive solution.

An optimal combination of reserves dispatch and wind power curtailment would result in a cost efficient solution when compared to reserve scheduling approach. So now the question is, how to create an optimal combination of the reserves and the wind power curtailment in the presence of the excessive wind power compared to the predicted power power? In the next sections, the problem statement is described and the main contributions of the thesis are briefly discussed.

1-2 Research Goal and Main Contributions of the Thesis

The goal of this thesis is to develop cost efficient stochastic optimization strategies to find out an optimal combination of the reserves and the wind power curtailment in order to reduce the total system costs.

In the path of reaching this research goal, two methods were proposed to find out an optimal combination of reserve dispatch and wind power curtailment, in order to provide a cost-efficient solution to reduce the overall system costs compared to the reserve scheduling approach. The major contributions of this thesis are:

1. **Convex Combination Approach**

   For the problem of stochastic unit commitment, integrated reserve scheduling and the wind power curtailment, a mathematical model which takes a convex combination of reserves and wind power curtailment terms was proposed. By taking the advantage of the proposed modeling approach, an optimization problem which integrates reserve scheduling and the wind power curtailment was formed. This problem was solved using a mixed integer chance constrained stochastic optimization framework.

2. **Mixed Logical Dynamical Approach**

   Another way to develop a cost-efficient solution is to use the mixed logic dynamical (MLD) approach to find out an optimal combination of the reserves and the wind power curtailment. In this method, the combination of reserves and wind power curtailment was modeled using the MLD framework. Similar to the convex combination approach, a chance-constrained stochastic optimization problem was formulated and solved. This optimization framework determines the optimal generation unit to provide reserves and the amount of reserves and the optimal wind power plants to curtail wind power and amount wind power to be curtailed.

The stochastic reserve scheduling approach which has only reserves without wind power curtailment was also simulated along with proposed approaches to demonstrate and compare the performance of the proposed approaches. The wind power generation in the grid was considered to be higher when compared to the predicted wind power. As a result, only down-spinning reserves were considered and the up-spinning reserves were neglected as a part of reserve scheduling approach. So the reserve scheduling approach has reduced number of decision variables when compared to the actual reserve scheduling approach. All the three optimization algorithms namely convex combination approach, mixed logical dynamical system approach and reserve scheduling approaches were applied to a case study. Finally, the performance parameters such as computational complexity and total system costs of the proposed algorithms were compared with reserve scheduling algorithm to analyze the trade-offs of the proposed optimization algorithms.

1-3 Structure of the Thesis

The road map of this thesis is etched in Figure 1-1. This thesis is divided into five chapters. The first chapter presents motivation and the research goals of the thesis. Further, it also presents the the main contributions of the thesis along with the organization and the road map of the thesis. The contents of the next chapters are summarized below:

Chapter-02

This chapter provides an overview on the unit commitment problem. Further, it provides a compact description on the modeling of power flow in the smart grids followed by the AC power and the DC power flow algorithms. In the next section, various optimization techniques which are used to solve the problem of unit commitment problem are explained. Later on, the comparison between the different optimization techniques to solve the unit commitment problem is provided. The unit commitment problem along with the renewable energy resources is given. The problems associated with the integration of the renewable energy into the smart grids are addressed. Finally, the problem of reserve scheduling to deal with the uncertain wind power is explained.

Chapter-03

This chapter provides a detailed explanation of the proposed methodologies to find out an optimal combination of wind power curtailment and down-spinning reserves. As an initial step, the power flow models of the grid are presented. In the next section, the convex combination model which integrates wind power curtailment and reserves followed by an optimization problem of the convex combination approach are presented in a lucid manner. In the next section, a way to deal with the probabilistic chance constraints is presented. In the following section, the concepts of logic and auxiliary variables are introduced. In the final section, the MLD modeling approach and the optimization problem associated with the MLD modeling approach are explained in a detailed manner.

Chapter-04

In this chapter, a detailed explanation on the simulation set-up is provided. The proposed optimization techniques are applied to a case study. The case study was divided into two parts. They are static demand case and dynamic demand case. The results of these cases were drafted and analyzed to demonstrate the effectiveness of the proposed algorithms. Finally, the conclusions based on the performance of the proposed optimization techniques are given.

Chapter-05

In this chapter, a conclusion for the problem of integrating reserves and wind power curtailment is provided. In the next section, four different future research directions are provided for the improvement of the uncertain wind power problem in the smart grids.

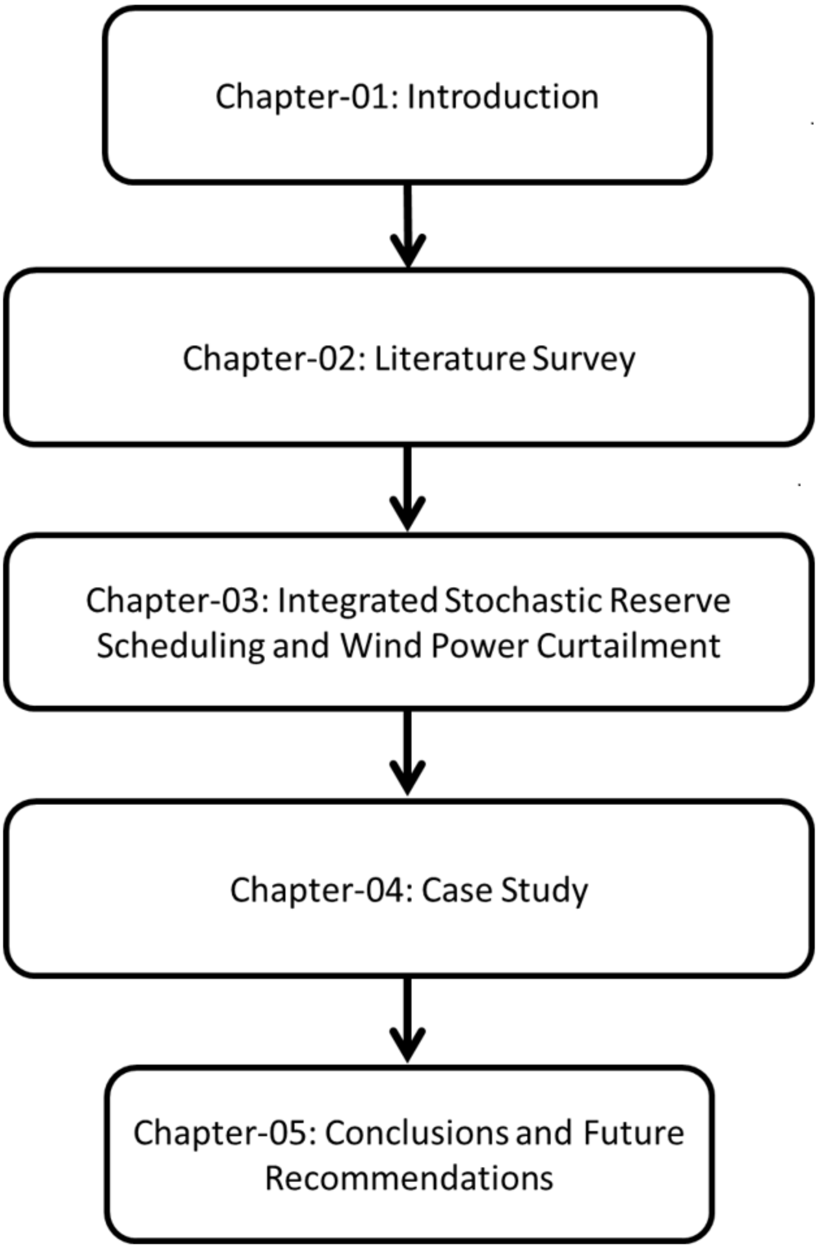


Figure 1-1: Structure of the thesis.

Chapter 2

Literature Survey

The unit commitment problem integrated with the reserve scheduling is explained in a detailed manner. Initially, unit commitment problem is defined, followed by the development of the power flow models. These power flow models are developed by building the equivalent π circuits and resulting equations of the power flow models were solved using the AC power flow calculus. The DC power flow framework has been introduced to overcome the computational complexity problems of the AC power flow model. Later on, the objective function and the constraints of the unit commitment problem are detailed, while in the next sections various optimization techniques to solve the unit commitment are explained and compared. The unit commitment problem with renewable energy sources and the challenges associated to this kind of problem are given. The robust unit commitment strategies are explained to deal with uncertain wind power in the smart grids.

2-1 Unit Commitment

We have seen a drastic improvement in the modern day power system. Right from the isolated systems, power systems have evolved into highly complex interconnected systems. These complex and interconnected power systems are highly reliable [9]. But at the same time, they pose many challenges in terms of operational planning and economics. The power systems are classified into three subsystems. They are generation systems, distribution systems, and the transmission systems [10]. Each of these three subsystems have their own operational constraints and attributes which make the operational planning of power in the smart grids more challenging.

The power systems experience a varying load with respect to the time of the day and day of the week based on requirements of the population. To suffice the varying demand, the system operators tend to switch on and switch off the generators based on the varying demand. A prior decision has to be taken on the commitment of the generation units to reduce the generation costs of the system. The process of making the prior decisions on which units to switch off and switch on is called unit commitment [2]. The generation units need heat as input to generate the electricity. The amount of heat required to generate every megawatt of electricity differs based upon the generation units. The fossil fuels such as coal, diesel etc. are used to generate the heat in the machines. The amount of fuel required to generate one megawatt of electricity differs for every generation unit. As a result, the cost of each megawatt of electricity differs from unit to unit. These costs significantly influence the unit commitment problem.

As the name suggests, unit commitment is an hourly commitment schedule of the generation units in the planning horizon. It is determined in a day ahead manner to minimize the generation costs of the units. The horizon of planning would be from hours to weeks. The unit commitment problem is divided into two main sub-problems as shown in Figure 2-1. They are:

- Determine the optimal units to commit based on the varying load.
- Minimize the generation costs of the system over the optimization horizon.

Each of the generation units has different costs, different maximum and minimum generation levels, and different ramp-up and ramp-down limits. The unit commitment algorithm considers all these costs and the constraints to minimize the total costs of the system. To understand the unit commitment problem in a detailed manner, we will first define what a grid is. In the next step, the power flow model in the smart grids is presented. Later on, we explain the AC power flow and the DC power flow models to optimize the power production in the grid.

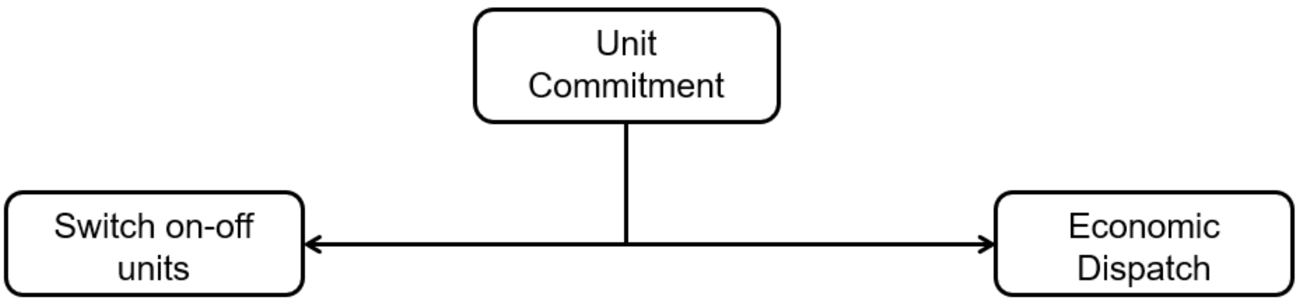


Figure 2-1: Subproblems of the unit commitment problem.

2-1-1 Grid Models

A grid is set of interconnected elements put together [1]. By connecting the equivalent π circuits as shown in the Figure 2-2, the mathematical relationship between the voltages, the impedances and the currents can be transformed into the equations in the next page [1]:

$$
\begin{aligned}
y_{11}v_1 + y_{12}v_2 + ...y_{1n}v_n &= i_1 && \text{(2-1)}\\
.... & && \text{(2-2)}\\
.... & && \text{(2-3)}\\
y_{n1}v_1 + y_{n2}v_2 + ...y_{nn}v_n &= i_n && \text{(2-4)}
\end{aligned}
$$

where y_{ij} denotes the complex admittance, v_i denotes the complex nodal voltages and i_i denotes the complex current at the bus. The matrix form of the equations mentioned above is:

$$\underline{Y} \times \underline{v} = \underline{i} \tag{2-5}$$

where $\underline{Y}$ is the admittance matrix, while v is the vector of the voltages.

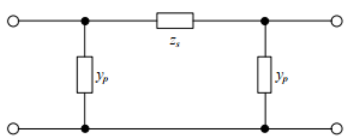
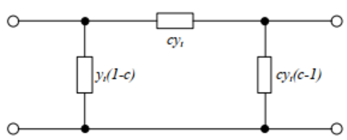
Network Element	p.u. Impedance/ Admittance	Equivalent π Circuit
Transmission Line	$\underline{z}_s = (R' + jwL')l\frac{S_r}{V_b^2}$ $\underline{y}_p = \frac{1}{2}(G' + jwC')l\frac{V_b^2}{S_r}$ R' : resistance [Ω/km] L' : inductance [H/km] G' : conductance [S/km] C' : capacitance [F/km] l : length [km]	
Transformer	$\underline{z}_t = \underline{v}_{sc}\frac{S_r}{S_b}$ $\underline{y}_t = \frac{1}{\underline{z}_t}$ $c = \frac{V_{r1}}{V_{b1}}\frac{V_{b2}}{V_{r2}}$ $\underline{v}_{sc}$: relative short circuit voltage referenced to primary side c : per unit ratio	

Figure 2-2: Network Elements and their respective π circuits [1].

In order to build the $\underline{Y}$ matrix, the diagonal elements are considered to be the net impedances connected to the bus i, while the off-diagonal elements denote the negative admittance between the buses i and j. The apparent power at n number of buses is given by [11]:

$$\underline{s} = p + iq = v_{\text{diag}}i^* \tag{2-6}$$

By substitution of equation (2-5) into equation (2-6), we obtain:

$$s = \underline{v_{\text{diag}}} \times \underline{Y}^* \times \underline{v}^* \tag{2-7}$$

The apparent power at every bus i is also given by:

$$s_i = \underline{v_i} \sum_{j=1}^{n} \underline{y_{ij}^*} \times \underline{v_j^*} \tag{2-8}$$

where $\underline{y_{ij}} = g_{ij} - jb_{ij}$, $\underline{v_i} = v_i e^{-i\theta}$ and $\underline{v_j} = v_i e^{-j\theta}$. From the equations above, we formulate the following non-linear power flow equation:

$$p_i = v_i \sum_{j=1}^{n} v_j (g_{ij} cos(\theta_i - \theta_j) + b_{ij} sin(\theta_i - \theta_j)) \tag{2-9}$$

$$q_i = v_i \sum_{j=1}^{n} v_j (g_{ij} sin(\theta_i - \theta_j) + b_{ij} cos(\theta_i - \theta_j)) \tag{2-10}$$

where the term p_i represents the active power at the i^{th}, while the term q_i denotes the reactive power at the i^{th} bus. The terms v_i and θ_i denote the voltage and the voltage angle respectively at the i^{th} bus. The equations (2-9) and (2-10) are highly non-linear and are classified as AC power flow equations. In order to solve the equations (2-9) and (2-10), at least two of the variables have to be fixed for each node. The active and the reactive power should be separated [12]. The obtained equations are solved using the AC power flow calculus.

AC Power Flow Calculus

The most commonly used algorithm to solve the non-linear equations is Newton-Raphson algorithm. In this algorithm, the nonlinear equations are linearized using Taylor series approximations. In every iteration the nonlinear equations are linearized to solve the AC power flow problem. The non-linear problem is linearized and can be written as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial p}{\partial v} V \\ \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial v} V \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \frac{\Delta V}{V} \end{bmatrix} \tag{2-11}$$

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = J \begin{bmatrix} \Delta \theta \\ \frac{\Delta V}{V} \end{bmatrix} \tag{2-12}$$

where J represents the Jacobian matrix, the solution is computed by taking the inverse of the Jacobian matrix and updating the voltage angles and magnitudes. The voltage updates can be stopped when convergence is reached. This method has quadratic convergence. It is computationally intensive and iterative in nature [12]. In order to overcome the disadvantages of the AC power flow, the DC power flow formulation has been introduced.

2-1-2 DC Power Flow

The AC power flow models are analyzed to be computationally complex [13]. If there are n nodes in the grid, then there will be 2n nonlinear equations to be solved in the AC power flow algorithm. Moreover, the convergence is not assured in the AC power flow algorithm [13]. In order to reduce the computational intensity and to assure convergence the DC power flow model was introduced. The DC power flow model is a linearized version of the AC power flow model. The DC power flow algorithm focuses more on active power flow while it neglects the voltage support, reactive power, and transmission losses. There are various assumptions made in the DC power flow algorithm which make the problem linear and simpler to solve. They are [14]:

- The difference of the voltage angle is small, i.e. $sin(\theta_1 - \theta_2) = \theta_1 - \theta_2$.
- All the voltage profiles are flat, i.e. the voltages at all the nodes are equal to 1 p.u. (per unit).
- Line resistances are negligible, i.e. $X_l >> R$, this implies that the transmission lines are lossless.

However, these assumptions are not always realistic. In most of the cases, the voltage profiles are not flat. The X/R ratio condition is not very realistic as well, so all these assumptions have an effect on the accuracy of the solution of DC power flow models [14].

DC Power Flow Formulation

The power flow through a transmission line between two nodes s and r is given by [14]:

$$P_L = \frac{V_s V_r}{X_L} sin(\theta_{sr}) \tag{2-13}$$

with V_s and V_r to be the complex voltages at the nodes s ands r respectively, where X_L represents the impedance of the transmission line and θ_{sr} is phase angle. Based upon the assumptions of the DC power flow model, the equation (2-13) turns out to be:

$$P_L = \frac{\theta_{sr}}{X_L} = B_L(\theta_{sr}) \tag{2-14}$$

The system can be modeled with a set of linear equations give below [13]:

$$\Delta P_L = \sum_{r=1}^{n} B_{sr}(\theta_s - \theta_r) \tag{2-15}$$

Finally, the equation can be modified as:

$$\Delta P = B' \Delta\theta \tag{2-16}$$

The solution of the equation (2-16) can be obtained in a single step, which is very simple and fast when compared to AC power flow. This makes DC power flow model to be more computationally efficient when compared to AC power flow model.

Advantages of DC Power Flow

The advantages of the DC power flow over the Newton- Raphson methods are [12]:

- The dimension of the B matrix is half the size of the original problem.
- The solution to DC power flow problem is not iterative, it can be calculated in a single step.
- The B' matrix is independent of the systems states, it is calculated only once based on the system's topology.

The first two advantages make the DC power flow algorithm 7 to 10 times faster than the AC power flow. The biggest advantage is that the solution of DC power flow is non-iterative. As a result, the computation time of the system is reduced. The DC power flow equations give the power injections at every node. These power injections can be transformed into the power flow in the lines. These transformations can be observed in the third chapter.

2-1-3 Optimization Problem: Unit Commitment

Economical operation of the power systems is extremely important to minimize the total system costs and at the same time minimize the fossil fuel consumption. In the power systems, the net demand of the system fluctuates a lot. The objective of the unit commitment is to find an economical and feasible solution for N_{g} number of generation units over a period of T time steps in the presence of the varying demand.

Cost Function

The objective function consists of the start-up and shut-down costs and the generation costs for each of the generation units. The objective function is give by equation [15]:

$$\sum_{t=1}^{T}\sum_{i=1}^{N_{\mathrm{g}}}((a_i + b_i P_{i,t}^{\mathrm{g}} + c_i (P_{i,t}^{\mathrm{g}})^2) + SU_{i,t} + SD_{i,t}) \tag{2-17}$$

where the term $P_{i,t}^{\mathrm{g}}$ denotes the output power of the system, while the terms b_i and c_i represent the linear and quadratic cost coefficient of the i^{th} generation unit respectively. The term a_i denotes the constant fuel consumption of the i^{th} generation unit. The terms $SU_{i,t}$ and $SD_{i,t}$ denote the start-up and the shut-down cost of the generation units. Two questions which need to be answered are:

1. Why are the system costs quadratic in nature?

2. Why does a generation unit need start-up costs?

The fuel cost curve of a thermal generation is nonlinear. This can be observed in Figure 2-3. The nonlinear curve can be approximated with a quadratic function as given in the equation (2-17) by leaving back the start-up and shut-down costs of the system. The terms b_i, c_i and a_i are found experimentally to approximate the nonlinear curve [2] as shown in Figure 2-3. The order of the quadratic function can still be reduced to linear terms by the piecewise approximation of the quadratic curve [16].

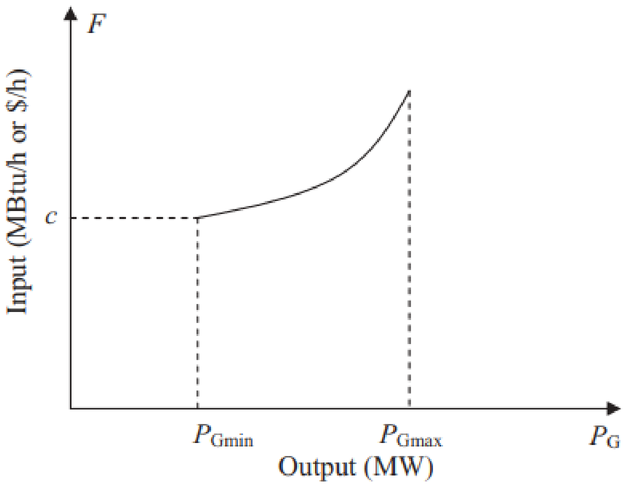


Figure 2-3: The fuel cost curve of a thermal generation unit [2].

Now coming to the second question, the generation units require a certain amount of energy to bring them on-line. This is because pressure and temperature build up slowly in the generation units [2]. This is the reason that the generation units need start-up costs. The constraints of the optimization problem are:

Constraints of Unit Commitment:

1. Power Balance Constraints:

$$\sum_{i=1}^{N_{\text{g}}} (P^{\text{g}}_{i,t}) u^{\text{g}}_{i,t} = P^{\text{d}}_{i,t} \tag{2-18}$$

2. Maximum and minimum generation limit constraints

$$\mathrm{P}_i^{\text{min}} \leq P^{\text{g}}_{i,t} \leq \mathrm{P}_i^{\text{max}} \tag{2-19}$$

3. Ramping limits of the system constraints:

$$-P^{\text{g}}_{\text{down,i}} \leq P^{\text{g}}_{i,t} - P^{\text{g}}_{i,t-1} \leq P^{\text{g}}_{\text{up},i} \tag{2-20}$$

4. Line Limits of the system constraints:

$$-P_{\text{line}} \leq P^{\text{f}}_{i,t} \leq P_{\text{line}} \tag{2-21}$$

5. Minimum and maximum up and down time of the generation units.

The first constraint is the coupled constraint with a binary variable and a continuous variable coupled together to form a bilinear term. This constraint ensures that the load is met with enough power. The second constraint ensures that the generation levels should be within the limits of the generation unit. The maximum and minimum generation levels depend upon the generation units, and are found out experimentally [2] so that a safe and stable operation of the generations units is ensured. The third constraint says that the ramping of the system should be within the limits. The fourth constraint says that the power flow in the lines should be within the line limits of the system. The fifth constraint ensures that any generation unit cannot switch off and switch on in two consecutive time steps. Each of the generation units takes minimum time to switch off and switch on. The optimization problem with constraints is presented in this section. Various optimization techniques to solve the unit commitment problem are discussed in the next paragraph.

2-1-4 Overview of the Optimization Techniques in Unit Commitment

There has been an enriched literature presented in the past such as genetic algorithm [17], dynamic programming [18], priority list method [19], particle swarm optimization [20], mixed integer programming [21], Lagrangian relaxation [22], heuristic methods [23], simulated annealing [24] and ant colony optimization [25] to deal with the unit commitment problem. In this section, most of the optimization problems are explained in a compact manner to understand the advantages and the disadvantages of theses methods.

Lagrangian relaxation is one of the approaches which is used to solve the non-linear integer unit commitment problem. In the unit commitment problem, there are two types of constraints. The first type of constraints are the coupled constraints [26] corresponding to the demand, reserves etc. The second type of constraints are called the local constraints corresponding to the generation units. The coupled constraints are relaxed by using the Lagrangian multipliers. The problem is decomposed into N_{g} number of subproblems which can be solved by the dynamic programming. The term N_{g} represents the number of generation units in the grid. There are different methods to select the Lagrangian multipliers and to apply this method to the practical systems [27]. However, using the Lagrangian relaxation method would result in the suboptimal solutions and as the size of the problem increases and the probability of convergence will decrease.

The genetic algorithm or the evolutionary programming algorithm is the global optimization technique which could be used to solve the unit commitment problem. The genetic algorithm represents a class of stochastic search techniques which is inspired by Darwin's theory that the fittest will survive [28]. There are four stages in a genetic algorithm. They are initialization, selection of the off-spring, evaluation of the off-spring and stopping. In the initial stages, the coding function of the genetic algorithm transforms the decision variables (both the binary variables and the continuous variables) into binary strings [28]. Then the genetic algorithm goes ahead with its operation and creation of the offspring. Next, the decoding function transforms the binary strings into the decision variables. In the third stage, the objective function is calculated from the decision variables and later on evaluated for the fitness function. There are two drawbacks with this genetic algorithm. The first one is that the size of the optimization problem and the computation time of the optimization problem are high and sometimes solutions might not converge. The second one is incorporating the constraints to the optimization problem. A hybrid method which combines the priority list with the evolutionary programming to deal with the unit commitment of large-scale systems was introduced to minimize the computation time. In the genetic algorithm, the initialization would be random, by using the priority list. The generation units with minimum cost are given priority to minimize the overall system costs. Four different methods were proposed for mutations or the off-spring creation [29]. These four methods are compared with each other to analyze the advantages and the disadvantages of the approaches. Now coming to the second problem, the penalty terms were added to the cost function to minimize the constraints violations. The solutions with constraint violations would result in very high system costs.

There many other optimization techniques such as PSO, heuristic methods, ant-colony optimization etc. The main drawbacks of these optimization techniques are computational complexity and convergence [28]. In order to avoid this computational complexity, the unit commitment problem is solved by using the mixed integer quadratic programming or mixed integer linear programming. There are various advantages of the mixed integer programming techniques when compared to other optimization techniques [16]. They are:

- The solution time is drastically reduced.
- The non-linear constraints can be linearized in a simpler manner.
- This method is also applicable to very large systems.

The coupled constraints and the start-up costs are mathematically represented in an efficient manner to decrease the computation time of the system [30]. This kind of mathematical representation would result in an enhanced solution in less time.

2-2 Unit Commitment with Renewable Energy Sources

In order to deal with the problems such depleting fossil fuels, increasing energy demand and increase in the global warming, many countries introduced renewable energy resources (RES) into the grid. The unit commitment problem is a day ahead scheduling process, so the renewable energy production has to be predicted in a day ahead manner to run the unit commitment. But a majority of the grids have wind power as a source of renewable energy. There are many sophisticated ways and enriched literature presented on the wind energy forecasting such as time-series methods [7], neural networks [6] and hybrid methods [31]. But forecasting wind energy with 100% accuracy is impossible because of it's chaotic nature. This would result in two problems. Firstly, if the actual wind power is less than the predicted wind power then the demand or the load cannot be met with enough power. Secondly, in a scenario where the actual wind power is greater than the predicted wind power there would be the frequency deviations. To avoid these two problems, the concept of reserves has been introduced to deal with uncertain wind power in the grid.

2-2-1 Modeling of the reserves

The reserves are introduced to deal with generation load mismatches in the grid. The generation load mismatches can arise either because of the uncertain wind power or the loss of a generator or a load in the grid [32]. The reserves are carried to suffice the unintended and unexpected generation deficit. The load shedding happens in the case of the unexpected generation deficit. The secondary frequency control or the automatic frequency control is activated in order to adjust the power production from the generation units. The frequency deviation is compensated and the tie line power is brought back to scheduled value by this secondary frequency control. This secondary frequency control determines the new set points to the generation units in order to mitigate the frequency deviation by either up-spinning or down-spinning the generators. This secondary frequency control acts in a distributed manner. The weighing vectors d^{ds} and d^{us} which represent down-spinning and up-spinning respectively are introduced for the generation units. By adding these vectors to the generation units, the excessive or deficit power is being compensated. If a generation unit is not participating in the secondary frequency control, then it's corresponding values of d^{ds}, d^{us} become zero. The sum of these distribution weighing vectors is always one to make sure that distribution reserves are equal to the uncertain wind power [33]. The reserves can be modeled as:

$$P_{\mathbf{inj},t} = C_{\mathrm{g}}(P_t^{\mathrm{g}} + R_t^{\mathrm{rs}}) + C_{\mathrm{w}}P_t^{\mathrm{w}} - C_{\mathrm{d}}P_t^{\mathrm{d}} \tag{2-22}$$

where $C_{\mathrm{g}} \in \mathbb{R}^{N_{\mathrm{b}}\times N_{\mathrm{g}}}, C_{\mathrm{w}} \in \mathbb{R}^{N_{\mathrm{b}}\times N_{\mathrm{w}}}$and $C_{\mathrm{d}} \in \mathbb{R}^{N_{\mathrm{b}}\times N_{\mathrm{d}}}$ are the matrices which denote the interconnections between the load sinks, the generators and the wind power units with network buses. The term R_t^{rs} represents the linear correction term and given as the linear function of the total generation-load mismatch. By considering uncertainty only in the wind power forecasting then the linear correction term [33] can be modeled as:

$$R_t^{\mathrm{rs}} = -d_t^{\mathrm{ds}}|P_{w_t}|_+ + d_t^{\mathrm{us}}|P_{w_t}|_- \tag{2-23}$$

where

$$|P_{w_t}|_+ = \begin{cases} \Delta P_{w_t} \text{ if } \Delta P_{w_t} \geq 0 \\ 0 \text{ otherwise} \end{cases} \tag{2-24}$$

$$|P_{w_t}|_- = \begin{cases} \Delta P_{w_t} \text{ if } \Delta P_{w_t} \leq 0 \\ 0 \text{ otherwise} \end{cases} \tag{2-25}$$

and

$$\Delta P_{w_t} = P_{w_t} - P_{w_t}^f \tag{2-26}$$

where P_{w_t} is the actual wind power, while $P_{w_t}^f$ represents the forecast of the wind power. If $\Delta P_{w_t} \geq 0$ then the down-spinning reserves should be provided. This means the amount of production should be decreased from the generation units. In the case of $\Delta P_{w_t} \leq 0$ the up-spinning reserves should be provided, this means the amount of power production should be increased in order to reduce the frequency deviation. The the frequency deviation arises because of difference between the actual wind power and the wind power forecast. The reserves are optimized by minimizing these distribution vectors. A detailed optimization problem is given in the appendix of the thesis.

2-2-2 Optimal Reserve Scheduling

In order provide optimal reserves, we need to identify various reasons for load shedding or the frequency deviations. The various reasons for the load shedding and frequency deviations are listed in [34]. They are:

1. Having a situation where the load and the wind power deviate greater than the system reserve levels.
2. Having a generator trip and the load and the wind power variations greater than the system reserve levels.
3. Having a generator trip and the load and the wind power variations after some time of the previous generator trip.

The optimal reserves were provided on the basis of the individual and the combinations of the reasons of the load shedding and frequency deviations. The optimal reserves were provided based on the probability distribution of generator trip and wind power forecast error. The load and the wind power were considered to be uncorrelated and the prediction error was considered to be gaussian [34]. The net uncertainty was taken as a combination of probabilistic individual uncertainties. Finally, the reserves were provided by considering the various combinations of probabilistic uncertainties. There are two issues with this approach. The first issue is that the error of the wind power is not Gaussian. The second issue is the way they model the reserves is very complex. The reserves can be modeled as a convex combination of the generator outage and the wind power prediction error which is very simple [3]. A scenario based two-stage stochastic optimization problem has been proposed to deal with the uncertain wind power. In the first stage, the power generation from the slow generators is optimized. In the second stage, the power from fast generators considering the uncertain wind power is optimized in the grid. A two-stage optimization strategy which takes an optimal trade-off between the robust reserve scheduling and the stochastic reserve scheduling has been developed [35], to analyze the trade-off between the robust and stochastic programming approaches. The two-stage stochastic optimization problems are broken down by using two methods. They are benders decomposition [36] and Lagrangian relaxation [30].

The two-stage stochastic optimization problem always results in suboptimal solutions [37]. To avoid the suboptimal solutions, a single stage chance constrained multi-objective stochastic program has been proposed to obtain optimal reserves dispatch in the grid. The authors of [38] have also provided an adaptive robust optimization technique to avoid the suboptimal solutions of the two-stage stochastic optimization problems. In the chance constrained optimization problem, the chance constraints are transformed into a scenario based problem. In this method a randomization approach is used to generate various scenarios of the uncertain variable. The number of scenarios depends upon the decision variables, as the decision variables of the unit commitment and the reserve scheduling are very high, the solution will be intractable. So a framework has been developed in [33] to provide the probabilistic guarantee on the tractability by considering the uncertainty to be a hyper-rectangle [3, 39]. The boundaries of the hyper-rectangle are minimized along with optimization problem. This approach have resulted in a tractable solution.

2-3 Summary

The unit commitment problem followed by the power flow models in the smart grids were introduced in this chapter. The disadvantages of non-linear power flow models i.e. the AC power models were explained. The DC power flow model was introduced to overcome the disadvantages of the AC power flow models. Later on, the objective function and the constraints of the unit commitment models were explained. In the next section, the unit commitment problem with renewable energy resources was given. The problems associated with the integration of the renewable energy resources were detailed. The reserve scheduling was introduced to deal with the uncertain wind power problem in the smart grids. Finally, various optimization techniques for the optimal reserve scheduling problem were analyzed to understand the drawbacks and the advantages of each of these methods.

Chapter 3

Integrated Stochastic Reserve Scheduling and Wind Power Curtailment

The theoretical frameworks to deal with stochastic unit commitment with integrated reserve scheduling and wind power curtailment problem with uncertain generation resources (Renewable Energy Resources) are explained in this chapter. Two methodologies were developed to deal with the problem of uncertain wind power in the grids. Both these methodologies were developed using the DC power flow models. In the first method, we consider the convex combination of reserve scheduling and wind power curtailment. In the second method, the convex combination model was transformed into the MLD model to determine the optimal action between the reserve scheduling of the generation unit and the wind power curtailment from the wind power plant.

The rest of the chapter is organized in the following manner. In the first section, we describe the problem setup and mathematical model of the DC power flow framework. In the next section, the problem formulation of the stochastic unit commitment with reserve scheduling and wind power curtailment using the convex combination approach is presented. In the next section, the convex combination model was transformed into the MLD model. Later on, the optimization problem corresponding to the MLD model is explained. Finally, the last section provides summary of the chapter.

3-1 Problem Setup

This section provides information on the DC power flow model, which is considered to be the steady-state model commonly used in power planning of smart grids. The DC power flow model is obtained by linearizing the AC power flow model. The DC power flow models are used to set up the problem of the combination of reserve scheduling and wind power curtailment. In this section, certain assumptions of this thesis are explained.

3-1-1 Definitions and Assumptions

A grid comprising of N_l lines, N_w wind farms, N_g conventional generators, N_b buses and N_d load sinks were considered in the optimization problem. Without loss of generality, a single wind power plant is considered in the grid, i.e. $N_\mathrm{w} = 1$. The assumptions of this thesis were:

- The DC power flow model based on [40] is used to formulate the optimization problem.
- The actual wind power penetration into the grid is always greater than the estimated wind power.
- Only uncertain parameter in the grid is the wind power, there is no load uncertainty in the grid.
- $N - 1$ security constraints are not considered [41].
- Wind power is assumed to be a controllable parameter.

The first assumption is to reduce the computational intensity of optimization [40]. The AC power flow model has more number of variables compared to the DC power flow model. The number of worst case scenarios of uncertain variables to be generated depends upon the number of decision variables. If the AC power flow model is used in that case the number of decision variables are high which leads to an increase in the number of scenarios. This might lead to an increase in computational complexity and result in an intractable solution. To avoid these problems, the DC power flow framework was used to formulate the problem. The second assumption is valid as wind power curtailment is only possible when there is excessive wind power than the estimated wind power. The third assumption is to focus on reserves and wind power curtailment in the presence of wind power uncertainty in the grids. The fourth assumption refers to the outages in the smart grids, the $N - 1$ security criteria can be modeled using the procedure mentioned in [41]. The outages are not considered in this thesis. The final assumption to consider wind power as a controllable parameter is valid since many sophisticated control algorithms were already developed to regulate the power generation from wind turbines [42, 43].

3-1-2 Modeling of the Power Flow in the Grids

The details of the DC power flow model are provided along with the assumptions made in the DC power flow model. The assumptions of the DC power flow model are:

- The voltage at every bus in the network remains constant as 1 per unit (p.u).
- The active power losses are neglected which means the line is assumed to be lossless.
- The voltage angle at every bus is very small, which means $\sin(\theta_{pq}) = \theta_{pq}$, where θ_{pq} is the voltage angle across the buses p and q.

The power flows from the generators to the load sinks through transmission lines. The active power flow in a transmission line between two buses p and q is given as:

$$P^{\mathrm{f}}_{pq} = b^{\mathrm{f}}_{pq}(\theta_p - \theta_q) \tag{3-1}$$

where θ_p, θ_q are the voltage angles at buses p and q respectively, while the term b^{f}_{pq} denotes the imaginary part of the admittance of the line connecting the buses p and q. By encrypting the equation above in a matrix form, we obtain:

$$P^{\mathrm{f}} = B^{\mathrm{f}}\theta \tag{3-2}$$

where $P^{\mathrm{f}} \in \mathbb{R}^{N_{\mathrm{l}}}$ and $\theta \in \mathbb{R}^{N_{\mathrm{b}}}$ are vectors which represent the power flow of each line and the voltage angle at every bus respectively. The power injection matrix is given by:

$$P_{\mathrm{inj}} = B_{\mathrm{bus}}\theta \tag{3-3}$$

where $P_{\mathrm{inj}} \in \mathbb{R}^{N_{\mathrm{b}}}$ denotes the power injection vector which consists of the power injection from every bus. The term $B_{\mathrm{bus}} \in \mathbb{R}^{N_{\mathrm{b}} \times N_{\mathrm{b}}}$ denotes the admittance matrix of the network. To express the power in the lines as a function of the power injections at the nodes, the term θ should be eliminated as follows:

$$\theta = (B_{\mathrm{bus}})^{-1} P_{\mathrm{inj}} \tag{3-4}$$

But, B_{bus} cannot be inverted as this is singular. In order to make the matrix B_{bus} invertible, a row and a column corresponding to slack bus and reference angle are eliminated from the B_{bus} matrix. By eliminating a row and a column, a new matrix called B_{dc} is obtained which is invertible. Also an element in the P_{inj} corresponding to the reference bus is removed. This gives a new power injection matrix $\tilde{P}_{\mathrm{inj}}$. The equation now becomes:

$$\tilde{\theta} = (B_{\mathrm{dc}})^{-1} \tilde{P}_{\mathrm{inj}} \tag{3-5}$$

Now substituting (3-5) into equation (3-2) we obtain:

$$\theta = \begin{bmatrix} (B_{\mathrm{dc}})^{-1}\tilde{P}_{\mathrm{inj}} \\ 0 \end{bmatrix} \implies P^{\mathrm{f}} = B^{\mathrm{f}} \begin{bmatrix} (B_{\mathrm{dc}})^{-1}\tilde{P}_{\mathrm{inj}} \\ 0 \end{bmatrix} \tag{3-6}$$

The power injection vector [44] can also be written as:

$$P_{\mathbf{inj},t} = C_{\mathrm{g}} P^{\mathrm{g}}_t + C_{\mathrm{w}} P^{\mathrm{w}}_t - C_{\mathrm{d}} P^{\mathrm{d}}_t \tag{3-7}$$

where $C_{\mathrm{g}} \in \mathbb{R}^{N_{\mathrm{b}} \times N_{\mathrm{g}}}, C_{\mathrm{w}} \in \mathbb{R}^{N_{\mathrm{b}} \times N_{\mathrm{w}}}$and $C_{\mathrm{d}} \in \mathbb{R}^{N_{\mathrm{b}} \times N_{\mathrm{d}}}$ are the matrices which denote the interconnections between the load sinks, the generators and the wind power units with network buses. The element (i, j) of the above mentioned matrices is "1" if and only if the generator i (the wind power and the demand power respectively) is connected to j^{th} bus of the grid. The terms $P^{\mathrm{g}}_t \in \mathbb{R}^{\mathrm{N_g}}$, $P^{\mathrm{w}}_t \in \mathbb{R}^{N_{\mathrm{w}}}$ and $P^{\mathrm{d}}_t \in \mathbb{R}^{N_{\mathrm{d}}}$ correspond to the power from the generation units, the wind power and the demand power respectively. The power flow modeling in the presence of the wind power forecast is seen, the extra variables to tackle the uncertain wind power are modeled in the next sections.

3-2 Convex Combination Approach

The main reasons for the generation load mismatches are:

- The difference in the actual wind power and the predicted wind power.
- The generator outages or load losses.

The above mentioned cases might lead to frequency deviations in the grid. The secondary frequency control or the automatic generation control (AGC) is activated to minimize the frequency deviations in the grids. The AGC control will adjust the production from the generation units to compensate for the frequency deviations in the grid and bring back the tie line exchange to the scheduled value. This AGC scheme works in a distributed manner as explained in the literature survey. As a result the generation units change their values and attain new steady state values to compensate the active power imbalance in the grid. The AGC control scheme can deal with both up-spinning reserves when the actual wind power is less than the forecast value and down-spinning reserves when the actual wind power is greater than the estimated wind power. The process of providing up-spinning and down-spinning results is called reserve scheduling. A schematic diagram of reserve scheduling is given in Figure 3-1.

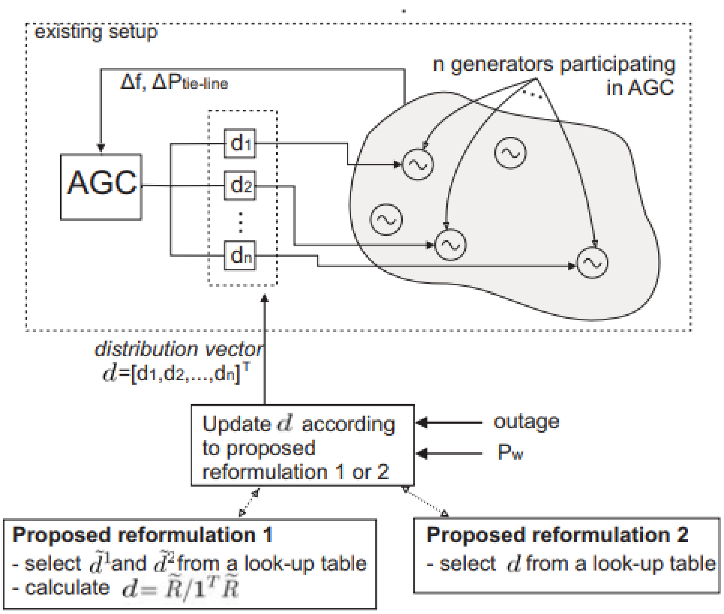


Figure 3-1: The schematic diagram of the reserve scheduling [3].

If the actual wind power is less than the predicted wind power then reserves regulation is only the way to deal with less wind power. But if the actual wind power is greater than the predicted wind power, there is wind power curtailment (to control the wind power) as an alternative to reserve scheduling.

3-2-1 Modeling of Reserve Scheduling and Wind Power Curtailment

There are two ways to deal with the frequency deviations caused by the excessive wind power in the grid rather than the estimated wind power. They are:

- Down spinning of the reserves (reserve regulation)
- Wind power curtailment (wind power spillage)

An optimal combination of reserve regulation and wind power curtailment would result in an enhanced solution in terms of total system costs compared to the reserve scheduling approach. A control scheme similar to the AGC control algorithm called the convex combination approach has been developed to obtain an optimal combination of the reserves and wind power curtailment. A schematic diagram of this control approach can been seen in Figure 3-2. As an initial step, a mathematical model which unifies the reserves and wind power curtailment is developed. It is:

$$\Delta P_t^{\mathrm{w}} = \sum_{i=1}^{N_{\mathrm{g}}} R_{i,t}^{\mathrm{ds}} + \sum_{i=1}^{N_{\mathrm{w}}} P_{i,t}^{\mathrm{wc}} \tag{3-8}$$

the term ΔP_t^{w} denotes the difference between the actual wind power and the estimated wind power, $R_{i,t}^{\mathrm{ds}}$ represents amount of the down-spinning from every generation unit i at time-step t. The term $P_{i,t}^{\mathrm{wc}}$ denotes the wind power curtailment from every wind power unit i at time step t. The equation (3-8) encodes that the uncertain wind power in the grid can be compensated with the convex combination of reserves regulation and wind power curtailment. The deviation of actual wind power from the predicted wind power is be given by:

$$\Delta P_t^{\mathrm{w}} = P_t^{\mathrm{w}} - P_t^{\mathrm{w,f}} \tag{3-9}$$

where $P_t^{\mathrm{w,f}}$ is the wind power forecast and P_t^{w} is the actual wind penetration into the grid at time step t. Based on the second assumption of this thesis, the actual wind power in the grid is greater than the estimated wind power. This leads to the following equation:

$$\Delta P_t^{\mathrm{w}} \geq 0 \tag{3-10}$$

The power balance equation with a day ahead estimated wind power is given by:

$$\sum_{i=1}^{N_{\mathrm{g}}} P_{i,t}^{\mathrm{g}} + \sum_{i=1}^{N_{\mathrm{w}}} P_{i,t}^{\mathrm{w,f}} - \sum_{i=1}^{N_{\mathrm{d}}} P_{i,t}^{\mathrm{d}} = 0 \tag{3-11}$$

where $P_{i,t}^{\mathrm{g}}$ denotes power from every generation unit i at time step t, while the terms $P_{i,t}^{\mathrm{w,f}}$, $P_{i,t}^{\mathrm{d}}$ denote the power from the i^{th} wind power plant and the power from demand unit i respectively at time step t. But, this equation will not hold in the presence of wind power uncertainty, so this equation along with the convex combination model would be:

$$\sum_{i=1}^{N_\mathrm{g}}(P_{i,t}^{\mathrm{g}} - R_{i,t}^{\mathrm{ds}}) + \sum_{i=1}^{N_\mathrm{w}}(P_{i,t}^{\mathrm{w}} - P_{i,t}^{\mathrm{w}}) - \sum_{i=1}^{N_\mathrm{d}} P_{i,t}^{\mathrm{d}} = 0 \tag{3-12}$$

where the terms $P_{i,t}^{\mathrm{w}}$ and $R_{i,t}^{\mathrm{ds}}$ denote the amount of the wind power curtailment and amount of the down-spinning reserves respectively.

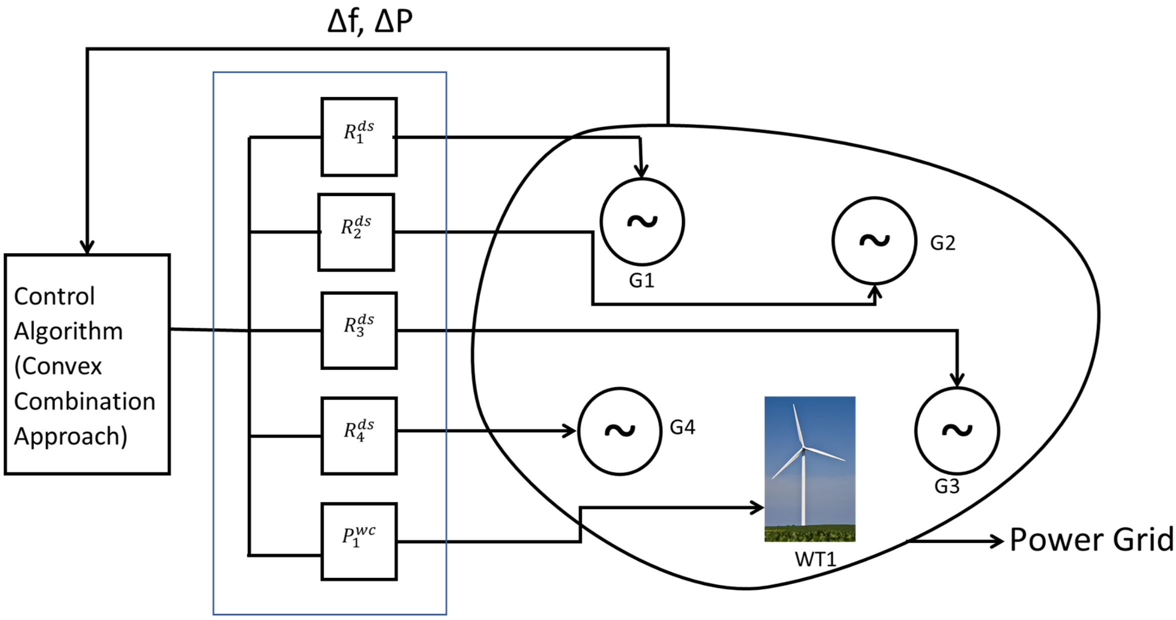


Figure 3-2: The schematic diagram of the convex combination approach. The terms G1, G2, G3, G4 represents the four generation units. The term of WT1 denotes the wind turbines.

After developing the convex combination approach model, the next task is to focus on the optimization problem of the convex combination approach.

3-2-2 Optimization Scheme: Convex Combination Approach

A power system with N_b buses, N_l lines, N_w wind farms, N_g conventional generators and N_d load sinks was considered to form the optimization problem. The vector of decision variables at every time step t will be:

$$x_{i,t} = [P_{i,t,s}^{\mathrm{g}}, \gamma_{i,t,s}^{\mathrm{g}}, z_{i,t,s}^{\mathrm{g}}, R_{i,t,s}^{\mathrm{ds}}, C_{i,t,s}^{\mathrm{su}}, C_{i,t,s}^{\mathrm{su}}] \tag{3-13}$$

where $P_{i,t,s}^{\mathrm{g}}$ denotes the scheduled power from every generation unit i at time step t for every scenario s, while the term $\gamma_{i,t,s}^{\mathrm{g}}$ denotes the binary variable to control the on-off status of every generation unit. The term $z_{i,t,s}^{\mathrm{g}}$ represents the auxiliary variables to model the minimum up and down times of every generation unit. The variable $R_{i,t,s}^{\mathrm{ds}}$ denotes the amount of down-spinning of the generation unit in the presence of excessive wind power generation when compared to the predicted wind power in the grid. If a particular generation unit doesn't participate in control then the term $R^{\mathrm{ds}}_{i,t,s}$ corresponding to the generation unit would be zero. The term $C^{\mathrm{su}}_{i,t,s}$ denote effective start-up cost of every generation unit. The last term, $P^{\mathrm{wc}}_{i,t,s}$ represents the amount of the wind power curtailment from every wind power plant. All the decision variables are defined for every time step t, where $t \in \{1, 2, 3....N_t\}$. The value $N_t = 24$ corresponds to a day ahead optimization problem.

Cost-Function of the Optimization

The total system costs are divided into three parts, they are:

1. The production costs of the system. Motivated by [40, 45], the production costs are considered to be in quadratic form.

2. The reserve costs of the system. By following [45], these reserve costs are considered to be linear.

3. The wind power curtailment costs. Similar to reserve costs, the wind power curtailment costs are also considered to be linear. The system operators are penalized for curtailing wind power. Hence the system also has wind power curtailment costs.

The final cost function of the optimization problem is:

$$\min_{x_{i,t}} \; (\sum_{t=1}^{N_t}\sum_{i=1}^{N_{\mathrm{g}}} f_{\mathrm{c}}(P^{\mathrm{g}}_{i,t,s}) + \sum_{s=1}^{N_s} p_s(\sum_{t=1}^{N_t}\sum_{i=1}^{N_{\mathrm{g}}}(f_{\mathrm{cc}}(R^{\mathrm{ds}}_{i,t,s}))) + \sum_{s=1}^{N_s} p_s(\sum_{t=1}^{N_t}\sum_{i=1}^{N_{\mathrm{w}}}(f_{\mathrm{wc}}(P^{\mathrm{wc}}_{i,t,s}))) \tag{3-14}$$

where

$$f_{\mathrm{c}} = (a_i + b_i P^{\mathrm{g}}_{i,t,s} + c_i(P^{\mathrm{g}}_{i,t,s})^2 + C^{\mathrm{su}}_{i,t,s}) \tag{3-15}$$

$$f_{\mathrm{cc}} = (C^{\mathrm{rs}}_i(R^{\mathrm{ds}}_{i,t,s})) \tag{3-16}$$

$$f_{\mathrm{wc}} = (C^{\mathrm{wc}}_i(P^{\mathrm{wc}}_{i,t,s})) \tag{3-17}$$

where the function f_{c} denotes quadratic cost function which has to be minimized for the economic dispatch from every generation unit i. The terms c_i and b_i denote the quadratic costs and the linear costs respectively of every megawatt of production per hour. The function f_{cc} represents the linear reserve costs of the system to be minimized, while the term C^{rs}_i denotes the reserve costs of each megawatt per hour of the i^{th} generation unit. The term p_s denotes the probability of each scenario of the uncertain wind power. Finally, the function f_{wc} denotes the linear cost function of the wind power curtailment. The term C^{wc}_i denotes the costs of every megawatt of wind power curtailed per hour.

Constraints of the optimization

The constraints of the optimization problem are:

- The most important constraint is power balance of the network:

$$\sum_{i=1}^{N_\mathrm{g}} P_{i,t,s}^{\mathrm{g}} + \sum_{i=1}^{N_\mathrm{w}} P_{i,t,s}^{\mathrm{w,f}} - \sum_{i=1}^{N_\mathrm{d}} P_{i,t}^{\mathrm{d}} = 0 \tag{3-18}$$

  where $P_{i,t}^{\mathrm{d}}$ denotes the i^{th} demand power of the grid at time step t. This constraint ensures that the combination of power from conventional generators and wind power should be equal to the total demand of the system when the actual wind power is equal to the wind power forecast ($P_{i,t}^{\mathrm{w}} = P_{i,t}^{\mathrm{w,f}}$).

- The power generation from every conventional generator should be within the generator limits. This is written as:

$$[P_{\mathrm{min},i}^{\mathrm{g}}]\gamma_{i,t,s}^{\mathrm{g}} \leq P_{i,t,s}^{\mathrm{g}} \leq [P_{\mathrm{max},i}^{\mathrm{g}}]\gamma_{i,t,s}^{\mathrm{g}} \tag{3-19}$$

  where $P_{\mathrm{min},i}^{\mathrm{g}}$ denotes the minimum power generation of every generation unit (i) and $P_{\mathrm{max}}^{\mathrm{g}}$ denotes maximum power of every generation unit (i). The term $\gamma_{i,t}^{\mathrm{g}}$ denotes the on-off status of every generation unit.

- The power flow in the transmission lines should be within the line limits of the grid.

$$-P_{\mathrm{line}} \leq P_{i,t,s}^{\mathrm{f}} \leq P_{\mathrm{line}} \tag{3-20}$$

  The power in each line depends upon the power injection vector. The relationship between the power injection vector and the matrix comprising of the power flow in the lines is given by equations (3-6) and (3-7).

- Every conventional generation unit should ramp-up and ramp-down within the limits.

$$-P_{\mathrm{down,i}}^{\mathrm{g}} \leq P_{i,t,s}^{\mathrm{g}} - P_{i,t-1,s}^{\mathrm{g}} \leq P_{\mathrm{up},i}^{\mathrm{g}} \tag{3-21}$$

  where $P_{\mathrm{down},i}^{\mathrm{g}}$ and $P_{\mathrm{up},i}^{\mathrm{g}}$ denote the ramp-up and ramp-down limits of the generation unit i.

- The effective start up costs should be greater than or equal to zero at every time-step t. The can be mathematically written as:

$$C_{i,t,s}^{su} \geq 0 \tag{3-22}$$

One more constraint on the start-up costs is:

$$\lambda_i^{su}(\gamma_{i,t,s}^{\mathrm{g}} - \gamma_{i,t-1,s}^{\mathrm{g}}) \leq C_{i,t,s}^{\mathrm{su}} \tag{3-23}$$

where λ_i^{su} represents the start-up costs of the every generation unit (i). This constraint implies that the effective start up costs would be zero unless the generation unit changes it's status from off to on.

- The probabilistic constraints:

$$\mathrm{Pr}\Bigg(- \mathrm{P}_i^{\mathrm{line}} \leq P_{i,t,s}^{\mathrm{f}} \leq \mathrm{P}_i^{\mathrm{line}}, \tag{3-24}$$

$$[\mathrm{P}_{\mathrm{min},i}^{\mathrm{g}}]\gamma_{i,t,s} \leq P_{i,t,s}^{\mathrm{g}} + R_{i,t,s}^{\mathrm{ds}}, \leq [\mathrm{P}_{\mathrm{max},i}^{\mathrm{g}}]\gamma_{i,t,s}, \tag{3-25}$$

$$0 \leq R_{i,t,s}^{\mathrm{ds}} \leq \mathrm{R}_{\mathrm{max},i}^{\mathrm{ds}}, \tag{3-26}$$

$$0 \leq P_{i,t,s}^{\mathrm{wc}} \leq \mathrm{P}_{\mathrm{max},i}^{\mathrm{wc}} \Bigg) \geq 1 - \epsilon \tag{3-27}$$

The constraint (3-24) ensures the probabilistic guarantee on the standard transmission capacity of the grid. The equation (3-25) encrypts the generation schedule along with reserve scheduling should be within limits of the generation. The next constraint (3-26) encodes the probabilistic limits on reserve regulation on the individual generation units. The last constraint (3-27) of the probabilistic constraints encodes the limits on the wind power curtailment.

- The power balance constraint in the presence of wind power uncertainty, reserve scheduling and wind power curtailment can be written as:

$$\sum_{i=1}^{N_{\mathrm{g}}}(P_{i,t,s}^{\mathrm{g}} - R_{i,t,s}^{\mathrm{ds}}) + \sum_{i=1}^{N_{\mathrm{w}}}(P_{i,t,s}^{\mathrm{w}} - P_{i,t,s}^{\mathrm{wc}}) - \sum_{i=1}^{N_{\mathrm{d}}} P_{i,t}^{\mathrm{d}} = 0 \tag{3-28}$$

- The next constraint ensures that the amount of down-spinning and the wind power curtailment in combination should be equal to the uncertain wind power in the grid.

$$\sum_{i=0}^{N_{\mathrm{g}}} R_{i,t,s}^{\mathrm{ds}} + \sum_{i=0}^{N_{\mathrm{w}}} P_{i,t,s}^{\mathrm{wc}} = \Delta P_{i,t,s}^{\mathrm{w}} \tag{3-29}$$

- The constraint in the minimum up and down time of the generation units for every generation unit i is given as:

$$0 \leq z^{\mathrm{g}}_{i,t,s} \leq 1 \tag{3-30}$$

$$\gamma^{\mathrm{g}}_{i,t,s} - (\gamma^{\mathrm{g}}_{i,t-1,s}) \leq z^{\mathrm{g}}_{i,t,s} \tag{3-31}$$

$$\sum_{j=t-\Delta t_{\mathrm{up}}+1}^{t} z^{\mathrm{g}}_{i,j,s} \leq \gamma^{\mathrm{g}}_{i,j,s}, \quad \text{for} \;\; t \geq \Delta t_{\mathrm{up}} \tag{3-32}$$

$$\sum_{j=t+1}^{t+\Delta t_{\mathrm{down}}} z^{\mathrm{g}}_{i,j,s} \leq 1 - \gamma^{\mathrm{g}}_{i,j,s} \;\; \text{for} \;\; t \leq N_{\mathrm{t}} - \Delta t_{\mathrm{down}} \tag{3-33}$$

The first constraint says that the auxiliary variables should be positive, the second one gives information about minimum time taken by the generation units to change their status from on to off or vice-verse.

Finally, the optimization problem proposed is a chance-constrained, multi-objective and mixed integer quadratic program. The chance constraints from equation (3-24) to (3-27) are not joint constraints. Each of these constraints have violation level ϵ. As the optimization problem consists of chance constraints, the goal now is to see how they can be solved.

3-2-3 Chance Constraints

The optimization problem formed in the above section has chance constraints. So we need to know how to deal with the chance constrained optimization problems. The general form of the chance constrained optimization problems is [46]:

$$\min_{x_{i,t}} J \quad \text{subject to} \tag{3-34}$$

$$\mathbb{P}(\delta \in \Delta | A(\delta)x + Bu + C(\delta)) \geq 0 \tag{3-35}$$

where J denotes the cost function to be minimized, while $\delta \in \Delta$ represents the uncertain wind power in the grid. The term x represents the decision variables, while the term u denotes the binary variables of the system. We assume that Δ is endowed with σ algebra Ω [47], where $\mathbb{P}$ denotes the probability measure over Ω. In order to avoid arbitrary assumptions on $\mathbb{P}$, we transform the problem of chance constraint into an equivalent scenario based optimization technique using a randomization approach inspired from [48].

The idea of the scenario approach is to generate a finite number of scenarios [46] or the instances of the uncertain variable. The optimization problem has to be solved by considering the constraints to be hard constraints of the optimization problem for each of the scenarios generated. The authors of [49] have proposed a lower bound on the number of scenarios N_{s} to be generated to obtain the probabilistic guarantees ϵ, with a confidence parameter $1-\beta$. The number of scenarios or the samples to be generated should follow the rule:

$$N_{\text{s}} \geq \frac{2}{\epsilon}(N_{\text{d}}N_t + \ln(\frac{1}{\beta})) \tag{3-36}$$

where N_{s} denotes the number of scenarios and N_{t} denotes the time horizon of optimization and N_{d} denotes the decision variables for optimization at every time step. The allowed violation levels and the confidence parameter were considered to be 5% and (10^{-5}) respectively. The number of scenarios increase linearly with respect to increase in decision variables, this would hamper the applicability of the theory to large scale systems. With mentioned parameters and the decision variables, a large number of scenarios have to be generated to suffice the lower bound of the scenarios. There are two problems with high number of scenarios. These high number of scenarios are:

- Impractical or not realistic.
- Might result in an intractable solution.

To avoid these two problems, we have considered 100 scenarios of uncertain wind power at every time step and 2400 scenarios through out the optimization horizon. This would result in a tractable solution with less computation time. Finally, the algorithm of the optimization problem is:

Algorithm 1 Convex Combination Approach

1: Initialize the on-off status of all the generation units i.e. $\gamma_{i,t,s} \in \{0, 1\}$
2: Fix the bounds of the uncertainty, $\Delta \in [\underline{\Delta}, \overline{\Delta}]$
3: Generate various scenarios of the uncertain variable i.e. $\Delta^1, \Delta^2......\Delta^s$
4: Define the vector of the optimization variables $x_{i,t} = [P^{\text{g}}_{i,t,s}, \gamma^{\text{g}}_{i,t,s}, z^{\text{g}}_{i,t,s}, R^{\text{ds}}_{i,t,s}, C^{\text{su}}_{i,t,s}, C^{\text{su}}_{i,t,s}]$
5: Minimize the cost function represented in the equation (3-14) subjected to constraints from the equation (3-18) to equation (3-33) for every scenario of the uncertain variable.
6: Obtain the solution with check for the feasibility of the solution.

The term Δ denotes the uncertain variable in the system, while the terms $\underline{\Delta}, \overline{\Delta}$ denote the upper and the lower bounds of the uncertainty. The optimization scheme using the convex combination approach is defined in this section. In the next sections, we defined another optimization scheme to enhance the performance of the optimization and minimize the costs of the system.

3-3 Mixed Logical Dynamical Approach

Using the MLD approach as explained in [50], the optimization problem would determine the optimal generation units to provide reserves and the optimal wind power plant to determine the wind power curtailment. This might result in an enhanced solution when compared to the convex combination approach. In this section, we transform the convex combination model of the wind power curtailment and the reserves regulation into the MLD framework.

3-3-1 Modeling using the MLD Framework

To elaborate the MLD framework that has been used as part of this thesis, we consider a function $f(.)$ defined over a bounded set. The upper and the lower bounds of the sets are considered to be M and m. In the case of binary decision variable $\delta \in \{0,1\}$ the following statements hold [50]:

$$[f(x) \leq 0] \Leftrightarrow [\delta = 1] \text{ is equivalent to} \begin{cases} f(x) \leq M(1-\delta) \\ f(x) \geq \epsilon + (m-\epsilon)\delta \end{cases} \tag{3-37}$$

where ϵ is a very small tolerance value called machine precision, which is used to change the strict inequality into non-strict inequality. The product of two binary variables δ_1 and δ_2 can be replaced by an axillary binary variable $\delta_3 \triangleq \delta_1\delta_2$, this can be verified from [50, 51]. Then the following statements hold [50]:

$$\delta_3 = \delta_3\delta_1 \text{ is equivalent to} \begin{cases} -\delta_1 + \delta_3 \leq 0 \\ -\delta_2 + \delta_3 \leq 0 \\ \delta_1 + \delta_2 - \delta_3 \leq 0 \end{cases} \tag{3-38}$$

The final one is multiplication of a function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ with a binary variable. The product of binary variable and the function can be replaced by an auxiliary variable $z \triangleq \delta f(x)$. This means the term $z = 0$ when $\delta = 0$, and $z = f(x)$ when $\delta = 1$. An equivalent representation would be [50]:

$$z = \delta f(x) \text{ is equivalent to} \begin{cases} z \leq M\delta \\ z \geq m\delta \\ z \leq f(x) - m(1-\delta) \\ z \geq f(x) - M(1-\delta) \end{cases} \tag{3-39}$$

In the above mentioned rules, we use only equation (3-39) to transform the convex combination model into the MLD model. To elaborate the transformation of the convex combination into the MLD framework, we introduce two logic variables:

- $\delta \in \{0,1\}$
- $\mu \in \{0,1\}$

At the same time, we introduce two auxiliary variables called x^{ds} and y^{wc}. These two auxiliary variables represent the amount of reserve regulation and the amount of wind power curtailment respectively. By applying equation (3-39) for the reserves regulation, we obtain:

$$x_i^{\mathrm{ds}} = \delta_i R_i^{\mathrm{ds}} \text{ is equivalent to} \begin{cases} x_i^{\mathrm{ds}} \leq M_i\delta_i \\ x_i^{\mathrm{ds}} \geq m_i\delta_i \\ x_i^{\mathrm{ds}} \leq R_i^{\mathrm{ds}} - m_i(1-\delta_i) \\ x_i^{\mathrm{ds}} \geq R_i^{\mathrm{ds}} - M_i(1-\delta_i) \end{cases} \tag{3-40}$$

where R_i^{ds} denotes the amount of reserve dispatch from every generation unit. The terms M_i and m_i denote the maximum and minimum values of the reserves from the i^{th} generation unit. The binary variable δ_i represents a switch which is used to control the "on-off" behavior of the reserves of the i^{th} generation unit. The term x_i^{ds} is the auxiliary variable which represents the amount of reserve regulation from the i^{th} generation unit. By applying the equation (3-39) to the wind power curtailment, the following equations are obtained:

$$y_i^{\text{wc}} = \mu_i P_i^{\text{wc}} \text{ is equivalent to} \begin{cases} y_i^{\text{wc}} \leq M_i \mu_i \\ y_i^{\text{wc}} \geq m_i \mu_i \\ y_i^{\text{wc}} \leq P_i^{\text{wc}} - m_i(1-\mu_i) \\ y_i^{\text{wc}} \geq P_i^{\text{wc}} - M_i(1-\mu_i) \end{cases} \tag{3-41}$$

where M_i and m_i denote the maximum and the minimum values of wind power curtailment respectively. The term P_i^{wc} denotes the amount of wind power curtailment of every wind power plant i. The term μ_i is the logic variable to control the "on-off" behavior of the wind power curtailment of every wind power plant i. The term y_i is considered to be the auxiliary variable of the wind power curtailment. The power balance equation in this MLD approach would be:

$$\sum_{i=1}^{N_{\text{g}}} (P_{i,t}^{\text{g}} - x_{i,t}^{\text{ds}}) + \sum_{i=1}^{N_{\text{w}}} (P_{i,t}^{\text{w}} - y_{i,t}^{\text{wc}}) - \sum_{i=1}^{N_{\text{d}}} P_{i,t}^{\text{d}} = 0 \tag{3-42}$$

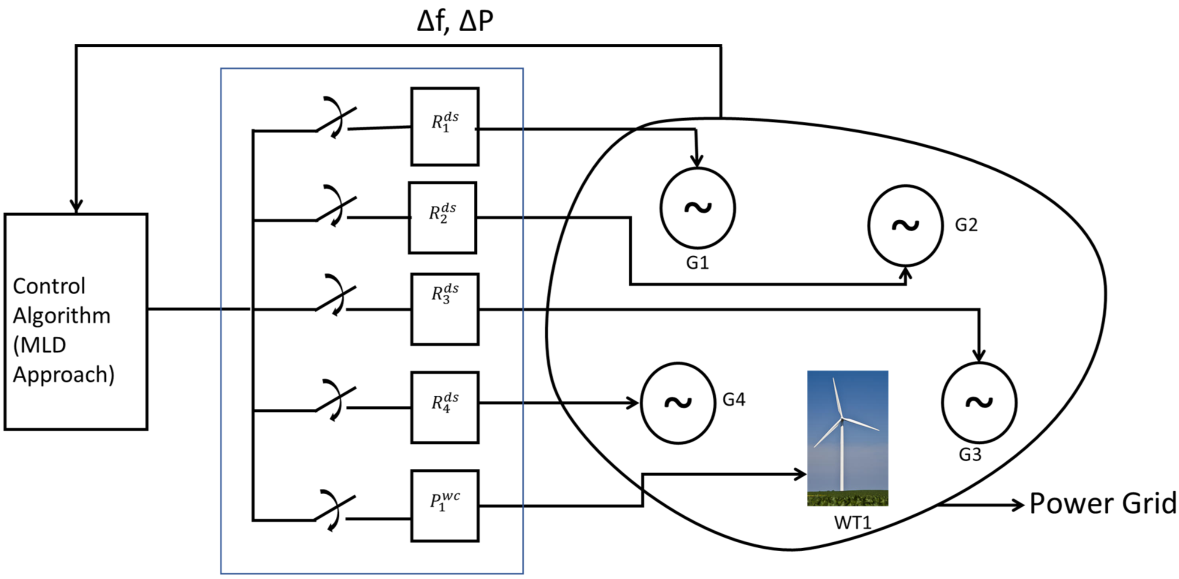


Figure 3-3: The schematic diagram of the MLD approach. The terms G1, G2, G3, G4 represents the four generation units. The term of WT1 denotes the wind turbines.

The convex combination model is transformed into the MLD model. The next focus is on the optimization problem corresponding to the MLD model.

3-3-2 Optimization Scheme: MLD Approach

The optimization problem in the MLD approach is similar to the convex combination approach, but a few variables were changed in the cost function and a few extra constraints were added to the optimization problem. The optimization variables in the MLD approach are:

$$x_{i,t,s} = [P^{\mathrm{g}}_{i,t,s}, \gamma^{\mathrm{g}}_{i,t,s}, z^{\mathrm{g}}_{i,t,s}, R^{\mathrm{ds}}_{i,t,s}, C^{\mathrm{su}}_{i,t,s}, \delta_{i,t,s}, \mu_{i,t,s}, y^{\mathrm{wc}}_{i,t,s}, x^{\mathrm{ds}}_{i,t,s}] \tag{3-43}$$

The extra optimization variables when compared to the previous optimization problem are $\delta_{i,t,s}, \mu_{i,t,s} x_{i,t,s}, y_{i,t,s}$. These extra optimization variables appear as a result of modeling reserves and wind power curtailment using the MLD approach. The cost function of the optimization problem is:

$$\min_{x_{i,t,s}} \left(\sum_{t=1}^{N_t}\sum_{i=1}^{N_{\mathrm{g}}} f_{\mathrm{c}}(P^{\mathrm{g}}_{i,t}) + \sum_{s=1}^{N_s} p_s\left(\sum_{t=1}^{N_t}\sum_{i=1}^{N_{\mathrm{g}}}(f_{\mathrm{cc}}(x^{\mathrm{ds}}_{i,t,s}))\right) + \sum_{s=1}^{N_s} p_s\left(\sum_{t=1}^{N_t}\sum_{i=1}^{N_{\mathrm{w}}}(f_{\mathrm{wc}}(y^{\mathrm{wc}}_{i,t,s}))\right)\right) \tag{3-44}$$

The functions and the terms used in the optimization are explained in the convex combination approach. The terms $x^{\mathrm{ds}}_{i,t,s}$ and $y^{\mathrm{wc}}_{i,t,s}$ represent the auxiliary variables of the reserves and the wind power curtailment respectively. In this optimization scheme of the MLD approach, we have a few modified constraints and few extra constraints because of the MLD model.

Modified and Extra Constraints of the MLD approach

- The power balance equation in this MLD approach would be:

$$\sum_{i=1}^{N_{\mathrm{g}}}(P^{\mathrm{g}}_{i,t,s} - x^{\mathrm{ds}}_{i,t,s}) + \sum_{i=1}^{N_{\mathrm{w}}}(P^{\mathrm{w}}_{i,t} - y^{\mathrm{wc}}_{i,t,s}) - \sum_{i=1}^{N_{\mathrm{d}}} P^{\mathrm{d}}_{i,t} = 0 \tag{3-45}$$

- The constraint on the amount of down-spinning and wind power curtailment

$$\sum_{i=0}^{N_{\mathrm{g}}} x^{\mathrm{ds}}_{i,t,s} + \sum_{i=0}^{N_{\mathrm{w}}} y^{\mathrm{wc}}_{i,t,s} = \Delta P^{\mathrm{w}}_{i,t,s} \tag{3-46}$$

  The sum of the auxiliary variables should be equal to the amount of the uncertain wind power in the grid.

- The probabilistic constraints:

$$\Pr\Bigg(-\mathrm{P}_i^{\mathrm{line}} \leq P_{i,t,s}^{\mathrm{f}} \leq \mathrm{P}_i^{\mathrm{line}}, \tag{3-47}$$

$$[\mathrm{P}_{\min,i}^{\mathrm{g}}]\gamma_{i,t,s} \leq P_{i,t,s}^{\mathrm{g}} + x_{i,t,s}^{\mathrm{ds}}, \leq [\mathrm{P}_{\max,i}^{\mathrm{g}}]\gamma_{i,t,s}, \tag{3-48}$$

$$0 \leq x_{i,t,s}^{\mathrm{ds}} \leq \mathrm{R}_{\max,i}^{\mathrm{ds}}, \tag{3-49}$$

$$0 \leq y_{i,t,s}^{\mathrm{wc}} \leq \mathrm{P}_{\max,i}^{\mathrm{wc}} \Bigg) \geq 1 - \epsilon \tag{3-50}$$

- The MLD constraints:

$$y_{i,t,s} \leq M_i \mu_{i,t,s} \tag{3-51}$$

$$y_{i,t,s} \geq m_i \mu_{i,t,s} \tag{3-52}$$

$$y_{i,t,s} \leq P_{i,t,s}^{\mathrm{wc}} - m_i(1 - \mu_{i,t,s}) \tag{3-53}$$

$$y_{i,t,s} \geq P_{i,t,s}^{\mathrm{wc}} - M_i(1 - \mu_{i,t,s}) \tag{3-54}$$

$$x_{i,t,s} \leq M_i \delta_{i,t,s} \tag{3-55}$$

$$x_{i,t,s} \geq m_i \delta_{i,t,s} \tag{3-56}$$

$$x_{i,t,s} \leq R_{i,t,s}^{\mathrm{ds}} - m_i(1 - \delta_{i,t,s}) \tag{3-57}$$

$$x_{i,t,s} \geq R_{i,t,s}^{\mathrm{ds}} - M_i(1 - \delta_{i,t,s}) \tag{3-58}$$

The first set of constraints from equation (3-51) to equation(3-54) denote the MLD constraints that corresponds to the reserve scheduling. The constraints from equation (3-55) to equation (3-58) represent the MLD constraints that correspond to the wind power curtailment. Finally, the algorithm of the MLD approach is given as:

Algorithm 2 Mixed Logical Dynamical Approach

1: Initialize the on-off status of all the generation units i.e. $\gamma_{i,t,s} \in \{0, 1\}$
2: Define the auxiliary variables $x_{i,t,s}^{\mathrm{ds}}$ and $y_{i,t,s}^{\mathrm{wc}}$
3: Fix the bounds of the uncertainty, $\Delta \in [\underline{\Delta}, \overline{\Delta}]$
4: Generate various scenarios of the uncertain variable i.e. $\Delta^1, \Delta^2......\Delta^s$
5: Define the vector of the optimization variables $x_{i,t,s} = [P_{i,t,s}^{\mathrm{g}}, \gamma_{i,t,s}^{\mathrm{g}}, z_{i,t,s}^{\mathrm{g}}, R_{i,t,s}^{\mathrm{ds}}, C_{i,t,s}^{\mathrm{su}}, \delta_{i,t,s}, \mu_{i,t,s}, y_{i,t,s}^{\mathrm{wc}}, x_{i,t,s}^{\mathrm{ds}}]$
6: Modify and add the constraints similar to the optimization problem
7: Minimize the cost function represented in the equation (3-44) subjected to constraints from the equation (3-18) to equation (3-58) for every scenario of the uncertain variable.
8: Obtain the solution with and check for the feasibility of the solution.

3-4 Summary

The theoretical developments were explained in a detailed manner to deal with the problem of uncertain wind power. The theoretical frameworks were formulated based on the DC power flow model. As an initial step, the DC power flow model and the assumptions of the DC power flow model were explained. Later on, two methods were developed to solve the problem of uncertain wind power penetration in the smart grids are explained in a detailed manner. They are:

- Convex Combination Approach
- MLD Approach

The optimization schemes and the constraints of optimization in both approaches were explained in a detailed manner. The theoretical developments are applied on a case study in the next chapter to analyze the advantages and the disadvantages of the each of these methods.

Chapter 4

Case Study

The main goal of this chapter is to describe the results of the proposed theoretical developments in detail. The theoretical developments which were proposed in the previous chapter were applied to an IEEE-30 bus network. The theoretical frameworks were applied to a static demand case as an initial step. Later on, the dynamic demand case was also simulated. In the first section, the details of the simulation set-up are explained. In the next sections, results of the proposed optimization techniques are compared with the reserve-scheduling in-order to illustrate the performance of the proposed algorithms. Finally, the results are discussed in a detailed manner to realize the advantages and disadvantages of the proposed theoretical developments.

4-1 Simulation Setup

There were few assumptions made in the previous chapter, they were:

- The day ahead wind and the demand profiles are known in advance.
- There is only one wind power plant in the grid.
- The only uncertain parameter in the grid is the wind power.

The day ahead wind power forecast throughout the optimization horizon is shown in Figure 4-1. The sampling time of the wind power is one hour. These wind power values were taken from the PJM website [52]. In Figure 4-1, length of each bar represents the amount of the wind power at every time step. The estimated wind power is never equal to the actual wind power as the wind power is highly chaotic. As mentioned in the chapter-02, the chaotic nature of the wind power does not allow the prediction methods to capture the pattern of the wind power with 100% accuracy. Therefore, the actual wind power can be modeled as a combination of the wind power forecast and an additive uncertainty. The uncertainty is unknown, but it is bounded. In order to approximate the uncertainty, various scenarios of the uncertainty are generated using the Monte Carlo simulations. The various scenarios of the actual wind power in the grid are generated using the Monte Carlo simulations. All the scenarios at every time step are shown in Figure 4-2.

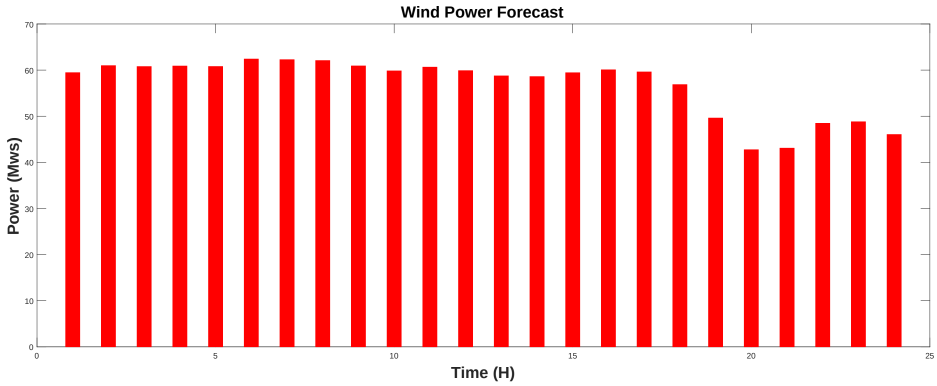


Figure 4-1: The day ahead wind power forecast at an interval of one hour.

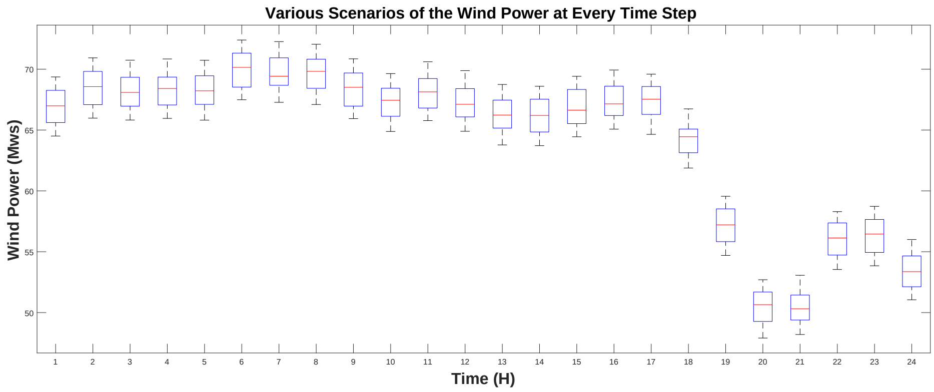


Figure 4-2: Various scenarios of the actual wind power penetration into the grid. The box at every time step represents various possible realizations of the wind power. The red line indicates the median value. The edges of the box at every time step correspond to the 25^{th} and the 75^{th} percentile values of the wind power scenarios.

After generation of scenarios, the next step is to implement optimization strategies on a case study. An IEEE-30 bus network was used to implement the proposed optimization strategies along with the reserve scheduling. The one-line diagram of the IEEE-30 bus network is shown in Figure 4-3. There are six generation units and twenty load profiles in the network. The wind power plant was modeled as a single in-feed at bus number six, as the wind power has a single output. The location of the generation units in the grid and the maximum and the minimum generation limits of the generation units are given in Table 4-1. The generation costs per megawatt of every generation unit are given in Table 4-2. The extra information such as the line limits of the grid, the ramping limits of the generation units and the reserve levels of the generation units was taken from Matpower [5].

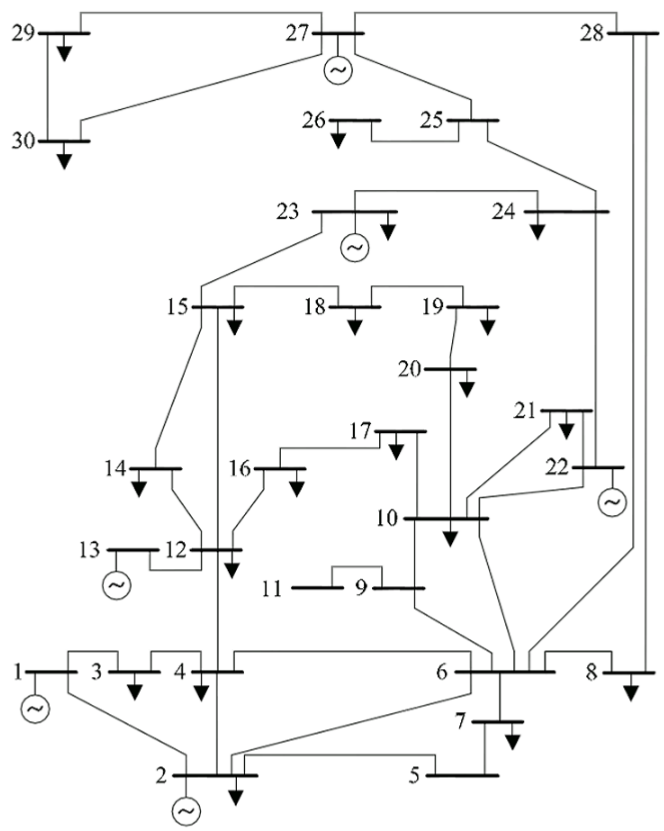


Figure 4-3: The one-line diagram of IEEE-30 bus network [4].

Generation Unit (N_g)	Bus Number	Maximum Generation(Mws)	Minimum Generation (Mws)
1	1	80	0
2	2	80	0
3	22	50	0
4	27	55	0
5	23	30	0
6	13	40	0

Table 4-1: The maximum and minimum generation levels and the bus number of the generation units [5].

Generator	Linear Costs (Euros)	Quadratic Costs (Euros)	Reserve Costs (Euros)
1	300	200.00	414.00
2	300	175.00	396.00
22	300	625.00	551.30
27	300	83.40	311.300
23	300	250.00	337.50
13	300	250.00	360.00

Table 4-2: The linear costs, quadratic costs and the reserve costs of every generation unit per megawatt [5].

Number of Scenarios

The scenarios of the actual wind power were generated using the Monte Carlo simulations. Now the question is, "How many scenarios should be generated to represent the uncertainty?" The lower bound of scenarios depends on the number of decision variables (N_{d}), the allowed level of constraints violations (ϵ) and the confidence parameter (β). The mathematical representation of the number of scenarios is:

$$N_{\mathrm{s}} = \frac{2}{\epsilon}(N_{\mathrm{d}} N_t + \ln(\frac{1}{\beta})) \tag{4-1}$$

where N_{s} denotes the number of scenarios and N_{t} denotes the time horizon of the optimization. The allowed violation levels and the confidence parameter were considered to be 5% and (10^{-5}) respectively to ensure minimum constraint violation. The number of scenarios for all three optimization methods differ as the number of decision variables are different for each optimization method.

Number	**Optimization Algorithm**	$N_{\mathrm{d}} \times N_t$	N_s
1	Reserve Scheduling Approach	30×24	$29,260$
2	Convex Combination Approach	31×24	$30,221$
3	MLD Approach	45×24	$43,661$

Table 4-3: The number of scenarios to be generated for different optimization methods.

Table 4-3 provides information on the number of scenarios to be generated for every optimization method. However, there are some problems associated with these high number of scenarios. They are impractical or not realistic and might result in an intractable solution. To avoid these two problems, only 100 scenarios at every time step and 2400 scenarios throughout the optimization horizon were considered. The uncertainty is assumed to be bounded between 0 and 10, this can be mathematically represented as:

$$\Delta P_t^{\mathrm{w}} \in [0, 10] \tag{4-2}$$

where ΔP_t^{w} denotes the wind power forecast error. The Matlab optimization interface called YALMIP was used in the optimization. The solver named Gurobi was used to solve the mixed integer programming problem. The simulations were executed on an i-07 processor with 8GB RAM. The results of the computation time of all the three optimization problems might be biased because of the optimization interface YALMIP.

4-2 Case Study A: Static Demand

The net demand of the system remains constant throughout the optimization horizon in a static demand case as shown in Figure 4-4. The length of each bar denotes the magnitude of net demand power of the network at every time step. The length of all the bars is constant as the net demand power is constant throughout the optimization horizon.

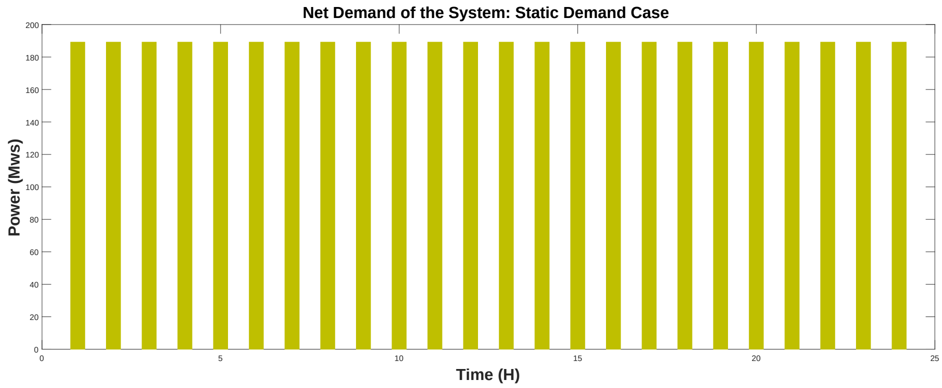


Figure 4-4: The net demand of the system through out the optimization horizon for the static demand case.

The Total System Costs

The total system costs include the generation, the effective start-up, the reserves and the wind power curtailment costs of the system. The main motive of the thesis is to reduce the total system costs, so the total system costs of the formulated optimization techniques are compared with the existing reserve scheduling approach to illustrate the performance of the proposed algorithms.

Methods	**Total costs (Euros)**
Reserve Scheduling Approach	2.20183×10^9
Convex Combination Approach	2.17901×10^9
MLD Approach	2.17901×10^9

Table 4-4: The total costs of the system for all the three different optimization algorithms.

The total system costs of all three optimization methods are shown in Table 4-4. It can be observed from the Table 4-4 that the MLD and the convex combination approaches have outperformed the reserve scheduling approach by 1.104%. However, the total system costs are the same for the MLD and the convex combination approaches. To analyze this effect, the averaged total system costs for different optimization techniques at every time steps were monitored carefully from Figure 4-5.

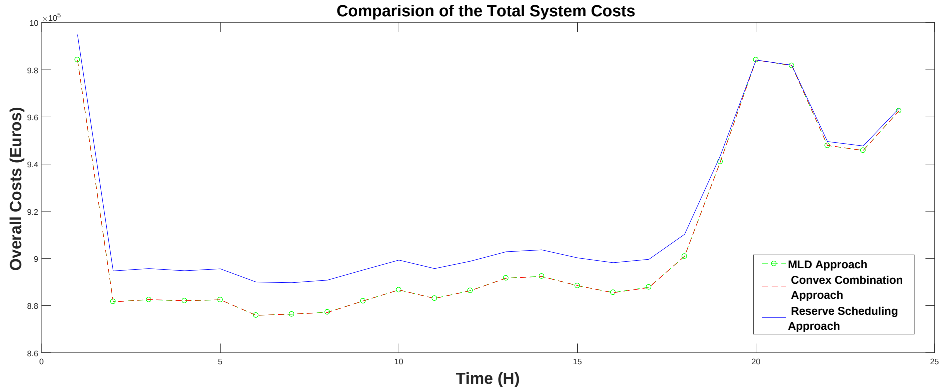


Figure 4-5: Comparison of the average total system costs of all three optimization methods.

The total system costs are very high at the initial time step in all three optimization methods because of the effective startup costs of the system. Initially, all the generation units were considered to be in the off status. Once the optimization starts, all the generation units change their status from off to on in order to satisfy the demand power. This results in the higher total system costs at the initial time step. The total system costs are high again from time step 19 to time step 24 because of the drop in the wind power integration (drop in wind power can be seen in the Figure 4-1). The total system costs are the same for all the three approaches between the time steps 20 and 22. To understand this phenomenon, the relative costs of the MLD and the convex combination approaches with respect to the reserve scheduling approach are plotted in Figures 4-6 and 4-7.

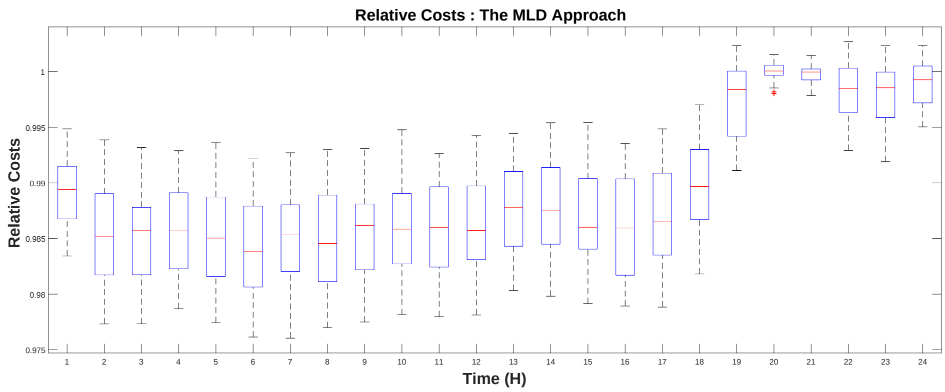


Figure 4-6: The relative costs of the MLD approach with respect the reserve scheduling approach. The red line indicates the median value. The edges of the box at every time step correspond to the 25^{th} and the 75^{th} percentile values of the total system system costs. The red marks denote the outliers of the data.

The relative costs of the MLD and the convex combination approaches are less than one from the time step 1 to time step 18 as shown in the Figures 4-6 and 4-7. However, the relative costs are a bit high from time step 19 to time step 24 because of the drop in the wind power between these time steps. It can be observed that in a few scenarios the convex combination and the MLD approaches perform better than reserve scheduling approach, while

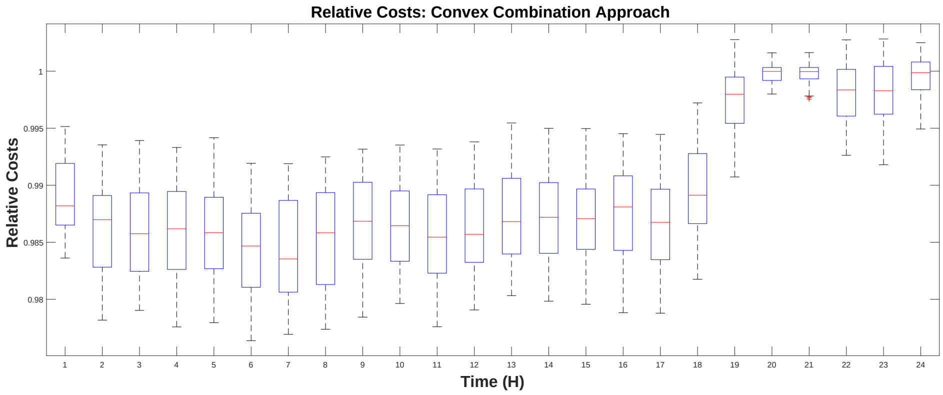


Figure 4-7: The relative costs of the convex combination approach with respect the reserve scheduling approach. The red line indicates the median value. The edges of the box at every time step correspond to the 25^{th} and the 75^{th} percentile values of the total system costs. The red lines denote the outliers of the data.

in the other scenarios vice-versa happen. This kind of behavior is observed because of the randomization approach which is used to generate scenarios. When the magnitude of the uncertainty is high then the total system costs are high. This happen because of the very high wind power curtailment costs, while the reserve costs in the reserve scheduling are relatively low. But on averaging of the scenarios, all three optimization approaches have the same optimal operational costs between the time steps 20 and 22. To understand this similar performance of all three optimizations approaches between the time steps 20 and 22, the averaged generator dispatch and the reserves dispatch along with the wind power curtailment were analyzed in depth.

Generator Dispatch

The active power dispatch of the reserve scheduling, the convex combination and the MLD approaches are represented in Figures 4-8, 4-9 and 4-10 respectively. The height of each bar denotes the amount of power generated from that particular generation unit.

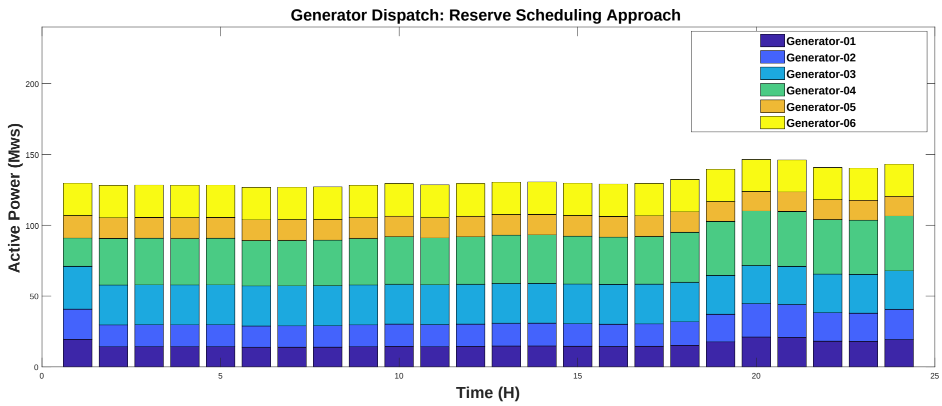


Figure 4-8: The active power dispatch from the different generation units in the reserve scheduling approach. The different colors indicate the power from the different generation units respectively. The corresponding color from of each generation units can be observed from the graph.

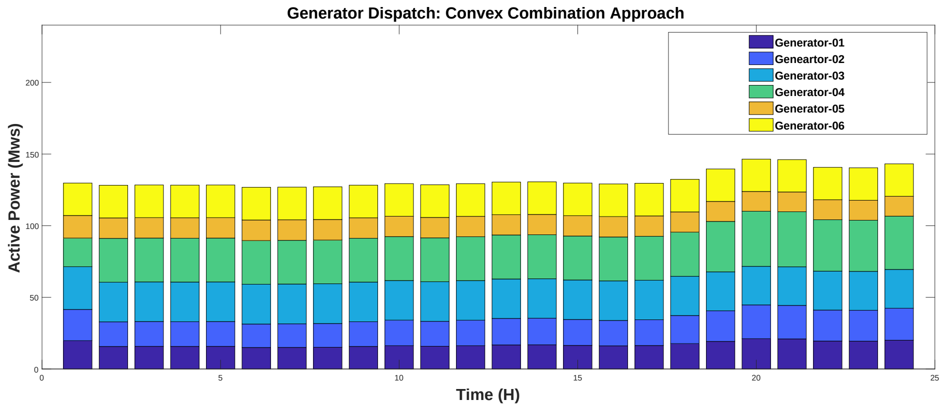


Figure 4-9: The active power dispatch from the different generation units in the convex combination approach. The different colors indicate the power from the different generation units respectively. The corresponding color from of each generation units can be observed from the graph.

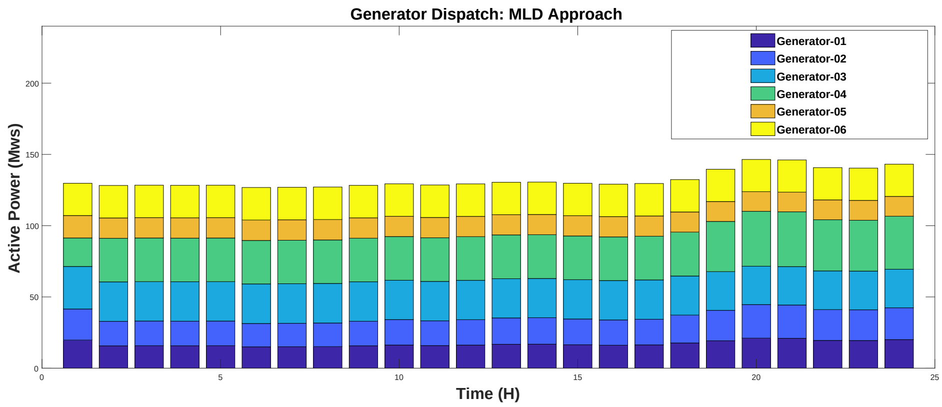


Figure 4-10: The active power dispatch from the different generation units in the MLD approach. The different colors indicate the power from the different generation units respectively. The corresponding color from of each generation units can be observed from the graph.

An important observation is that all the three optimization techniques have a similar generator dispatch. The generation levels are relatively high from time step 19 to time step 24 due to the drop in the wind power at these time steps. The generator-04 accounts for the variance in the wind power, because the generation costs of the generator-04 are lower when compared to the other generation units. Another observation is at every time step of the optimization horizon, all the generators are contributing for the demand power. The wind power generation in combination with the spatial relation of the wind power plant (i.e. how the wind power plant is connected to other generation units and the demand power) would be the reason for all the generation units to be in on status through out the optimization horizon. The wind power would be unsymmetrically contributing for the demand. As a result, none of these demand units would have been satisfied with wind power. So all the generation units were in on status to suffice the demand power.

Reserves Dispatch and Wind Power Curtailment

A combination of reserves and wind power curtailment is used to deal with the uncertainty of the wind power generation. In the case of reserve scheduling approach, only the reserves are regulated, while the other two proposed approaches have wind power curtailment along with the reserve regulation.

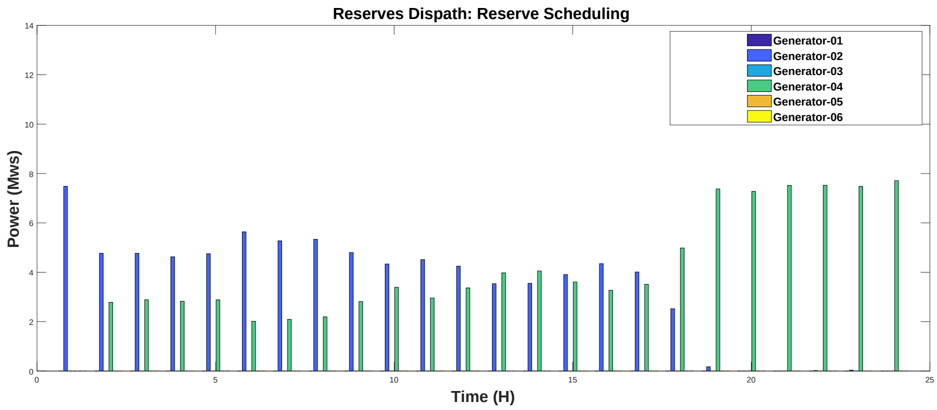


Figure 4-11: The reserves dispatch from the different generation units in the reserve scheduling approach. The height of each bar denotes the amount of the reserves dispatch from every generation unit. If there is no color bar corresponding to the generator that means there are no reserves from that particular generation unit.

The reserves distribution of the reserve scheduling approach are shown in Figure 4-11. Only two generation units, generator-02 (at the 2^{nd} bus) and generator-04 (at the 27^{th} bus) are contributing for the reserve distribution. The reserves are distributed in an asymmetrical manner. To analyze the asymmetrical distribution of the reserves, it is logical to observe the reserve levels along with the generation levels. The power dispatch of generator-04 is relatively high from the 19^{th} time step. At the same time, down-spinning reserves costs of generator-04 are low. This leads to higher reserves dispatch from generator-04 from the time step 19. It can be observed that the reserves distribution depends upon the generation levels, the reserve costs and the spatial relations of the generators. These are the main reasons for the asymmetrical distribution of the reserves as shown in Figure 4-11.

A combination of the reserves dispatch and the wind power curtailed of the convex combination approach is shown in Figure 4-12. Only the wind power is curtailed and there are no reserves dispatched from the generation units from time-step 01 to time step 19. The reason is analogous to the reserve scheduling case, the wind power is very high when compared to the generators dispatch in these time-steps. At the same time, from time step 19, the generation levels of the generator-04 are high and the wind power generation drops down. This could be the reason for the combination of the reserve dispatch and the wind power curtailed between time steps 19 to 24. Another reason for the high wind power curtailment could be the spatial relation of the wind power plant as mentioned in one of the previous paragraphs.

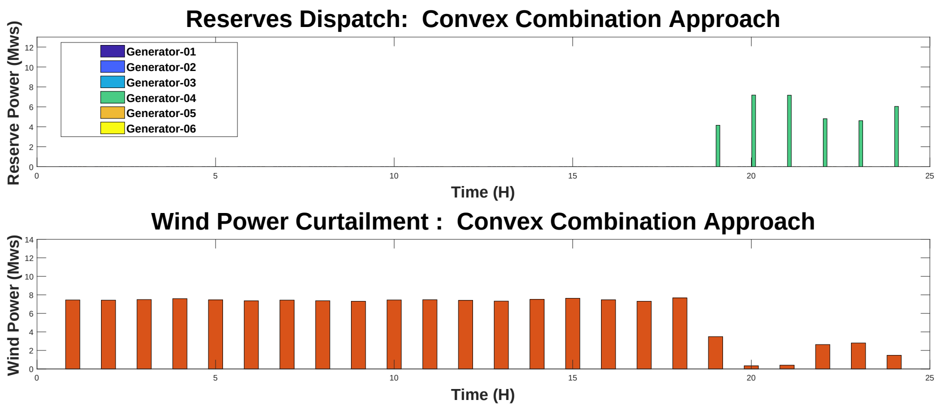


Figure 4-12: The reserves dispatch and the wind power curtailment of the convex combination approach. The height of each bar denotes the amount of the reserves dispatch from every generation unit. If the there is no color bar corresponding to the generation unit, then it means there are no reserves from that particular generation unit. The second graph corresponds to the wind power curtailment and the height of each bar denotes the amount of wind power curtailed at every time step.

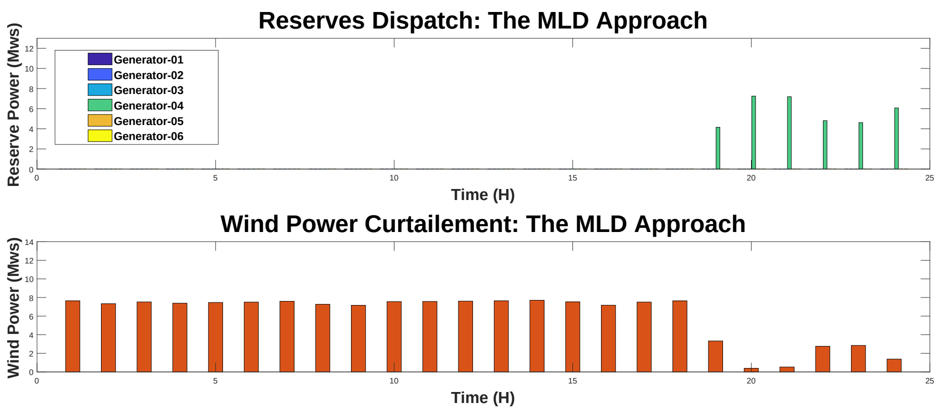


Figure 4-13: The reserves dispatch and the wind power curtailment of the MLD approach. The height of each bar denotes the amount of the reserves dispatch from every generation unit. If the there is no color bar corresponding to the generation unit, then it means there are no reserves from that particular generation unit. The second graph corresponds to the wind power curtailment and the height of each bar denotes the amount of wind power curtailed at every time step.

The reserves distribution from the generation units and the wind power curtailment of the MLD approach can be seen in Figure 4-13. An interesting observation is that the reserves distribution and the wind power curtailment of the MLD approach are the same as the convex combination approach. This means the system has an inherent switching behavior and an extra switch is not required as proposed in the MLD approach. The similar inherent switching behavior of the reserves is observed by the authors of [53]. To conclude, the switching behavior of the reserves and the wind power curtailment depends upon three important factors, they are:

- The spatial relations.
- The costs of the reserves and the wind power curtailment.
- The generation levels and the wind power penetration into the grid.

The generation levels and the wind power levels play a key role in the optimal switching of the generation units to dispatch reserves and the wind power units to curtail the wind power. Due to the above mentioned reasons, the total system costs of the convex combination approach and the MLD approach are exactly the same.

Computation Time

An overview of the computation time of all three optimization methods is provided by Table 4-5. It can be observed that the computation time of the MLD approach is significantly higher than the other two approaches. This is due to an increase in a number of the optimization variables in the MLD approach. Another observation is that the computation time of the reserve scheduling and the convex combination method are almost the same. This is because of the similar number of the decision or the optimization variables in both approaches.

Methods	**Computation time (secs)**
Reserve Scheduling Approach	5.70
Convex Combination Approach	5.77
MLD Approach	6.21

Table 4-5: The table provides comparison of the computation time of all the three optimization methods.

4-3 Case Study B: Dynamic Demand

The net demand profile of the grid varies with respect to time throughout the optimization horizon in the dynamic demand case as shown in Figure 4-14. The length of each bar denotes the magnitude of the net demand power of the network. It is assumed that the demand profile is known in advance and all the PQ buses (Load Buses) are multiplied with this demand profile to get a net time-varying demand of the network.

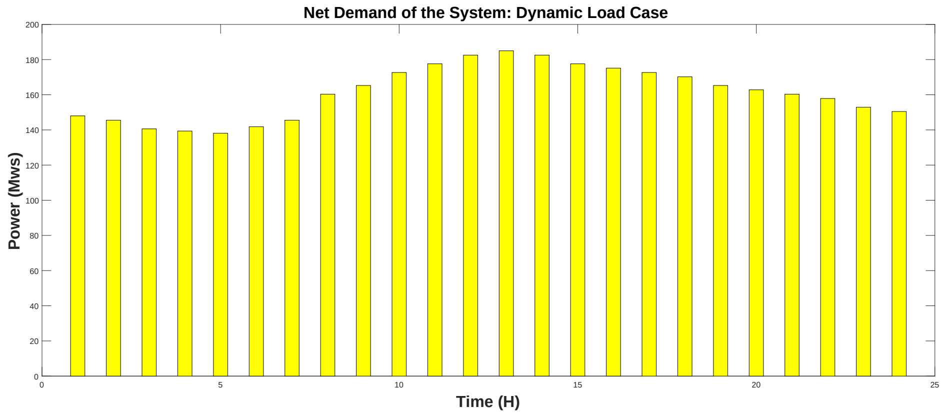


Figure 4-14: The net demand of the system through out the optimization horizon for the dynamic demand case.

The Total System Costs

The total system costs of the convex combination and the MLD approaches are compared with the reserve scheduling approach to analyze the performance of the proposed theoretical developments in the dynamic demand case.

Methods	Total costs (Euros)
Reserve Scheduling Approach	1.13120×10^9
Convex Combination Approach	1.12354×10^9
MLD Approach	1.12354×10^9

Table 4-6: The total costs of the system for all the three different optimization algorithms.

The MLD and the convex combination approaches perform slightly better than the reserve-scheduling approach. There is an improvement of 0.64% in the MLD and the convex combination approaches when compared to the reserve scheduling approach. The total system costs of the MLD and the convex combination approaches are same. An overview of the average total systems costs throughout the optimization horizon is shown in Figure 4-15. The relative costs of the MLD and the convex combination approaches with respect to the reserve scheduling approach are shown in Figures 4-16 and 4-17.

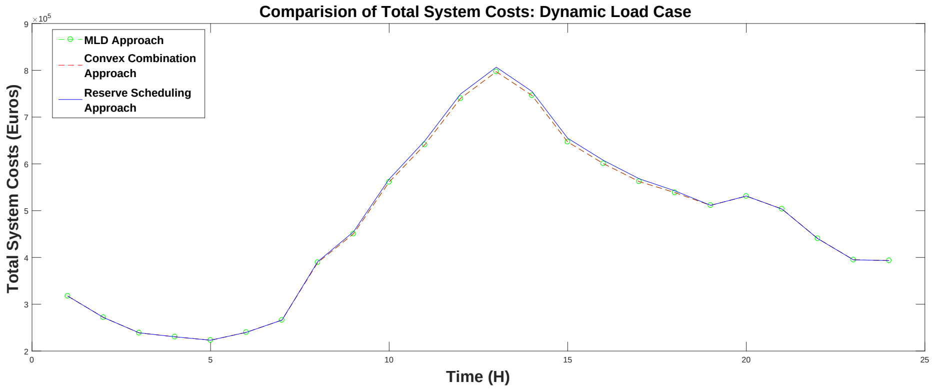


Figure 4-15: Comparison of the average total system costs of all three optimization methods of the dynamic demand case.

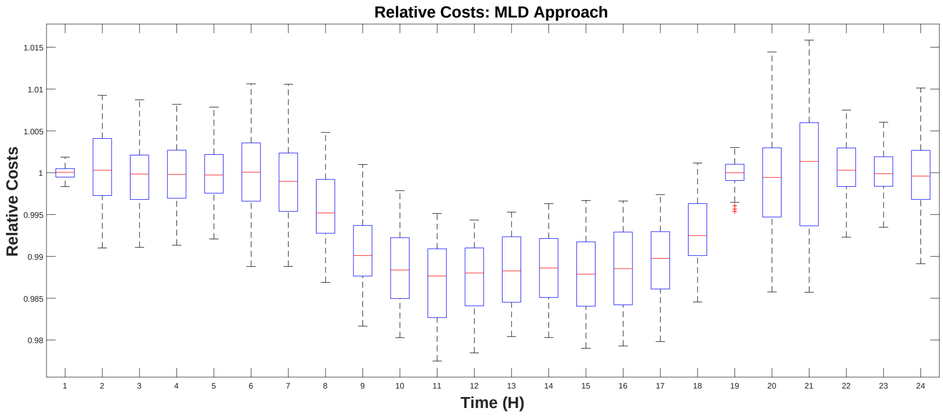


Figure 4-16: The relative costs of the convex combination approach with respect the reserve scheduling approach for the dynamic demand case. The red line indicates the median value. The edges of the box at every time step corresponds to the 25^{th} and the 75^{th} percentile values of the total system costs. The red lines denote the outliers of the data.

The averaged total system costs are low at the initial step. The total system costs increase with increase in the demand of the system. However, the total system costs are high when the demand is low from the time step 19 to end of the optimization horizon. This is because of the drop in the wind power. We can see a spike in the total system costs at time step 20, this is due to the drop in the wind power in the grid. The relative costs of the MLD and the convex combination approaches are less than one between time steps 10 and 16, when both load and wind power are high. In the other cases, in the few scenarios the reserve scheduling performs better and for another few scenarios vice-versa happen. This could because of the randomization approach to generate the wind power scenarios. The magnitude of the uncertainty differs at each scenario, so reserve scheduling might be good for few scenarios and the vice-versa in other scenarios based on the magnitude of the uncertain wind power. To analyze the above mentioned phenomena, the generator and the reserves dispatches along with wind power curtailment are examined throughly.

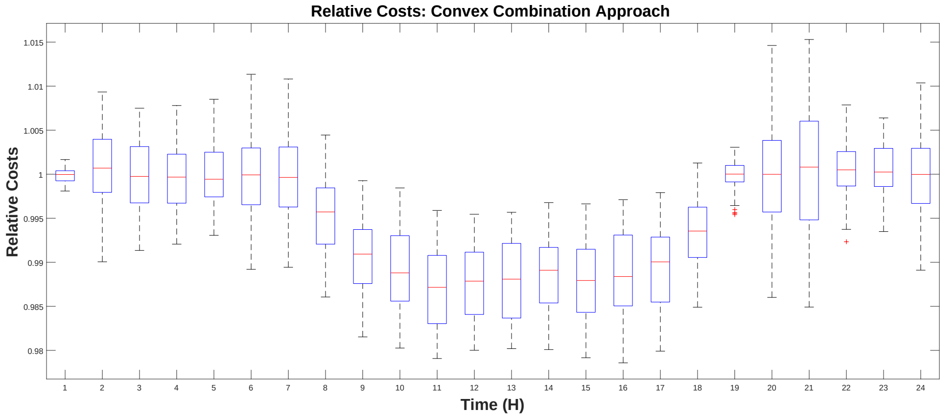


Figure 4-17: The relative costs of the MLD approach with respect the reserve scheduling approach for the dynamic demand case. The red line indicates the median value. The edges of the box at every time step corresponds to the 25^{th} and the 75^{th} percentile values of the total system costs. The red lines denote the outliers of the data.

Generator Dispatch

The active power dispatch from all the three optimization methods were similar. From Figures 4-18, 4-19 and 4-20, it can be observed that the generator-03 is used to provide the power for the increased demand during the peak hours. The power dispatch from the generator-03 is generally low because of the high generation costs. However, in the peak demand hours, extra power is taken from the generator-03 to satisfy the net demand of the system. The power dispatch from the generator-04 is high, as the generation costs of this unit are low. The generator-04 contributes for the variance in the wind power due to low generation costs, spatial relations and the time-varying nature of the demand.

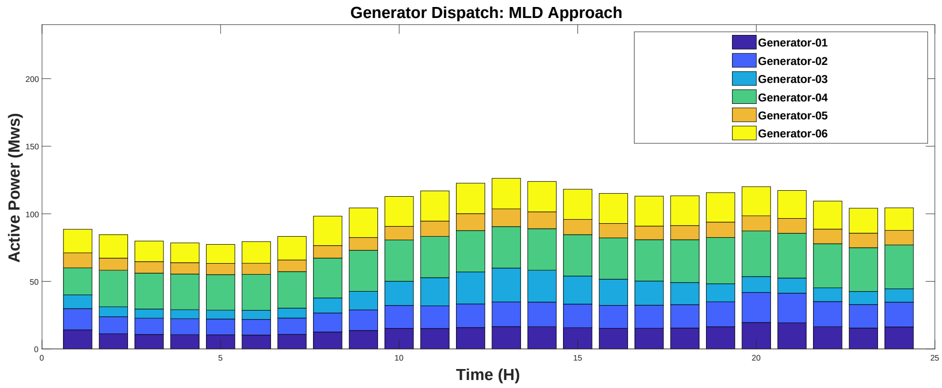


Figure 4-18: The active power dispatch from the different generation units in the MLD approach. The different colors indicate the power from the different generation units respectively. The corresponding color from of each generation units can be observed from the graph.

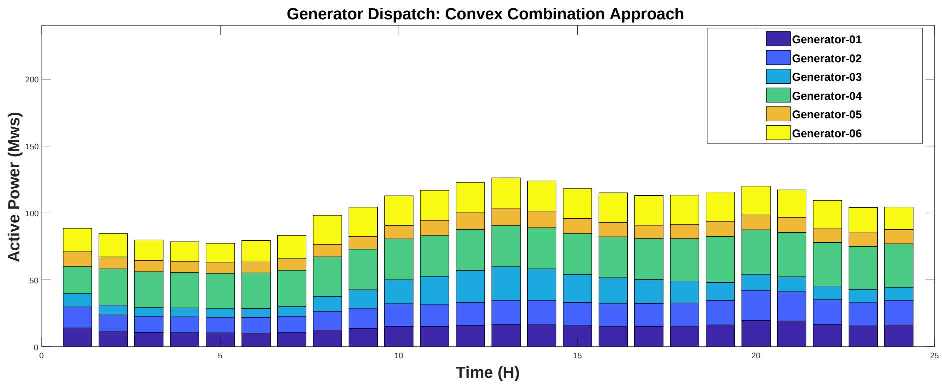


Figure 4-19: The active power dispatch from the different generation units in the MLD approach. The different colors indicate the power from the different generation units respectively. The corresponding color from of each generation units can be observed from the graph.

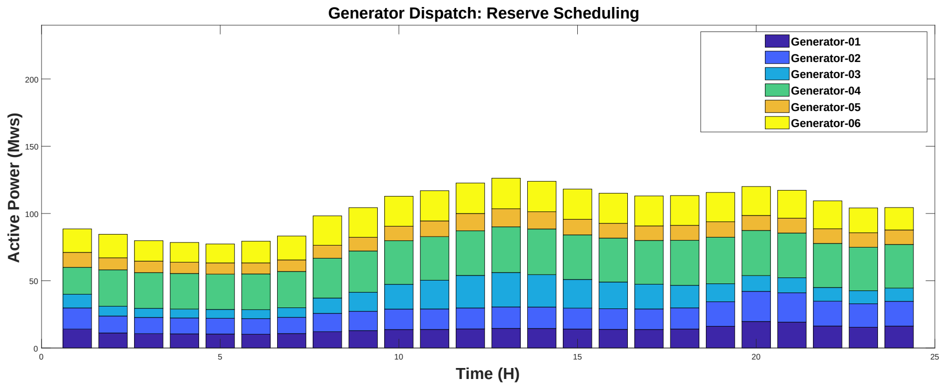


Figure 4-20: The active power dispatch from the different generation units in the MLD approach. The different colors indicate the power from the different generation units respectively. The corresponding color from of each generation units can be observed from the graph.

Reserves Dispatch and Wind Power Curtailment

The reserves dispatch of reserve scheduling approach is shown in Figure 4-21. It can be seen that the three generation units are contributing to the reserves dispatch at different time steps in an asymmetric manner. Especially, three generation units, generator-02, generator-05 and generator-04 contribute for the reserves during the start and end of the peak demand hours. Even though the cost of the reserves from the generator-04 are low, this generator doesn't contribute to the reserves of the system at initial time steps. Later on, during the peak load hours, the generator-04 contributes for the reserves. An asymmetrical distribution of the reserves can be seen in the Figure 4-21. One of the reasons for this kind of behavior could be the spatial relationships. A particular demand unit might be affected by the drop in the wind power, but the generator which has the lowest reserve costs might not contribute to that particular demand. In these kinds of scenarios, the generator which contributes more to that demand power also contributes to the reserves. The main causes for the wind power curtailment and the asymmetrical reserves dispatch are spatial relations, generator dispatch, and the wind power penetration into the grid.

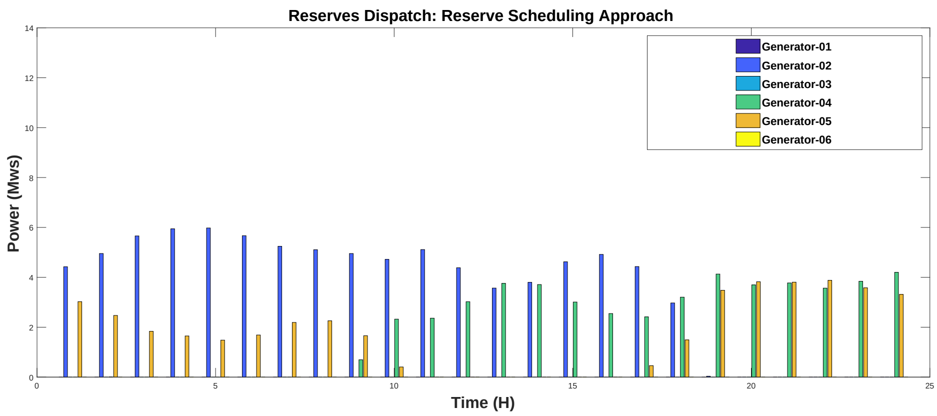


Figure 4-21: The reserves dispatch from the different generation units of the reserve scheduling for the dynamic demand. The height of each bar denotes the amount of the reserves dispatch from every generation unit. If the there is nor bar corresponding to the color, it means the generator is not contributing for the reserves.

As mentioned above, the main reasons for the wind power curtailment and the asymmetrical reserves dispatch are spatial relations, generator dispatch, and the wind power penetration into the grid. The wind power curtailment and the reserve scheduling of the convex combination approach and the MLD approach can be seen from Figures 4-22 and 4-23. From these Figures, it can be observed that the wind power curtailment and the reserves generated are same for both the convex combination and the MLD approaches. As mentioned in the static demand case, switching depends upon the generation levels, the costs of the reserves and in addition the spatial relations. So no need of an extra switch similar to the MLD approach, the convex combination approach performs similarly to the MLD approach with less computation time.

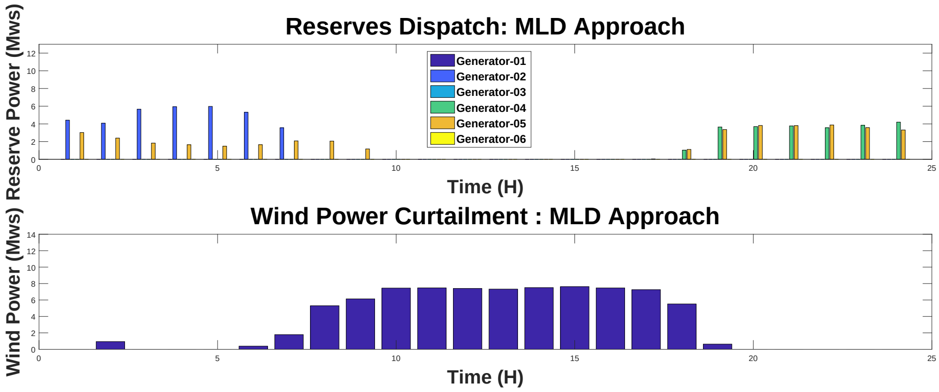


Figure 4-22: The reserves dispatch and the wind power curtailment of the MLD approach for the dynamic demand case. The height of each bar denotes the amount of the reserves dispatch from every generation unit. If the there is color nor bar corresponding to the generation unit, then it means there are no reserves from that particular generation unit. The second graph corresponds to the wind power curtailment. The height of each bar denotes the amount of wind power curtailed at every time step.

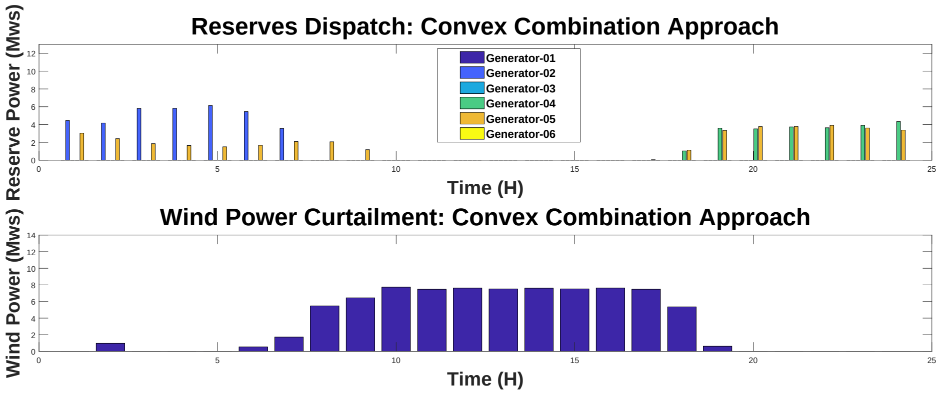


Figure 4-23: The reserves dispatch and the wind power curtailment of the convex combination approach for the dynamic demand case. The height of each bar denotes the amount of the reserves dispatch from every generation unit. If the there is nor color bar corresponding to the generation unit, then it means there are no reserves from that particular generation unit. The second graph corresponds to the wind power curtailment. The height of each bar denotes the amount of wind power curtailed at every time step.

Computation Time

Table 4-7 provides an overview of the computation time of all three optimization methods. It can be observed that the computation time of the MLD approach is significantly higher than the other two approaches. This is due to an increase in the number of the optimization variables in the MLD approach which can be observed from Table 4-3. The computation time is higher when compared to the static load case due to time-varying nature of the demand power.

Methods	Computation time (secs)
Reserve Scheduling Approach	6.70
Convex Combination Approach	6.74
MLD Approach	7.21

Table 4-7: The table provides comparison of the computation time of all the three optimization methods.

4-4 Summary

The proposed optimization strategies were applied to two case studies, the first is the static demand case and the second one is the dynamic demand case. In both case studies, the MLD and the convex combination approaches have outperformed the reserve scheduling approach in terms of the total system costs. The total system costs of the MLD approach and the convex combination approach were same. To analyze the similarity of the costs the generator dispatch and the reserves dispatch along with the wind power curtailment were examined. The generator dispatch was similar for all the three optimization approaches, while the there was a difference in the reserves dispatch.

However, the reserves dispatch and the wind power curtailment were same for the MLD approach and the convex combination approach. This is the reason why both the approaches have the same total system costs. The convex combination approach has an inherent switching behavior. The switching of the reserves depends upon the generator dispatch, wind power penetration into the grid and the spatial relations between the demand units, the generators and the wind power plants. So, an extra switch similar to the MLD approach is not required for the operational conditions considered in this thesis. The MLD approach just increases the computation time of the system and won't improve the performance of the system when compared to convex combination approach. So the convex combination approach would be preferable over the MLD approach.

Chapter 5

Conclusions and Future Recommendations

5-1 Conclusions

Two methods were proposed to unify the reserve scheduling and the wind power curtailment to tackle with the problem of uncertain wind power generation in the smart grids. The DC power flow framework was used to formulate the optimization problem. An approach presented in [3] has been modified to unify the reserve scheduling and the wind power curtailment. Later on, two different CCPs (chance constrained programs) were proposed to create an optimal trade-off between the reserves regulation and the wind power curtailment in the smart grids.

As an initial step, reserves and wind power curtailment were unified using the convex combination model. Later on, a chance constrained optimization scheme was proposed to create an optimal combination of the reserve regulation and the wind power curtailment. The convex combination model which unifies the wind power curtailment and the reserve scheduling was transformed into a MLD model following the rules presented in [50]. Similar to the convex combination approach, a chance constrained optimization program corresponding to the MLD model was proposed to find an optimal combination of the wind power curtailment and the reserve scheduling.

Both proposed methods have chance constraints. The problem of chance constraints was transformed into a scenario based stochastic optimization problem following the approach presented in [48]. The scenarios to be generated depend upon the prior feasibility guarantees. This would result in a very high number of scenarios to be generated. However, a large number of scenarios are highly impractical and will make the solution intractable. To avoid these two problems, only 100 scenarios were considered at every time step and 2400 scenarios throughout the optimization horizon. This assumption reduced the computation time and the solution of both approaches was tractable.

The proposed theoretical developments have been implemented on an IEEE-30 bus network. They were compared with the reserve scheduling approach to demonstrate the performance of the proposed algorithms. The proposed methodologies were implemented on two case studies. The first case was the static demand case where the net demand of the system was constant through out the optimization horizon. The proposed methods had outperformed the reserve scheduling approach. However, the computation time was higher in both the approaches as they have more number of decision variables when compared to the reserve scheduling. The second case was the dynamic demand case where the net demand of the system was varying with respect to the time. In the dynamic demand case also the reserve scheduling was outperformed by the proposed methods.

However, the performance of convex combination and MLD approaches was exactly the same in both case studies. To analyze this, the generator dispatch, reserves dispatch and the wind power curtailed were observed in both the approaches. They were also exactly the same. The switching of reserves and wind power curtailment depend on various parameters such as the costs of reserves and wind power curtailment, the spatial relations, the generator dispatch and the wind power generated in the grid. So an extra switch as proposed in the MLD approach is not necessary to minimize the total system costs for the operational conditions considered in this thesis.

5-2 Future Recommendations

There are various future research directions. They are:

1. **Multiple Wind Power Plants**

   In this thesis, only one wind turbine was considered in the grid with low wind power generation. This work could be extended by adding multiple wind power plants which would suffice at least 40% of the demand in the grid. The proposed optimization techniques can be applied to the multiple wind turbines concept and then compare the performance with the reserve scheduling approach. The joint PDF (probability density function) of the wind farms can be considered to generate the net wind power scenarios using the Monte Carlo simulations.

2. **AC Power Flow**

   The proposed optimization techniques could be applied to the AC power flow algorithm by adding a few extra variables such as voltage, reactive power flow to the optimization problem. The resulting problem would be non-linear but this could be transformed into a convex problem. The convex problem can be solved using the SDP (semidefinite programming) method. But still, tractability of the solution can be an issue. The methods proposed in [54] can be used to achieve the tractability of the proposed solution.

3. **Forecasting of Wind Power and Demand Power**

   Wind power and demand can be estimated using many techniques such as SARIMA, ANN (artificial neural networks), SARIMAX etc. The effect of exogenous variables on the wind power forecasting would be an interesting area of research. However, wind power and demand are correlated as they are influenced by the exogenous factors such as temperature, atmospheric pressure etc. This could also be implemented in the optimization problem by considering a joint PDF of the wind power and demand to generate various scenarios of the uncertain wind power and the demand.

4. **MPC for Power Control of Wind Farms**

   The proposed optimization techniques can be high-level control strategies, while a MPC (Model Predictive control) scheme can be as a low level controller to track the reference of the wind power provided by the high level control schemes. The wind farm model is highly non-linear. So implementing an NMPC (non-linear model predictive control) would be computationally heavy. However, this non-linear model of the wind farm can be transformed into a piecewise affine model as suggested in [42]. This piecewise affine model can be transformed into an MLD model. A MPC with MIQP optimization can be implemented to track the reference provided by the high-level control algorithms.

Appendix A

Reserve Scheduling

The actual wind power penetration into the grid is not always equal to the estimated wind power. So a term called $R^{\mathrm{ds}}_{i,t}$ have been introduced as a correction term to counter the wind power mismatch. This correction term is modeled as a linear function which is expressed as the difference between the actual wind power and the estimated wind power. This can be mathematically represented as;

$$R^{ds}_{i,t} = d^{\mathrm{ds}}_{i,t}(P^{\mathrm{wf}}_t - P^{\mathrm{w}}_t) \tag{A-1}$$

where P^{w}_t, P^{wf}_t denote the actual wind power penetration and the estimated wind power respectively, $d^{\mathrm{ds}}_{i,t}$ is denotes of the amount of down-spinning from the all the generation units in the grid. By considering N_{l} lines, N_{w} wind farms, N_{g} conventional generators, N_{b} buses, and finally N_{d} load sinks, the vector of the decision variables in the optimization problem is;

$$x_{i,t} = [P^{\mathrm{g}}_{i,t}, \gamma^{\mathrm{g}}_{i,t}, z^{\mathrm{g}}_{i,t}, d^{\mathrm{ds}}_{i,t}, C^{\mathrm{su}}_{i,t}, R^{\mathrm{ds}}_{i,t}] \tag{A-2}$$

where $P^{\mathrm{g}}_{i,t}$ denotes the scheduled power coming from each and every generator (i) at time-step t, $\gamma^{g}_{i,t}$ denotes the binary variable to control the on-off status of all the generation units. The term $d^{\mathrm{ds}}_{i,t}$ denotes the amount of the down-spinning from every generation unit. The term $C^{\mathrm{su}}_{i,t}$ denotes the effective start-up costs each generation at time-step t. Finally, the term $z^{\mathrm{g}}_{i,t}$ denotes the auxiliary variable of to model the minimum-up and minimum down times of the every generation unit.

Cost-Function

The cost function is divided into two parts, they are the generation costs of the convectional generation units, and the reserve-regulation costs. The cost function can be written as;

$$\min_{x_{i,t}} \; \left(\sum_{t=1}^{N_t}\sum_{i=1}^{N_{\mathrm{g}}} f_c(P^{\mathrm{g}}_{i,t}) + \sum_{s=1}^{N_s} p_s\left(\sum_{t=1}^{N_t}\sum_{i=1}^{N_{\mathrm{g}}}(f_{cc}(R_{i,t,s}))\right)\right) \tag{A-3}$$

where

$$f_c = (a_i + b_i P^{\mathrm{g}}_{i,t,s} + c_i(P^{\mathrm{g}}_{i,t,s})^2 + C^{\mathrm{su}}_{i,t,s}) \tag{A-4}$$
$$f_{cc} = (C^{\mathrm{rs}}_i(R_{i,t,s})) \tag{A-5}$$

where the function f_c denotes the economic dispatch from every generation unit i, the terms c_i and b_i denote the quadratic costs and the linear costs of the generation from every generation unit i. The term $C^{\mathrm{su}}_{i,t,s}$ denotes the effective start costs of every conventional generation unit i. The function f_{cc} is represents the reserve scheduling costs of the system, the term C^{rs}_i denote the costs of the the reserve scheduling from every generator i respectively. The total optimization horizon is denoted by N_t and it is equal to 24 hours. The term p_s represents the probability of each and every scenario of the uncertain wind power realization.

Constraints of the optimization

The constraints in the optimization problem are:

- The most important constraint is power balance of the network, this means the demand should meet with enough power at every time step t and there is no excessive power in the grid. It can be written as;

$$\sum_{i=1}^{N_{\mathrm{g}}} P^{\mathrm{g}}_{i,t,s} + \sum_{i=1}^{N_{\mathrm{w}}} P^{\mathrm{w,f}}_{i,t,s} - \sum_{i=1}^{N_{\mathrm{d}}} P^{\mathrm{d}}_{i,t,s} = 0 \tag{A-6}$$

- The power generation from every conventional generator should be within the limits, it can be written as;

$$[P^{\mathrm{g}}_{\mathrm{min},i}]\gamma^{\mathrm{g}}_{i,t,s} \leq P^{\mathrm{g}}_{i,t} \leq [P^{\mathrm{g}}_{\mathrm{max},i}]\gamma^{\mathrm{g}}_{i,t,s} \tag{A-7}$$

  where $P^{\mathrm{g}}_{\mathrm{min},i}$ denotes the minimum power generation of every generation unit (i), $P^{\mathrm{g}}_{\mathrm{max}}$ denote maximum power of every generation unit (i). The term $\gamma^{\mathrm{g}}_{i,t}$ denotes the on-off status of every generation unit (i).

- This constraint implies that the power flow in the transmission lines should be within the line limits of the grid.

$$-P_{\mathrm{line}} \leq P^{\mathrm{f}}_{i,t,s} \leq P_{\mathrm{line}} \tag{A-8}$$

- Every conventional generation unit should ramp-up and ramp-down within the limits.

$$-P^{\mathrm{g}}_{\mathrm{down,i}} \leq P^{\mathrm{g}}_{i,t} - P^{\mathrm{g}}_{i,t-1} \leq P^{\mathrm{g}}_{\mathrm{up},i} \tag{A-9}$$

  where $P^{\mathrm{g}}_{\mathrm{down},i}$, $P^{\mathrm{g}}_{\mathrm{up},i}$ denote the ramp-up and ramp-down limits of the generation unit i.

- The effective start up costs should be greater than or equal to zero at every time-step t. The can be mathematically written as;

$$C^{su}_{i,t,s} \geq 0 \tag{A-10}$$

One more constraint on the start-up costs is;

$$\lambda^{su}(\gamma^{\mathrm{g}}_{i,t,s} - \gamma^{\mathrm{g}}_{i,t-1,s}) \leq C^{\mathrm{su}}_{t} \tag{A-11}$$

where λ^{su}_{i} represents the start-up costs of the every generation unit (i). This constraint implies that the effective start up costs would be zero unless the generation unit changes it's status from off to on.

- The next constraints are probabilistic constraints, these constraints exist because of the uncertainty in the wind power, they can be written as;

$$\mathrm{Pr}\Bigg(\Delta P^{\mathrm{w}}_{i,t}\| - \mathrm{P}^{\mathrm{line}}_{i,t} \leq P^{\mathrm{f}}_{i,t,s} \leq \mathrm{P}^{\mathrm{line}}_{i,t}, \tag{A-12}$$

$$[\mathrm{P}^{\mathrm{g}}_{\mathrm{min},i}]\gamma_{i,t,s} \leq P^{g}_{i,t,s} + R^{s}_{i,t,s} \leq [\mathrm{P}^{\mathrm{g}}_{\mathrm{max},i}]\gamma_{i,t,s} \tag{A-13}$$

$$0 \leq R^{\mathrm{s}}_{i,t,s} \leq \mathrm{R}^{\mathrm{max}}_{i}. \Bigg) \geq 1 - \epsilon \tag{A-14}$$

The first constraint among the probabilistic constraints encodes that probabilistic guarantee on the standard transmission capacity of the grid. The second one encodes that the generation schedule along with the reserve scheduling should be within the limits of the generation. The final one encodes the limits on the reserve regulation on the individual generation units.

- The power balance constraint in the presence of the wind uncertainty and the reserve scheduling can be written as;

$$\sum_{i=1}^{N_{\mathrm{g}}} (P^{\mathrm{g}}_{i,t} + R^{\mathrm{s}}_{i,t}) + \sum_{i=1}^{N_{\mathrm{w}}} P^{\mathrm{w}}_{i,t} - \sum_{i=1}^{N_{\mathrm{d}}} P^{\mathrm{d}}_{i,t} = 0 \tag{A-15}$$

- The next constraint say that the sum of all the vectors contributing for the down spinning should be equal to one.

$$\sum_{i=0}^{N_{\mathrm{g}}} d^{\mathrm{ds}}_{i,t} = 1; \qquad d^{\mathrm{ds}}_{i,t} \geq 0 \tag{A-16}$$

- The minimum up and down time of the generation units constraint,

$$0 \leq z^{\mathrm{g}}_{i,t,s} \leq 1 \tag{A-17}$$

$$\gamma^{\mathrm{g}}_{i,t,s} - (\gamma^{\mathrm{g}}_{i,t-1,s}) \leq z^{\mathrm{g}}_{i,t,s} \tag{A-18}$$

$$\sum_{j=t-\Delta t_{\mathrm{up}}+1}^{t} z^{\mathrm{g}}_{i,j,s} \leq \gamma^{\mathrm{g}}_{i,j,s}, \quad \text{for} \;\; t \geq \Delta t_{\mathrm{up}} \tag{A-19}$$

$$\sum_{j=t+1}^{t+\Delta t_{\mathrm{down}}} z^{\mathrm{g}}_{i,j,s} \leq 1 - \gamma^{\mathrm{g}}_{i,j,s} \;\; \text{for} \;\; t \leq N_{\mathrm{t}} - \Delta t_{\mathrm{down}} \tag{A-20}$$

the first constraint says that the auxiliary variables should be positive, the second one says about minimum time taken by the generation units to change their status from on to off or vice-verse.

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