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Behavioral-Based Pedestrian Modeling Approach: Formulation, Sensitivity Analysis, and Calibration

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Abstract

Pedestrians are among the travelers most vulnerable to collisions that are associated with high fatality and injury rates. The increasing rate of urbanization and mixed land-use construction make walking (along with other non-motorized travel) a predominant transportation mode with a wide variety of behaviors expected. Because of the inherent safety concerns seen in pedestrian transportation infrastructures, especially those with conflicting multimodal movements expected (crosswalks, transit platforms, etc.), it is important that pedestrian behavior is modeled as a risk-taking stochastic behavior that may lead to errors and thus collision formation. In previous work, the complexity and cost associated with building pedestrian models in a cognitive-based environment weighted down the construction of simulation tools that can capture pedestrian-involved collisions, including those seen in shared space environments. In this paper, a tool that will help evaluate the safety of pedestrian traffic is initiated: an extended modeling framework of pedestrian walking behavior is adopted while incorporating different physiological, physical, and decision-making elements. The focus is on operational decisions (i.e., path choices defined by longitudinal and lateral trajectories) with a pre-specified set of origins and destinations. The model relies on the prospect theory paradigm where pedestrians evaluate their acceleration and directional alternatives while considering the possibility of colliding with other "particles." Using a genetic algorithm method, the new model is calibrated using detailed trajectory data. This model can be extended to model the interactions between a variety of different modes that are present in different mixed land-use environments.

Keywords

pedestrians, bicycles, human factors, analysis, modeling and forecasting, safety, simulation

Pedestrians are vulnerable elements of transportation systems. Pedestrians often interact with other modes of transportation, including bicycles/bicyclists, transit vehicles, personal vehicles, obstacles/platforms, and other pedestrians (*1*). During such interactions, pedestrians are more exposed to collision/fall risks and often have less physical protection than the other modes. The limited protection and the high exposure to collisions and falls (i.e., incidents) result in more severe injuries or fatalities (*2*). Pedestrians are further exposed to higher risks of injuries and fatalities in urban areas. In addition, urban travelers often use multiple modes to commute, including walking for at least a portion of their trips. In 2016, 76% of pedestrian fatalities were in urban areas (*3*). In Washington DC, U.S., pedestrian fatalities accounted for 56% of total collision fatalities in 2015 (*4*). Of all roadway fatalities in large U.S. cities (> 500,000 in population size), pedestrian collisions as a median account for 12.1% of total traffic fatalities (*4*). In the current urban

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landscape, the most critical and unsafe time for pedestrians is when they are crossing the roadway. In 2011, of all pedestrian fatalities from single-vehicle crashes, 76.6% are associated with pedestrian crossing in contrast to 16.1% where the pedestrian is moving parallel to the vehicular traffic (*5*). The risk of a collision involving a pedestrian is then increased as a result of speed, the adopted traffic control schemes, the more complex maneuvering involved, and distractions. This safety issue has motivated a focus on pedestrian interactions while accounting for behavioral and cognitive dimensions including distractions, risk-taking attitudes, uncertainty, and sensitivity associated with the surrounding environment. The traffic or statistical models to analyze pedestrian scenarios are often limited in scope and need adaptation to incorporate both safety and mobility performance measures.

Trying to model pedestrian interactions in a cognitive framework poses multiple challenges: (1) Heterogeneity challenge: the pedestrians' trajectories are not homogeneous and can differ dramatically depending on the pedestrians' risk-taking attitudes; (2) Formulation challenge: translating decision-making theories (i.e., judgement phase) into trajectories (i.e., execution phase) is computationally "expensive" with multiple behavioral and physiological characteristics to be taken into account; (3) Calibration and validation challenge: collecting trajectory data at the moments that lead up to a pedestrian contact or collision is challenging in nature. Accordingly, a model that captures risk that leads to collisions is essential. Such a type of model can utilize collision-free trajectory data where pedestrians try to avoid collisions. Such trajectory data is available, while trajectory data with multiple collisions may not be readily available for calibration and validation purposes. It should be noted that collisions may be between pedestrians or between pedestrians and other modes of transportation (i.e., bicycles, personal vehicles, buses, etc.). This paper focuses on pedestrian-only traffic modeling with the possibility of extending the modeling framework to account for mixed traffic flow in the future.

Research into this field is essential to develop pedestrian/crowd management and control systems that can accommodate different pedestrians' interactions while preventing unsafe walking behaviors. For walking to be a practical choice, pedestrians must feel safe and protected by the existing transportation infrastructure. To build this infrastructure, designers and engineers must take into consideration the way in which pedestrians interact, and engineer features that will help to reduce pedestrian injuries. In other words, the objective of this paper is to develop a pedestrian flow model in a risk-taking environment that will assist engineers and planners in creating safer and smarter urban transportation networks. Toward realizing such an objective, a behavior-based formulation is offered and tested for feasibility. The formulation is then translated into a mathematical model that: (i) is computationally inexpensive; (ii) includes the psychological pedestrian decision-making process (based on prospect theory [PT]); and (iii) is adaptable to model multimodal mixed environment scenarios in the future.

Given such objectives and approach, this paper is organized as follows. The next section presents a brief review of literature on the main studies that address (i) pedestrian safety and (ii) pedestrian walking behavior, especially those focusing on the decision-making component of such behavior. The section after that offers the formulation paradigm along with the corresponding sensitivity analysis. The following section presents the calibration method and results. The penultimate section offers the numerical simulation and analyzes the corresponding output. The final section concludes this paper, with possible future research directions.

Background

There are several approaches to dissecting pedestrian interactions. A limited number of these approaches, however, account for physical contact and safety issues. Often researchers study pedestrian safety empirically or via exposure statistics to reduce the observed fatalities/injuries of pedestrians with other objects (*6, 7*). Another approach is risk analysis through physical modeling (*8*). This method has a greater scope as it can be applied to interactions that result in collisions and those that do not.

Focusing on pedestrian flow modeling, there is a wealth of literature on pedestrian walking behavior. Such literature is based on the work of Yuan and Daamen (*9*). The physical walking models, also called the "human movement models" are classified into five categories: (i) force-based models; (ii) optimization-based models; (iii) cellular automata (CA) models; (iv) utility-based models; (v) agency-models, and (vi) data-driven approaches (models). The force-based models' seminal paper is that of Helbing and Monar (*10*). This paper first introduced the force-based models under the name of a social-force model: pedestrians' interactions are represented mainly by internal attraction forces (i.e., based on the pedestrian's destination) and external repulsion forces (based on the location of other pedestrians and obstacles). Additional social potential fields can supplement the model, thus offering flexibility in relation to adding different factors (through different fields/potentials) (*11*). However, the additive aspect of the forces and the non-realistic representation of contacts/collisions through Newtonian physics remain a subject of criticism (*12*). The optimization-based models consider the pedestrians' movement as an outcome of an optimization exercise on an energy or utility scale (i.e., least effort concept applied to pedestrians' movement). However, the interpretation of the parameters in such models is non-intuitive and this type of model cannot represent behavior in some crowded situations such as panic situations (*13*). CA models are based on the discretization of space and the representation of the movement of pedestrians occupying each space unit through basic two-dimensional rules (random and/or deterministic rules) (*14*). CA models may be classified as floor field models (where additional layers may capture the route choice decisions of pedestrians) or lattice gas models (*15, 16*). Despite their computational simplicity, the main issue in these models is the non-natural movements generated and the non-ability to reproduce some more complex crowd dynamics such as those observed in narrow bottleneck situations (*17*). Discrete choice models are mostly based on random-utility theory: rationality is assumed where pedestrians try to maximize a given utility based on the operational alternatives presented in front of them (i.e., speed and direction alternatives) (*18*). Despite these models capturing a behavioral decision-making dimension rarely seen in other models, the corresponding framework may be too rigid since it assumes transitivity and consistency in behavior. Accordingly, some modifications on such modeling frameworks are suggested, including the use of bounded rationality theory (*19*). These models remain problematic in relation to implementation and calibration requirements; such requirements limit the use of discrete choice models in building simulation tools for prediction purposes; discrete choice models are mostly used to analyze pedestrian traffic in specific situations. Finally, "agency models" (i.e., agent-based models) consider pedestrians as "proactive and autonomous" entities (i.e., agents). The dynamics governing the movement of such agents may range from rule-based dynamics or collision avoidance/evading maneuvers' dynamics (*9, 20*). Despite being suitable to replicate multiple traffic dynamics, agent-based models are complex to calibrate and do not capture risk-taking behavior in an explicit manner. Some efforts to incorporate some of these dynamics, especially the evasive maneuvers' dynamics, into different aforementioned modeling frameworks have also been attempted with different levels of success (*21–27*). Finally, the "data-driven approaches" models of Yuan and Daamen are classified as "data-in-the-loop" models and "data-in-the-model" models. The data-in-the-loop models are simply based on real data that are fed in to analyze and predict collective movements of pedestrians through smoothing, interpolation, and integration techniques. Such data-driven approaches may generate artificial behaviors since they are mostly based on sensing technologies (video sensing, GPS sensing, blue-tooth sensing) rather than actual behavioral modeling. The data-in-the-model models are based on real data fed into a calibration module to estimate parameters of one of the aforementioned modeling types. The calibrated models are then used to predict and analyze pedestrian's traffic dynamics. In other words, the data-driven models of Yuan and Daamen are simply an application of one type of the aforementioned modeling approaches (*9*).

Based on the above review, the main two modeling approaches that capture risk-taking behaviors are social force (SF) models (through the interplay of the repulsion and the attraction force parameters) and discrete-choice models (through the rationality or the bounded rationality paradigm while weighing different alternatives). Given that SF models cannot capture the contact/collision dynamics through the Newtonian Force framework, the authors believe that decision-making theories are a feasible approach to formulate a new model that can account for cognitive and behavioral dimensions such as uncertainty and risk. However, instead of relying on bounded rationality theory, this research is based on PT as a natural extension to random utility maximization. Moreover, this paper presents an implementation approach to transform such theoretical framework into an implementable pedestrian traffic simulation tool that may be calibrated and validated.

In PT, the interpretations of gains and losses and the resulting physical reactions are at the core of the decision-making processes that lead to different risk-taking attitudes. These attitudes may constitute the fundamental cause behind pedestrian's acceptance of contact. It is important that pedestrian models include a comprehensive decision-making process to accurately account for human errors. Utilizing PT will enhance modeling accuracy and provide space to account for pedestrians' misinterpretations. A subjective utility-based formulation will calculate immediate future risks and will be the base of a generalized model that can be applied in multiple pedestrian movement scenarios.

Economics-Based Modeling Paradigm and Base Sensitivity Analysis

The economic-based modeling paradigm is focused on introducing a novel modeling framework based on the concept of "rational" theory to characterize the behaviors of pedestrians with the possibility of allowing collisions with different objects (*28, 29*). The rational theory assumes that humans do have a "value" associated with different alternatives and they are trying to maximize such value. As in the discrete choice models, the alternatives can be the direction of movement and the speed of movement. Unlike the traditional bounded rationality approaches, the main intellectual merit behind introducing this new framework is to allow risk-taking tendencies with subjective estimation of such risk (i.e., loss associated with collisions and relative gain associated with accelerating versus decelerating). It should be noted that this modeling research is not offered as a critique to existing modeling efforts. It is noted that collective pedestrian traffic phenomena (such as lane formation and zipper effects) may not be created as an outcome of individual interactions; similarity in decision-making logic and pedestrian modeling should not be interpreted as similar behavior, as adapting a decision-making theory to account for collective processes is a challenging task. It is still unclear if the behavior of the sum of individuals can be aggregated to a crowd behavior model at this stage of this research.

With such limitations being acknowledged, under the construct of rationality, it is considered that pedestrians are subjectively aware of their physiological and information-processing capabilities and vulnerabilities, and behave according to those capabilities. Given these capabilities, the underlying decision-making logic is linked to alternatives that may be characterized by a **directional** component and a **movement** component.

Starting with the **directional component** of the model, pedestrians should choose their direction of movement in a two-dimensional environment (at this stage of this research work). This paper assumes that pedestrians decide their direction of movement based on the value function. The value function to determine the direction can have different forms (e.g., exponential constant relative risk aversion [ECRRA] form, constant relative risk aversion [CRRA] form, and PT form). Assuming a PT value function form where the desired direction is specified to be directly ahead:

$$U^{\theta}_{PT}(a_n) = \eta_i \cos(\theta_i) \left[\frac{\left(\frac{v_i}{v_{d,i}}\right)}{\left(1 + \left(\frac{v_i}{v_{d,i}}\right)\right)^{\left(\frac{\xi_i - 1}{2}\right)}} \right] \tag{1}$$

where

i indicates a given pedestrian number,

θ is the angle between the line connecting the pedestrians to the destination and the direction of travel, and

v_d is the desired speed.

The desired speed is interpreted as the speed aimed for by a pedestrian during free-flow conditions and in passing conditions. Accordingly, when modeling the accelerations of pedestrians, the desired speed is the maximum speed adopted by a decision-maker even during a passing maneuver. The existence of different desired speeds in different walking scenarios is acknowledged, but is not accounted for at this stage of the formulation.

Focusing on the form of the value function, η and ξ are parameters to be calibrated (η is an amplitude parameter that represents the sensitivity to the choice of direction and ξ is a measure of non-linearity in the PT value function). Finally, v is the maximum achievable speed at direction θ given the choice of acceleration $a_{n,i}$ (or a_i at this stage ignoring added inter-driver heterogeneity—subject to reaction time RT_i and relaxation time τ_i). This model recognizes that the value function is at its maximum (at the value of 1) when traveling toward the destination at the desired speed. The value function may be standardized to be between different values (example, between -1 and $+1$). Each of the aforementioned parameters should be calibrated based on individual characteristics to reflect user heterogeneity. Figure 1 illustrates the above value function for different values of θ and $^{v}/_{v_d}$.

To complement the directional component, the proposed **movement component** has its roots in the car-following model of Hamdar et al. (*30*). The original model by Hamdar et al. is focused on vehicle maneuvers within a lane. The new modeling paradigm, on the other hand, utilizes Hamdar et al.'s approach to determine the possible maximum speed for each direction of travel. For this purpose, pedestrians' maximum speed should be determined based on pedestrian-pedestrian interactions.

Following the original Hamdar et al.'s model and considering U^{θ}_{PT} (the value function), the total utility function can be calculated for pedestrians:

$$U^{\theta}(a_n) = \max_j \left\{ \left(1 - p^{\theta}_{n,j}\right) U^{\theta}_{PT}(a_n) - p^{\theta}_{n,j} w_c k(v, \Delta v, j) \right\} \tag{2}$$

where

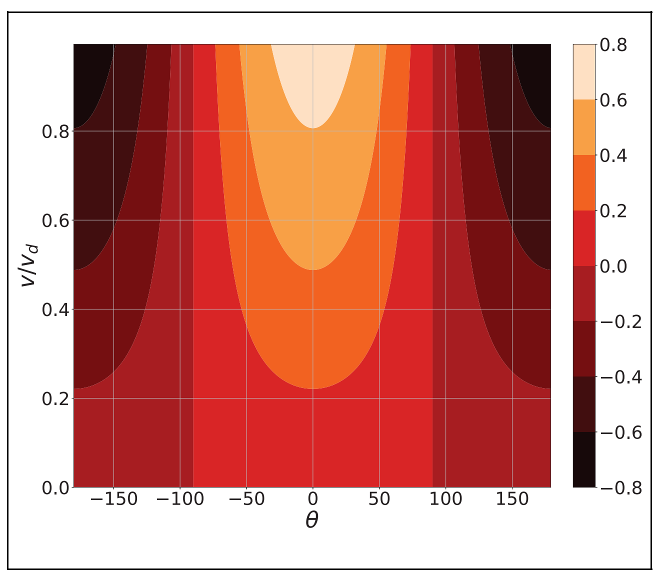

Figure 1. Value function of Equation 1 for different values of θ **and** $^{v}/_{v_d}$.

j denotes the type of interaction (with another pedestrian or another obstacle), and
$p^{\theta}_{n,j}$, w_c, and $k(v, \Delta v, j)$ denote the collision probability, collision weighting parameter, and collision seriousness term, respectively.
Note that $k(v, \Delta v, j)$ is different for different combinations of pedestrian-pedestrian or pedestrian-object interactions. For example, it is expected that pedestrians put more weight on pedestrian-to-wall collisions compared with pedestrian-pedestrian collisions (depending on the characteristics of the wall or the pedestrian).

To elaborate further, Equations 1 and 2 constitute the basis to calculate the utility of moving at a certain direction (given the θ component) at a given speed (given the a_n value along a given direction relative to a reference axis that provides the speed change from the previous time-step, i.e., the reference speed). The collision seriousness term k may be a function of speed, relative speed, and type of conflicting object. However, at this stage, this term is set to be equal to 1, since all pedestrians are moving in the same direction and since there is no interaction considered between different types of traffic object (e.g., pedestrian versus bicycle versus personal vehicle). As for the collision weighing parameter, it is a parameter that will be calibrated based on trajectory available and is specific to each pedestrian, capturing inter-pedestrian heterogeneity. The type of conflict between pedestrians (i.e., crossing versus side conflict) is assumed not to affect the collision weighing parameter. The authors hypothesize that the change of the collision weighing parameter is minimal at this stage from time-step to time-step, but is significantly different across pedestrians as it is more related to the pedestrians' characteristics.

With the above assumptions, the formulation presented still considers the stochastic response adopted by pedestrians by generating different probabilistic forms associated with the utility of movements (Equation 2). Among these forms, the logistic functional form (depending on the value function error distribution) specified by Hamdar may be used to calculate the probability density function (*31*):

$$g^{\theta} = \begin{cases} \dfrac{e^{\beta_{PT} U^{\theta}(a_n)}}{\int_{a_{min}}^{a_{max}} e^{\beta_{PT} U^{\theta}\left(a'\right)} da'} & a_{min} < a_n < a_{max} \\ 0 & Otherwise \end{cases} \tag{3}$$

where
β_{PT} reflects the sensitivity of choice to the utility $U^{\theta}(a_n)$. Once the pedestrian selects the acceleration, the speed can be calculated for the direction of θ (see Equation 1 for more details).

For implementation purposes, at this stage of the study, a start is made with the basic assumption by calculating the probability of collision based on the possibility of trajectory intersection between pedestrian I and a pedestrian j in the vicinity of pedestrian i. The vicinity of the pedestrian is currently defined as the total number of pedestrians surrounding pedestrian i from all directions (i.e., exhaustive approach). In particular,

$$p^{\theta}_{i,j} = \begin{cases} 1 & \text{if the trajectories of pedestrians } i \text{ and } j \text{ intesect} \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Accordingly, for a given pedestrian i, the total collision probability is set to:

$$p^{\theta}_{i} = \sum\nolimits_{j=1}^{N} \left(\frac{p^{\theta}_{i,j}}{N} \right) \tag{5}$$

where
N is the total number of pedestrians numbered from $j = 1$ in the vicinity of pedestrian i at a given time-step.

The vicinity of a pedestrian may be represented by a given circle encompassing all surrounding pedestrians at this stage, but this term may be described differently by different researchers. Equation 5 assumes that all conflicts are weighted similarly by a given pedestrian regardless of the type of conflict (i.e., merging versus crossing conflict) and the proximity of the conflict. This constitutes a starting point given that pedestrians function in a limited decision space and given that pedestrians travel at relatively low speeds to take into account longer anticipation horizons. This assumption will be tested in this paper through the calibration exercise. It should be noted that, despite the simplistic form of Equations 4 and 5, the proposed collision probabilities take into account the speed/acceleration and the direction of movement of surrounding pedestrians in the next anticipation horizon: the number of trajectory intersections depends on the current directions and speeds of the surrounding pedestrians as estimated by the pedestrian in motion (i.e., speed and direction of surrounding pedestrians follow a normal distribution with the mean and standard deviation being the current speed and distribution, respectively). In other words, since a pedestrian-only scenario with low speeds and unidirectional movement is being dealt with, the offered concise yet calibrated collision probability formulation is feasible.

Based the above, it is now possible to find the velocity v_i (the term i is dropped for added clarity) associated with acceleration a_i leading to the highest utility term.

First, the derivatives of both PT and the total utilities with respect to the velocities should be computed:

$$\frac{dU_{PT}^{\theta}}{dv} = -\left[\frac{\left(1/v_d\right)\left(1+\left(v/v_d\right)\right)^{\left(\frac{\xi-1}{2}\right)} - \frac{\xi-1}{2}\left(1/v_d\right)\left(1+\left(v/v_d\right)\right)^{\left(\frac{\xi-3}{2}\right)}}{\left(1+\left(v/v_d\right)\right)^{(\xi-1)}}\right] \sin\left[\frac{\left(v/v_d\right)}{\left(1+\left(v/v_d\right)\right)^{\left(\frac{\xi-1}{2}\right)}}\right] \tag{6}$$

For added simplicity, the term $k(v, \Delta v, j)$ is assumed to be 1 in Equation 2. The first derivative with respect to v of the total utility term becomes:

$$\frac{dU_i^{\theta}}{dv} = \left(1 - p_i^{\theta}\right)\frac{dU_{PT}^{\theta}}{dv} \tag{7}$$

As for the second derivative,

$$\frac{d^2U_i^{\theta}}{dv^2} = \left(1 - p_i^{\theta}\right)\frac{d^2U_{PT}^{\theta}}{dv^2} \tag{8}$$

Given such values, and since the $g(.)$ function of Equation 3 can be approximated, a normal distribution and the Wiener implementation process may be adopted. The velocity and directional means are the (combined) mean $\mu_{v_i}^{\theta}$ that maximizes the utility term U_i^{θ}. As for the variance, it can be approximated as:

$$\sigma_{v_i}^{\theta} = -1.0 \Big/ \left(\beta_{PT}\frac{dU_i^{\theta}}{dv}\right) \tag{9}$$

At every time-step, and with a given relaxation time, the velocity directional term may be computed as:

$$v_i(t+\Delta t) = \mu_{v_i}^{\theta} + \sigma_{v_i}^{\theta} * y(t+\Delta t) \tag{10}$$

where

$y(t + \Delta t)$ is the standard Wiener process.

Toward illustrating the behavioral patterns reproduced by the model, a base sensitivity analysis is performed, modifying key parameters in the model: the collision weight, w_c, the sensitivity term β_{PT}, and linearity term ξ. These parameters are key in the calibration process, as will be shown in the following section. The behavioral patterns are mainly illustrating by the model functions g^{θ} of Equation 3, U^{θ} of Equation 2, and U_{PT}^{θ} of Equation 1. The collision probability p_i^{θ} is set at 0.1. The results are shown in Figure 2.

A sensitivity analysis helps determine the parametric values that will re-create reasonable pedestrian behavioral patterns while analyzing the associated meaning of each term incorporated in the model being studied. Comparing Figure 2, *a–c*, with Figure 2, *g–i*, it can be deduced that the impact of the sensitivity term to the surrounding stimuli (i.e., β_{PT}) is more pronounced that the impact of the non-linearity term ζ. Increasing sensitivity will produce a more limited angular choice set, as may been seen if comparing Figure 2*a* with Figure 2*c*. In other words, pedestrians will have a more limited choice in relation to the angles they choose from. On the other hand, the added sensitivity will result in higher speed values (concentration of the probability distribution function in an area with higher v/v_d ratios) but with less variance/volatility in such speeds. In other words, higher sensitivity reduces the disturbances and limits the pedestrian behavior to a more straightforward path choice with more concentrated speed set. Such a result may be expected, as being sensitive to the surroundings limits pedestrian choices and reduces pedestrian behavior to a more deterministic approach (assuming that the surrounding stimuli is stable). On the other hand, as the non-linearity term is increased, the main impact seems to be on the choice of velocities (rather than the choice of angles) that tend to go to lower values with higher variations in the available speed choice sets.

In relation to the collision parameter w_c, it is clear that increasing the corresponding values from 1 to 100 (i.e., Figure 2*d* versus Figure 2*e*) affects the associated pedestrian behavior observed. Such impact is almost negligible when increasing w_c from 100 to 1,000. Since the collision weight does affect the avoidance behavior of pedestrians in the offered modeling framework, this observation leads to the hypothesis that w_c is closer to 100 than to 1,000 (or even 10,000). This hypothesis leads to the use of such a range when choosing the initial w_c values in the calibration exercise. More importantly, the w_c values in the work of Hamdar et al. when analyzing driver behavior were found to be in the area of 10,000 (*32*). This means that drivers put a much higher weight on the loss associated with a collision with another car/vehicle compared with the weight pedestrians put on colliding with another pedestrian. Such a result illustrates the advantage of this modeling framework quantifying the risk-tendencies of drivers or pedestrians when being in near-collision scenarios. Further insights are given on the suitability of the aforementioned parametric values when performing a more elaborate calibration.

It should be noted that different value functions will lead to different outcomes in relation to behavioral interpretations. However, regardless of the form of the value function, (1) the suggested modeling framework in which such function is incorporated, (2) the implementation and calibration of this framework, and (3) the analysis of the associated findings are among the proposed contributions made in this research work.

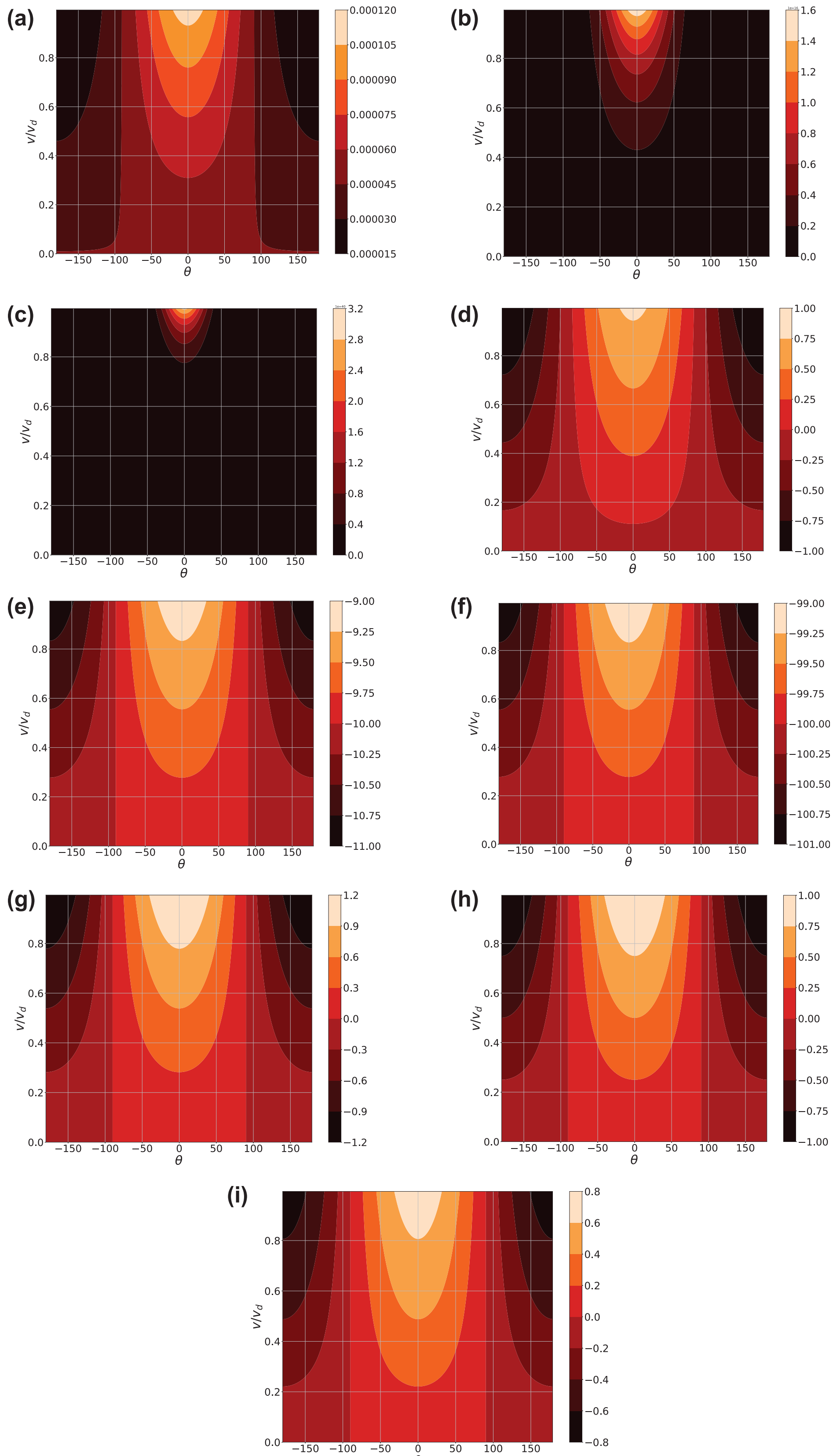

Figure 2. Parametric sensitivity analysis applied on the model functions with different values of θ **and** $^{v}/_{v_d}$.

Calibration and Simulation Validation

There have been multiple recent efforts to calibrate pedestrian walking behavior at the microscopic level using high-resolution trajectory data. These efforts mainly differed in relation to the estimation methods adopted (e.g., maximum likelihood estimation versus heuristic-based estimation) and in relation to the objective error being minimized (e.g., absolute error terms in location/speed versus relative or mixed error terms in location/speeds). For example, Zeng et al. calibrated the parameters of a modified version of an SF model using the maximum likelihood estimation (*33*). Trajectory data were collected at a signalized intersection in Nagoya City, Japan. On the other hand, Hussein and Sayed used a heuristic-based calibration approach leveraging a genetic algorithm (GA) to calibrate a pedestrian-agent-based simulation model (*34, 35*). Trajectory data in different walking environments (e.g., crowded intersection versus bridge walkway) are extracted and the cost function to minimize was a distance-based function incorporated in a mixed error form (*34, 35*). Given the complex nature of the formulation adopted in this paper, the maximum likelihood function could not be derived, and a heuristic-based approach is needed. This approach is in line with multiple recent calibration efforts (*36–40*). However, such calibration approach is utilized with this prospect-based risk-taking modeling framework for the first time; the ability to reach an acceptable error value while capturing inter-pedestrian heterogeneity may be perceived as a contribution.

The objective of the calibration (i.e., the objective function) is to minimize the location trajectory error; in other words, the error is based on the Euclidean distance between the simulated location and the actual observed location at every time-step. For such purposes, the Euclidean distance (which is also an absolute error measure) between simulated pedestrians' coordinates and their corresponding real coordinates should be minimized while changing the parameters mentioned in the previous modeling section (Economics-Based Modeling Paradigm and Base Sensitivity Analysis). Given such an approach, the next sub-section presents the data used, and the calibration formulation is introduced afterward.

Data Description

The trajectories for a narrow bottleneck scenario were provided based on experiments conducted at the Delft University of Technology (TU Delft) (*41*). Such a scenario is chosen since it covers different traffic regimes and the associated microscopic behaviors (i.e., congested and non-congested regimes). In the narrow bottleneck experiment, the walking area consisted of an area 5 m long by 4 m wide, followed by an area 5 m long by 1 m wide. In other words, pedestrians are instructed to pass through a corridor that has a 1 m width and a 5 m length. The narrow bottleneck experiment conducted at TU Delft recorded the trajectories of 1,154 pedestrians for 915 s. Every data point consisted of an x-y coordinate recorded every 0.1 s. In total, 178,195 data points were saved. At the bottleneck data (entrance of the corridor), the mean speed was 0.68 m/s with a standard deviation of 0.4 m/s.

Genetic-Algorithm (GA)-Based Calibration

This research aims at having a microscopic calibration with emphasis on the non-homogenous traffic dynamics that stem from individual interactions between pedestrians. Accordingly, the parameters introduced in the modeling section (Economics-Based Modeling Paradigm and Base Sensitivity Analysis). should be chosen to minimize an error term. Since the model suggested is non-linear and stochastic in nature, the corresponding optimization problem is also nonlinear in form and needs to be solved numerically. For such reasons, a GA is used for calibration purposes and is based on the GA method used by Hamdar et al. to calibrate a PT-based car-following model (*43*). Such an algorithm will directly compare the trajectories observed with the trajectories obtained from the microscopic simulation while pre-specifying the boundary (initial) conditions and the final direction of travel. A key aspect remains the identification of the neighboring pedestrians. In this research, it is considered that all neighbors within a 5 m radius of a given pedestrian affect this pedestrian's behavior and are thus incorporated in calculating the different individual utilities of the different directional accelerations chosen. The second key aspect is the error function. To allow for consistency, at the beginning of the simulation, the locations/coordinates are initialized to be equal to the observed values. The calibration performance measure is then the coordinate location. This value can have a direct translation to the velocity, gap, and relative velocity error. The free-flow speed is calculated by finding the maximum speed adopted by a pedestrian during the experiment.

An absolute location error term is selected as the type of error to be minimized. In other words, the calibration objective function is the Euclidean distance between the simulated pedestrian's location and the observed (actual data-based) pedestrian's location. This selection is made because such error is not sensitive to the type of congestion regime a pedestrian is encountering. For example, if using the difference in velocity as the base of the objective function to be minimized, the absolute error is more sensitive to changes in the empirical data when speeds are higher, and the relative error is more sensitive to changes

Table 1. Average Absolute Error Statistics (in meters)

Statistic	Value
Maximum	0.059
Minimum	0.019
Average	0.029
Median	0.025
Standard deviation	0.009

at lower speeds. Specifically, the location error is calculated at every time-step for a given pedestrian. At the next time-step, the location of the pedestrian is reset to their actual location. The future position of the pedestrian is then calculated and compared with their actual position while considering the surrounding pedestrians/obstacles. This will give each pedestrian an absolute error term at every time-step of the simulation. The final objective function to be minimized is the average error based on the entire observation period.

In addition to the objective function—also known as the fitness function—the genetic algorithm involves using chromosomes to represent sets of goal calibration parameters. During each chromosome generation, the mixed error fitness is calculated, and greedy selection is used to pick the parameters with the 10 best fitness scores. These become parents that are then used to generate chromosomes which are combined to create children. During this combination, a crossover point is chosen using random selection, and genes, except for the chromosome with the single best fitness score, are randomly mutated with a probability rate of 10%. At the beginning, a fixed number of generations are assessed, and the process ends when the fitness score drops below 10 cm or there is no improvement during 20 successive chromosome generations.

Calibration and Simulation Results

The calibration methodology described earlier has been applied to 830 pedestrians of the 1,154 pedestrians observed at the TU Delft experiment. The results are offered in Table 1 and Figure 3.

The error statistics and its distribution are encouraging as the values found are below the 10 cm threshold with a single peak around the 3 cm value. Given such results, Figure 4 presents the distributions of the different parameters discussed in the section presented earlier (Economics-Based Modeling Paradigm and Base Sensitivity Analysis). It is clear that a significant level of inter-pedestrian heterogeneity is captured through the offered calibration exercise. It is also noticed that the

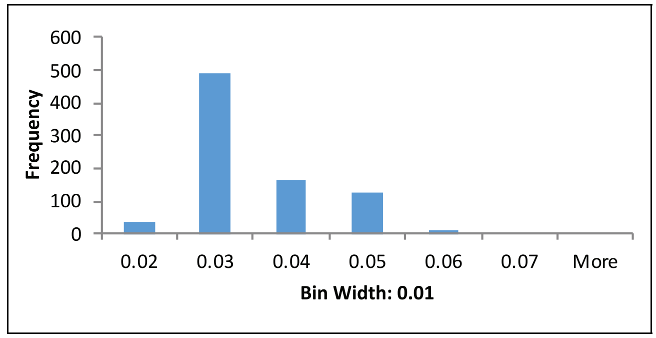

Figure 3. Error distribution for all pedestrians in the calibration exercise.

distributions found do not exhibit clear peaks (i.e., a predominant value for each parameter) as is normally observed when calibrating vehicular traffic models. Another interesting observation is possibly seen in Figure 4*c*. The non-linearity term ζ has a bi-modal distribution and thus there might be two walking regimes that should be further investigated. The first regime may be associated with more conservative walkers and the second regime may be associated with more aggressive stable walkers.

To make sure that the parameters observed are all contributions to the behavioral patterns to be replicated by this model, the correlation between the four parameters presented in Table 2 is studied. The results show that limited correlation exists between the calibrated parameters, and thus it is suggested that the modeling construct offered in this paper is feasible and produces realistic behavioral patterns.

For validation purposes, simulation is used to further investigate the suitability of the modeling and calibration approaches adopted. The pedestrian model is implemented using a python code. The simulation time-step is 0.1 s. Each pedestrian reacts in a continuous manner (every 0.1 s) with a delay equal to the reaction time. The pedestrian maximum accelerations/decelerations and directional change are governed by the Wiener process, mainly through a correlation time parameter. The maximum directional change reported was 3° per second (within the range provided in Liu et al.) (*43*). The maximum absolute acceleration value reported is 2.5 m/s^2 (*44–46*).

Starting with the microscopic measures, an attempt is made to re-create the trajectories observed through simulating the first 100 pedestrians detected in the TU Delft experiment. These trajectories are offered in Figure 5.

The PT-based pedestrian model is able to reproduce similar trajectory patterns to those observed at the TU

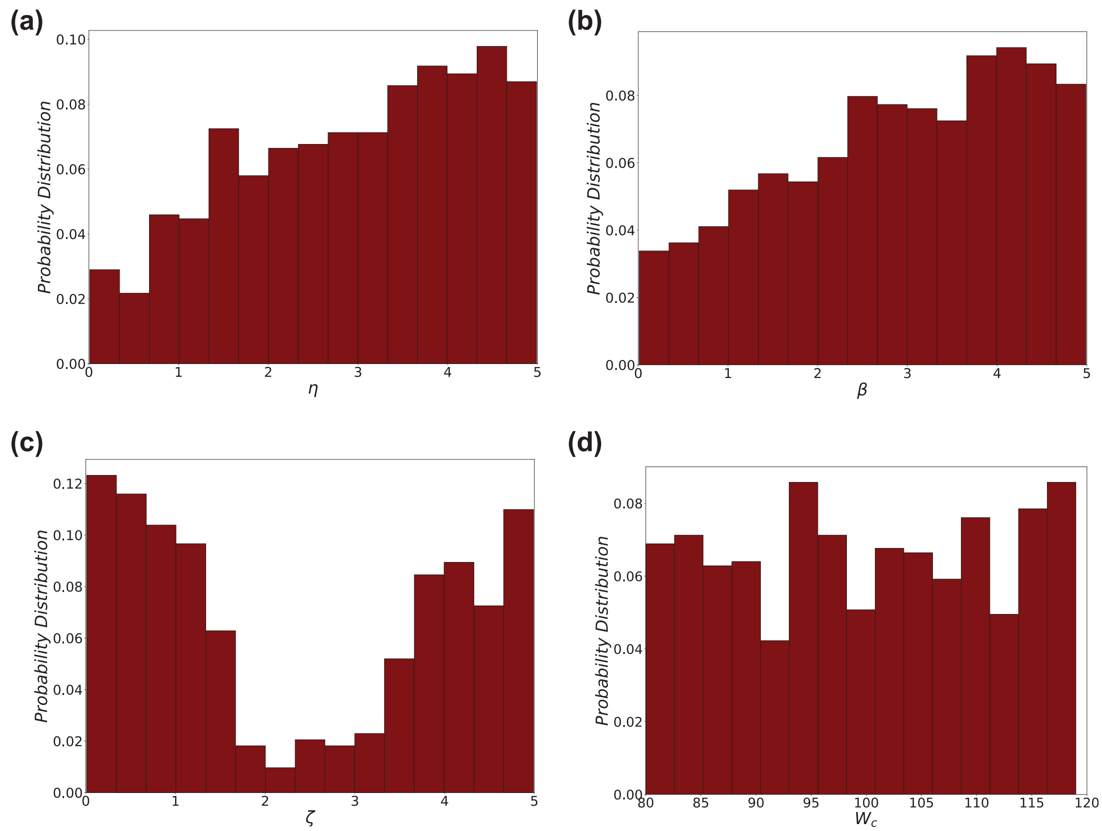

Figure 4. Calibrated parameters' distributions.

Table 2. Correlation Matrix Between Calibrated Parameters

	η	ζ	w_c	β_{PT}
η	1			
ζ	0.123320338	1		
w_c	−0.06667759	−0.01717079	1	
β_{PT}	−0.02528499	0.134601975	−0.04694932	1

Delft experiment. In particular, the layer/lane formation patterns inside the bottleneck area (i.e., concentration of trajectory lines at the upper, middle, and lower sections of the bottleneck) may be observed in both the observed and the simulated trajectories. Moreover, the funneling behavior seen at the entrance of the bottleneck is re-created. However, the lateral variation associated with the steps taken by each pedestrian is not clearly seen in the simulated trajectories. In addition, the lane formation/zipper effects seen within the bottleneck walls are not as clearly formed if comparing the simulation results with the observed trajectories. Despite the encouraging trajectory patterns re-created, further investigations may be needed to look into the walking behavior re-created by the proposed model especially in crowded situations.

In relation to macroscopic traffic measures, the focus at this stage is on the macroscopic speed-density relationship at the entrance of the bottleneck. The speed-density data points are aggregated for 30 s durations in the area preceding the bottleneck. In other words, the densities are extracted for the x-coordinates ranging between 5 m and 10 m and for the y-coordinates ranging between 0 m and 4 m (as displayed in Figure 5). The results are offered in Figure 6.

For both simulated and observed flows, the free-flow speed ranges between 1.5 and 2 m/s; as the density increases, the speed decreases as expected. However, the speed rate of decrease seems to accelerate at a density level of 2 ped/m^2. The main difference pertains to the more congested states of traffic. At higher density levels (i.e., densities greater than 4 ped/m^2), the simulated flow often exhibits slower speed patterns if compared with those observed in the experiment. This type of result may be associated with the consideration of multiple surrounding pedestrians when computing the collision probability in the proposed model. Such considerations may reproduce slower speed patterns especially in crowded situations. In addition, the simulated model does not produce data points that may

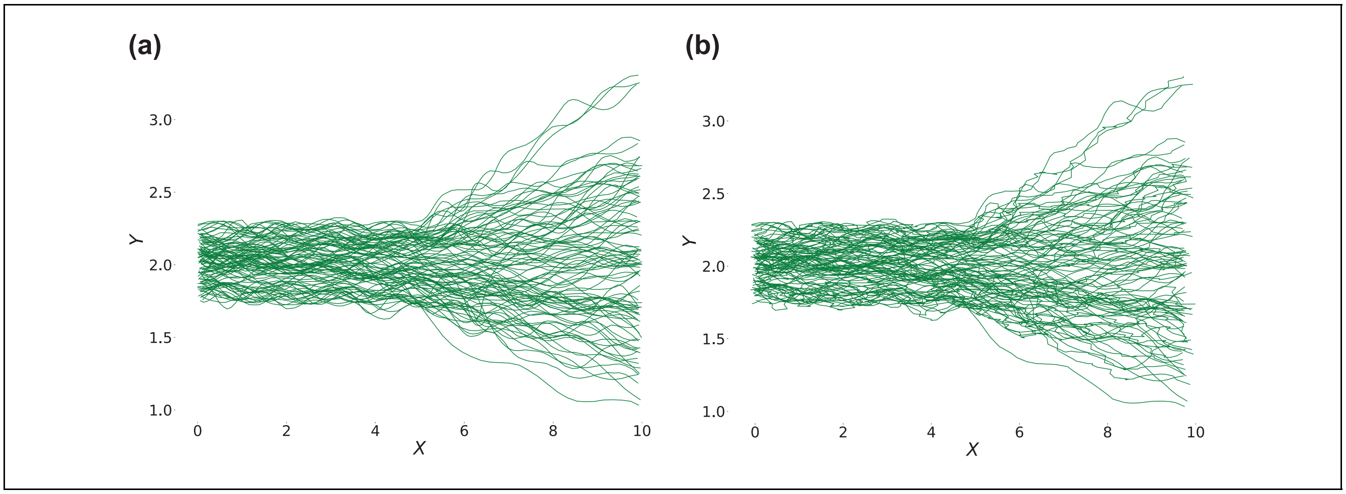

Figure 5. Observed (*a*) versus simulated and (*b*) trajectories of the narrow bottleneck scenario TU Delft experiment—first 100 pedestrians.

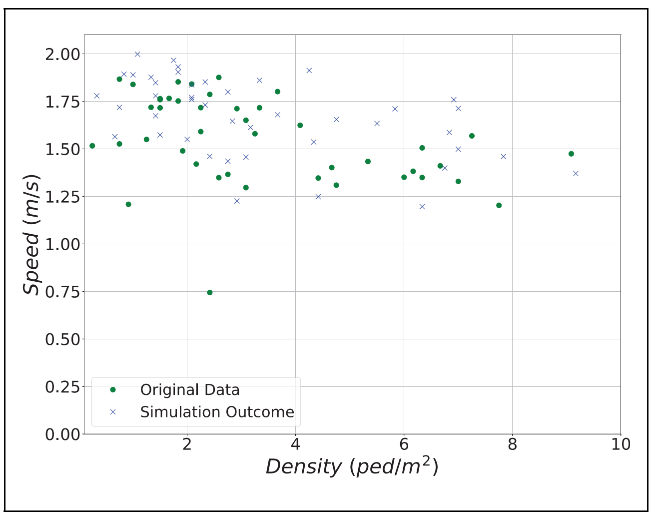

Figure 6. Speed-density relationship at $X = 7$ m for simulated versus observed (original) pedestrian flow at TU Delft narrow bottleneck experiment.

correspond to outliers in relation to the speed-density relationship (i.e., data points associated with low densities and low speeds). The outliers may be related to experimental observations (e.g., human subject participants slowing down with no pedestrians in the surrounding space, possibly because of distraction) but not to a pedestrian walking behavior reproduced by a given microscopic model.

Conclusions

This paper offers a new modeling paradigm of pedestrian walking behavior using PT. With such a decision-making paradigm, an explicit incorporation of a collision probability is made possible. Moreover, a value function allows translating the objective directional pedestrians' gains in speed into subjective perceived gains. This value function is key to identifying the risk averseness or the risk-taking tendencies of pedestrians from a cognitive perspective (i.e., judgement phase). In line with such formulation, the newly presented formulation was then translated into an implementable Wiener process for a faster, more efficient simulation exercise. Once implemented, the model was calibrated using GA methodology, then validated in relation to trajectory re-creation and a resulting speed-density relationship. The data used for calibration and validation purposes are trajectory pedestrian observations provided by TU Delft in a narrow bottleneck scenario.

Given the aforementioned summary, the following two questions may be asked: 1) What is the contribution of another pedestrian walking behavior model if compared with the many existing models such as the CA pedestrian model, the SF model, or the pedestrian discrete choice model (that has the same random utility construct of the offered PT pedestrian model)? 2) Does the model re-create existing walking behaviors and crowd dynamics? Answering these questions defines the contribution made in this paper.

Answering Question 1: Existing pedestrian flow models are mostly physics-based or mechanistic in nature (e.g., the CA model, and the SF model and its variants) and does not have an underlying decision-making theory supporting them. Using such models does not result in parameters or parametric values that are straightforward to interpret from a pedestrian behavior perspective. The CA models have been developed further to account for behavioral observations while incorporating decision-making considerations. Such development does not lead, however, to a modeling paradigm that may account for collision formation while interpreting the associated findings from a risk-taking cognitive perspective (*47, 48*). The discrete choice models and the rule-based information processing models attempted to answer such issues, but were not offered in an implementable efficient simulation form. In other words, discrete choice models (as is the case when analyzing acceleration models) are more suited to analyzing behaviors than to creating a traffic simulation tool for design and prediction purposes. The offered modeling approach of the PT behavioral paradigm may be able to address such problem. Given such a contribution, it should be noted that the offered modeling paradigm is not intended as a critique reducing the significance of existing pedestrian walking models. These models are of extreme importance to the traffic flow community despite some of their limitations that this paper tried to address.

The other contribution of the offered paradigm is that it is built as an extension to a unidirectional PT driver acceleration model. In other words, deriving the Wiener process for both a directional and a speed value set of alternatives makes it possible to extend the formulation further and investigate different forms of collision probabilities and subjective value functions. Moreover, this work is a first step toward re-creating a generalizable decision-making model for pedestrians, cyclists, and drivers that is efficient in relation to computational complexity and that can be calibrated against individual trajectory data.

The final advantage of the offered model lies in the possibility of re-creating collisions/contact between the decision-makers being modeled. Even though safety in pedestrian traffic is more associated with turbulences (lateral and longitudinal waves possibly leading to stampede and "crush" scenarios) rather than contact, safety in mixed traffic may be related to the contact between pedestrians and other modes of travels such as cyclists and motorized vehicles. Accordingly, re-producing collisions through a "subjective" collision probability and collision weight factors may be of interest for analyzing pedestrian safety in mixed right-of-way scenarios (e.g., crossing a road, pedestrians and cars sharing a given urban space). Different value functions—possibly reducing the associated error term—may be further investigated, resulting in different behavioral analysis, assessment, and evaluation.

Answering Question 2: the formulation of a behavioral-based pedestrian model with possible insights into the cognitive dimensions leading to unsafe walking scenarios may be appreciated by traffic safety and human factor researchers. However, from a traffic flow theory perspective, the models created should be calibrated/validated against trajectories and macroscopic traffic data before being used for simulation purposes. Even though the formulation offered in this paper is analytically non-tractable, the adoption of the Wiener process makes it possible to use heuristic-based approaches to calibrate the corresponding parameters. In other words, the use of the GA method (heuristic) makes it possible to find the parametric values that minimize the difference between the observed trajectories and the simulated trajectories (i.e., the objective function to be minimized in the Euclidean distance between the actual and simulated location of a pedestrian at every time-step). It is acknowledged that analytically tractable calibration methods (e.g., maximum likelihood methods) allow testing for the statistical fit of each parameter's estimate and the associated robustness. However, at this stage of the research, the authors believe that the calibration exercise presented in this paper and the associated validation simulation effort show encouraging results. These results constitute a motivation for the extension and improvement of the offered modeling approach, while capturing both inter- and intra-pedestrian heterogeneities.

Author Contributions

The authors confirm contribution to the paper as follows: study conception and design: Prof. Hamdar, Prof. Talebpour and Prof. Treiber; data collection: Prof. Daamen and Prof. Knoop; analysis and interpretation of results: Pof. Hamdar, Prof. Knoop and Prof. Talebpour; draft manuscript preparation: Ms. D'sa, Prof. Hamdar and Prof. Talebpour. All authors reviewed the results and approved the final version of the manuscript.

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