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Quantifying the hierarchy of public transport networks

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***Abstract*—Public transport networks constitute critical infrastructure in urban systems. Public transport networks are characterised by their hierarchical structure, yet methods to quantify their underlying hierarchy are lacking. We propose a metric for quantifying the hierarchy in public transport networks which incorporates topological as well as passenger flow information. Our proposed metric consists of three components which jointly define the relative hierarchical position of nodes across the network while the distribution of hierarchy defines the hierarchy of the network itself. We apply the metric to the case studies of Amsterdam and Rotterdam to demonstrate its usefulness in comparing different network states both within and across networks. Using this metric, we identify different patterns in network structures for network states and different spatial distributions of hierarchy between networks. Furthermore, by dividing the network into functional levels, we identify a multi-layer hierarchical structure that describes the functionality of the network. The potential application of this metric relates to the assessment of network development scenarios, evaluating bottlenecks and analysing the network vulnerability. Furthermore, the metric is potentially suitable for assessing different network structures such as aviation or maritime networks.**

***Index Terms*—transportation, networks, hierarchical, degree**

I. Introduction

Public Transit Networks (PTN) play a pivotal role in the evolution of cities. As cities transform from monocentric to a more diverse and complex polycentric organisation of urban areas [1], [2], PTN help meet the needs of a diverse set of citizens. The future of urban mobility in large cities is intimately linked to the organisation of public transport passenger flows across the city [3]. In cities where PTN is extensively used, the daily movement of people gives rise to a natural hierarchy of mobility patterns [4].

The hierarchical organisation of a PTN plays an important part in determining how the network can be designed in the most efficient way [5]–[7]. Several PTN around the world have multiple functional layers and interchange hubs serve as key nodes in allowing travelling across different network layers. Multiple network indicators, including measures of hierarchical ordering [6], [8], [9] only reveal the importance of individual elements without the context of their placement in the network itself [10]–[12]. We lack a quantitative metric to specifically evaluate nodes in a PTN based on their value to overall network structure. Moreover, when comparing different networks across cities (e.g. [13], [14]), the hierarchical structure of a network is left out of the comparison. Even though the influence of transfers has been incorporated into topological metrics, this predominantly relates to the requirement for transfers in the shortest path rather than based on actual traffic flowing through specific transfer locations in the PTN [15]. Transfer flows have also been used as part of a method for identifying connections to be prioritised in the timetable synchronisation problem [16]. Recently, a method for identifying hierarchical structure based on passenger transfer flow pattern has been proposed [17], focusing on inter-line flows as a criterion. Notwithstanding, a measure for evaluating the hierarchical structure in a network incorporating the function of transfer locations is generally lacking.

In this study, we propose a metric incorporating both topological properties and passenger demand data to understand functional hierarchy in PTN. This metric allows incorporating information concerning both the macroscopic flow distribution characteristics as well as structural network properties. We apply the proposed metric to evaluate two case study networks, i.e. the PTN of Amsterdam and Rotterdam, to demonstrate how this metric shows different outcomes for different network structures and how these differences may be interpreted in terms of network's functional efficiency. We conclude by outlining general potential applications of the metric and discuss the implications for decision making regarding the development of PTN. Identifying the hierarchical position of an element is relevant for a wide range of different types of networks [18] and especially relevant for planning for urban mobility [4]. Hence this gives rise to the following question:

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"How can we measure network hierarchy?" In developing such a measure, we want to ensure that it is easy to calculate and communicate so that it is useful for planners and can support decision-making in the development of PTNs.

II. Methodology

To develop a comprehensive metric for hierarchy in PTN, we first devise a general definition of hierarchy. Based on a combination of topological hierarchy [10]–[12] and flow-based hierarchy metrics [19]–[23], we characterise a node's hierarchy based on the strength (*topological influence*) and diversity of its connections (*redundancy*) with its direct neighbours in the network and its capacity to function as a transfer hub (*transfer potential*). In the following, we first define the three components of the proposed metric and thereafter present how this is integrated into a single metric of hierarchical degree. Finally, depending on the distribution of the hierarchical degree in the network, we determine the overall hierarchy of the PTN.

Components of hierarchical degree

Following our definition above, the metric comprises of the following three components: *topological influence*, *redundancy* and *transfer potential*. The three elements have a complementary function. The first and second components are based on a representation of the Service-space, often known as $\mathbb{P}$-space in the literature (see e.g. [24]) whereas the last component is based on the Infrastructure-space commonly known as $\mathbb{L}$-space in the literature (see e.g. [25]). Being connected to a diverse range of nodes means a node can act as a transfer hub depending on the strength of flows with its neighbours in the network, who in turn have much less connections among them. Functioning as a transfer hub relates to the share of passengers transferring at the specific node and the number of directions one can transfer to. Moreover, the hierarchical degree is only calculated for nodes with more than two directions to travel to since otherwise it is unable to function as a transfer location. In other words, the degree of the nodes in the Infrastructure-space should be more than *two* or else the hierarchical degree should be set to zero in this context. Thus,

$$H_i = 0, \quad \text{if} \quad k_i^{\mathbb{L}} \leq 2, \tag{1}$$

which holds for all three elements. In (1), $k_i^{\mathbb{L}}$ represents the degree of node i in the Infrastructure-space. If the degree of the nodes relates to the Service-space, we note this as $k_i^{\mathbb{P}}$. Throughout the subsequent paragraphs, we elaborate upon the different components of the proposed metric.

A: Topological influence

The first component relates to the topological influence of a node in the network, incorporating the function of nodes it is directly connected to. We derive the value for this element from the eigenvector centrality (see [26]) with a small adjustment to normalise for the network size. The eigenvector centrality is commonly used by search engines and the analysis of social and citation networks to reflect the relative importance by valuing connections to other important nodes more highly than connections to less important nodes.

The normalisation of the eigenvector centrality is done in order to facilitate cross-network comparison as larger networks tend to have a more spread centrality with generally lower eigenvector centrality values. Furthermore, normalising the eigenvector centrality results with a confined range for output values of the topological influence within the interval $[0, 1]$ where a value of 0 indicates the node is separated from the network while a value of 1 indicates the node is the most influential node in the network. As indicated above, we apply the Service-space representation in order to capture the importance of direct lines between nodes rather than the node being a direct neighbour which would have been the case if we were to use the Infrastructure-space representation. Formally, we describe it as e_i^A the **topological influence** of node *i*, based on connections to other **influential nodes** where,

$$e_i^A = \begin{cases} \dfrac{x_i}{x_{max}}, & \text{if} \quad k_i^{\mathbb{L}} > 2; \\ 0, & \text{otherwise}, \end{cases} \quad \forall i, \tag{2}$$

based on the definition of the eigenvector centrality (x_i) [26] as,

$$x_i = \frac{1}{\lambda} \sum_{j=1}^{N} a_{ij} x_j. \tag{3}$$

In (3), a_{ij} is an element of the adjacency matrix of the network describing whether two nodes i and j are connected $(a_{ij} = 1)$ or not $(a_{ij} = 0)$, and x_j is the eigenvector centrality of node j. λ is a constant value signifying the largest eigenvalue of the adjacency matrix.

B: (Non-)Redundancy

The second component of our proposed metric is concerned with evaluating the significance of a node for facilitating transfers, i.e. whether a node is required for connecting its directly connected nodes (neighbours). If there are lesser direct connections between neighbours of a node, the more hierarchical the node is. This is measured using the clustering coefficient [27], [28].

We determine the clustering coefficient value of a node to test its redundancy in the Service-space. We employ again the Service-space as this indicates which of the directly connected nodes share a mutual line between them and require no transfer at the node in question. Consequently, a higher value of the clustering coefficient indicates many of the directly connected nodes share a line and only few transfers have to be made at the node in question. We subtract the clustering coefficient from one, so that a lower value is attained if the clustering coefficient is higher. We denote the second term e_i^B as the **non-redundancy** of node i, based on **mutual connections** among neighbouring nodes where,

$$e_i^B = \begin{cases} 1 - c_i(k_i^{\mathbb{P}}), & \text{if} \quad k_i^{\mathbb{L}} > 2; \\ 0, & \text{otherwise}, \end{cases} \quad \forall i, \tag{4}$$

where the definition of the clustering coefficient is [29]

$$c_i(k_i) = \frac{n_i}{k_i^{\mathbb{P}}(k_i^{\mathbb{P}}-1)/2} \qquad \forall i. \tag{5}$$

Here, n_i indicates the number of links between neighbouring nodes in the Service-space[1].

C: Transfer potential

The third part of the metric is based on the share of transferring passengers at a station. Transferring in this sense means changing lines, including possibly changing to a different mode. This element is fundamentally different from the prior elements as this incorporates passenger demand data regarding transfers at a node. Consequently, an empirical demand aspect of hierarchy in PTN is reflected in this component by including the share of transfers that take place at a certain node, in addition to the transfer potential resulting from topological features. We multiply the share of transferring passengers by the share of potential transfer directions. The latter is calculated as the degree minus two (to subtract both origin and destination) divided by the degree in the Infrastructure-space. This definition of transferring directions relates to the fact that transferring here relates to getting off the vehicle and the vehicle generally has an in- and outgoing link. We define e_i^C as the **transfer potential** based on **transfer passenger share** and **transfer directions** for node i where,

$$e_i^C = \begin{cases} \dfrac{log(P_i^{transfer})}{log(P_{max}^{transfer})}\dfrac{k_i^{\mathbb{L}}-2}{k_i}, & \text{if} \quad k_i^{\mathbb{L}} > 2; \\ 0, & \text{otherwise}, \end{cases} \qquad \forall i. \tag{6}$$

In (6), $P_i^{transfer}$ is the number of passengers transferring at node *i*, $P_{max}^{transfer}$ is the the maximum number of passengers transferring at any node in the system. Thus, we use $P_{max}^{transfer}$ to normalise. The logarithm is used because many nodes take just a small share of transferring passengers compared to the highest scoring node. Therefore, using the logarithm allows to better differentiate among nodes that have a small fraction of the transfer passenger share but are an important part of the system compared to their neighbours.

An integrated metric of node hierarchy

We integrate the three components by multiplying them so that a high score for one of the elements does not undervalue the other elements across comparisons. Combining all three components, we obtain the hierarchical metric as follows,

$$H_i = e_i^A e_i^B e_i^C, \quad \forall i, \ \{H_i \in \mathbb{R} : 0 \leq H_i < 1\}. \tag{7}$$

Using the components above, we now define the hierarchical degree as,

$$H_i = \begin{cases} \dfrac{x_i}{x_{max}}(1-c_i(k_i))\dfrac{log(P_i^{transfer})}{log(P_{max}^{transfer})}\dfrac{k_i-2}{k_i} & \text{if } k_i > 2; \\ \\ 0, & \text{otherwise}, \end{cases} \qquad \forall i,. \tag{8}$$

[1]A link in the Service-space corresponds to a line in the Infrastructure-space

The hierarchical degree can take values within the range [0,1]. By analysing the Gini-distribution of the hierarchical degree, we can obtain a final metric of network hierarchy. The Gini coefficient is selected here to reflect the distributional efforts of nodal hierarchy. Gini coefficient values are also bounded to $[0, 1]$ where a value of 0 corresponds to an egalitarian network where all nodes are of equal position whereas a value of 1 implies that a single node is of utmost importance and all the rest are secondary.

Multi-level network representation

In order to subdivide the nodes into different functional levels, three sub-levels are distinguished here. We have chosen for three levels as a significant share of the nodes are not hierarchical at all ($H_i = 0$) according to our definition. We differentiate between low and high hierarchy for nodes with a H metric value which is non-zero using three levels which are the high-, low- and non-hierarchical categories. We consider a node scoring zero as non-hierarchical, while we consider nodes with a hierarchical degree greater than 0.125 (which can be obtained in case each of the components has a value of 0.5) as high in hierarchy and finally, we consider values in the interval $(0, 0.125)$ as low in hierarchy.

III. Results

Case study description

We illustrate the proposed metric for the Dutch cities of Amsterdam and Rotterdam. For the Amsterdam PTN, we report the results for the network state before and after summer 2018 when the network was re-designed in conjunction with the opening of the North-South line (NZL). In addition, we also report results for scenarios including possible future network extensions. The Rotterdam PTN allows to assess the transfer-ability of the proposed metric and its ability to offer insights by comparing networks in different cities.

The data we use for this study is retrieved from open source topological data provided by both municipal organisations through this project collaboration between TU Delft and *RoyalHaskoningDHV* complemented by passenger data (which is not allowed to be shared due to privacy concerns). The former are imported from a NDOV data-set which includes node, line and link data and is a specific format used by Dutch transport organisations. The latter are imported from regional data-sets to estimate the flows and transfers in the network.

To determine the number of transfers in the network, we use a static assignment model (see e.g. [30], [31]) based on an *all-or-nothing* assignment algorithm. This assignment algorithm is fast in terms of computation time but has its limitations related to the lack of variety in route choice preferences and not accounting for congestion on links ([32]).

Amsterdam PTN

Fig. 1 shows the cumulative distribution function (CDF) of the hierarchical degree H in the (current) Amsterdam PTN. For reference, the CDF of the Rotterdam PTN is also plotted. For Amsterdam PTN, more than 75% of the nodes are considered non-hierarchical while over 20% of the nodes are in the range $(0, 0.1]$. In comparison, in the Rotterdam PTN, more than 85% of the nodes are considered non-hierarchical while over 12.5% of the nodes are in the range $(0, 0.1]$. In addition, we observe a much lower share for 0 scoring nodes in the Amsterdam PTN. For higher values ($H_i > 0.1$) the share of nodes between the networks is on-par.

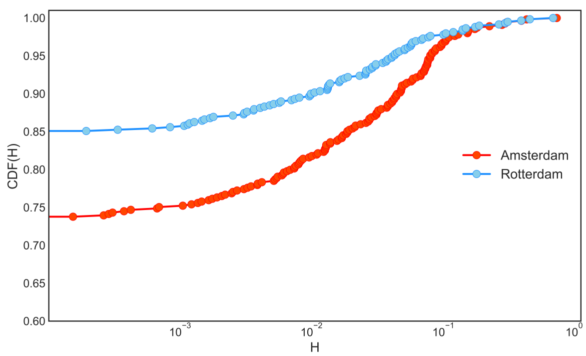

Fig. 1. Cumulative distribution of hierarchy in the Amsterdam & Rotterdam PTN

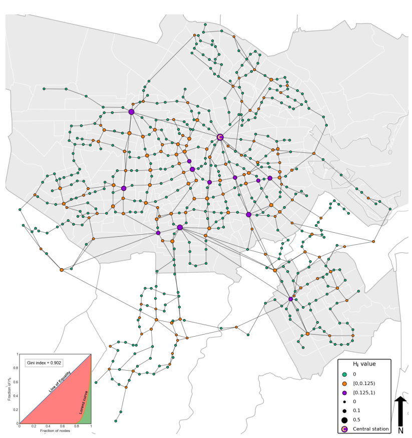

Fig. 2. Overview of nodes in the Amsterdam PTN

In Fig. 2, we illustrate the geographical distribution of the node hierarchy in the network. Hierarchical nodes are spread throughout the network with the exception of the northern districts which are situated on the other bank of the IJ-river. High scoring nodes include in addition to the central station also additional train stations located around four to five kilometres from the central station itself. It is notable that the majority of the nodes have a low hierarchical value. In order to quantify the distributional effects of hierarchy across the network, we use the Gini-coefficient (see e.g. [33]–[35] of the hierarchy for which we use the definition from the reference [36],

$$G = 1 - \sum_{k=1}^{e} (X_k - X_{k-1})(Y_k + Y_{k+1}). \quad (9)$$

$$\{G \in \mathbb{R} : 0 \leq G \leq 1\}$$

The Gini-coefficient of the node hierarchy in the Amsterdam PTN is 0.902. For clarity, we visualise the corresponding Lorentz curve of Gini coefficient in the bottom right corner of the inset Fig. 2. We perform a correlation analysis for the different components of the hierarchy metric in the Amsterdam PTN. Table I summarises the correlation between each of the components and the hierarchical degree. Note that even though all are given equal weights in H, e^C correlates most with the hierarchical degree when calculated over all nodes, suggesting that it is slightly more informative of the overall hierarchical degree. In this table, the values for the Rotterdam which are discussed in a subsequent section are shown in parentheses.

TABLE I
CORRELATION ANALYSIS FOR THE AMSTERDAM PTN WITH ROTTERDAM VALUES IN PARENTHESES FOR COMPARISON

Correlation	$\mathbf{H_i}$	$\mathbf{e^A}$	$\mathbf{e^B}$
$\mathbf{e^A}$	0.777 (0.773)		
$\mathbf{e^B}$	0.646 (0.652)	0.722 (0.531)	
$\mathbf{e^C}$	0.794 (0.753)	0.605 (0.467)	0.661 (0.722)

For the scenario reflecting the previous network state (before the opening of the NZL in 2018), we find that the H_i scores are generally lower than in the current network. The values for the hierarchical degree in the current network are more equally distributed than they have been in the past. The hierarchy is more spread out throughout the network as more nodes became important in the current situation. Further analysis reveals that this has been mainly driven by an increase in the topological influence which has led to a more even spread of hierarchy among the high scoring nodes. The Gini-coefficient for the network pre-NZL is 0.891 which is a little lower than for the current network.

Scenario Analysis

For future scenarios, an overview of the results for each scenario is shown in table II. In the second column, we evaluate the effect of each scenario in terms of its alignment with the current policy which is focused on becoming more robust and relieving the traffic load on the central station by enhancing other parts of the network. The evaluation ranges from a very negative effect (- -) to a very positive effect (++) and anything in between. In the last column, we provide an indication of the implementation costs, based on the additional infrastructure required, length of the route and necessity of additional civil engineering works. These values are just rough indications and range from very high implementation costs (- -) to relatively low implementation costs (++). The R2 scenario is perceived to be the most desirable in terms of the hierarchical structure but it is also associated with the most expensive implementation costs. This analysis illustrates that the proposed hierarchical metric and the changes therein can be augmented in the substantiation for policies towards an expansion of the metro network.

Rotterdam PTN results

TABLE II
COMPARISON OF THE FUTURE SCENARIOS FOR THE AMSTERDAM PTN

Scenario	**New route**	**Desirable hierarchical change**	**Implementation costs**
G1	Isolatorweg - Centraal Station	+ -	+
G2	Isolatorweg - Noorderpark	+ -	-
R1	Schiphol Airport - Centraal Station	+	-
R2	Schiphol Airport - Muiderpoort	++	- -

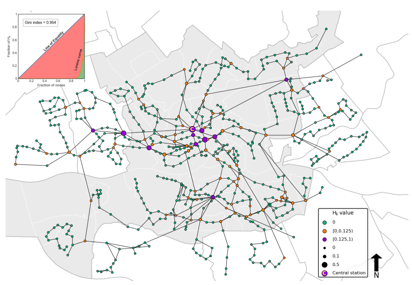

Fig. 3. Overview of nodes in the Rotterdam PTN

In Fig. 3 we show the geographical distribution of node hierarchy across the Rotterdam PTN, including the corresponding Gini-coefficient plot. It is evident that unlike in the case of Amsterdam, most hierarchical nodes are located in the centre of the city. High scoring nodes are located in close proximity to the central station while train stations further away from the central station score lower in general. Furthermore, the northern, eastern and southern areas of the city have very few hierarchical nodes compared to Amsterdam, where the network refurbishment in 2018 improved the distribution of hierarchy across the city.

With a Gini-coefficient of 0.954 for the Rotterdam PTN, the unequal distribution of hierarchy in the network indicates that the network load is functionally serviced only by the central part of the city. For different elements in the Rotterdam PTN, we conduct a correlation analysis to provide an overview of how the different elements and the hierarchical degree correlate, which is shown in table I. The correlation for both the topological influence and the non-redundancy with the hierarchical degree for the Rotterdam PTN is similar to the values obtained for the Amsterdam network but the correlation between the transfer potential and the hierarchical degree is considerably lower. This could be explained by some nodes in the Rotterdam PTN being located further away from the centre and having a relatively low hierarchical degree, while functioning as an important transfer hub for regional passengers. These nodes do generally have a lower influence as many of the influential nodes are all located in the city centre and the peripheral transfer hubs are not connected well to the centre.

This is further confirmed by the correlation between the topological influence and the transfer potential, which is much lower for the Rotterdam PTN than for the Amsterdam PTN. This is possibly caused by influential nodes being located in the city centre while transfer hubs are also located towards the edges of the network. The correlation between the topological influence and redundancy is also much lower for the Rotterdam PTN than for the Amsterdam one. A possible explanation for this observation could be that many of the influential nodes in the city centre are also redundant due to many mutual connections among them. Lastly, the correlation between the non-redundancy and transfer potential appears to be higher for the Rotterdam PTN which indicates that important transfer hubs in the Rotterdam PTN are less redundant than in Amsterdam, offering fewer locations for transfers.

IV. DISCUSSION AND CONCLUSION

In this article, we propose a metric for quantifying hierarchy in PTN and apply it to different case study networks. By combining different dimensions of the definition of hierarchy, the proposed metric consolidates different functionality pertaining to topological influence, redundancy and transfer potential into one measure.

The case studies demonstrate how the hierarchical metric provides some unique insights into the hierarchical structures of the Amsterdam and Rotterdam PTN, allowing for the comparison and evaluation of temporal network variants for a given city as well as comparing different cities. We find that the spatial distribution of the hierarchical metric differs for Amsterdam and Rotterdam. The Amsterdam PTN appears to have better connections between decentralised nodes by means of a (nearly complete) ring structure, which relieves the pressure on the centre of the network. For the Rotterdam network, no such ring structure exists which reinforces our observation of how a lot of the hierarchical nodes are located in the centre of the network. Even though the hierarchical coefficient of the Rotterdam PTN is higher than the coefficient of Amsterdam, the network structure of Amsterdam appears to be more balanced and robust in this case, by having a more scattered layout of its hierarchical nodes. This analysis suggests that the Gini coefficient has only limited value in explaining the distributional effects of node hierarchy because of its inability to incorporate spatial information reflecting to the underlying structure. Future research may design means to quantify the degree of spatial distribution in the node hierarchy values.

Potential applications of the proposed metric include the assessment of impact of proposed network changes on hierarchical distribution and the identification of bottlenecks. Notwithstanding, the increase in a hierarchy for a network could mean that only a few nodes increase in hierarchy (the rich getting richer) while it could also mean that second order nodes increase in function, relieving bottlenecks. Therefore, only evaluating the changes in hierarchical degree provides a limited understanding of the effects of a scenario on the network. A more in depth approach to a scenario in order to evaluate its strategic prospects should be applied for which the hierarchy metric can be used as a baseline indicator. For example, the on-going discussions related to the extension of existing metro lines in Amsterdam will benefit from investigating their hierarchical consequences using the approach proposed in this study.

In this study we apply a predominantly node-based approach where additional insights could be achieved using a link- or line-based approach for hierarchy. Furthermore, future research may underpin the identification of layers in network hierarchy and enhance the analysis with empirical data such as information on transferring flows obtained from smart-card journeys [37], [38]. This will allow enhancing the analysis with detailed temporal empirical data, as performed for example in the context of timetable design and passenger delay estimations [16], [39] . Future research into how hierarchical levels in PTN impact network vulnerability is a crucial topic to address, especially in the context of network developments [40]. Finally, applying this metric approach to different network structures such as maritime or aviation networks could be an endeavour of future research in assessing the suitability of this network analysis method in general.

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