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Statistical Evaluation of the Identified Structural Parameters of an idling Offshore Wind Turbine

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Abstract. With the increased need for renewable energy, new offshore wind farms are being developed at an unprecedented scale. However, as the costs of offshore wind energy are still too high, design optimization and new innovations are required for lowering its cost. The design of modern day offshore wind turbines relies on numerical models for estimating ultimate and fatigue loads of the turbines. The dynamic behavior and the resulting structural loading of the turbines is determined for a large part by its structural properties, such as the natural frequencies and damping ratios. Hence, it is important to obtain accurate estimates of these modal properties. For this purpose stochastic subspace identification (SSI), in combination with clustering and statistical evaluation methods, is used to obtain the variance of the identified modal properties of an installed 3.6MW offshore wind turbine in idling conditions. It is found that one is able to obtain confidence intervals for the means of eigenfrequencies and damping ratios of the fore-aft and side-side modes of the wind turbine.

1. Introduction

With an increasing awareness that the world cannot sustain the ever growing use of fossil fuels on the long run, a transition has started towards the use of renewable energy sources. Wind energy has great potential of meeting this sustainable energy need, where especially offshore wind energy resources appear to be suited for generating large quantities of wind energy. Other than their onshore relatives, offshore wind farms generate significantly less public opposition, which is often due to noise, visual pollution and the already intensive use of the land itself.

Unfortunately, the development and construction of large scale offshore wind farms is not yet economically viable without subsidy schemes from governments. In order for offshore wind energy to become one of the main energy sources of the future, significant efforts need to be made for lowering the cost of offshore wind power. Part of this cost-saving should be realized by lowering the capital expenses, for instance by optimizing the structural designs of offshore wind turbines and by eliminating undesired over-conservatism in the design.

In the design of wind turbines there are strong assumptions made on the individual damping contributions from soil, hydrodynamic and structural damping. As the aerodynamic damping directly follows from the (nonlinear) aerodynamics in the aero-elastic models used for design and analysis, this is not set as a specific input variable, but rather follows from the operational conditions and applied aerodynamic models. Due to the uncertainties on the actual values of the individual damping contributions, conservative estimates are used, that are often lumped into the structural damping ratios of the structural model of the wind turbine. With these conservative assumptions in the design phase, it could be the case that the actual system is significantly over-designed. Therefore, research has to be done for finding the true parameters of the actual, installed, offshore wind turbine. With better knowledge of the true system parameters, future designs can be less conservative, which may lead to lower overall costs. In addition, lifetime extensions and/or upgrades can potentially lower the cost of electricity of existing wind farms.

An interest has been developed, over the last couple of years, into estimating the structural parameters of offshore wind turbines using large sets of measurement data obtained from actual installed offshore wind turbines (OWTs). Classical experimental techniques, such as Experimental Modal Analysis (EMA) [1], are seldom used as they require that one measures the input forces to the system, such that the transfer functions of the system can be determined. It's not hard to imagine that it is physically impossible to measure the exact input forces on a structure as large and complex as an offshore wind turbine. Not to mention that these forces actually follow from a complex interplay between structure, wind and waves.

Hence, generally two experimental approaches, used to experimentally obtain modal properties of an OWT, can be distinguished. Firstly, by forcing a stop of the turbine and quickly pitching the blades of the turbine, the trust force is removed, thereby obtaining an inverse step-function type of excitation [2, 3]. Obviously, the input is not a perfect inverse step, as one has to take into account the maximum pitch speed and the fact that the turbine is still excited by unknown aero- and hydrodynamic forces. Secondly, one can accept the fact that there are unknown inputs and go for an "output-only" identification, using Operational Modal Analysis (OMA) methods [4, 5, 6, 7, 8], an idea that was already used over twenty years ago on a vertical axis onshore turbine [9]. However, a number of assumptions are made in these approaches, such as a linear-time-invariant system and white-noise excitation, which are in practise only partly met.

A novel approach is presented in this paper, in which an operational modal analysis (OMA) technique, based on stochastic subspace identification (SSI), is combined with automated clustering techniques and statistical analysis of the results, in order to obtain the confidence intervals of the means of the eigenfrequencies and the corresponding damping ratios.

Firstly, the different methods that are combined are briefly discussed in section 2. Secondly, the measurment data obtained from the Burbo Bank wind farm and some pre-processing steps are discussed in section 3. This is followed by a presentation of, and discussion on, the obtained results in section 4. Finally, from the results a number of conclusions are drawn in section 5.

2. Operational modal analysis, clustering and statistical analysis

2.1. Operational modal analysis and stochastic subspace identification

The Stochastic Subspace Identification method is an evolution of the Subspace Identification (SI), which is widely used in system and control engineering. It is able to find numerically reliable state-space models (with a predefined model order) for dynamic systems directly from measured data. In fact, it can be shown that unbiased estimates are obtained if one has an infinite amount of data points [10]. As the identification is performed in presence of measurement and system noise, the so called SSI Past Outputs Multivariable Output-Error StatesPace (PO-MOESP) method is used for this purpose, as was also done in [8].

One of the main features of this method, and the reason for using it here, is that one is able to improve the identified model by combining multiple data sets obtained in similar conditions, as is depicted in figure 1. Due to continuously changing environmental conditions (as is the case for wind turbines) it is virtually impossible to get a single, long enough, measurement with constant conditions. Hence, by using multiple short measurements with similar conditions, one is able to use the different data batches to get the "best" possible identified model. These methods

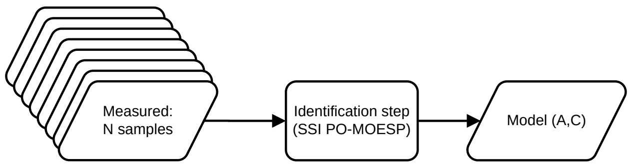


Figure 1: Multiple data sets are used to estimate a single model using the PO-MOESP method.

are available in the LTI System Identification Toolbox [11] developed at the Delft Center for Systems and Control, which was for that reason also used in this research. As the purpose of the current paper is not to outline the SSI method, and its variant that was used for this research, the reader is referred to [12, 13, 10].

2.2. Applied approaches for obtaining confidence intervals

In earlier work, presented in [8], a statistical analysis was performed by evaluating the variance of the eigenfrequencies and associated damping ratio's over the different identified model orders. However, using different model orders for obtaining an estimate of the variance of the results is not the best approach imaginable. This is due to the fact that one is in fact comparing different models obtained from the same data, instead of comparing similar models identified from different batches of data. Hence, the former can be interpreted as a convergence study into the model order of the identified models.

This work employs two different methods for obtaining the desired statistical results. Firstly, use is made of a bootstrap-method in order to estimate the variance of the system parameters as seen in [14] and discussed in section 2.2.1. Although the bootstrap is a precise method, it is not guaranteed to be accurate, which means that a biased result might be found. The second approach taken uses multiple data sets, where use is made of a so-called *leave-one out analysis*. This allows to find estimates of the confidence interval, while still taken as much data as possible into account for the identification step, as is discussed in section 2.2.2.

2.2.1. Bootstrapping approach

In the bootstrapping approach applied here (see figure 2), a subset of the available measured time series (i.e. P samples, as depicted in the figure) corresponding to a specific set of environmental and operational conditions are used to identify the best model possible, as was briefly discussed in section 2.1. Using the identified model, one is able to synthesize new time series of "measurement" data (synthesized samples), using a random noise excitation source. Note that one can synthesize as many samples as desired, with the desired "measurement" length, as the identified model is a Linear-Time-Invariant (LTI) model and hence, one no longer has to be aware of changing environmental and operational conditions. By performing an identification for all generated data samples (assuming n samples: $1 \leq l \leq n$), one obtains $n \times s$ identified models ($\boldsymbol{A}_{l,m}$, $\boldsymbol{C}_{l,m}$ state space matrices), where m ($2 \leq m \leq s$) defines the model order. In order to find the poles and modes of these identified models, the eigenvalue problems are solved.

$$\begin{aligned} \left(\boldsymbol{A}_{l,m} - \tilde{\lambda}_{l,m,k}\boldsymbol{I}\right)\tilde{\boldsymbol{x}}_{l,m,k} &= \boldsymbol{0} \\ \tilde{\boldsymbol{\phi}}_{l,m,k} &= \boldsymbol{C}_{l,m}\tilde{\boldsymbol{x}}_{l,m,k}, \end{aligned} \tag{1}$$

where $\tilde{\boldsymbol{\phi}}_{l,m,k}$ denotes the k^{th} mode and $\tilde{\lambda}_{l,m,k}$ the associated pole. By comparing the poles obtained from sample l for the different model orders, the physical (i.e. stable) poles and modes ($\lambda_{l,m,k}$, $\boldsymbol{\phi}_{l,m,k}$) can be separated from the noise poles, as is discussed in section 2.3.1. Now, as only the poles corresponding to a certain model order ($m = 40$) are used for the statistical analysis, one ends up with n sets of modes and poles ($\boldsymbol{\Lambda}_l$, $\boldsymbol{\Phi}_l$). The clustering step is discussed in more detail in section 2.3.2. Note that one can determine the eigenfrequencies and modal damping ratios direct from the complex poles [15]. Now, by determining the mean and standard deviation within the Fore-Aft and Side-Side clusters one can easily compute the confidence intervals of the means of the eigenfrequencies and corresponding modal damping ratios.

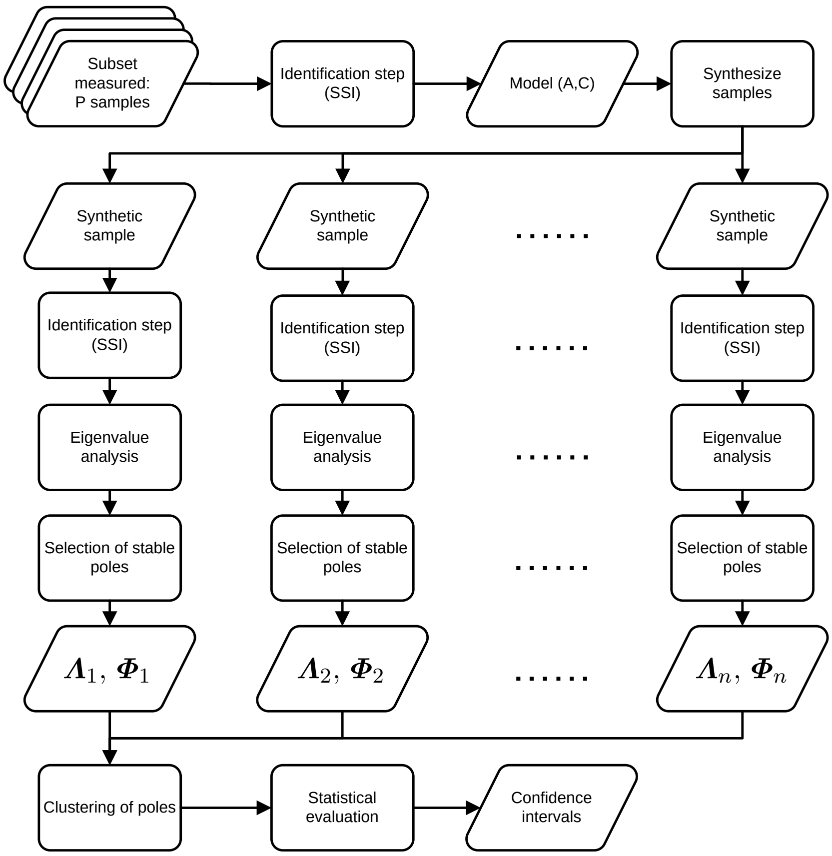


Figure 2: Flowchart of the applied Bootstrapping approach.

A great advantage of the bootstrapping approach is that (in theory) one only requires a single measurement to perform the statistical analysis. Obviously, the limited amount of data will affect the quality of the model identified, which could thus introduce a bias in the results.

2.2.2. Leave-one-out analysis

The leave-one-out approach is visualized in figure 3, where one can clearly see that the second part of the procedure is equal to the second part of the bootstrap procedure (see figure 2), hence for the explanation on this part the reader is referred to section 2.2.1. The first steps in the approach are different though. Firstly, instead of identifying one single model from all available data batches, n subsets of data batches ($N - M$ samples) are selected randomly from the full set. Subsequently, the identification step, using the SSI PO-MOESP method, eigenvalue analysis and selection of stable poles (as described in section 2.2.1) are performed for all these n subsets. By subsequently applying the hierarchical clustering (section 2.3.2) and statistical analysis, the confidence intervals for the mean of the damping and eigenfrequencies can be obtained.

A clear downside of this approach is that one requires large amounts of data in order to have

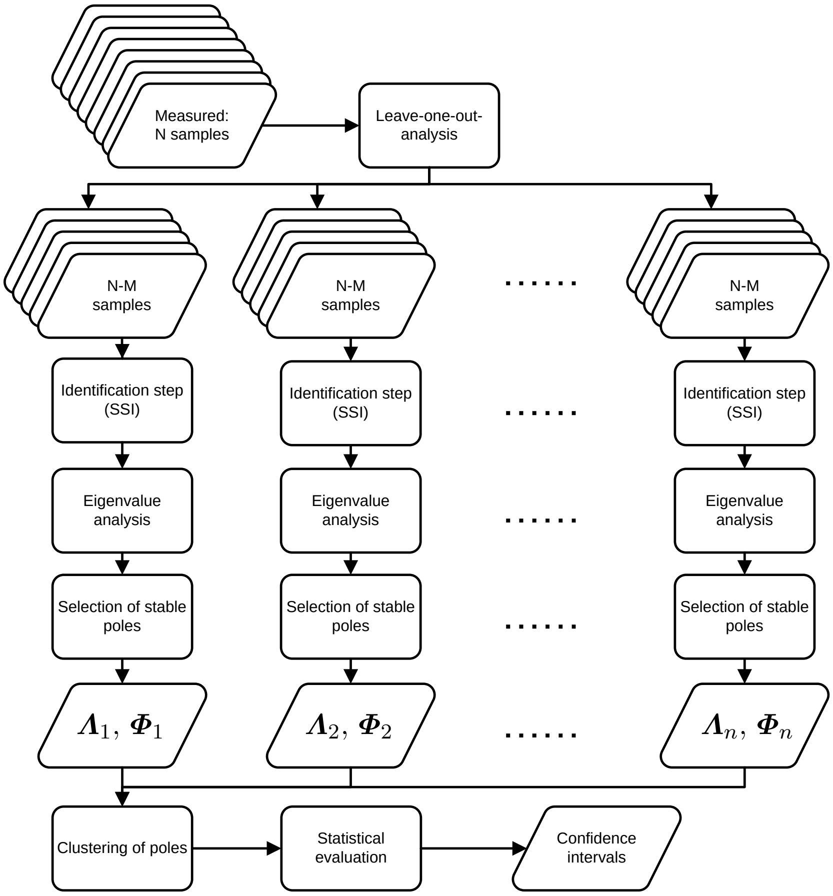


Figure 3: Flowchart of the approach based on the *leave-one-out* analysis.

sufficient subsets (i.e.) samples for the statistical analysis. The advantage however, is that the method is able to find unbiased results.

2.3. Automated clustering of modal results

Clustering methods are used for two reasons. Firstly, a clusterings algorithm is applied on the stability plot, in order to identify the physical poles from the mathematical or noisy poles. Secondly, clustering is needed to identify similar poles which come forward out of the identification routines. In this research two clustering methods have been applied.

2.3.1. Selection of stable poles

The first clustering (or selection) method applied is the *Moving window*, which is applied to find the stable poles from the stability plot of a single data sample as is shown in figure 4. Here the frequencies of the identified poles are shown for the different model orders. For each of the "tails" in the plot the mean of the poles is determined, after which it is checked whether the pole at each model order falls within a certain range w.r.t. to this mean.

2.3.2. The hierarchical clustering method

The second method is the hierarchical clustering method, this method is used for identifying and binning similar poles (and modes) from the identifications performed on different data batches, thereby enabling the statistical analysis using the method proposed in [16]. The distance

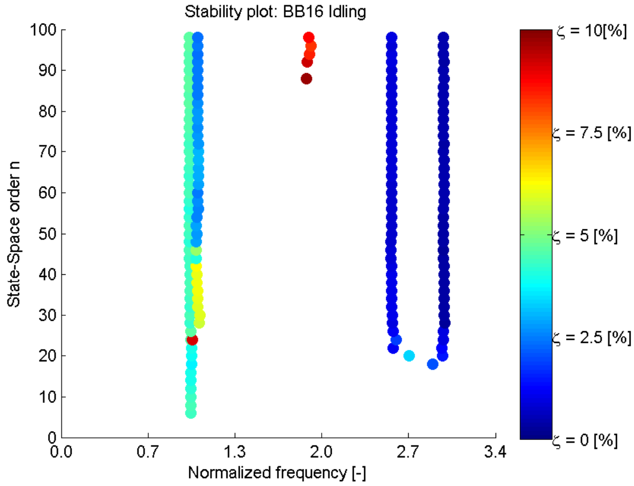


Figure 4: Example of a stability diagram obtained from measurement data acquired at an average wind speed of 1.7 m/s. Note that the damping values are shown in percentage of critical. (source: [8])

norm used in the hierarchical clustering method presented in [16] is adjusted to correct for the closely spaced frequencies of the identified poles. By squaring the modeshape similarity in this distance norm, more emphasis is put on differences in mode shapes. The adjusted distance norm is found in Eq. (2).

$$d_{i,j} = \frac{|f_i - f_j|}{f_j} + (1 - MAC_{i,j})^2 \,, \tag{2}$$

where $d_{i,j}$ denotes the distance between poles i and j, f_i, f_j denote the eigenfrequencies and the MAC denotes the Modal Assurance Criterion, which shows the correlation between modeshape i and j by means of an indicator that can take a value between 0 (no correlation) and 1 (perfect correlation) [17]. Note that the clustering is performed by collecting poles which are "close" to each other (i.e. below a certain predefined tolerance: $d_{i,j} < tol$) in a cluster.

3. Measurement data from the Burbo Banks Wind Farm

Firstly, it is introduced from which wind farm and turbine the used measurement data is obtained. In addition to this, a brief introduction to the different measurement channels that were available is given and the pre-processing steps before the identification are discussed.

3.1. Turbine # 16 in the Burbo Banks wind farm

The data obtained from the idling Burbo Bank turbine number 16 (BB16) are used in this paper. The Burbo Bank windfarm, owned by DONG Energy, is an operational offshore wind farm of the west coast of England, close to Liverpool in the Irish sea. It consists of 25 Siemens SWT-3.6-107 turbines, each with a rated power of 3.6 MW and equipped with a rotor that has a diameter of 107m. The wind farm has been inaugurated on 18 Oktober 2007 and has ever since been providing electricity for about 80 thousand households. The site, its location and BB16 are shown in figure 5. It was chosen to use the data from turbine number 16 as it is equipped with additional instrumentation for monitoring and analysis purposes. Considering the position of the wind farm it is reasonable to expect that the wind and the wave excitations are mainly from the west side, which is the oceanic side of the wind farm. The available information for the sea conditions indicate a typical wave period of $T_w = 7 - 10[s]$ with an averaged significant

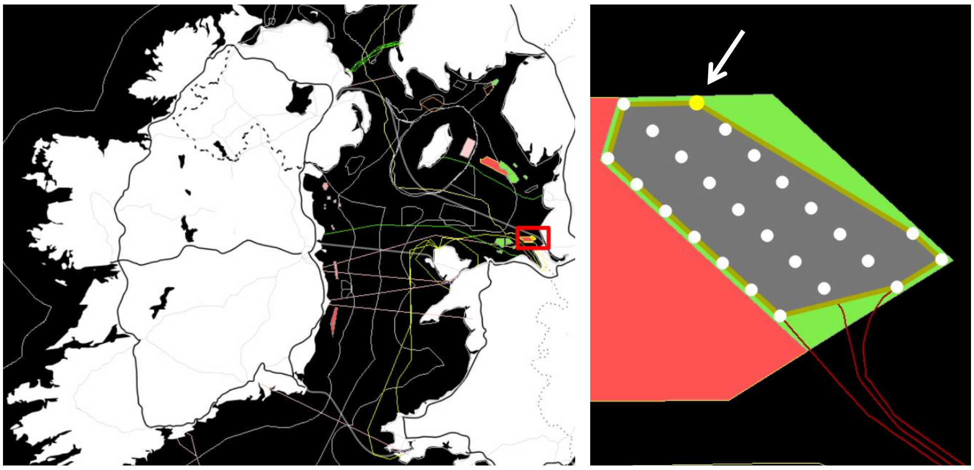


Figure 5: Location of the Burbo Banks offshore wind farm directly of the coast of Liverpool, United Kingdom (left), where the measurement data was obtained from turbine 16 (indicated on the right).

wave height of $H_w = 5[m]$. Maximum waves height of $H_{max} = 10[m]$ are registered in extreme conditions. As a consequence the highest wave excitations are expected in the frequency range of $f_w = 0.1 - 0.15[Hz]$.

3.2. Selection of measurement data

Several sensors were available on the offshore wind turbine. Firstly, a bi-directional accelerometer was installed in the back-end of the nacelle. In addition a number of strain gauges at the tower top, which measure the bending moments and the torsional moment, and strain gauges at the tower bottom, that measure the bending moments, were available.

The measurement data from these sensors is sampled at $25[Hz]$ and stored. As this wind farm has already been operating for a couple of years, large amounts of data are available for analysis. Using the synchronized measurements of the generator speed, several blocks where found where the turbine is idling. The turbines can be in idling mode if the wind speeds are too low, or due to planned maintenance and/or system malfunctions, therefore measurement data in different environmental conditions could be extracted from the database. Note that for the identification results presented, data has been filtered out for average wind speeds between 0 and 5 m/s and pitch angles $\geq 78°$ and only data from the bi-directional accelerometer has been taken into account.

4. Results on test turbine

For both the leave-one-out and the bootstrap method, set out in section 2, the resulting eigenfrequencies and damping ratios corresponding to the first two modes of the idling turbine are shown in figures 6a and 6b and table 1. The distribution of the identified eigenfrequencies and damping values for the Fore-Aft modes are visualized using the boxplots in figure 6a. Similar for the Side-Side modes, the results for both methods are shown in figure 6b. From these figures it is clear that a very narrow spread is found in case of the eigenfrequencies, for instance all the samples for the Fore-Aft mode are within a band of $\pm 2.5\%$ in the case of the multi-sample analysis.

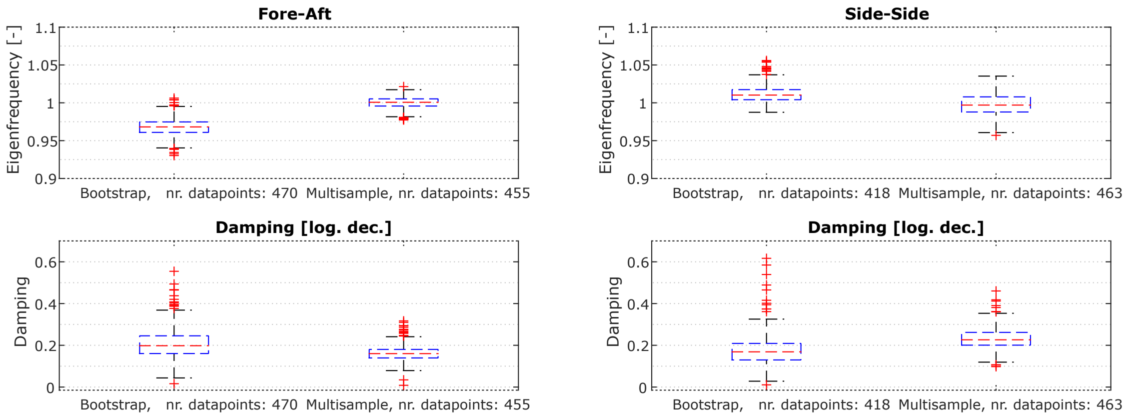
Note that in order to make the distinction between the two modes one must perform the clustering as was discussed in section 2.3. Also note that number of identified poles within each cluster does not coincide with the number of data sets. This indicates the difficulty of

Table 1: Resulting mean $\mu_\star$ and standard deviation $\sigma_\star$ of the (normalized) eigenfrequencies ω [-] and damping ratios ζ in logarithmic decrement [-] for the Fore-Aft and Side-Side modes.

Fore-Aft	μ_ω [-]	σ_ω [-]	μ_ζ [-]	σ_ζ [-]	nr points
Bootstrap	0.9679	0.0113	0.2083	0.0708	470
Multi-sample	1.0000	0.0078	0.1617	0.0360	455

Side-Side	μ_ω [-]	σ_ω [-]	μ_ζ [-]	σ_ζ [-]	nr points
Bootstrap	1.0112	0.0110	0.1772	0.0737	418
Multi-sample	0.9973	0.0137	0.2317	0.0510	463

automatically clustering the modes, as one expects to find one Fore-Aft and one Side-Side mode from each data set. Thereby showing that outliers have already been eliminated during the hierarchical clustering process.



(a) Fore-Aft mode (b) Side-Side mode

Figure 6: Identified normalized eigenfrequencies [-] and damping ratios in logarithmic decrement [-], for both bootstrap and multi-sample analysis, clustered to Fore-Aft and Side-Side modes.

Table 2: 95 % Confidence intervals for the mean values of the normalized eigenfrequencies ω [-] and damping ratios ζ in logaritmic decrement, for both the Fore-Aft and Side-Side mode.

Method	*Fore-Aft* μ_ω [-]	*Fore-Aft* μ_ζ [-]
Bootstrap	$[0.9669, 0.9690]$	$[0.2019, 0.2147]$
Multi-sample	$[0.9993, 1.0007]$	$[0.1584, 0.1650]$
Method	*Side-Side* μ_ω [-]	*Side-Side* μ_ζ [-]
Bootstrap	$[1.0101, 1.0123]$	$[0.1701, 0.1842]$
Multi-sample	$[0.9960, 0.9986]$	$[0.2270, 0.2363]$

The number of iterations used in the bootstrap procedure is chosen such that such that the number of identified poles within each cluster, for the different identification routines, is within the same range. This was done such that the two methods could be compared with one another. The resulting means and standard deviations corresponding to the performed identifications are shown in table 1. These statistical properties are used to produce the 95% confidence intervals, as shown in figure 2.

Note that, although the ranges of the confidence intervals listed in table 2 are comparable, there are clear offsets between confidence intervals obtained using the Bootstrap and multi-sample approaches. This clearly indicates that both methods show a similar precision, although not the same accuracy. It is also clear that the offsets between the confidence intervals for the identified eigenfrequencies are relatively small, whereas they are relatively large for the damping ratios obtained. This also indicates that it is harder to obtain reasonably accurate estimates of the damping ratios, than it is to obtain these for the eigenfrequencies.

As was stated in the brief introduction of the SSI method, it can obtain unbiased estimates in case one uses infinite amounts of data. Note that it is of course practically impossible to collect and process infinite amounts of data, but as the multi-sample approach used much more data for obtaining the confidence interval, it can be concluded that this method has generated a more trustworthy result. Hence, the results from the bootstrap show a clear bias, which is due to the fact that significantly less measurement data was used in the identification. In conclusion, in the case that only a small amount data is available, the bootstrap method is able to provide a reasonable estimate of the spread in damping values and eigenfrequencies obtained. Only for more accurate results, the number of data sets should be increased in the identification routine and a multi-sample approach is to be advised.

5. Conclusions and recommendations

Firstly it was found that, due to the fact that the modes of interest are closely spaced, both in terms of frequency and damping, more emphasize has to be put on the mode shape correlation in the hierarchical clustering. Nonetheless, the number of data points in the clusters were lower than the total amount of samples used. Hence, the clustering methods and/or distance norm applied are to be improved in order to accurately cluster all modes and enable an accurate, efficient and fully automatic processing of the identification results.

Secondly, when comparing the results from the multiple sample analysis to the bootstrap approach, it is clear that the spread found in the multiple sample analysis is of similar precision as that of the bootstrap results. Next to this, it was found that the accuracy of the multi-sample analysis over the bootstrap result is higher. This could be explained by the fact that all data samples where used in the multiple sample approach and only a subset of the data available was used to update the initial identification performed in the bootstrap analysis. Nonetheless it should be noted that the bootstrap is a precise method, but it is not guaranteed to be accurate, which means that a biased result might be found, as was the case here.

It is shown that one is able to find the confidence interval for the mean of the damping ratios of the first two modes of the wind turbine. Note that for the eigenfrequencies the obtained spread in results is very small, thereby indicating that one is able to accurately estimate the eigenfrequencies, whereas improvements are to be made for the estimation of the damping ratios. It is believed that by vastly increasing the number of measurements used for the analysis, together with a more precise selection of the different operational conditions (in terms of for instance wind speeds, operational states and/or pitch angle), the differences in operational conditions within the data used can be minimized. This is believed to reduce the spread in the results found.

Future work will be to perform these analyses for all possible operational conditions, thereby obtaining a mapping between damping ratios and eigenfrequencies on the one hand and the operational conditions on the other, such that these results can be used for validating and/or updating of aero-elastic design models. Following this approach, one can start identifying the assumptions which lead to over-conservatism in design, such that these lessons can be taken into account for future wind farms. At the same time, by taking into account the actual structural properties of installed wind turbines, reassessments of their lifetime can be performed.

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