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L.E.J. Brouwer, fifty years later

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Preface L.E.J. Brouwer, fifty years later

1. Introduction

Luitzen Egbertus Jan (Bertus) Brouwer (27 February 1881–2 December 1966) was one of the world's leading mathematicians and logicians, not afraid to be controversial. His revolutionary insights changed and enriched the landscape of mathematics, topology and logic in an ever-lasting way.

Let us briefly elaborate on his accomplishments, starting with the pillar of his legacy centred around topology.

Early in his career, in a period of roughly two years, Brouwer proved a number of fundamental theorems in topology that gave answers to famous old problems and opened up new areas of research. Because of these discoveries he is seen as the creator of modern topology. His most important results were: the fixed point theorem, the simplicial approximation theorem, the definition of a topological dimension function, the definition of degree of a map, and the topological invariance of domain.

The fixed point theorem states that for every n, every continuous map $f: \mathbb{I}^n \to \mathbb{I}^n$ has a fixed point. Here \mathbb{I}^n is the *n*-cube in \mathbb{R}^n . The list of its applications is endless. It is, for example, essential in topology, differential equations, differential geometry and game theory. In economics, the theorem is the heart of the proof of the existence of general equilibria in market economies. The Brouwer degree of a map $f: \mathbb{S}^n \to \mathbb{S}^n$, where \mathbb{S}^n denotes the *n*-sphere, is a direct generalization of the winding number of analysis. It was applied in the proof that \mathbb{S}^n admits a non-vanishing tangent vector field if and only if n is odd. The simplicial approximation theorem is a fundamental result for algebraic topology. It roughly says that every continuous map between geometric realizations of simplicial complexes is, after further subdivisions, homotopic to a simplicial map. Indeed, it was Brouwer who gave the first modern definition of homotopy. The invariance of domain theorem says that if U is an open subset of some \mathbb{R}^n and $f: U \to \mathbb{R}^n$ is an injective continuous map then f(U) is an open subset of \mathbb{R}^n . It is very strongly connected to Brouwer's work in dimension theory, where, building on work of Poincaré, he gave the first formal definition of the dimension of a topological space. Topology is being studied all over the world now. At present it covers a wide range of sub-disciplines: from set-theoretic and general topology via topological dynamics, continuum theory and dimension theory to geometric and algebraic topology. All these have, to a varying extent, their origins in Brouwer's work.

Brouwer's mathematical work was not restricted to topology, but also extended to set theory, measure theory and complex analysis. Brouwer's work in topology and adjacent fields has been collected in [1].

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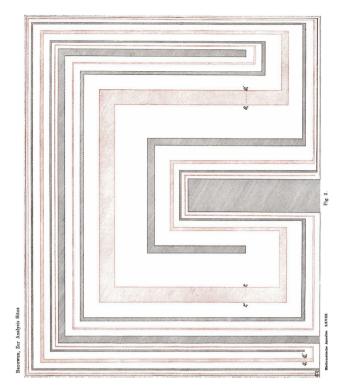


Fig. 1. The Lakes of Wada, part 1.

As an example of Brouwer's creativity in topology, we reproduce here his Lakes of Wada: three lakes on an island that share one and only one border. See Figs. 1 and 2, presumably drawn by Brouwer himself. These drawings were reproduced from his publication *Analysis Situs*, Math. Ann., 68 (1910), 422–434, with permission by Springer Nature.¹

The other pillar of Brouwer's accomplishments is centred around the foundations of mathematics, including its philosophical aspects. In this area, mathematical logic for short, he was controversial and revolutionary, with the audacity and the courage to doubt even the most undisputed logical principles, recasting them in a new light. This affected our appreciation of the natural numbers, as well as the continuum of real numbers, leading Hermann Weyl to his eloquent comment: '... und Brouwer, das ist die Revolution!'²

In fact, Brouwer found that the law of the excluded third, *tertium non datur*, "p or not p", should not always be taken for granted, and likewise the principle of the double negation, "not not p is p". Brouwer's work in this area, about Philosophy and Foundations of Mathematics, has been edited and collected in his Collected Works, Volume 1: Philosophy and Foundations of Mathematics [2].

¹ Remarkably, from a publishing point of view, as mentioned in van Dalen [7], p.143, Brouwer's paper is the only one in the Mathematische Annalen containing coloured figures, leading to extended discussions between author, editor and printer.

² Weyl, Das Kontinuum, 1921, pp. 55–56, GA II, p.157.

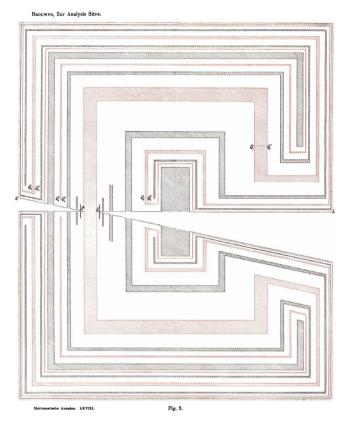


Fig. 2. The Lakes of Wada, part 2.

The result of Brouwer's scrutiny of established logical principles was the emergence of a deeper logic, the *intuitionism*, in a framework of a 'constructive' approach of mathematics, logic, and even informatics (computer science) as it nowadays has turned out. In Brouwer's innovating insight a proof is a construction that establishes correctness. This insight has led to the *proof interpretation* of Heyting and Kolmogorov, further elaborated in the type theories of De Bruijn, Martin-Löf and others. They paved the way to verification of mathematical proofs and complex systems. Numerous automatic systems, ubiquitous in our daily life, from microchips and remote controls to large-scale railway and air traffic control systems, have been proved reliable and safe using tools that have been created as applications. In the last decade these developments have led to the ground-breaking programme of 'univalent foundations', championed at IAS, Princeton, by the late Vladimir Voevodsky (Fields medallist 2002, deceased September 30, 2017), a programme in the research forefront of type theory and verification of proofs and systems.

End of the year 2016 fifty years had passed after the dramatic traffic accident at December 2, 1966 that ended the life of Bertus Brouwer.

His ideas and insights are nowadays more alive and vigorous than ever. A witness of his impressive scientific off-spring is the genealogic tree in Fig. 7. In view of this recognition, in honourful commemoration of his passing away half a century ago, and also aiming at a wider

distribution of his work highlighting his influence on current research in mathematics, philosophy and theoretical computer science, initiatives were taken by the Dutch KWG (Koninklijk Wiskundig Genootschap, Royal Dutch Mathematical Society) to organize some suitable events dedicated to Brouwer's life and work (https://www.wiskgenoot.nl/). These activities concerned the following (see Fig. 8):

- 1. The Brouwer Archive, collected by Dirk van Dalen, was on September 2, 2016 officially and festively transferred to be hosted in the North-Holland Archive in Haarlem (https://n oord-hollandsarchief.nl/). Brouwer spent a considerable part of his youth in Haarlem. This public symposium afternoon, attended by 160 participants, featured several talks about Brouwer and his work, and was titled L.E.J. Brouwer - Haarlems wiskundig wonderkind (L.E.J. Brouwer - Haarlem's mathematical prodigy). The event was held in the Hodshonhuis Haarlem, the location of the KHMW, Koninklijke Hollandse Maatschappij der Wetenschappen, Royal Holland Society of Sciences and Humanities. The symposium was organized by KWG, in cooperation with the KHMW. See Fig. 11. The programme consisted of the following lectures: Wellcome by Louise Gunning-Schepers (President of the KHMW) Jaap Korevaar, Persoonlijke herinneringen aan L.E.J. Brouwer (personal memories to L.E.J. Brouwer) Dirk van Dalen, Brouwer, introvert geleerde en onversaagd activist (Brouwer, introvert scientist and bold activist) Alexander Rinnooy Kan, Sporen van Brouwer (Traces of Brouwer) Dirk van Dalen, Over het archief van Brouwer (About the Brouwer Archive) *Symbolische overdracht van het archief (Symbolic transfer of the Archive)* Closure by Erik van den Ban (President of the KWG, and symposium chair)
- December 9, 2016: One-day symposium *L.E.J. Brouwer, fifty years later*, at the Science Park in Amsterdam, organized and sponsored by KWG, KNAW, UvA, VUA, KdV, ILLC, CWI, GQT. The very well attended symposium (180 participants) was chaired by Teun Koetsier (Vrije Universiteit Amsterdam) and featured the following lectures. See Figs. 3– 5.

Wellcome by Erik van den Ban, President KWG
Opening symposium by Jos Baeten, Director CWI
Sergei Artemov, Intuitionism and Knowledge
Raf Bocklandt, Reflections on a cup of coffee
Mark van Atten, On two problems with the Theory of the Creating Subject
Alexander Dranishnikov, On topology of robot motion planning
Saul Kripke, Free choice sequences: A temporal interpretation compatible with acceptance of classical mathematics
Yiannis Moschovakis, Intuitionism and effective descriptive set theory
Michael Rathjen, Indefiniteness, definiteness and semi-intuitionistic theories of sets
Dirk van Dalen, Where did it all begin?
Closure symposium by Sonja Smets, Director ILLC

3. Moreover, the editorial board of the journal Indagationes Mathematicae and the board of

5. Moreover, the entorial board of the journal indegationes Mathematicae and the board of the Royal Dutch Mathematical Society (KWG) decided to devote a special issue of the journal to the work of Brouwer, again titled *L.E.J. Brouwer, fifty years later*. The resulting collection, realized as a so-called Virtual Special Issue distributed in some physical journal issues, counts 35 articles, see the Table of Contents at the end of this Preface, comprising together with this Preface circa 880 pages. The editors of this VSI are in Fig. 6.

For the development of Brouwer's scientific and personal life, the origins of his ideas and his correspondence with the scientific community, we refer to van Dalen [7-10] and van Stigt [14,15]. Brouwer did much of his work at his Cabin in Blaricum, where he entertained numerous leading mathematicians. See Figs. 9 and 10.

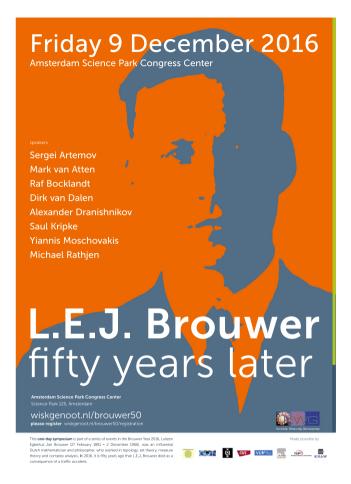


Fig. 3. Poster of the Brouwer Symposium, hosted by CWI, Amsterdam 2016. (Design Kitty Molenaar.).

For further background and references, see van Atten [5,6], van Dalen [11,12], Koetsier [3] and van Mill [13]. For an earlier collection of papers witnessing Brouwer's influence in logic and the foundations of mathematics we mention the Proceedings of the L.E.J. Brouwer Centenary Symposium, held in Noordwijkerhout 8–13 June, 1981, edited by Anne Troelstra and Dirk van Dalen, see [4].

2. Acknowledgements

- 1. We are grateful to several institutions and persons who were instrumental in facilitating and enabling the present 'Brouwer Volume', and its preparations. At the start of the work on this Volume in 2016 some well-attended scientific events were leading up to composing this Volume.
- With respect to the persons contributing to the scientific content of this Volume, our foremost thanks are to the authors, for creating their insightful and well-crafted papers, in a rich tapestry of interwoven strands generated by and often in direct continuation of the works of Bertus Brouwer.



Fig. 4. Lecturers Brouwer symposium *L.E.J. Brouwer, fifty years later.* Left to right: Michael Rathjen, Raf Bocklandt, Sergei Artemov, bust of Brouwer by J.G. Wertheim, Mark van Atten, Alexander Dranishnikov, Dirk van Dalen, Yiannis Moschovakis. Lecturer Saul Kripke is not on this picture, but is present on another photo taken this day, see Fig. 5. (Photograph Minnie Middelberg.).

We are also grateful to numerous colleagues who were helping out with reviews of the articles, providing many helpful corrections and advisory remarks. Several authors performed a double duty, reviewing in some cases the contribution of fellow authors.

- 3. We mention in particular many helpful suggestions by Marc Bezem (University of Bergen, Norway), and Albert Visser (Utrecht University, the Netherlands) during our editing process, helping us considerably with the selection of suitable reviewers.
- 4. Our thanks also extend to our contacts at this Journal's Bureau, Yvonne Philippo and colleagues, for their efficient and smooth cooperation in the production of this Volume. This includes our Editor-in-Chief Henk Broer for his encouragement and guidance throughout.
- 5. We are grateful for the indispensable support and sponsoring of the following institutions, organizations and companies:
 - (a) Stichting Compositio
 - (b) GQT, the Geometry and Quantum Theory cluster
 - (c) (UvA) Universiteit van Amsterdam
 - (d) (CWI) Centrum Wiskunde & Informatica
 - (e) (VUA) Vrije Universiteit Amsterdam
 - (f) Elsevier
 - (g) (NWO) Nederlandse Organisatie voor Wetenschappelijk Onderzoek



Fig. 5. Saul Kripke and Dirk and Dokie van Dalen, some 50 years after their first meeting at MIT; this time at the symposium *L.E.J. Brouwer, fifty years later*, December 9, 2016, Amsterdam. (Photograph Minnie Middelberg.).



Fig. 6. Editors special issue *L.E.J. Brouwer, fifty years later.* Left to right: Jan Willem Klop, Geurt Jongbloed, bust of Brouwer by J.G. Wertheim, Jan van Mill, Dirk van Dalen. (Photograph Minnie Middelberg.).

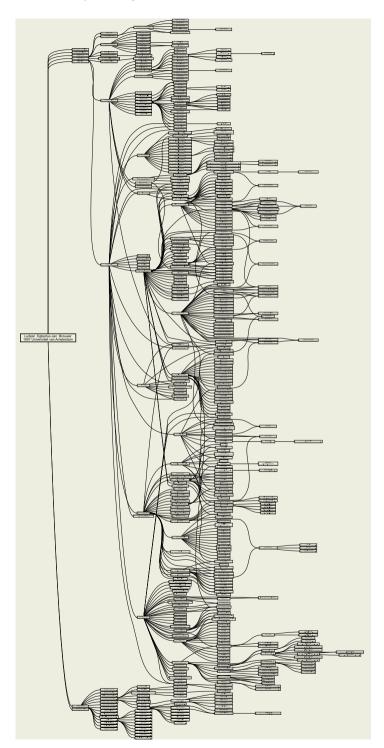


Fig. 7. Scientific genealogy of Brouwer, composed by the *Mathematics Genealogy Project*, Department of Mathematics, North Dakota State University. Names are readable in the digital version after zooming in.



Fig. 8. Brouwer as newly appointed professor in the Oudemanhuispoort, 1912, University of Amsterdam. (Photograph Brouwer Archive.).

- (h) (KNAW) Koninklijke Nederlandse Akademie van Wetenschappen
- (i) (KWG) Koninklijk Wiskundig Genootschap
- 6. Of the institutions as mentioned we are in particular grateful to our contact persons and coorganizers of the symposia: Erik van den Ban (KWG), Eric Opdam (KdV), Yde Venema, Sonja Smets (ILLC), Jos Baeten, Joke Blom, Daniëlle Kollerie, Peter Hildering, Annette



Fig. 9. Brouwers Hut (Cabin), in Blaricum, near Amsterdam. (Photograph Dokie van Dalen.).

Kik, Minnie Middelberg (CWI), Teun Koetsier (VUA), Rien Kaashoek (VUA), Evelien Wallet, Monique Onderwater (UvA).

- 7. Thanks to Dr. Mitch Keller of the Mathematics Genealogy Project, Department of Mathematics, North Dakota State University, for manufacturing and providing us with the tree of scientific descendants of Brouwer, see Fig. 7.
- 8. Our Brouwer volume starts with an article of Brouwer himself, *On the natural concept of dimension.* See also the Table of Contents. This paper was first published in German in Crelle's Journal, the Journal für die reine und angewandte Mathematik, 142 (1913), pages 146–152. It contains the first formal definition of dimension. We are indebted to the editors of the Journal für die reine und angewandte Mathematik for allowing us to reproduce it here, and to Robbert Fokkink (Delft University of Technology) for translating it into English. We are also indebted to Catriona Byrne, the Mathematics Editorial Director of Springer, and Oda Siqveland of Springer Nature, for their help in obtaining Springer's permission to include the drawings of Brouwer's Lakes of Wada in this editorial.

3. In memoriam

- 1. One of our intended authors was Sol Feferman, deceased on July 26, 2016. He was preparing an article for this Volume together with Michael Rathjen. We are thankful that Feferman's ideas are represented in an extensive and honourful way in the article of Michael Rathjen, in this Volume.
- 2. We also acknowledge in gratitude the work of Walter van Stigt, deceased November 24, 2015, in his early cooperation (the academic year 1976/1977) at Utrecht University with



Fig. 10. Brouwer working at his Cabin around 1924. (Photograph Brouwer Archive.).

Dirk van Dalen, exploring, documenting and safe-guarding the legacy of Brouwer as to its biographical aspects, leading van Stigt to his book [15], *Brouwer's Intuitionism*, Elsevier, Amsterdam, 1991.

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References

- [1] Hans Freudenthal, LEJ Brouwer Collected Works: Geometry, Analysis, Topology and Mechanics, Elsevier, 2014.
- [2] Arend Heyting, Collected Works 1. Philosophy and Foundations of Mathematics, North Holland, Elsevier, 1975.
- [3] Teun Koetsier, Arthur Schopenhauer and LEJ Brouwer, a Comparison Combined Proceedings for the Sixth and Seventh Midwest History of Mathematics Conferences, 1999.
- [4] Anne S. Troelstra, Dirk van Dalen (Eds.), The L. E. J. Brouwer Centenary Symposium, in: Studies in Logic and the Foundations of Mathematics, vol. 110, North-Holland Publishing Co., Amsterdam-New York, 1982.
- [5] Mark van Atten, Luitzen Egbertus Jan Brouwer, in: Edward N. Zalta (Ed.), The Stanford Encyclopedia of Philosophy, winter 2017 ed., Metaphysics Research Lab, Stanford University, 2017.
- [6] Mark van Atten, Robert Tragesser, Mysticism and mathematics: Brouwer, Gödel, and the common core thesis, in: Klarheit in Religionsdingen, 2003, pp. 145–160.
- [7] Dirk van Dalen, Mystic, Geometer, and Intuitionist, The Clarendon Press, Oxford University Press, New York, 1999, The life of L. E. J. Brouwer. Vol. 1, The dawning revolution. Volume 2, Hope and Disillusion.
- [8] Dirk van Dalen, LEJ brouwer 1881– 1966, in: Een biografie. Het heldere licht van de wiskunde, Bert Bakker, Amsterdam, 2001.
- [9] Dirk van Dalen, The Selected Correspondence of LEJ Brouwer, Springer Science & Business Media, 2011.
- [10] Dirk van Dalen, LEJ Brouwer–Topologist, Intuitionist, Philosopher: How Mathematics is Rooted in Life, Springer Science & Business Media, 2012.

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- [11] Dirk van Dalen, Poincaré and Brouwer on intuition and logic, Nieuw Arch. Wiskd. (5) 13 (3) (2012) 191–195.
- [12] Dirk van Dalen, Snapshots of Brouwer's life and work, Nieuw Arch. Wiskd. (5) 17 (4) (2016) 250–259.
- [13] Jan van Mill, Brouwers dimensionsgrad: controverse en verwarring, Nieuw Arch. Wiskd. (5) 14 (2) (2013) 130– 138.
- [14] Walter P. van Stigt, The rejected parts of Brouwer's dissertation on the foundations of mathematics, Hist. Math. 6 (4) (1979) 385–404.
- [15] Walter P. van Stigt, Brouwer's Intuitionism, North-Holland, Amsterdam, 1990.

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