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# Interpreting repeated CPT in unsaturated soils

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## ABSTRACT

Cone penetration tests, CPTs, are extensively used in the Netherlands to assess the stability of fourteen thousand kilometres of dykes protecting the country from flooding. On the regional dykes, site testing is planned and executed only from spring to autumn. The data collected in the drier season of the year must be used then in safety factor calculation for dyke stability with reference to the worst expected conditions, including the highest weights and the highest water pressures over the year. Inferring reliable values of the shear strength in a different season implies understanding the unsaturated response of the dyke material and the effect of variable water content on the CPT response. In previous studies referring to CPTs in unsaturated soils, it was observed that both the cone resistance and the sleeve friction depend on suction, however, only the cone resistance was used to determine the shear strength in combination with water content or suction probes installed into the ground. In this contribution, we analyse an extensive set of data, coming from repeated CPTs performed over one year on the Maasdijk near Oijen in the Netherlands. The data are elaborated to investigate whether the entire set of data can be exploited to try to derive the water content and the constant water content shear strength at the same time, if the test is repeated in different seasons.

**Keywords:** unsaturated soils, seasonal field monitoring, CPT.

## 1. Introduction

Cone penetration tests, CPTs, are extensively used in the Netherlands to assess the stability of fourteen thousand kilometres of dykes protecting the country from flooding. On the regional dykes, site testing is planned and executed from spring to autumn. In winter, testing is forbidden to avoid extra load on the dykes and reduce the risk of collapse. However, dyke stability must be assessed with reference to the worst combination of loads and water pressures, which typically corresponds to the wettest condition of the year experienced in the winter season. This practice implies that a reduction factor on the shear strength derived from CPTs in summer should be applied, not to overestimate the available shear strength when the highest environmental loads are applied to the dyke body.

Inferring reliable values of the shear strength in a different season implies understanding the unsaturated response of the dyke material and the effect of variable water content on the CPT response. The literature on the interpretation of CPTs in unsaturated soils is rather scarce. Attempts have been made in the past to derive the strength from the cone tip resistance in unsaturated soils with analytical solutions (e.g., Russel, 2004; Russel and Khalili, 2006; Miller et al., 2018), tests in the calibration chamber (e.g., Pournaghiazar, 2011; Miller et al., 2018) and combined laboratory and modelling procedures (e.g., Chao et al., 2023). In all cases, the CPT data interpretation was based on the knowledge of either the water content or the suction of the soil at the depth of the investigation.

Additionally, it was observed that also sleeve friction depends on suction, possibly to a lesser extent compared to the cone resistance (Miller et al., 2018). In spite of this observation, the attention in the past has been kept focussed on the cone tip resistance and the sleeve friction measurement has been mostly disregarded.

In the practice, especially for extensive investigation, neither the water content nor the suction profiles are known at the time of a standard CPT investigation, which hinders the interpretation of the experimental information. In this contribution, we analyse an extensive set of data, coming from repeated CPTs performed on the Maasdijk near Oijen in the Netherlands over one year. The investigation was promoted by Rijkswaterstaat, the Dutch Ministry of Infrastructure and Water Management, and performed by Deltares over one year. The description of the whole set of tests on two investigated sites, including CPT data, suction and water content monitoring in the field, and laboratory tests can be found in the technical report by van Duinen (2021).

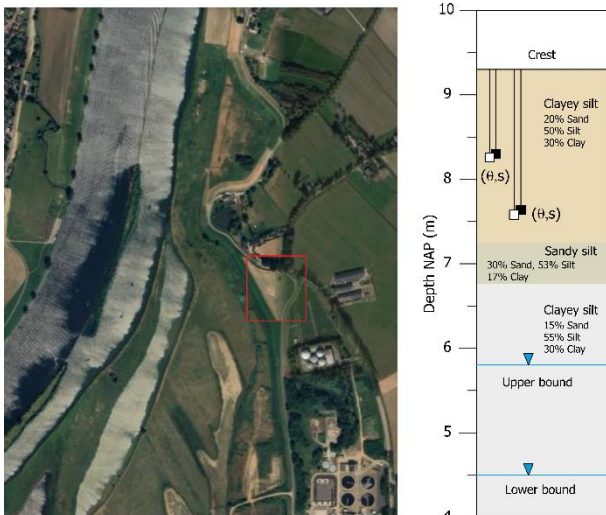
One set of data was chosen to evaluate how the cone resistance,  $q_c$ , and the sleeve friction,  $f_s$ , varied with water content over one year. The constant water content strength, herein referred to as “undrained” for the sake of simplicity, was derived from  $q_c$  with the aid of laboratory tests performed at Delft University of Technology (TUDelft). The data are elaborated to investigate whether and how the entire set of data, including the sleeve friction, could be exploited to derive information on the water content and the undrained shear strength at the same time, which could help in more reliable prediction of the available strength of the unsaturated soil in a different season.

## 2. Field data

Field investigation was conducted from the crest of the Maasdijk near Oijen, in the central-eastern part of the Netherlands (Fig. 1). The elevation reference used for height is the Normaal Amsterdam Peil (NAP). As shown in the map, a floodplain is situated in front of the primary dyke with a width of approximately 380 m. The crest of the dyke, where the CPTs were performed, is rather wide, which implies a nearly one-dimensional water exchange with the atmosphere.

The dyke was constructed in the 1950s and is made of clay and sandy clay. The subsoil layers are reported in Fig. 1 and consist of a brown clayey silt layer about 2 meters thick ( $I_p = 13.1\%$ ), a sandy silt layer 1 m thick ( $I_p = 5.2\%$ ) and a grey clayey silt layer 1.5 m thick ( $I_p = 16.1\%$ ). The position of the water table oscillates over the year between 4.5 m and 5.8 m NAP.

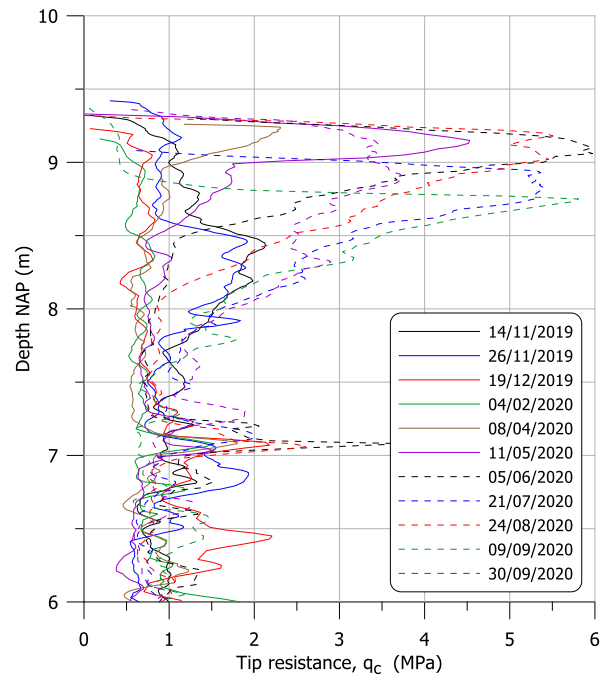
To the scope of the original study, the dyke was instrumented with suction and water content probes. While the latter continued working for the entire year of investigation, the suction probes, unfortunately, cavitated one after the other at the start of the summer. The position of the monitoring sensors for volumetric water content and matric suction is also displayed in Fig. 1.



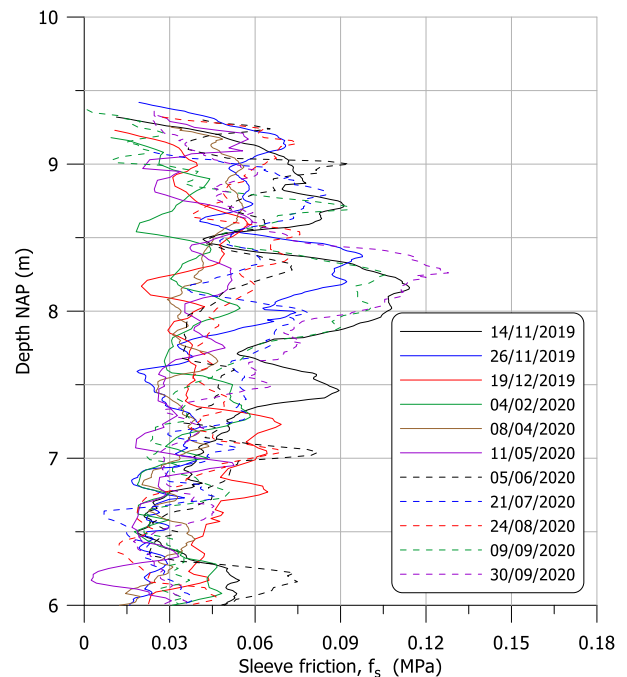
**Figure 1.** Location of the test site at Maasdijk near Oijen

Cone penetration tests (CPTs) were repeatedly performed over one year, from September 2019 to October 2020, to assess variations in measurements across the seasons (van Duinen, 2021). Two series of CPTs were conducted in each testing campaign within an area of about 15 m by 5 m. The tip resistance,  $q_c$ , and sleeve friction,  $f_s$ , from one of the two testing series at different times over depth are plotted in Fig. 2. and Fig. 3, respectively.

The daily precipitation recorded during the testing period by the weather station at Megen, located 6 km east of Oijen, (station code 903, KNMI) is plotted in Fig. 4. The precipitation data clearly distinguish a wet season, starting from October 2019 to March 2020 and the beginning of the average dry season afterwards until October 2020. Sporadic summer rainfall events could also be observed in July, August and late September.

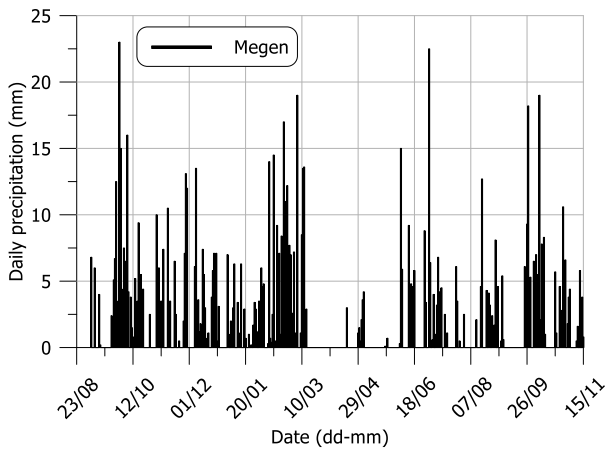


**Figure 2.** Profile of tip resistance from CPT at different times



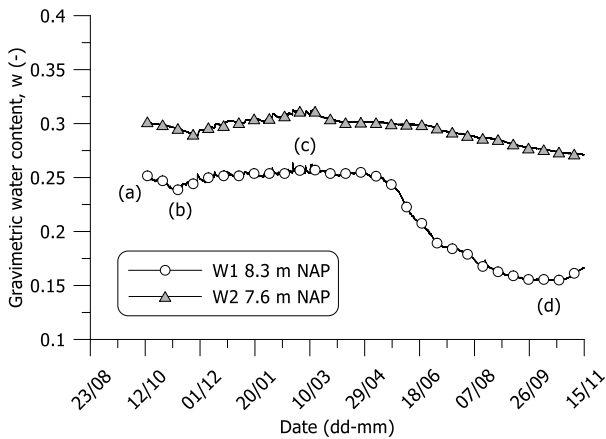
**Figure 3.** Profile of sleeve friction from CPT at different times

The CPT data in Fig. 2 and Fig. 3 highlight interaction of the subsoil with the atmosphere over a depth of about 2.5 m. The tip resistance during the wet period from December 2019 to March 2020 gave values in the range of 0.5 MPa to 2 MPa, with a general trend of tip resistance reduction. However, in April 2020, the tip resistance of the first meter (8.5 - 9.5 m NAP) started to increase. By the end of September 2020, the tip resistance of the first two meters had almost increased six-fold from 1 MPa to 6 MPa. A similar trend can be observed in the sleeve friction measurements, which decreased during the wet season and increased in the dry season although with a more scattered variation compared to the tip resistance.

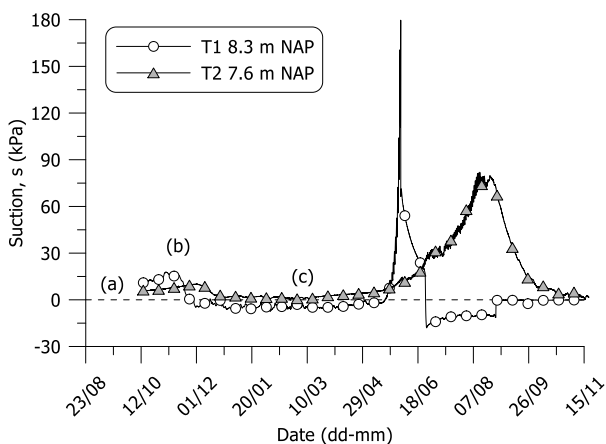


**Figure 4.** Daily precipitation registered from the weather station at Megén

Field measurements were integrated with in-situ monitoring sensors to describe the unsaturated state of the subsoil. Water content reflectometers CS616 of Campbell Scientific and tensiometers T5 of Meter Group were installed to record the volumetric water content,  $\theta$ , and the matric suction,  $s$ , respectively. Four pairs of sensors were installed at different depths, from 8.3 m to 6.2 m NAP.



**Figure 5.** Gravimetric water content in the field at 8.3 m and 7.6 m NAP during the measurement campaign



**Figure 6.** Suction in the field at 8.3 m and 7.6 m NAP during the measurement campaign

Figure 5 and Fig. 6 report the gravimetric water content,  $w$ , and suction from the measurement of the most surficial sensors located at 8.3 m and 7.6 m NAP, most interested by the unsaturated conditions. The gravimetric water content was derived from the in situ measurement based on the information from the samples retrieved from the site. The monitoring data at the shallow depth of 8.3 m NAP showed a good correlation with the precipitation data. The gravimetric water content reduced during the short dry period, September 2019 (point a to point b), before increasing towards fully saturated conditions ( $w = 0.26$ ) during the subsequent wet period (point b to point c). In the long dry period (point c to point d), the water content decreased drastically from 0.26 to 0.15.

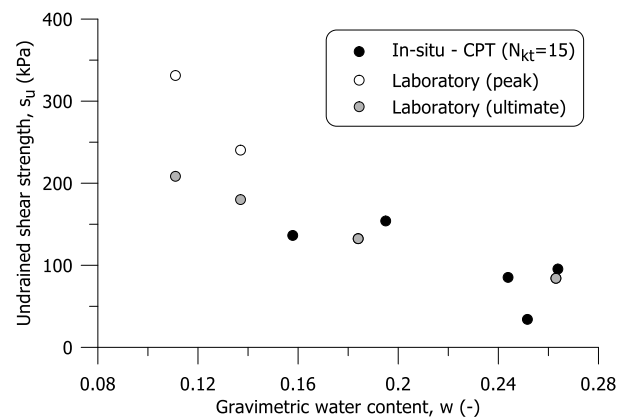
Similarly to the water content sensors, the tensiometers showed a maximum suction of about 15 kPa in September 2019, before the intense precipitation in late autumn and winter 2019 caused almost null suction in the upper part of the soil. A clear increase in suction starting from spring 2020 can be observed in Fig. 6 with the most surficial tensiometer, at 8.3 m NAP, reaching a maximum suction of about 180 kPa in early June before cavitation and the sensor at 7.6 m NAP cavitating at about 80 kPa later in August.

### 3. Laboratory data

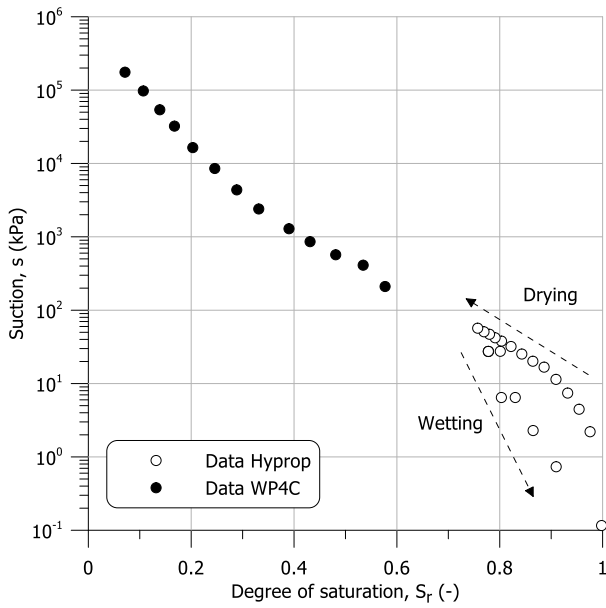
The site investigation was accompanied by laboratory testing of samples retrieved from the site at the start of the monitored period. Constant water content, undrained, triaxial tests were performed on undisturbed samples at different water contents, to condition the transformation factor,  $N_{kt}$ , to be used in the derivation of the shear strength from  $q_c$  measurements. A detailed description of the tests and their interpretation can be found in the paper by Chao et al. (2023).

In Fig. 7 the laboratory data are compared to selected data derived from CPT measurements, adopting a cone factor  $N_{kt} = 15$  to transform the cone tip resistance,  $q_c$ .

To complete the information in view of data elaboration, the water retention properties of the soil were investigated using the Hyprop® and the Dew Point Potentiometer WP4C® (Meter Group, 2015, 2024), including wetting and drying cycles, and the data are reported in Fig. 8.



**Figure 7.** Undrained shear strength from triaxial data in the laboratory and CPT data in the field



**Figure 8.** Water retention data from Hyprop® and WP4C®

To transform measured water content on site into suction, also after the tensiometers cavitated, the data in Fig. 8 were fitted with a modified van Genuchten's model as in Romero and Jommi (2008)

$$S_r = C(s) \left[ \frac{1}{1+(\alpha s)^n} \right]^m; C(s) = 1 - \frac{\ln\left[1+\frac{s}{a}\right]}{\ln(2)}, \quad (1)$$

The fitting parameters for the main drying and wetting curve and scanning curve are reported in Table 1.

**Table 1.** Input parameters for the water retention model

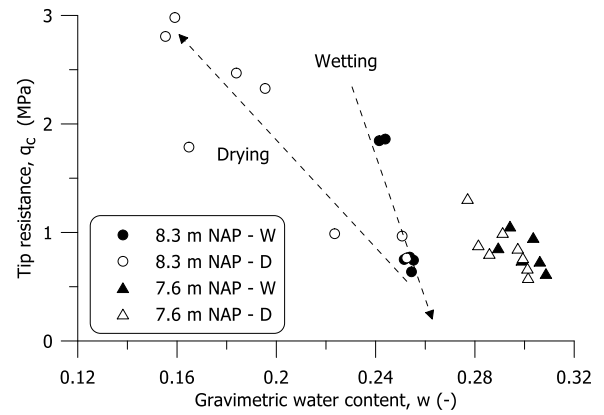
Path	$\alpha$ (MPa <sup>-1</sup> )	$m$ (-)	$n$ (-)	$a$ (MPa)	$k_s$ (MPa <sup>-1</sup> )
Drying	25	0.22	1.20	1000	-
Wetting	15	0.60	0.45	1000	-
Scanning	-	-	-	-	10 <sup>-5</sup>

#### 4. Data analysis

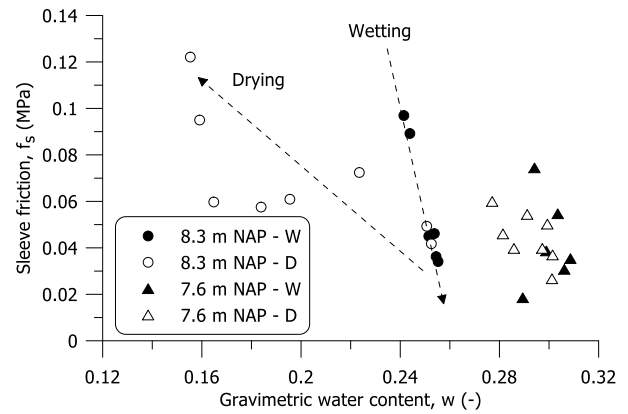
The tip resistance,  $q_c$ , and the sleeve friction,  $f_s$ , at the two instrumented depths are reported as a function of the water content over the site investigation period in Fig. 9 and Fig. 10, respectively.

Not surprisingly, the most surficial measurements, at 8.3 m NAP, indicate a clear decrease in the tip resistance with the increase in the water content during the wet season (October–March), and a subsequent increase during the dry season. At 7.6 m NAP, the same trend can be appreciated, although the modest change in water content during the year,  $0.28 < w < 0.32$  is reflected in modest variation in the CPT response.

The sleeve friction shows a similar trend, although the data appear more scattered, compared to the tip resistance data.



**Figure 9.** Evolution of tip resistance with gravimetric water content

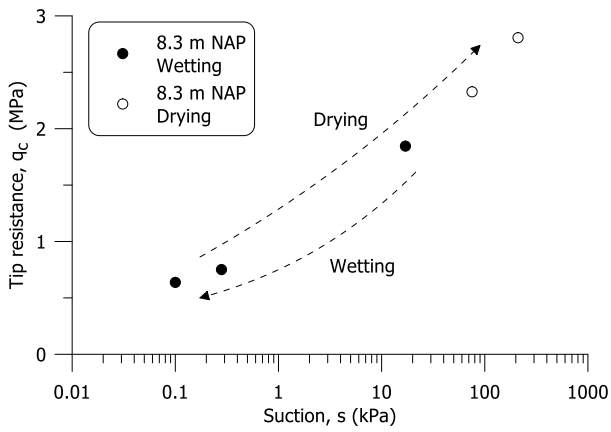


**Figure 10.** Evolution of sleeve friction with gravimetric water content

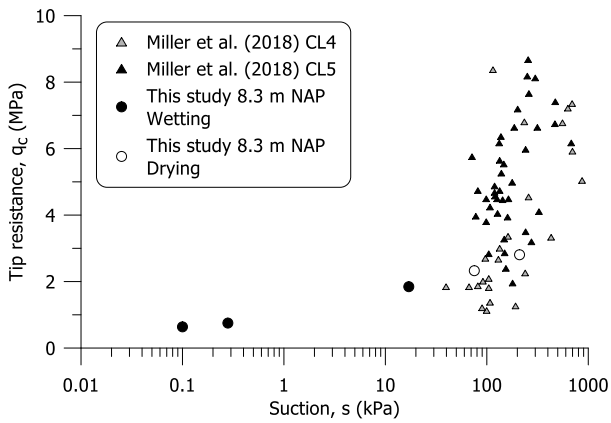
The scatter may be partially attributable to the natural variability of the construction soil, given that the CPTs were performed in a small area, but obviously not at the same point. Nonetheless, it is known from the literature that sleeve friction suffers from some difficulty in the measurement already in saturated conditions (Lunne and Andersen 2007; Robertson 2009).

More interestingly, the response is significantly hysteretic if the data are plotted as a function of the water content. The evidence is disappointing from the practical viewpoint because it implies that knowledge of the water content at the time of the measurement, either from water content sensors installed in the field or from direct water content determination, would not be enough to infer the available undrained shear strength.

To better analyse the data, the water content was transformed into suction, relying on the water retention data and performing a simulation of the previous hydraulic history, including the observed drying-wetting periods. Representative states were considered: for the wetting period, 14 November 2019 ( $S_r = 0.93$ ), 19 December 2019 ( $S_r = 0.95$ ), and 9 March 2020 ( $S_r = 0.97$ ); for the dry period, 29 June 2020 ( $S_r = 0.78$ ) and 30 September 2020 ( $S_r = 0.63$ ). The water content detected on the chosen dates was transformed into suction by modelling the water retention data in Fig. 8 (Chao et al., 2023). The dependence of tip resistance on the hydraulic history, measured by the current suction is reported in Fig. 11.



**Figure 11.** Derived dependence of the measured tip resistance and the estimated suction from the retention model



**Figure 12.** Comparison between tip resistance and suction from this study and databased from Miller et al. (2018) (data redrawn from Miller et al. (2018))

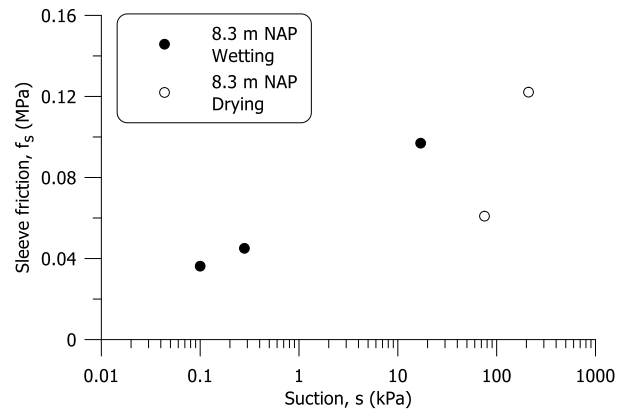
The data are also compared in Fig. 12 to a recent tip resistance, water content and suction database, compiled by Miller et al. (2018) from both field investigation and tests in a calibration chamber. The data from this study are compared with Class 4 (CL4) and Class 5 (CL5) soil types reported by Miller et al. (2018) with similar plasticity indexes of the soil at Oijen.

Although the suction values experienced at Oijen are in general smaller compared to the ones included in the database, in the overlapping range, from about 20 kPa to about 200 kPa, the present data fall in the expected range.

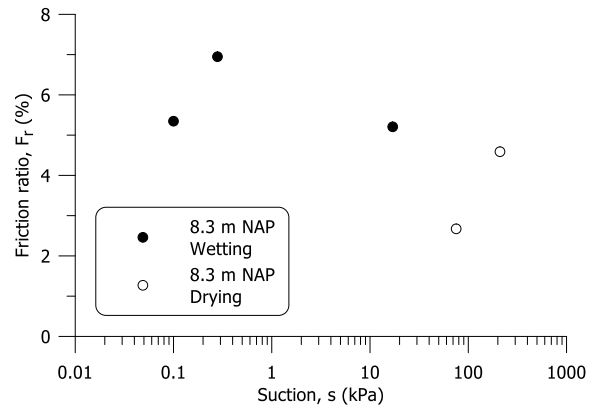
It is worth noting that hysteresis flattens down and almost disappears if the tip resistance values are plotted as a function of suction. The observation indicates that suction, rather than water content, could be used in the current application to predict seasonal variations in the undrained shear strength over time. However, unfortunately, well-known practical difficulties arise in measuring suction in the field, confirmed by cavitation of tensiometers at an early stage of the summer season.

## 5. Sleeve friction and friction ratio

Very scarce information can be found in the literature on the evolution of sleeve friction in unsaturated soils. The sleeve friction data and friction ratio,  $F_r$ , from the Oijen test site, are plotted as a function of suction in Fig. 13 and Fig. 14, respectively.



**Figure 13.** Derived dependence of the measured sleeve friction and the estimated suction from the retention model

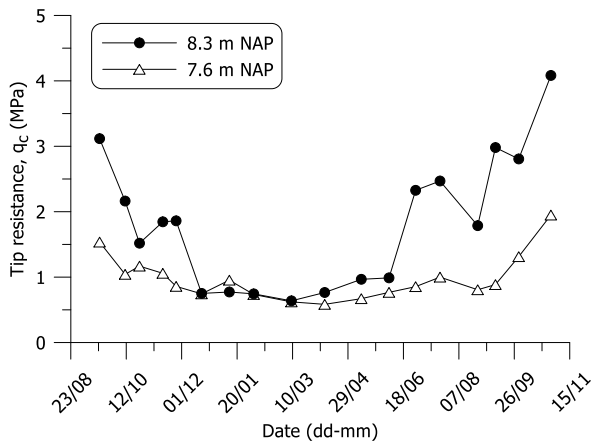


**Figure 14.** Derived dependence of the calculated friction ratio from field measurements and the estimated suction from the retention model

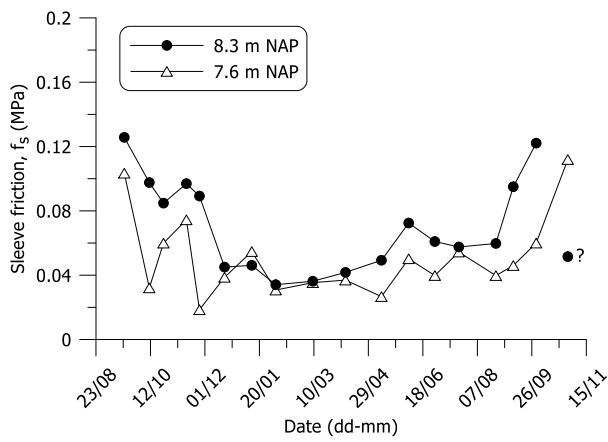
The data correspond to the same set in Fig. 11. As expected, also the sleeve friction increases with suction. However, the friction ratio shows the opposite trend. Higher values of friction ratio correspond to wetter conditions, progressively decreasing with drying. The result shows smaller sensitivity of sleeve friction to suction, compared to the tip resistance. Similar observations were reported by Miller et al. (2018) from field measurements at the Goldsby test site. However, a more erratic variation in friction ratio with water content was reported by the same authors for the North Base site.

To better understand the trend, the entire set of data for  $q_c$  and  $f_s$  is reported as a function of time in Fig. 15 and Fig. 16, respectively. Both measurements mirror the water content data in Fig. 5. Mid-October the wet season starts, and both  $q_c$  and  $f_s$  start decreasing. The lowest values are recorded in mid-March, at the end of the continuous rainy season. April marks the transition towards the summer season, which brings to drier conditions. In the year analysed, the period between June and July was quite rainy, which temporarily decreased both  $q_c$  and  $f_s$  recordings, before they sharply increased again in the dry period covering August and September.

The comparison between  $q_c$  and  $f_s$  in the time scale reveals once more that the cone resistance appears more sensitive to water content compared to the sleeve friction.



**Figure 15.** Time variation of the tip resistance at 8.3 m NAP and 7.6 m NAP during the measurement campaign



**Figure 16.** Time variation of the sleeve friction at 8.3 m NAP and 7.6 m NAP during the measurement campaign

In order to understand the reason for the reduced sensitivity of sleeve friction to water content variations, reference can be made to the literature reporting the results of laboratory tests aimed at investigating the behaviour of unsaturated soils-steel interfaces, especially in comparison with the shear resistance of the unsaturated soil itself.

Among the works looking into the behaviour of unsaturated soils – steel interface, two are worth mentioning, namely Hamid and Miller (2009) and Liu and Vanapalli (2018). The two works agree on some general conclusions which can be summarised as:

- (i) Suction contributes to the peak interface strength, with a more pronounced effect for rough steel surfaces;
- (ii) After the peak, the resistance decreases sharply and small relative displacements are enough to tend to the residual post-peak strength;
- (iii) The residual strength at the interface is not significantly influenced by suction, compared to the strength of the soil;
- (iv) More than suction, the normal stress acting on the interface determines the shear failure mechanism, with high normal stresses bringing towards a soil-soil failure, rather than soil-interface failure, as already discussed, e.g., by Heerema (1979) or Tsubakihara et al. (1993) for cohesive soils.

The data from Liu and Vanapalli (2018) also suggest that dilatancy plays a key role in the peak strength, with the shear mechanism at the interface being less dilative than the parent soil shear failure. To explain the evidence, besides hypotheses on the reduced role of menisci on the shear mechanism at the interface, the authors quote the results of an extensive experimental investigation on residual strength at high suction performed by Vaunat and coworkers (Vaunat et al., 2006; Vaunat et al., 2007; Merchan et al., 2008). The authors show that decreasing water content tends to aggregate the soil fabric and increase its stiffness, which suggests that increasing suction tends to switch the interface interaction from predominantly “adhesive” to mostly “frictional”. Eventually, it is worthwhile observing that the interface shear mechanism would benefit from water deficit in the soil only if menisci were created on the steel surface during penetration. However, it is reasonable to assume that the relative displacement between the sleeve and the soil would decrease the beneficial effect quite quickly.

The previous results and considerations may explain why suction is clearly beneficial on the tip resistance, while it is less effective on the sleeve friction, as observed with reference to the data in Fig. 15 and Fig. 16.

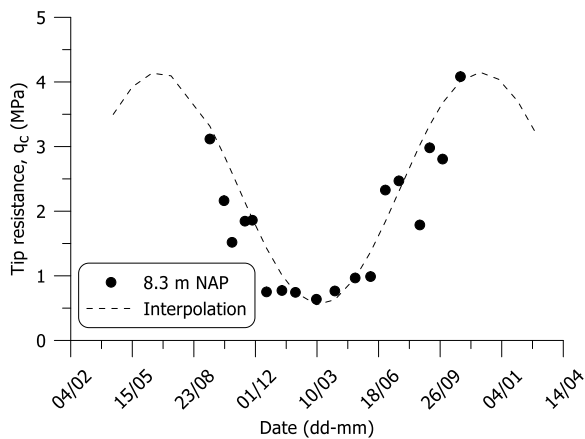
A consequence of the previous results is that the friction ratio,  $F_r$ , must reduce at increasing suction, and explains the rationale behind the elaboration in Fig. 14. It is worthwhile observing that the conclusion remains valid until the failure mechanism develops as interface shear, at moderate normal stresses. For high normal stresses at the interface, if a soil-soil shear mechanism is activated due to interlocking on the rough surface, the conclusion is expected to be no longer valid.

## 6. Discussion and conclusions

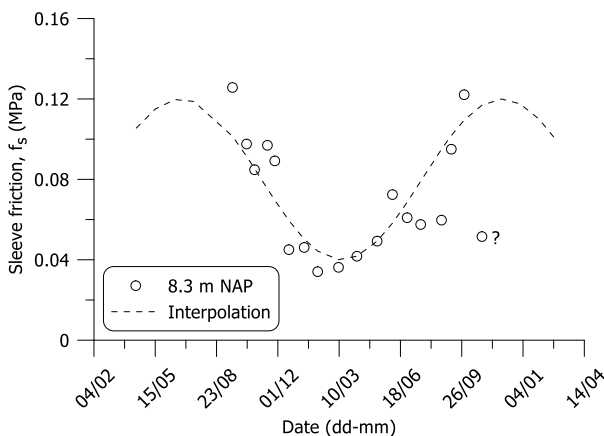
The previous evidence suggests that in surficial unsaturated soils, the variation of friction ratio with suction could be exploited as a proxy for missing measurement of suction. The idea is exemplified in the following, with reference to the current data set.

In Fig. 17 and Fig. 18, the available data for  $q_c$  and  $f_s$  are interpolated with harmonic functions of time, assuming a harmonic seasonal variation of the soil state, as a first approximation. It is worthwhile observing that the chosen functions have no predictive capability at all. Instead, they are blind interpolations, adopted only to the scope of exploring the possibility of better exploiting the entire available data set. The two chosen functions have the same phase. Once more, although realistic, this is only a preliminary assumption, which should be verified by enlarging the database or performing a dedicated experimental investigation.

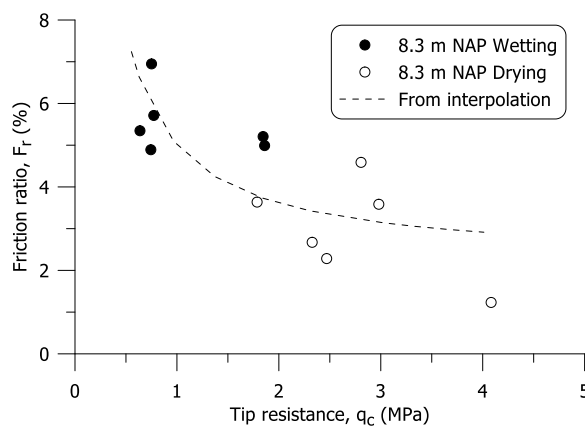
Once the two interpolation functions are fixed, their ratio allows to infer the friction ratio,  $F_r$ , as a function of time, which is plotted in Fig. 19. The comparison of available values from the field investigation and the interpolation function, match reasonably well considering the arbitrariness of the original interpolation functions and their quality in reproducing the data over time. Nonetheless, the approach seems promising, especially in view of an expeditious evaluation in the field.



**Figure 17.** Harmonic interpolation of tip resistance data in the investigated time period



**Figure 18.** Harmonic interpolation of sleeve friction data in the investigated time period



**Figure 19.** Comparison between the friction ratio from field measurement and the predicted one from harmonic interpolation of tip resistance and sleeve friction

For the proposal to become effective in the practice, various steps are still needed. On the one hand, the database should be enriched with more controlled data; on the other hand, a dedicated experimental investigation would be worthwhile. Moreover, the reliability of the derivation should be better proven, especially in view of the dependence of the sleeve friction measurement on the roughness of the cone. Relying on repeated measurements over time would require extremely careful calibration and execution of the tests.

## Acknowledgements

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## References

- Chao, C. Y., S., Muraro, W., Broere, and C. Jommi. “Experimental observation and constitutive modelling of the shear strength of a natural unsaturated soil”. In: *E3S Web of Conferences* 382, 02001, 2023. <https://doi.org/10.1051/e3sconf/202338202001>
- Hamid, T.B., and G.A. Miller. “Shear strength of unsaturated soil interfaces” *Can Geotech. J.*, 46, pp. 595-606, 2009. <https://doi.org/10.1139/T09-002>
- Heerema, E.P. “Relationship between wall friction, displacement velocity and horizontal stress in clay and sand for pile driveability analysis” *Ground Engineering*, 12(1), pp. 55-65, 1979. [https://doi.org/10.1016/0148-9062\(79\)91368-8](https://doi.org/10.1016/0148-9062(79)91368-8)
- Liu, Y., and S.K. Vanapalli. “Testing of unsaturated soil-steel interface shear strength”. *Proc 7th International Conference on Unsaturated Soils (Hong Kong)*, 2018. <https://www.issmge.org/publications/publication/testing-of-unsaturated-soil-steel-interface-shear-strength>
- Lunne, T., and K. H. Andersen. “Soft clay shear strength parameters for deepwater geotechnical design”, In: *Proceedings of the 6th International Offshore Site Investigation and Geotechnics Conference: Confronting New Challenges and Sharing Knowledge*, London, UK, 2007, pp. 151-176.
- Merchán, V., J., Vaunat, E., Romero, and T. Meca. “Experimental study of the influence of suction on the residual friction angle of clays”, In: *Proceeding of the 1st European Conference on Unsaturated Soils, E-UNSAT 2008*, Durham, UK, 2008, pp. 423-428.
- Meter Group, “User Manual Hyprop”, 2015, [Hyprop\\_Manual.pdf\(metergroup.com\)](Hyprop_Manual.pdf(metergroup.com))
- Meter Group, “WP4C Dewpoint Potential Meter Manual”, 2024, [20588\\_WP4C\\_Manual\\_Web.pdf\(metergroup.com\)](20588_WP4C_Manual_Web.pdf(metergroup.com))
- Miller, G. A., N. K., Tan, R. W., Collins, and K. K. Muraleetharan. “Cone penetration testing in unsaturated soils”, *Transportation Geotechnics*, 17, pp. 85-99, 2018. <https://doi.org/10.1016/j.trgeo.2018.09.008>
- Pournaghiazar, M. “The cone penetration test in unsaturated sands”, PhD Dissertation, The University of New South Wales, Australia, 2011. <https://doi.org/10.26190/unsworks/15490>
- Robertson, P. K. “Interpretation of cone penetration tests—a unified approach”, *Canadian Geotechnical Journal*, 46(11), pp. 1337-1355, 2009. <https://doi.org/10.1139/T09-065>
- Romero, E., and C. Jommi. “An insight into the role of hydraulic history on the volume changes of anisotropic clayey soils”, *Water Resources Research*, 44, pp. 1-16, 2008. <https://doi.org/10.1029/2007WR006558>
- Russel, A.R. “Cavity Expansion in Unsaturated Soils”, PhD Dissertation, The University of New South Wales, Australia, 2004. <https://doi.org/10.26190/unsworks/22215>
- Russel, A.R., and N. Khalili. “On the problem of cavity expansion in unsaturated soils”, *Comput. Mech.*, 37: pp. 311–330, 2006. <https://doi.org/10.1007/s00466-005-0672-7>
- Tsubakihara, Y., H., Kishida, and T. Nishiyama. “Friction between cohesive soils and steel”, *Soils and Foundations* 33(2), pp.145-156, 1993. [https://doi.org/10.3208/sandf1972.33.2\\_145](https://doi.org/10.3208/sandf1972.33.2_145)



van Duinen, A. "Shear Strength of Initially Unsaturated Soil. Results measurement sites Westervoort and Oijen", Deltares, Delft, The Netherlands, 2021.

Vaunat, J., C., Amador, E., Romero, and E. Djeran-Maigre. "Residual strength of a low plasticity clay at high suctions". In: Proceedings of the 4th International Conference on Unsaturated Soils, Carefree, Arizona, United States, 2006, pp. 1279-1289. [https://doi.org/10.1061/40802\(189\)105](https://doi.org/10.1061/40802(189)105)

Vaunat, J., V., Merchán, E., Romero, and J. Pineda. "Residual strength of clays at high suctions". In: Theoretical and Numerical Unsaturated Soil Mechanics, 2007, pp. 151-163. [https://doi.org/10.1007/3-540-69876-0\\_17](https://doi.org/10.1007/3-540-69876-0_17)

Young, H., and A.R. Russell. "Cone penetration tests in unsaturated silty sands", Canadian Geotech. J. 53, pp. 431-444, 2016. <https://doi.org/10.1139/cgj-2015-0142>