Reliability assessment of ultra-deep oil and gas wellbore casing using data statistics and numerical simulations

Yang, Shangyu; Zhang, Renren; Wang, Jianjun; Li, Xinhong; Fan, Heng; Yang, Ming

DOI
10.1016/j.jlp.2020.104369

Publication date
2020

Document Version
Final published version

Published in
Journal of Loss Prevention in the Process Industries

Citation (APA)
Yang, S., Zhang, R., Wang, J., Li, X., Fan, H., & Yang, M. (2020). Reliability assessment of ultra-deep oil and gas wellbore casing using data statistics and numerical simulations. *Journal of Loss Prevention in the Process Industries*, *69*, Article 104369. https://doi.org/10.1016/j.jlp.2020.104369

Journal of Loss Prevention in the Process Industries 69 (2021) 104369

Contents lists available at ScienceDirect

Journal of Loss Prevention in the Process Industries

journal homepage: http://www.elsevier.com/locate/jlp

Reliability assessment of ultra-deep oil and gas wellbore casing using data statistics and numerical simulations

Shangyu Yang [a], Renren Zhang [b,*], Jianjun Wang [a], Xinhong Li [b], Heng Fan [c], Ming Yang [d]

[a] *China CNPC Tubular Goods Research Institute, State Key Laboratory of Performance and Structural Safety for Petroleum Tubular Goods and Equipment Materials, Xi'an, 710077, China*
[b] *School of Resources Engineering, Xi'an University of Architecture and Technology, No.13 Yanta Road, Xi'an, 710055, China*
[c] *School of Electronic Engineering, Xi'an Shiyou University, Shaanxi, Xi'an, 710077, China*
[d] *Safety and Security Science Section, Faculty of Technology, Policy, and Management, Delft University of Technology, the Netherlands*

ARTICLE INFO

Keywords:
Reliability assessment
Wellbore casing
Monte-carlo
Data statistics
Numerical model

ABSTRACT

Ultra-deep oil and gas wells have become a new development trend in onshore oil and gas exploitation. However, Ultra-deep oil and gas wellbore casing is with high failure risk due to the harsh environment. It is essential to evaluate the reliability of wellbore casing. This paper assesses the operational reliability of wellbore casing using data statistics and numerical simulation. Firstly, the theoretical model for reliability analysis of wellbore casing is established, and the variables in the model are determined, including rock mechanics, cement ring, and casing string strength factors. Subsequently, considering the random distribution of model variables, many statistics and analyses are performed to determine the distribution parameters of the model variables. Eventually, Monte Carlo based numerical simulations are carried out to obtain the residual strength distribution and the reliability of wellbore casing. The production casing in the ultra-deep well with a depth of 6.5 km in China as an industrial case is used to illustrate the present study. It is observed that this study can be useful to guide a more accurate assessment of the reliability of ultra-deep wellbore casing.

1. Introduction

There are many potential hazard factors in the production process of ultra-deep oil and gas wells, which pose a severe threat to wellbore casing integrity (Feng, 2014; 2020). The decrease of string strength caused by string corrosion may result in the failure of wellbore casing (Zhu et al., 2019), string fracture (Liang et al., 2012), and string wear and deformation (Liu et al., 2017; Qiao et al., 2019). Wellbore casing integrity is the prerequisite for normal production of oil and gas wells, and its failure will lead to severe consequences, e.g., gas release from the wellbore and shutdown. Thus, it is necessary to assess wellbore casing reliability for reducing the production accident risk of ultra-deep oil and gas wells (Li et al., 2019, 2020).

Currently, considerable efforts were made in the field of risk and reliability of oil & gas strings. A reliability model based on the non-probabilistic reliability theory is proposed to study the uncertainty of variables and calculate the reliability of casing under actual load (Xu et al., 2015; Zhao et al., 2017). Abimbola and Khan (2016) presented a novel dynamic risk analysis methodology that is applicable at different stages of drilling operations. Abimbola et al. (2014) conducted a real-time barriers failure probability assessment of offshore drilling operations involving subsurface Blowout Preventer. Abaei et al. (2018) proposed a Bayesian Network-based methodology for reliability assessment of marine floating structure and predicting optimum design point of the mooring system. Sule et al. (2018) presented a reliability assessment of a managed pressure drilling operation by investigating the kick control operation of constant bottom-hole pressure technique of managed pressure drilling. Taleb-Berrouane et al. (2020) developed a hybrid reliability assessment model strengthening SPN with BN capabilities, and it enables the analysis of continuous input data without the necessity of time-slice discretization process. Considering the randomness of variables and their parameter distribution, Zhuo et al. (2018) established the limit state equations to evaluate the reliability of the casing system. Liao et al. (2010) built a risk evaluation method based on the reliability theory to calculate the reliability of the casing. Fan et al. (2016) used a partial coefficient method to develop a design method for casing string reliability during the drilling and completion stages.

To evaluate the reliability of the wellbore casing more accurately, a

---

* Corresponding author.
E-mail address: zhang_renren@126.com (R. Zhang).

https://doi.org/10.1016/j.jlp.2020.104369
Received 29 September 2020; Received in revised form 26 November 2020; Accepted 5 December 2020

large number of data statistics of reliability model variables are carried out. Monte Carlo method is a traditional but accurate reliability analysis method, which can be combined with other evaluation methods easily. He et al. (2018) combined BP neural network with the Monte Carlo method to evaluate the reliability of the gas storage unit. Azarkish and Rashki (2019) conducted a reliability-based sensitivity analysis of a shell and tube heat exchanger using Monte Carlo simulation. Chakraborty et al. (2020) proposed a Monte-Carlo Markov Chain simulation approach for evaluating coverage-area reliability of mobile wireless sensor networks with multistate nodes. Adumene et al. (2020) integrated Bayesian Network – Markov Mixture technique with Monte Carlo simulation for dynamic safety assessment of the assets under the influence of MIC. Talebberrouane et al. (2016) presented a methodology based on Petri net with probabilistic analysis using Monte Carlo simulation for long term availability of safety critical system. Xi (2017) conducted a reliability analysis of the prestressed aqueduct by integrating the Monte Carlo method into finite element code. Wang et al. (2019) applied the Monte Carlo method to analyze the reliability of the subset. Monte Carlo method is used in the present study to evaluate the reliability of wellbore casing in ultra-deep oil and gas wells. The probability distributions of model variables are determined by a large number of data statistic. It can overcome the limitations of the previous studies.

There were a lot of publications reported on risk and reliability of oil and gas facilities. However, these studies mainly focus on the establishment of reliability model or methodology, and the empirical probability distributions of reliability parameters were used in quantitative calculation and assessment. Due to the variation of operational conditions of oil and gas casing, the empirical value of model variables may introduce the high uncertainties in reliability calculation results. Therefore, this paper aims to develop a novel methodology for reliability evaluation of wellbore casing. The uniqueness of the present work is the determination of the model variables by extensive data statistics. A reliability model based on Monte Carlo and finite element method is established. The data reported in the literature and provided by oil companies is used to extract the probability distribution of reliability model variables. Eventually, the methodology is applied to the production casing in the ultra-deep well with a depth of 6.5 km in Changning-Weiyuan of China.

The remainder of the paper is organized as follows: Section 2 presents the proposed methodology framework, including analytical theory and models. Section 3 presents data statistics and analysis of model variables. Section 4 presents a case of an ultra-deep well to tested proposed methodology. Section 4 gives the conclusions of this work.

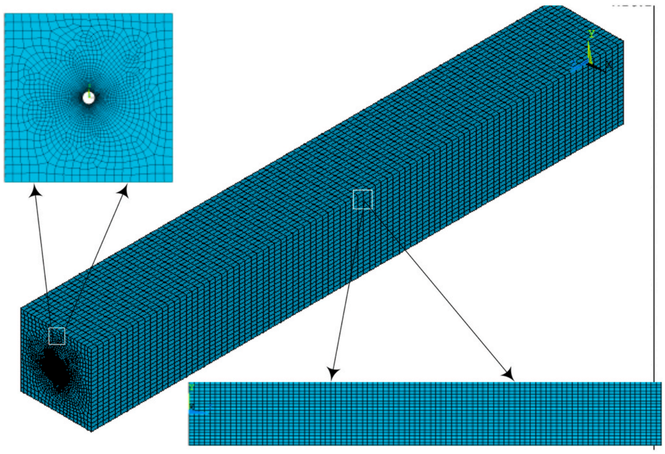


Fig. 1. Finite element model.

2. Reliability model of wellbore casing

The reliability models of wellbore casing in ultra-deep gas and oil well are proposed in this part, which contain a theoretical model based on reliability theory and numerical simulation model. Firstly, the used reliability theory based on the limit-state equation is introduced in detail. Then, the influence variables of wellbore casing are determined, and the reliability theory model is established; Finally, the numerical model of wellbore casing is established, combining the Monte Carlo method with finite element code.

2.1. *Reliability theory*

Reliability refers to the ability or possibility of a component or product to complete a specified task under characteristic conditions (Wang et al., 2019). Generally, the failure risk can be expressed as reliability or failure probability (Guan et al., 2010), and its expression is follows.

$$\begin{cases} P_r = P[Z = g(x_1, x_2, \cdots, x_n) \geq 0] \\ P_f = P[Z = g(x_1, x_2, \cdots, x_n) < 0] \end{cases} \tag{1}$$

where P_r and P_f are success and failure probabilities, respectively; x_1, x_2, …, x_n, are the random affecting variables of wellbore casing.

If the random variables affecting wellbore casing can be expressed as a continuous function, then $Z = g\ (x_1, x_2, \ldots, x_\mathrm{n})$ is the probability distribution function.

$$P_r + P_f = 1 \tag{2}$$

2.2. *The established reliability model*

2.2.1. *Limit state function*

The variables x_i that affect wellbore casing state can be divided into two categories. The first category is the random variable L that affects the external load of wellbore casing. The second category is the random variable S that affects the strength of wellbore casing. The functions of the two types of variables are expressed as follows.

$$\begin{cases} L = L(x_{L_1}, x_{L_2}, \cdots, x_{L_n}) \\ S = S(x_{S_1}, x_{S_2}, \cdots, x_{S_n}) \end{cases} \tag{3}$$

where $x_{L\mathrm{i}}$ is a random variable related to wellbore casing load, and $x_{S\mathrm{i}}$ is a random variable related to wellbore casing strength.

Then, multiple variables can be simplified into two random variables, as shown in Eq. (4).

$$Z = L - S \tag{4}$$

It is assumed that strength and load are two independent random variables, which follow a specific probability distribution. Then the probability density functions are presented by $p_{L(\mathrm{x})}$ and $p_{\mathrm{S(x)}}$, respectively. The reliability of the casing can be expressed as follows:

$$P_r = P(Z > 0) = P(L - S > 0) \tag{5}$$

Based on the probability distribution of external load variables and the performance parameters of string, the reliability model of the string is established according to limit state principle, and the variables include rock mechanics and cement ring factors.

$$Z = g(S_0, P, P_1, P_2, P_3, E_3, \nu_3, E_2, \nu_2, E_1, \nu_1, D, T) \tag{6}$$

where S_0 is #P110 yield strength, MPa; P is internal pressure during drilling and completion, MPa; P_1 is maximum horizontal principal stress at a certain depth, MPa; P_2 is minimum horizontal principal stress at a certain depth, MPa; P_3 is the vertical horizontal principal stress of a certain depth formation, MPa; E_3 is the elastic modulus of a certain depth formation, G; ν_3 is Poisson's ratio at a certain depth; E_2 is elastic modulus of cement, G; ν_2 is Poisson's ratio of cement ring; D_2 is the

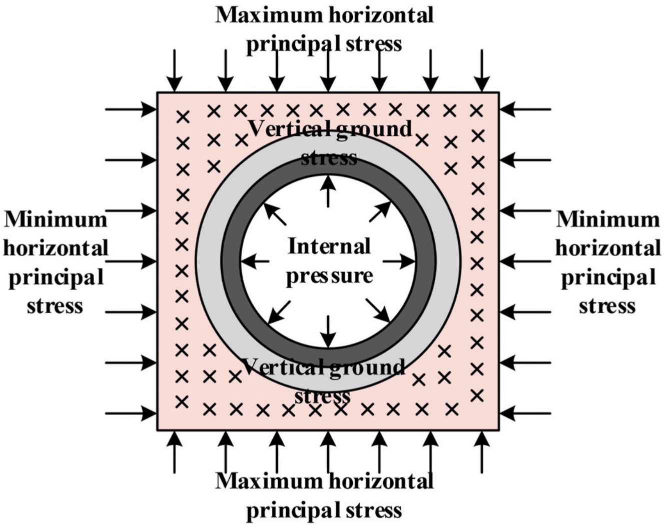


Fig. 2. The simplified model of casing loading.

thickness of cement, mm; E_1 is #P110 elastic modulus, G; ν_1 is #P110 Poisson's ratio; D is #P110 outer diameter, mm; T is #P110 wall thickness, mm.

The string is in a safe state when Z is greater than zero. The occurrence probability of a safe state is the reliability of string, denoted as P_r; The string is unsafe when Z is less than zero. The occurrence probability of an unsafe state is the failure probability of string, denoted as P_f, as shown in Eq. (9).

$$\begin{cases} P_r = P[Z = g(S_0, P, P_1, P_2, P_3, E_3, \nu_3, E_2, \nu_2, E_1, \nu_1, D, T) \geq 0] \\ P_f = P[Z = g(S_0, P, P_1, P_2, P_3, E_3, \nu_3, E_2, \nu_2, E_1, \nu_1, D, T) < 0] \end{cases} \tag{7}$$

2.2.2. *Numerical model*

As a popular finite element code, ANSYS is commonly used in the field of structural reliability analysis. This study uses ANSYS to model the reliability of production casing in the ultra-deep well, including cement and formation, as shown in Fig. 1. In the pre-processing part of the model, the related variables and parameters of formation, cement, and casing are defined by the statistical data or nominal values of API standard. The probability distributions of model variables can be integrated into the finite element model to perform a quantitative calculation. In the grid division part, the mapping grid division is adopted, and the grid density is adjusted. Calculation results show that with the gradual encryption of the grid, the stress value increases first, then remains unchanged, and then decreases. In this paper, the grid density at constant stage stress value is selected to ensure the relative accuracy of results; The common contact surface between the cement and formation is set as surface contact. In the loading part, considering the pressure of cement slurry on casing string during drilling and completion phase, a corresponding internal pressure is exerted on the inner wall of the casing; considering the geological influence to the wellbore, three-dimensional stress of relative depth is applied around the wellbore, which contains the maximum principal ground stress, the minimum principal ground stress and the vertical ground stress. Then the simplified loading diagram is shown in Fig. 2. The environmental loadings are applied at 1000 mm from the wellbore centre.

3. Determining parameters of model variables

The safety of wellbore casing is mainly affected by the external load and string strength. The external load is mainly affected by the formation and cement ring factors. String strength contains Modulus elasticity, Poisson's ratio, Wall thickness and Outer diameter. However, the external load and casing strength are subject to uncertainty and randomness, fluctuating within a specific range. This issue was solved by empirical probability density functions in the previous studies (), although it may not be accurate and precise. Thus, this paper attempts to perform a statistical analysis of external loads and string strength to accurately determine their distribution types and parameters to accurately analyze wellbore casing reliability. The specific variable analysis is as follows.

Table 1
Rock elastic modulus data.

Number	Elastic Modulus (GPa)	Number	Elastic Modulus (GPa)	Number	Elastic Modulus (GPa)	Number	Elastic Modulus (GPa)	Number	Elastic Modulus (GPa)
1	13.36	29	22	57	32.4	85	31.8	113	22
2	18.87	30	13	58	30.1	86	33.3	114	41
3	16.75	31	25	59	20.7	87	6.83	115	11.2
4	20.7	32	23	60	37.2	88	34.5	116	30
5	34.92	33	25	61	31.8	89	37.24	117	33
6	35.42	34	47.8	62	30.3	90	35.76	118	–
7	36.05	35	41.9	63	29.8	91	36.5	119	10.93
8	42.47	36	31.6	64	30.8	92	49	120	21
9	41.41	37	47.7	65	19.7	93	59	121	25
10	37.5	38	9.76	66	15.7	94	21	122	13.36
11	33.36	39	17	67	38.3	95	25	123	27.75
12	42.86	40	9.13	68	23.3	96	22	124	–
13	43.83	41	13	69	26.6	97	27	125	29.56
14	50.38	42	15	70	13.1	98	19	126	42.99
15	39.89	43	39	71	7.31	99	20	127	20.41
16	43.17	44	24	72	13.4	100	22	128	32.12
17	44.33	45	46	73	30.3	101	23	129	42.1
18	48.43	46	12	74	30.1	102	20.7	130	–
19	56.38	47	22	75	20.4	103	24.2	131	41
20	29.3	48	19	76	5.91	104	27.8	132	49
21	34.7	49	36	77	32.1	105	27	133	23.5
22	30.11	50	34	78	19	106	64	134	65.6
23	37.3	51	23.5	79	43.1	107	10.53	135	77
24	18.98	52	49	80	30.34	108	66	136	78.2
25	38.36	53	59	81	49.9	109	36.12	137	80.9
26	37.24	54	21.36	82	44	110	12.51	138	66.94
27	19.18	55	25.4	83	47	111	17.64	139	55.96
28	33.21	56	28.1	84	38.4	112	22.6	140	54

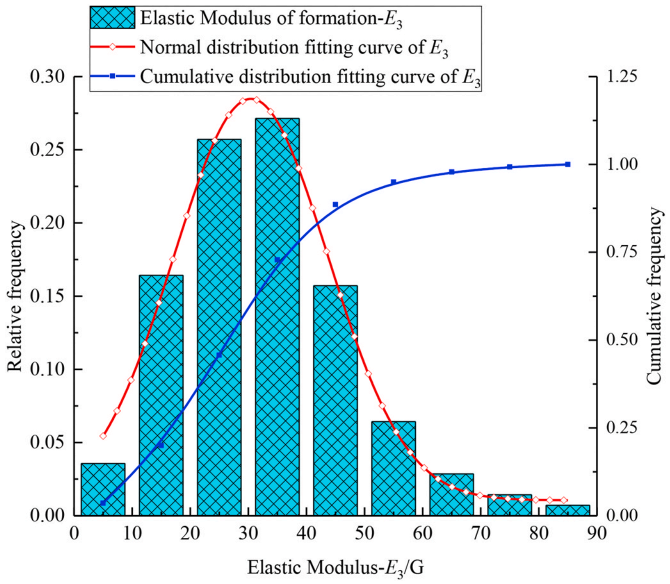


Fig. 3. Probability distribution and fitting curve of shale elastic modulus.

3.1. *Variables of geological rock*

Geological variables affecting wellbore casing are determined by the uncertainty of stratigraphic lithology and random of in-situ stress. This article uses big data statistics and parameter analysis to determine the parameters of the model variables. Firstly, a number of field data about geological variables are collected, then many analyses are performed to obtain data features. Finally, the distribution law of collected data is found, and distribution parameters can be obtained by Gaussian fitting. The data analysis process is as follows.

3.1.1. *Elastic modulus of rock*

As shown in Table 1, 140 elastic modulus data of shale rock are collected by literature and field of Changning-Weiyuan area.

Then, statistical analysis was performed based on the above data. The result shows that the shale rock has a strong elastic modulus variability in Weiyuan-Changning area, with a maximum value of 80.90 G and a minimum value of 5.91 G. Its coefficient of variation is 0.4735, the average value is 35.0210 G, a standard deviation is 15.3661 G. And the 95% mean confidence interval for the level is [29.8856, 35.0210].

Gaussian fitting is performed on the data in Table 1. The fitting curve is shown in Fig. 3. Fig. 3 presents the probability distribution and the fitting curve of shale rock elastic modulus. The normal distribution curve equation of shale elastic modulus by fitting is obtained.

$$P_{E_3} = 0.1052 + \frac{9.1397}{26.5880 \times (\pi/2)^{1/2}} \times e^{-2\times\left(\frac{E_3 - 30.4432}{30.68}\right)^2}, R^2 = 0.9917 \tag{8}$$

Polynomial fitting is performed on the cumulative distribution probability of elastic modulus, and the resulting curve equation is shown as Eq. (9).

$$P'_{E_3} = -0.1701 + 0.03236E_3 - 2.2062{E_3}^2, R^2 = 0.9878, \tag{9}$$

3.1.2. *Poisson's ratio of rock*

As shown in Table 2, 148 Poisson's ratio data of shale rock are found by literature and the field of Changning-Weiyuan area.

Then, the data of Poisson's ratio are statistic and analyzed. The analysis result presents, Poisson's ratio variability of shale in the Weiyuan-Changning area is low, with the maximum value of 0.38, the minimum value of 0.10. Its coefficient of variation is 0.22669; the average value is 0.22; standard deviation is 0.049. The confidence level of 95% is [0.21, 0.22].

Gaussian fitting is performed on the data in Table 2. The fitting results are shown in Fig. 4, presenting the probability distribution. The fitting curve of the shale Poisson's ratio is obtained. The normal distribution curve equation of shale Poisson's ratio by fitting is shown as Eq.

Table 2
Shale Poisson ratio data.

Number	Poisson's ratio	Number	Poisson's ratio	Number	Poisson's ratio	Number	Poisson's ratio	Number	Poisson's ratio
1	0.38	31	0.25	61	0.22	91	0.20	121	0.18
2	0.35	32	0.25	62	0.22	92	0.20	122	0.18
3	0.35	33	0.25	63	0.22	93	0.20	123	0.18
4	0.33	34	0.25	64	0.22	94	0.20	124	0.18
5	0.32	35	0.25	65	0.22	95	0.20	125	0.18
6	0.32	36	0.25	66	0.21	96	0.20	126	0.18
7	0.31	37	0.24	67	0.21	97	0.20	127	0.18
8	0.30	38	0.24	68	0.21	98	0.20	128	0.18
9	0.30	39	0.24	69	0.21	99	0.20	129	0.18
10	0.30	40	0.24	70	0.21	100	0.19	130	0.18
11	0.30	41	0.23	71	0.21	101	0.19	131	0.18
12	0.30	42	0.23	72	0.21	102	0.19	132	0.18
13	0.30	43	0.23	73	0.21	103	0.19	133	0.17
14	0.29	44	0.23	74	0.21	104	0.19	134	0.16
15	0.29	45	0.23	75	0.21	105	0.19	135	0.16
16	0.29	46	0.23	76	0.21	106	0.19	136	0.15
17	0.29	47	0.23	77	0.21	107	0.19	137	0.15
18	0.28	48	0.23	78	0.21	108	0.19	138	0.15
19	0.28	49	0.23	79	0.21	109	0.19	139	0.15
20	0.27	50	0.23	80	0.21	110	0.19	140	0.15
21	0.27	51	0.23	81	0.20	111	0.18	141	0.14
22	0.27	52	0.23	82	0.20	112	0.18	142	0.14
23	0.27	53	0.22	83	0.20	113	0.18	143	0.14
24	0.26	54	0.22	84	0.20	114	0.18	144	0.12
25	0.26	55	0.22	85	0.20	115	0.18	145	0.12
26	0.26	56	0.22	86	0.20	116	0.18	146	0.12
27	0.26	57	0.22	87	0.20	117	0.18	147	0.12
28	0.25	58	0.22	88	0.20	118	0.18	148	0.10
29	0.25	59	0.22	89	0.20	119	0.18		
30	0.25	60	0.22	90	0.20	120	0.18		

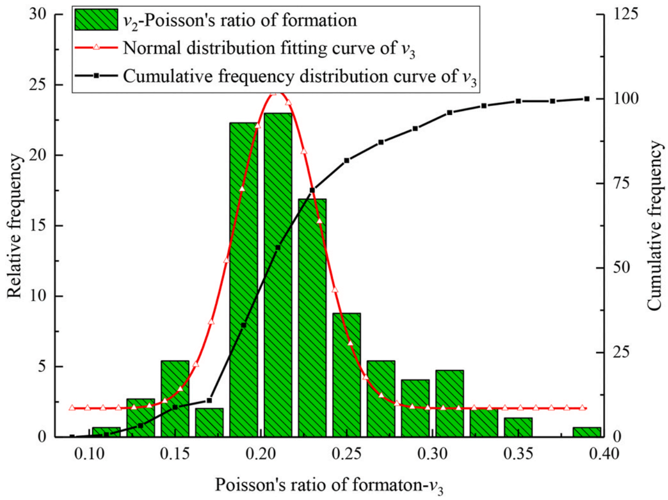


Fig. 4. Shale Poisson's ratio probability distribution and fitting curve.

(10).

$$P_{\nu_3} = 1.937 + \frac{1.38}{0.055 \times (\pi/2)^{1/2}} \times e^{-2\times\left(\frac{\nu_3-0.21}{0.055}\right)^2}, R^2 = 0.9619 \tag{10}$$

A polynomial fitting is performed on the cumulative distribution probability of Poisson's ratio, and the resulting curve equation is obtained.

$$P'_{\nu_3} = -43.744 + 64.778\nu_3 + 3125.314{\nu_3}^2 - 6008.274{\nu_3}^3, R^2 = 0.9741 \tag{11}$$

3.1.3. Three-dimensional stress of formation

Table 3 presents the in-suit stress data. There are large data of three-dimensional stress of formation vary from 1.3 km to 2.4 km.

According to the statistical analysis in Table 3, the in-situ stress relationship at the same depth in the Weiyuan-Changning area is, maximum horizontal in-situ stress > vertical in-situ stress > minimum horizontal in-situ stress. This relationship is in line with the real situation and can support the subsequent in-situ stress prediction.

Then, the data in Table 3 are performed linear fitting analysis, as shown in Fig. 5. And the linear regression equations were obtained as shown following.

$$P_1 = 17.1205 + 0.0197h, R^2 = 0.7259 \tag{12}$$

$$P_2 = 1.4873 + 0.0186h, R^2 = 0.7953 \tag{13}$$

$$P_3 = 1.1942 + 0.0229h, R^2 = 0.9202 \tag{14}$$

3.2. Variables of cement ring

The cement ring is an essential barrier to wellbore components. Its geometric properties contain the outer diameter and thickness; its mechanical properties contain Elastic modulus and Poisson's ratio. According to the statistics of on-site construction data and distribution parameters of cement ring in API, all performance parameters of the cement ring follow the normal distribution, and the distribution parameters are shown in Table 4.

According to the tolerance range of cement ring performance parameters specified by API standard (1994), its normal distribution curves can be obtained by the above table. Taking the Elastic modulus of the cement ring is 109 MPa, Poisson's ratio of 0.25 as an example, its normal distribution curves can be seen in Fig. 6.

3.3. Variables of wellbore casing

The randomness of casing properties is a critical parameter that affects its strength, including geometric parameters (such as outer diam-

Table 3
Ground stress date.

Depth (m)	Maximum ground stress (MPa)	Minimum ground stress (MPa)	Vertical ground stress (MPa)	Depth (m)	Maximum ground stress (MPa)	Minimum ground stress (MPa)	Vertical ground stress (MPa)
1300	–	–	40	2420	76.3	55.14	61.96
1464	–	26	–	2422	70.13	52.94	63
1520	40.4	28.03	34.96	2424	82.97	57.45	62
1520	55.78	32.51	34.96	2428	60	58.16	64
1525	47.12	29	35.08	2441.63	75	45	57
1525	47.69	28.975	30	2479.59	71.758	53.525	63.488
1525	47.123	28.975	35.075	2496	59	50	60
1526	47.153	29.994	35.098	2580	60	55	50
1530	48.17	30.19	37.96	2598.795	66.476	48	60.433
1530	48.17	30.19	37.96	2627.51	–	49.193	
1545	48.09	32.89	35.54	2627.51	66.48	49.19	60.43
1545	41.73	26.98	35.54	2675	78.79	49	61.525
1545	49.46	27.89	35.54	2675	76.2	48.55	61.525
1545	48.11	33.5	35.54	2686.59	67.97	46.309	61.792
1575	48	29	35	2686.59	67.97	46.31	61.79
1580	47.08	37.13	31.28	2723.85	68.913	46.85	62.649
1580	49.57	30.49	38.75	2723.85	68.91	46.85	62.65
1600	48.17	30.19		2855	72.23	49.11	65.67
1745	48	–	35	2855	72.23	49.11	65.67
1925	63	32	–	2950	65	45	
1925	63	32	–	3025	–	46.7	
2000	46	35	48	3031	–	50	
2039	40	45	–	3178	70	54	
2177	48	29	45	3200	–	–	66
2203	45.4	34.9	–	3200	–	–	66
2300	–	–	55	3356.5	91.63	73.6	81.81
2300	–	–	56	3488	93.72	76.56	85.32
2325.78	67.307	50.195	59.55	3500	–	–	75
2348	62.1	47.7	–	3519	88.3	69.6	
2385.25	69.028	51.478	61.072	3600	76.3	70	
2397	79.6	55.7	–	3700	–	–	90

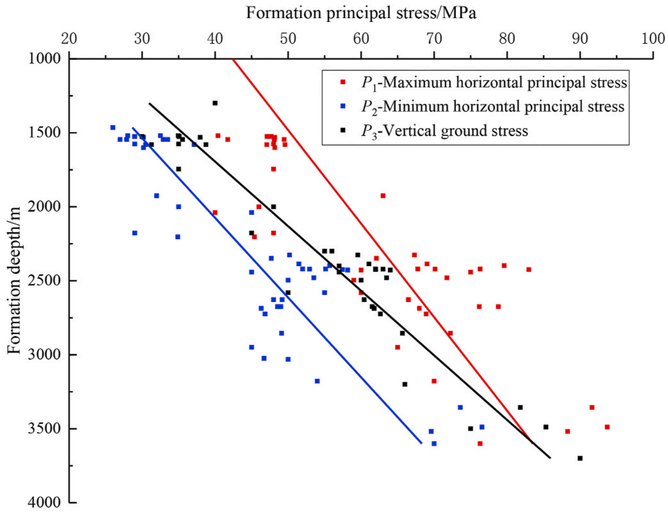


Fig. 5. Three-dimensional stress distribution regulation of formation.

Table 4
Random distribution of variables parameters about cement ring.

Parameter names	Mean value/Nominal value	Coefficient of variation
Elastic Modulus	1	0.1
Poisson’s ratio	1	0.025
Outer meter	1.005	0.01
Thickness	1	0.018

eter and wall thickness) and mechanical parameters (such as elastic modulus, Poisson’s ratio, minimum yield strength). The nominal values of outer diameter and a wall thickness of casing were stipulated by API standard, and the yield strength of casing at different rigidities was specified with a minimum value. That is, the default elastic modulus is 207 G, and Poisson’s ratio is 0.3. Due to manufacturing, transportation and other reasons, the actual string attribute value does not equal the nominal absolute value. There is a specific deviation. Galambos and Ravindra (1973) conducted a statistical analysis of the same string parameters and found that string mechanical parameters all obey the normal distribution. The normal distribution function is as follows.

$$f(x)=\frac{1}{\sqrt{2\pi}\sigma_x}\exp\left[-\frac{(x-\mu_x)^2}{2\sigma_x^2}\right], Cov=\frac{\sigma_x}{\mu_x} \tag{15}$$

where x is attribute parameters of string, σ_x is their mean square error, μ_x is mean value, and Cov is and coefficient of variation.

The random distribution and parameters of string were obtained by API standard, as Table 5. Then the normal distribution curves of elastic modulus, Poisson’s ratio and yield strength of P110 casing can be drawn, as Fig. 7.

4. Reliability calculation of wellbore casing

Taking the anonymous gas well with a depth of 6500 m in the Changning-Weiyuan block as an example, the reliability of wellbore casing and the sensitivity of the variables were studied. Through the statistics and calculation of in-situ stress, three-dimensional stresses of formation are 135.32 MPa, 124.19 MPa, and 113.09 MPa, respectively. According to the actual drilling fluid density, the internal pressure of wellbore casing is 79.62 MPa. Wellbore casing thickness is 12.65 mm. Moreover, the yield limit of the string is 828 MPa. According to the actual project, the outer diameter of cement ring is 241 mm. The stress and strain of the output string are obtained by simulation, as shown in Fig. 8.

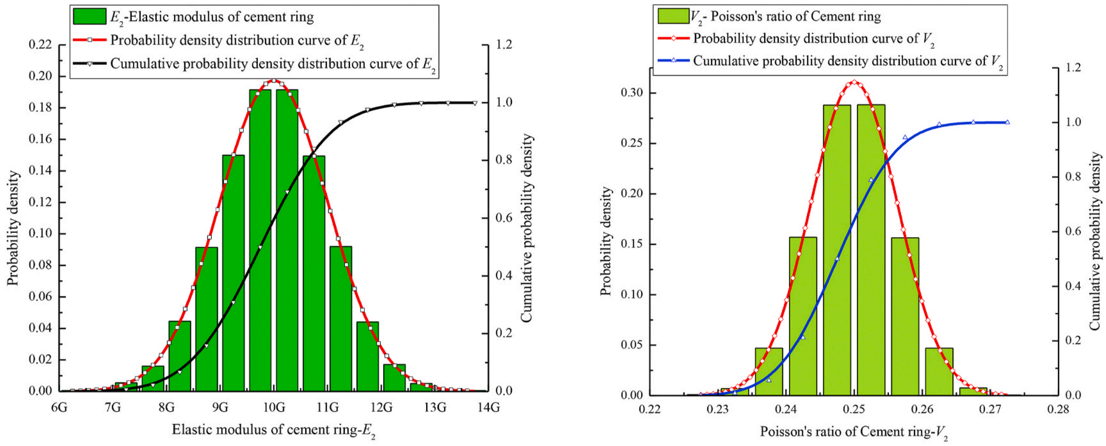
Fig. 8 shows that this model has good convergence and can simulate the stress of wellbore casing string under actual working conditions. The reliability calculation of wellbore casing and variables sensitivity analysis are as follows.

4.1. Reliability calculation of wellbore casing

According to the string strength calculation standard, the safety factors of casing string at different depths are analyzed and calculated in the Changning-Weiyuan area. The lookup table shows that the rated tensile strength of #P110 is 5149.3 KN and its safety design factor is range from 1.6 to 1.8; the rated compressive strength of #P110 is 94.423 MPa, and its safety design factor is range from 1 to 1.35; the

Table 5
Random distribution of variables parameters about casing mechanical properties.

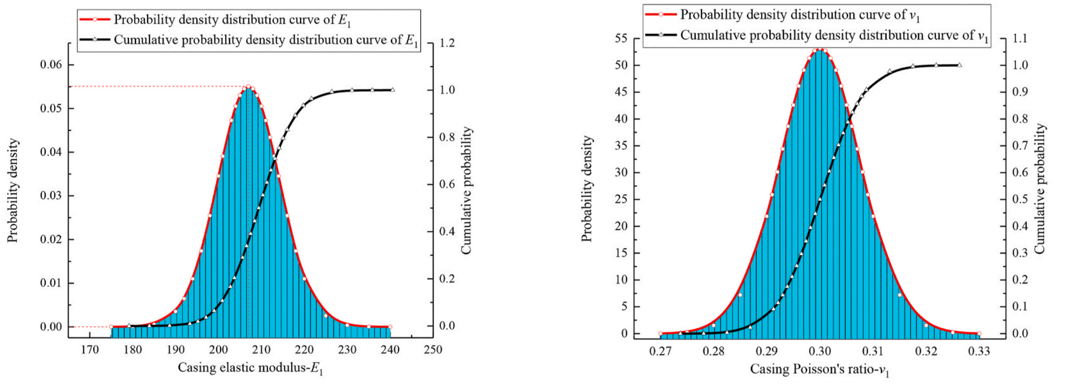
Casing performance	Elastic Modulus	Poisson’s ratio	Yield Strength	Outer meter	Thickness
Mean value/ Nominal value	1.00	1.00	1.09	1.0025	1.00
Coefficient of variation	0.035	0.025	0.022	0.0019	0.31



(a) Normal distribution diagram of elastic modulus

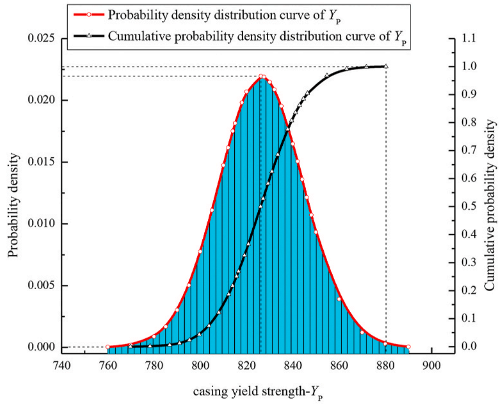
(b) Normal distribution diagram of Poisson's ratio

Fig. 6. Random distribution of performance parameters of cement ring.



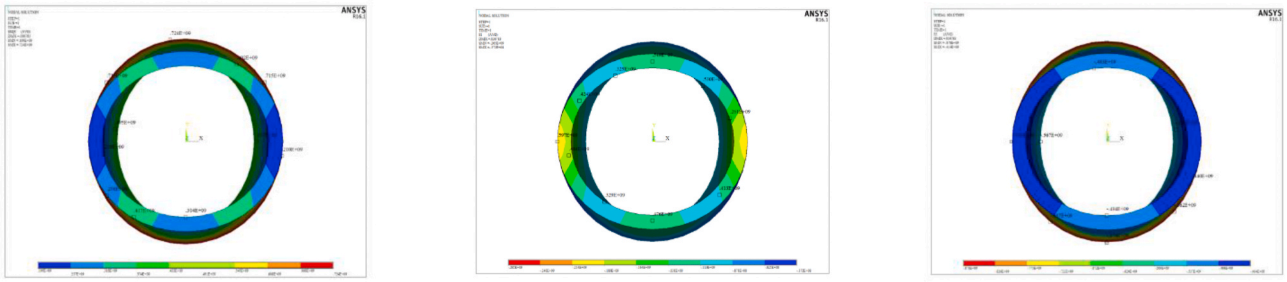
Normal distribution curve of Elastic modulus of P110 casing

Normal distribution curve of Poisson's ratio of P110 casing



Normal distribution curve of Yield Strength of P110 casing

Fig. 7. Random distribution curves of casing mechanical properties.



(a) Stress-strain cloud diagram

(b) Compressive stress-strain

(c) Tensile stress-strain

Fig. 8. Stress and strain cloud diagram of casing string.

rated crush strength of #P110 is 89.8 MPa, and its safety design factor is range from 1 to 1.125. With reference to API RP 2 A-WSD-2014 standard (2014) and pipe strength design data (2005), the calculation formula of pipe strength can be obtained.

The calculation formulas of actual tensile strength and safety factor are as follows.

$$P_t = 0.85 \times \rho_1 g h \times \frac{1}{1000}$$
$$n_t = \frac{P_t}{P_T} \tag{16}$$

The calculation formulas of actual compressive strength and safety factor are as follows.

$$P_{in} = 0.00981 \times (\rho_2 - \rho) h$$
$$n_{in} = \frac{P_{in}}{P_{IN}} \tag{17}$$

The calculations formula of actual crushing strength and safety factor are as follows.

$$P_c = 0.00981 \times (1 - k) \rho_2 h$$
$$n_c = \frac{P_c}{P_C} \tag{18}$$

where n_t, n_{in} and n_c present safety factor of string tensile, internal pressure and crushing strength, respectively; P_T, P_{IN} and P_C present specified string tensile, internal pressure and crushing strength, KN/MPa, respectively; P_t, P_{in} and P_c present actual string tensile, internal

Table 6
Calculated safety factor of casing strength at different depths.

Depth/km	Safety factor			
	Tensile	Compressive	Crushing strength	Yield strength
1	11.87	7.98	20.09	4.62
2	5.90	3.99	10.04	3.28
3	3.90	2.66	6.70	2.37
4	2.97	2.00	5.02	1.87
4.5	2.64	1.77	4.46	1.69
5	2.37	1.60	4.02	1.53
5.5	2.16	1.45	3.65	1.42
6	1.98	1.33	3.34	1.31
6.5	1.82	1.23	3.09	1.20
7	1.70	1.14	2.87	1.11

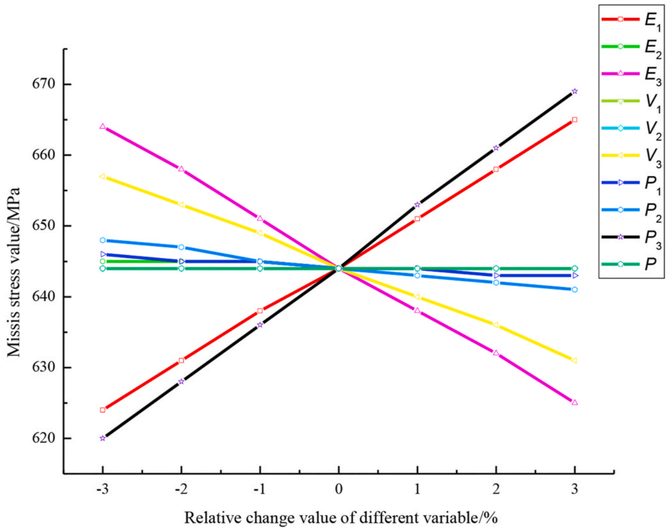


Fig. 9. Curves of variables and strings stress.

pressure and crushing strength, KN/MPa, respectively; ρ_1, ρ_2 and ρ are the density of string, cement slurry and formation water, respectively, g/cm^3; g is the gravity acceleration, Kg/N; h is the depth of formation, m.

According to formulas (16) to (18), the string safety factors of tensile strength, internal pressure, crushing strength and yield strength at different formation depths are calculated, respectively. And the safety factor of the casing string is shown in Table 6.

Based on the API standard about the string design and considering the high formation pressure of drilling and production conditions in ultra-deep gas wells, the design safety factors of crushing strength, tensile safety factor and compressive are 1.125, 1.8 and 1.20, respectively. It can be seen from Table 6 that the formation depth satisfying the safety factor of string strength is 6.5 km. In order to more intuitively analyze the relationship of string safety factor and formation depth, the change curves of string safety factor were drawn, as shown in Fig. 11. From the figure, we can see that the strength safety factor of string decreases as the formation depth increases. Moreover, the string value of

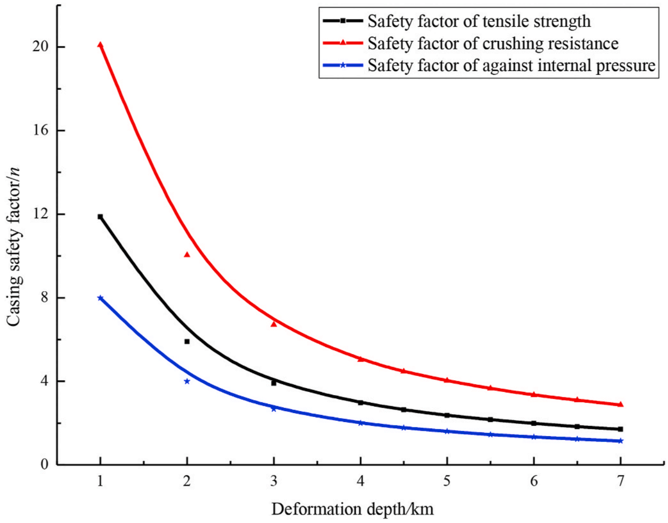


Fig. 11. Strength safety factors in different depth.

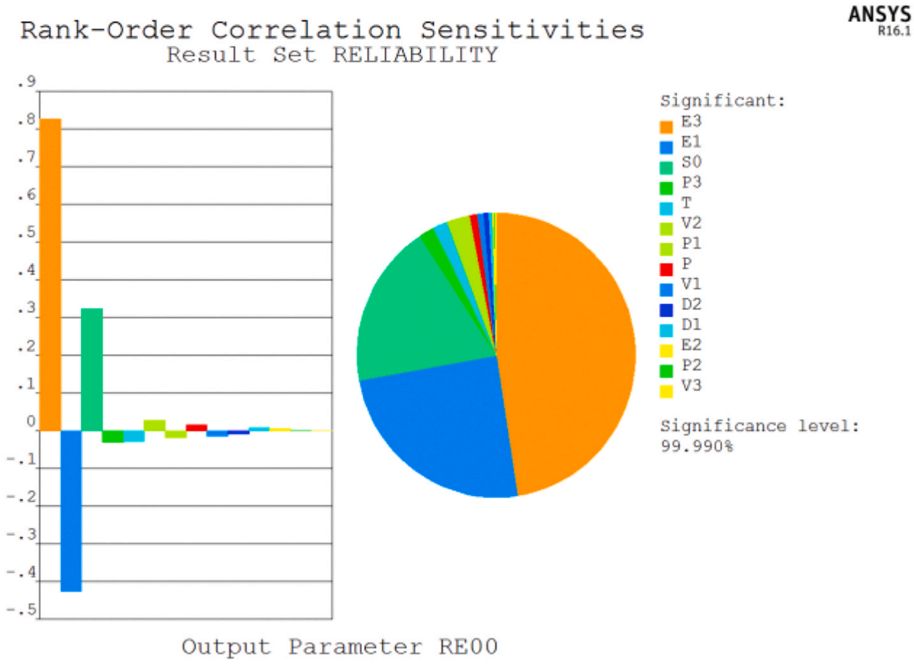


Fig. 10. Sensitivity analysis of different variables.

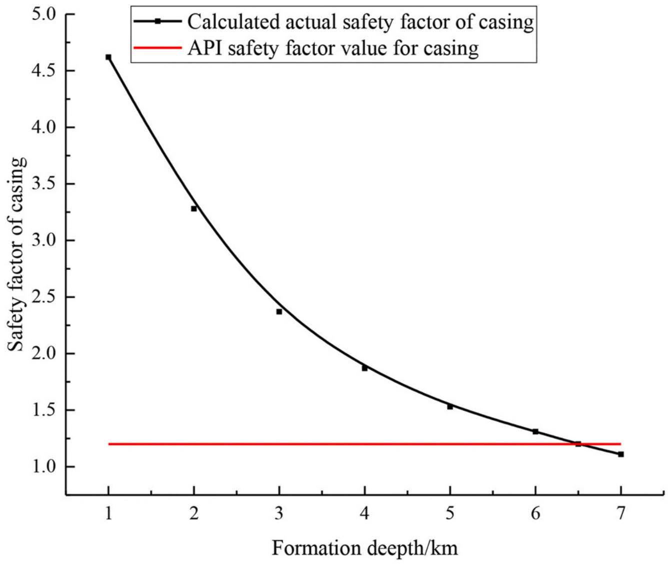


Fig. 12. Yield safety factor in different depth.

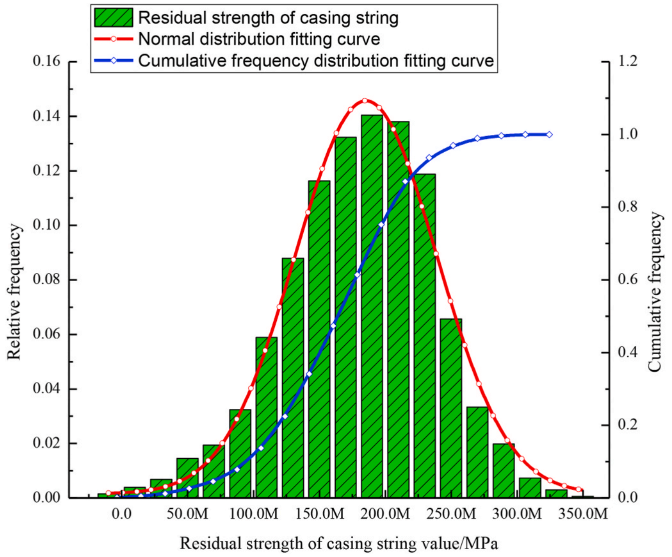


Fig. 13. Normal distribution and fitting curve of remaining strength of strings.

the crushing strength safety factor is greater than the tensile safety factor, which is greater than the compressive safety factor.

The change curve of formation depth and string yield safety factor was obtained by numerical simulation, as shown in Fig. 12. According to API standard, the safety factor of string yield is 1.20, and the formation depth meeting the safety factor of string strength is about 6.5 km. Then, the formation depth of 6.5 km is taken for detailed analysis to determine the variables value and their distribution parameters. Next, inputting them into the reliability model of string and taking multiple Monte Carlo simulations, then the residual strength and distribution of string at 6.5 km are obtained as follows.

Fitting the residual strength of casing string in Fig. 13, then its normal distribution and cumulative frequency fitting formula can be obtained, as following equations.

Table 7
Confidence intervals for strings residual strength and safety factors in different levels at 6.5 km.

Confidence/ %	Intensity interval/ MPa	The value range of safety factor	Reliability index/%
90	[160.408 , 200.912]	[1.240 , 1.320]	99.9233
91	[159.794 , 201.526]	[1.239 , 1.322]	99.9198
92	[159.181 , 202.139]	[1.238 , 1.323]	99.9158
93	[158.444 , 202.876]	[1.237 , 1.324]	99.9114
94	[157.585 , 203.735]	[1.235 , 1.326]	99.0626
95	[156.603 , 204.717]	[1.233 , 1.328]	99.9002
96	[155.498 , 205.821]	[1.231 , 1.331]	99.8928
97	[154.026 , 207.294]	[1.229 , 1.334]	99.8832
98	[152.062 , 209.258]	[1.225 , 1.338]	99.8697
99	[148.993 , 212.326]	[1.220 , 1.345]	99.8466

$$P_{S_0} = 0.00157 + \frac{1.94135 \times 10^7}{1.07482 \times 10^8 \times (\pi/2)^{1/2}} \exp\left[-2\frac{\left(S_0 - 1.85295 \times 10^8\right)^2}{1.17226} \right], R^2 = 0.9877 \tag{19}$$

$$P_{S_0}{}' = -2.60443 \times 10^9 S_0 + 4.8519 \times 10^{-17} {S_0}^2 - 9.47204 \times 10^{-26} {S_0}^3, R^2 = 0.9932, \tag{20}$$

Fitting coefficient R^2 is about 99%, indicating that the curve fitting effect is better, which can be used as a reference for calculating the reliability of string in 6.5 km. Further, the confidence interval of remaining strength and the range of string safety factors at a depth of 6.5 km is obtained at confidence level ranging from 90% to 99%, as shown in Table 7.

The above table shows that the higher the confidence level, the greater the range of string safety factor, but the reliability index decreases, that is, the reliability decreases. And the safety factor of string ranging from 90% to 99% confidence level is from 1.220 to 1.345. Referring to API standard, the string yield safety design factor is 1.20, then the casing string at 6.5 km can meet safety design requirements for strings of ultra-deep oil and gas wells in the Changning-Weiyuan area.

4.2. Sensitivity analysis of model variables

In this paper, the single variable method is used to analyze the factors affecting the reliability of the casing string. Furthermore, based on the research of a 1% variation of the variable value, the relationship between all variables and the strength of string is drawn to analyze variable sensitivity. In addition, the output sensitivity of model variables by numerical simulation is shown in Fig. 10.

Fig. 9 shows that model variables of cement ring have little effect on the change of string strength, that is, the sensitivity of E_2 and V_2 is very low; the variables related to formation have a great influence on the change of string strength, i.e., the sensitivity of E_3 and V_3 is very high; the string elastic modulus E_1 and the formation vertical principal stress P_3 have a great influence on the change of string strength, namely their sensitivity are very high. The formation principal stress P_1, P_2 has a slight change on string strength, namely their sensitivity is low; string internal pressure P and Poisson's ratio V_1 has little effect on the change of string strength, and the sensitivity is very low. The analysis results are consistent with the software simulation results, and it can also be obtained from Fig. 10 that wall thickness *T* has a slight effect on the string, and its sensitivity is low.

5. Conclusion

This paper uses data statistic and FEM simulation to conduct reliability assessment of wellbore casing reliability in the ultra-deep oil and gas well. The variables distribution parameter in the theoretical model of reliability is determined by data statistics. The numerical simulation based on Monte Carlo is established to calculate the wellbore casing reliability of ultra-deep well and obtain the sensitive variables affecting wellbore casing reliability. The most advantage of the proposed methodology is obtaining data of model variables more realistic and calculating string reliability more accurately.

A real field case study is presented to illustrate the proposed methodology. Thirteen model variables affecting casing reliability belong to the normal distribution by statistical analysis of large data and API standard, including the influencing factors of the casing, cement ring, and formation. Then the reliability of wellbore casing and variables sensitivity was obtained by the Monte Carlo method and single variable method respectively, as follows.

- Through reliability analysis, the residual strength and its parameter distribution of casing string are obtained. Further, the safety factor and its value range of casing string in different confidence intervals are calculated, ranging from 1.220 to 1.345 at 6.5 km. The reliability of case string can meet the requirements of the safety design of casing string, and the casing string is safe at a depth of 6.5 km.
- Through variables sensitivity analysis, diameter-thickness ratio, cement thickness, string elastic modulus, Poisson's ratio, and formation, three-dimensional stress are inversely proportional to string stress. That is, the above variable factors are required to control its quality from the source and focus on their monitoring in production to reduce the failure probability of wellbore casing.

Case analysis results show that the proposed methodology can be applied to the reliability analysis of wellbore casing more accurately. Moreover, it can provide extensive data and theoretical support for other ultra-deep oil and gas well. However, this methodology also has certain limitations, which requires more time and effort in the formation data statistics module. Thus, the focus of the next step work should be to establish a database of in-situ stress distribution in different oil and gas well production areas to facilitate the establishment of reliability models.

Acknowledgement

The project is supported by National key Technology R&D Program of China (2019YFF0217504), Special Scientific Research Project of Education Department of Shaanxi Province, China (20JK0729; 18JK0608), National Natural Science Foundation of China (U1762211), CNPC Scientific Research and Technology Development Project (2020B-4020) and Key research and development project in Shaanxi Province (2018GY-172).

References

Abimbola, M., Khan, F., 2016. Development of an integrated tool for risk analysis of drilling operations. Process Saf. Environ. Protect. 102, 421–430.

Abimbola, M., Khan, F., Khakzad, N., 2014. Dynamic safety risk analysis of offshore drilling. J. Loss Prev. Process. Ind. 30, 74–85.

Abaei, M.M., Abbassi, R., Garaniya, V., Chai, S., Khan, F., 2018. Reliability assessment of marine floating structures using Bayesian network. Appl. Ocean Res. 76, 51–60.

Azarkish, H., Rashki, M., 2019. Reliability and reliability-based sensitivity analysis of shell and tube heat exchangers using Monte Carlo Simulation. Appl. Therm. Eng. 159, 113842.

Adumene, S., Khan, F., Adedigba, S., 2020. Operational safety assessment of offshore pipeline with multiple MIC defects. Comput. Chem. Eng. 106819.

American Petroleum Institute, 1994. API Bulletin 5C3-1994. Bulletinon Formulas and Calculations for Casing, Tubing, Drill Pipe and Linepipe Properties. API Production Department, Dallas.

Api RP 2A-WSD, 2014. Recommended Practice for Planning, Designing and Constructing Fixed Offshore Platforms, Working Stress Design. API, USA.

Chakraborty, S., Goyal, N.K., Mahapatra, S., Soh, S., 2020. A Monte-Carlo Markov chain approach for coverage-area reliability of mobile wireless sensor networks with multistate nodes. Reliab. Eng. Syst. Saf. 193, 106662.

Feng, Y., Fu, A., Wang, J., Wang, P., Li, D., Yin, C., Liu, H., 2020. Failure control and integrity technologies of tubing/casing string under complicated working conditions: research progress and prospect. Nat. Gas. Ind. 40 (2), 106–114.

Feng, Y., Han, L., Zhang, F., Bai, Z., Liu, W., 2014. Research progress and prospect of oil and gas well tubing string integrity technology. Nat. Gas. Ind. 34 (11), 73–81.

Fan, H., Yan, X., Feng, Y., Wang, J., Yang, X., Wang, P., 2016. Design method of practical reliability of casing based on partial coefficient method. J. China Univ. Petrol. 37, 807–814, 06.

Guan, Z., Yan, L., Ma, G., 2010. Assessment method of casing failure risk based on structure reliability theory. Acta Pet. Sin. 31, 161–164, 01.

Galambos, T.V., Ravindra, M.K., 1973. Tentative load and resistance factor design criteria for steel buildings. Washington University, St. Louis (MO). Research report no. 18.

He, L., Li, Z., Li, J., Wen, K., Yu, W., Gong, J., 2018. Reliability evaluation of gas storage unit based on BP neural network and Monte Carlo method. Oil Gas Storage Transp. 37 (10), 1104–1113.

Liang, E., Li, Z., Zhao, J., 2012. Model for collapsing strength calculation of worn casing. Petrol. Drill Tech. 40, 41–45, 02.

Liu, W., Tao, C., Wan, Y., 2017. Numerical simulation of deformation failure mechanism of horizontal fracturing casing in tight oil reservoir. Petrol. Sci. Bullet. 466–477, 02 (04).

Li, X., Han, Z., Zhang, R., Abbassi, R., Chang, D., 2020. An integrated methodology to manage risk factors of aging urban oil and gas pipelines. J. Loss Prev. Process. Ind. 104154.

Li, X., Chen, G., Khan, F., Xu, C., 2019. Dynamic risk assessment of subsea pipelines leak using precursor data. Ocean. Eng. 178, 156–169.

Liao, H., Guan, Z., Yan, Z., 2010. Casing failure risk assessment method based on reliability theory. J. China Univ. Petrol. 31, 161–164, 01.

Qiao, L., Tian, Z., Zeng, B., Yang, H., Fu, P., 2019. Multi-factor coupling casing deformation for shale horizontal wells. Fault-Block Oil Gas Field 26, 107–110, 01.

Sule, I., Khan, F., Butt, S., Yang, M., 2018. Kick control reliability analysis of managed pressure drilling operation. J. Loss Prev. Process. Ind. 52, 7–20.

Taleb-Berrouane, M., Khan, F., Amyotte, P., 2020. Bayesian Stochastic Petri Nets (BSPN)-A new modelling tool for dynamic safety and reliability analysis. Reliab. Eng. Syst. Saf. 193, 106587.

Talebberrouane, M., Khan, F., Lounis, Z., 2016. Availability analysis of safety critical systems using advanced fault tree and stochastic Petri net formalisms. J. Loss Prev. Process. Ind. 44, 193–203.

Wang, Z., Broccardo, M., Song, J., 2019. Hamiltonian Monte Carlo methods for Subset Simulation in reliability analysis. Struct. Saf. 76, 51–67.

Xi, X., 2017. Reliability analysis of prestressed aqueduct based on Monte Carlo method in ANSYS. Sichuan Build. Mater. 43 (12), 173–174.

Xu, L., Zhou, J., Li, J., Jiang, S., 2015. A new model for reliability analysis of casing strength. China Petrol. Mech. 43 (11), 41–45.

Zhao, L., Yan, Y., Han, W., Zhao, L., Yan, X., 2017. Reliability assessment of casing strength based on interval model. J. Saf. Sci. Technol. 13, 63–67, 06.

Zhuo, Z., Wu, Z., Yue, J., 2018. Reliability assessment method of casing in active wells based on superposition of multiple factors. J. Chongqing Univ. Sci. Technol. (Nat. Sci. Edn.) 20, 37–40, 04.

Zhu, S., Li, J., Fu, A., Chen, Y., Du, M., 2019. Research progress on corrosion failure and prevention of casing damage during the production of oil and gas. Surf. Technol. 48, 28–48, 05.