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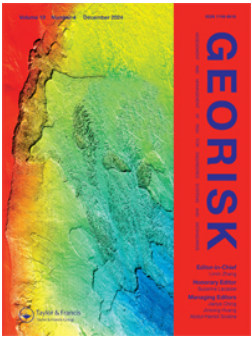
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# Identifying unaccounted capacity of ground anchors through Bayesian updating: a case study

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## ABSTRACT

Ground anchors are crucial components in various construction and engineering applications. They play a critical role in retaining structures and, therefore, design guidelines have established the necessity of comprehensive testing campaigns to derive the anchor characteristic resistance. The latter is a specified percentile within a presumed statistical distribution. In principle, a limited number of investigation tests cannot be used to estimate the characteristic values. To overcome this limitation, in a simplified way, the design codes suggest reducing the resistance found in experimental results by a factor to estimate the anchor characteristic resistance to be used in the design. In this paper, the authors propose a new approach for interpreting ground anchor test results and determining the statistical distribution of ground anchor resistance. The approach is based on the use of Bayesian updating, formulated as a structural reliability problem, and on the definition of a simplified phenomenological model relating the imposed load and the measured anchor (creep) displacements. This distribution can be used to determine a “proven” anchor characteristic resistance, which can then be used to update the anchor design.

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## KEYWORDS

Ground anchors; field testing; Bayesian updating; structural reliability; creep

## 1. Introduction

Ground anchors are essential components in various construction and engineering applications, especially in the case of retaining structures. Recognising the critical function of anchors, design guidelines have established the necessity of comprehensive testing for ground anchors (e.g. EN 1997; CEN 2004, and FHWA-IF-99-015; Sabatini, Pass, and Bachus, Robert 1999).

While the guidelines may use different terminologies and test procedures, the basic philosophy of the main type of tests remains the same: a very limited number of anchors (2-3) must be tested to failure for each distinct set of ground conditions/working loads (“investigation tests” according to the EN 1997 terminology), whereas ideally, all anchors should be tested for acceptability, i.e. they should demonstrate the ability to sustain a given proof load (“acceptance tests” according to EN 1997).

The deformations and the stability of geotechnical structures are affected by uncertainties in factors such as geometrical parameters, material properties, soil spatial variability and loads (Guo et al. 2023; Kormi et al. 2019; Li, Zhang, and Yuan 2024; Zhang et al.

2022). To account for these uncertainties, the current design approaches rely on the use of characteristic values (e.g. EN 1990; CEN 2002). These characteristic values, in principle, represent specified percentiles within a presumed statistical distribution of a given quantity (in this case the anchor resistance). In this perspective, the limited number of investigation tests, do not allow to derive reliable statistical distributions, and in theory cannot be used to estimate the characteristic values using simple frequentist statistics. Pragmatically, to derive the anchor characteristic resistance, the design codes prescribe to reduce the experimental results (either the minimum or the average depending on the number of tests) by a factor.

In this paper, the authors propose a methodology that allows to derive the statistical distribution of ground anchor resistance without requiring a large number of investigation tests. This distribution can be used to determine an anchor characteristic resistance, which can then be used to update the anchor capacity evaluation. If the approach reveals unaccounted capacity, this can be used, for example, to reduce the grouted length of the anchor or to increase the anchor spacing, implying the reduction of costs and raw material consumption, while granting the anchor

reliability. Alternatively, in the context of existing structures, this unaccounted capacity could be used to change the use of retaining structures (e.g. applying larger loads) or to extend the residual service life.

The proposed methodology is based on “Bayesian updating with equality information” as developed by Straub (2011). Therefore, it requires the use of a model capable of reproducing the anchor response and the probabilistic representation of the model parameters. Then, Bayesian inference is used to evaluate the posterior probability density function of the model, given measurement data (Beck and Katafygiotis 1998 and Katafygiotis and Beck 1998). The model posterior allows to derive the distributions of geotechnical anchor resistance (grout-soil interface failure) and, from that, the characteristic value.

The methodology will be applied in relation to a case study described in Section 2. In Section 3 the anchor response model is introduced and Bayesian updating is used. Finally, in Section 4 the results coming from the Bayesian updating will be critically compared with the ones derived from the standard design practice.

## 2. Case study

For the case study, two different campaigns of ground anchor investigation tests performed in two different locations (Site 1 and Site 2) in the Port of Rotterdam will be discussed. For the sake of clarity, the following subsections will provide some indication on: (i) the geotechnical conditions of the two sites (Section 2.1), (ii)

details of the tests (Section 2.2) and (iii) test results (Section 2.3).

### 2.1. Site description

To characterise the sites, in both Site 1 and Site 2 a series of cone penetration tests (CPTs) was performed. The results, in terms of variation of cone tip resistance ( $q_c$ ) along depth are reported in Figure 1 (Figure 1(a,b) refer to Site 1 and 2, respectively). In both cases, the CPT results highlight the presence of two granular soil layers separated by a fine-grained soil layer. The results in each site are rather repeatable, highlighting that in the considered sites the spatial variability in the soil properties is limited.

### 2.2. Test methodology

The details of the tests, in terms of anchor type geometry and proof load ( $P_p$ ), are reported in Table 1. The value of proof load was related to the expected geotechnical resistance of the anchor grout body and estimated by using the  $q_c$ -based design approach proposed in the Dutch code CUR 166. In all the cases, the grouted length is positioned, below approximately 25 m depth, in the deep granular soil layer (“Kreftenheye formation”), as is shown in Figure 1 (the black rectangles represents the anchor grouted body).

By following the procedure described in EN-ISO 24477-5, the load ( $P$ ) is applied in cycles of increasing amplitude. At the maximum load of each cycle, the

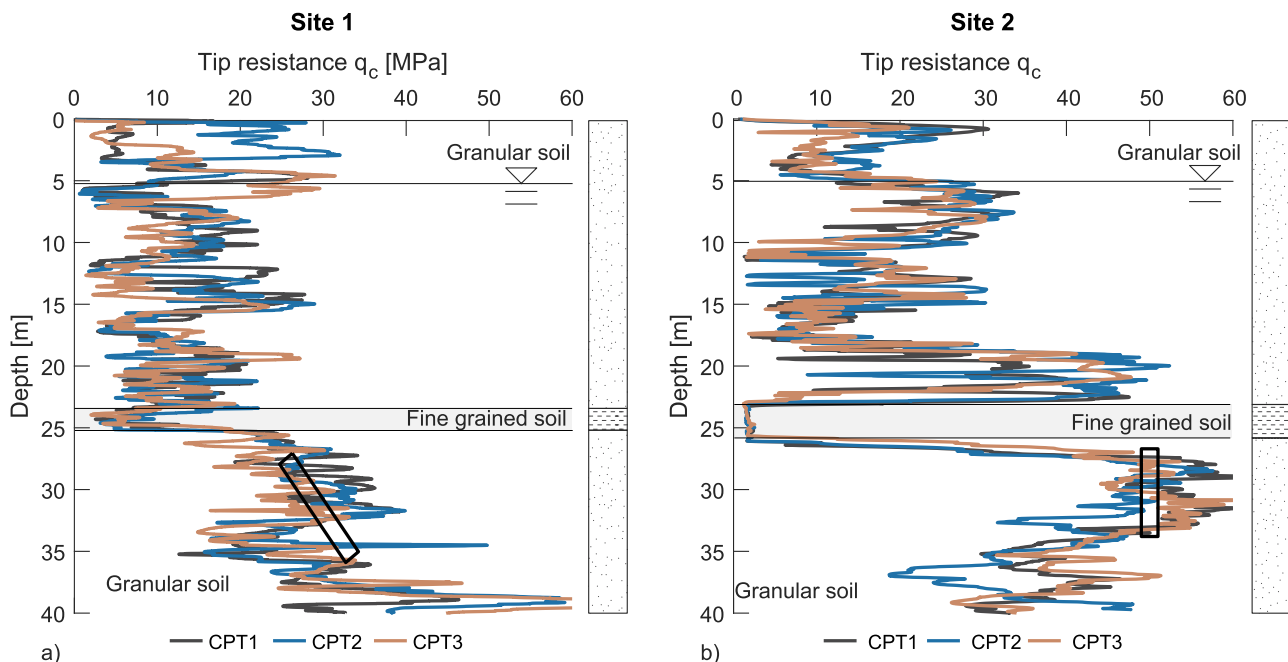


Figure 1. Results of cone penetration tests in Site 1 (a) and Site 2 (b).

**Table 1.** Summary of anchor test details.

	Test	Anchor type	Anchor length [m]	Grouted length [m]	Grouted diameter [m]	Inclination with hor. [°]	$P_p$ (kN)
Site 1	A	Self-drilling	46.6	12.4	0.335	40	5140
	B	Self-drilling	46.6	12.4	0.335	40	5140
Site 2	C	Self-drilling	26.7	7.1	0.4	90	2500
	D	Self-drilling	26.7	7.1	0.4	90	2500
	E	Self-drilling	26.7	7.1	0.4	90	2500

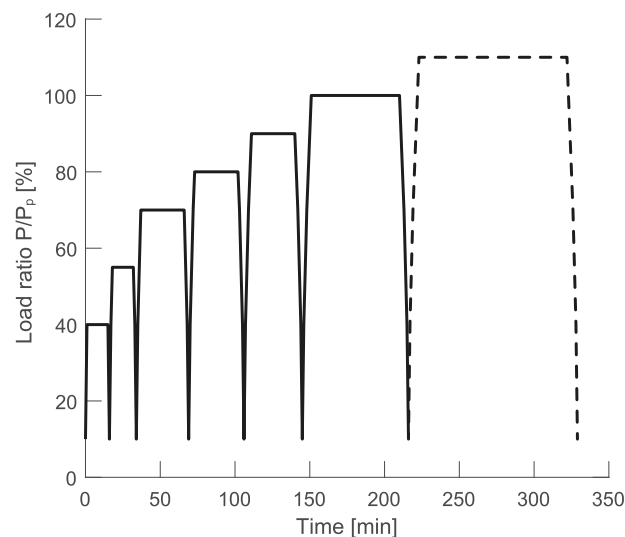
load is maintained constant for a specified period of time (creep phase). The imposed load history is represented in Figure 2 (solid line). For anchors D and E an additional load cycle was performed (dashed line).

Displacements are measured at the end of every load/unload step and periodically during creep, according to standard steps. It is worth mentioning that the standard test procedures (ISO 22477-5:2018) do not prescribe neither the measurements of displacements during the loading/unloading nor a specific loading rate.

### 2.3. Test results

As was also observed by Ostermayer (1975), ground anchors exhibit a rate dependent response (creep), even when they are embedded in a granular soil layer. This is due to the time needed for the rearrangement of the sand grain sub-structure (di Prisco and Imposimato 1996, Lago et al., 2022). For this reason, test results are usually interpreted in terms of creep rate ( $k_s$ ), defined as the slope of the linear branch of a displacement-time curve plotted in a semilogarithmic plane (Figure 3, relative to the final step of test A).

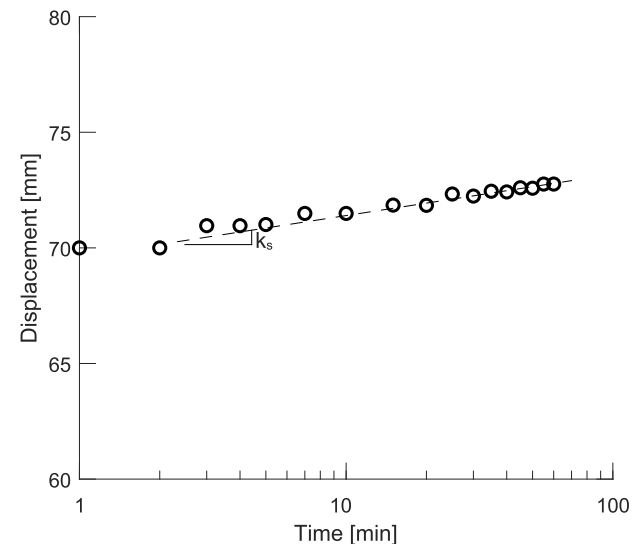
The results of the 5 tests in terms of variation of  $k_s$  with  $P$  are reported in Figure 4 (Figure 4(a) and Figure 4(b) are relative to Site 1 and 2, respectively).

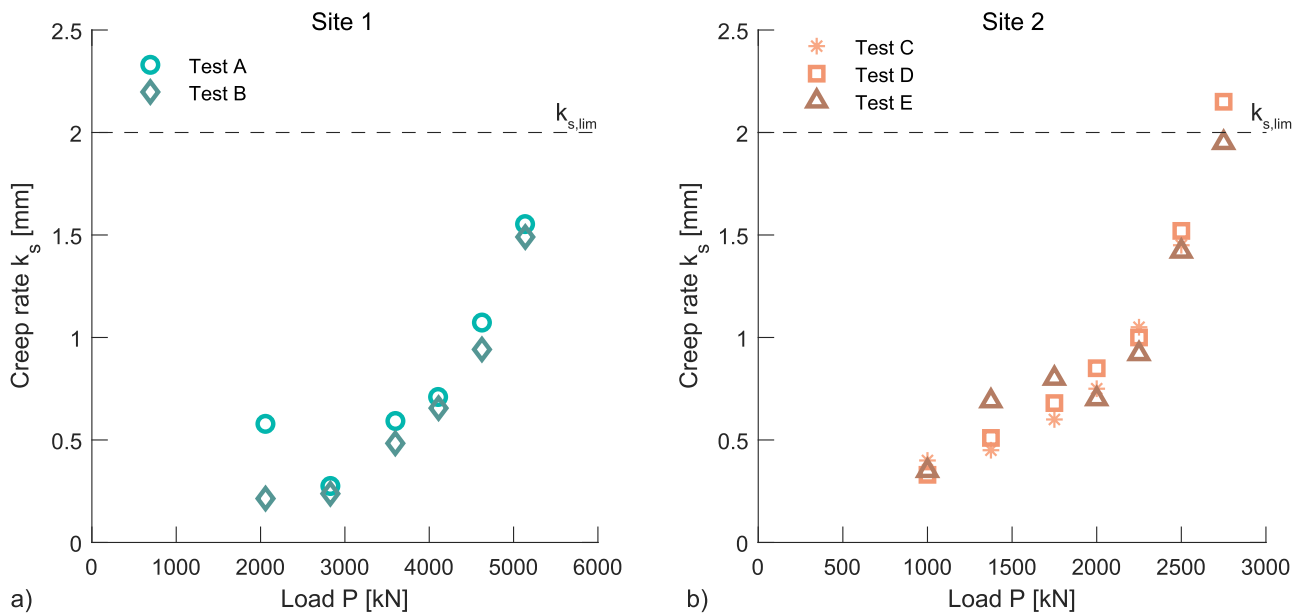

**Figure 2.** Imposed load history (dashed cycle only for anchors D and E).

As expected, an increase in the load is associated with an increase in the  $k_s$  value. The results of the tests performed in the different sites are aligned, and therefore it can be assumed that both spatial variability in the soil and the uncertainties associated with the anchor installation are practically negligible. The slight discrepancy in the results observed at low  $P$  values is likely to be due to slightly different rate of application of the load. These conclusions are however not general and refer to the specific conditions of the investigated site.

Since anchor response is rate-dependent, from a theoretical point of view (Pisanò and di Prisco 2016; di Prisco and Flessati 2021), it is not possible to use standard definitions for the failure load (i.e. the value of load for which an infinitesimal load increment is associated with an infinite displacement increment). In case of a rate-dependent response, during creep, the onset of instability should be identified (Pisanò and di Prisco 2016), by following Lyapunov's theory of stability (Lyapunov 1892), with an increase in the displacement rate.

Nevertheless, in the current practice (EN-ISO 24477-5), by following what is recommended by Ostermayer (1975), "failure" is identified with the value of load ( $P_{lim}$ ) corresponding to a limit threshold value of  $k_s$  ( $k_{s,lim}$ ). According to the anchor test standard (EN-ISO 24477-5), in case  $k_{s,lim}$  is not exceeded during the


**Figure 3.** Definition of the creep rate (data relative to the final step of test A).



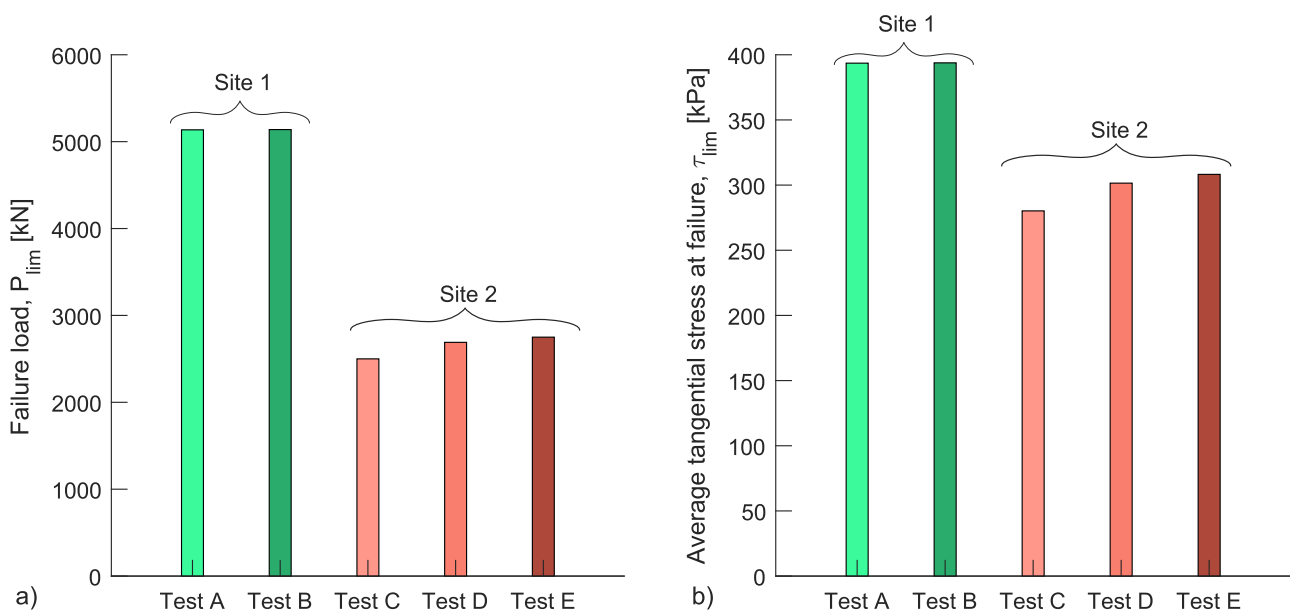
**Figure 4.** Test results in terms of evolution of  $k_s$  with  $P$ : (a) Site 1 and (b) Site 2.

test, failure load has to be identified as the maximum test load.

Different creep threshold values are adopted in different European countries (a discussion is reported in Merrifield et al. 2013), but in the following, since both sites are located in the Netherlands, the value proposed in the Dutch code CUR 166 will be employed ( $k_{s,lim} = 2$  mm, dashed line in Figure 4). It is worth mentioning that this limit value is assumed to always be the same, independent of the anchor length and soil characteristics.

As can be derived from Figure 4, both Test A and Test B do not exceed  $k_{s,lim}$ . By following what is proposed in EN-ISO 24477-5,  $P_{lim}$  is then identified as the maximum imposed load ( $P_{lim} = P_p = 5140$  kN). Also in case of Test C,  $k_{s,lim}$  was not exceeded (failure load is then  $P_{lim} = P_p = 2500$  kN) and this is the reason for which in Test D and Test E an additional loading phase (up to  $P = 1.1P_p = 2750$  kN) was considered. For the sake of clarity, the failure loads are plotted in Figure 5(a).

For the sake of completeness, in Figure 5(b) the results are also plotted in terms of average tangential



**Figure 5.** (a) Failure loads and (b) soil-grout average interface tangential stresses at failure.

stresses along the soil-grout shaft at failure ( $\tau_{lim} = P_{lim}/\pi D l$ , being  $D$  and  $l$  the grouted diameter and length, respectively). For the sake of simplicity, this value is calculated by assuming the tangential stresses along the anchor free length to be negligible.

The values of  $\tau_{lim}$  obtained for Site 1 are significantly larger than the one of Site 2. Since (i) the two set of tests were performed at approximately the same depth and (ii) the soil internal friction angle is almost constant for very large  $q_c$  values (Robertson and Campanella 1983), this difference in  $\tau_{lim}$  can be explained by the anchor inclination: when the anchor is vertical (Site 2), lower normal stresses along the grouted anchor shaft are expected and, therefore,  $\tau_{lim}$  is expected to be lower.

According to the current Dutch design standard (CUR 166), the characteristic value of  $P_{lim}$  ( $P_{lim,k}$ ) can be estimated from the experimental test results by using the following expression (valid for a number of test lower or equal than 3):

$$P_{lim,k} = \frac{P_{lim,min}}{\xi} \quad (1)$$

where  $P_{lim,min}$  is the minimum value of  $P_{lim}$  obtained during the tests and  $\xi$  is a reduction factor. The value of  $\xi$  depends on the number of investigation tests performed. A discussion on the practical consequences of the  $\xi$  value is reported in Section 4.

### 3. Determination of anchor characteristic resistance through Bayesian updating

#### 3.1. Model definition

As previously mentioned, the approach proposed by the authors, based on Bayesian updating, requires the introduction of an anchor model, capable of providing, in a continuous manner, the output variable (in this case creep rate) from the test input data (load and anchor geometry).

The novel model introduced by the authors does not have the ambition of upscaling the micro-mechanical behaviour (i.e. stress and strain distribution and their spatial evolution) to the macroscopic one (i.e. the evolution of displacements at constant load). The model is rather phenomenological and defines a simplified relationship between the (macroscopic quantities)  $k_s$  and  $\tau$  ( $P/\pi D l$ ):

$$k_s = \frac{1}{E_a} \frac{\tau}{1 - \tau/\tau_a} + k_{s,min} \quad (2)$$

being  $1/E_a$  the initial curve slope,  $\tau_a$  is the theoretical value of  $\tau$  for which  $k_s$  becomes infinite (Figure 6) and  $k_{s,min}$  the minimum creep rate. According to

Ostermayer (1975),  $k_{s,min}$  is mainly associated with partial debonding at grout-steel interface, creep of cement grouting and relaxation of tendon steel.

The three model parameters ( $E_a$ ,  $\tau_a$  and  $k_{s,min}$ ) will be calibrated, by following the Bayesian updating procedure, on the experimental creep measurements. It is worth mentioning that, in general, the inference of the three model parameters are expected to be influenced by both anchor geometry (geometry, depth and inclination) and soil properties. A discussion of these dependencies is not part of this paper.

#### 3.2. Determination of model parameters through Bayesian updating

The approach used by the authors for the Bayesian updating is the one originally proposed in Straub and Papaioannou (2015) and then subsequently improved in Betz et al. (2018). According to this approach, the Bayesian updating problem is reformulated as a structural reliability problem (i.e. through the exceedance of limit states). The “failure domain” ( $g$ ) is defined through the following limit state function:

$$g = \ln(p) - \ln(c \cdot L(\boldsymbol{\theta}|\mathbf{m})) \quad (3)$$

where  $p$  is a uniformly distributed random variable defined between 0 and 1,  $L$  is the likelihood (where  $\boldsymbol{\theta}$  and  $\mathbf{m}$  are vectors containing the model parameters and the measurements, respectively), and  $c$  is a scaling factor, progressively adapted so that it is always equal to the maximum likelihood. The main advantage of this approach, named aBUS (adaptive Bayesian Updating with Structural reliability methods), is that it is very versatile since it allows the use of all the numerical

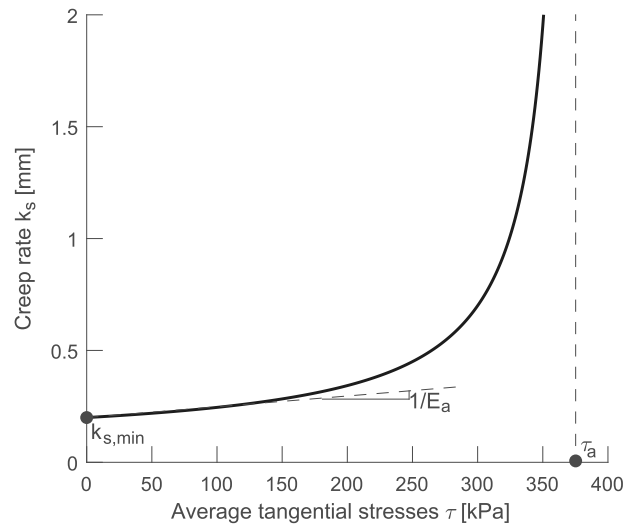
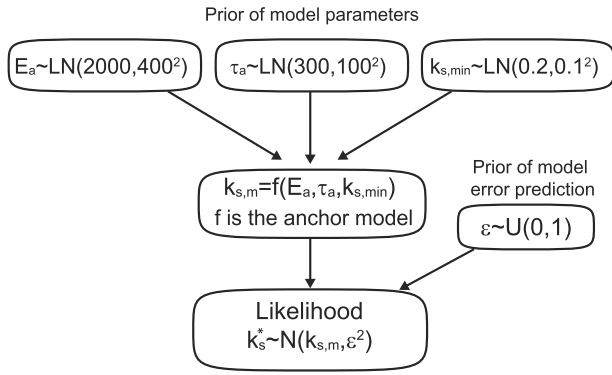


Figure 6. Example of model results for,  $E_a = 3000$  kPa/mm,  $\tau_a = 375$  kPa and  $k_{s,min} = 0.2$  mm.



**Figure 7.** Schematic of the statistical model, U stands for uniform and LN for lognormal.

strategies developed for structural reliability problems for solving the Bayesian updating, making it extremely robust and efficient. The numerical procedure used by the authors is the one introduced in Betz et al. (2018). This is based on the use of subset simulation (SuS), in which a Markov Chain Monte Carlo (MCMC) approach is used. To improve the efficiency of the algorithm, the standard deviation of the MCMC proposal distribution is adapted during the iterations to grant that the acceptance rate ( $\alpha$ ) is equal to the optimum one ( $\alpha_{opt} = 0.44$ ; Betz et al. 2018).

The use of aBUS-SuS (Figure 7) requires:

- an analytical model representative of physical behaviour, correlating the test input to the measurements (Section 3.1);
- likelihood definition and prior distribution of the model prediction error ( $\epsilon$ ) to be calibrated (Section 3.2.1);
- prior distribution of the parameters ( $E_a$ ,  $\tau_a$  and  $k_{s,min}$ ) (Section 3.2.2);
- number of samples to be drawn from the posterior distribution ( $N$ );
- probability of intermediate subsets ( $p_t$ );
- target acceptance rate ( $\alpha_{target}$ ).

In all the cases considered in this paper,  $N = 10,000$ ,  $p_t = 0.1$  and  $\alpha_{target} = 0.44$ . The results demonstrating that the chosen  $N$  and  $p_t$  values do not influence the solution are reported in the Appendix.

### 3.2.1. Likelihood

The likelihood is expressed as follows:

$$L = \frac{1}{\sqrt{(2\pi)^M |\Sigma|}} \exp \left[ -\frac{1}{2} (\mathbf{k}_{s,m}(\boldsymbol{\theta}, \tau^*) - \mathbf{k}_s^*)^T \Sigma^{-1} (\mathbf{k}_{s,m}(\boldsymbol{\theta}, \tau^*) - \mathbf{k}_s^*)^T \right] \quad (4)$$

where

- $\mathbf{k}_s^*$  is a  $M$  (the number of measurements) long vector containing the values of creep derived by interpreting the experimental test result;
- $\mathbf{k}_{s,m}$  is a  $M$  long vector containing the value of  $k_s$  calculated by means of Equation (1), in which the values of  $\tau$  are equal to the ones imposed in the experimental tests ( $\tau^*$ ) and  $\boldsymbol{\theta} = [E_a \quad \tau_a \quad k_{s,min}]^T$  are the parameters to be updated;
- $\Sigma$  is the  $M \times M$  covariance matrix. The entries along the main diagonal are assumed to be equal to  $\epsilon^2$ , whereas all the other entries are assumed to be nil, meaning that the residuals are assumed uncorrelated.  $\epsilon$  is not known at the beginning and it is treated as a parameter to be updated. Its prior distribution is assumed to be uniform between 0 and 1. From a practical point of view,  $\epsilon$  is a term, independent between anchors, accounting for the anchor-to-anchor variability: it accounts for the spatial variability in the soil, the uncertainties associated with the anchor installation and load rate application effects. It is worth mentioning that to account spatial variability for, a minimum number of anchor test should be performed. According to EN 1997 this minimum number is three. In Appendix, it is shown that this assumption does not affect the results in terms of characteristic value of limit tangential stress.

For Site 1 Test A and test B results are considered together. However, the first result for Test A (Figure 4,  $P = 2000\text{kN}$ ) was considered to be an outlier and was disregarded, therefore  $M = 11$ . For Site 2 all the results of tests C, D and E were considered together and  $M = 20$ .

### 3.2.2. Prior distributions of model parameters

For all the three model parameters, lognormal prior distributions were chosen. For the sake of simplicity, the prior distributions assume the model parameters to be independent. However, since in the proposed approach both marginal distribution and correlation are updated, if any correlation is present, this will be found by the approach. As was previously mentioned  $k_{s,min}$  is mainly associated with partial debonding at grout-steel interface (Ostermayer 1975) and, therefore, it can be assumed independent on soil properties. In contrast, both  $E_a$  and  $\tau_a$  are expected to be dependent on soil properties and anchor installation method.

The mean and the standard deviation values used in the prior distributions are reported in Table 2. To analyse the influence of the prior distributions on the characteristic value of limit tangential stress, the authors performed a sensitivity study, by considering reasonable upper and lower limits. The results demonstrating that in these ranges of values the influence of the prior



**Table 2.** Values for the lognormal prior distributions.

	Mean	Standard deviation
$E_a$ [kPa/mm]	2000	400
$\tau_a$ [kPa]	300	100
$k_{s,min}$ [mm]	0.2	0.1

distributions on the characteristic value of limit tangential stress is negligible are reported in Appendix.

### 3.3. Results

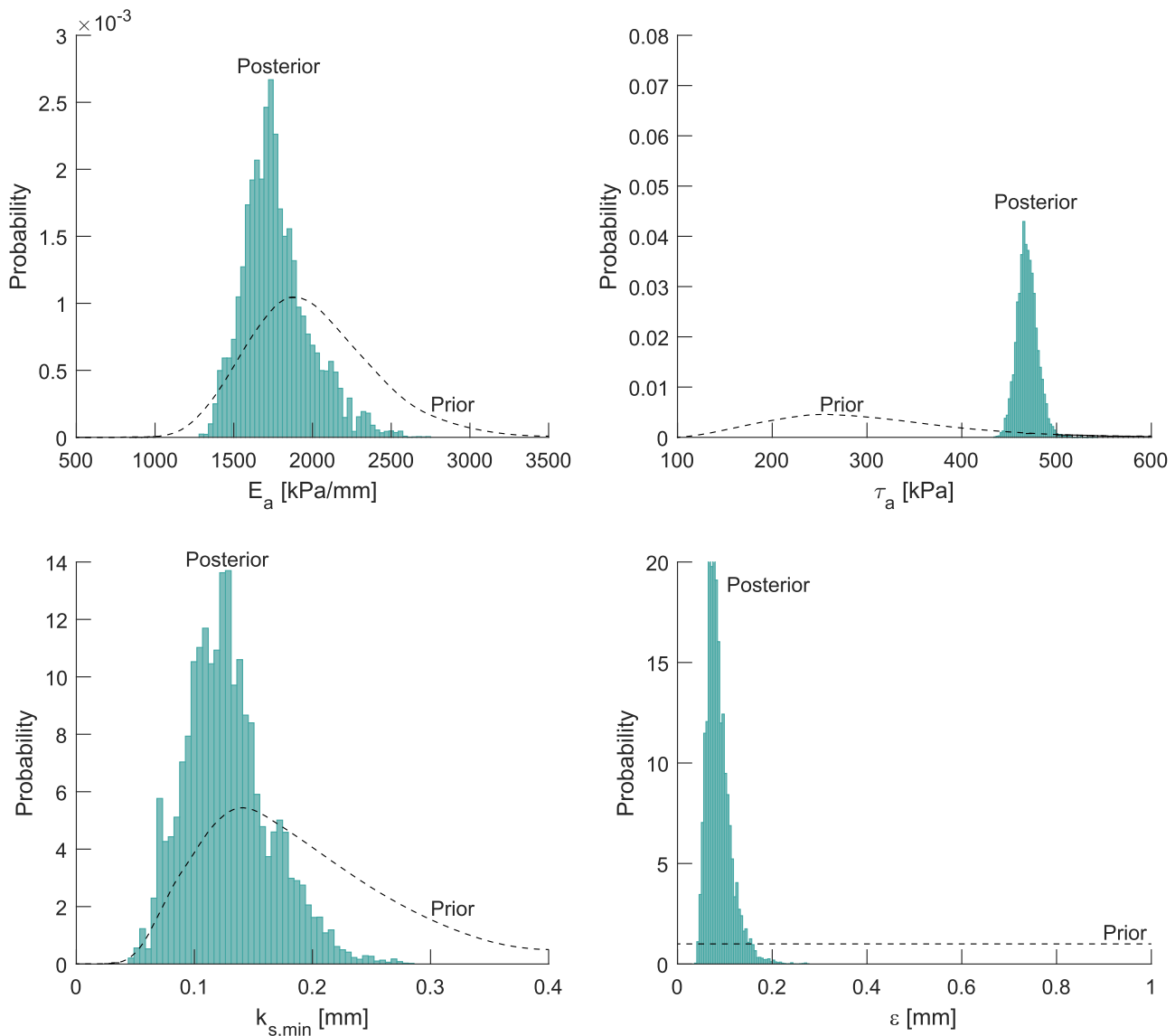
The output of the approach used for the Bayesian updating are the posterior distributions of the model parameters (Section 3.3.1). These can be introduced in the model of Section 3.1 to derive both the mean and the credible interval of the model predictions (Section 3.3.2).

#### 3.3.1. Posterior model parameter distributions

Updating takes place on the joint probability distribution of the parameters: both marginal distributions and the correlation between the parameters are updated.

The posterior marginal distributions of  $E_a$ ,  $\tau_a$ ,  $k_{s,min}$  and  $\varepsilon$  obtained by using aBuS-SuS are compared in Figure 8 with the corresponding prior distributions for Site 1 and the correlation of the updated variables are reported in Figure 9. The corresponding results for Site 2 are reported in Figure 10 and Figure 11. Correlation between variables can be noted for  $E_a$  and  $\tau_a$ , whereas the others seem to be uncorrelated.

For the sake of completeness, the output of aBuS-SuS are also summarised in Table 3 in terms of mean and standard deviation.


**Figure 8.** Prior and posterior marginal distributions for Site 1.

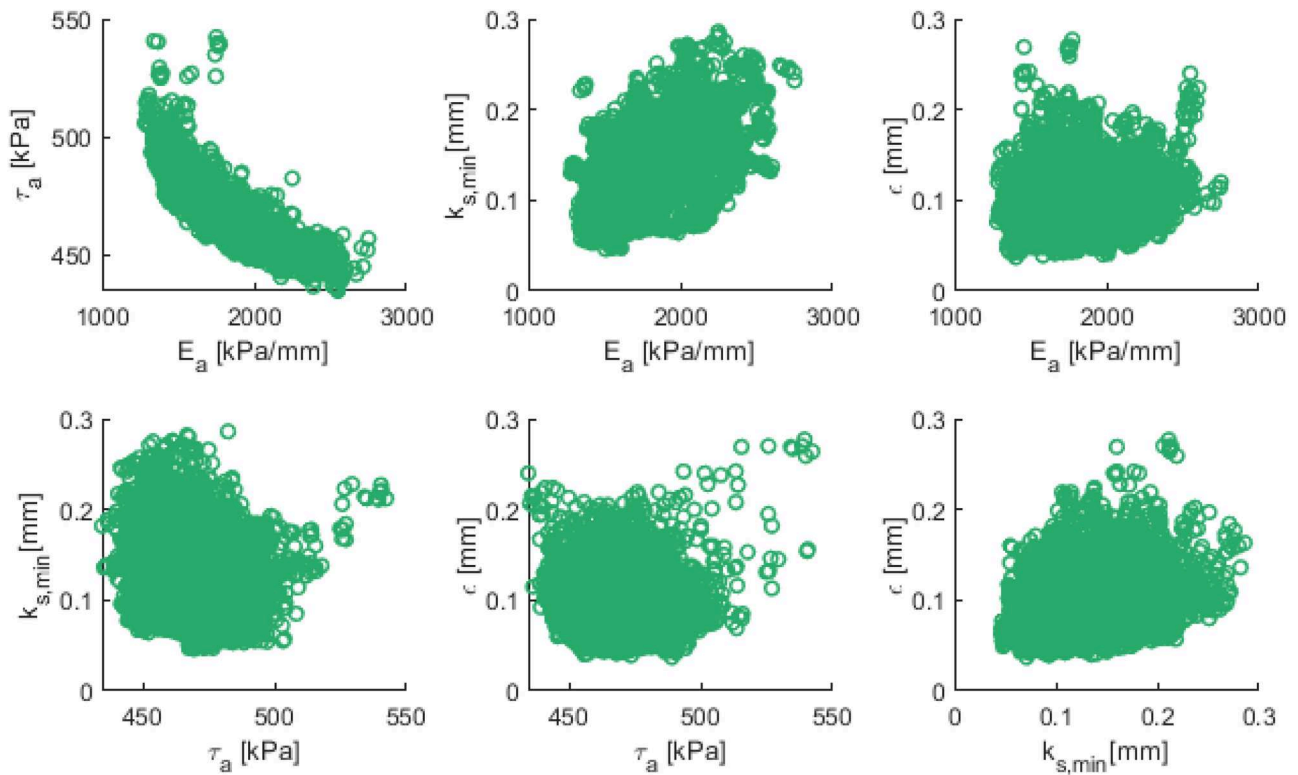


Figure 9. Correlation between updated variables of Site 1.

### 3.3.2. Posterior model results

The joint posterior distributions presented in the previous section can be introduced in the model described by Equation (1). The comparison between experimental results and model predictions are reported in Figure 12 (Figure 12(a,b) refer to Site 1 and 2, respectively).

In particular, the model predictions are reported in terms of mean value and the 90% credible intervals of both the model posterior and the posterior predictive. The model posterior refers only to the model itself, given the uncertainty in model parameters, while the posterior predictive describes the uncertainty in displacements in case one wants to predict the displacements of a new anchor at the same site. The posterior predictive includes all uncertainties that arise from the data spread (model, spatial variability, loading rate effect, etc.)

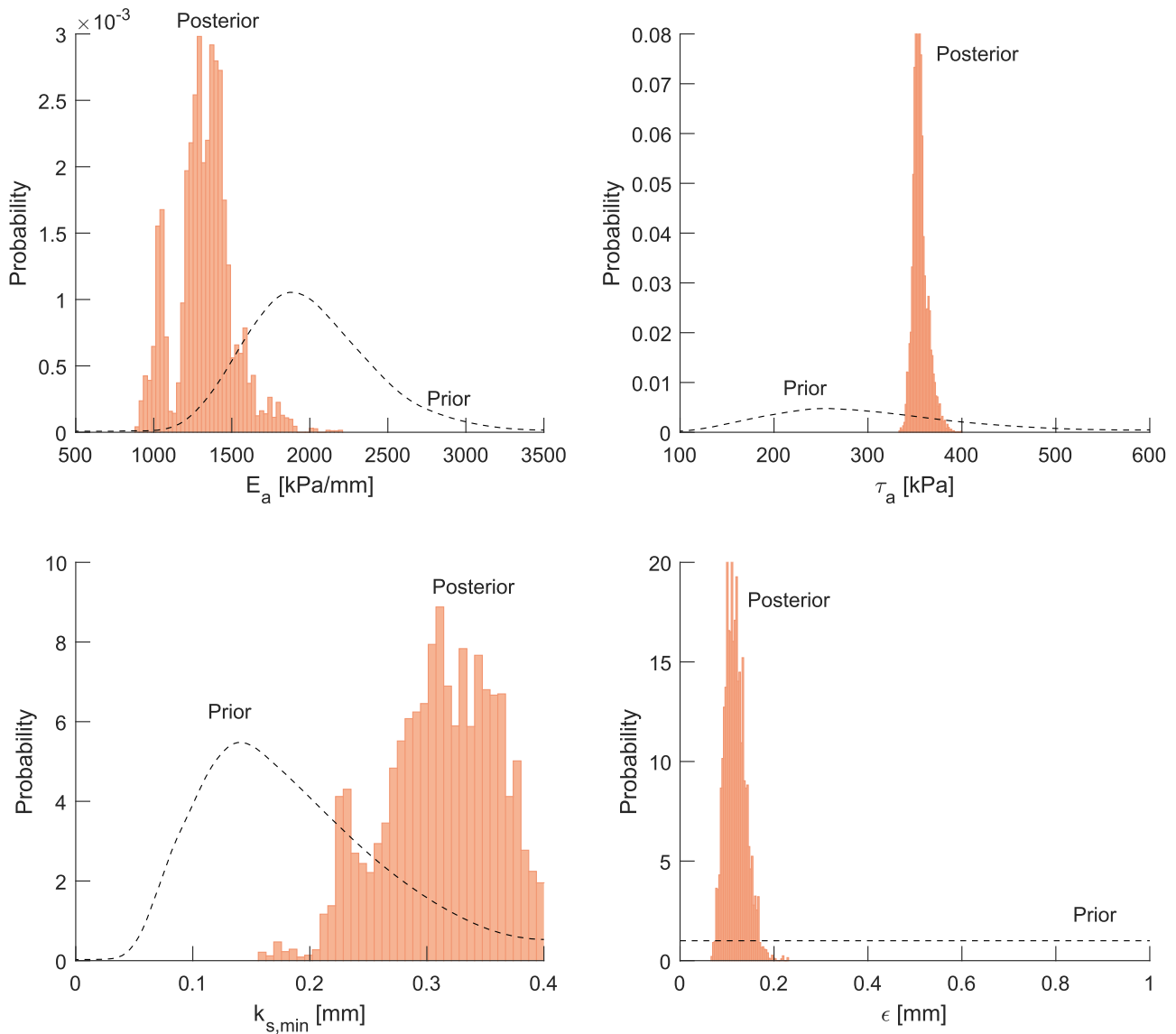
### 3.4. Evaluation of the anchor characteristic resistance

The posterior parameter distributions and the proposed model can be used to derive the distribution of limit tangential stresses, i.e. the ones for which  $k_s = 2$  mm. These distributions are reported in Figure 13 (Figure 13(a) and Figure 13(b) are relative to Site 1 and 2, respectively). It is essentially a section of the posterior predictive credible interval along the  $k_s = 2$  mm line. The dashed

lines in Figure 13(a,b) represent the 5th percentile. By following the philosophy of EN1997, the limit tangential stress is interpreted as a material strength parameter (for the soil) and the 5th percentile is assumed to be corresponding to the characteristic value of  $\tau_{lim}$  ( $\tau_{lim,k}$ ). For Site 1 and Site 2, these values are 402 and 299 kPa, respectively. The corresponding  $P_{lim,k}$  are equal to 5246 and 2668 kN, respectively (Figure 13(c,d)). In the following section these values are compared with those obtained by following the design standard.

## 4. Critical discussion of the obtained results in the light of the design standard

As was previously mentioned (Section 2.3), according to the current Dutch design standard (CUR 166) the anchor characteristic resistance depends on the partial (reduction) factor  $\xi$ . This latter ranges in between 1.39 and 1.13, depending on the number of tests conducted (1.32 and 1.30 for 2 or 3 tests, respectively). In the case where all anchors are tested to failure,  $\xi$  can be assumed to be 1. This simplified approach is based on the idea that increasing the number of tested anchors reduces the uncertainties associated with soil properties and heterogeneity. As was previously mentioned, in this case, both CPT and anchor test results (Figure 1 and Figure 4) are quite repeatable, highlighting a practically



**Figure 10.** Prior and posterior marginal distributions for Site 2.

negligible influence of soil spatial variability and installation method on the results.

The values of  $P_{lim,k}$  and  $\tau_{lim,k}$  obtained by using  $\xi = 1.32$  (representative for the number of test performed) and  $\xi = 1$  (as an ideal upper bound) are reported in Table 4.

In case  $\xi = 1.32$  (i.e. when a small number of anchors are tested), the characteristic values derived from the design standard approach are significantly lower than those obtained by using the Bayesian updating procedure (the difference is approximately equal to 30%). This suggests that, the same anchor performance ( $P_{lim,k}$ ) can be achieved with shorter anchors. In particular, for Site 1 and 2, the anchor length can be reduced by 3 and 2 m, respectively, leading to a more economic and sustainable anchor design while granting the same design resistance.

In case  $\xi = 1$ , the characteristic values calculated by following the design standard are similar (but slightly lower) to those derived by using the Bayesian updating procedure. Therefore, in this case the potential reduction in terms of anchor length (for the same anchor performance) is significantly lower (25 and 45 cm for Site 1 and 2, respectively). However, it is worth mentioning that  $\xi = 1$  is possible only in case all the anchors are tested. In the case of very large constructions requiring numerous anchors, this corresponds to a very large number of tests on anchors. In such cases, the use of the Bayesian updating procedure can optimise the allocation of resources. In other words, in this second case, the optimisation has to be intended in a broader sense, including not only the anchor design itself, but also the *in situ* testing programme. It is worth noting that according to EN1997, the minimum number of tests is 3. In case the

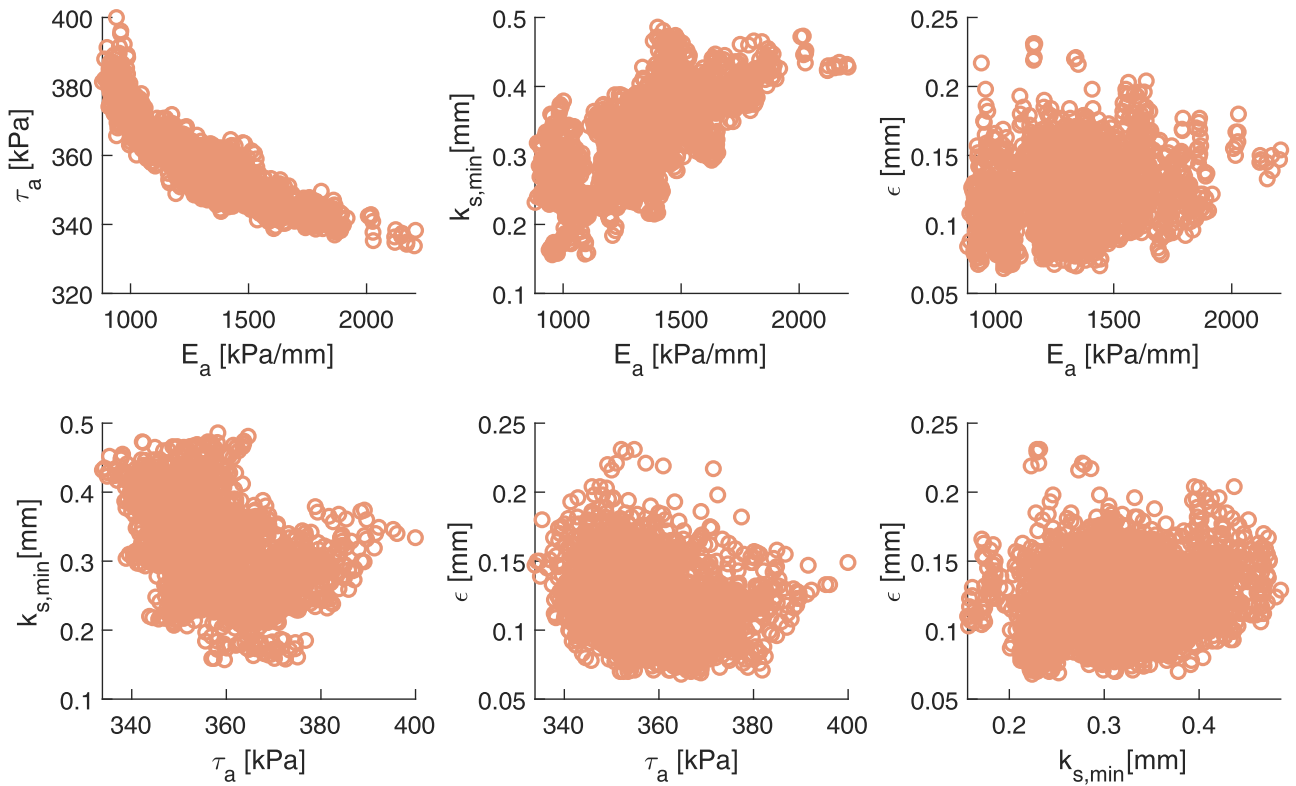


Figure 11. Correlation between updated variables of Site 2.

Table 3. Summary of the posterior marginal distributions.

	Site 1		Site 2	
	Mean	Standard deviation	Mean	Standard deviation
$E_a$ [kPa/mm]	1772	24	1320	185
$\tau_a$ [kPa]	469	10.7	356	7.8
$k_{s,min}$ [mm]	0.13	0.04	0.31	0.05
$\epsilon$ [mm]	0.09	0.03	0.12	0.02

site is characterised by large spatial variability (not the case of the sites considered in this paper), this number is not sufficient. In those cases, the spread in the results of anchor tests (and of posterior distribution of model parameters) can be used as a measure of the site heterogeneity.

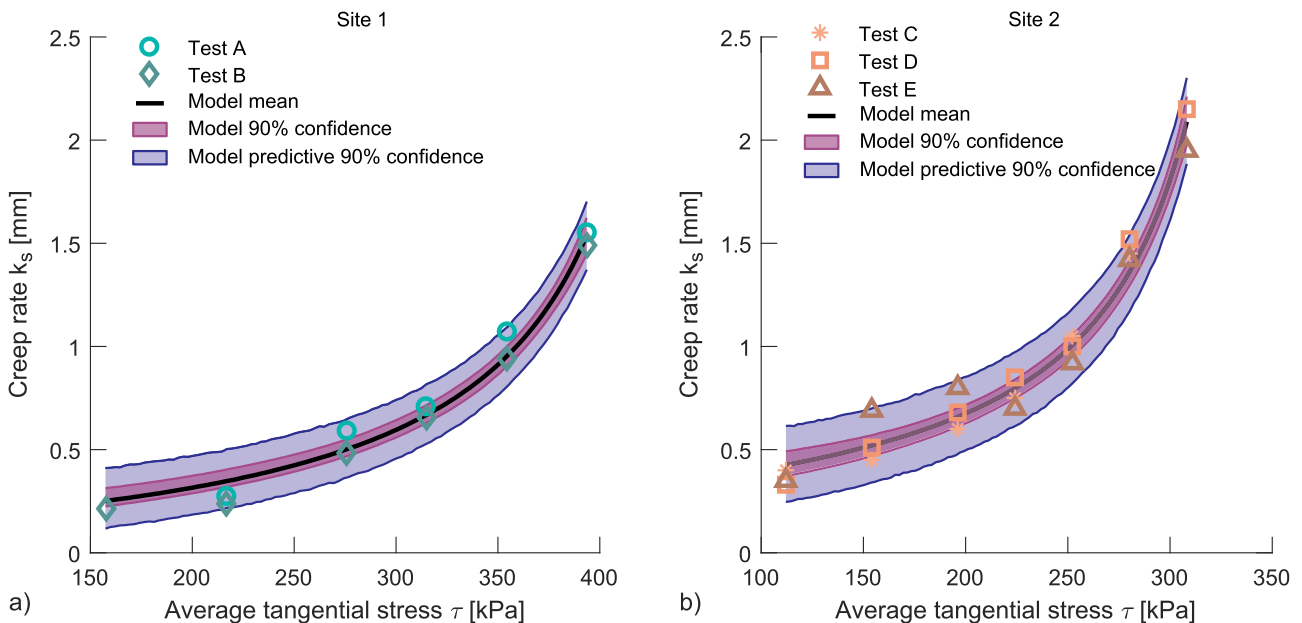
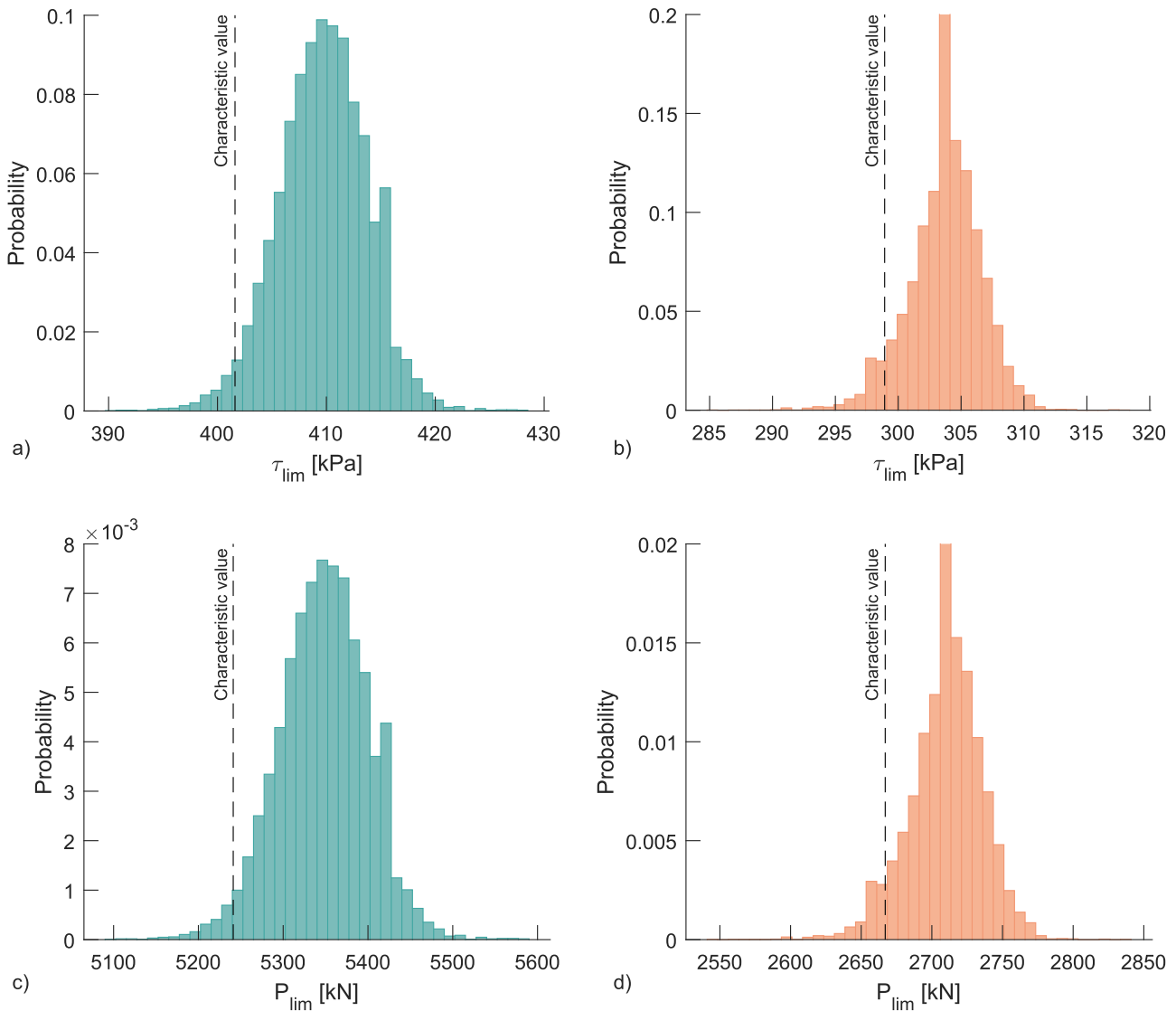


Figure 12. Posterior model distribution: (a) Site 1 and (b) Site 2.



**Figure 13.** Distribution of the tangential stresses at failure for (a) Site 1 and (b) Site 2 and distributions of limit loads for (c) Site 1 and (d) Site 2.

**Table 4.** Summary of the anchor characteristic resistance calculated by following CUR 166 and with the proposed approach.

	CUR 166			
	Site 1		Site 2	
$P_{lim,k}$ [kN]	$\xi = 1.32$ 3900	$\xi = 1$ 5140	$\xi = 1.32$ 1900	$\xi = 1$ 2500
$\tau_{lim,k}$ [kPa]	299	394	213	280
Proposed approach				
	Site 1		Site 2	
$P_{lim,k}$ [kN]	5246		2668	
$\tau_{lim,k}$ [kPa]	402		299	

## 5. Conclusions

In this paper, the authors show how the results of a limited number of *in situ* experimental tests can be used to derive the distribution and the characteristic values of

resistance of grout anchors. The proposed methodology is based on equality updating via Bayesian inference and requires the definition of a model allowing to relate, in a continuous way, the imposed load and the measured creep. The approach used for the Bayesian updating is based on the formulation of the problem as a structural reliability problem and on the use of subset simulations.

For the sake of simplicity, the authors introduced a very simple phenomenological model for reproducing the anchor response. However, the whole methodology is expected to be valid even if more complex (e.g. micro to macro) models are introduced to reproduce the anchor response.

The procedure proposed by the authors is general and requires minimal computational time (on the order of tens of seconds). As a result, it can be easily used in anchor design to address the limitations and

over-conservatism of semi-probabilistic “design by testing” approaches based on partial safety factors, which are not tailored to specific site conditions or cases. The use of the proposed methodology is exemplified on two experimental campaigns in which failure tests on ground anchors were performed in two different locations in the Port of Rotterdam. By comparing the results in terms of characteristic values of limit tangential stress along the anchor grouted shaft obtained from the new methodology and the ones obtained by using the current design standard, we observe an increase up to 30% of the anchor characteristic resistance values. The limit tangential stresses are calculated by assuming the interaction between the free length of the anchor and the surrounding soil to be negligible. In principle, this assumption has to be verified during the *in situ* experimental tests, for instance by including local strain measurements (e.g. fibreglass) along the whole anchor length.

In contrast to the standard procedures, according to which the estimation of the anchor resistance is essentially based only on the final test measurement, the approach proposed by the authors allows to use in a more efficient way all the data gathered during the tests.

In principle, the proposed approach can be used for all type of anchor tests. However, tests performed at load values significantly lower than the failure load are less informative and the model predictions might be less accurate, leading to an underestimation of the anchor resistance characteristic value. Though particularly interesting from a design point of view, this aspect is not covered in this paper, making it a promising area for future developments. The developed code is freely available at <https://github.com/LFlessati/AnchorUpdating>.

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## Disclosure statement

No potential conflict of interest was reported by the author(s).

## References

- Beck, J. L., and L. S. Katafygiotis. 1998. “Updating Models and Their Uncertainties. I: Bayesian Statistical Framework.” *Journal of Engineering Mechanics* 124 (4): 455–461. [https://doi.org/10.1061/\(asce\)0733-9399\(1998\)124:4\(455\)](https://doi.org/10.1061/(asce)0733-9399(1998)124:4(455)).
- Betz, W., I. Papaioannou, J. L. Beck, and D. Straub. 2018. “Bayesian Inference with Subset Simulation: Strategies and Improvements.” *Computer Methods in Applied Mechanics and Engineering* 331:72–93. <https://doi.org/10.1016/j.cma.2017.11.021>
- CEN. 2002. “EN 1990: Eurocode – Basis of Structural Design.” *Comité Européen de Normalization*.
- CEN. 2004. “EN 1997-1: Geotechnical Design: Part 1: General Rules.” *Comité Européen de Normalization*.
- CUR 166. 2005. Damwandconstructies, available at Civieltechnisch Centrum Uitvoering Research en Regelgeving: P.O.Box 420, 2800 AK Gouda (NL).
- Dal Lago, B., L. Flessati, P. Marveggio, P. Martinelli, G. Fraraccio, C. di Prisco, and M. di Prisco. 2022. “Experimental Tests on Shallow Foundations of Onshore Wind Turbine Towers.” *Structural Concrete* 23 (5): 2986–3006. <https://doi.org/10.1002/suco.202100655>
- di Prisco, C., and L. Flessati. 2021. “Progressive Failure in Elastic–Viscoplastic Media: From Theory to Practice.” *Géotechnique* 71 (2): 153–169. <https://doi.org/10.1680/jgeot.19.P.045>
- di Prisco, C., and S. Imposimato. 1996. “Time Dependent Mechanical Behaviour of Loose Sands.” *Mechanics of Cohesive-frictional Materials* 1 (1): 45–73. [https://doi.org/10.1002/\(SICI\)1099-1484\(199601\)1:1<45::AID-CFM3>3.0.CO;2-O](https://doi.org/10.1002/(SICI)1099-1484(199601)1:1<45::AID-CFM3>3.0.CO;2-O)
- EN-ISO 224477-5. 2018. “Geotechnical Investigation and Testing — Testing of Geotechnical Structures – Part 5: Testing of Grouted Anchors”, Technical Committee ISO/TC 182 Geotechnics and Technical committee CEN/TC 341 Geotechnical investigation and testing.
- Guo, W., X. Tan, J. Zhang, X. Lin, X. Dong, and X. Hou. 2023. “System Reliability and Sensitivity Analysis of Lateral Loaded Pile Considering Soil’s Spatial Variability.” *Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards* 17 (4): 651–665. <https://doi.org/10.1080/17499518.2023.2174264>.
- Katafygiotis, L. S., and J. L. Beck. 1998. “Updating Models and Their Uncertainties. II: Model Identifiability.” *Journal of Engineering Mechanics* 124 (4): 463–467. [https://doi.org/10.1061/\(asce\)0733-9399\(1998\)124:4\(463\)](https://doi.org/10.1061/(asce)0733-9399(1998)124:4(463)).
- Kormi, T., A. Hentati, M. Selmi, and N. BelHadj Ali. 2019. “Reliability-based Assessment of Foundations Under HM Combined Loading Using Random Finite Element Method.” *Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards* 14 (4): 293–307. <https://doi.org/10.1080/17499518.2019.1690151>.
- Li, B., L. Zhang, and J. Yuan. 2024. “A Simple Monte Carlo Simulation Method for Geotechnical Reliability-Based Design.” *Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards* 18 (3): 558–569. <https://doi.org/10.1080/17499518.2023.2222368>
- Littlejohn, G. S. 1980. “Design Estimation of the Ultimate Load-Holding Capacity of Ground Anchors.” *Ground Engineering* 13 (8): 1–11.
- Lyapunov, A. M. 1892. “The General Problem of the Stability of Motion.” Doctoral dissertation, University of Kharkov, Kharkov Mathematical Society, Kharkov, Russia (in Russian).
- Merrifield, C., O. Møller, B. Simpson, and E. Farrell. 2013. “European Practice in Ground Anchor Design Related to the Framework of EC7.” Proceedings of the 18th

- International Conference on Soil Mechanics and Geotechnical Engineering, Paris, France, September 2013.
- Ostermayer, H. 1975. "Construction, Carrying Behaviour and Creep Characteristics of Ground Anchors." In *Diaphragm Walls & Anchorages*, edited by Institution of Civil Engineers, 141–151. London: Thomas Telford.
- Pisanò, F., and C. di Prisco. 2016. "A Stability Criterion for Elasto-Viscoplastic Constitutive Relationships." *International Journal for Numerical and Analytical Methods in Geomechanics* 40 (1): 141–156. <https://doi.org/10.1002/nag.2395>
- Robertson, P. K., and R. G. Campanella. 1983. "Interpretation of Cone Penetration Tests – Part I (Sand)." *Canadian Geotechnical Journal* 20 (4): 718–733. <https://doi.org/10.1139/t83-078>
- Sabatini, P. J., D. G. Pass, and C. Bachus, Robert. 1999. *Ground Anchors and Anchored Systems United States*. Federal: Highway Administration. Office of Bridge Technology.
- Straub, D. 2011. "Reliability Updating with Equality Information." *Probabilistic Engineering Mechanics* 26 (2): 254–258. <https://doi.org/10.1016/j.probingmech.2010.08.003>
- Straub, D., and I. Papaioannou. 2015. "Bayesian Updating with Structural Reliability Methods." *Journal of Engineering Mechanics* 141 (3): 04014134. [https://doi.org/10.1061/\(ASCE\)EM.1943-7889.0000839](https://doi.org/10.1061/(ASCE)EM.1943-7889.0000839)
- Zhang, J., Z. Wang, J. Hu, S. Xiao, and W. Shang. 2022. "Bayesian Machine Learning-Based Method for Prediction of Slope Failure Time." *Journal of Rock Mechanics and Geotechnical Engineering* 14 (4): 1188–1199. <https://doi.org/10.1016/j.jrmge.2021.09.010>

## Appendix

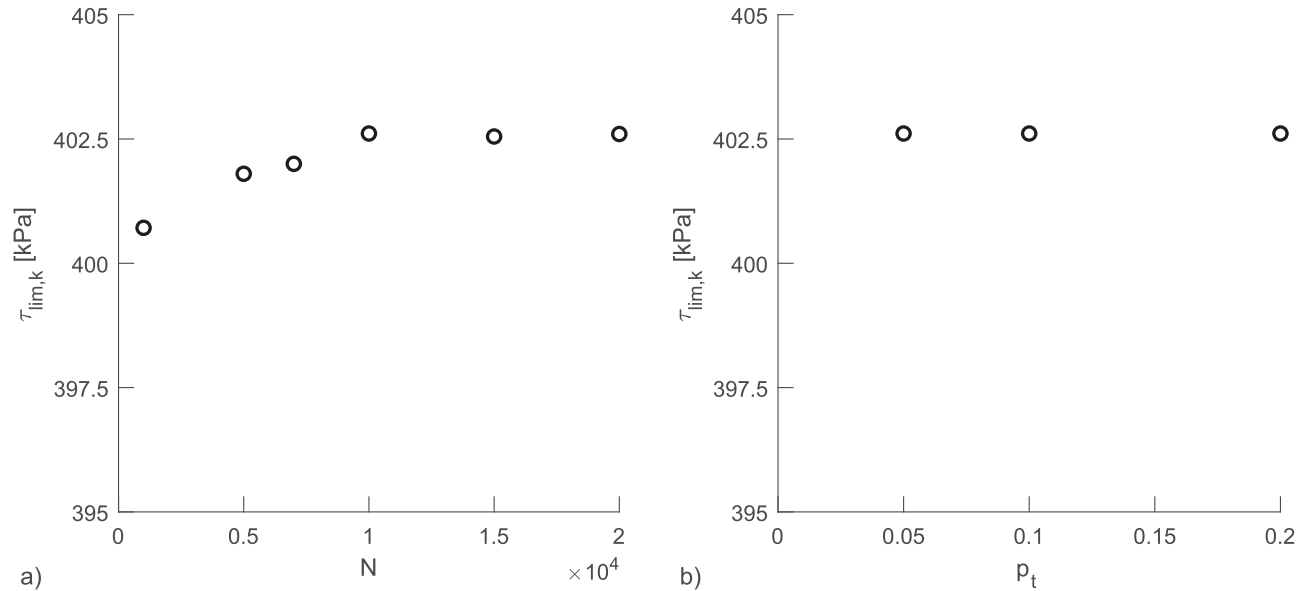
The influence of the input parameters for aBuS-SuS on the characteristic value of limit average tangential stress along the anchor shaft is discussed in Figure A1–Figure A5. In particular, the influence of the numerical parameters  $N$  and  $p_t$  is discussed in Figure A1(a) and Figure A2(b), the influence of the prior distribution of the model parameters in Figure A2–Figure A4 and the influence of the variance of the predicted error in Figure A5. For the sake of brevity, only Site 1 is taken into account.

Regarding the model parameter priors, the values listed in Table 2 were used as a baseline, along with variations where these values were increased by 50% and decreased by the same amount. It is worth noting that the mean values of the priors of  $k_{s,min}$  and  $E_a$  in Table 2 were chosen using their definitions from the experimental results of Figure 4, whereas, the mean value of the prior of  $\tau_a$  is estimated by assuming a frictional behaviour of the soil-grout interface:

$$\tau_a = K \sigma'_v \tan \phi' \quad (5)$$

in which the friction angle  $\phi' = 43^\circ$  (estimated from CPT data with the approach proposed in Robertson and Campanella (1983)),  $\sigma'_v$  the vertical effective stresses are calculated accounting for the anchor depth and the non-dimensional factor  $K$  allowing to pass from vertical effective stresses to the normal stresses acting along the grouted length is assumed equal to 1.4, the lower bound value suggested in Littlejohn (1980).

The numerical solution in terms of  $\tau_{lim,k}$  is practically unaffected testifying both the robustness of the employed approach and the validity of the results.



**Figure A1.** (a) influence of  $N$  on  $\tau_{lim,k}$  and (b) influence of  $p_t$  on  $\tau_{lim,k}$ .



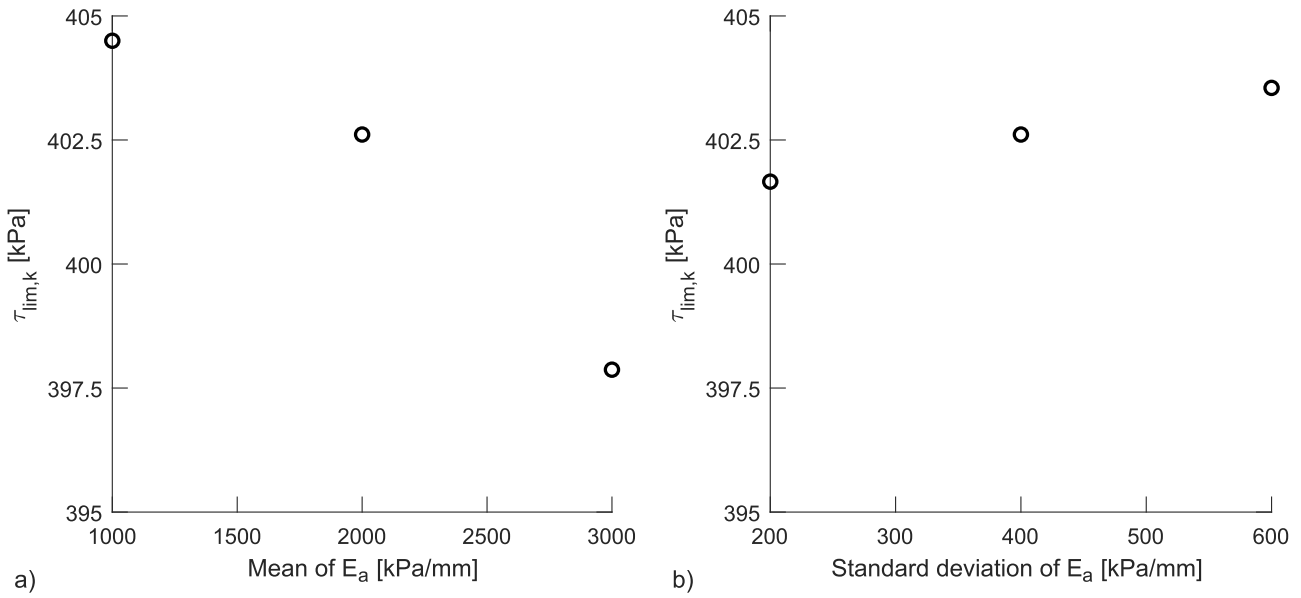


Figure A2. . Influence of the prior distribution of  $E_a$  on  $\tau_{lim,k}$ : (a) mean and (b) standard deviation.

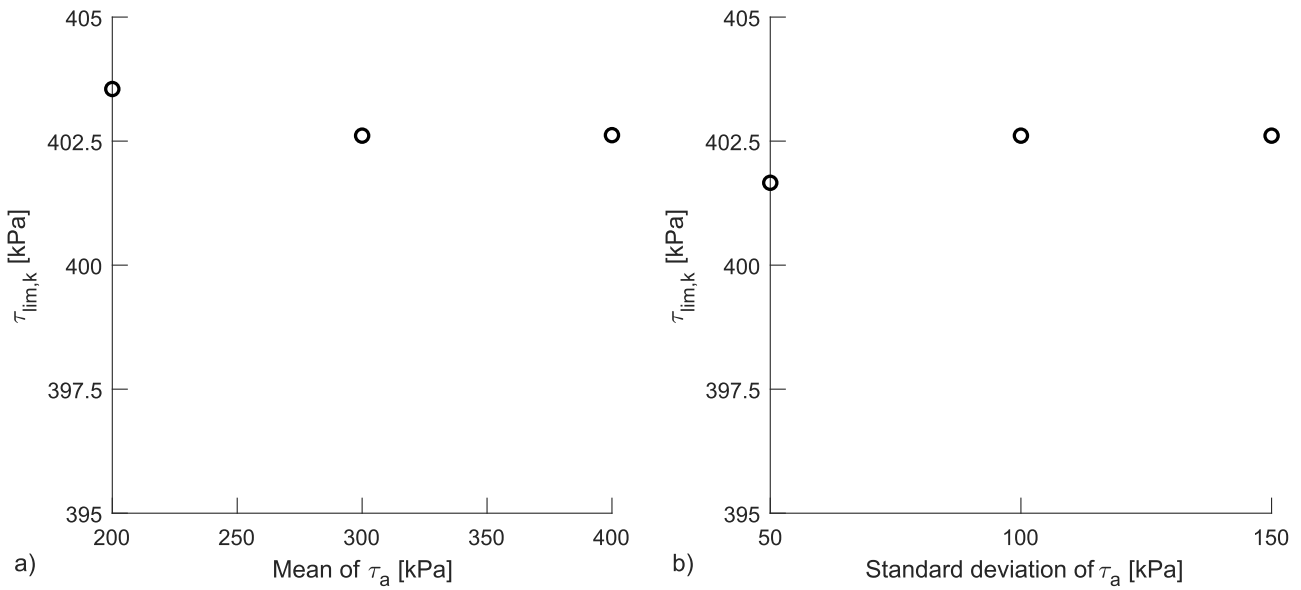


Figure A3. . Influence of the prior distribution of  $\tau_a$  on  $\tau_{lim,k}$ : (a) mean and (b) standard deviation.

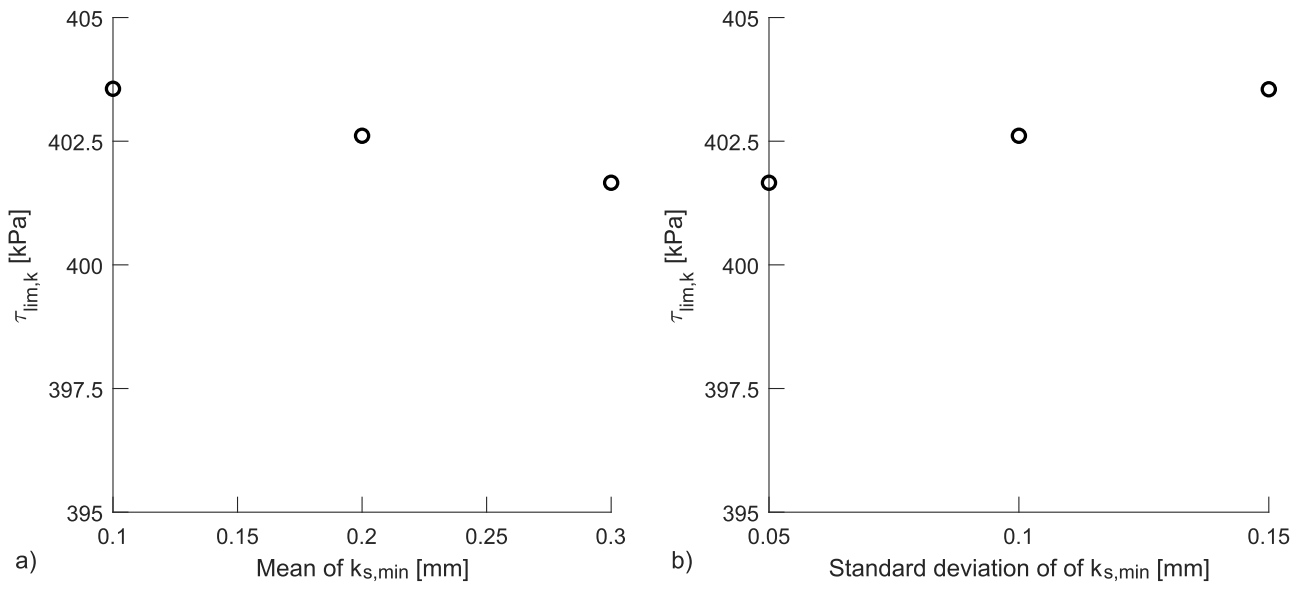


Figure A4. . Influence of the prior distribution of  $k_{s,min}$  on  $\tau_{lim,k}$ : (a) mean and (b) standard deviation.

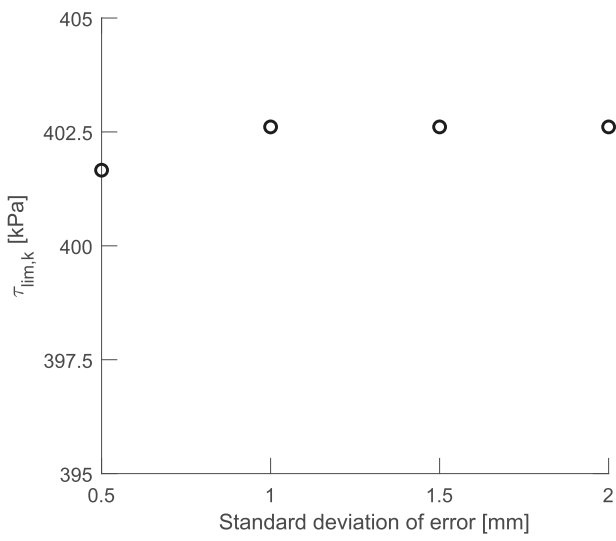


Figure A5. . Influence of the error on  $\tau_{lim,k}$ .