

## Extended CSDT model for shear capacity assessments of bridge deck slabs

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9

10 **Abstract**

11

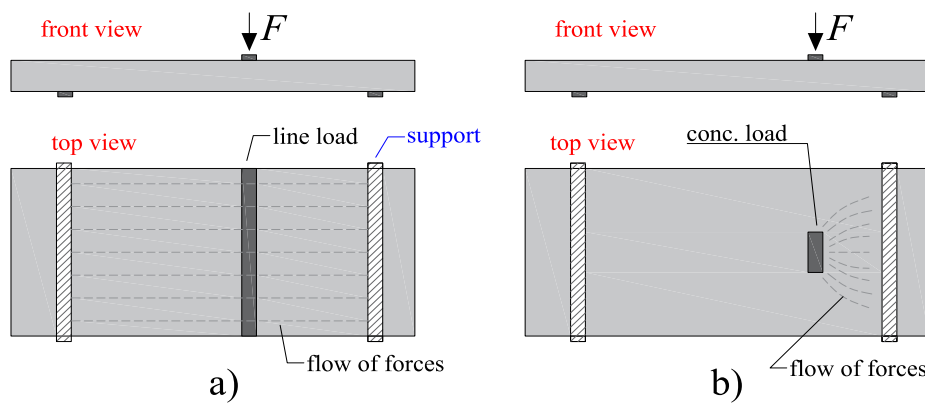
12 The shear strength evaluation of reinforced concrete bridge deck slabs by accurate models can  
13 indicate strength reserves and avoid costly operations necessary for the extension of their lifetime.  
14 This article introduces an approach that extends the Critical Shear Displacement Theory model  
15 (CSDT) for reaching higher levels of approximation of the shear strength for slabs subjected to  
16 concentrated loads close to the support. A database with 141 tests of wide reinforced concrete  
17 members under concentrated loads close to the support failing in one-way shear was built. The tests  
18 represented typical loads in bridge slabs and were assessed through a combination of CSDT with  
19 different models of effective shear width. In other analyses, the entire database with 214 test results  
20 of slabs failing by different mechanisms was evaluated and a general effective shear width model was  
21 proposed (GESW). The best results, which are a function of the effective shear width model used,  
22 reached a mean ratio between experimental and predicted shear capacities of 1.06 with a coefficient  
23 of variation of 14%, which is similar to that reported by some studies including linear and non-linear  
24 finite element analyses. Furthermore, this level of accuracy was insensitive to the shear slenderness  
25 and support conditions of the tests. The extended CSDT predicted the shear capacity of bridge deck  
26 slabs in preliminary analyses more precise than semi-empirical models provided in the current design  
27 codes, and the level of accuracy is comparable to methods using Linear Finite Element Analyses  
28 (LFEA). Moreover, our proposed combination of the CSDT with a general effective shear width  
29 model (GESW) provides reasonable levels of accuracy for slabs under concentrated loads regardless  
30 of the failure mode of the tests. Therefore, the proposed approaches can be applied to bridge deck  
31 slabs, which are subjected to a variety of loading and support conditions.

32 **Keywords:** Bridge deck slabs; Critical shear displacement theory; Database; Effective shear width;  
33 Reinforced concrete; Shear strength;

34

35 **1 INTRODUCTION**

36 The shear capacity of bridge deck slabs attracted attention from several researchers and bridge  
37 owners in Europe in the last decade since a large number of these structures built between 1960 and  
38 1980 have reached the end of their originally devised service life [1–4]. A number of these bridges  
39 do not rate sufficiently for shear according to the currently governing codes, despite no signal of  
40 distress. This result indicated that widely accepted semi-empirical approaches of design could be  
41 overly conservative. Since conservative predictions of shear strength could indicate the need for  
42 replacement or retrofitting of these structures, the identification of more accurate approaches for  
43 predicting the shear capacity of bridge deck slabs involves an economic and environmental issue,  
44 beyond the user's safety. Apart from that, the design of wide reinforced concrete members prioritizes  
45 solutions without shear reinforcement, since installing shear reinforcement is not cost-effective and  
46 may result in reinforcement congestion. Therefore, also in design, the use of precise one-way shear  
47 models can be essential to ensure adequate safety levels for members without stirrups.



48  
49 Figure 1 - Slabs loaded (a) over the entire width analyses by de Sousa et al. [5] and b) under  
50 concentrated loads in non-symmetrical conditions subjected to one-way shear failures.

51 In a previous study on wide beams and one-way slabs loaded over the entire width [5] (Figure  
52 1a), it was identified that the Critical Shear Displacement Theory model (CSDT [6,7]) showed the  
53 best levels of accuracy and precision compared to many semi-empirical and mechanical models of  
54 shear strength, with the mean ratio between experimental and predicted shear capacities of 1.15 and  
55 COV of 16%. Different from previous publications [8,9], de Sousa et al. [5] applied the analyses for  
56 both slender and non-slender members, in addition to different support and loading conditions.  
57 Therefore, it was decided to further assess the CSDT model for slabs under concentrated loads in  
58 non-symmetrical conditions (Figure 1b), with emphasis on the one-way shear capacity.

59 Although the number of studies on shear in reinforced concrete members increased  
60 significantly in the last decade, most of them were focused on the level of precision by semi-empirical

61 code models of shear strength [3,10–13]. In publications that include mechanical-based models [2,14–  
62 16], the analyses focused on one kind of support conditions and hence, covered a reduced number of  
63 tests. At the same time, only a limited number of studies addressed the fact the slabs under  
64 concentrated loads may show a transitional failure mode between one-way and two-way shear [17].  
65 As a consequence, if the governing failure mode is unknown, the use of a one-way shear model to  
66 assess members whose governing failure mode is punching shear may lead to unsafe predictions of  
67 shear strength. Therefore, we identified the need for a more comprehensive study, covering slabs  
68 under different support conditions, assessed by a mechanical-based model such as the CSDT and  
69 accounting for different failure modes that may take place.

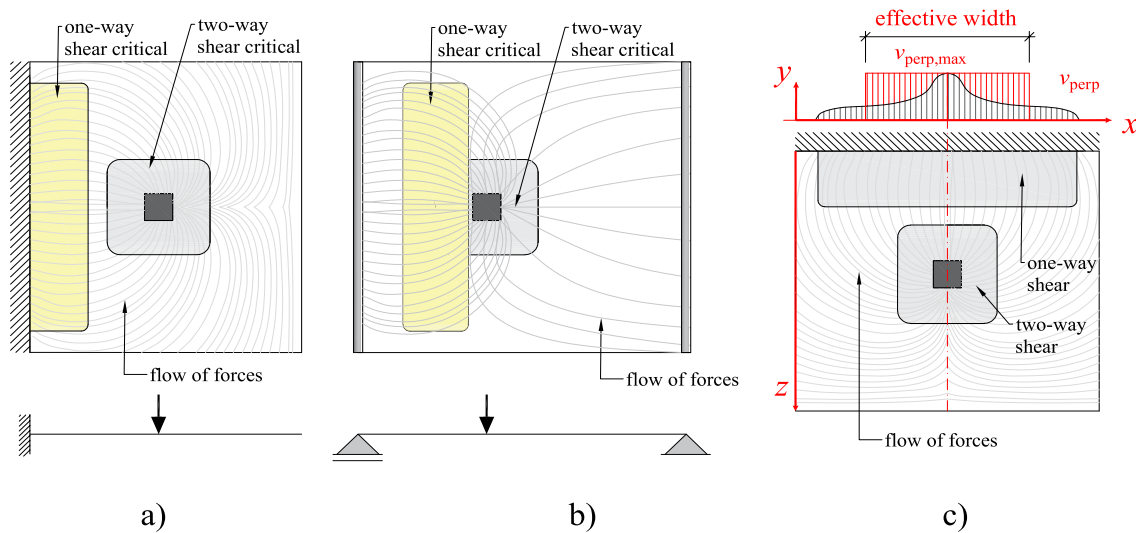
70 In this study, the application of the Critical Shear Displacement Theory Model (CSDT) [6,7]  
71 is extended to the assessment of one-way shear capacity of wide reinforced concrete members under  
72 concentrated loads in non-symmetrical conditions (Figure 1b). Different from previous studies, we  
73 covered a variety of support conditions (cantilevers, simply supported and continuous members;  
74 members under different loading conditions such as single loads and double loads close to the support;  
75 and we provided recommendations when the governing failure mode is known or unknown.

76 The literature was reviewed in order to discuss the influence of the shear slenderness over the  
77 governing failure mode of slabs. Furthermore, models to account the slab behavior under concentrated  
78 loads and approaches to account improved shear capacities for loads close to the support are described  
79 and assessed in the paper. Different databases were used to derive and validate each recommendation  
80 for cases where the governing failure mode is known or unknown and how to account for the higher  
81 shear strength for slabs under concentrated loads close to the support. The application limits and  
82 benefits of each recommendation are highlighted in the paper, which also compares the results with  
83 well-established models from the literature.

84 **2 LITERATURE REVIEW**

85 **2.1 Shear failure modes**

86 One-way shear failure and two-way shear failure or punching can be critical in bridge deck  
87 slabs without shear reinforcement [2,17]. The critical failure mode can vary according to the gradient  
88 of shear forces close to concentrated loads [18]. For slabs loaded over the entire width, the shear force  
89 per unit length is almost constant over the shear span if the self-weight is neglected. On the other  
90 hand, for flat slabs under concentric loads, the gradient of unitary shear forces (shear force per unit  
91 length of the critical perimeter) becomes higher near the loaded region, since the perimeter of the  
92 shear transfer is reduced [18]. Some studies suggest the combination of shear field analyses with one-  
93 way and two-way shear models for the determination of the critical failure mode [2,19], whereas  
94 others already highlight that some tests can show the same capacity for one-way and two-way shear  
95 [20]. This means that the ratio between the one-way shear effects ( $V_{exp}$ ) from the acting punching load  
96 ( $P_{exp}$ ) with the calculated one-way shear capacity ( $V_{calc}$ ) is very similar to the ratio between the acting  
97 punching load ( $P_{exp}$ ) with the calculated punching capacity ( $P_{calc}$ ). Since the most critical failure mode  
98 may change according to the geometry of the load, slab, and support conditions [14], the check of  
99 both failure modes is essential for the assessment of existing structures, where a precise estimation  
100 of the shear capacity is required [20].



101 a) b) c)  
102 Figure 2 - Critical regions of one-way and two-way shear for a) cantilever (adapted from Reißer [21])  
103 and b) simply supported members; c) effective width definition for one-way shear analyses (adapted  
104 from Reißer [21])

105 Figure 2 shows the complex transition between these two failure modes. For cantilever slabs  
106 under concentrated loads, for instance, regions of critical one-way and two-way shear can be better  
107 differentiated for large shear spans (Figure 2a), whereas for simply supported slabs, such regions

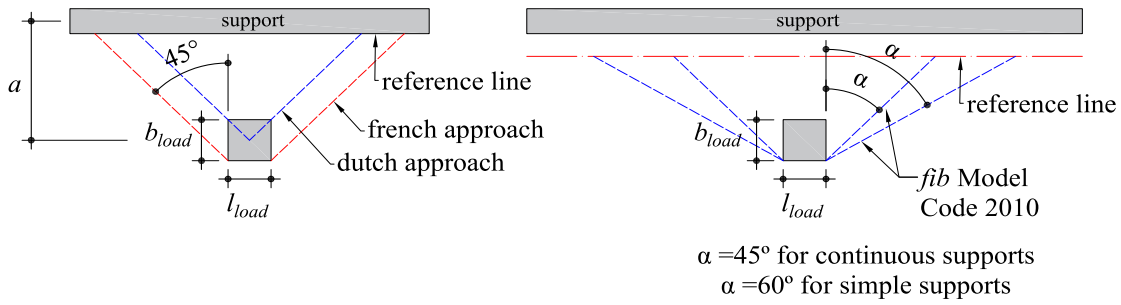
108 intercept each other (Figure 2b). Different studies have agreed on the existence of a trend for the  
109 punching failure mode to become critical for higher shear slenderness [17,20–23].

110 Attention should be drawn to the fact that both one-way shear expressions and punching shear  
111 expressions were derived and calibrated using lab tests designed with idealized boundary conditions.  
112 For instance, one-way shear expressions were derived based on simply supported beam tests with  
113 point loads [24]; and punching shear expressions were based on punching tests on idealized slab-  
114 column connections. One-way slabs typically have boundary conditions and failure modes between  
115 the two types of failure modes, therefore, none of these two types of expressions were developed for  
116 such structures.

## 117 **2.2 Effective shear width**

118 When slabs are subjected to concentrated loads, an effective shear width needs to be defined  
119 together with a one-way shear model, since not the full slab width carries the same shear stress [2,25].  
120 Figure 2c shows the profile of shear stresses over the support as well as the distribution of shear  
121 stresses around the load [2,3,22,25]. Integrating the shear stress  $v_{perp}$  over the width results in the  
122 sectional shear at failure. However, for design, deriving the shear stress distribution over the support  
123 is not practical, and therefore a uniform shear stress is commonly considered over a reduced width,  
124 which is the effective shear width (Figure 2c). The integral of the maximum shear stress  $v_{perp,max}$  over  
125 the effective width should theoretically approach the integral of the shear stress  $v_{perp}$  over the full  
126 width. The values of  $v_{perp,max}$  can be determined by linear elastic finite element (LEFE) analysis with  
127 shell elements adjusting the shear modulus  $G$  and the Poisson ratio  $\nu$  to account for cracking and load  
128 redistribution [2,11,19,26,27]. However, the relevant section may vary according to the shear model  
129 (between  $d$  and  $d/2$  away from discontinuities or at the support) and according to the support and  
130 loading conditions [20].

131 In practice, the effective width is usually defined based on a method of horizontal load  
132 spreading from the concentrated load to the support or a section parallel to the support (Figure 3).  
133 However, some publications already highlighted that the French method (as shown in Figure 3) could  
134 overestimate the effective width in more than 30% for tests with shear slenderness higher than 5 [28].  
135 Physically, this horizontal load spreading can be influenced by factors such as the reinforcement ratio  
136 in the transverse direction [1,3], available member width [3,29], and size of the concentrated load  
137 [21].



138

139 Figure 3 – Models of effective shear width used in design guides with respective reference lines.

140 Table 1 shows an overview of expressions for the effective width in the one-way shear strength  
 141 of reinforced concrete members under concentrated loads at the slab mid-width. For loads close to  
 142 the edge, however, the effective shear width are equal to  $b_r + b_{eff}/2$ , where  $b_r$  is the distance from the  
 143 load axe to the free edge of one-way slabs. Table 1 displays some replaced design code models, e.g.  
 144 the Brazilian code from 1980 [30], since the current codes do not provide recommendations related  
 145 to the effective shear width. According to the table, most code provisions [30–34] and some proposed  
 146 in the literature [22,35,36] assume the effective width increases for larger shear spans. This idea relies  
 147 on the yield line theory [37] and experimental investigations [38], which account for shear forces  
 148 spreading on elastic plates under concentrated loads, also confirmed partially by LEFE analyses [2].  
 149 In summary, most available models of effective shear width do not take into account the change in  
 150 the governing failure mode according to the position of the load [32,34] or were calibrated for specific  
 151 supporting conditions [15]. More consistent models of effective shear width, on the other hand,  
 152 usually require LEFE analyses [2].

153 Table 1 - Overview of analytical models that predict the effective width in analyses of one-way  
 154 shear strength of wide RC members under concentrated loads close to the support.

|                                    |  |
|------------------------------------|--|
| Old Dutch approach [31] (replaced) | $b_{eff1} = b_{load} + 2 \cdot a_v \quad (1)$  |
| French [32]                        | $b_{eff2} = l_{load} + 2 \cdot (b_{load} + a_v) \quad (2)$   |
| Brazilian code [30] (replaced)     | $b_0 = b_{load} + h \quad (3)$ <p>For cantilever members:</p> $b_{NBR} = b_0 + 0.5 \cdot a \cdot \left(1 - \frac{b_0}{l}\right) \leq \max(b_{slab}; a + 0.5 \cdot b_{NBR}) \quad (4)$ <p>For other static systems:</p> $b_{NBR} = b_0 + a \cdot \left(1 - \frac{b_0}{l}\right) \leq \max(b_{slab}; a + 0.5 \cdot b_{NBR}) \quad (5)$ |
| German guideline [39] (replaced)   | $t = b_{load} + 2 \cdot h_1 + h \quad (6)$ <p>For cantilever members:</p>  |



|                              |   |
|------------------------------|---|
|                              | $b_{H240} = \begin{cases} 0.2\ell_k + 0.3a & \text{for: } 0.2\ell_k < a < \ell_k; t_y < 0.2\ell_k; t_x < 0.2\ell_k \\ t_y + 0.3a & \text{for: } 0.2\ell_k < a < \ell_k; 0.2\ell_k < t_y < 0.4\ell_k; t_x < 0.2\ell_k \end{cases} \quad (7)$ <p>For simply supported members:</p> $b_{H240} = t_y + 0.5a \quad \text{for: } 0 < a < \ell, t_y \leq 0.8\ell, t_x \leq \ell \quad (8)$ <p>For loads close to simple support of continuous members:</p> $b_{H240} = t_y + 0.4a \quad \text{for: } 0.2\ell < a < \ell, t_y \leq 0.4 \cdot \ell, t_x \leq 0.2\ell \quad (9)$ <p>For loads close to continuous supports</p> $b_{H240} = t_y + 0.3a \quad \text{for: } 0.2\ell < a < \ell, t_y \leq 0.4\ell, t_x \leq 0.2\ell \quad (10)$ |
| Swedish Code [40] (replaced) | $b_{BBK} = \max \begin{cases} b_{load} + 7 \cdot d_l \\ 0.65 \cdot (b_{load} + l_{load}) + 10.65 \cdot d_l \end{cases} \quad (11)$  |
| fib Model Code 2010 [34]     | $b_{effMC} = l_{load} + 2 \cdot (b_{load} + a_v - \min(d_l; a_v / 2)) \cdot \tan \theta \quad (12)$ $\theta = \begin{cases} 45^\circ, \text{ cantilever of continuous members} \\ 60^\circ, \text{ if load is close to simple support} \end{cases}$   |
| Zheng et al. [41]            | $b_{Zh} = l_{load} + l_{span} \cdot (1 - r_{cp}) \cdot \tan \Phi \quad (13)$ $r_{cp} = \frac{b_{load}}{l_{span}} \leq 0.4 \quad (14)$ $\Phi (^\circ) = 23.3 \cdot r_{cp} + 35.1 \quad (15)$   |
| Bauer [35]                   | $b_{eff,Bauer} = l_{load} + b_{eff1} \quad (16)$  |
| Vidaković and Halvonik [36]  | <p>For cantilever members:</p> $b_{eff,VH} = l_{load} + 2 \cdot (b_{load} + \min(2 \cdot d_l; a_v)) \quad (17)$   |
| Reißen and Hegger [42,43]    | <p>For simply supported members:</p> $b_{eff} = b \cdot \lambda_b \cdot \lambda_{\rho q} \cdot \lambda_l \cdot \lambda_{a/d} \quad (18)$ $\lambda_b = 1.2 - 0.12 \cdot b \leq 1, \text{ for } b \leq 5.5 \text{ m}$ $\lambda_{\rho q} = 0.74 + 2.2 \cdot \rho_q^{0.4}, \text{ for } 0 \leq \rho_q \leq 0.7\%$ $\lambda_l = 0.81 + 0.045 \cdot l \leq 1.04, \text{ for } l \geq 2 \text{ m}$ $\lambda_{a/d} = 1.8 - 0.19 \cdot a/d, \text{ for } 2.91 \leq a/d \leq 5.41 \quad (19)$   |
| Reißen [21]                  | $b_{Reißen} = 7 \cdot d_{l,load} + k_{bf} \cdot l_{load} \quad (20)$ <p>With: <math>d_{l,load} \leq 0.40 \text{ m}</math></p> $k_{bf} = -\frac{5}{8} \cdot \max(a_1; a_2) / d + \frac{9}{4}, \begin{cases} \leq 1 \\ \geq 0.5 \end{cases} \quad (21)$   |
| Rombach and Velasco [44]     | <p>For LEFE analyses:</p> $b_{RV} = 0.6 + 0.95 \cdot h + 1.15 \cdot a \quad (22)$   |
| Natário et al. [2]           | <p>For LEFE analyses on cantilever slabs:</p> $b_{eff} = F_{applied} / v_{avg,4d} \quad (23)$   |
| Shu et al. [45]              | <p>For NLFE analyses on cantilever slabs:</p>   |

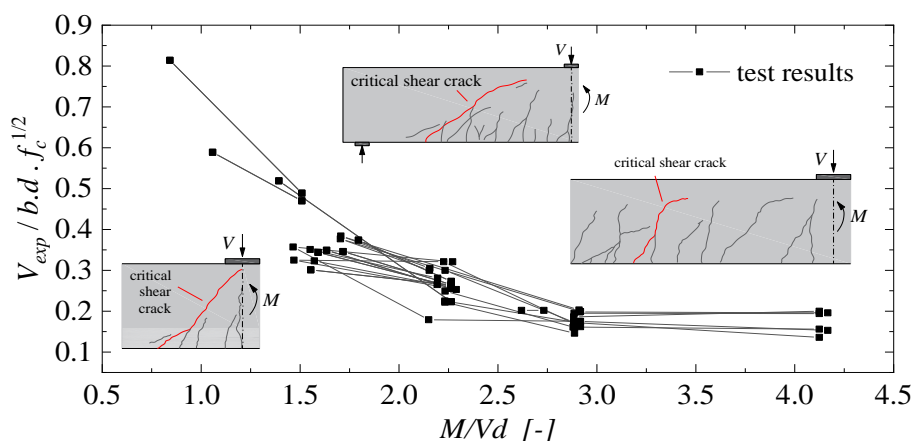
$$b_{eff,Shu} = b_w \cdot \beta_{w1} = b_w \cdot \frac{v_{E,avg}}{v_{R,code}} \quad (24)$$

155

156

## 157 2.3 Failure modes and shear transfer mechanisms in one-way shear

158 Since Kani [46] and Leonhardt and Walther [47], it has been known that different shear failure  
 159 modes can occur as a function of the shear slenderness  $M/Vd$  and that shear strength increases  
 160 considerably for short members. Figure 4 shows the way the nominal shear strength of wide  
 161 reinforced concrete members (width-to-effective depth  $b/d > 1$ ) increases as the shear slenderness  
 162 decreases for tests under concentrated loads (CL) [5]. The figure also shows how the critical shear  
 163 crack shape changes according to the shear slenderness [48]. For concentrated loads close to the  
 164 support, or shear slenderness  $M/Vd < 2.5$ , direct load transfer may occur by compressive struts  
 165 improving the shear capacity. Such members are usually called non-slender members or deep beams  
 166 for beam-shaped members. The higher concentration of compressive stresses between load and  
 167 support usually leads to the crushing of concrete at failure [49]. This failure mode is called shear-  
 168 compression failure [7].



169

170 Figure 4 - Shear slenderness effect on the one-way shear behavior of wide reinforced concrete  
 171 members without stirrups. Adapted from de Sousa et al. [5].

172 Commonly, the same shear strength model derived for flexure-shear failures is used for the  
 173 design and verification of shear strength of non-slender members through the application of a factor  
 174 that reduces the acting shear force  $V_{Ed}$  or improves the shear capacity  $V_R$  in a critical section, as  
 175 suggested in NEN 1992-1-1:2005 [50] and *fib* Model Code 2010 [34]. The shear reduction factor  $\beta$   
 176 from NEN-EN 1992-1-1:2005 [50] first considered only the bending moment effect on the  
 177 compression chord or cantilever action [51]. This means that only the effect of lower crack openings  
 178 and large compression chord depth were taken into account. In fact, the shear strength enhancement  
 179 for non-slender members is caused by a combination of the following mechanisms: (i) higher  
 180 compression chord capacity (cantilever action [18,51]) due to the large compression zone depth [49]  
 181 and (ii) direct load transfer that occurs by compression arch beyond the inclined cracking load (or

182 strut if it has a straight shape), also named arching action [17,52]. In the literature, both mechanisms  
 183 are cited as the source of improved arching action [2].

184 Table 2 shows a summary of the main expressions suggested by different references to account  
 185 for the increase in shear capacity for loads close to supports – past codes used the shear span to depth  
 186 ratio  $a/d$  as the main parameter. The model proposed by Reißer [21] was calibrated for the European  
 187 code shear model and took into account the ratio  $\max\{a_1;a_2\}/d$  ( $a_1$  and  $a_2$  refer to the distances from  
 188 the section of zero bending moment to the support and load axes, respectively) in such a way that it  
 189 provides precise estimations of strength for both simply supported and continuous members. In this  
 190 text, the ratio  $\max\{a_1;a_2\}/d$  has the same meaning as the shear slenderness  $M/Vd$ . Since the influence  
 191 of the shear slenderness is already taken into account in the shear models from *fib* Model Code 2010  
 192 and SIA 262:2013 by the calculations of the internal forces, the  $\beta$  factor takes into account the  
 193 improved arching action only by the clear shear span-to-effective depth ratio  $a_v/d$  as a more  
 194 conservative approach.

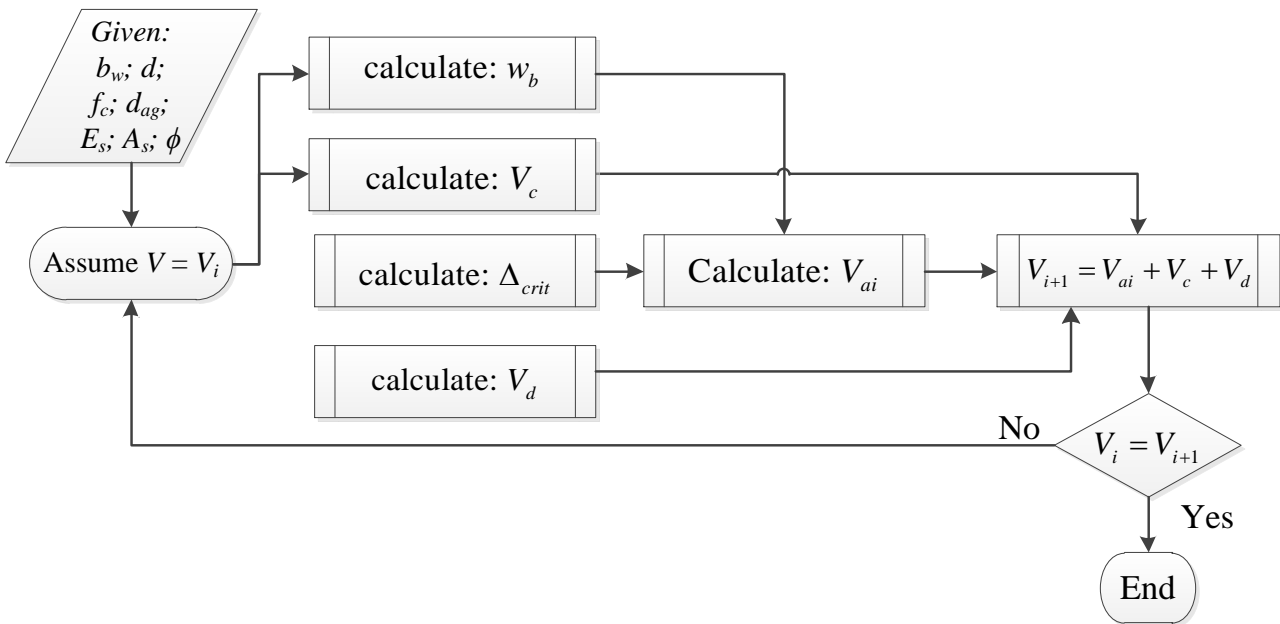
195 Table 2 - Expressions for reducing the acting shear load  $V_E$  for non-slender members according to  
 196 different references.

| Reference  | Model   |
|--|---|
| ABNT NBR 6118:2014 [53] – Brazilian code<br>DIN 1045:1988 [54] – German code | $\beta = \frac{a}{2 \cdot d}$ (25)  |
| DIN 1045-1:2001 [55]– German code  | $\beta = \frac{x}{2 \cdot d}$ (26)<br>$x$ measured from load axis to support edge                       |
| NEN-EN 1992-1-1:2005 [50] – European code                                    | $\beta_{EC} = \frac{a_v}{2 \cdot d} \begin{cases} \leq 1.00 \\ \geq 0.25 \end{cases}$ (27)              |
| <i>fib</i> Model Code 2010 [34]  | $\beta_{MC} = \frac{a_v}{2 \cdot d} \begin{cases} \leq 1.00 \\ \geq 0.50 \end{cases}$ (28)              |
| SIA 262:2013 [56] – Swiss code   | $\beta_{SIA} = \frac{a_v}{2 \cdot d}$ (29)  |
| Reißer [21]  | $\beta_{R16} = \frac{\max\{a_1;a_2\}}{2.8 \cdot d} \begin{cases} \leq 1.0 \\ \geq 0.4 \end{cases}$ (30) |
| Natário et al. [2]   | $\beta_{Nat14} = \frac{a_v}{2.75 \cdot d} \begin{cases} \leq 1.00 \\ \geq 0.50 \end{cases}$ (31)        |
| Yang et al. [57]   | $\beta[M/Vd] = \frac{M}{Vd \cdot 2} \leq 1$ (32)  |

197

198 **2.4 Critical Shear Displacement Theory Model**

199 The Critical Shear Displacement Theory (CSDT) [6,7] assumes that a critical inclined crack  
 200 starts from a major flexural crack, which will lead to collapse when the shear displacement  $\Delta$  of the  
 201 crack reaches a critical value and causes a secondary crack (dowel crack) along the reinforcement. A  
 202 dowel crack causes the detachment of the tensile reinforcement from the concrete along the shear  
 203 span, which significantly reduces the lateral confinement on the crack and the member flexural  
 204 stiffness [7]. Due to the opening of the main crack, an additional vertical shear displacement is  
 205 required for the recovery of the previous shear stress level in the crack, which feeds the growth of  
 206 flexure-shear cracks and leads to a brittle collapse of the member [7].



207  
 208 Figure 5 - Flowchart of the calculations using the CSDT model

209 The CSDT assumes that the shear capacity of RC members without stirrups is resisted by (i)  
 210 compression chord capacity [58], (ii) dowel action [59], and (iii) aggregate interlock [60]. The  
 211 contribution of the residual tensile strength of concrete is neglected at failure [7], and the aggregate  
 212 interlock contribution is a function of the crack width  $w_b$  at the level of the tensile reinforcement and  
 213 derived from the shear displacement  $\Delta$  [61]. Figure 5 and Table 3 show, respectively, a flowchart of  
 214 the calculations for the prediction of shear capacity and the base equations used.

215  
 216  
 217  
 218

Table 3 – Expressions used in the CSDT [6]

| Model   | Expression   |
|---|--|
| General [6]   | $V_u = V_c + V_{ai} + V_d$ (33)  |
| Compression chord [58]  | $V_c = \frac{2}{3} \frac{z_c}{z} V = \frac{d - s_{cr}}{d + s_{cr}} V$ (34)   |
| Aggregate interlock [6]   | $V_{ai} = \text{either of } \begin{cases} R_{ai} \sigma_{pu} \int_0^{s_{cr}} b (A_x(\Delta, w) - \mu A_y(\Delta, w)) ds \\ R_{ai} f_c^{0.56} s_{cr} b \frac{0.03}{w_b - 0.01} (-978\Delta^2 + 85\Delta - 0.27) \\ R_{ai} \int_0^{s_{cr}} \tau_{ai}(\Delta, w) b ds \end{cases}$ (35) |
| Dowel action [59]   | $V_d = 1.64 b_n \phi^3 \sqrt{f_c}$ , $f_c$ in [MPa] (36)   |
| Factors   | Expression   |
| Height of fully developed crack   | $s_{cr} = \left(1 + \rho_l n_e - \sqrt{2\rho_l n_e + (\rho_l n_e)^2}\right) d$ (37)  |
| Critical shear displacement   | $\Delta_{cr} = \frac{25d}{30610\phi} + 0.0022 \leq 0.025$ mm (38)  |
| Crack width at the bottom of the crack                                  | $w_b = \frac{M}{z A_s E_s} l_{cr,m}$ (39)  |
| Reduction factor for aggregate interlock for high-strength concrete [6] | $R_{ai} = 0.85 \cdot \sqrt{\left(\frac{7.2}{f_c - 40} + 1\right)^2 - 1} + 0.34$ with $f_c$ in MPa and $f_c > 65$ MPa (40)  |

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222

### 223 3 DATABASES

224 This study assumes that checking both shear-critical failure modes, one-way and two-way  
225 shear, is essential to identify the governing failure modes of existing bridge deck slabs. Therefore, a  
226 careful classification of the failure modes of tests from the literature is of paramount importance to  
227 understand the limits of application of the available one-way and two-way shear models. Moreover,  
228 this classification allows a fairer assessment of the precision of one-way and two-way shear models,  
229 as well as models of effective shear width for slabs under concentrated loads.

230 This study will discuss the results of three database subsets which are published in the public  
231 domain [62]: (i) wide beams and one-way slabs loaded over the entire width failing in one-way shear  
232 (Database A); (ii) slabs under a single concentrated load failing in one-way shear, two-way shear or  
233 a combination of both (Database B0) and; (iii) slabs subjected to double loads close to the line support  
234 (Database C).

#### 235 3.1 Database filtering and organization

236 The Database B0 includes 214 test results of slabs under single concentrated loads that were  
237 classified according to the main failure mode in (i) wide beam shear or one-way shear (WB), (ii)  
238 punching shear (P) and (iii) transition mode between wide beam shear and punching shear (WB/P).  
239 Since this study focuses on the one-way shear model, tests with signs of punching failure were  
240 initially removed from the database B0, which resulted in the database B1 (141 tests). This filtering  
241 was based on (i) the cracking pattern of the members, when available in the original references and  
242 (ii) the classification reported by other authors [10,63], which was also based on the cracking pattern  
243 and (iii) in the classification of Natário [20], who combined shear fields from LEFE analyses with  
244 one-way shear and punching shear models according to the Critical Shear Crack Theory (CSCT)  
245 [2,19,20]. The main criteria used for the removal of members because of a punching failure in this  
246 study were (i) absence of a critical shear crack visible on the edge of the members and (ii) position at  
247 which the critical shear crack intercepted the middle depth of the member when the cut view was

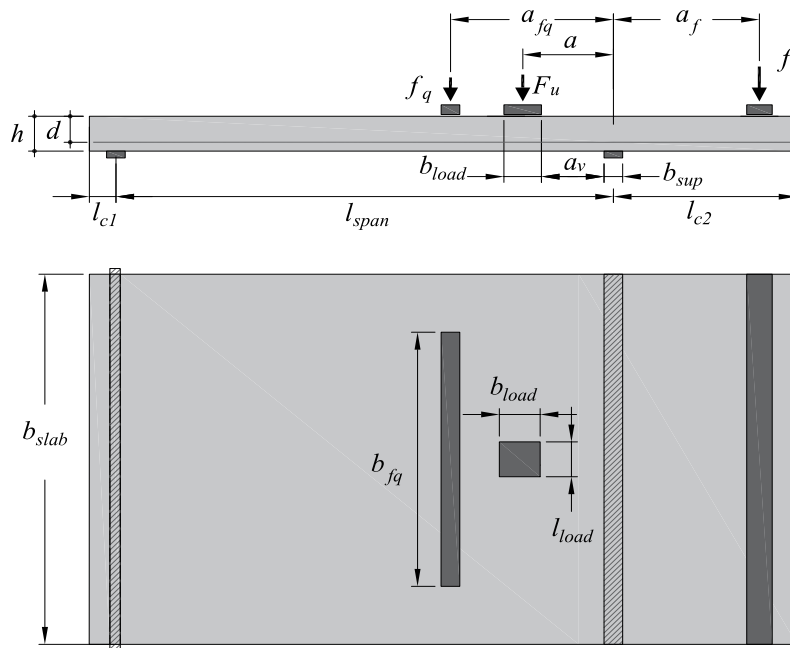
248 available. When the internal cracking pattern was not shown in the references, it was considered a  
249 punching failure if the cracking pattern was predominantly formed by radial and tangential cracks or  
250 if a conical crack could be seen.

251 The database B1 of slabs under concentrated loads after removing punching tests  
252 comprehends 141 test results from the following references: Cullington et al. [64], Lantsoght [63],  
253 Reußen [21], Lubell [65], Bui et al. [66], Regan [67], Regan and Rezai-Jarobi [68], Vaz Rodriguez  
254 et al. [69], Rombach and Latte [70,71], Natário et al. [2,72], Rombach and Henze [73], and Vida et  
255 al. [74]. The database entries include the effect of self-weight on the calculated shear capacities and  
256 on the shear slenderness parameters for continuous members.

257 The database B1, whose organization was inspired by those of Lantsoght et al. [10], Reußen  
258 [21] and Henze et al. [11], has been published in the public domain [62] and includes 46 tests on  
259 cantilever members (CT), 33 tests with concentrated loads close to the internal support of continuous  
260 members (CS), and 62 tests with concentrated loads close to the simple supports (SS). It also includes  
261 two modes of one-way shear failures, namely shear-compression failures for non-slender members,  
262 or shear slenderness  $M/Vd < 2.5$  (55 tests  $\equiv$  39 %), and flexure-shear failures for slender members, or  
263 shear slenderness  $M/Vd \geq 2.5$  (86 tests  $\equiv$  61%).

264 Figure 6 displays the main geometrical loading parameters in the database for members with  
265 continuity over line supports and subjected to a combination of concentrated loads and line loads.  
266 The same definitions have been used for other structural systems.

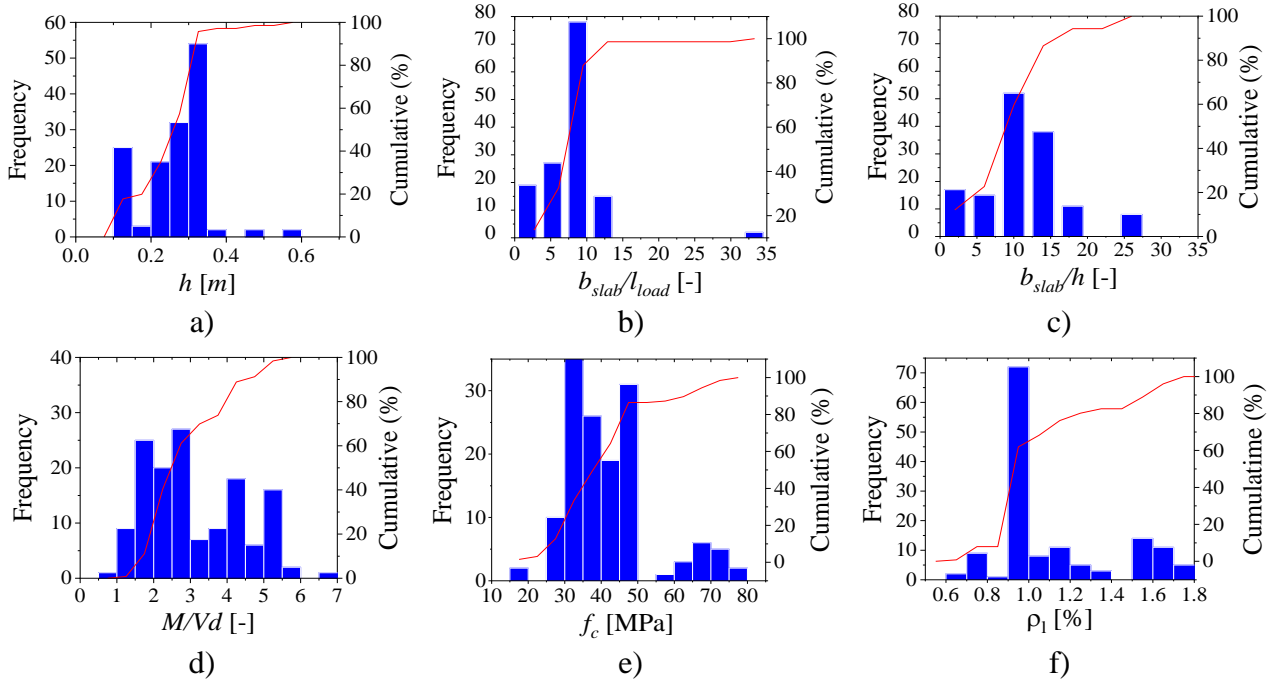




267

268 Figure 6 - Geometrical parameters of wide members with continuity over the support.

269 Figure 7 shows the distributions of the parameters related to the tests included in the database  
 270 B1. Similar to beam-databases [75,76], most experiments were performed for members of thicknesses  
 271 less than 600 mm (Figure 7a) and on wide members whose ratio between the slab width and load  
 272 dimensions in the width direction was higher than 5 (Figure 7b). The full width of members with  
 273  $b_{slab}/l_{load} < 5$  was probably activated in the test, depending on the distance from the load to the support.  
 274 However, as some models of effective width are overly conservative, some predictions may indicate  
 275 that the full width was not mobilized. Figure 7c shows that the  $b_{slab}/h$  aspect ratio (Figure 11c) was  
 276 higher than 5 in more than 75% of the tests, and Figure 7d highlights the number of tests in the  
 277 database performed with a shear slenderness  $M/Vd$  between 2 and 3. This range indicates that a  
 278 considerable number of tests were subjected to a transitional failure mode between shear-compression  
 279 and flexure-shear. Figure 7e show that 16 tests from the database have a concrete compressive  
 280 strength larger than 60 MPa and, hence, the level of accuracy for members with reduced aggregate  
 281 interlock may be assessed. Figure 7f shows that the longitudinal reinforcement ratio ranges between  
 282 0.6 and 1.8%, where the larger ratios may not be representative of those used in bridge deck slabs.

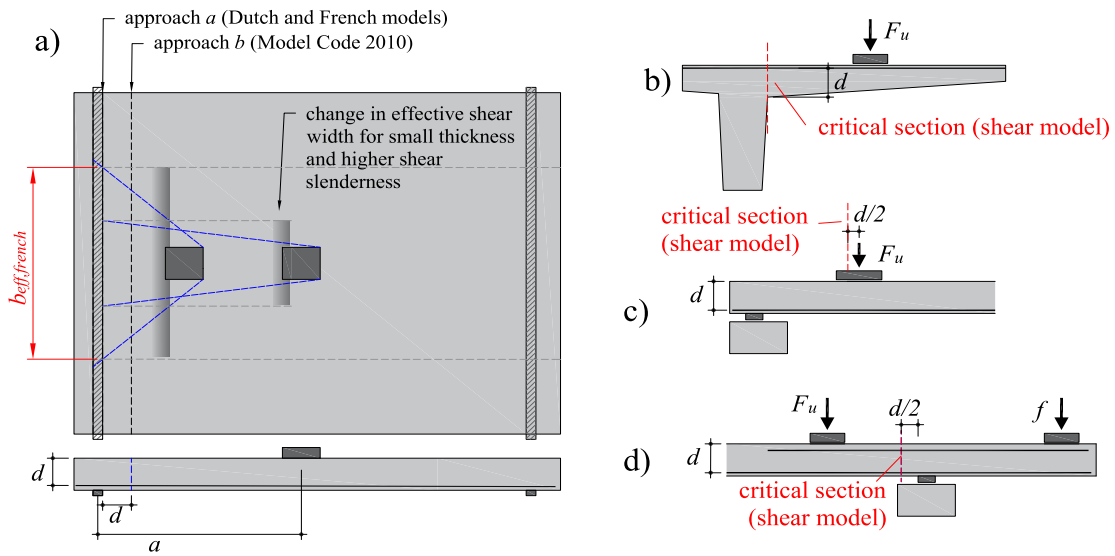


283 Figure 7 – Distribution of parameters in the database B1 for the following parameters: a) thickness  
 284 of the slab at the support edge, b) ratio of slab width-to-load dimension in the width direction, c) ratio  
 285 of slab width-to-effective depth, d) shear slenderness; e) concrete compressive strength and f)  
 286 longitudinal reinforcement ratio.

287 **4 PROPOSED RECOMMENDATIONS**

288 **4.1 Section for internal forces calculations**

289 Since most mechanical based models of shear strength were derived for shear slenderness  
290  $M/Vd$  higher than 2.5, the assumption of the section far from  $d$  or  $d/2$  from the highest bending  
291 moment axes [6,77] or from geometrical discontinuity [34] does not play an important influence.  
292 However, when using these models for lower slenderness ( $M/Vd < 2.5$ ), the location of this section  
293 assumes a major influence. Because of this, a previous investigation was made in order to identify  
294 the section that could balance precision and safety for the ratio  $V_{exp}/V_{cal}$  in both ranges of shear  
295 slenderness and for different support conditions (Figure 8b,c,d). Assuming that the shear capacity is  
296 reduced due to an increase in the opening of the critical shear crack [6,77,78], the control section for  
297 the calculations of the internal forces  $M_{Ed}$  and  $V_{Ed}$  remain at sections close to the higher bending  
298 moment for all models. However, the critical section at the support edge of cantilever slabs was used  
299 instead of the section at  $d$  or  $d/2$  from the support edge in order to reach better predictions for these  
300 support conditions [5].



301  
302 Figure 8 – a) reference lines to calculate the effective shear width in French model [32] and  
303 proposed approach; critical sections used for b) cantilever members, c) simply supported members,  
304 and d) continuous members.

305 **4.2 Arching action**

306 This study proposes to combine the CSDT result with a semi-empirical coefficient  $\beta$  based on  
307 the ratio  $a_v/d$  (Equation (41)) to extend the CSDT model to predict the shear capacity of non-slender  
308 members without additional iterative calculations:

309

$$\beta_{\text{Prop}} = \frac{a_v}{2.5 \cdot d} \begin{cases} \leq 1.00 \\ \geq 0.40 \end{cases} \quad (41)$$

310

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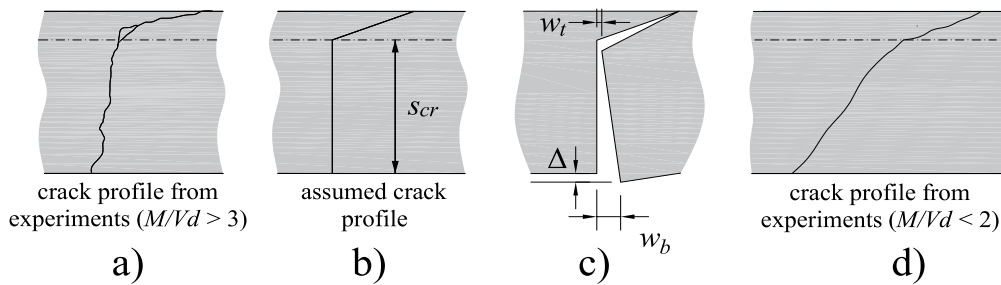
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The combination of the CSDT with reduction factor  $\beta$  for non-slender members should be understood as an engineering approach comparable to empirical simplifications used by most design codes [34,50] and strain-based models [2]. Theoretically, this approach is not exact because the shear failure mechanism for non-slender members is different from that for slender ones: the shape and relative contribution of the main shear-transfer mechanisms vary significantly when the shear slenderness decreases since the vertical branch of the assumed crack profile of the critical shear crack becomes not representative anymore (Figure 9b).



317

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Figure 9 - a) and b) Crack profile simplification for specimens with  $M/Vd > 3$ , c) main parameters of CSDT, and d) crack profile for non-slender members ( $M/Vd < 2$ ).

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For lower shear slenderness, the inclination of the major flexural crack increases in such a way that the contribution of the aggregate interlock decreases significantly, while the contribution of the compression chord  $V_c$  increases according to internal equilibrium [79]. The use of strut-and-tie models for continuous members with maximum shear slenderness  $M/Vd < 2$  may better represent the problem [80]: plane sections do not remain plane, and shear strains become dominant for those members [81]. However, this approach may not be practical for the slabs studied since the problem is strongly three-dimensional. As such, for practical purposes, we consider the choice of including  $\beta$  as adequate.

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### 4.3 Effective shear width

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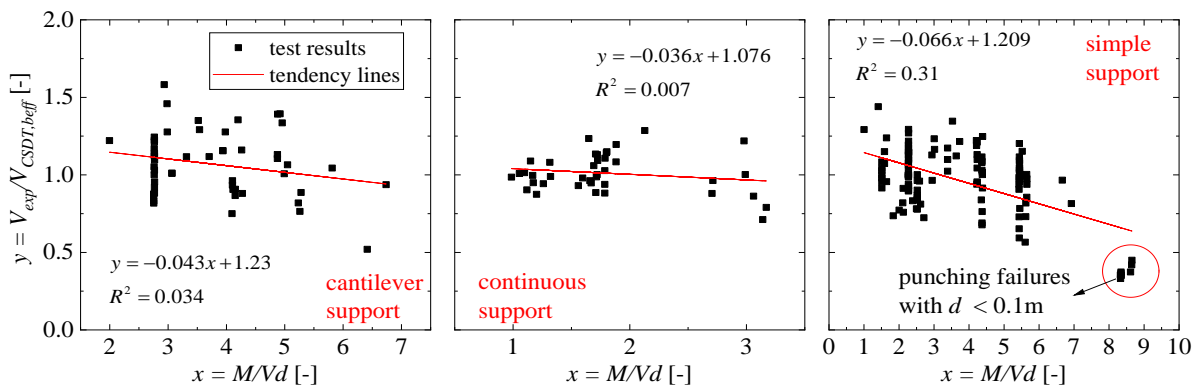
In design and assessment of existing structures, two kinds of analyses may occur (i) the governing failure mode is unknown, and a conservative prediction of the shear capacity may be adequate for preliminary design, and (ii) a more precise estimation of the shear capacity is required, usually in the assessment of existing structures preliminarily rated as critical in shear [82]. In the latter case, a detailed analysis of the governing failure mode would be essential to determine the shear capacity, which requires LEFE analyses combined with a mechanical-based model, such as conducted

335 by Natário [20] or using one-way and two-way shear models adjusted to slabs under concentrated  
 336 loads in non-symmetrical conditions.

337 Since the governing failure mode of the tests in the database B1 is known, we proposed in this  
 338 study two kinds of analyses. The first group of analyses investigates the accuracy of different effective  
 339 shear width models combined with the CSDT model using a database with the governing failure mode  
 340 known (one-way shear – Database B1). From these analyses, we derive recommendations for the  
 341 assessment of existing structures when the governing failure mode is known (one-way shear), and  
 342 precise estimation of the shear capacity is the main purpose.

343 The second group of analyses aims to assess the shear capacity of slabs when the governing  
 344 failure mode is unknown (Database B0). This means that one-way or two-way shear failures were  
 345 included in the analyses. In order to provide consistent predictions of shear capacity regardless of the  
 346 critical failure mode and covering different support conditions, the General Effective Shear width  
 347 model (GEWS) was developed accounting that if punching failure governs, the predicted one-way  
 348 shear capacity should be decreased by predicting a smaller effective shear width.

349 The idea of the GESW model is to provide a simple alternative to assess the shear capacity of  
 350 slabs using only a one-way shear model combined with an effective shear width. The proposed model  
 351 is based on the French effective shear width model [32] adjusted by a correction factor  $\alpha$ . This factor  
 352 considers that increasing the shear slenderness ( $\lambda=M/Vd$ ) or decreasing the effective depth of the  
 353 reinforcement, the punching shear failure becomes governing. Therefore, a reduced effective shear  
 354 width should be predicted for slabs on which punching shear may be critical. The values of  $\alpha$  were  
 355 derived based on regression analyses to improve the average and coefficient of variation of the ratio  
 356  $V_{exp}/V_{calc}$  with the CSDT model combined with the French model of effective shear width. These  
 357 regression analyses were organized according to the support conditions of the tests (Figure 10).



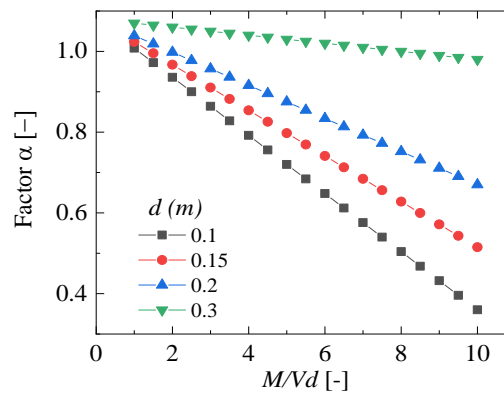
358  
 359 Figure 10 - Ratio of  $V_{exp} / V_{cal}$  of the CSDT combined with the original French effective shear width  
 360 model and  $\beta_{proposed}$  to account improved arching action for loads close to the support.

361 Based on the literature review (Section 2) and parameters influence of  $V_{exp}/V_{cal}$  according to  
 362 the Database B0 similar to showed in Figure 10, we identified that the shear slenderness parameter  
 363  $M/Vd$  would be the most important parameter to be considered in the GESW for all support  
 364 conditions. Table 4 shows the equations for the proposed model of effective shear width. Figure 8a  
 365 and Figure 11 illustrates this idea for simply supported slabs of small thickness. In Table 4, we  
 366 considered the effective depth  $d$  only for simply supported members by two reasons: (i) the thickness  
 367 variation is small in the database for other support conditions and (ii) to improve the predictions of  
 368 tests with punching failure and effective depth lower than 0.1 m (Figure 10). At this point, we  
 369 highlighted that this approach seeks to provide a model for design or preliminary assessment of  
 370 existing structures. When higher levels of approximation are required, the use of one-way and two-  
 371 way shear models is essential to determine the governing failure mode, as we will discuss in the next  
 372 sections.

373 Table 4 - General effective shear width model proposed (GESW) according to the support conditions,  
 374 shear slenderness  $\lambda=M/Vd$ , and effective depth  $d$  of the longitudinal reinforcement.

|                    |  |
|--------------------|--|
| General model      | $b_{GESWM} = b_{eff, French} \cdot \alpha$ (42)  |
| Cantilever slabs   | $b_{GESWM} = b_{eff, French} \cdot (-0.05 \cdot \lambda + 1.05)$ (43)                  |
| Simple support     | $b_{GESWM} = b_{eff, French} \cdot [(0.31 \cdot d - 0.103) \cdot \lambda + 1.08]$ (44) |
| Continuous support | $b_{GESWM} = b_{eff, French} \cdot (-0.072 \cdot \lambda + 1.08)$ (45)                 |

375



376

377 Figure 11 – Variation of the factor  $\alpha$  according to the shear slenderness and effective depth of  
 378 reinforcement for the simply supported slabs.

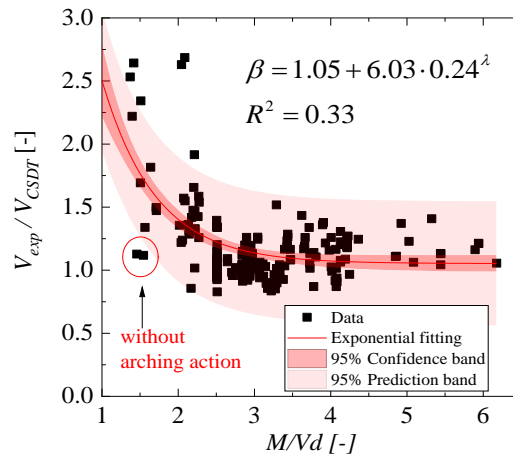
## 379 5 RESULTS

380 This section addresses a comparison between the experimental shear strengths from the  
381 databases A, B0, B1 and C [62], and those predicted by the CSDT model. Firstly, the level of accuracy  
382 (average value - AVG) and precision (coefficient of variation - COV) of the  $V_{exp}/V_{cal}$  ratio for a  
383 database of wide members loaded over the entire width was assessed according to the shear  
384 slenderness, with no influence of the effective shear width model (Database A -Section 5.1). In a  
385 second step, the analyses involved the database of slabs under single concentrated loads failing in  
386 one-way shear (Database B1 - Sections 5.2, 5.3 and 5.4). Then we compared the predicted one-way  
387 shear capacities with the experimental ones for 8 tests of slabs subjected to double concentrated loads  
388 parallel to the support (Database C - Section 5.5). Finally, we discuss the results of analyses conducted  
389 for the overall database of slabs under single loads (Database B0- Sections 5.6) using one-way and  
390 two-way shear models.

### 391 5.1 Members loaded over the full width – Proposal for $\beta_{arching}$

392 This analyses aims to assess only the proposed model regarding the improved arching action  
393 for non-slender members, without the influence of the effective shear width models. For this study, a  
394 database of wide beams and one-way slabs loaded over the entire was used (Database A). This  
395 database is published in the public domain [62] and covers different support conditions and a  
396 comprehensive range of shear slendernesses. The database includes 36 tests with  $M/Vd \leq 2.5$  and 146  
397 tests with  $M/Vd > 2.5$ .

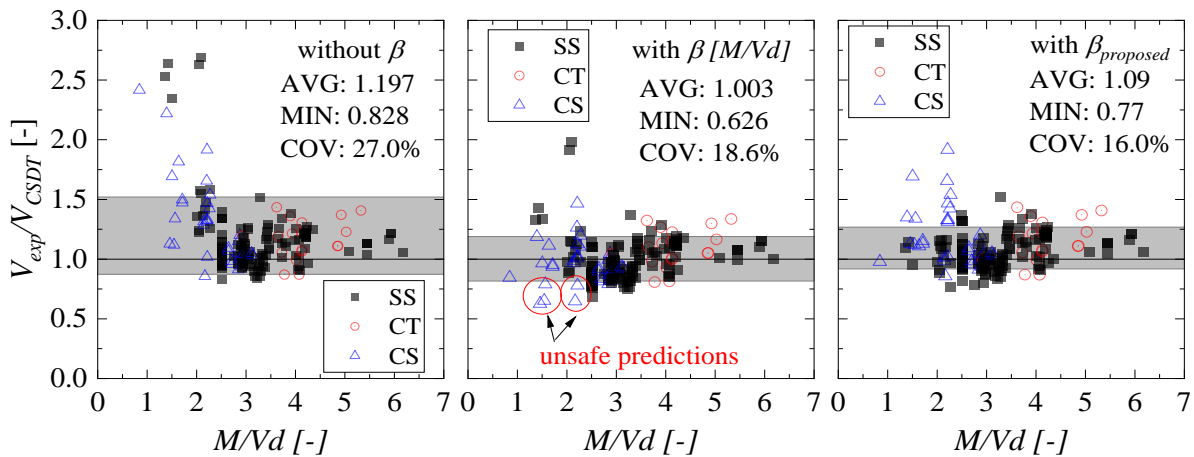
398 Figure 12 shows a  $\beta$  factor derived based on a regression analysis with exponential adjustment  
399 according to the shear slenderness  $\lambda=M/Vd$ . This graph highlighted that the scatter between predicted  
400 and calculated shear strengths in the range of shear slenderness lower than 3 is considerably higher  
401 compared to the other range. This occurs because the arching action is highly influenced by the  
402 cracking pattern, which shows a higher variability for short slenderness [57]. Since the CSDT model  
403 already takes into account the shear slenderness by the calculations of  $M_{Ed}$  and  $V_{Ed}$ , the  $\beta$  factor based  
404 on the ratio  $M/Vd$  could lead to overly optimistic predictions of resistance, mainly when arching  
405 action does not play an influence as a result of the occurring cracking pattern (see test without arching  
406 action in Figure 12). Because of this, some authors proposed to adopt the inclined cracking load  
407 instead of the ultimate shear load as the failure criterion since this parameter shows a considerably  
408 lower scatter [57]. However, as most references do not report the inclined cracking load for slab tests  
409 as this cracking is harder to observe in slabs under concentrated loads than in beam members, the  
410 ultimate shear load was considered in the regression analyses.



411

412 Figure 12 – Alternative  $\beta$  factor derived based on exponential fitting between experimental and  
 413 predicted shear strengths. Note:  $\lambda = M/Vd$ .

414 Figure 13 shows the  $V_{exp}/V_{calc}$  ratio according to the shear slenderness by including or not  
 415 different approaches for improved arching action for non-slender members. The gray ranges represent  
 416  $\pm 1$  standard deviation from the mean value.



417

418 Figure 13 - Effect of factor  $\beta$  on the statistics of  $V_{exp}/V_{cal}$  for tests loaded over the entire width (line  
 419 loads). (CS = continuous support; CT = cantilever support and SS = simple support).

420 According to Figure 13, applying an improved arching action factor with the CSDT reduces  
 421 the coefficient of variation from 24.7% to 18.6% when using  $\beta[M/Vd]$  (Figure 12), and to 16.0% when  
 422 using  $\beta_{proposed}$  (Equation (41)). Any approach shows a wider scatter between experimental and  
 423 predicted shear capacities for continuous members (CS), due to the higher variability in the position  
 424 of the critical shear crack. Although the theoretical critical section was at  $d/2$  from the position with  
 425 the maximum bending moment, this procedure is still conservative for most tests. Table 5 shows that  
 426 the average (AVG)  $V_{exp}/V_{cal}$  ratio ranged from 1.197 to 1.093 with the proposed factor  $\beta_{prop}$ . In Table  
 427 5,  $V_{exp,red}$  refers to the experimental shear capacity reduced by the different parameters  $\beta$ . The lower  
 428 scatter between experimental and predicted shear capacities occurred with the  $\beta_{proposed}$  and  $\beta_{EC}$ .



429 Table 5 - Statistical results of the predicted to calculated shear strengths according to different  
 430 approaches to account the arching action for non-slender members.

|          | $\frac{V_{exp}}{V_{CSDT}}$ | $\frac{V_{exp,red}}{V_{CSDT}}$ | $\frac{V_{exp,red}}{V_{CSDT}}$      | $\frac{V_{exp,red}}{V_{CSDT}}$    |
|----------|----------------------------|--------------------------------|-------------------------------------|-----------------------------------|
| Approach | <i>Without</i><br>$\beta$  | <i>With</i><br>$\beta_{EC}$    | <i>With</i><br>$\beta_{Figure\ 12}$ | <i>With</i><br>$\beta_{proposed}$ |
| AVG      | 1.197                      | 1.134                          | 1.003                               | 1.093                             |
| MIN      | 0.828                      | 0.828                          | 0.626                               | 0.770                             |
| COV      | 0.270                      | 0.172                          | 0.186                               | 0.160                             |

431

## 432 5.2 Effective shear width models

433 The database B1 gathered according to the descriptions in Section 3 [62] was used in the next  
 434 analyses of the level of accuracy of the CSDT combined with different approaches for the effective  
 435 shear width. Table 6 and Table 7 show statistical results from the  $V_{exp}/V_{calc}$  ratio for different ranges  
 436 of shear slenderness  $\lambda=M/Vd$ . The results are shown as a function of the ratio  $M/Vd$  instead of the  
 437 ratio  $a_v/d$  since the former is a more useful parameter to distinguish members subjected to shear-  
 438 compression failure from those subjected to flexure-shear failures, mainly for continuous slabs.  
 439  $\beta_{proposed}$ , which accounts for improved arching action for non-slender members, was adopted in most  
 440 analyses. Since some tests did not fulfill conditions related to load dimensions for use the effective  
 441 width from the German guidelines [39], Equations (6) to (10) in Table 1, this model was not evaluated.  
 442 In the same way, the model provided by Halvonik et al. [15] was not evaluated since this was purposed  
 443 only for cantilever specimens.

444 According to Table 6, older design code models of effective shear width, such as the Brazilian  
 445 model [30], lead to overly conservative predictions in most cases (mean  $V_{exp,red}/V_{CSDT}= 1.772$ ). The  
 446 Swedish provisions [40], on the other hand, lead to unsafe predictions, with average ratios of  $V_{exp}/V_{cal}$   
 447 of 0.706 and 0.981 in the different ranges of shear slenderness evaluated. The *fib* model of effective  
 448 shear width leads to average values of  $V_{exp}/V_{cal}$  of 1.092 and 1.192 for non-slender and slender  
 449 members, respectively. The best accuracy and precision over the different slenderness ranges assessed  
 450 were achieved by the French model of effective shear width [32], thus indicating, on average, that the  
 451 French approach provides reasonable predictions of effective shear width for slabs failing in one-way  
 452 shear.

453

454 Table 6 - Statistics of  $V_{exp}/V_{calc}$  according to the range of shear slenderness  $\lambda = M/Vd$  and effective  
 455 width model provided in design codes.

|            |     |                   | $\frac{V_{exp,red}}{V_{CSDT}}$ | $\frac{V_{exp,red}}{V_{CSDT}}$ | $\frac{V_{exp,red}}{V_{CSDT}}$ | $\frac{V_{exp,red}}{V_{CSDT}}$ |
|------------|-----|-------------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|
|            |     | $b_{eff}$         | <b>ABNT</b>                    | <b>Swedish</b>                 | <b>French</b>                  | <b>Fib</b>                     |
| $\lambda$  | N   | $\beta_{arching}$ | <b>Prop</b>                    | <b>Prop</b>                    | <b>Prop</b>                    | <b>Prop</b>                    |
| <2.5       | 55  | AVG               | 1.401                          | 0.706                          | 1.044                          | 1.092                          |
|            |     | MIN               | 0.869                          | 0.481                          | 0.773                          | 0.758                          |
|            |     | COV               | 21.9%                          | 23.5%                          | 11.4%                          | 20.8%                          |
| $\geq 2.5$ | 86  | AVG               | 2.009                          | 0.981                          | 1.070                          | 1.192                          |
|            |     | MIN               | 0.818                          | 0.438                          | 0.723                          | 0.513                          |
|            |     | COV               | 40.8%                          | 18.7%                          | 15.8%                          | 23.7%                          |
| All        | 141 | AVG               | 1.772                          | 0.873                          | 1.060                          | 1.153                          |
|            |     | MIN               | 0.818                          | 0.438                          | 0.723                          | 0.513                          |
|            |     | COV               | 41.2%                          | 25.4%                          | 14.3%                          | 23.0%                          |

456

457

458 Table 7 - Statistics of  $V_{exp}/V_{calc}$  according to the range of shear slenderness  $\lambda = M/Vd$  and effective  
 459 width models suggested in the literature.

|            |     |                   | $\frac{V_{exp,red}}{V_{CSDT}}$ | $\frac{V_{exp}}{V_{CSDT}}$ | $\frac{V_{exp,red}}{V_{CSDT}}$ | $\frac{V_{exp,red}}{V_{CSDT}}$ |
|------------|-----|-------------------|--------------------------------|----------------------------|--------------------------------|--------------------------------|
|            |     | $b_{eff}$         | <b>Zheng</b>                   | <b>Reissen</b>             | <b>Bauer</b>                   | <b>Prop (GESW)</b>             |
| $\lambda$  | N   | $\beta_{arching}$ | <b>Prop</b>                    | -                          | <b>Prop</b>                    | <b>Prop</b>                    |
| < 2.5      | 55  | AVG               | 0.755                          | 1.601                      | 1.401                          | 1.043                          |
|            |     | MIN               | 0.512                          | 1.113                      | 0.869                          | 0.748                          |
|            |     | COV               | 22.7%                          | 17.7%                      | 21.9%                          | 12.2%                          |
| $\geq 2.5$ | 86  | AVG               | 1.302                          | 1.638                      | 2.009                          | 1.295                          |
|            |     | MIN               | 0.459                          | 0.983                      | 0.818                          | 0.830                          |
|            |     | COV               | 34.0%                          | 29.6%                      | 40.8%                          | 19.2%                          |
| All        | 141 | AVG               | 1.089                          | 1.624                      | 1.772                          | 1.197                          |
|            |     | MIN               | 0.459                          | 0.983                      | 0.818                          | 0.748                          |
|            |     | COV               | 41.3%                          | 25.7%                      | 41.2%                          | 20.3%                          |

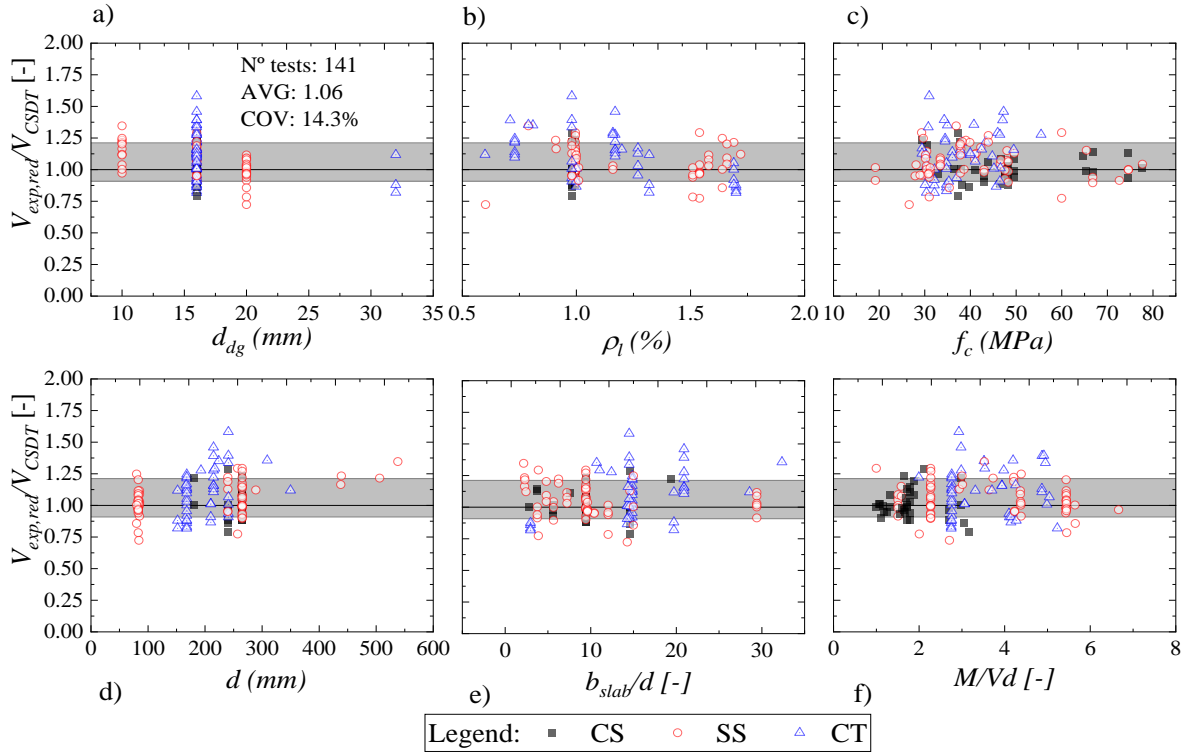
460

461 Table 7 shows that the average value of  $V_{exp}/V_{cal}$  ranged from 1.089 to 1.772 with the  
 462 approaches studied for the definition of an effective shear width. The effective width from Zheng et  
 463 al. [41] provided a wider scatter between experimental and predicted shear capacities (COV > 30%  
 464 on average) and a  $V_{exp}/V_{cal}$  mean value much lower than 1 for non-slender members. Since the  
 465 approach of Reissen [21] includes the effect of the improved arching action on non-slender members  
 466 in the effective width model by using the factor  $k_{bf}$  (Equation (21) in Table 1), the experimental shear  
 467 capacities were not reduced for these calculations. The model provided conservative predictions of

468 shear strength for all tests assessed and a 25.7% coefficient of variation. Bauer and Muller's approach  
469 [83] resulted in the most conservative predictions for slender members ( $V_{exp}/V_{cal} = 2.009$ ), but with a  
470 wider scatter (COV = 40.8%). The proposed GESW model provided good AVG (1.197) and COV  
471 (20.3%) values compared to the other models. Comparing Table 6 and Table 7, both GESW model  
472 and French model provide an accurate estimation of the test results. However, the GESW turns out  
473 to be slightly more conservative for Database B1 as it was derived to assess slabs under both failure  
474 modes (one-way shear and two-way shear).

475 **5.3 Sensitivity of parameters**

476 Since the first purpose is to derive recommendations for precise predictions of shear strength  
 477 when the one-way shear failure mode is governing (Database B1), the CSDT combined with the  
 478 French effective shear width model and  $\beta_{prop}$  was further assessed with parameter studies (Figure 14).



479  
 480 Figure 14 -  $V_{exp}/V_{cal}$  ratio as a function of the main mechanical and geometrical parameters for wide  
 481 members subjected to concentrated loads close to the support with predominant one-way shear  
 482 failure: a) aggregate size  $d_g$ , b) longitudinal reinforcement ratio  $\rho_l$ , c) concrete compressive strength  
 483  $f_c$ , d) effective depth  $d$ , e) width-to-effective depth ratio  $b/d$ , and (e) shear slenderness  $M/Vd$ . (CS =  
 484 continuous support; CT = cantilever support and SS = simple support).

485 Figure 14 shows the ratio of  $V_{exp,red}/V_{cal}$  as a function of different parameters. The results  
 486 indicate no significant influence of the aggregate size (Figure 14a) and reinforcement ratio (Figure  
 487 14b) on the predictions of shear strength with the studied approach. Wider scatter in some regions  
 488 (see Figure 14a) for 16 mm aggregate size can be assigned to a higher number of tests. Figure 14c  
 489 shows that the CSDT provides accurate and precise predictions of shear strength for members of high  
 490 strength concrete ( $f_c > 65$  MPa), for which a lower contribution of the aggregate interlock is accounted  
 491 for by the parameter  $R_{ai}$  from the CSDT. This approach also handled well the range of thicknesses  
 492 studied (Figure 14d). Although the range of thickness studied is not representative of solid slab  
 493 bridges [10], the available results are of interest for slab-between-girders bridges. Moreover, the

494 studied approach enabled accurate predictions, regardless of the shear slenderness parameter  $M/Vd$   
 495 (Figure 14f).

#### 496 5.4 Comparison with design code provisions

497 Table 8 shows a comparison of different code-based approaches for the one-way shear capacity  
 498 of the experiments gathered in database B1 described in Section 3. The French model is used for  
 499 determining the effective width in combination with code provisions from Europe [50,56] and North  
 500 America [84]. The *fib* Model Code 2010 [34] is the only code which includes guidance for improved  
 501 arching action and effective width for concentrated loads close to the support. The same factor  $\beta_{prop}$   
 502 for use with the CSDT was adopted in combination with the Swiss code SIA 262:2013 model [56]  
 503 and with the AASHTO code provisions for bridges [84].

504 Table 8 - Statistics of  $V_{exp}/V_{calc}$  according to the range of shear slenderness  $\lambda = M/Vd$  for different  
 505 design code approaches.

|           |     |                   | $\frac{V_{exp,red}}{V_{CSDT}}$ | $\frac{V_{exp,red}}{V_{CSDT}}$ | $\frac{V_{exp,red}}{V_{AASHTO}}$ | $\frac{V_{exp,red}}{V_{CEN04}}$ | $\frac{V_{exp,red}}{V_{MC}}$ | $\frac{V_{exp,red}}{V_{SIA262}}$ |
|-----------|-----|-------------------|--------------------------------|--------------------------------|----------------------------------|---------------------------------|------------------------------|----------------------------------|
|           |     | $b_{eff}$         | <b>French</b>                  | <b>GESW</b>                    | <b>French</b>                    | <b>French</b>                   | <b>fib MC</b>                | <b>French</b>                    |
| $\lambda$ | N   | $\beta_{arching}$ | <b>Prop</b>                    | <b>Prop</b>                    | <b>Prop</b>                      | <b>CEN (2005)</b>               | <b>fib MC</b>                | <b>Prop</b>                      |
| <2.5      | 55  | AVG               | 1.044                          | 1.043                          | 1.421                            | 1.704                           | 1.762                        | 1.096                            |
|           |     | MIN               | 0.773                          | 0.748                          | 0.986                            | 1.095                           | 1.176                        | 0.819                            |
|           |     | COV               | 11.4%                          | 12.2%                          | 16.5%                            | 18.0%                           | 22.6%                        | 14.8%                            |
| >2.5      | 86  | AVG               | 1.070                          | 1.295                          | 1.476                            | 1.122                           | 1.479                        | 1.037                            |
|           |     | MIN               | 0.723                          | 0.830                          | 0.918                            | 0.606                           | 0.749                        | 0.733                            |
|           |     | COV               | 15.8%                          | 19.2%                          | 33.3%                            | 24.0%                           | 29.0%                        | 16.4%                            |
| All       | 141 | AVG               | 1.060                          | 1.197                          | 1.454                            | 1.349                           | 1.589                        | 1.060                            |
|           |     | MIN               | 0.723                          | 0.748                          | 0.918                            | 0.606                           | 0.749                        | 0.733                            |
|           |     | COV               | 14.3%                          | 20.3%                          | 28.2%                            | 29.8%                           | 27.6%                        | 15.9%                            |

506

507 Table 8 shows that the code provisions studied provided a mean ratio of  $V_{exp}/V_{cal}$  between  
 508 1.043 and 1.704 for  $\lambda < 2.5$ . The most precise results were achieved by the French effective shear  
 509 width model combined with the one-way shear model based on the CSDT, as proposed in this study  
 510 when the governing failure mode is known and is one-way shear. The predictions with the AASHTO  
 511 code provisions were more conservative, with a mean ratio between experimental and calculated  
 512 shear strengths higher than 1.4 on both ranges of shear slenderness studied.

513 Table 8 also shows that the average ratio of  $V_{exp}/V_{cal}$  ranged from 1.060 to 1.589 for a shear  
514 slenderness higher than 2.5. Remarkably, the Swiss code provisions reached the same level of  
515 accuracy and precision of CSDT when combined with the French model of effective shear width and  
516 use of  $\beta_{prop}$ . Although these models (CSDT and SIA 262:2013) were derived in different ways, this  
517 result occurs because both models rely on some similar ideas, such as the higher influence of  
518 aggregate interlock in the shear strength and the decrease of the shear strength for increasing shear  
519 slenderness. Since both AASHTO models and *fib* Model Code Models were derived based on the  
520 Simplified Modified Compression Field Theory (SMCFT [78]), the statistical differences can be  
521 attributed to the models to account for improved arching action and the effective shear width used.

## 522 **5.5 Test with double loads**

523 The number of tests with double loads parallel to line supports is very limited. There are only  
524 8 tests in the literature conducted by Rombach and Henze [85], Vaz Rodrigues et al. [19] and Reißer  
525 et al. [3]. Most of these tests were conducted on cantilever slabs (7/8). The test with 4 loads close to  
526 the line support conducted by Vaz Rodrigues [19] showed a punching failure and was not analyzed  
527 in this study because it showed a transitional failure between the one-way and two-way shear. Table  
528 9 shows the statistics of the ratio between experimental and predicted one-way shear resistances for  
529 these tests. In summary, the level of accuracy of the CSDT model combined with the French effective  
530 shear width was close to that of slabs subjected to a single load. However, additional tests are needed  
531 to confirm these findings. The most unsafe prediction in Table 9 ( $V_{exp}/V_{pred} = 0.89$ ) occurred for the  
532 only test with a ratio  $a_v/d < 2.5$ . Therefore, the proposed factor to consider arching action ( $\beta_{prop}$ ) may  
533 have been too optimistic for this type of loading.

534

535

536 Table 9 - Statistical of the experimental to calculated shear strengths for tests with double loads  
 537 close to a line support.

| Authors                  | Test      | $\frac{V_{exp,red}}{V_{CSDT,beff\ French}}$ |
|--------------------------|-----------|---|
| Vaz Rodrigues et al [19] | DR1b      | 1.26  |
|                          | DR2a      | 1.03  |
|                          | DR2b      | 1.07  |
| Rombach & Henze [85]     | 2d x 2    | 0.89  |
|                          | 3d x 2    | 0.99  |
|                          | 4d x 2    | 1.06  |
|                          | 5d x 2    | 1.01  |
| Reißen et al. [3]        | MS35BB-22 | 1.04  |
| AVG                      |           | 1.04  |
| COV (%)                  |           | 9.78  |

538

## 539 5.6 General approach for one-way and two-way shear

540 An alternative approach to assessing the one-way shear models applicable to members with  
 541 possible punching failure is to decrease the effective shear width accordingly with the shear  
 542 slenderness, as discussed in the proposed GESW model (Section 4). In this study, we assumed that  
 543 the French effective shear width should be multiplied by the parameter  $\alpha$  (Equation (42)).

544 Table 10 shows the statistics of the ratio between experimental and predicted shear capacities  
 545 with one-way and two-way shear models according to the failure mode for the database with 214 test  
 546 results of slabs under single concentrated loads (Database B0). For the punching shear provisions, the  
 547 proposed model from prEN 1992-1-1:2018 [86] was used (based on the CSCT), while for the one-  
 548 way shear models we combined the CSCT models with the French and GESW models. Table 10  
 549 shows that the level of precision reached with the CSCT combined with the GESW model is very  
 550 similar for both failure modes, while the other approaches provide precise estimations only for their  
 551 respective failure modes.

552 Table 10 - Comparison of predictions with the CSDT for one-way shear and the punching shear  
 553 provisions from prEN 1992-1-1:2018 [86] according to the failure mode.

| Failure mode | N°  |                        | $\frac{P_{exp}}{P_{EC18}}$ | $\frac{V_{exp,red}}{V_{CSDT}}$ | $\frac{V_{exp,red}}{V_{CSDT}}$ |
|--------------|-----|------------------------|----------------------------|--------------------------------|--------------------------------|
|              |     | <i>b<sub>eff</sub></i> | -                          | French                         | GESW                           |
| P            | 51  | AVG                    | 1.092                      | 0.808                          | 1.044                          |
|              |     | MIN                    | 0.724                      | 0.331                          | 0.712                          |
|              |     | COV                    | 20.0%                      | 31.6%                          | 21.2%                          |
| WB           | 141 | AVG                    | 1.219                      | 1.060                          | 1.197                          |
|              |     | MIN                    | 0.466                      | 0.723                          | 0.748                          |
|              |     | COV                    | 31.9%                      | 14.3%                          | 20.3%                          |
| WB/P         | 22  | AVG                    | 1.220                      | 1.007                          | 1.121                          |
|              |     | MIN                    | 0.942                      | 0.712                          | 0.833                          |
|              |     | COV                    | 21.5%                      | 13.4%                          | 15.6%                          |
| All          | 214 | AVG                    | 1.189                      | 0.994                          | 1.153                          |
|              |     | MIN                    | 0.466                      | 0.331                          | 0.712                          |
|              |     | COV                    | 29.2%                      | 21.0%                          | 20.8%                          |

554

555



## 556 6 DISCUSSIONS

557 Previous publications on the field of one-way slabs under concentrated loads usually  
558 concentrate on the accuracy of semi-empirical models applied to reduced databases [10,11]. When  
559 mechanical-based models are investigated, usually the analyses concentrate on one kind of support  
560 condition [15]. Most of them neglect the governing failure mode of the tests [11,87]. Therefore, a gap  
561 of more comprehensive studies is realized related to the shear capacity of slabs under concentrated  
562 loads failing in different modes.

563 Tests with a presumed punching failure were initially removed from the database B1. Only  
564 members with predominant one-way shear failure were used in the first statistical analyses. Therefore,  
565 part of the higher level of accuracy in Section 5.2 with the French effective shear width can be  
566 attributed to the improved database selection. However, we have highlighted that the classification of  
567 the failure modes for some members may not be an easy task. For such cases, the experiments must  
568 be classified as governed by a mixed failure between one-way shear and two-way shear, as made in  
569 previous publications [10]. In these tests, both conical cracks at the top/bottom face and flexure-shear  
570 cracks at the edges of the slab arise at failure. Some studies have claimed that one-way and two-way  
571 shear capacities can be very similar in terms of strength ratio ( $V_{exp}/V_{calc}$  similar to  $P_{exp}/P_{calc}$ ) [20],  
572 which was also verified in this study for some tests during the classification of the failure modes.  
573 Particularly, this is also in line with the ACI 318-19 punching provisions [88], where the punching  
574 capacity is assumed to be governed by one-way shear when the load becomes very rectangular.

575 Since most mechanical models, such as the CSDT [6], CSCT [77], and SMCFT [78] were  
576 derived from flexure-shear failures, one could question their possible extension to non-slender  
577 members, whose predominant failure mode is a shear-compression failure. In fact, some studies, as  
578 well as the current ACI 318-19 [88], have highlighted those members should be assessed by strut-  
579 and-tie models, instead of sectional strain-based models [89,90]. However, most engineers have  
580 raised the possibility of covering a more extensive range of cases with the same model. In particular,  
581 for the shear assessment of existing RC slab bridges, there is a need for uniform approaches that allow

582 checking all cross-sections and load positions in a preprogrammed way. Such an approach requires  
583 the checking of models for non-slender members in an approach similar to the one suggested in design  
584 guides, e.g., NEN 1992-1-1:2005 [50] and *fib* Model Code 2010 [34], i.e. based on the reduction of  
585 the acting shear load close to the support. We have highlighted that such analyses should be used only  
586 as a first assessment of structures without stirrups. As such, they are in line with the need for a  
587 preprogrammed method for the assessment of a large number of existing RC slab bridges.

588         The level of accuracy reached by the CSDT with our method for arching action and the French  
589 model of effective width is similar to that obtained by the CSCT [2], but removes the need for finite  
590 element calculations. The proposed CSDT extension excels due to its easy application. The overall  
591  $V_{exp}/V_{cal}$  average ratio with the CSDT is 1.06, with a 14.3% coefficient of variation for a set with 141  
592 test results. Comparatively, Natário [2] achieved a 1.12  $V_{exp}/V_{calc}$  average ratio with 11% COV for  
593 simply supported members (62 tests) and 1.07 AVG and 16% COV for cantilever members (27 tests).  
594 However, Natário's study did not include continuous members or members with combinations of  
595 loads (concentrated loads combined with line loads). The database B0 has 33 tests with loads close  
596 to continuous supports. The mean ratio between experimental and predicted shear capacities by the  
597 CSDT model with the French effective shear width and  $\beta_{prop}$  is 1.01 with a COV of 11.3%. Therefore,  
598 this study comprehends a larger variety of support and loading conditions. The narrow scatter between  
599 experimental and predicted shear capacities with the CSDT demonstrates its accuracy and precision  
600 in assessing the one-way shear capacity of wide RC members under concentrated loads, such as slab  
601 bridges.

602         Different from other studies [15,28], we have identified that the use of the French approach  
603 for determining the effective shear width provides reasonable levels of accuracy combined with the  
604 one-way shear strength model based on the CSDT. Regarding studies on simply supported members  
605 in which the French model leads to unsafe predictions of the shear strength [15,28], our analysis  
606 indicates that these experiments presented signals of punching failures and, therefore, should also be  
607 evaluated by two-way shear strength models to reach more precise predictions.

608           Notably, the CSDT combined with the GESW model provides homogeneous levels of  
609 precision in predicting the shear capacity for specimens with one-way and two-way shear failures in  
610 the database B0, capturing well the complex transition between these two failure modes. The reason  
611 for this observation is that the precision and accuracy of the predictions with the GESW model were  
612 similar between different failure modes, shear slenderness, and support conditions. In addition, the  
613 level of precision was considerably better than that obtained with current semi-empirical code models  
614 [10,87], for which COVs are usually larger than 35%. Therefore, in a programmed approach of  
615 assessment, the CSDT combined with the GESW model may be used as the only model to check  
616 shear failures when the governing failure mode is unknown.

617 **7 CONCLUSIONS**

618 This study presents an extension of the Critical Shear Displacement Theory model for wide  
619 members under concentrated loads. Different databases were used to assess (i) the proposed arching  
620 action factor, (ii) the accuracy and precision of the CSDT combined with different models of effective  
621 shear width for slabs under single concentrated loads; (iii) the accuracy of the CSDT model to assess  
622 members with double concentrated loads parallel to the support and (iv) to assess slabs that showed  
623 different failures modes in shear. The following can be concluded:

624 1. The model for improved arching action for non-slender members can be combined with the  
625 CSDT as a first step for the determination of their shear strength. This approach was validated against  
626 databases of wide members loaded over the entire width, as well as for slabs under concentrated loads  
627 failing in one-way shear.

628 2. The CSDT, combined with the effective width model from the French design guides [32],  
629 provides accurate results of shear strength for wide members with predominant one-way shear failure,  
630 regardless of the shear slenderness and support conditions. The same level of precision was reached  
631 for slabs under double concentrated loads parallel to the support.

632 3. The level of accuracy of our proposed approach based on the CSDT combined with the  
633 French effective shear width was higher than that of most design code models, regardless of the  
634 parameters analyzed. Since our approach requires only analytical calculations (without finite element  
635 analysis), it can easily be implemented in the daily engineering practice for first levels of  
636 approximation.

637 4. Despite the simplicity of the French effective width model, it seems to represent well most  
638 one-way shear tests investigated. However, for members with punching failure, the approach may  
639 lead to unsafe predictions of shear strength, as verified in this study. Since the governing failure mode  
640 may not be known in preliminary analyses, both failure modes must be checked in the daily  
641 engineering practice for higher levels of approximation.

642 5. The most general effective shear width model (GESW) leads to good levels of accuracy for  
643 slabs under concentrated loads since it deals with both one-way and two-way shear failures.  
644 Moreover, the proposed approach addresses in a novel manner the transition between one-way shear  
645 and two-way shear failures of slabs under concentrated loads. However, it should be highlighted that  
646 this approach should be used only for preliminary designs and global assessment of a large number  
647 of assets, since it does not determine the governing failure mode physically. Apart from that, further  
648 studies are required in order to include the effects of other parameters, such as the slab width  $b$  in the  
649 transition between one-way and two-way shear failures.

650 6. This study shows that traditional models of effective shear width and punching shear do not  
651 provide precise predictions of shear strength when the critical failure mode is other than that assumed  
652 by the model (Section 5.6). Because of this, adjustments are required on each model to extend the  
653 applications of them for both failure modes. In this study, different approaches are shown to assess  
654 the shear capacity when the governing failure mode is known or unknown. The proposed approaches  
655 apply to wide beams and slabs under different support conditions (simple, continuous, and cantilever  
656 support), different loading conditions (loaded over the entire width or concentrated on the width  
657 direction).

## 658 **DECLARATION OF CONFLIT INTEREST**

659 The authors declare that they have no known competing financial interest or personal  
660 relationships that could have appeared to influence the work reported in this paper.

## 661 **CREDIT AUTHORSHIP CONTRIBUTION STATEMENT**

662 Alex de Sousa: Conceptualization, Methodology, Resources, Data curation, Writing - original  
663 draft, preparation. Eva Lantsoght: Conceptualization, Supervision, Writing - review & editing.  
664 Yuguang Yang: Supervision, Writing - review & editing. Mounir El Debs: Supervision, Project  
665 administration, Funding acquisition & manuscript review.

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671

## 672 NOTATION

| Notation      | Description  |
|---------------|--|
| $a$           | shear span: distance between the center of the support and the center of the load                      |
| $a_v$         | clear shear span: distance between face of support and face of load                                    |
| $b$           | width of the structural member   |
| $b_n$         | clear width of the structural member   |
| $d$           | effective depth of longitudinal reinforcement  |
| $d_t$         | effective depth of transverse reinforcement  |
| $d_g$         | maximum aggregate size   |
| $f_c$         | concrete compressive strength  |
| $f_{ck}$      | characteristic concrete compressive strength   |
| $f_{cm}$      | mean value of cylinder compressive strength of concrete  |
| $f_y$         | yield strength of reinforcement  |
| $k_c$         | slope of stress line, $k_c = 1.28$ according to [91]   |
| $m_{Ed}$      | design (factored) moment per unit length in critical section   |
| $m_{Rd}$      | plastic design (factored) moment per unit length in critical section                                   |
| $n_e$ or $n$  | ratio between elastic modulus of steel and concrete  |
| $l_{cr,m}$    | spacing of two neighboring major cracks  |
| $s_{cr,CSDT}$ | height of fully developed crack  |
| $s_{rm}$      | crack spacing of primary cracks  |
| $w$           | crack width  |
| $w_b$         | crack width at the bottom of the crack   |
| $x$           | neutral axis depth   |
| $z$           | length of internal lever arm or effective shear depth according to <i>fib</i> MC 2010, taken as $0.9d$ |
| $A_x, A_y$    | projected areas of a cracked surface for a unit crack length in two directions                         |
| $A_s$         | longitudinal reinforcement area  |
| $A_g$         | gross area of concrete section   |
| $E_c$         | modulus of elasticity of concrete  |
| $E_s$         | elastic modulus of steel   |
| $G_c$         | modulus of shear deformation for un-cracked concrete chord   |
| $G_f$         | concrete fracture energy   |
| $M$           | cross-sectional bending moment   |
| $M_{Ed}$      | design sectional moment  |
| $N_{Ed}$      | design sectional axial load  |
| $P_{exp}$     | measured peak load in an experiment  |
| $P_{EN18}$    | Predicted punching capacity by prEN 1992-1-1:2018 [86]   |
| $V$           | shear force  |
| $V_{ai}$      | shear force transferred by aggregate interlock   |
| $V_c$         | shear force transferred in concrete compression zone   |
| $V_d$         | shear force transferred by dowel action  |
| $V_{Ed}$      | design shear force   |
| $V_{exp}$     | Experimental shear force strength from the database tests  |
| $V_{exp,red}$ | Experimental shear force reduced by the parameter $\beta$  |
| $V_{cal}$     | Calculated shear force strength  |
| $V_{AASHTO}$  | one-way shear capacity calculated according to AASHTO  |
| $V_{CEN}$     | one-way shear capacity calculated according to NEN 1992-1-1:2005                                       |
| $V_{ACI-19}$  | one-way shear capacity calculated according to ACI 318-19  |
| $V_{MC}$      | one-way shear capacity calculated according to Model Code 2010   |

|                 |   |
|-----------------|---|
| $V_{SIA}$       | one-way shear capacity calculated according to SIA 262:2013   |
| $V_{CSDT}$      | one-way shear capacity calculated according to CSDT   |
| $\alpha_e$      | modular ratio ( $E_s/E_c$ )   |
| $\beta$         | reduction factor for the contribution of loads close to the support to the shear force  |
| $\gamma_c$      | partial safety factor for concrete  |
| $\Delta$        | shear displacement at crack   |
| $\Delta_{cr}$   | critical shear displacement   |
| $\Delta_e$      | distance between neutral axis and center of internal lever arm $z$  |
| $\varepsilon_s$ | steel strain  |
| $\varepsilon_x$ | longitudinal strain at mid-depth of the effective shear depth   |
| $\phi$          | rebar diameter  |
| $\mu_{CSDT}$    | friction coefficient for contact area between aggregate particles and matrix with $\mu = 0.4$ proposed according to Walraven [92] |
| $\rho_s$        | longitudinal reinforcement ratio  |
| $\sigma$        | normal stress   |
| $\sigma_{pu}$   | crushing (yielding) strength of matrix, or contact stress at cracked surface  |
| $\tau$          | shear stress  |
| $\tau_{ai}$     | shear stress transferred by aggregate interlock   |
| $\tau_{Rd}$     | design shear capacity of concrete   |
| $\tau_c$        | concrete shear capacity   |
| AVG             | Average value   |
| COV             | coefficient of variation  |
| CSCT            | Critical Shear Crack Theory   |
| CSDT            | Critical Shear Displacement Theory  |
| SMCFT           | Simplified Modified Compression Field Theory  |
| MIN             | Minimum value   |



674 **REFERENCES**

- 675 [1] Lantsoght EOL, Van Der Veen C, Walraven JC. Shear in One-Way Slabs under  
676 Concentrated Load Close to Support. *ACI Struct J* 2013;110.
- 677 [2] Natário F, Fernández Ruiz M, Muttoni A. Shear strength of RC slabs under concentrated  
678 loads near clamped linear supports. *Eng Struct* 2014;76:10–23.  
679 <https://doi.org/10.1016/j.engstruct.2014.06.036>.
- 680 [3] Reißen K, Classen M, Hegger J. Shear in reinforced concrete slabs-Experimental  
681 investigations in the effective shear width of one-way slabs under concentrated loads and  
682 with different degrees of rotational restraint. *Struct Concr* 2018;19:36–48.  
683 <https://doi.org/10.1002/suco.201700067>.
- 684 [4] Henze L, Harter M, Rombach GA. Querkrafttragfähigkeit von Stahlbetonplatten ohne  
685 Querkraftbewehrung unter konzentrierten Einzellasten (Shear capacity of reinforced concrete  
686 slabs without transverse reinforcement under concentrated loads Part 2: Proposal for a new  
687 design model). *Beton- Und Stahlbetonbau* 2019. <https://doi.org/10.1002/best.201800106>.
- 688 [5] de Sousa AMD, Lantsoght EOL, El Debs MK. One-way shear strength of wide reinforced  
689 concrete members without stirrups. *Struct Concr* 2020:1–25.  
690 <https://doi.org/10.1002/suco.202000034>.
- 691 [6] Yang Y, den Uijl J, Walraven J. Critical shear displacement theory: on the way to extending  
692 the scope of shear design and assessment for members without shear reinforcement. *Struct*  
693 *Concr* 2016;17:790–8. <https://doi.org/10.1002/suco.201500135>.
- 694 [7] Yang Y, Walraven J, Uijl J den. Shear Behavior of Reinforced Concrete Beams without  
695 Transverse Reinforcement Based on Critical Shear Displacement. *J Struct Eng*  
696 2017;143:04016146. [https://doi.org/10.1061/\(ASCE\)ST.1943-541X.0001608](https://doi.org/10.1061/(ASCE)ST.1943-541X.0001608).
- 697 [8] Gurutzeaga M, Oller E, Ribas C, Cladera A, Marí A. Influence of the longitudinal  
698 reinforcement on the shear strength of one-way concrete slabs. *Mater Struct Constr*  
699 2015;48:2597–612. <https://doi.org/10.1617/s11527-014-0340-5>.
- 700 [9] Conforti A, Minelli F, Plizzari GA. Influence of width-to-effective depth ratio on shear  
701 strength of reinforced concrete elements without web reinforcement. *ACI Struct J*  
702 2017;114:995–1006. <https://doi.org/10.14359/51689681>.
- 703 [10] Lantsoght EOL, van der Veen C, Walraven JC, de Boer A. Database of wide concrete  
704 members failing in shear. *Mag Concr Res* 2015;67:33–52.  
705 <https://doi.org/10.1680/macr.14.00137>.
- 706 [11] Henze L, Rombach GA, Harter M. New approach for shear design of reinforced concrete  
707 slabs under concentrated loads based on tests and statistical analysis. *Eng Struct*  
708 2020;219:110795. <https://doi.org/10.1016/j.engstruct.2020.110795>.
- 709 [12] Reißen K, Hegger J. Experimentelle Untersuchungen zum Querkrafttragverhalten von  
710 auskragenden Fahrbahnplatten unter Radlasten. *Beton- Und Stahlbetonbau* 2013;108:315–24.  
711 <https://doi.org/10.1002/best.201200072>.
- 712 [13] Reißen K, Hegger J. Experimentelle Untersuchungen zur mitwirkenden Breite für Querkraft  
713 von einfeldrigen Fahrbahnplatten. *Beton- Und Stahlbetonbau* 2013;108:96–103.  
714 <https://doi.org/10.1002/best.201200064>.
- 715 [14] Lantsoght EOL, van der Veen C, Walraven J, de Boer A. Experimental investigation on shear  
716 capacity of reinforced concrete slabs with plain bars and slabs on elastomeric bearings. *Eng*

- 717 Struct 2015;103:1–14. <https://doi.org/10.1016/J.ENGSTRUCT.2015.08.028>.
- 718 [15] Halvonik J, Vidaković A, Vida R. Shear Capacity of Clamped Deck Slabs Subjected to a  
719 Concentrated Load. *J Bridg Eng* 2020;25:04020037.  
720 [https://doi.org/10.1061/\(ASCE\)BE.1943-5592.0001564](https://doi.org/10.1061/(ASCE)BE.1943-5592.0001564).
- 721 [16] Bui TT, Abouri S, Limam A, NaNa WSA, Tedoldi B, Roure T. Experimental investigation of  
722 shear strength of full-scale concrete slabs subjected to concentrated loads in nuclear  
723 buildings. *Eng Struct* 2017;131:405–20. <https://doi.org/10.1016/j.engstruct.2016.10.045>.
- 724 [17] Lantsoght EOL, van der Veen C, Walraven JC, de Boer A. Transition from one-way to two-  
725 way shear in slabs under concentrated loads. *Mag Concr Res* 2015;67:909–22.  
726 <https://doi.org/10.1680/mac.14.00124>.
- 727 [18] Muttoni A, Fernandez Ruiz M. Shear in slabs and beams: should they be treated in the same  
728 way? *Fédération Int. du Bét. Bull. N°57*, 2010, p. 105–28.
- 729 [19] Vaz Rodrigues R, Fernández Ruiz M, Muttoni A. Shear strength of R/C bridge cantilever  
730 slabs. *Eng Struct* 2008;30:3024–33. <https://doi.org/10.1016/j.engstruct.2008.04.017>.
- 731 [20] Natário F. Static and Fatigue Shear Strength of Reinforced Concrete Slabs Under  
732 Concentrated Loads Near Linear Support. Thesis (Docteur ès Sciences) - Faculté de  
733 L'environnement Naturel, Architectural et Construit Laboratoire de Construction en Béton,  
734 École Polytechnique Fédérale de Lausanne, 2015. <https://doi.org/10.5075/epfl-thesis-6670>.
- 735 [21] Reißen K. Zum Querkrafttragverhalten von einachsig gespannten Stahlbe-  
736 tonplatten ohne Querkraftbewehrung unter Einzellasten. PhD Thesis (Doctor of Engineering), Faculty of  
737 Civil Engineering, RWTH Aachen University, 2016.
- 738 [22] Henze L. Querkrafttragverhalten von Stahlbeton-Fahrbahnplatten. PhD Thesis, Institute for  
739 Concrete Structures, Technische Universität Hamburg (TUHH), 2019.
- 740 [23] Doorgeest J. Transition Between One-way Shear and Punching Shear. Master of Science  
741 Thesis, Delft University of Technology, 2012.
- 742 [24] Regan PE. Shear Resistance of Members without Shear Reinforcement; Proposal for CEB  
743 Model Code MC90 1987:1–28.
- 744 [25] Lantsoght EOL, Van Der Veen C, De Boer A, Walraven JC. Transverse load redistribution  
745 and effective shear width in reinforced concrete slabs. *Heron* 2015;60:145–79.
- 746 [26] Lantsoght EOL, de Boer A, van der Veen C. Distribution of peak shear stress in finite  
747 element models of reinforced concrete slabs. *Eng Struct* 2017;148:571–83.  
748 <https://doi.org/10.1016/j.engstruct.2017.07.005>.
- 749 [27] Sagaseta J, Tassinari L, Fernández Ruiz M, Muttoni A. Punching of flat slabs supported on  
750 rectangular columns. *Eng Struct* 2014;77:17–33.  
751 <https://doi.org/10.1016/j.engstruct.2014.07.007>.
- 752 [28] Reißen K, Hegger J. Experimental investigations on the effective width for shear.  
753 Maintenance, Monit. Safety, Risk Resil. Bridg. Bridg. Networks - Proc. 8th Int. Conf. Bridg.  
754 Maintenance, Saf. Manag. IABMAS 2016, 2016, p. 2340–8.
- 755 [29] Lantsoght EOL, de Boer A, van der Veen C, Walraven JC. Effective shear width of concrete  
756 slab bridges. *Proc Inst Civ Eng - Bridg Eng* 2015;168:287–98.  
757 <https://doi.org/10.1680/bren.13.00027>.
- 758 [30] ABNT NBR 6118. NBR 6118: Design and execution of reinforced concrete buildings (in  
759 portuguese). Rio de Janeiro: Associação Brasileira de Normas Técnicas (ABNT); 1980.

- 760 [31] NORMCOMISSIE 351001. NEN 6720 Technische Grondslagen voor Bouwvoorschriften  
761 1995.
- 762 [32] FD P 18-717. Eurocode 2 - Calcul des structures en béton - Guide d'application des normes  
763 NF EN 1992 2013.
- 764 [33] Grasser E, Thielen G. Hilfsmittel zur Berechnung der Schnittgrößen und Formänderungen  
765 von Stahlbetontragwerken : nach DIN 1045, Ausg. Juli 1988. 3<sup>o</sup> revised. Berlin: Beuth  
766 Verlag GmbH; 1991.
- 767 [34] Fédération Internationale du Béton (fib). fib Model Code for Concrete Structures 2010. vol.  
768 Vol. 1-2. Lausanne, Switzerland: Ernst & Sohn - fédération internationale du béton, Bulletin  
769 65; 2012.
- 770 [35] Bauer T, Müller M. Straßenbrücken in Massivbauweise nach DIN- Fachbericht - Beispiele  
771 prüffähiger Standsicherheitsnachweise. 2. Auflage. Berlin: Bauwerk; 2003.
- 772 [36] Vidakovi A, Halvonik J. Shear resistance of clamped deck slabs assessed using design  
773 equations and FEM analysis. 13th Int. Conf. Mod. Build. Mater. Struct. Tech., 2019, p. 0–7.
- 774 [37] Johansen KW. Yield-Line Formulae for Slabs. CRC Press Yea; 2005.
- 775 [38] Goldbeck AT. The Influence of Total Width of the Effective Width of Reinforced-Concrete  
776 Slabs Subjected to Cental Concentrated Loading. ACI J Proc 1917;13:78–88.  
777 <https://doi.org/10.14359/15865>.
- 778 [39] DAfStb. Deutscher Ausschuss für Stahlbeton Heft 240: Hilfsmittel zur Berechnung der  
779 Schnittgrößen und Formänderungen von Stahlbetonbauwerken. (German Committee for  
780 Structural Concrete Book 240: Tools for calculation of internal forces and changes in shape  
781 of rein. Berlin / Germany: Beuth Verlag; 1991.
- 782 [40] BBK (Statens Betong Kommitte). Regulations for concrete structures – Part 1: Design. BBK,  
783 Stockholm, Sweden; 1979.
- 784 [41] Zheng Y, Taylor S, Robinson D, Cleland D. Investigation of ultimate strength of deck slabs  
785 in steel-concrete bridges. ACI Struct J 2010;107:82–91.
- 786 [42] Reißer K, Hegger J. Shear Capacity of Reinforced Concrete Slabs under Concentrated  
787 Loads. 8th IABSE Congr. 2012 "Innovative Infrastructures - Towar. Hum. Urban., Seoul,  
788 Südkorea.,: 2012. <https://doi.org/10.2749/222137912805111320>.
- 789 [43] Reißer K, Hegger J. Numerical investigations on the shear capacity of reinforced concrete  
790 slabs under concentrated loads. Res. Appl. Struct. Eng. Mech. Comput., CRC Press; 2013, p.  
791 1507–12. <https://doi.org/10.1201/b15963-271>.
- 792 [44] Rombach GA, Velasco RR. Schnittgrößen auskragender fahrbahnplatten infolge von  
793 radlasten nach DIN-fachbericht. Beton- Und Stahlbetonbau 2005;100:376–84.  
794 <https://doi.org/10.1002/best.200590093>.
- 795 [45] Shu J, Plos M, Zandi K, Ashraf A. Distribution of shear force: A multi-level assessment of a  
796 cantilever RC slab. Eng Struct 2019;190:345–59.  
797 <https://doi.org/10.1016/J.ENGSTRUCT.2019.04.045>.
- 798 [46] Kani GNJ. The Riddle of Shear Failure and its Solution. ACI J Proc 1964;61:441–68.  
799 <https://doi.org/10.14359/7791>.
- 800 [47] Leonhardt F, Walther R. Schubversuche an einfeldrigen Stahlbetonbalken mit und ohne  
801 Schubbewehrung [Shear tests on single-span reinforced concrete beams with and without  
802 shear reinforcement]. Berlin: Beuth; 1962.

- 803 [48] Cavagnis F, Fernández Ruiz M, Muttoni A. An analysis of the shear-transfer actions in  
804 reinforced concrete members without transverse reinforcement based on refined experimental  
805 measurements. *Struct Concr* 2018;19:49–64. <https://doi.org/10.1002/suco.201700145>.
- 806 [49] Bairán JM, Mendiña R, Marí A, Cladera A. Shear Strength of Non-Slender Reinforced  
807 Concrete Beams. *ACI Struct J* 2020;277–90. <https://doi.org/10.14359/51721369>.
- 808 [50] CEN. EN 1992-1-1: Eurocode 2: Design of concrete structures -Part 1-1: General rules and  
809 rules for buildings, EN 1992-1-1:2005 2005.
- 810 [51] Fernández Ruiz M, Muttoni A, Sagaseta J. Shear strength of concrete members without  
811 transverse reinforcement: A mechanical approach to consistently account for size and strain  
812 effects. *Eng Struct* 2015;99:360–72. <https://doi.org/10.1016/J.ENGSTRUCT.2015.05.007>.
- 813 [52] Kim D, Kim W, White RN. Arch Action in Reinforced Concrete Beams—A Rational  
814 Prediction of Shear Strength. *ACI Struct J* 1999;96:586–93. <https://doi.org/10.14359/695>.
- 815 [53] ABNT NBR 6118. NBR 6118: Design of concrete structures — Procedure (in portuguese).  
816 Rio de Janeiro: Associação Brasileira de Normas Técnicas (ABNT); 2014.
- 817 [54] DIN 1045. Beton und Stahlbeton - Bemessung und Ausführung. Berlin: Deutsches Institut  
818 für Normung (DIN); 1988.
- 819 [55] DIN 1045-1. Bemessung und Konstruktion von Stahlbeton- und Spannbetonbauteilen.  
820 Berlin: Deutsches Institut für Normung (DIN); 2001.
- 821 [56] SIA. Code 262 for concrete structures. Zürich: Swiss Society of Engineers and Architects;  
822 2013.
- 823 [57] Yang Y, Uijl JA den, Walraven JC. Residual Shear Capacity of 50 years Old RC Solid Slab  
824 Bridge Decks. *Int. IABSE Conf. Assessment, Upgrad. Refurb. Infrastructures.*, Rotterdam:  
825 2013, p. 538–9.
- 826 [58] Mörsch E. *Der Eisenbetonbau [Concrete-steel construction]*. New York.: Engineering News  
827 Publishing; 1909.
- 828 [59] Baumann T, Rusch H. Versuche zum studien der verdübe- lungswirkung der  
829 biegezugbewehrung eines stahlbetonbalkens [Exper- imental study on dowel action in  
830 reinforced concrete beam]. Deutscher Ausschuss für Stahlbeton (DAfStb), Ernst, Berlin (in  
831 German); 1970.
- 832 [60] Walraven JC. Fundamental Analysis of Aggregate Interlock. *J Struct Div ASCE*  
833 1981;107:2245–2270.
- 834 [61] Chana PS. Investigation of the mechanism of shear failure of reinforced concrete beams.  
835 *Mag Concr Res* 1987;39:196–204. <https://doi.org/10.1680/macr.1987.39.141.196>.
- 836 [62] Sousa AMD, Lantsoght EOL, El Debs MK. Database of wide beams and slabs under  
837 concentrated loads failing in different shear failure modes. Zenodo 2020.  
838 <https://doi.org/10.5281/zenodo.4059803>.
- 839 [63] Lantsoght EOL. Shear in Reinforced Concrete Slabs under Concentrated Loads Close to  
840 Supports. Ph.D. Thesis , Faculty of Civil Engineering and Geosciences, Delft University of  
841 Technology, 2013.
- 842 [64] Cullington DW, Daly AF, Hill ME. Assessment of reinforced concrete bridges: Collapse  
843 tests on Thurloxton underpass. *Bridg Manag* 1996;Vol. 3:667–74.
- 844 [65] Lubell AS. Shear in Wide Reinforced Concrete Members. PhD dissertation. University of

- 845 Toronto, Canadá, 2006.
- 846 [66] Bui TT, Limam A, Nana W-S-A, Ferrier E, Bost M, Bui Q-B. Evaluation of one-way shear  
847 behaviour of reinforced concrete slabs: experimental and numerical analysis. *Eur J Environ*  
848 *Civ Eng* 2017;1–27. <https://doi.org/10.1080/19648189.2017.1371646>.
- 849 [67] Regan PE. *Shear Resistance of Concrete Slabs at Concentrated Loads Close to Supports*.  
850 Polytechnic of Central London, UK, 1982.
- 851 [68] Regan PE, Rezai-Jorabi H. Shear Resistance of One-Way Slabs Under Concentrated Loads.  
852 *ACI Struct J* 1988;85:150–7. <https://doi.org/10.14359/2704>.
- 853 [69] Vaz Rodrigues R, Muttoni A, Burdet O. Large Scale Tests on Bridge Slabs Cantilevers  
854 Subjected to Traffic Loads. *Proc. 2nd Int. Congr. fib, Naples, Italy: 2006*.
- 855 [70] Rombach GA, Latte S. Shear resistance of bridge decks without shear reinforcement. *Proc.*  
856 *Int. fib Symp., Tailor Made Concrete Structures; 2008*, p. 519–25.  
857 <https://doi.org/doi:10.1201/9781439828410.ch86>.
- 858 [71] Rombach G, Latte S. Querkrafttragfähigkeit Von Fahrbahnplatten Ohne  
859 Querkraftbewehrung. *Beton- Und Stahlbetonbau* 2009;104:642–56.  
860 <https://doi.org/10.1002/best.200900029>.
- 861 [72] Natário F, Fernández Ruiz M, Muttoni A. Experimental investigation on fatigue of concrete  
862 cantilever bridge deck slabs subjected to concentrated loads. *Eng Struct* 2015;89:191–203.  
863 <https://doi.org/10.1016/J.ENGSTRUCT.2015.02.010>.
- 864 [73] Henze L, Rombach GA. Versuche zur Querkrafttragfähigkeit von Stahlbetonplatten unter  
865 auflagnahen Einzellasten; Versuchsbericht (Experimental investigations on shear capacity  
866 of concrete slabs with concentrated loads close to the support). Hamburg: 2017.  
867 <https://doi.org/10.15480/882.1443>.
- 868 [74] Vida R, Halvonik J. Experimentálne overovanie šmykovej odolnosti mostovkových dosiek  
869 (Experimental verification of shear resistance of bridge deck slabs). *Inžinierske*  
870 *Stavby/Inženýrské Stavby* 2018;2–6.
- 871 [75] Reineck K-H, Bentz EC, Fitik B, Kuchma DA, Bayrak O. ACI-DAfStb Database of Shear  
872 Tests on Slender Reinforced Concrete Beams without Stirrups. *ACI Struct J* 2013;110:867–  
873 76. <https://doi.org/10.14359/51685839>.
- 874 [76] Collins MP, Bentz EC, Sherwood EG. Where is shear reinforcement required? Review of  
875 research results and design procedures. *ACI Struct J* 2008;105:590–600.  
876 <https://doi.org/10.14359/19942>.
- 877 [77] Muttoni A, Ruiz MF. Shear Strength of Members without Transverse Reinforcement as  
878 Function of Critical Shear Crack Width. *ACI Struct J* 2008;105:163–72.  
879 <https://doi.org/10.14359/19731>.
- 880 [78] Bentz EC, Vecchio FJ, Collins MP. Simplified modified compression field theory for  
881 calculating shear strength of reinforced concrete elements. *ACI Struct J* 2006;103:614–24.  
882 <https://doi.org/10.14359/16438>.
- 883 [79] Cavagnis F, Fernández Ruiz M, Muttoni A. Shear failures in reinforced concrete members  
884 without transverse reinforcement: An analysis of the critical shear crack development on the  
885 basis of test results. *Eng Struct* 2015;103:157–73.  
886 <https://doi.org/10.1016/j.engstruct.2015.09.015>.
- 887 [80] Yang Y. Shear Behaviour of Reinforced Concrete Members without Shear Reinforcement: A

- 888 New Look at an Old Problem. Delft University of Technology, 2014.
- 889 [81] Mihaylov BI, Bentz EC, Collins MP. Two-parameter kinematic theory for shear behavior of  
890 deep beams. *ACI Struct J* 2013;110:447–55. <https://doi.org/10.14359/51685602>.
- 891 [82] Lantsoght E, de Boer A, van der Veen C. Levels of approximation for the shear assessment  
892 of reinforced concrete slab bridges. *Struct Concr* 2017;18:143–52.  
893 <https://doi.org/10.1002/suco.201600012>.
- 894 [83] Bauer T, Müller M, Blase T. Strassenbrücken in Massivbauweise nach DIN-Fachbericht :  
895 Beispiele prüffähiger Standsicherheitsnachweise ; Stahlbeton- und Spannbetonüberbau nach  
896 DIN-Fachbericht 101 und 102 ; mit CD-ROM. Bauwerk; 2005.
- 897 [84] AASHTO. AASHTO LRFD Bridge Design Specifications. 8th Editio. Washington D.C.:  
898 American Association of State Highway and Transportation Officials (AASHTO); 2017.
- 899 [85] Rombach G, Henze L. Querkrafttragfähigkeit von Stahlbetonplatten ohne  
900 Querkraftbewehrung unter konzentrierten Einzellasten. *Beton- Und Stahlbetonbau*  
901 2017;112:568–78. <https://doi.org/10.1002/best.201700040>.
- 902 [86] Muttoni A, Caldentey AP, Hegger J, Vill M, Shave JD, Menegotto M. PT-SC2-T1 D3BG -  
903 Background documents to prEN 1992-1-1:2018 2018.
- 904 [87] Reißer K, Hegger J. Database of shear tests on RC slabs without shear reinforcement under  
905 concentrated loads – Assessment of design rules according to Eurocode 2. 16th Eur. Bridg.  
906 Conf., Edinburgh, Scotland: 2015.
- 907 [88] ACI Committee 318. Building Code Requirements for Structural Concrete (ACI 318-19)  
908 2019:988.
- 909 [89] Vollum RL, Fang L. Shear enhancement near supports in RC beams. *Mag Concr Res*  
910 2015;67:443–58. <https://doi.org/10.1680/mac.14.00309>.
- 911 [90] Sagaseta J, Vollum RL. Shear design of short-span beams. *Mag Concr Res* 2010;62:267–82.  
912 <https://doi.org/10.1680/mac.2010.62.4.267>.
- 913 [91] Krips M. Rißbreitenbeschränkung im Stahlbeton und Spannbeton (Crack width restriction in  
914 reinforced concrete and prestressed concrete). Berlin: Ernst & Sohn; 1985.
- 915 [92] Walraven JC. Aggregate Interlock: a Theoretical and Experimental Analysis. PhD. Thesis,  
916 Delft University of Technology, 1980.
- 917