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Quantum simulation of a Fermi-Hubbard model using a semiconductor quantum dot array

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Interacting fermions on a lattice can develop strong quantum correlations, which lie at the heart of the classical intractability of many exotic phases of matter¹⁻³. Seminal efforts are underway in the control of artificial quantum systems, that can be made to emulate the underlying Fermi-Hubbard models⁴⁻⁶. Electrostatically confined conduction band electrons define interacting quantum coherent spin and charge degrees of freedom that allow all-electrical pure-state initialisation and readily adhere to an engineerable Fermi-Hubbard Hamiltonian⁷⁻¹⁷. Until now, however, the substantial electrostatic disorder inherent to solid state has made attempts at emulating Fermi-Hubbard physics on solid-state platforms few and far between^{18,19}. Here, we show that for gate-defined quantum dots, this disorder can

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20 **be suppressed in a controlled manner. Novel insights and a newly developed semi-automated**
21 **and scalable toolbox allow us to homogeneously and independently dial in the electron filling**
22 **and nearest-neighbour tunnel coupling. Bringing these ideas and tools to fruition, we real-**
23 **ize the first detailed characterization of the collective Coulomb blockade transition²⁰, which**
24 **is the finite-size analogue of the interaction-driven Mott metal-to-insulator transition¹. As**
25 **automation and device fabrication of semiconductor quantum dots continue to improve, the**
26 **ideas presented here show how quantum dots can be used to investigate the physics of ever**
27 **more complex many-body states.**

28 The potential for realizing novel electronic and magnetic properties of correlated-electron
29 phases in low-dimensional condensed-matter physics, in topics ranging from high- T_c supercon-
30 ductivity to electronic spin liquids¹⁻³, has prompted quantum simulation efforts across multiple
31 platforms^{4-6, 18, 19, 21, 22}. Theoretical and proof-of-principle experimental work has shown how emer-
32 gent spin physics²¹ and two-site Mott physics²² can be simulated on programmable quantum com-
33 puting platforms. These digital quantum simulation efforts promise universality, but come at the
34 cost of requiring large numbers of highly-controlled quantum bits with additional error-correction
35 overhead. Analog quantum simulation efforts, on the other hand, aim to directly implement well-
36 defined Hamiltonians. Such emulators are typically limited by the residual entropy of the initial-
37 ized system, restricting experimental correlations in span and strength⁶. Furthermore, scaling to
38 sufficiently homogeneous systems of larger size is not always straightforward^{4-6, 19}.

39 Semiconductor quantum dots form a scalable platform that is naturally described by a Fermi-

40 Hubbard model in the low-temperature, strong-interaction regime, when cooled down to dilution
41 temperatures⁷⁻¹⁰. As such, pure state initialization of highly-entangled states is possible even with-
42 out the use of adiabatic initialization schemes²³. Coherent evolution of excitations can span many
43 sites, as, contrary to what might be expected, > 20 coherent oscillations in charge or spin can be
44 observed on adjacent sites¹³⁻¹⁵. Furthermore, local control and read-out of both charge and spin
45 degrees of freedom have become matured areas of research, given the large ongoing effort of using
46 quantum dots as a platform for quantum information processing¹¹⁻¹⁷. In particular, excellent con-
47 trol of small on-site energy differences²⁴ or tunnel couplings^{14,15} has been shown at specific values
48 of electron filling and tuning.

49 Quantum simulation experiments can leverage many of these developments, trading off some
50 of the experimental difficulties involved in full coherent control for ease of scaling. Until now,
51 however, calibration routines for quantum dots have been quite inefficient and limited in scope. As
52 such, the effective control of larger parameter spaces as well as the calibration of larger samples
53 seem like insurmountable obstacles. What has been lacking, thus, is an efficient and scalable con-
54 trol paradigm for Hamiltonian engineering that extends to the collective Fermi-Hubbard parameter
55 regimes well beyond those required for qubit operation^{25,26}.

56 In this Letter, we demonstrate the simulation of Fermi-Hubbard physics using semiconduc-
57 tor quantum dots. We describe an experimental toolbox, validated by direct numerical simulations,
58 that allows for the independent tuning of filling and tunnel coupling as well as the measurement
59 of all interaction energies, and employ it to map out the accessible parameter space of a triple

60 quantum dot device with unprecedented detail and precision. As the tunnel couplings are homo-
61 geneously increased, we witness the delocalization transition between isolated Coulomb blockade
62 and collective Coulomb blockade, the finite-size analogue of the interaction-driven Mott transition.

63 The one-dimensional quantum dot array is electrostatically defined using voltages applied to
64 gate electrodes fabricated on the surface of a GaAs/AlGaAs heterostructure (Fig. 1), that selec-
65 tively deplete regions of the 85-nm-deep two-dimensional electron gas (2DEG) underneath. The
66 outermost dots can be (un)loaded from Fermi reservoirs on the sides, which have an effective
67 electron temperature of 70-75 mK (6.0-6.5 μeV). The three gates at the top are used to define a
68 sensing-dot channel, the conductance of which is sensitive to changes in the charge state of the
69 array and is directly read out using radio-frequency reflectometry.

70 The control of Fermi-Hubbard model parameters is achieved by modulation of the potential
71 landscape in the 2DEG using the seven bottom-most gate electrodes (Fig. 1). These gates come in
72 two flavours. Plunger gates P_i are designed to tune the single-particle energy offsets ϵ_i of individual
73 dots i , allowing us to set an overall chemical potential $\mu' = \langle \epsilon_i \rangle$ and add site-specific detuning
74 terms $\delta\epsilon_i$. Barrier gates B_{ij} allow for the modulation of tunnel couplings t_{ij} between the i th and
75 j th dot or Γ_i between an outer dot i and its adjacent Fermi reservoir, respectively. The interaction
76 energies are determined by the potential landscape realized to achieve this set $\{\mu', \delta\epsilon_i, t_{ij}, \Gamma_i\}$, and
77 comprise of the on-site Coulomb interaction terms U_i and inter-site Coulomb interaction terms
78 V_{ij} . With each dot filled to an even number of electrons, we can describe the addition of the
79 next two electrons per dot within an effective single-band extended Hubbard picture²⁷, using site-

80 and-spin-specific electronic creation and annihilation operators $c_{i\sigma}^\dagger$ and $c_{i\sigma}$ and dot occupations

81 $n_i = \sum_{\sigma} c_{i\sigma}^\dagger c_{i\sigma}$:

$$H = - \sum_i \epsilon_i n_i - \sum_{\langle i,j \rangle, \sigma} t_{ij} (c_{i\sigma}^\dagger c_{j\sigma} + \text{h.c.}) + \sum_i \frac{U_i}{2} n_i (n_i - 1) + \sum_{i,j} V_{ij} n_i n_j. \quad (1)$$

82 In practice, both P_i and B_{ij} gates exhibit cross-talk to all the ϵ_i and t_{ij} (with smaller effects
 83 on U_i and V_{ij}), and in addition must compensate for initial disorder. Setting Hamiltonian parame-
 84 ters experimentally therefore requires carefully chosen linear combinations of gate voltages. This
 85 idea is employed regularly in spin qubit experiments in order to change the on-site energies ϵ_i de-
 86 terministically over small ranges²⁴, but here we go further in important ways. Our experimental
 87 toolbox uses linear combinations of gate voltage changes $\{P_i, B_{ij}\}$ for the independent control of
 88 the Fermi-Hubbard parameters $\{\mu', \delta\epsilon_i, t_{ij}\}$ to within several $k_B T$ and over a wide range of fillings
 89 and tunnel couplings.

90 Fig. 2a-b shows the filling of the array with up to $N = 9$ electrons, three electrons per dot,
 91 while keeping the inter-dot tunneling terms small ($t_{ij} < V_{ij} < U_i$) and the tunnel couplings to the
 92 reservoirs roughly constant. The dark lines arise from steps in the charge detector conductance,
 93 indicating a transition in the number of electrons on one of the dots. The horizontal and diagonal
 94 lines indicate filling of one of the dots from the reservoir, whereas the vertical (polarization) lines
 95 indicate electron transitions between sites (not seen in Fig. 2b which shows only changes in N).
 96 To achieve this level of control required several new insights. As a start, we measure the cross-

107 talk between the seven gate voltages and the three dot detunings at multiple points in gate space,
108 allowing for the direct definition of virtual $\delta\epsilon_i$ gates that are accurate over a range of several meV
109 (see Methods and Extended Data Fig. 1). Furthermore, it allows us to define virtual barrier gates
100 that change specific tunnel couplings while keeping all dot detunings constant. In addition, we
101 achieve homogeneous filling of a quantum dot array (as in Fig. 2a) through non-homogeneous
102 changes in the ϵ_i , as the dots have to each overcome a different sum of local interaction energies
103 $U_i + \sum_{i \neq j} V_{ij}$. This is a consequence of the finite size of the array (only the middle dot has two
104 neighbours) and the inhomogeneity in interaction terms (see Methods and Extended Data Fig. 2-3).
105 Finally, as multiple electrons are added to the array, we use the virtual barrier gates described above
106 to counter the effect that changing plunger gate voltages (and the higher wave function overlap of
107 higher electron fillings) have on the tunnel couplings.

108 Having filled the array with a given number of electrons, we can quantitatively characterize
109 the various parameters in the Fermi-Hubbard model directly from relevant feature sizes in the
110 charge stability diagram as we detune away from uniform filling. The spacing between charge
111 addition lines of half-filled dot levels yields the on-site Coulomb interaction term U_i , whereas the
112 displacement of single charge addition lines upon filling another dot yields their inter-site Coulomb
113 coupling V_{ij} (see Fig. 2c and Methods for automation and protocols). Finally, we can extract the
114 interdot tunnel coupling t_{ij} at transitions where an added electron moves between adjacent sites
115 i and j (the polarization lines seen in Fig. 2a). The width of such transitions is determined by
116 the hybridization of the charge states on the two sites and is thus a measure of tunnel coupling.
117 We implement an iterative tuning process that allows for automated repeated measurements of the

118 polarization line width with changing virtual barrier gates and thus tunnel coupling. To account
119 for the only remaining cross-talk, between each virtual barrier gate and the other tunnel coupling,
120 we redefine the virtual barrier gates such that they influence their local tunnel coupling only, while
121 keeping all other parameters constant (see Fig. 2d and Extended Data Fig. 4).

122 We showcase the potential of well-controlled quantum dot arrays to emulate Fermi-Hubbard
123 physics by employing this newly developed toolbox for the realization of collective Coulomb
124 blockade (CCB) physics, validating the results through direct numerical Fermi-Hubbard model
125 calculations. Coulomb blockade (CB) is a purely classical effect that arises from the finite charg-
126 ing energies of each individual quantum dot, where the charge excitations at half filling are gapped
127 out, analogous to the Mott gap. When quantum tunneling effects between sites are turned on,
128 however, a much richer phase diagram appears. The CB of individual dots is destroyed as the
129 degeneracy of the peaks in the equilibrium charge addition spectrum is lifted and broadened into
130 minibands, giving way to collective Coulomb blockade²⁰ (see Fig. 3a and Extended Data Fig. 5
131 for simulated data of a simplified model). As tunnel couplings continue to increase relative to local
132 charging energies this gap will vanish in the thermodynamic limit, giving rise to a metallic state.
133 The CCB physics is best described by the equilibrium electron addition spectrum as a function
134 of filling and tunnel coupling, the two main experimental control parameters of the quantum dot
135 array.

136 The experimental phase diagram is mapped out by the independent control over electron fill-
137 ing and tunnel coupling strength over as large a range as possible (Methods). It is constructed con-

138 tinuously by linear interpolation of gate values in between 3 to 12 calibrated points per miniband
139 (Fig. 3b) where the on-site energies and tunnel couplings are well calibrated and the interaction
140 energies measured (see Extended Data Fig. 6). At low tunnel coupling, the miniband has a finite
141 width due to residual V_{ij} . The main effect of increased nearest-neighbour tunnel coupling on the
142 addition spectrum is a widening of the minibands at the expense of the collective gap at uniform
143 filling, analogous to the reduction of the Mott gap with increasing tunnel coupling. Along with tun-
144 nel coupling, also the inter-site Coulomb coupling V_{ij} increases (see Extended Data Fig. 6). The
145 gap between minibands continues to decrease with increasing tunnel coupling, but will be prohib-
146 ited from closing completely by the charging energy of what has essentially become one large dot,
147 a quantity inversely proportional to its large but finite total capacitance. The low and high tunnel
148 coupling regimes are also clearly distinguished in transport measurements through the quantum
149 dot array and in charge stability diagrams (see Extended Data Fig. 7). To test the validity of our
150 approach, we perform numerical calculations of the addition spectrum within each band based on
151 Eq. (1) and using experimental parameters that are either calibrated or measured (see Methods
152 and Extended Data Table 1-2). The agreement between measurement and numerical calculation in
153 Fig. 3b indeed validates the use of experimental tools for Hamiltonian engineering over the entire
154 measured diagram.

155 Putting these results in perspective, we are able to calibrate and characterize site-specific
156 quantum dot parameters up to values of tunnel coupling reaching $U/t = 7.1(4)$. The large en-
157 ergy scales obtained compared to temperature, $t/k_B T = 54(5)$, give access to the regime where
158 quantum correlations are strong¹⁻³. Extending this work to larger quantum dot arrays, whether

159 for the purpose of analog quantum simulation or quantum computation, requires further automa-
160 tion of our methods²⁸, and extensions to parallelize the calibration routines. Scalable gate layouts
161 for 1D arrays already exist²⁹, which together with the programmable disorders in on-site ener-
162 gies, can be mapped onto the physics of many-body localization³⁰. Further advances in connec-
163 tivity and homogeneity are underway in the pursuit of scalable quantum computing, including
164 square³¹ and triangular³² geometries, industrial-grade fabrication processes and magnetically quiet
165 ²⁸Si substrates³³, that open up further possibilities for quantum simulation experiments with quan-
166 tum dots.

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244 C.R. and W.W. grew the heterostructure, X.L. and S.D.S. performed the theoretical analyses with
245 X.L. carrying out the numerical simulations, T.H., T.F., X.L., L.J., S.D.S. and L.M.K.V. contributed
246 to the interpretation of the data, and T.H. wrote the manuscript (X.L. wrote part of the Methods),
247 with comments from T.F., X.L., S.D.S. and L.M.K.V.

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249 The authors declare no competing financial interests. Correspondence and requests for materials
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251 **Figure 1 | Gate-defined quantum dot array as a platform for quantum simulations of the**

252 **Fermi-Hubbard model**

253 Electron micrograph of a sample nominally identical to the one used for the measurements. The
254 bottom three circles indicate the triple dot array, whose Hamiltonian parameters derive from the
255 local potential landscape controlled by the seven bottom-most gates (B_{1L} to B_{3R}). The top circle
256 and arrow indicate the sensing dot channel, the radio-frequency reflectance of which is monitored
257 to enable real-time charge sensing. Crossed squares indicate distinct Fermi reservoirs that are
258 contacted using ohmic contacts. We describe a toolbox that allows for the control of the quantum
259 dot array at the level of the microscopic Fermi-Hubbard model. In particular, it allows for the
260 independent calibration of $\{\mu', \delta\epsilon_i, t_{ij}\}$ and the measurement of the Coulomb interaction terms
261 $\{V_{ij}, U_i\}$. Measurable observables for quantum dots include both local charge occupation and
262 global charge transport as well as local spin degrees of freedom and nearest-neighbour singlet-
263 triplet spin correlations (through spin-to-charge conversion protocols^{11,16,17}).

264 **Figure 2 | Hamiltonian engineering using a scalable toolbox of local control and measure-**
265 **ments**

266 **a** Charge stability diagram showing uniform filling of the array of up to three electrons per dot in
267 the vertical direction, using a combination of all seven gates (only P_1 values are shown) that equally
268 sweeps the local fillings n_i while keeping the tunnel couplings between dots and to the reservoirs
269 nominally identical. Lines correspond to charge transitions. **b** Theoretical charge stability diagram
270 of a triple-quantum-dot system in the classical limit ($t = 0$) exchanging particles with a reservoir
271 at $U/k_B T = 300$, analogous to the measurement in **a**. **c** As we focus on relevant sections of the
272 charge-stability diagram of the array, we calibrate all relative cross-capacitances of the seven-gate,
273 three dot-system, allowing for deterministic changes in ϵ_i and subsequent measurement of on-site
274 and inter-site Coulomb couplings. **d** Measurements of both tunnel couplings as a function of two
275 linear combinations of gate voltages, VB_{12} and VB_{23} , that keep either t_{23} or t_{12} (the full line de-
276 notes the average value) as well as the three on-site energies ϵ_i constant whilst increasing t_{12} or
277 t_{23} , respectively (an exponential fit to $\alpha \exp(VB_{ij}/\beta)$ is shown). Individual tunnel coupling data
278 points are taken at a rate of roughly 1 Hz and have typical fitting errors of several per cent (not
279 shown). Text in brackets denote the dominant charge states in the many-body eigenstate.

280 **Figure 3 | Collective Coulomb blockade physics in the Fermi-Hubbard phase space**

281 **a** Schematic representation of the charge addition spectrum of a Mott insulator at half filling and
282 a triple quantum dot array in Coulomb blockade (bottom) and those of a metallic phase at half
283 filling and a triple quantum dot array in collective Coulomb blockade (top). **b** The experimen-
284 tally accessible parameter space of the Fermi-Hubbard model for a triple quantum dot array as
285 a function of electron filling and nearest-neighbour tunnel coupling. Continuous charge sensing
286 measurements following the charging lines are shown, at calibrated gate values where the dots are
287 filled homogeneously (only ϵ_3 values are shown) and the t_{ij} 's are set to be roughly equal. Plotted
288 spacings between the bands are set by the Coulomb interaction terms measured at small tunnel
289 coupling. Red circles indicate extended Hubbard model calculations of the transitions. In the ver-
290 tical direction, they are set using the same measured $t_{avg} = (t_{12} + t_{23})/2$ as the experimental data.
291 In the horizontal direction, the simulations start from measured interaction energies with $\sim 10\%$
292 errors (see Methods, Extended Data Fig. 6 and Extended Data Table 1-2). Text in brackets denotes
293 electron filling.

294 **Methods**

295 **Materials and set-up** The triple quantum dot sample was fabricated on a GaAs/Al_{0.25}Ga_{0.75}As
296 heterostructure that was grown by molecular-beam epitaxy. The 85-nm-deep 2D electron gas has
297 an electron density of $2.0 \times 10^{11} \text{ cm}^{-2}$ and 4 K mobility of $5.6 \times 10^6 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$. All sample
298 structures were defined using electron-beam lithography, with metallic gates (Ti/Au) and ohmic
299 contacts (Ni/AuGe/Ni) deposited on the bare wafer in a lift-off process using electron-beam evap-
300 oration, similarly to the definition of metallic markers, leads and bonding pads, and with sample
301 mesas defined using a diluted Piranha wet etch. The plunger gates were connected to bias-tees on
302 the printed circuit board, allowing for fast sweeps and RF excitations to be applied in addition to
303 DC voltages. RF reflectometry³⁴ of the sensing dot channel conductance is done at 110.35 MHz
304 employing a homebuilt LC circuit on the printed circuit board. The sample was cooled down in
305 an Oxford Kelvinox 400HA dilution refrigerator to a base temperature of 45mK whilst applying
306 positive bias voltages to all gates. With the sample cold and the dots formed through application
307 of appropriate voltages to the metallic gates, read-out was performed by feeding the RF reflec-
308 tometry circuit a roughly -99 dBm carrier wave, the reflected signal of which is amplified at 4 K
309 and subsequently demodulated and measured using custom electronics. Using this technique on a
310 sensing dot is preferred to forming a quantum point contact, and yields measurement bandwidths
311 exceeding 1 MHz. The sensing dot position is asymmetric in order to obtain different sensitivities
312 to each of the three dots. Note that as an alternative to electrostatically defined charge sensors in
313 the 2DEG itself, dispersive read-out using the nanofabricated top gates would allow to measure
314 how much charges move in response to gate voltage changes³⁵. For more detailed methods please

315 see Baart *et. al.*¹⁶.

316 **Eliminating cross-talk through the definition of virtual gates** Changes in ϵ_i can be tracked di-
317 rectly by following transitions in the charge stability diagram and are found to depend linearly
318 on gate values for voltage changes up to several tens of millivolts. In general, small changes in
319 the energy offsets of each of the three dots will thus be achieved via a linear combination of voltage
320 changes on each of the seven gates: $\delta \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{pmatrix} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} & \alpha_{15} & \alpha_{16} & \alpha_{17} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} & \alpha_{25} & \alpha_{26} & \alpha_{27} \\ \alpha_{31} & \alpha_{31} & \alpha_{33} & \alpha_{34} & \alpha_{35} & \alpha_{36} & \alpha_{37} \end{pmatrix} \delta (P_1 P_2 P_3 B_{1L} B_{12} B_{23} B_{3R})^T$.
321 Of these 21 matrix elements, the three α_{ii} 's describe the coupling of the plungers P_i to the energy
322 offset ϵ_i of their respective dot i . The other 18 elements are cross-talks, whose values can eas-
323 ily be related to the α_{ii} 's through the slope of charge addition lines (see Extended Data Fig. 1a).
324 This leaves the relative weights of the α_{ii} 's and the absolute value of one of the elements to be
325 determined. As the difference between the single-particle energies of two dots stays fixed along a
326 polarization line, we can determine the relative weights from the slope of these lines (see Extended
327 Data Fig. 1b). The absolute value of α_{22} can be found using photon-assisted tunneling measure-
328 ments (see Extended Data Fig. 4). For the measurements presented in Fig. 3b, the matrix has been
329 measured multiple times for different fillings and tunnel couplings: the 'plunger' side α_{11} - α_{33} of
330 the matrix was measured 25 times in total and the 'barrier' part α_{14} - α_{37} 12 times (see Extended
331 Data Fig. 1c). In between these points, we used linear interpolation as function of measured tunnel
332 coupling to extract matrix elements when needed.

333 With all matrix elements known, the ϵ_i 's can be deterministically changed, a technique which is
334 extensively used throughout the results presented here in two main ways, (1) by measuring Hamil-

335 tonian parameters through direct interpretation of features in the addition spectrum and (2) through
336 the definition of ‘virtual gates’, both for plunger and barrier gates, that greatly simplify the tuning
337 process. For instance, the virtual gate for the energy offset of the leftmost dot, ϵ_1 , is defined by
338 a simple combination of plunger gates: $\delta \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix}^{-1} \begin{pmatrix} \delta\epsilon_1 \\ 0 \\ 0 \end{pmatrix}$. To form virtual barrier
339 gates we use $\delta B_{12} \rightarrow \delta V B_{12} = \delta(P_1, P_2, P_3, B_{12})$ with $\delta \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix} = -\delta B_{12} \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix}^{-1} \begin{pmatrix} \alpha_{14} \\ \alpha_{24} \\ \alpha_{34} \end{pmatrix}$,
340 which allows for making the barrier separating dots 1 and 2 more (or less) transparent without
341 changing the energy offsets ϵ_i of any of the dots, which is to say, stay at the same location in the
342 charge stability diagram. Linear combinations of this gate and its equivalent between dots 2 and 3
343 yield the two orthogonal control gates for changing t_{ij} , as used in Fig. 2d.

344 **Classically coupled dots and homogeneous filling** Isolated quantum dots are well described by a
345 classical capacitance model³⁶. This description is valid as long as tunnel coupling energies are neg-
346 ligible compared to capacitive (Coulomb) effects. In this case, the charge states s of the system are
347 simply described by the set of individual dot occupations (n_1, n_2, \dots) as the n_i ’s are good quantum
348 numbers. As has been shown previously⁹, one can map the classical capacitance model onto the
349 extended Hubbard model of Eq. 1 with omission of its tunneling terms, which is readily diagonal-
350 ized with eigenenergies $E(n_1, n_2, \dots) = -\sum_i \epsilon_i n_i + \sum_i \frac{U_i}{2} n_i (n_i - 1) + \sum_{i,j \neq i} V_{ij} n_i n_j$. Because
351 we experimentally probe changes in the equilibrium charge state of the array coupled to adjacent
352 electron reservoirs, typically kept at an equal and constant electrochemical potential μ and temper-
353 ature $k_B T$, we are interested in the charge addition spectrum $\frac{\partial \langle N \rangle}{\partial \mu}$, with $\langle N \rangle = k_B T \frac{\partial \ln \mathcal{Z}}{\partial \mu}$, $\mathcal{Z} =$
354 $\text{Tr}\{\exp[-(H - \mu N)/k_B T]\}$, where $N = \sum_i n_i$ is the total electron number and \mathcal{Z} is the grand
355 partition function. In this classical case and at constant chemical potential $\mu = 0$, the equations

356 for the charge addition spectrum $\frac{\partial \langle N \rangle}{\partial \mu} = \frac{\langle N^2 \rangle - \langle N \rangle^2}{k_B T}$ simplify to simple Boltzmann-weighted sums
 357 over the charge states s , namely $\mathcal{Z} = \sum_s \exp[-E_s/k_B T]$ and $\langle N^k \rangle = \frac{1}{\mathcal{Z}} \sum_s N_s^k \exp[-E_s/k_B T]$.
 358 Note that for the purpose of finding the charge transitions, any spin-degeneracy of the charge states
 359 can be ignored. The charge stability measurements shown in the main text effectively show two-
 360 dimensional slices of the charge addition spectrum as a function of changes in the ϵ_i 's.

361 The filling of the quantum dot array is controlled experimentally by changing the energy difference
 362 between the electronic states at the Fermi level of the reservoir and those of the dot array itself.
 363 The former can be done by applying a bias voltage to the relevant Fermi reservoir, the latter by
 364 applying voltages to top gates that influence the single-particle energies ϵ_i on the dots. Because
 365 the partition function is only sensitive to changes in $H - \mu N$, one can equivalently think about
 366 changes in the ϵ_i 's as influencing the chemical potential directly through $\delta(\mu N) = \delta(\sum_i \epsilon_i n_i)$,
 367 which at uniform filling, simplifies to $\delta\mu = \langle \delta\epsilon_i \rangle$. This allows for a different look at the gate
 368 control over a quantum dot array with M sites. Instead of thinking about M different ϵ_i 's, we can
 369 define one global chemical potential term $\mu' = \langle \epsilon_i \rangle$ and $M - 1$ energy differences $\delta_i = \epsilon_i - \mu'$,
 370 where the latter describe the setting of some (controllable) disorder potential landscape at a fixed
 371 chemical potential μ' .

372 In the case of a large and homogeneous system, changing all ϵ_i equally would uniformly and
 373 homogeneously fill all dots in the system. For the triple-quantum-dot sample described in the main
 374 text, however, both the finite size (e.g. only one of the three dots has two direct neighbours) and
 375 inhomogeneous interaction terms (e.g. $U_1 \neq U_2$) mean a different approach is needed: we have to

376 link up a set of well-defined points in the $(\epsilon_1, \epsilon_2, \epsilon_3)$ -space. In the case of $V_{ij} = 0$, and focussing
 377 on the regime from 0 to 2 electrons per site, the only obvious choice would be to identify and align
 378 points A (where the eight charge states (000) to (111) are degenerate) and point B (where (111) to
 379 (222) are degenerate) (see Extended Data Fig. 2a). These points are lined up by changing the on-
 380 site single particle energies by ratio of their on-site repulsions $\epsilon_i = \mu'U_i/\langle U \rangle$. Analogously, under
 381 finite V_{ij} , we use the ratio of the sum of all locally relevant interaction energies $W_i = U_i + \sum_{j \neq i} V_{ij}$
 382 as $\epsilon_i = \mu'W_i/\langle W \rangle$. Note, however, that the inter-site repulsion breaks particle-hole symmetry and
 383 moves states with more than one particle added to a homogeneously filled state to higher energy,
 384 meaning we can only find points with at most 4 degenerate states. We can align points C (where
 385 (000), (100), (010) and (001) are degenerate) and D (where (111), (211), (121) and (112) are
 386 degenerate) (see Extended Data Fig. 2b), or we can align points E (where (110), (101), (110) and
 387 (111) are degenerate) and F (where (221), (212), (221) and (222) are degenerate) (see Extended
 388 Data Fig. 2c), the two of which are particle-hole partners of the same total state.

389 Defining a miniband as the region in chemical potential where one uniform filling transitions to
 390 the next one (the first miniband is thus the transition region between (000) and (111)), it becomes
 391 clear that the inter-site Coulomb terms already widen the miniband at zero tunnel coupling. On top
 392 of this, too large a deviation in the site-specific energy offsets ϵ_i 's from the desired values (which
 393 amounts to disorder in the dot energies) can also increase the miniband width. For changes in
 394 $\delta\epsilon_1 = -\delta\epsilon_3$, this can be seen in Fig. 2a. For changes in $\delta\epsilon_2$, the width remains minimized as
 395 long as the $\delta\epsilon_2$ remains in the window between two well-defined points denoted by the crosses and
 396 diamonds of Extended Data Fig. 2 (see also Extended Data Fig. 3).

397 **Anti-crossing measurement and fit** Much of the day-to-day work in quantum dot arrays in general
398 and for the measurements described here in particular consists of the interpretation of features in
399 the charge stability diagram. In the case of well isolated dots with localized electrons ($t/U \ll 1$)
400 this essentially boils down to one-dot features (parallel lines) and two-dot features (anti-crossings
401 and associated polarization lines). Indeed, pattern recognition of anti-crossings is the crucial step
402 in the automated initial tuning of double quantum dots²⁸.

403 In general, the processing of a charge stability diagram (e.g. Fig. 2c) starts with finding charge
404 transitions in the raw sensor dot data using an edge finding algorithm. The results are filtered to
405 only leave edge sections with more than a threshold number of points. Next, we employ a k-means
406 algorithm to cluster the edges into line sections. Depending on the data, manual input might
407 be needed, either in the selection of relevant clusters or, sometimes, in the case of noisy data,
408 manual selection of points. In determining on-site interaction terms U_i , calculating the orthogonal
409 distance between two parallel lines suffices. In the case of an anti-crossing, we employ a 2D
410 fitting routine in a rotated frame $2 \begin{pmatrix} y \\ x \end{pmatrix} = \delta \begin{pmatrix} \epsilon_i + \epsilon_j \\ \epsilon_i - \epsilon_j \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \alpha_{ii} & \alpha_{ij} \\ \alpha_{ji} & \alpha_{jj} \end{pmatrix} \delta \begin{pmatrix} P_i \\ P_j \end{pmatrix}$, simultaneously
411 fitting both branches in a least squares sense to $y - y_0 = \pm \left(V_{ij}/2 + \sqrt{(x - x_0)^2 + t_{ij}^2} \right)$. Fitting
412 parameters are three of the matrix elements (corresponding to the angles of the two dot lines and the
413 polarization line), the two offsets x_0 and y_0 and the two energies V_{ij} and t_{ij} . Both the procedures to
414 find U_i and V_{ij} are limited to $t/U < 0.15$, as around this value for the tunnel coupling there are no
415 straight line sections in the charge addition diagram left where two well-defined localized charge
416 states meet. Further discussion on this can be found with Extended Data Fig. 6.

417 **Practical limits to achievable parameter space** As can be seen in Fig. 3b, there are limits to the
418 achievable parameter space in terms of electron filling and tunnel coupling for the device measured.
419 This is mostly due to the gate layout, which was designed for spin qubit experiments at fillings
420 around one electron per site and tunnel couplings up to several tens of μeV (red shaded area
421 in Fig. 3b). The chosen lithographic separation between the dots does not allow for sufficient
422 wavefunction overlap between singly-occupied sites to achieve much larger tunnel couplings. With
423 multiple electrons per dot, however, the wavefunctions are more extended and much larger tunnel
424 couplings are possible. Here, practical difficulties in compensating for cross-talk make it hard to
425 reach very small tunnel couplings.

426 **Verification through Fermi-Hubbard calculations: measuring miniband width** We perform
427 numerical simulations with two levels of detail. Extended Data Fig. 5 shows the collective
428 Coulomb blockade transition in a simplified model to illustrate the main concepts. Results from a
429 more detailed simulation are overlaid with the experimental data in Fig. 3b. We here elaborate on
430 these two approaches.

431 In the simplified model calculation, we ignored the inter-site Coulomb interactions $V_{ij}n_i n_j$, which
432 will split the peaks in the addition spectrum even at zero tunnel coupling, as discussed above. It is
433 included in the detailed model. Because it is difficult to experimentally fix the absolute chemical
434 potential over large areas of the parameter space due to nonlinearities in the gating effects, the ad-
435 dition spectrum in Fig. 3b was constructed by plotting the middle transition within each miniband
436 as a straight line at fixed ϵ_3 , and measuring the chemical potentials of adjacent transitions with

437 respect to those. As we can see from Extended Data Fig. 5b, such an approximation is justified at
 438 small $t/U (< 0.15)$, although it neglects any change in the interaction terms with increasing tunnel
 439 coupling. Furthermore, since the interaction parameters are non-constant over the experimental
 440 phase space (Extended Data Fig. 6), the detailed simulations take this into account. Finally, as
 441 also discussed above, it requires an inhomogeneous change in the site-specific energy offsets to
 442 homogeneously fill the array. In order to allow direct comparison to the experiment, we thus have
 443 to take the correct $\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{pmatrix}$ line to describe the filling (horizontal axis of Fig. 3b). Note that because
 444 of the non-constant interaction energies, this vector will generally differ with miniband number
 445 and tunnel coupling.

446 In order to find the correct filling vector and subsequently the position of the transitions, we use
 447 the following procedure for each data set at a particular tunnel coupling and miniband number: (i)
 448 When the system has $N = 3n$ electrons, its ground state is tuned to be the (n, n, n) state. (ii) The
 449 two critical points (both for n and $n' = n + 1$) at which the four states (n, n, n) , $(n \pm 1, n, n)$,
 450 $(n, n \pm 1, n)$, and $(n, n, n \pm 1)$ are degenerate are identified. (iii) Linking these points in the
 451 three-dimensional parameter space spanned by $(\epsilon_1, \epsilon_2, \epsilon_3)$ yields the filling line $\delta \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{pmatrix}$. (iv) The
 452 three charge transitions of the miniband are subsequently found to lie somewhere on this line. (v)
 453 This procedure yields a fixed width of the miniband, but leaves one degree of freedom unspecified,
 454 which is the relative position of the middle dot detuning relative to the outer dots, addressed in the
 455 next paragraph.

456 We illustrate this procedure for the data with the second largest tunnel couplings in the fourth

457 miniband in Fig. 3b in the main text, for which the following set of quantum dot parameters
 458 applies: $t = 0.29$, $U_1 = 2.26$, $U_2 = 2.70$, $U_3 = 2.48$, $V_{12} = 0.65$, $V_{23} = 0.57$, $V_{13} = 0.43$
 459 (all in meV). First of all, it is helpful to show the ‘uniform’ chemical potential μ that correspond
 460 to the specific ϵ_i ’s (a ‘global’ chemical potential μ can be regained through $\mu = \frac{1}{N} \sum_i \epsilon_i n_i$).
 461 Such a comparison is shown in Extended Data Table 1. We can see that in the three-dimensional
 462 parameter space the filling vector defined by $\delta \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{pmatrix}$ can be very different from the one defined
 463 by $\delta \begin{pmatrix} \mu \\ \mu \\ \mu \end{pmatrix}$. This shows that the distinction is important, and a simple simulation with a uniform
 464 chemical potential as in Extended Data Fig. 5b will not compare well with the experiment. Second,
 465 note that the simulations are done for the specific middle dot detuning denoted by the asterisk in
 466 Extended Data Fig. 2b and Extended Data Fig. 3b, whereas the experimental detuning will be in
 467 between that situation and the detuning denoted by the diamond in the same figures. This means
 468 that although the total width of the miniband will be fixed, the relative position of the middle
 469 transition between the outer transitions of each miniband (which we denote α and which will be
 470 close to 0.5) depends on the specific middle dot detuning. To overlay the simulation results on the
 471 experimental data, we used values of $\alpha = (0.5, 0.6, 0.65, 0.6)$ for the four minibands, respectively.
 472 Finally, Extended Data Table 2 gives an overview for the width of the fourth miniband at different
 473 tunnel couplings, as Fig. 3b in the main text only plots the data along the ϵ_3 direction. It can be
 474 seen that the theory compares well with the experiment along all three directions, which further
 475 corroborates the consistency of our measurements.

476 **Data availability statement** Source data for both main text and Extended Data figures are provided
 477 with the paper. Raw data and analysis files supporting the findings of this study³⁸ are available from

478 <https://doi.org/10.5281/zenodo.546675>.

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491 **Extended Data Figure 1 | Gate-to-dot cross-talk**

492 **a** Cross-talk measurement of gates P_1 and B_{12} on the left dot detuning. The slope of the charge
493 transition (fit in white) yields the relative effect ($\delta B_{12}/\delta P_1 = -\alpha_{11}/\alpha_{14}$) of the two gates on the
494 single-particle energy offset ϵ_1 of the leftmost dot. Note also the nonzero background in charge
495 sensor response we find in experiments, which is due to a direct coupling between the swept gate
496 voltages and the sensing dot conductivity. **b** Charge stability diagram showing the anti-crossing
497 (white) and polarization line (red) between the left and middle dot, yielding the relative effect
498 $\alpha_{11} = \alpha_{21} + (\delta P_2/\delta P_1)(\alpha_{22} - \alpha_{12})$ of the two plungers on their respective dots. Automated edge
499 finding and fitting procedures are outlined in Methods. **c** Measured matrix elements α_{ij}/α_{22} as a
500 function of tunnel coupling. No visual distinction is made between the measured matrix elements
501 at different electron filling. No error bars are shown, as the small uncertainty in the slope fits yields
502 errors smaller than marker size.

503 **Extended Data Figure 2 | Simulated classical charge addition spectra**

504 **a-c** Simulated charge addition spectra (see Methods) for a triple quantum dot at zero tunnel cou-
505 pling, $U_2 = 1.05U_1 = 0.95U_3$ and up to two particles per dot, connected to a reservoir at $\mu = 0$
506 and $k_B T = 0.02U$ (>10 times larger than for the experiments described in the main text), with
507 $V_{ij} = 0$ and $\delta_i = 0$ (**a**) or with $V_{12} = V_{13} = 2V_{23} = 0.2U$ and $\delta_i = 0$ (**b**) or $\delta_1 = \delta_3 = 0$ and
508 $\delta_2 = U/15$ (**c**). States are denoted by charge occupation $(n_1 n_2 n_3)$ and specific degeneracy points
509 A-F are referred to in Methods. The relation between ϵ_i and μ' specified in the bottom left boxes
510 applies to the vertical line at zero (horizontal) detuning.

511 **Extended Data Figure 3 | Miniband width and electron temperature**

512 **a** Measured charge stability diagrams of the 222-333 miniband as a function of homogeneous fill-
513 ing (only P_1 values are shown) and offset in the outer two dot energies by changing P_1 and P_3
514 in opposite directions, akin to the simulations of Extended Data Fig. 2c. **b** Similar measurement
515 as a function of the offset in the middle dot energy, controlled by P_2 . The P_1 values are some-
516 what different from **a** because these measurements were taken at slightly different tunnel coupling
517 tunings. The white diamond and asterisk indicate (roughly) the position of the same degeneracy
518 points as shown in Extended Data Fig. 2. **c** Broadening of a charge addition line due to the finite
519 temperature of the (rightmost) Fermi reservoir. A Fermi-Dirac fit of the transition is shown in red,
520 which yields an effective reservoir temperature of 72(1) mK.

521 **Extended Data Figure 4 | Determining lever arm and tunnel coupling**

522 **a** Example of a photon assisted tunneling (PAT) measurement, which at low tunnel couplings is the
523 measurement method of choice for both lever arm and tunnel coupling. Plotted is the difference in
524 charge sensor response between applying a microwave excitation or not as a function of detuning.
525 Dashed red line is a fit to the hybridized charge state spectrum of the double dot²⁴. The energy
526 difference between bonding and anti bonding states yields the minimum in frequency ($2t$) and the
527 slope away from the transition gives the lever arm between detuning voltages applied to the gates
528 and single-particle energy difference change between the two dots. The need to generate AC exci-
529 tations and transmit them without significant losses through coaxial cables in the fridge, however,
530 limits the maximum tunnel frequency we can accurately determine with this method to roughly
531 20 GHz (83 μeV). **b** Example of a polarization line width measurement, with fit in red. As an al-
532 ternative to PAT, one can determine the tunnel coupling by assessing the width of the polarization
533 line³⁷. The excess charge (say on the left dot) transition is broadened both by an effective electron
534 temperature and by the tunnel coupling. Charge sensor response is however not a direct measure-
535 ment of excess charge. Not only does there exist a finite cross-talk between the gate voltages and
536 the charge sensor response that leads to a finite slope away from the transition, we also typically
537 find a back-effect of the excess charge on the sensing dot, leading to a different slope on either side
538 of the transition. We fit the data with the following equation, taking this back-effect into account
539 to first order in excess charge: $V(\epsilon) = V_0 + \delta V Q(\epsilon) + \left[\frac{\delta V}{\delta \epsilon} \Big|_{Q=0} + \left(\frac{\delta V}{\delta \epsilon} \Big|_{Q=1} - \frac{\delta V}{\delta \epsilon} \Big|_{Q=0} \right) Q(\epsilon) \right] \epsilon$,
540 where $V(\epsilon)$ is the charge sensor response as a function of the detuning $\epsilon = \delta(\epsilon_i - \epsilon_j)$ away from to
541 the transition and V_0 , δV and $\frac{\delta V}{\delta \epsilon}$ are the background signal, sensitivity and gate-sensor coupling,

542 respectively. Note that ϵ is a linear combination of the swept gate voltages, taking the relevant
543 cross-capacitances and the lever arm into account. Excess charge on the left dot is described by
544 $Q(\epsilon) = \frac{1}{2} \left(1 + \frac{\epsilon}{\Omega} \tanh \left(\frac{\Omega}{2k_B T_e} \right) \right)$, with $\Omega = \sqrt{\epsilon^2 + 4t_{ij}^2}$ and effective temperature $k_B T_e \approx 6.5 \mu\text{eV}$
545 (1.6 GHz). **c** Excess charge as function of detuning for three different tunnel couplings, showing
546 that this characterization method works up to significantly larger tunnel couplings than PAT. **d**
547 Comparison of PAT and polarization line width measurements. The data is well explained by as-
548 suming a constant lever arm $\alpha_{22} = 83(1) \mu\text{eV/mV}$ between gate P_2 and the middle dot. Text in
549 brackets denote relevant charge states, error bars are 1σ fit uncertainties.

550 **Extended Data Figure 5 | Simulations of collective Coulomb blockade for the simplified**

551 **Hubbard model**

552 **a** Cartoon diagram of a triple dot system, which is a simplified version of the model used to de-
553 scribe the experiments in the main text. Specifically, we have set a uniform tunnel coupling t and
554 Hubbard U , while ignoring the inter-site Coulomb interaction term V_{ij} . We describe two levels per
555 dot with a level splitting Δ that separates the single-particle energies of the first and second orbital.
556 Each energy level is doubly degenerate due to the spin degrees of freedom. **b** Peaks in the electron
557 addition spectrum for the triple dot system in **a**. It is known that the classical Coulomb blockade
558 effect arises purely from the charging effects of the quantum dots. When electron tunneling be-
559 tween quantum dots is allowed, however, quantum fluctuations compete with the classical charging
560 effects and give rise to a rich phase diagram, which is known as collective Coulomb blockade²⁰.
561 The metal-insulator transition in such a system is best captured by the charge addition spectrum,
562 which is precisely what we measure in the experiment (Fig. 3b in the main text). The numbers in
563 **b** indicate the average electron numbers in the system when the chemical potential resides at the
564 respective gap. Here we use $\Delta/U = 0.2$, and $k_B T/U = 0.04$ (>20 times larger than for the experi-
565 ments described in the main text). **c-f** Line cuts for the addition spectrum in **b** at different values of
566 t/U . As we discussed in the main text, there will be three different regimes in this phase diagram:
567 at weak tunnel couplings the quantum dot states split into minibands but the isolated Coulomb
568 blockade of each individual dot is preserved; at intermediate tunnel couplings the Coulomb block-
569 ade of individual dots is lost, but the gap between minibands remains open; finally, in the large
570 tunnel coupling limit the gap between minibands can become comparable to temperature, and the

571 system will be in a metallic state. The same can be seen in these line cuts. At $t = 0$ we can see that
572 there are four critical chemical potentials μ at which electrons can be added to the triple dot. For
573 the present model, these four peaks occur at $\mu = 0, U, 2U + \Delta,$ and $3U + \Delta,$ respectively. Each
574 peak is triply degenerate, as the energy cost to add electrons to any of the three dots is identical.
575 For nonzero but small tunnel couplings (**d-e**) each triply degenerate peak at $t = 0$ starts to split
576 into a miniband, indicating the breakdown of Coulomb blockade in each dot. However, different
577 minibands are still separated by gaps that arise from a collective origin, reminiscent of the energy
578 gap in a Mott insulator. Finally, at sufficiently high tunnel couplings we find nonzero $\frac{\partial \langle N \rangle}{\partial \mu}$ at the
579 middle gap (**f**), indicating that Coulomb blockade is overwhelmed by temperature altogether.

580 **Extended Data Figure 6 | Characterizing model parameters**

581 **a** Simulated charge stability diagram for a triple dot system with parameters $t = 0.006$, $U_1 =$
582 3.98 , $U_2 = 3.48$, $U_3 = 2.70$, $V_{12} = 0.41$, $V_{23} = 0.35$, $V_{13} = 0.11$ (all energies in meV). As
583 described in Methods, the eigenstates can be obtained exactly in the $t = 0$ limit, as the eigenstates
584 of the triple dot system can be represented simply by the charge states $(n_1 n_2 n_3)$. In this regime,
585 one can show that on the ϵ_2 - ϵ_3 plane the border between the (111)/(112) region and the border
586 between the (111)/(110) region are exactly separated by an energy of U_3 . Similarly, the border
587 between the (111)/(121) region and the border between the (111)/(101) region are separated by
588 an energy of U_2 . In the presence of a nonzero but small tunnel coupling as is the case here, we
589 expect that such an estimate is still reasonable. Now that the tunnel coupling is nonzero, the ground
590 state of the system is no longer an exact charge state $(n_1 n_2 n_3)$, but generally a superposition of
591 different charge states. To retain a connection to the $t = 0$ limit, we keep labeling sections of the
592 charge stability as $(n_1 n_2 n_3)$, but with the distinction in mind that $(n_1 n_2 n_3)$ no longer denotes the
593 exact ground state, but instead the charge state with the largest weight in the actual ground state.
594 As a check, we can determine the values of U_2 and U_3 from the simulated charge stability diagram
595 using the method described above and find that $U_2 = 3.44$ meV and $U_3 = 2.71$ meV, respectively,
596 which is reasonably close to the corresponding model parameters. Since the data in Fig. 2c is
597 taken at $t/U = 0.002$, we can thus trust the extracted U . **b** Charge stability diagram for a triple
598 dot system with parameters $t = 0.17$, $U_1 = 2.92$, $U_2 = 2.39$, $U_3 = 2.53$, $V_{12} = 0.55$, $V_{23} =$
599 0.47 , $V_{13} = 0.27$ (all energies in meV). We find that the structure of the charge stability diagram
600 remains qualitatively the same as that in **a**, and if we again extract the values of U_2 and U_3 using the

601 same method, we find that $U_2 = 2.48 \text{ meV}$ and $U_3 = 2.56 \text{ meV}$, which still agrees reasonably well
602 with the original model parameters. Granted, at sufficiently large t/U the structure of the charge
603 stability diagram will change drastically, and the present method to extract model parameters is
604 bound to fail. However, as we never enter those regimes, our fitting method serves the purpose
605 of this experiment. **c-e** Calibrated tunnel couplings (**c**) and measured inter-site Coulomb (**d**) and
606 on-site Coulomb (**e**) terms at calibrated values of the average tunnel coupling, corresponding to
607 the experimental parameter space plot shown in Fig. 3b of the main text. Blue fill indicates data
608 from the first subband from 0 to 6 electrons, red fill data from the second subband from 6 to 12
609 electrons. Error bars are 1σ fit uncertainties.

610 **Extended Data Figure 7 | Isolated versus collective Coulomb blockade in charge and trans-**
611 **port**

612 **a (c)** Charge stability diagram around the (333) regime in the low (high) tunnel coupling regime,
613 using a combination of all seven gates (only P_1 values are shown) that change the local fillings
614 equally. To further investigate the distinct phases, we focus on the regime with around nine elec-
615 trons in total, corresponding to half-filling of the second band, and look at both charge sensing and
616 transport. In the localized phase ($t/U < 0.02$ in **a**), the charge stability diagram shows transition
617 lines following three distinct, well-defined directions, corresponding to the filling of the separate
618 lithographically defined dots. In the delocalized phase ($t/U > 0.15$ in **c**), this distinct nature is
619 all but lost, highlighting the incipient formation of a large single dot. The same effect can also
620 be seen in transport measurements, as we observe Coulomb diamond sizes as a function of fill-
621 ing. **b** Transport through the array following the zero-detuning line of Fig. 2b of the main text
622 as a function of applied bias (60% on leftmost and 40% on bottom right reservoir). In the (333)
623 state, this applied bias has to overcome the local (strong) Coulomb repulsion in order for current
624 to flow, similar to a Mott insulator whose Fermi energy resides inside the gap. Adjacent Coulomb
625 diamonds correspond to a Fermi-level inside the miniband and are significantly smaller, allowing
626 current to flow at much smaller bias voltages. **d** Similar data in the high tunnel coupling regime.
627 Whereas the individual nature of the dots is all but gone, global (weaker) Coulomb repulsion still
628 prohibits transport at small bias, as expected for the collective Coulomb blockade phase. The no-
629 tion of a large gap at half-filling is gone, and it is but the charging energy of the entire system that
630 prohibits transport to occur, regardless of filling. The dots are in collective Coulomb blockade, and

⁶³¹ its transport characteristics are similar to that of a small, metallic island.

632 **Extended Data Table 1 | Example of simulated transition points**

633 Transition points for a triple dot system with parameters $t = 0.29$, $U_1 = 2.26$, $U_2 = 2.70$, $U_3 =$
634 2.48 , $V_{12} = 0.65$, $V_{23} = 0.57$, $V_{13} = 0.43$ (all in meV). The label $N_1 \rightarrow N_2$ indicates that this
635 data is for the transition from a total of N_1 particles to N_2 particles. ϵ_i ($i = 1, 2, 3$) are the ‘local’
636 chemical potentials on each dot, while μ is the ‘uniform’ chemical potential. The last two columns
637 compare the experimental and theoretical total width of the fourth miniband. All energies are in
638 meV.

639 **Extended Data Table 2 | Comparison of experimental and theoretical miniband width**

640 Comparison of the experimental and theoretical width of the fourth miniband in Fig. 3b in the
641 main text at five calibrated values of the tunnel coupling. Theoretical widths take the interaction
642 energies measured at the specific tunnel coupling values into account (see Extended Data Fig. 6).
643 All energies are in meV.