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Barrier Lyapunov function-based fixed-time FTC for high-order nonlinear systems with predefined tracking accuracy

Xiaolin Wang · Jihui Xu · Maolong Lv · Lei Zhang · Zilong Zhao

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Abstract This article proposes a fixed-time adaptive fault-tolerant control methodology for a larger class of high-order (powers are positive odd integers) nonlinear systems subject to asymmetric time-varying state constraints and actuator faults. In contrast with the state-of-the-art control methodologies, the distinguishing features of this study lie in that: (a) high-order asymmetric time-varying tan-type barrier Lyapunov function (BLF) is devised such that the state variables can be convergent to the preassigned compact sets all the time provided their initial values remain therein, which not only

preserves the constraints satisfaction, but warrants the validity of the adopted neural network approximator; (b) the proposed control design ensures the tracking errors converge to specified residual sets within fixed time and makes the size of the convergence regions of tracking errors adjustable *a priori* by means of a new BLF-based tuning function and a projection operator; (c) a variable-separable lemma is delicately embedded into the control design to extract the control terms in a “linear-like” fashion which not only overcomes the difficulty that virtual control signals appear in a non-affine manner, but also solves the problem of actuator faults. Comparative simulations results finally validate the effectiveness of the proposed scheme.

Keywords High-order nonlinear systems · Fixed-time stability · High-order tan-type BLF · Predefined tracking accuracy

X. Wang · J. Xu (✉) · L. Zhang
Department of Equipment Management and UAV
Engineering College, Air Force Engineering University,
Shaanxi 710051, Xi’an, China
e-mail: changan6018@163.com

X. Wang
e-mail: wangxiaolin_95@163.com

L. Zhang
e-mail: jxzl80926@163.com

M. Lv (✉)
Air Traffic Control and Navigation College, Air Force
Engineering University, Shaanxi 710051, Xi’an, China
e-mail: M.Lyu@tudelft.nl

M. Lv
College of Aeronautics Engineering, Air Force
Engineering University, Shaanxi 710038, Xi’an, China

Z. Zhao
Faculty of Electrical Engineering, Mathematics and
Computer Science, Delft University of Technology, Van
Mourik Broekmanweg 6, 2628, XE Delft, The Netherlands
e-mail: Z.Zhao-8@tudelft.nl

1 Introduction

Many actual systems can be described as high-order nonlinear systems, such as connected vehicles dynamics (e.g., a tractor with several trailers, etc.), synchronous motors, or aircraft wing rock dynamics [1–5]. Compared with strict-feedback and pure-feedback systems [6–11], high-order nonlinear systems are more general in the sense that positive-odd-integer powers appear in the dynamics. It is well documented in the literature that high-order nonlinear systems are intrin-

sically more challenging than strict-feedback and pure-feedback systems, as feedback linearization and backstepping methods fail to work [12]. To handle such problem, the adding-one-power-integrator method was successfully proposed in [13] by introducing iteratively one high power integrator instead of a linear one. Although a number of control problems (i.e., global robust stabilization [14], practical tracking control [15], and asymptotic tracking control [16]) have been solved, the system nonlinearities considered in the above-mentioned results are required to satisfy the growth constraints, i.e., $|\psi_m(\cdot)| \leq (|\chi_1|^{r_1} + \dots + |\chi_m|^{r_m})\rho_m(\chi_1, \dots, \chi_m)$ with $\rho_m(\cdot)$ ($1 \leq m \leq n-1$) known nonnegative smooth functions. To remove this limitation, an intelligent tracking control method was firstly developed in [17] without involving any restrictive growth condition. The same strategy, together with the common Lyapunov function method, was extended to deal with switched high-order nonlinear systems in [18]. Most recently, a new error compensation method was further derived in [19] to improve the tracking accuracy. Despite those efforts above, these results rarely focus on the rate of convergence. To be specific, only the exponential convergence of tracking error is guaranteed in the aforementioned works, which reveals the convergence time tends to be infinite.

From a practical perspective, the rate of convergence is of great significance to the transient tracking performance [20, 21]. Recently, the finite-time tracking control for lower-/high-order nonlinear systems is investigated in [22–24], which makes the tracking error converge into the compact set within a finite time. Nevertheless, the convergence time achieved in [22–24] depends on the initial states of the system. This inevitably brings up a problem, that is, the convergence time cannot be accurately settled when the initial states of the system are unknown. To solve such problem, the fixed-time control [25–28] is proposed skillfully, by which the tracking error can converge into the compact set within a fixed time and its upper bound of convergence time is independent of system initial conditions. The authors in [29] established an adaptive fuzzy feedback fixed-time control mechanism for uncertain high-order nonlinear systems. However, the Lyapunov function in [25–29] will be augmented by additional estimation error variables when uncertainties exist in lower-/high-order model, leading to the result that the state tracking errors only can converge to a bounded compact set whose size cannot be set explicitly in advance (spec-

ified tracking accuracy) within fixed time. On the other hand, the above results neglect crucial aspects, such as state constraints, whose importance is explained hereafter.

Because state constraints generally exist in actual systems, ignoring these state constraints may deteriorate system performance and even endangers system safety [30, 31]. In order to prevent violations of state constraints, barrier Lyapunov function (BLF) is exploited to guarantee the non-violation of state constraints, while ensuring closed-loop stability [32]. Generally, the commonly seen forms include log-type BLF [33], tan-type BLF [34] and integral-type BLF [35]. Compared with common log-type BLF and integral-type BLF, the tan-type BLF can integrate constraint analysis into a general method, which can handle constrained/unconstrained situations [36–38]. Thus, the tan-type BLF has become a popular research topic. However, most of the existing results of tan-type BLF are concentrated on low-order nonlinear systems subject to full-state constraints and cannot be directly applied to high-order cases. On the other hand, the existing tan-type BLF is symmetric time-invariant, which cannot be used to handle asymmetric time-varying cases [34, 37, 38]. To our best knowledge, such issue has not been considered in the literature studies. In addition, the actuator faults are ubiquitous in engineering applications, which may cause the system instability and performance degradation [39–43]. Motivated by the above observations, the main innovative points of this work are listed as follows:

- (1) Compared with existing fixed-time control designs [25–29], our proposed method can not only ensure fixed-time convergence, but also can make the tracking errors converge to a compact set whose size can be specified/predefined *a priori*.
- (2) As opposed to the existing symmetric time-invariant tan-type BLFs [34, 37, 38], a new type of high-order tan-type BLF is proposed to guarantee that state variables are confined within some asymmetric time-varying sets all the time, and the initial conditions are inside of corresponding sets.
- (3) A new BLF-based tuning function and a projection operator are delicately embedded into the control design to make the size of the convergence regions of state tracking errors adjustable in the framework of fixed-time stability. A variable-separable lemma is utilized to extract the virtual control signals and actuator faults in a “linear-like” manner.

2 Preliminaries and problem formulation

2.1 Problem statement

Consider the uncertain high-order nonlinear systems with the following form:

$$\begin{cases} \dot{\xi}_i = \Psi_i(\bar{\xi}_i) + \Gamma_i(\bar{\xi}_i)\xi_{i+1}^{p_i}, \\ \dot{\xi}_n = \Psi_n(\xi) + \Gamma_n(\xi)u^{p_n}, \\ y = \xi_1, \quad 1 \leq i \leq n-1, \end{cases} \quad (1)$$

where $y \in \mathbb{R}$ is the system output; $u \in \mathbb{R}$ is the control input (to be designed); $\bar{\xi}_i = [\xi_1, \xi_2, \dots, \xi_i]^T \in \mathbb{R}^i$ and $\xi = [\xi_1, \xi_2, \dots, \xi_n]^T \in \mathbb{R}^n$ are the states. For $i = 1, \dots, n$, $\Psi_i(\cdot)$ and $\Gamma_i(\cdot)$ are unknown continuous functions, and p_i are positive odd integers. In this paper, we consider the system (1) to be the time-varying full-state constraints, i.e., $\xi_i(t)$ is required to remain in the set $\underline{\omega}_{ci}(t) < \xi_i(t) < \bar{\omega}_{ci}(t)$, where $\underline{\omega}_{ci}(t)$ and $\bar{\omega}_{ci}(t)$ such that $\underline{\omega}_{ci}(t) < \bar{\omega}_{ci}(t)$.

Remark 1 When $p_i = 1$, dynamics (1) reduces to the models of works in [6–9, 21, 22, 26, 27, 32, 44, 45] which means (1) includes most of the existing nonlinear systems as special cases. Besides, the nonlinear functions $\Psi_i(\cdot)$, here, are completely unknown, and they do not need to satisfy the growth constraints in [14–16], i.e., $|\Psi_i(\cdot)| \leq (|\xi_1|^{p_1} + \dots + |\xi_i|^{p_i})\rho_i(\xi_1, \dots, \xi_i)$ with $\rho_i(\cdot)(1 \leq i \leq n_i - 1)$ known nonnegative smooth functions.

The actuator fault is taken into account, which contains no fault, partial fault, bias fault and stuck fault. In line with [38] and [43], suppose the actuator output when fault occurs is expressed as

$$u = \mathcal{H}u_c + \eta_u, \quad (2)$$

where $0 \leq \mathcal{H} \leq 1$ denotes the efficiency factor of the actuator and η_u represents the unknown time-varying bias fault of the actuator. The actuator fault can represent the following fault patterns:

(1) $\mathcal{H} = 1$ and $\eta_u = 0$. In this case, the actuator is working normally, and free of failures and faults.

(2) $0 < \mathcal{H} < 1$ and $\eta_u = 0$. This indicates that the actuator is undergoing the failures/faults with partial loss of effectiveness (PLOE).

(3) $\mathcal{H} = 0$. The case is said to be the failure with total loss of effectiveness (TLOE). This means that no matter the control u_c takes any values, u is stuck at an unknown value.

Assumption 1 It is assumed that the efficiency factor \mathcal{H} satisfies $0 < \underline{\mathcal{H}} \leq \mathcal{H} < 1$ for the PLOE faults, where $\underline{\mathcal{H}}$ is a known constant. In addition, the bias fault is assumed to be bounded by $\eta_u \leq \bar{\eta}$ with $\bar{\eta}$ being the known positive constant.

Remark 2 Please note that the actuator fault (2) in our paper has been extensively used in the literature [10, 39–43]. However, the common point in aforementioned works is to require the systems to be in low-order form, making the above methods fail due to the existing high powers, i.e., $(\mathcal{H}u_c + \eta_u)^{p_n}$. A new design must be sought beyond the available schemes.

Without loss of generality, the following standard assumptions from the literature are made.

Assumption 2 For a continuously differentiable desired trajectory $y_r(t)$, there exists a known positive function $y_0(t)$, i.e., $|y_r(t)| \leq y_0(t)$, $|\dot{y}_r(t)| \leq \dot{y}_0(t)$, moreover $y_0(t) < \bar{\omega}_{ci}(t)$.

Assumption 3 The signs of $\Gamma_i(\cdot)$ are known and there exist known positive constants $\underline{\Gamma}_i > 0$ and $\bar{\Gamma}_i > 0$, ($1 \leq i \leq n$) such that $\underline{\Gamma}_i \leq \Gamma_i(\cdot) \leq \bar{\Gamma}_i$.

Remark 3 Assumption 2 is standard and it is essential for the state constrained systems to select suitable bounded reference signal $y_r(t)$ to be tracked. Thus, Assumption 2 is reasonable, and we can always provide a robust estimate about the upper bound of $y_r(t)$. Assumption 3 is given to ensure the controllability of dynamics (1).

The control objective is to design a fixed-time singularity-free fault-tolerant controller such that:

(1) The asymmetric and time-varying state constraints under fault-tolerant control are not transgressed;

(2) The tracking errors can converge into the specified compact sets within fixed time.

The following lemmas are useful for deriving the main results.

Lemma 1 [5] For any $\chi_1, \chi_2 \in \mathbb{R}$ and positive odd integer b , there exist real-valued functions $r_1(\cdot, \cdot)$ and $r_2(\cdot, \cdot)$ such that

$$(\chi_1 + \chi_2)^b = \ell(\chi_1, \chi_2)\chi_1^b + \nu(\chi_1, \chi_2)\chi_2^b,$$

where $\ell(\chi_1, \chi_2) \in [\underline{\ell}, \bar{\ell}]$ with $\underline{\ell} = 1 - d$ and $\bar{\ell} = 1 + d$, where $d = \sum_{k=1}^b \frac{b!}{k!(b-k)!} \frac{b-k}{b} l^{\frac{b}{b-k}}$ is an arbitrary constant taking value in $(0, 1)$ for some appropriately small constant l , $|\nu(\chi_1, \chi_2)| \leq \bar{\nu}(d) = \sum_{k=1}^b \frac{b!}{k!(b-k)!} \frac{k}{b} l^{-\frac{b}{k}}$ with $\bar{\nu}(d)$ being a positive constant.

Lemma 2 [17] Let $\zeta_1 \in \mathbb{R}$, $\zeta_2 \in \mathbb{R}$ and r_1 and r_2 be positive constants. For any $\varpi > 0$, it holds that

$$|\zeta_1|^{r_1} |\zeta_2|^{r_2} \leq \frac{r_1 \varpi |\zeta_1|^{r_1+r_2}}{r_1 + r_2} + \frac{r_2 \varpi^{-r_1/r_2} |\zeta_2|^{r_1+r_2}}{r_1 + r_2}.$$

Lemma 3 [27] Consider the nonlinear system

$$\dot{x} = f(x(t)), x(0) = x_0, \tag{3}$$

if there exists a Lyapunov function $V(x)$ for the system (3) such that

$$\dot{V}(x) \leq -\alpha V^p(x) - \beta V^q(x),$$

where $\alpha, \beta > 0$; $p > 1$; $0 < q < 1$, then the origin of the system (3) is fixed-time stable within

$$T \leq T_{\max} := \frac{1}{\alpha(p-1)} + \frac{1}{\beta(1-q)}.$$

2.2 Radial basis function neural network

In theory, RBFNN can approximate any nonlinear function with arbitrary precision by choosing appropriate parameters [44], which is formulated as

$$\Psi(\chi) = w^T \theta + \delta(\chi), \tag{4}$$

where $\delta(\chi)$ is the minimum approximation error of the RBFNN, the weight vector is expressed as

$$w^* = \arg \min_{w \in \mathbb{R}^n} \left\{ \Psi(\chi) - w^T \theta \right\},$$

$\theta = [\theta_1(\chi), \dots, \theta_n(\chi)]^T$ represents the basis function vector with

$$\theta_i(\chi) = \exp \left[-\frac{(\chi - \iota_i)^T (\chi - \iota_i)}{\mathfrak{N}_i^T \mathfrak{N}_i} \right], i = 1, \dots, n,$$

where n is the number of the RBFNN nodes, $\iota_i = [i_{i1}, i_{i2}, \dots, i_{iq}]^T$ and \mathfrak{N}_i are the center and width of the Gaussian function, respectively. It is worth noting that, if the i th node is viewed to be activated, $\theta_i(\chi)$ has to be in the compact set $\Omega_\theta = \{\theta_i(\chi) | 1 > \theta_i(\chi) \geq \varepsilon_{\min} > 0\}$ with ε_{\min} denoting the user-defined activation threshold. If the value of x is in the active region, i.e., $\|\chi - \iota_i\|^2 \leq -\mathfrak{N}_i^T \mathfrak{N}_i \ln(\varepsilon_{\min})$,

the node i is called to be activated. Therefore, we can conclude that the smaller the value of ε_{\min} is, the larger the size of active region is. However, the amount of computation will increase dramatically if the value of ε_{\min} is chosen very small, which reveals that the importance of the selection of ε_{\min} .

3 Fixed-time adaptive controller design

A novel fixed-time singularity-free fault-tolerant control scheme is proposed for high-order nonlinear systems. A RBFNN $w_i^T \theta_i(Z_i)$ with $w_i \in R^i$, $\theta_i(Z_i) \in R^i$ is utilized to handle the approximation of the unknown function $\bar{f}_i(Z_i)$ that will be specified latter. We use $\delta_i(Z_i)$ to denote the approximation error which is bounded by $\bar{\delta}$, i.e., $|\delta_i(Z_i)| \leq \bar{\delta}$. By lumping w_i and $\theta_i(Z_i)$ into the following vectors:

$$\Xi_i = [w_i^T, \delta_i(Z_i)]^T, \tag{5}$$

$$\vartheta_i(Z_i) = [\theta_i^T(Z_i), 1]^T, \tag{6}$$

and we further have

$$\bar{\Psi}_i(Z_i) = w_i^T \theta_i(Z_i) + \delta_i(Z_i) = \Xi_i^T \vartheta_i(Z_i). \tag{7}$$

To quantify the control objective, we define a constant $q > 0$ as $q = \max_{1 \leq i \leq n} \{p_i\}$ and consider the following change of coordinates:

$$\begin{cases} s_1 = \xi_1 - y_r, \\ s_i = \xi_i - \xi_{i,c}, i = 2, 3, \dots, n, \end{cases} \tag{8}$$

where $\xi_{i,c}$ represents the virtual control law which will be specified later. Before performing the controller design, define the following switch functions:

$$\varpi_i(s_i(t)) := \begin{cases} \varpi_{bi}(s_i(t)), & \text{if } s_i(t) > 0, \\ \varpi_{ai}(s_i(t)), & \text{if } s_i(t) \leq 0. \end{cases} \tag{9}$$

$$\psi_i(s_i(t)) := \begin{cases} 1, & \text{if } |s_i(t)| > \beta_i, \\ 0, & \text{if } |s_i(t)| \leq \beta_i. \end{cases} \tag{10}$$

$$\text{sg}_i(s_i(t)) := \begin{cases} \frac{s_i^{q-p_i+1}(t)}{|s_i^{q-p_i+1}(t)|}, & \text{if } |s_i(t)| > \beta_i, \\ \frac{s_i^{q-p_i+1}(t)}{[\beta_i^2 - s_i^2(t)]^{2(q-p_i+1)} + |s_i^{q-p_i+1}(t)|}, & \text{if } |s_i(t)| \leq \beta_i. \end{cases} \tag{11}$$

where $\varpi_i(s_i(t))$ is the asymmetric time-varying tracking error constraints, $i \in \{1, 2, \dots, n\}$.

From the definitions of (10) and (11), one has

$$\text{sg}_i(s_i(t))\psi_i(s_i(t)) = \begin{cases} \frac{s_i^{q-p_i+1}(t)}{|s_i^{q-p_i+1}(t)|}, & \text{if } |s_i(t)| > \beta_i, \\ 0, & \text{if } |s_i(t)| \leq \beta_i, \end{cases} \quad (12)$$

and

$$[\psi_i(s_i(t))]^m = \psi_i(s_i(t)), \quad (13)$$

where m represents positive integer.

Step 1: Using (1) and (8), differentiating s_1 with respect to time yields

$$\dot{s}_1 = \Psi_1(\xi_1) + \Gamma_1(\xi_1)\xi_2^{p_1} - \dot{y}_r. \quad (14)$$

Consider the high-order tan-type BLF as

$$V_1 = \frac{2\varpi_1^{q-p_1+2}(t)}{\pi(q-p_1+2)} \tan\left(\frac{\pi\zeta_1^{q-p_1+2}(t)}{2\varpi_1^{q-p_1+2}(t)}\right) \psi_1. \quad (15)$$

where $\zeta_1 = |s_1| - \beta_1$. From (9), we know that $\varpi_{b1}(t)$ and $\varpi_{a1}(t)$ represents the lower and upper bound of tracking error, respectively, with $\varpi_{b1}(t) > \beta_1 > 0$, $\varpi_{a1}(t) < -\beta_1 < 0$.

Remark 4 If $\varpi_1(t)$ approaches to infinity, the term $\frac{2\varpi_1^{q-p_1+2}(t)}{\pi(q-p_1+2)} \tan\left(\frac{\pi\zeta_1^{q-p_1+2}(t)}{2\varpi_1^{q-p_1+2}(t)}\right)$ will tend to $\frac{\zeta_1^{q-p_1+2}}{q-p_1+2}$, and thus the proposed high-order tan-type BLF can deal with both the constrained case and the unconstrained case. The conventional BLF in [30,31,33] can only handle the constrained case due to the fact $\lim_{\varpi_1 \rightarrow \infty} \frac{1}{q-p_1+2} \times \log\left(\frac{\varpi_1^{q-p_1+2}}{\varpi_1^{q-p_1+2} - \zeta_1^{q-p_1+2}}\right) = 0$.

Remark 5 It is well known that there have been many studies on the tan-type BLF constraint control problem [34,37,38], but they only consider the symmetric time-invariant scenarios. However, due to the influence of faults, asymmetric time-varying constraints are more common in actual systems, so the fault-tolerant control of asymmetric time-varying constraints is more practical. That is the reason why we construct asymmetric time-varying BLF in our paper.

Remark 6 In (15), the switch function ψ_1 is applied such that the bounds of steady-state tracking errors can be preset as β_1 . By utilizing a smooth switch $\text{sg}_1(s_1(t))$, the chattering phenomenon is effectively

eliminated, and thus the transient performance of system is improved.

In view of (14) and (15), the time derivative of V_1 can be expressed as

$$\begin{aligned} \dot{V}_1 &= \frac{2\varpi_1^{q-p_1+1}(t)\dot{\varpi}_1(t)}{\pi} \tan\left(\frac{\pi\zeta_1^{q-p_1+2}(t)}{2\varpi_1^{q-p_1+2}(t)}\right) \psi_1 \\ &\quad + l_1(\bar{\Psi}_1(Z_1) + \Gamma_1(\xi_1)\xi_2^{p_1}) \text{sg}_1(s_1)\psi_1 \\ &\quad - l_1 \frac{\dot{\varpi}_1(t)}{\varpi_1(t)} \zeta_1(t) \psi_1, \end{aligned} \quad (16)$$

where $\bar{\Psi}_1(Z_1) = \Psi_1(\xi_1) - \dot{y}_r$, $l_1 = \zeta_1^{q-p_1+1}(t)\Theta_1^{-1}$, $\Theta_1 = \cos^2\left(\frac{\pi\zeta_1^{q-p_1+2}(t)}{2\varpi_1^{q-p_1+2}(t)}\right)$ and $Z_1 = [\xi_1, \dot{y}_r]^T$. Noting (7), it can be known that (16) can be rewritten as

$$\begin{aligned} \dot{V}_1 &= \frac{2\varpi_1^{q-p_1+1}(t)\dot{\varpi}_1(t)}{\pi} \tan\left(\frac{\pi\zeta_1^{q-p_1+2}(t)}{2\varpi_1^{q-p_1+2}(t)}\right) \psi_1 \\ &\quad + l_1\left(\Xi_1^T \vartheta_1(Z_1) + \Gamma_1(\xi_1)\xi_2^{p_1}\right) \text{sg}_1(s_1)\psi_1 \\ &\quad - l_1 \frac{\dot{\varpi}_1(t)}{\varpi_1(t)} \zeta_1(t) \psi_1 \\ &\leq \frac{2\varpi_1^{q-p_1+1}(t)\dot{\varpi}_1(t)}{\pi} \tan\left(\frac{\pi\zeta_1^{q-p_1+2}(t)}{2\varpi_1^{q-p_1+2}(t)}\right) \psi_1 \\ &\quad + l_1(\Phi\rho_1 + \Gamma_1(\xi_1)\xi_2^{p_1}) \text{sg}_1(s_1)\psi_1 \\ &\quad - l_1 \frac{\dot{\varpi}_1(t)}{\varpi_1(t)} \zeta_1(t) \psi_1, \end{aligned} \quad (17)$$

where $\Phi = \sqrt{\max\{w_i^T w_i + \delta^2\}}$, $i = 1, 2, \dots, n$, $\rho_1 = \text{sg}_1(s_1)\sqrt{\vartheta_1^T(Z_1)\vartheta_1(Z_1) + \lambda_0}$ with $\lambda_0 > 0$ being the parameter to be designed. We are now in the position to handle the term $\xi_2^{p_1}$ in (17) through Lemma 1 as

$$\xi_2^{p_1} = (s_2 + \xi_{2,c})^{p_1} \leq \bar{v}_1 |s_2^{p_1}| + l_1 \xi_{2,c}^{p_1}. \quad (18)$$

Substitute (18) into (17), and the derivative of V_1 is

$$\begin{aligned} \dot{V}_1 &\leq \frac{2\varpi_1^{q-p_1+1}(t)\dot{\varpi}_1(t)}{\pi} \tan\left(\frac{\pi\zeta_1^{q-p_1+2}(t)}{2\varpi_1^{q-p_1+2}(t)}\right) \psi_1 \\ &\quad - l_1 \frac{\dot{\varpi}_1(t)}{\varpi_1(t)} \zeta_1(t) \psi_1 + \frac{1}{\Theta_1} \bar{\Gamma}_1(\xi_1) \bar{v}_1 |\zeta_1^{q-p_1+1} s_2^{p_1}| \psi_1 \\ &\quad + l_1 \left(\Phi\rho_1 + \Gamma_1(\xi_1)l_1 \xi_{2,c}^{p_1}\right) \text{sg}_1(s_1)\psi_1. \end{aligned} \quad (19)$$

According to Lemma 1, one has

$$\begin{aligned} & \bar{\Gamma}_1(x_1)\bar{v}_1|\zeta_1^{q-p_1+1}s_2^{p_1}|\psi_1 \\ & \leq \bar{\Gamma}_1(x_1)\bar{v}_1\left(\frac{q-p_1+1}{q}\phi_1^{\frac{q-p_1+1}{q}}\zeta_1^{q+1}+\frac{p_1}{q+1}\phi_1^{-\frac{p_1}{q+1}}s_2^{q+1}\right)\psi_1 \\ & \leq \bar{\Gamma}_1(x_1)\bar{v}_1\left(\phi_1^{\frac{q-p_1+1}{q}}\zeta_1^{q+1}+\phi_1^{-\frac{p_1}{q+1}}s_2^{q+1}\right)\psi_1. \end{aligned} \tag{20}$$

Substitute (20) into (19) yields

$$\begin{aligned} \dot{V}_1 & \leq \frac{1}{\Theta_1}\bar{\Gamma}_1(\xi_1)\bar{v}_1\left(\phi_1^{\frac{q-p_1+1}{q}}\zeta_1^{q+1}+\phi_1^{-\frac{p_1}{q+1}}s_2^{q+1}\right)\psi_1 \\ & + \frac{2\varpi_1^{q-p_1+1}(t)\dot{\varpi}_1(t)}{\pi}\tan\left(\frac{\pi\zeta_1^{q-p_1+2}(t)}{2\varpi_1^{q-p_1+2}(t)}\right)\psi_1 \\ & + l_1\left(\Phi\rho_1+\Gamma_1(\xi_1)\ell_1\xi_{2,c}^{p_1}\right)\text{sg}_1(s_1)\psi_1 \\ & - l_1\frac{\dot{\varpi}_1(t)}{\varpi_1(t)}\zeta_1(t)\psi_1. \end{aligned} \tag{21}$$

Design the singularity-free switching controller $\xi_{2,c}$ as

$$\begin{aligned} \xi_{2,c} & = -\left[\frac{1}{\Gamma_1(\xi_1)\ell_1}\left[\frac{a_1}{l_1}\tan^p\left(\frac{\pi\zeta_1^{q-p_1+2}(t)}{2\varpi_1^{q-p_1+2}(t)}\right)\right]\text{sg}_1(s_1)\right. \\ & + (\beta_2+1)\text{sg}_1(s_1)+\frac{1}{l_1\Theta_1}\bar{\Gamma}_1(\xi_1)\bar{v}_1\phi_1^{\frac{q-p_1+1}{q}}\zeta_1^{q+1} \\ & + \frac{2\varpi_1^{q-p_1+1}(t)\dot{\varpi}_1(t)}{l_1\pi}\tan\left(\frac{\pi\zeta_1^{q-p_1+2}(t)}{2\varpi_1^{q-p_1+2}(t)}\right) \\ & + \tilde{h}_1\zeta_1+\frac{c_1}{l_1}\tan^q\left(\frac{\pi\zeta_1^{q-p_1+2}(t)}{2\varpi_1^{q-p_1+2}(t)}\right)\text{sg}_1(s_1) \\ & \left. + \hat{\Phi}\rho_1+\frac{l_1}{4}\text{sg}_1(s_1)+\kappa^2\rho_1\gamma_1\right]^{\frac{1}{p_1}}, \end{aligned} \tag{22}$$

where $c_1, a_1, \kappa > 0$ are the parameters to be designed. The time-varying gain term \tilde{h}_1 is chosen as

$$\tilde{h}_1 = \sqrt{\left(\frac{\dot{\varpi}_{b1}}{\varpi_{b1}}\right)^2 + \left(\frac{\dot{\varpi}_{a1}}{\varpi_{a1}}\right)^2} + o_1 \tag{23}$$

in which $o_1 > 0$ is the parameter to be designed. The switched tuning function γ_1 is chosen as

$$\gamma_1 = \begin{cases} \tan^q\left(\frac{\pi\zeta_1^{q-p_1+2}(t)}{2\varpi_1^{q-p_1+2}(t)}\right)\rho_1\text{sg}_1(s_1), & \text{if } |s_1(t)| > \beta_1, \\ 0, & \text{if } |s_1(t)| \leq \beta_1. \end{cases} \tag{24}$$

Remark 7 In (22), when $\zeta_1 \rightarrow 0$, it has $\sin^q\left(\frac{\pi\zeta_1^{q-p_1+2}(t)}{2\varpi_1^{q-p_1+2}(t)}\right) \sim \left(\frac{\pi\zeta_1^{q-p_1+2}(t)}{2\varpi_1^{q-p_1+2}(t)}\right)^q$. According to the L'Hospital's rule, one has

$$\begin{aligned} & \lim_{\zeta_1 \rightarrow 0} \frac{\tan^q\left(\frac{\pi\zeta_1^{q-p_1+2}}{2\varpi_1^{q-p_1+2}}\right)}{l_1} \\ & = \lim_{\zeta_1 \rightarrow 0} \frac{\sin^q\left(\frac{\pi\zeta_1^{q-p_1+2}}{2\varpi_1^{q-p_1+2}}\right)\cos^{2-q}\left(\frac{\pi\zeta_1^{q-p_1+2}}{2\varpi_1^{q-p_1+2}}\right)}{\zeta_1^{q-p_1+2}} \\ & = \lim_{\zeta_1 \rightarrow 0} \frac{\left(\frac{\pi\zeta_1^{q-p_1+2}}{2\varpi_1^{q-p_1+2}}\right)^q}{\zeta_1^{q-p_1+2}} = 0, \end{aligned}$$

that is to say, there is no singularity problem if $\varpi_1(t) \neq 0$ in our proposed fixed-time controller. Furthermore, we note that the inequation $\tan\left(\frac{\pi\zeta_1^{q-p_1+2}}{2\varpi_1^{q-p_1+2}}\right) > 0$ holds since $\zeta_1 < \varpi_1$; thus, it is feasible to design the tuning function (24).

In view of (23), it can be derived that

$$-l_1\tilde{h}_1\zeta_1-l_1\zeta_1(t)\frac{\dot{k}_1(t)}{k_1(t)} = -\left(\tilde{h}_1+\frac{\dot{k}_1(t)}{k_1(t)}\right)\zeta_1^{\frac{q-p_1+2}{\Theta_1}} < 0. \tag{25}$$

Substituting (22) and (25) into (21) reaches

$$\begin{aligned} \dot{V}_1 & \leq -c_1\tan^q\left(\frac{\pi\zeta_1^{q-p_1+2}(t)}{2\varpi_1^{q-p_1+2}(t)}\right)\psi_1-l_1(\beta_2+1)\psi_1 \\ & - a_1\tan^p\left(\frac{\pi\zeta_1^{q-p_1+2}(t)}{2\varpi_1^{q-p_1+2}(t)}\right)\psi_1+\tilde{\Phi}\gamma_1-\kappa^2\gamma_1^2 \\ & + \frac{1}{\Theta_1}\bar{\Gamma}_1(\xi_1)\bar{v}_1\phi_1^{-\frac{p_1}{q+1}}s_2^{q+1}\psi_1-\frac{l_1^2}{4}\psi_1. \end{aligned} \tag{26}$$

Step i ($i \in \{2, \dots, n-1\}$): The design process for step i follows recursively from step 1. From (1) and (8), the time derivative of variable s_i can be written as

$$\dot{s}_i = \bar{\Psi}_i(Z_i) + \Gamma_i(\bar{\xi}_i)\xi_{i+1}^{p_i} - \frac{\partial \xi_{i,c}}{\partial \hat{\Phi}}\dot{\hat{\Phi}}, \tag{27}$$

where $\bar{\Psi}_i(Z_i) = \Psi_i(\bar{\xi}_i) - \sum_{j=1}^{i-1} \frac{\partial \xi_{j+1,c}}{\partial \xi_j}\dot{\xi}_j - \frac{\partial \xi_{i,c}}{\partial y_r}\dot{y}_r$ and $Z_i = [\xi_1, \xi_2, \dots, \xi_i, \dot{y}_r]^T$. Consider the high-order

tan-type BLF as follows:

$$V_i = V_{i-1} + \frac{2\varpi_i^{q-p_i+2}(t)}{\pi(q-p_i+2)} \tan\left(\frac{\pi\zeta_i^{q-p_i+2}(t)}{2\varpi_i^{q-p_i+2}(t)}\right) \psi_i. \quad (28)$$

where $\zeta_i = |s_i| - \beta_i$. From (9), we know that $\varpi_{bi}(t)$ and $\varpi_{ai}(t)$ represent the lower and upper bound of tracking error, respectively, with $\varpi_{bi}(t) > \beta_i > 0$, $\varpi_{ai}(t) < -\beta_i < 0$. Let $l_i = \frac{\zeta_i^{q-p_i+1}(t)}{\Theta_i}$ and $\Theta_i = \cos^2\left(\frac{\pi\zeta_i^{q-p_i+2}(t)}{2\varpi_i^{q-p_i+2}(t)}\right)$, then the time derivative of V_i is

$$\begin{aligned} \dot{V}_i \leq & \dot{V}_{i-1} + l_i \left(\Phi \rho_i + \Gamma_i(\bar{\xi}_i) \xi_{i+1}^{p_i} - \frac{\partial \xi_{i,c}}{\partial \hat{\Phi}} \dot{\hat{\Phi}} \right) \text{sg}_i(s_i) \psi_i \\ & + \frac{2\varpi_i^{q-p_i+1}(t) \dot{\varpi}_i(t)}{\pi} \tan\left(\frac{\pi\zeta_i^{q-p_i+2}(t)}{2\varpi_i^{q-p_i+2}(t)}\right) \psi_i \\ & - l_i \frac{\dot{\varpi}_i(t)}{\varpi_i(t)} \zeta_i(t) \psi_i, \end{aligned} \quad (29)$$

where $\rho_i = \text{sg}_i(s_i) \sqrt{\vartheta_i^T(Z_i) \vartheta_i(Z_i) + \lambda_0}$ with $\lambda_0 > 0$ being the parameter to be designed. Along similar lines as (18)–(20), one has

$$\begin{aligned} \dot{V}_i \leq & \dot{V}_{i-1} + l_i \left(\Phi \rho_i + \Gamma_i(\bar{\xi}_i) l_i \xi_{i+1,c}^{p_i} - \frac{\partial \xi_{i,c}}{\partial \hat{\Phi}} \dot{\hat{\Phi}} \right) \text{sg}_i(s_i) \psi_i \\ & + \frac{1}{\Theta_i} \bar{\Gamma}_i(\bar{\xi}_i) \bar{\nu}_i \left(\phi_i^{\frac{q-p_i+1}{q}} \zeta_i^{q+1} + \phi_i^{-\frac{p_i}{q+1}} s_{i+1}^{q+1} \right) \psi_i \\ & + \frac{2\varpi_i^{q-p_i+1}(t) \dot{\varpi}_i(t)}{\pi} \tan\left(\frac{\pi\zeta_i^{q-p_i+2}(t)}{2\varpi_i^{q-p_i+2}(t)}\right) \psi_i \\ & - l_i \frac{\dot{\varpi}_i(t)}{\varpi_i(t)} \zeta_i(t) \psi_i. \end{aligned} \quad (30)$$

Design the singularity-free switching controller $\xi_{i+1,c}$ as

$$\begin{aligned} \xi_{i+1,c} = & \left[\frac{1}{\Gamma_i(\bar{\xi}_i) l_i} \left[\frac{1}{l_i \psi_i \Theta_{i-1}} \bar{\Gamma}_{i-1}(\bar{\xi}_{i-1}) \bar{\nu}_{i-1} \phi_{i-1}^{-\frac{p_i-1}{q+1}} \zeta_i^{q+1} \psi_{i-1} \right. \right. \\ & + \frac{c_i}{l_i} \tan^q\left(\frac{\pi\zeta_i^{q-p_i+2}(t)}{2\varpi_i^{q-p_i+2}(t)}\right) \text{sg}_i(s_i) + \frac{1}{l_i} \beta_i^2 \psi_i \text{sg}_i(s_i) \\ & + \frac{a_i}{l_i} \tan^p\left(\frac{\pi\zeta_i^{q-p_i+2}(t)}{2\varpi_i^{q-p_i+2}(t)}\right) \text{sg}_i(s_i) + \kappa^2 \rho_i (\gamma_{i-1} + \gamma_i) \\ & + \hat{\Phi} \rho_i + \hat{h}_i \zeta_i + \frac{l_i}{4} \text{sg}_i(s_i) + \frac{1}{l_i \Theta_i} \bar{\Gamma}_i(\bar{\xi}_i) \bar{\nu}_i \phi_i^{\frac{q-p_i+1}{q}} \zeta_i^{q+1} \\ & + \frac{2\varpi_i^{q-p_i+1}(t) \dot{\varpi}_i(t)}{l_i \pi} \tan\left(\frac{\pi\zeta_i^{q-p_i+2}(t)}{2\varpi_i^{q-p_i+2}(t)}\right) \psi_i \\ & \left. \left. + (\beta_{i+1} + 1) \text{sg}_i(s_i) + \frac{l_i}{4} \text{sg}_i(s_i) \left(\frac{\partial \xi_{i,c}}{\partial \hat{\Phi}} \right)^2 \right] \right]^{\frac{1}{p_i}} \end{aligned} \quad (31)$$

here the time-varying gain term \hat{h}_i is chosen as

$$\hat{h}_i = \sqrt{\left(\frac{\dot{\varpi}_{bi}}{\varpi_{bi}}\right)^2 + \left(\frac{\dot{\varpi}_{ai}}{\varpi_{ai}}\right)^2} + o_i \quad (32)$$

in which $o_i > 0$ is the parameter to be designed. The switched tuning function γ_i is chosen as

$$\gamma_i = \begin{cases} \tan^q\left(\frac{\pi\zeta_i^{q-p_i+2}(t)}{2\varpi_i^{q-p_i+2}(t)}\right) \rho_i \text{sg}_i(s_i) + \gamma_{i-1}, & \text{if } |s_i(t)| > \beta_i, \\ \gamma_{i-1}, & \text{if } |s_i(t)| \leq \beta_i. \end{cases} \quad (33)$$

Substituting (31) to (30) and using the fact that $-l_i \hat{h}_i \zeta_i - l_i \zeta_i(t) \frac{\dot{\varpi}_i(t)}{\varpi_i(t)} = -\left(\hat{h}_i + \frac{\dot{\varpi}_i(t)}{\varpi_i(t)}\right) \frac{\zeta_i^{q-p_i+2}}{\Theta_i} < 0$, we can further have

$$\begin{aligned} \dot{V}_i \leq & - \sum_{m=1}^i c_m \tan^q\left(\frac{\pi\zeta_m^{q-p_m+2}(t)}{2\varpi_m^{q-p_m+2}(t)}\right) \psi_m + \tilde{\Phi} \gamma_i - \frac{l_i^2}{4} \psi_i \\ & - \sum_{m=1}^i a_m \tan^p\left(\frac{\pi\zeta_m^{q-p_m+2}(t)}{2\varpi_m^{q-p_m+2}(t)}\right) \psi_m + \wp_i + \Im_i \\ & + l_i (-\beta_{i+1} - 1) \psi_i + \frac{1}{\Theta_i} \bar{\Gamma}_i(\bar{\xi}_i) \bar{\nu}_i \phi_i^{-\frac{p_i}{q+1}} s_{i+1}^{q+1} \psi_i, \end{aligned} \quad (34)$$

where

$$\begin{aligned} \wp_i = & - \frac{l_{i-1}^2}{4} \psi_{i-1} + l_{i-1} (-\beta_i - 1) \psi_{i-1} - \beta_i^2 \psi_i \\ \Im_i = & - \kappa^2 \gamma_i^2 - \sum_{m=2}^i l_m \psi_m \text{sg}_m(s_m) \frac{\partial \xi_{m+1,c}}{\partial \hat{\Phi}} \dot{\hat{\Phi}} \\ & - \sum_{m=2}^i \frac{l_m^2}{4} \psi_m \left(\frac{\partial \xi_{m+1,c}}{\partial \hat{\Phi}} \right)^2. \end{aligned} \quad (35)$$

It should be worth noting that $\wp_i < 0$ is always satisfied. If $0 \leq \beta_i + 1$, the inequation $l_{i-1} (-\beta_i - 1) \psi_{i-1} \leq 0$ holds, if $0 > \beta_i + 1$, the inequation $l_{i-1} (-\beta_i - 1) \psi_{i-1} \leq \frac{l_{i-1}^2}{4} \psi_{i-1} + (-\beta_i - 1)^2 \leq \frac{l_{i-1}^2}{4} \psi_{i-1} + \beta_i^2 \psi_i$ holds and thus $\wp_i < 0$.

Step n: Invoking (1) and (8), the dynamics of \dot{s}_n are given by

$$\dot{s}_n = \bar{\Psi}_n(Z_n) + \Gamma_n(\xi) \ell_n \mathcal{H}^{p_n} u_c^{p_n} - \frac{\partial \xi_{i,c}}{\partial \hat{\Phi}} \dot{\hat{\Phi}}, \quad (36)$$

where $\bar{\Psi}_n(Z_n) = \Psi_n(\xi) - \sum_{i=1}^{n-1} \frac{\partial \xi_{i+1,c}}{\partial \xi_i} \dot{\xi}_i - \frac{\partial \xi_{n,c}}{\partial y_r} \dot{y}_r + \Gamma_n(\xi) \nu_n \eta_u^{p_n}$ and $Z_n = [\xi_1, \xi_2, \dots, \xi_n, \dot{y}_r]^T$. Take the

high-order tan-type BLF as

$$V_n = \frac{2\varpi_n^{q-p_n+2}(t)}{\pi(q-p_n+2)} \tan\left(\frac{\pi\zeta_n^{q-p_n+2}(t)}{2\varpi_n^{q-p_n+2}(t)}\right) \psi_n + \frac{1}{2\kappa} \tilde{\Phi}^2 + V_{n-1}. \tag{37}$$

where $\zeta_n = |s_n| - \beta_n \cdot \kappa$ is the parameter to be designed. $\hat{\Phi}$ is the estimate of Φ with $\tilde{\Phi} = \Phi - \hat{\Phi}$ denoting the estimation error. Similarly to step i , define $l_n = \frac{\zeta_n^{q-p_n+1}(t)}{\Theta_n}$ and $\Theta_n = \cos^2\left(\frac{\pi\zeta_n^{q-p_n+2}(t)}{2\varpi_n^{q-p_n+2}(t)}\right)$, the time derivative of V_n can be given by

$$\begin{aligned} \dot{V}_n \leq & \dot{V}_{n-1} + \frac{2\varpi_n^{q-p_n+1}(t)\dot{\varpi}_n(t)}{\pi} \tan\left(\frac{\pi\zeta_n^{q-p_n+2}(t)}{2\varpi_n^{q-p_n+2}(t)}\right) \psi_n \\ & + l_n \left(\hat{\Phi} \rho_n + \Gamma_n(\xi) \ell_n \mathcal{H}^{p_n} u_c^{p_n} - \frac{\partial \xi_{i,c}}{\partial \hat{\Phi}} \dot{\hat{\Phi}} \right) \text{sg}_n(s_n) \psi_n \\ & - l_n \frac{\dot{\varpi}_n(t)}{\varpi_n(t)} \zeta_n(t) \psi_n - \frac{1}{\kappa} \dot{\tilde{\Phi}} \dot{\hat{\Phi}}, \end{aligned} \tag{38}$$

where $\rho_n = \text{sg}_n(\zeta_n) \sqrt{\vartheta_n^T(Z_n) \vartheta_n(Z_n) + \lambda_0}$ with $\lambda_0 > 0$ being the parameter to be designed. Design the singularity-free switching fault-tolerant controller u_c and adaptation law $\hat{\Phi}$ as

$$\begin{aligned} u_c = & - \left[\frac{1}{\Gamma_n(\xi) \ell_n \mathcal{H}^{p_n}} \left[\frac{c_n}{l_n} \tan^q\left(\frac{\pi\zeta_n^{q-p_n+2}(t)}{2\varpi_n^{q-p_n+2}(t)}\right) \text{sg}_n(s_n) \right. \right. \\ & + \hat{\Phi} \rho_n + \frac{1}{l_n \psi_n \Theta_{n-1}} \bar{\Gamma}_{n-1}(\bar{\xi}_{n-1}) \bar{\nu}_{n-1} \phi_{n-1}^{-\frac{p_n-1}{q+1}} \zeta_n^{q+1} \psi_{n-1} \\ & + \frac{a_n}{l_n} \tan^p\left(\frac{\pi\zeta_n^{q-p_n+2}(t)}{2\varpi_n^{q-p_n+2}(t)}\right) \text{sg}_n(s_n) + \frac{1}{l_n} \beta_n^2 \psi_n \text{sg}_n(s_n) \\ & + \left. \frac{2\varpi_n^{q-p_n+1}(t)\dot{\varpi}_n(t)}{l_n \pi} \tan\left(\frac{\pi\zeta_n^{q-p_n+2}(t)}{2\varpi_n^{q-p_n+2}(t)}\right) \psi_n + \hat{h}_n \zeta_n \right. \\ & \left. + \frac{l_n}{4} \text{sg}_n(s_n) \left(\frac{\partial \xi_{i,c}}{\partial \hat{\Phi}} \right)^2 + \kappa^2 \rho_n (\gamma_{n-1} + \gamma_n) \right] \frac{1}{p_n}, \end{aligned} \tag{39}$$

$$\dot{\hat{\Phi}} = \text{Proj}(\kappa \gamma_n) = \begin{cases} \kappa \gamma_n, & \text{if } |s_n(t)| > \beta_n, \\ 0, & \text{if } |s_n(t)| \leq \beta_n. \end{cases} \tag{40}$$

where $\hat{\Phi}(0) \in \Omega$, and the time-varying gain term \hat{h}_n is chosen as

$$\hat{h}_n = \sqrt{\left(\frac{\dot{\varpi}_{bn}}{\varpi_{bn}}\right)^2 + \left(\frac{\dot{\varpi}_{an}}{\varpi_{an}}\right)^2} + o_n \tag{41}$$

in which $o_n > 0$ is the parameter to be designed. The switched tuning function is chosen as

$$\gamma_n = \begin{cases} \tan^q\left(\frac{\pi\zeta_n^{q-p_n+2}(t)}{2\varpi_n^{q-p_n+2}(t)}\right) \rho_n \text{sg}_n(s_n) + \gamma_{n-1}, & \text{if } |s_n(t)| > \beta_n, \\ \gamma_{n-1}, & \text{if } |s_n(t)| \leq \beta_n. \end{cases} \tag{42}$$

Using (34) and substituting (39) and (40) into (38) yields

$$\begin{aligned} \dot{V}_n \leq & \frac{l_n \psi_n}{\tan^q\left(\frac{\pi\zeta_n^{q-p_n+2}(t)}{2\varpi_n^{q-p_n+2}(t)}\right)} \frac{\tilde{\Phi}}{\kappa} [\kappa \gamma_n - \text{Proj}(\kappa \gamma_n)] + \wp_n \\ & - \sum_{i=1}^n a_i \tan^p\left(\frac{\pi\zeta_i^{q-p_i+2}(t)}{2\varpi_i^{q-p_i+2}(t)}\right) \psi_i + \Im_n \\ & - \sum_{i=1}^n c_i \tan^q\left(\frac{\pi\zeta_i^{q-p_i+2}(t)}{2\varpi_i^{q-p_i+2}(t)}\right) \psi_i, \end{aligned} \tag{43}$$

where

$$\begin{aligned} \wp_n = & - \frac{l_{n-1}^2}{4} \psi_{n-1} + l_{n-1} (-\beta_n - 1) \psi_{n-1} - \beta_n^2 \psi_n, \\ \Im_n = & - \kappa^2 \gamma_n^2 - \sum_{i=2}^n \frac{l_i^2}{4} \psi_i \text{sg}_i(s_i) \left(\frac{\partial \xi_{i,c}}{\partial \hat{\Phi}} \right)^2 \\ & - \sum_{i=2}^n l_i \psi_i \text{sg}_i(s_i) \frac{\partial \xi_{i,c}}{\partial \hat{\Phi}} \dot{\hat{\Phi}}. \end{aligned} \tag{44}$$

Similar to the analysis procedure in [44,45], according to the property of projection operator (40), one has $\tilde{\Phi} \kappa^{-1} [\kappa \gamma_n - \text{Proj}(\kappa \gamma_n)] \leq 0$ with $\text{Proj}(\kappa \gamma_n) \leq \kappa \gamma_n$ (see Appendices E in [46]). The projection operator guarantees that the estimate $\hat{\Phi}$ is always in a compact set Ω , i.e., $\hat{\Phi} \in \Omega$. Along with the fact that $\Phi \in \Omega$, the estimation error $\tilde{\Phi}$ is ensured bounded. With the addition of the fact $\frac{l_n \psi_n}{\tan^q\left(\frac{\pi\zeta_n^{q-p_n+2}(t)}{2\varpi_n^{q-p_n+2}(t)}\right)} \geq 0$. Incorporating the

Young's inequation, one arrives

$$- \sum_{i=2}^n l_i \psi_i \text{sg}_i(s_i) \frac{\partial \xi_{i,c}}{\partial \hat{\Phi}} \dot{\hat{\Phi}} \leq - \kappa^2 \gamma_n^2 - \sum_{i=2}^n \frac{l_i^2}{4} \psi_i \left(\frac{\partial \xi_{i,c}}{\partial \hat{\Phi}} \right)^2, \tag{45}$$

and therefore, the inequation $\mathfrak{S}_n \leq 0$ always holds. Then, we can derive that

$$\begin{aligned} \dot{V}_n \leq & - \sum_{i=1}^n c_i \tan^q \left(\frac{\pi \zeta_i^{q-p_i+2}(t)}{2\omega_i^{q-p_i+2}(t)} \right) \psi_i - \sum_{i=1}^n a_i \\ & \times \tan^p \left(\frac{\pi \zeta_i^{q-p_i+2}(t)}{2\omega_i^{q-p_i+2}(t)} \right) \psi_i. \end{aligned} \quad (46)$$

4 Stability analysis

At this point, the main result of this paper is given in Theorem 1.

Theorem 1 Consider the high-order nonlinear systems (1) subject to actuator faults (2), under Assumptions 1–3, the virtual controllers (22) and (31), the state-constrained fault-tolerant controller (39), and the adaption laws (40). By choosing appropriate design parameters, there exists a settling time independent of initial states such that tracking errors converge into the user-defined intervals and all signals of the closed-loop system are fixed-time stable and the time-varying state constraints will never be violated.

Proof Consider the total high-order BLFs as

$$\bar{V} = \sum_{i=1}^n \frac{2\omega_i^{q-p_i+2}(t)}{\pi(q-p_i+2)} \tan \left(\frac{\pi \zeta_i^{q-p_i+2}(t)}{2\omega_i^{q-p_i+2}(t)} \right) \psi_i. \quad (47)$$

Invoking (18), (19) and (21), it yields that

$$\begin{aligned} \dot{\bar{V}} \leq & - \sum_{i=1}^n c_i \tan^q \left(\frac{\pi \zeta_i^{q-p_i+2}(t)}{2\omega_i^{q-p_i+2}(t)} \right) \psi_i + \tilde{\Phi} \gamma_n \\ & - \sum_{i=1}^n a_i \tan^p \left(\frac{\pi \zeta_i^{q-p_i+2}(t)}{2\omega_i^{q-p_i+2}(t)} \right) \psi_i. \end{aligned} \quad (48)$$

Substituting (45) into (48), we get

$$\begin{aligned} \tilde{\Phi} \gamma_n &= \tilde{\Phi} \sum_{i=1}^n \tan^q \left(\frac{\pi \zeta_i^{q-p_i+2}(t)}{2\omega_i^{q-p_i+2}(t)} \right) \psi_i \sqrt{\vartheta_i^T \vartheta_i + \lambda_0} \\ &\leq \omega_{im} \sqrt{2n(2+\lambda_0)} \psi_i^{1-q} \left(\frac{\pi(q-p_i+2)}{2\omega_i^{q-p_i+2}(t)} \right)^q \bar{V}^q, \end{aligned} \quad (49)$$

where ω_{im} stands for the upper bound of estimation error $\tilde{\omega}_i$. Then, the time derivative of \bar{V}_i becomes

$$\dot{\bar{V}}_i \leq -a_1 V_i^p(t) - b_1 V_i^q(t), \quad (50)$$

where $a_1 = \frac{\pi(q-p_i+2)a_i}{2\omega_i^{q-p_i+2}(t)}$, $b_1 = \min_{i \in \{1,2,\dots,n\}} \left\{ \frac{\pi(q-p_i+2)c_i}{2\omega_i^{q-p_i+2}(t)} - \omega_{im} \sqrt{2n(2+\lambda_0)} \psi_i^{1-q} \left(\frac{\pi(q-p_i+2)}{2\omega_i^{q-p_i+2}(t)} \right)^q \right\}$. With the results obtained in (46), it leads to that the inequations $V_n \leq 0$ hold; therefore, we can derive that ζ_i and $\tilde{\Phi}$ are bounded. Due to that $\zeta_i = |s_i| - \beta_i$ and the value of β_i is user-defined, s_i is bounded. Since $\hat{\Phi} = \Phi - \tilde{\Phi}$ and Φ is a constant, we have $\hat{\Phi}$ is bounded. Since ξ_i ; $V \in L_\infty$. Therefore, all the signals of closed-loop system are bounded. In view of Lemma 2 and previous analysis, we can conclude that the tracking errors s_1 will asymptotically converge to β within fixed time $T = \frac{1}{a_1(p-1)} + \frac{2}{b_1(1-q)}$. The proof is thus completed. \square

Remark 8 Although some existing works such as [44] and [45] have dealt with the predefined accuracy issue, these works require the systems to be in low-order form and cannot preserve the fixed-time stability. Compared with the existing FTC strategies [39–43], our proposed method can not only achieve fixed-time convergence, but also guarantee the asymmetric time-varying state constraints. In other words, our proposed method includes existing approaches as special cases.

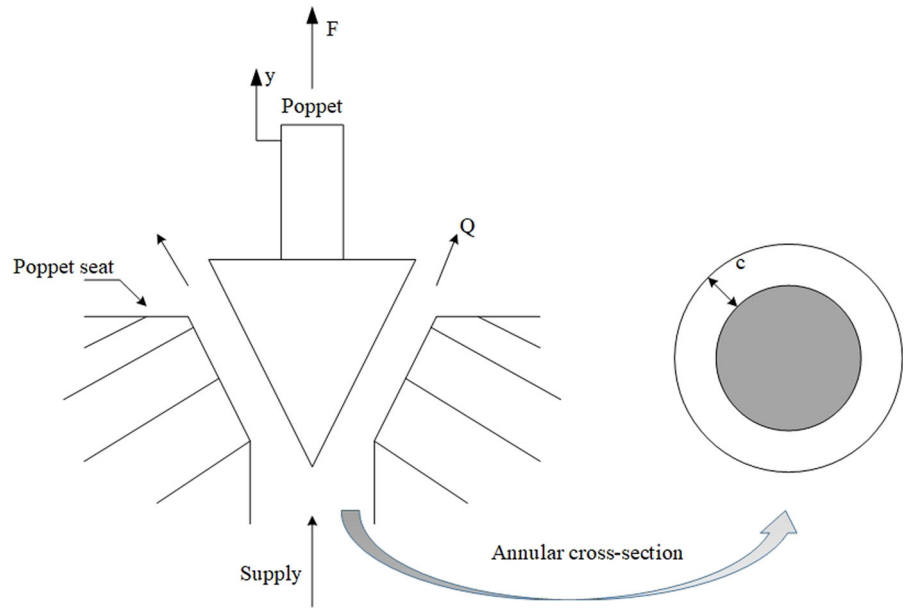
Remark 9 In Theorem 1, we can prove that the tracking error s_1 asymptotically converges to a predefined interval β_1 and similar analysis can be applied to prove that other state errors s_i also converge to prescribed interval β_i . With this result, along with the adaptation law designed in (40), we have $\lim_{t \rightarrow \infty} \dot{\hat{\Phi}} = 0$ and $\hat{\Phi}$ ultimately converges to a constant. Such result is also demonstrated by the evolutions in Fig. 4b.

5 Simulation example

To verify the effectiveness of the proposed scheme in practical application, a poppet valve system which is one of the most commonly used components in hydraulic systems is considered as shown in Fig. 1 [1, pp. 54].

A poppet valve is typically utilized to control the timing and quantity of gas or vapor flow into an engine,

Fig. 1 Poppet valve system



and its behavior can be modeled by the annular leakage equation. The input force F drives the poppet to move for regulating the volumetric flow rate $Q_{vol} = \lambda c^3$ of oil from the high-pressure to the low-pressure chamber, where $\lambda = \frac{\pi r}{6\mu L} \Delta P$ is a lumped coefficient, $c = \alpha y$ is the effective clearance of the annular passage with α a constant and y the displacement of poppet, and where r, μ and L are constants independent of the axial motion of poppet, and ΔP is the pressure drop between two chambers. The dynamics of oil volume V in upper chamber is given by

$$\dot{V} = Q_{vol} - R(t), \tag{51}$$

where R is the lumped reduction rate of oil attributed to consumption and other leakages. Likewise, the equation of motion of the poppet is

$$m\ddot{y} = -k\dot{y}(t) + T(t) + F(t), \tag{52}$$

where m is the mass of the poppet, k is the viscous friction coefficient, T is the lumped elastic force, and F represents the input force. At this point, let us introduce the following notation substitutions:

$$\xi_1 = V, \xi_2 = y, \xi_3 = \dot{y}, u = F, \tag{53}$$

Then, the dynamic of systems (52) and (53) comes down to

$$\dot{\xi}_1 = \Gamma_1 \xi_2^3 + \Psi_1, \dot{\xi}_2 = \xi_3, \dot{\xi}_3 = \Gamma_3 u + \Psi_3, \tag{54}$$

where $\Gamma_1 = \lambda \alpha^3, \Psi_1 = -R, \Gamma_3 = 1/m, \Psi_3 = \frac{1}{m}[T - k\xi_3]$ with $m = 7.5\text{kg}, k = 2.5\text{N/m}, R =$

$5\text{L/min}, \Delta P = 10\text{N/m}^2, T = 5\text{N}, \mu = 2.5, L = 5, r = 1.25, \alpha = 4.5$. The reference signal is $y_r(t) = \sin(t) + 0.5 \cos(2t)$. While conducting the simulation, the initial state values are chosen as $y_r(0) = 0, \xi_1(0) = [0, 0, 0]^T$ and $\hat{\phi}(0) = 0.5$, and the design parameters are chosen as $c_1 = 7.5, c_2 = 8.2, c_3 = 9, a_1 = 5.5, a_2 = 6, a_3 = 7, \kappa = 1, o_1 = 1.15, o_2 = 1.2, o_3 = 1.05, p = 1.1, q = 0.9, \beta_1 = 0.03$. Consider the actuator fault $u = \mathcal{H}u_c + \eta_u$ with $\mathcal{H} = \begin{cases} 1, & t \leq 10, \\ 0.6, & t > 10, \end{cases}$

and $\eta_u = \begin{cases} 0, & t \leq 10, \\ 1 + 0.5\sin(t), & t > 10, \end{cases}$. The system states are restrained in $\underline{w}_{ci}(t) < \xi_i(t) < \bar{w}_{ci}(t)$ with $\underline{w}_{c1}(t) = 0.1\cos(3t) - 1.6, \bar{w}_{c1}(t) = 0.1\sin(3t) + 1, \underline{w}_{c2}(t) = 0.2\cos(5t) - 2, \bar{w}_{c2}(t) = 0.2\sin(5t) + 2, \underline{w}_{c3}(t) = 2\sin(2t) - 7.5, \bar{w}_{c3}(t) = 1.4\cos(2t) + 31$, and the lower and upper bound of dynamics tracking error are given as $\underline{w}_{b1}(t) = 0.1\cos(3t) - 0.65, \underline{w}_{a1}(t) = 0.1\sin(3t) + 0.13, \underline{w}_{b2}(t) = 0.2\cos(5t) - 1.2, \underline{w}_{a2}(t) = 0.2\sin(5t) + 1.5, \underline{w}_{b3}(t) = 2\sin(2t) - 4.3, \underline{w}_{a3}(t) = 1.4\cos(2t) + 3.3$. The simulation results are shown in Figs. 2, 3 and 4, where the standard adaptive control scheme as in [17] is taken as a means of comparison. In order to expound the advantages of tracking performances, several performance indices, i.e., the integral time absolute error (ITAE) $\left[\int_0^T t |s_1(t)| dt \right]$, the root-mean-square error (RMSE) $\left[\frac{1}{T} \int_0^T s_1^2(t) dt \right]^{\frac{1}{2}}$ and the mean absolute error (MAE)

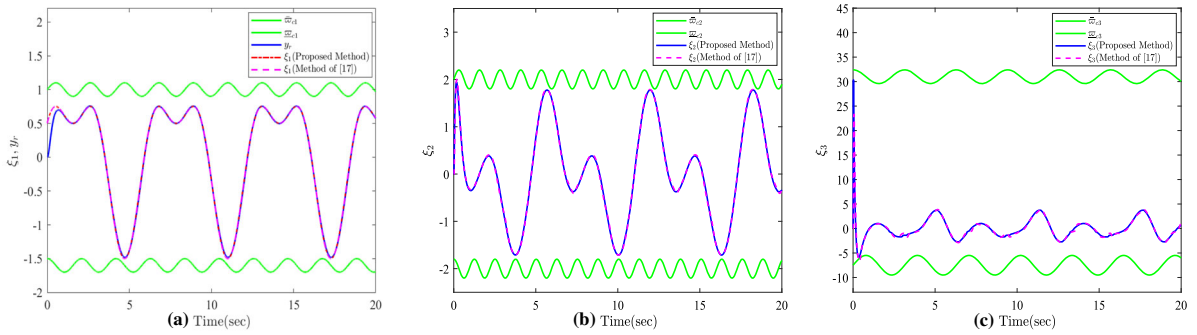


Fig. 2 a Evolution of y_1 and y_r , b evolution of ξ_2 under two schemes, c evolution of ξ_3 under two schemes

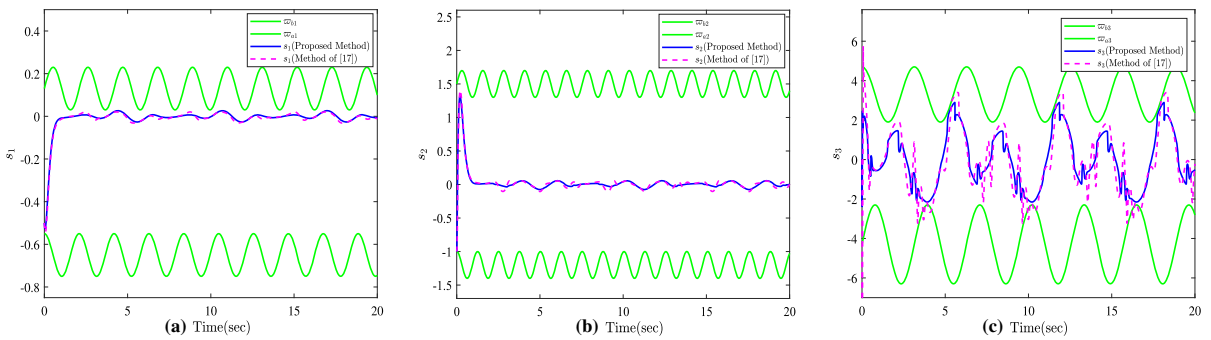


Fig. 3 a Profile of s_1 under two schemes, b profile of s_2 under two schemes, c profile of s_3 under two schemes

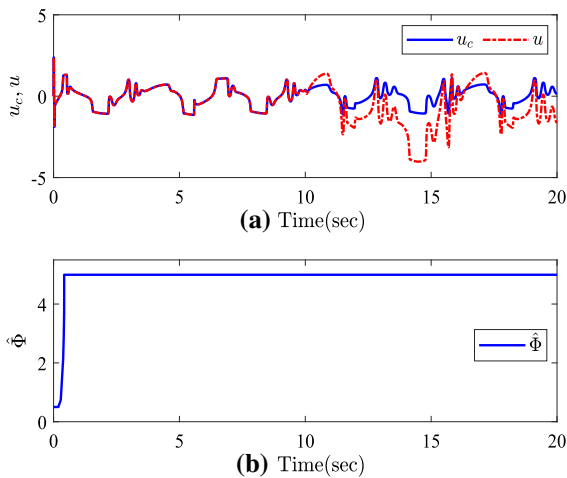


Fig. 4 a The fault-tolerant controller u_c and actual control input u , b the adaptation law $\hat{\phi}$

Table 1 Performance indices under two schemes

Performance indices	Proposed method	Ref. [17]
ITAE	1.7598	3.1642
RMSE	0.1635	0.4843
MAE	0.0286	0.0324

closely and the tracking error converges to a specified interval $\beta_1 = 0.03\text{ft/s}$ within fixed time, and the proposed control scheme possesses a better performance than the method in [17]. Figures 2b, c and 3b, c show the profiles of ξ_2 , ξ_3 , s_2 and s_3 . The asymmetric and time-varying constraints are guaranteed via the proposed control scheme, while the system state constraint is violated with the control method in [17]. Besides, compared with the existing results, the fluctuation of tracking error is smaller by utilizing the proposed method. The boundness of the controller and adaptive parameters is illustrated in Fig. 4. As shown in Fig. 4, the proposed controller owns good robustness to the actuator faults.

$\left[\frac{1}{T} \int_0^T |s_1(t)| dt\right]$ are introduced, and the calculation result is summarized in Table 1.

It can be perceived from Figs. 2a, 3a and Table 1 that the system output y tracks the desired trajectory y_r

6 Conclusions

An asymmetric time-varying barrier Lyapunov functions-based nonsingular fixed-time adaptive FTC method is constructed for high-order nonlinear systems in the presence of asymmetric time-varying state constraints and actuator faults. By constructing a new tuning function and a projection operator, the size of the convergence regions of state tracking errors in our case can be adjustable. The singularity and chattering problems are avoided by utilizing the switch function and smooth sign function. An interesting problem to be investigated in the future is how to solve the adaptive control problem for the high-order nonlinear systems in the presence of time-varying actuator fault. Besides, how to reduce the complexity of controller is still a very challenging problem [47–49], and this may be regarded as a possible future research topic.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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