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Dynamic railway timetable rescheduling for multiple connected disruptions

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ABSTRACT

Unexpected disruptions occur in the railways on a daily basis, which are typically handled manually by experienced traffic controllers with the support of predefined contingency plans. When several disruptions occur simultaneously, it is rather hard for traffic controllers to make rescheduling decisions, because (1) the predefined contingency plans corresponding to these disruptions may conflict with each other and (2) no predefined contingency plan considering the combined effects of multiple disruptions is available. This paper proposes a Mixed Integer Linear Programming (MILP) model to reschedule the timetable in case of multiple disruptions that occur at different geographic locations but have overlapping periods and are pairwise connected by at least one train line. The dispatching measures of retiming, reordering, cancelling, adding stops and flexible short-turning are formulated in the MILP model that also considers the rolling stock circulations at terminal stations and platform capacity. We develop two approaches for rescheduling the timetable in a dynamic environment: the sequential approach and the combined approach. In the sequential approach, a single-disruption rescheduling model is applied to handle each new disruption with the last solution as reference. In the combined approach, the multiple-disruption rescheduling model is applied every time an extra disruption occurs by considering all ongoing disruptions. A rolling-horizon solution method to the multiple-disruption model has been developed to handle long multiple connected disruptions in a more efficient way. The sequential and combined approaches have been tested on real-life instances on a subnetwork of the Dutch railways with 38 stations and 10 train lines operating half-hourly in each direction. In a few cases, the sequential approach did not find feasible solutions, while the combined approach obtained the solutions for all considered cases. Besides, the combined approach was able to find solutions with less cancelled train services and/or train delays than the sequential approach. For long disruptions, the proposed rolling-horizon method was able to generate high-quality rescheduling solutions in an acceptable time.

1. Introduction

Railways play a significant role in passenger transportation. For example, there are approximately 1.1 million trips by train every day in the Netherlands (ProRail, 2017). Thus, reliable train services are important. However, railway operations are often disturbed by unexpected events like extreme weather, accidents and infrastructure failures, which are getting worse in recent years. On the Dutch railways, the number of unplanned disruptions occurring each year was 1846 in 2011 and increased to 4085 in 2017 (RijdendeTreinen, 2018). Such disruptions usually last for a few hours, causing considerable negative impact on passengers

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and imposing much extra workload on personnel. In practice, disruptions are still handled manually by traffic controllers who make dispatching decisions (e.g. cancelling, delaying and short-turning) with pre-designed contingency plans as guidelines, while each contingency plan corresponds to one disruption at a specific location. When disruptions occur simultaneously at different locations, the contingency plans corresponding to them may conflict with each other. Under these circumstances, traffic controllers have to adjust the timetable based on their own experiences without any guidelines, which leads to time-consuming and suboptimal solutions (Ghaemi et al., 2017b). Therefore, it is necessary to develop a more efficient way of handling multiple disruptions, which has not been dealt with in the literature so far.

In this paper, we are concerned with unplanned disruptions that cause complete track blockages between stations for hours. Our focus is on rescheduling the timetable in case of multiple complete track blockages where each is connected to another by at least one train line. In these cases, train services should be adapted to multiple time–space disruption windows that are located in different locations and may start/end at different time instants. The main challenge is that the service adjustments towards one disruption window may influence the one towards another disruption window, and vice versa.

To solve this challenge, we put forward a multiple-disruption rescheduling model based on the single-disruption rescheduling model of Zhu and Goverde (2019). The single-disruption rescheduling model applies delaying, reordering, cancelling, flexible short-turning and flexible stopping, and considers station capacity as well as trains turning at terminal stations. Short-turning means that a train ends its operation at a station before the blocked tracks and the corresponding rolling stock turns to operate another train in the opposite direction. Flexible short-turning means that for each train a full choice of short-turn station candidates is given, and the model decides the optimal station and time of short-turning the train. Flexible stopping means that for each train the scheduled stops can be skipped and extra stops can be added. Except skipping stops, the other characteristics are all kept in the multiple-disruption rescheduling model that aims to minimize service cancellations and deviations from the planned timetable.

To deal with multiple connected disruptions in a dynamic environment, two approaches are proposed, which are a sequential approach based on a single-disruption rescheduling model, and a combined approach based on a multiple-disruption rescheduling model.

The contributions of this paper are summarized as follows:

- We develop a multiple-disruption timetable rescheduling model for multiple complete track blockages that are pairwise connected by at least one train line.
- We propose two approaches, the sequential approach and the combined approach, to deal with multiple connected disruptions in a dynamic environment.
- We propose a new rolling horizon solution method to generate high-quality solutions for long multiple connected disruptions in an acceptable time.
- The sequential and combined approaches are tested on real-life instances on a subnetwork of the Dutch railways.
- It is shown that the combined approach is able to handle more kinds of multiple disruption scenarios and generate better solutions than the sequential approach.

The remainder of this paper is organized as follows. Section 2 gives a literature review on timetable rescheduling models for railway disruptions. Section 3 introduces the sequential approach and the combined approach, and the differences between the two rescheduling models used in these approaches, the single-disruption model and the multiple-disruption model. Section 4 gives the detailed mathematical formulation of the multiple-disruption model. In Section 6, a case study is carried out to explore the performance of the sequential or combined approach. Finally, Section 7 concludes the paper.

2. Literature review

A typical consequence of a disruption is that the tracks between two stations are partially or completely blocked. In case of partial blockages, some trains can still use the remainder track as in Zhan et al. (2016) where a partial blockage is considered in a double-track railway line. In case of complete blockages, the consequence becomes more serious that no trains can run through the blocked area at all during the disruption period. This problem has been dealt with more widely in the literature compared to partial blockages, see Meng and Zhou (2011), Narayanaswami and Rangaraj (2013), Zhan et al. (2015), Ghaemi et al. (2017a, 2018) and Zhu and Goverde (2019). There are also models focusing on both partial and complete blockages, including Cadarso et al. (2013), Louwerse and Huisman (2014), Veelenturf et al. (2015) and Binder et al. (2017).

Different dispatching measures are used to reschedule the timetable during disruptions. Meng and Zhou (2011) allow retiming trains, while Narayanaswami and Rangaraj (2013) allow both retiming and reordering trains. In both papers, the considered disruption durations are at most 1 h. For longer disruptions that last for a few hours, cancelling trains is necessary, because it helps to reduce train delays that may propagate to the network beyond the disrupted area. Zhan et al. (2015, 2016) use retiming, reordering and cancelling trains focusing on Chinese railways where seat reservations are needed. Under this circumstance, short-turning trains is not applied in their models, which however is a common strategy used in the systems without seat reservations, e.g., metro systems and some European railway systems. The models allowing short-turning trains include Louwerse and Huisman (2014), Veelenturf et al. (2015), Ghaemi et al. (2017a, 2018) and Zhu and Goverde (2019). In general, the last stop of a train before the blocked tracks is fixed as the station where the train can short-turn, as in Louwerse and Huisman (2014) and Veelenturf et al. (2015). However, a train may be completely cancelled rather than short-turned, if the short-turn station lacks capacity. To reduce the possibility of a train being completely cancelled, Ghaemi et al. (2017a, 2018) allow a train to short-turn at either of the last two stations before the blocked tracks, while Zhu and Goverde (2019) introduce more flexibility by allowing a train to short-turn at one of all possible short-turn stations that are before the blocked tracks. Another way of reducing cancelled trains is rerouting trains. The trains that originally plan to run through the blocked tracks can be rerouted through another corridors to reach the destinations, while part/all of the intermediate stations in the planned paths may change. This strategy is applied in both Veelenturf et al. (2015) and Binder et al. (2017). When passengers are taken into account, particular strategies are used to mitigate the negative impact on passengers, such as adding additional trains, adding extra stops and skipping stops. Both Cadarso et al. (2013) and Binder et al. (2017) allow adding additional trains. Veelenturf et al. (2017) allow adding stops, while Zhu and Goverde (2019) allow both adding stops and skipping stops.

Most papers assume that the disruption duration is known at the beginning of the disruption and will not change over time. However in practice, a disruption could either end earlier or extend than expected (Zilko et al., 2016). A few papers deal with uncertain disruptions. Zhan et al. (2016) propose a rolling horizon framework where the timetable is rescheduled gradually with renewed disruption durations taken into account. Meng and Zhou (2011) propose a stochastic programming model that takes the uncertainty of the disruption duration into account. The model reschedules the timetable dynamically by a rolling horizon approach.

In the real world, multiple disruptions occur on a daily basis, while how to deal with them is rarely considered in the existing literature. Veelenturf et al. (2015) proposed a model said to be applicable to multiple track blockages, but no results were given. Van Aken et al. (2017) designed alternative timetables for handling multiple planned disruptions (i.e. infrastructure maintenance possessions). They focus on periodic timetables for full-day possessions, and as such do not consider the transitions between the original timetable and the rescheduled timetable, and vice versa. For shorter disruptions, such transitions have to be taken into account. According to Ghaemi et al. (2017b), a disruption consists of three phases: the transition phase from the planned timetable to the disruption timetable, the stable phase where the disruption timetable is implemented, and the recovery phase from the disruption timetable to the planned timetable. Veelenturf et al. (2015) and Zhu and Goverde (2019) consider all phases of one single disruption.

This paper proposes a Mixed Integer Linear Programming (MILP) model to reschedule the timetable in case of multiple disruptions that are pairwise connected by at least one train line. The multiple-disruption rescheduling model considers all phases of each disruption that causes complete track blockage for hours. Retiming, reordering, cancelling, adding stops and flexible short-turning are all formulated into the model that also takes into account rolling stock circulations and station capacity. Two approaches are developed to deal with multiple disruptions in a dynamic environment. A sequential approach applies a single-disruption rescheduling model to handle each new disruption with the previous rescheduled timetable as the reference. A combined approach applies a multiple-disruption rescheduling model to handle each new disruption considering the combined effects of all ongoing disruptions.

3. Problem description

In this paper, multiple connected disruptions are defined as two or more disruptions that

- have overlapping periods,
- occur at different geographic locations,
- may start/end at different time instants, and
- are pairwise connected by at least one train line.

Two disruptions having overlapping periods means that a disruption occurs when another disruption is ongoing. To be more specific, suppose a disruption i starts at time t^i_{start} and will end at t^i_{end} ($t^i_{\text{start}} < t^i_{\text{end}}$), and another disruption j starts at time t^j_{start} and will end at t^j_{end} ($t^j_{\text{start}} < t^j_{\text{end}}$). Then, the durations of these two disruptions are overlapping, if $t^i_{\text{start}} \leq t^j_{\text{start}} < t^i_{\text{end}}$ or $t^j_{\text{start}} \leq t^i_{\text{start}} < t^j_{\text{end}}$. It is possible that the disruption periods of two disruptions are *not* overlapping, but a train is influenced by a first disruption, and then later will be affected by a second disruption that started after the other disruption already ended. In this situation, during the first disruption it is unknown that there will be a second disruption occurring later. Therefore, these two disruptions can only be seen as two separate disruptions, and they can still be handled by either the sequential approach or the combined approach proposed in this paper. We require overlapping periods as one of the criteria to define multiple connected disruptions of which the combined effects can be actually taken into account during timetable rescheduling.

Fig. 1 illustrates different kinds of multiple disruptions.

In each case of Fig. 1, three train lines are operated in a triangle network where the stops served by each train line are indicated by circles. In case a, the first disruption occurs between 8:00 and 9:45 and affects train line 1, while the second disruption occurs between 8:20 and 10:15 at a different location and affects both train line 1 and train line 2. These two disruptions occur at different locations, have overlapping period, and are connected by train line 1, which are thus regarded as multiple connected disruptions. If the second disruption occurs between 16:00 and 17:45 as in case b, then these two disruptions are separate disruptions, since they do not have overlapping period. If the second disruption occurs at a different location as in case c, although these two disruptions have an overlapping period, they are not regarded as multiple connected disruptions, because they are not connected by any train line. Compared to case a, cases b and c are more easily handled using the method of Zhu and Goverde (2019), because there are no/few interactions among the timetable adjustments towards each disruption.

In this paper, our focus is on handling multiple connected disruptions. To this end, two approaches are proposed. One is the sequential approach that uses the single-disruption rescheduling model to solve each disruption sequentially. Another is the combined approach that applies the multiple-disruption model to handle each extra disruption with all ongoing disruptions taken into account. The introductions to these two approaches are given as follows.

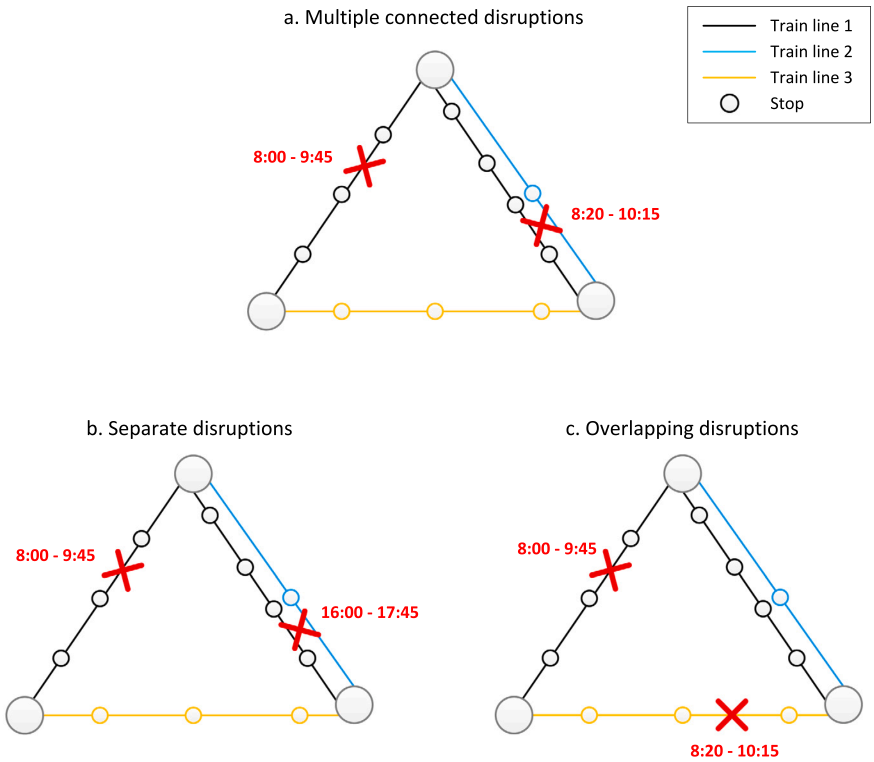

Fig. 1. Examples of multiple disruptions.

3.1. The sequential approach

The schematic layout of the sequential approach is shown in Fig. 2, where the single-disruption rescheduling model is applied every time a new disruption emerges. This can be considered as the straightforward extension to multiple disruptions that traffic controllers would apply to keep the complexity manageable for manual decision making. As this model can deal with one disruption at a time only, it uses the previous solution as reference when handling the disruption that starts later. This means that (1) the train services that are previously decided to be cancelled will remain cancelled; (2) the train departures/arrivals that are previously decided to be delayed can no longer occur before those time instants, as early departures/arrivals are not allowed; and (3) the short-turnings of the trains that do not run through the new track blockage will remain.

3.2. The combined approach

The schematic layout of the combined approach is shown in Fig. 3. Here, the single-disruption rescheduling model is applied for the 1st disruption only and the multiple-disruption rescheduling model is applied every time an extra disruption emerges. When handling later disruptions, the multiple-disruption rescheduling model makes service adjustments by taking all ongoing disruptions into account and respecting the train arrivals and departures that have already been realized according to the previous rescheduled timetable.

In this paper, the sequential approach is based on the single-disruption rescheduling model of Zhu and Goverde (2019) by removing the measure of skipping stops and replacing the objective with the one of the multiple-disruption model that is introduced in Section 4.

3.3. Differences between the single-disruption model and the multiple-disruption model

Compared to the single-disruption rescheduling model, the multiple-disruption rescheduling model additionally considers the interactions among the dispatching decisions towards different disruptions. These interactions mainly occur among short-turning decisions. Recall that short-turning means that a train ends its operation at a station before the blocked tracks and the corresponding rolling stock turns to be used by another train in the opposite direction. With the following example, we explain the differences between the single-disruption model and the multiple-disruption model.

In Fig. 4, four blue trains, $\mathrm{tr}_1, \mathrm{tr}_3, \mathrm{tr}_5$, and tr_7, operate from station A to station I; and four yellow trains, $\mathrm{tr}_2, \mathrm{tr}_4, \mathrm{tr}_6$, and tr_8, operate from station I to station A. Between stations F and G, a disruption occurs for a certain period, which is illustrated by a grey rectangle.

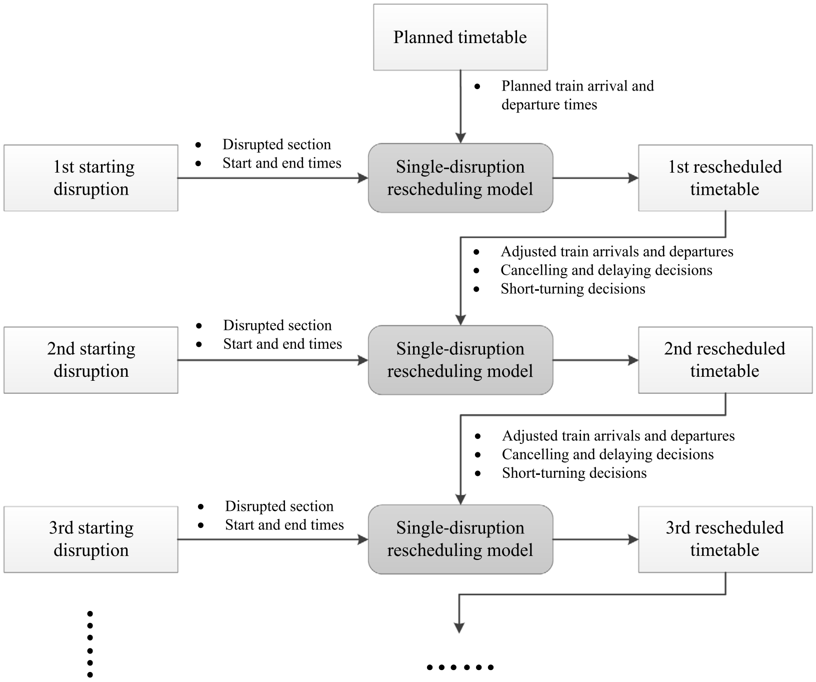

Fig. 2. Dealing with multiple connected disruptions by the sequential approach.

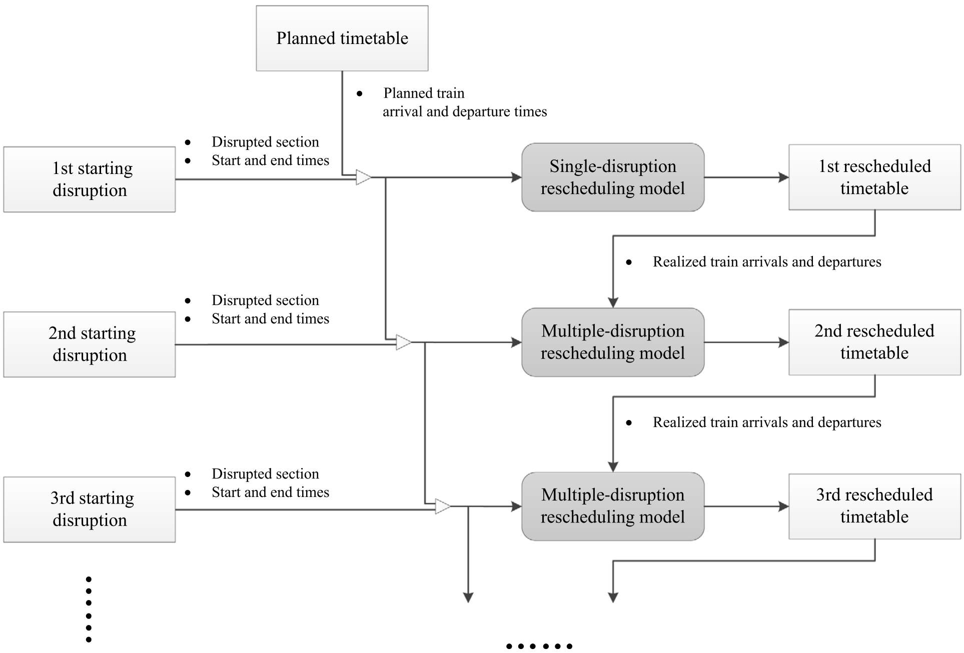

Fig. 3. Dealing with multiple connected disruptions by the combined approach.

Due to the disruption, two blue trains, tr_1 and tr_3, are short-turned at station F to take over the operations of two yellow trains, tr_4 and tr_6, from station F to station A; while these yellow trains are short-turned at station G to take over the operations of these blue trains from station G to station I.

Suppose a little bit later another disruption occurs between stations C and D as Fig. 5 shows. Then more short-turnings will happen even to the same train, and some short-turnings are interdependent. For example, train tr_3 is now short-turned at both stations C and F. Moreover, the short-turning between trains tr_2 and tr_3 at station D enables the short-turning between trains tr_3

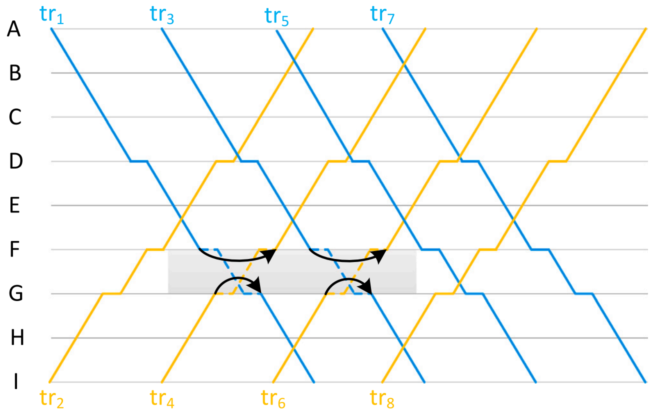

Fig. 4. Example of single-disruption rescheduling solution. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

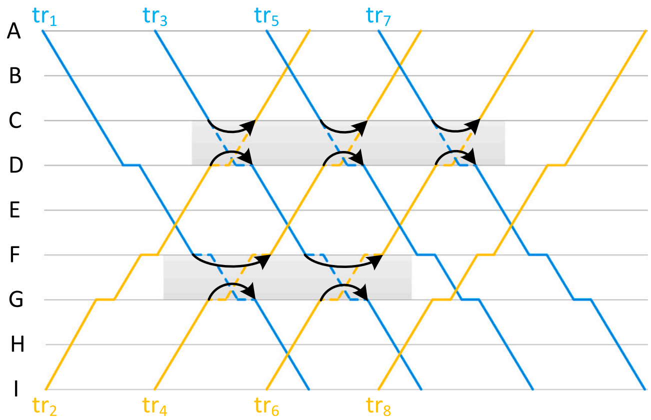

Fig. 5. Example of multiple-disruption rescheduling solution. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

and tr_6 at station F, which in turn enables the short-turning between trains tr_6 and tr_7 at station D. This indicates that during such multiple connected disruptions, a train might be short-turned at a station at each side of *each* disrupted section, and the short-turning at one station may affect the short-turning at another station. These are not considered in the single-disruption model, but should be formulated in the multiple-disruption model.

4. The multiple-disruption rescheduling model

4.1. Definitions

The multiple-disruption rescheduling model is based on an event-activity network formulation. Train departures (arrivals) are formulated as departure (arrival) *events*, which are contained in the set E_{de} (E_{ar}). Each event $e \in E_{\mathrm{de}} \cup E_{\mathrm{ar}}$ is associated with the original scheduled time o_e, station st_e, train line tl_e, train number tr_e, and operation direction dr_e. A train line indicates the origin, the destination, all intermediate stops between the origin and the destination, and the operation frequency (e.g. every 30 min).

Directed arcs between events are called *activities*. Running activities $(e, e') \in A_{\mathrm{run}}$ describe train running between adjacent stations:

$$A_{\mathrm{run}} = \left\{ (e, e') \in E_{\mathrm{de}} \times E_{\mathrm{ar}} \,:\, tr_e = tr_{e'}, \text{ and } tr_e \text{ goes directly from station } st_e \text{ to } st_{e'} \right\}.$$

Dwell (pass-through) activities $(e, e') \in A_{\mathrm{dwell}}$ (A_{pass}) describe trains dwelling at (passing through) stations:

$$A_{\mathrm{dwell}} = \left\{ (e, e') \in E_{\mathrm{ar}} \times E_{\mathrm{de}} \,:\, tr_e = tr_{e'}, st_e = st_{e'}, \text{ and } o_e < o_{e'} \right\},$$
$$A_{\mathrm{pass}} = \left\{ (e, e') \in E_{\mathrm{ar}} \times E_{\mathrm{de}} \,:\, tr_e = tr_{e'}, st_e = st_{e'}, \text{ and } o_e = o_{e'} \right\}.$$

Short-turn activities $(e, e') \in A_{\mathrm{turn}}$ describe trains turning at stations before blocked tracks to operate the trains in the opposite directions from the same train line:

$$A_{\mathrm{turn}} = \left\{ a = (e, e') \in E^{\mathrm{turn}}_{\mathrm{ar}} \times E^{\mathrm{turn}}_{\mathrm{de}} \,:\, tl_e = tl_{e'}, tr_e \neq tr_{e'}, dr_e \neq dr_{e'}, st_e = st_{e'}, \text{ and } o_{e'} + D - o_e \geq L_a \right\},$$

in which the arrival (departure) events are defined by the set $E_{\mathrm{ar}}^{\mathrm{turn}}$ ($E_{\mathrm{de}}^{\mathrm{turn}}$), D is the maximum delay allowed to an event, and L_a represents the minimum duration required for a short-turn activity a. We allow a short-turn activity a to be created from an arrival event e to a departure event e' that was originally planned to occur earlier than e considering that the rescheduled time of e' may be later than the rescheduled time of e so that the short-turning between them could be possible.

We also use the original OD turn activities A_{odturn} to describe trains turning at the destinations to the opposite trains from the same train line, and headway activities A_{head} to describe the headways of following or crossing trains. Zhu and Goverde (2019) already describe the sets A_{odturn}, A_{head}, and the constraints or decision variables corresponding to them (i.e. the constraints or decision variables about OD turns and reordering trains), which are used also in the multiple-disruption model exactly the same. Hence, for details we refer to Zhu and Goverde (2019). For the multiple-disruption model, we introduce constraints of cancelling, delaying, flexible short-turning trains and station capacity, as well as the decision variables used in these constraints. Recall that flexible short-turning means that each train is given a full choice of short-turning station candidates, and the model decides the optimal station and time of short-turning the train. In particular, a train may short-turn a station earlier if the capacity of a later short-turning station is insufficient.

We will use the following decision variables:

x_e : continuous variable deciding the rescheduled time of event e,
d_e : continuous variable deciding the delay of event e,
c_e : binary variable with value 1 indicating that e is cancelled, and 0 otherwise,
y_e : binary variable with value 1 indicating that station st_e is a short-turn station of train tr_e, and 0 otherwise,
m_a : binary variable with value 1 indicating that a short-turn activity $a \in A_{\mathrm{turn}}$ is selected, and 0 otherwise.
$u_{e,i}$: binary variable with value 1 indicating that train tr_e occupies the ith platform of station $st_e, e \in E_{\mathrm{ar}}$, and 0 otherwise.
$v_{e,j}$: binary variable with value 1 indicating that train tr_e occupies the jth pass-through track of station $st_e, e \in E_{\mathrm{ar}}$, and 0 otherwise.

For the notation of parameters and sets we refer to Appendix A.

4.2. Objective

The objective is minimizing train service cancellations and deviations from the planned timetable,

$$\text{minimize} \sum_{e \in E_{\mathrm{ar}}} w c_e + \sum_{e \in E_{\mathrm{ar}} \cup E_{\mathrm{de}}} d_e, \tag{1}$$

where E_{ar} (E_{de}) is the set of arrival (departure) events, and w is a fixed penalty for each cancelled service. A service refers to a train run between two adjacent stations. This objective aims to minimize the impact of the disruption to the rest of the network.

4.3. Constraints

4.3.1. Cancelling and delaying trains

For each train departure or arrival event e, the rescheduled time x_e is relevant to the delaying decision d_e and the cancelling decision c_e as follows:

$$M_1 c_e \le x_e - o_e \le M_1, \qquad e \in E_{\mathrm{ar}} \cup E_{\mathrm{de}}, \tag{2}$$

$$x_e - o_e = d_e + M_1 c_e, \qquad e \in E_{\mathrm{ar}} \cup E_{\mathrm{de}}, \tag{3}$$

$$d_e \ge 0, \qquad e \in E_{\mathrm{ar}} \cup E_{\mathrm{de}}, \tag{4}$$

$$d_e \le D, \qquad e \in \left(E_{\mathrm{ar}} \cup E_{\mathrm{de}}\right) \backslash E^{\mathrm{NMdelay}}. \tag{5}$$

Constraint (2) states that the rescheduled time of a cancelled event e (i.e. $c_e = 1$) is the original scheduled time o_e plus M_1 that is set to 1440 min here. Constraint (3) states that the rescheduled time of an non-cancelled event is the original scheduled time plus the delayed time. Constraints (4) and (5) require that the delay of an event is non-negative, and should be no larger than D minutes if the event does not belong to E^{NMdelay}. The set E^{NMdelay} contains all events that are not imposed with the upper delay limit. These events correspond to the trains that have already started from the origins before a disruption starts. These trains can no longer be cancelled and short-turning them could also be impossible due to rolling stock or station capacity shortage, and thus they have to dwell at the last possible stations before the blocked tracks until the disruption ends.

4.3.2. Avoiding trains entering any disrupted section

Suppose the current emerging disruption is the nth disruption ($n \ge 2$), then trains are forbidden to enter the blocked tracks due to any disruption i ($1 \le i \le n$) during the corresponding disruption period that starts at time t_{start}^i and ends at time t_{end}^i:

$$x_e \ge t_{\mathrm{end}}^i (1 - c_e), \qquad e \in E_{\mathrm{de}}, st_e = st_{\mathrm{en}}^{i,tr_e}, t_{\mathrm{start}}^i \le o_e < t_{\mathrm{end}}^i, 1 \le i \le n, \tag{6}$$

where $st_{\mathrm{en}}^{i,dr_e}$ represents the entry station of the ith disrupted section in direction dr_e that is either upstream or downstream. For instance in Fig. 5, for the downstream blue train tr_3: the entry station of the 1st disrupted section (i.e. section F–G) is F and the

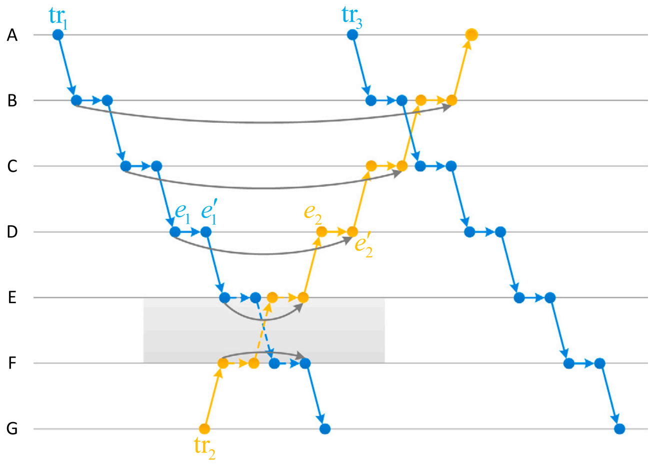

Fig. 6. Example of short-turning options under one single disruption.

entry station of the 2nd disrupted section (i.e. section C–D) is C, while for the upstream yellow train tr_4: the entry station of the 1st disrupted section is G and the entry station of the 2nd disrupted section is D. It is assumed that the duration of a disruption is known at the beginning of the disruption and will not change over time.

4.3.3. Operation consistency for trains without short-turning possibilities

For each train, the operation consistency of the two events that constitute the same running activity is always kept (i.e. both events are cancelled/kept simultaneously):

$$c_{e'} - c_e = 0, \qquad (e, e') \in A_{\text{run}}, \tag{7}$$

and the operation consistency of the two events that constitute the same dwell/pass-through activity is always kept if neither of them is relevant to any short-turn activity (i.e. no short-turning possibility):

$$c_e - c_{e'} = 0, \qquad (e, e') \in A_{\text{station}}, e \in E_{\text{ar}} \backslash E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}} \backslash E_{\text{de}}^{\text{turn}}, \tag{8}$$

where the set of station activities $A_{\text{station}} = A_{\text{dwell}} \cup A_{\text{pass}}$. Recall that $E_{\text{ar}}^{\text{turn}}$ ($E_{\text{de}}^{\text{turn}}$) is the set of arrival (departure) events that are the tails (heads) of short-turn activities. The tail (head) of an activity a is the event that a starts from (points to).

4.3.4. Breaking operation consistency for trains with short-turning possibilities

If a train is short-turned at a station, the operation consistency of its arrival and departure events at the station must be broken. For example in Fig. 6 where section E–F is completely blocked and five possible short-turn activities (grey arcs) are created between trains tr_1 and tr_2. If possible the short-turn activity at station D is selected, then for train tr_1 the arrival event e_1 must be kept while the departure event e_1' must be cancelled, and for train tr_2 the arrival event e_2 must be cancelled while the departure event e_2' must be kept. In this case, $e_1 \in E_{\text{ar}}^{\text{turn}}, e_1' \in E_{\text{de}} \backslash E_{\text{de}}^{\text{turn}}$, and $e_2 \in E_{\text{ar}} \backslash E_{\text{ar}}^{\text{turn}}, e_2' \in E_{\text{de}}^{\text{turn}}$. To decide whether to break the operation consistency of such two events e and e' that form a station activity and only *one* of them has a short-turning possibility, the following constraints are established:

$$c_e \le c_{e'}, \qquad (e, e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}} \backslash E_{\text{de}}^{\text{turn}}, \tag{9}$$

$$c_{e'} \le c_e + y_e, \qquad (e, e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}} \backslash E_{\text{de}}^{\text{turn}}, \tag{10}$$

$$c_{e'} \ge y_e, \qquad (e, e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}} \backslash E_{\text{de}}^{\text{turn}}, \tag{11}$$

$$c_{e'} \le c_e, \qquad (e, e') \in A_{\text{station}}, e \in E_{\text{ar}} \backslash E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, \tag{12}$$

$$c_e \le c_{e'} + y_{e'}, \qquad (e, e') \in A_{\text{station}}, e \in E_{\text{ar}} \backslash E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, \tag{13}$$

$$c_e \ge y_{e'}, \qquad (e, e') \in A_{\text{station}}, e \in E_{\text{ar}} \backslash E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, \tag{14}$$

where y_e is a binary decision variable with value 1 indicating that station st_e is chosen as the short-turn station of train tr_e, and 0 otherwise. If station st_e is *not* chosen as the short-turn station of arriving train tr_e (i.e. $y_e = 0$), then constraints (9) and (10) ensure that the operation consistency of events e and e' are kept; otherwise, constraint (11) requires event e' that does not have a short-turning possibility to be cancelled. Constraints (12)–(14) are similar but consider the different case where the departure event e' has a short-turning possibility while the arrival event e does not.

A train could be affected by two or more disruptions such as train tr_2 shown in Fig. 7 where another section B–C is also disrupted. In this case, more short-turning activities are created due to the extra disruption, and in particular events e_2 and e_2' both have

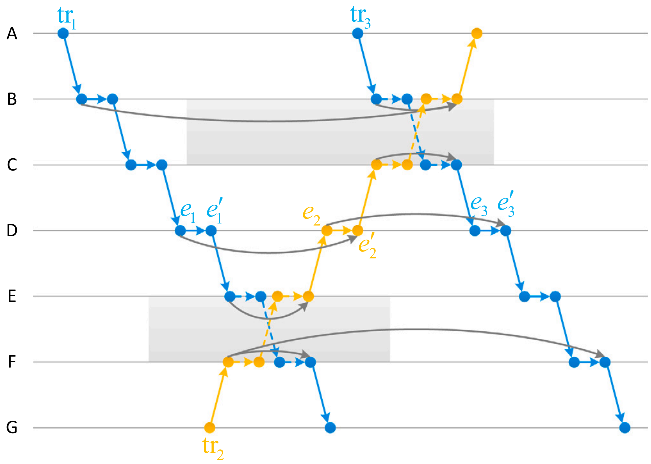

Fig. 7. Example of short-turning options under two connected disruptions.

short-turning possibilities, $e_2 \in E_{\text{ar}}^{\text{turn}}, e_2' \in E_{\text{de}}^{\text{turn}}$, but at most one of them will make it. To decide whether to break the operation consistency of such two events e and e' that form a station activity and *both* of them have short-turning possibilities, the following constraints are established:

$$c_e - c_{e'} = y_{e'} - y_e, \qquad (e,e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, \tag{15}$$

$$y_e + y_{e'} \leq 1, \qquad (e,e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, \tag{16}$$

$$c_{e'} \geq y_e, \qquad (e,e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, \tag{17}$$

$$c_e \leq 1 - y_e, \qquad (e,e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, \tag{18}$$

$$c_e \geq y_{e'}, \qquad (e,e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, \tag{19}$$

$$c_{e'} \leq 1 - y_{e'}, \qquad (e,e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}},. \tag{20}$$

Constraint (15) ensures that if y_e and $y_{e'}$ are both equal to 0, then the operation consistency of events e and e' must be kept. Constraint (16) requires that at most one of y_e and $y_{e'}$ can be 1. Constraints (17) and (18) ensure that if $y_e = 1$ and $y_{e'} = 0$, then event e' must be cancelled, whereas event e must be kept due to the short-turning. Constraints (19) and (20) ensure that if $y_e = 0$ and $y_{e'} = 1$, then event e must be cancelled, whereas event e' must be kept due to the short-turning.

4.3.5. Limiting the short-turning stations for each train

At each side of the ith disrupted section, at least one short-turn station is chosen for a train if its operation in the disrupted section is cancelled:

$$\sum_{e:tr_e=tr} y_e \geq c_{e'}, \quad tr \in TR_{\text{turn}}^i, e \in E_{\text{ar}}^{i,\text{turn}}, e' \in E_{\text{de}}, tr_{e'} = tr, st_{e'} = st_{\text{en}}^{i,dr_{e'}}, 1 \leq i \leq n, \tag{21}$$

$$\sum_{e':tr_{e'}=tr} y_{e'} \geq c_e, \; tr \in TR_{\text{turn}}^i, e' \in E_{\text{de}}^{i,\text{turn}}, e \in E_{\text{ar}}, tr_e = tr, st_e = st_{\text{ex}}^{i,dr_e}, 1 \leq i \leq n, \tag{22}$$

where $E_{\text{ar}}^{i,\text{turn}} \subset E_{\text{ar}}^{\text{turn}}$ ($E_{\text{de}}^{i,\text{turn}} \subset E_{\text{de}}^{\text{turn}}$) is the set of arrival (departure) events relevant to the short-turn activities corresponding to the ith disruption, TR_{turn}^i is the set of trains corresponding to the events in $E_{\text{ar}}^{i,\text{turn}} \cup E_{\text{de}}^{i,\text{turn}}$, and $st_{\text{en}}^{i,dr_{e'}}$ (st_{ex}^{i,dr_e}) represents the entry (exit) station of the ith disrupted section in direction $dr_{e'}$ (dr_e). In (21) and (22), we use "≥" instead of "=" because the short-turn activities relevant to one train could correspond to different disruptions. In other words, it is possible that an event $e \in E_{\text{ar}}^{i,\text{turn}} \cap E_{\text{ar}}^{j,\text{turn}}$ (or $e' \in E_{\text{de}}^{i,\text{turn}} \cap E_{\text{de}}^{j,\text{turn}}$), while $i \neq j, 1 \leq i,j \leq n$. For example in Fig. 7, the short-turn activity from train tr_2 to train tr_1 at station F corresponds to the first disruption and also corresponds to the second disruption as an early short-turning. In this case, the arrival event of train tr_2 at station F must belong to both $E_{\text{ar}}^{1,\text{turn}}$ and $E_{\text{ar}}^{2,\text{turn}}$, and the departure event of train tr_1 at station F must belong to both $E_{\text{de}}^{1,\text{turn}}$ and $E_{\text{de}}^{2,\text{turn}}$.

At one side of all disrupted sections, the number of short-turn stations chosen for a train cannot be larger than the number of its departure (arrival) events that originally occur at the entry (exit) stations of these disrupted sections but were cancelled.

$$\sum_{e:tr_e=tr} y_e \leq \sum_{e'} c_{e'}, \quad tr \in TR_{\text{turn}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}, tr_{e'} = tr, st_{e'} \in ST_{\text{en}}^{dr_{e'}}, 1 \leq i \leq n, \tag{23}$$

$$\sum_{e':tr_{e'}=tr} y_{e'} \leq \sum_{e} c_e, \; tr \in TR_{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, e \in E_{\text{ar}}, tr_e = tr, st_e \in ST_{\text{ex}}^{dr_e}, 1 \leq i \leq n, \tag{24}$$

where $ST_{\text{en}}^{dr_{e'}} = \bigcup_{i=1}^{n} st_{\text{en}}^{i,dr_{e'}}$ and $ST_{\text{ex}}^{dr_e} = \bigcup_{i=1}^{n} st_{\text{ex}}^{i,dr_e}$. Constraint (23) ensures that at one side of all disrupted sections, the number of short-turn stations chosen for train tr is not larger than the number of its departure events that originally occurred at the entry stations of the disrupted sections but were cancelled. Constraint (24) ensures that at the other end of all disrupted sections, the number of short-turn stations chosen for train tr is not larger than the number of its arrival events that originally occurred at the exit stations of these disrupted sections but were cancelled.

4.3.6. Selecting short-turn activities

For each train, at most one short-turn activity will be selected at a short-turn station. This is formulated by

$$\sum_{\substack{a\in A_{\text{turn}},\\ tail(a)=e}} m_a = c_{e'} - c_e, \qquad (e,e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}} \backslash E_{\text{de}}^{\text{turn}}, \tag{25}$$

$$\sum_{\substack{a\in A_{\text{turn}},\\ head(a)=e'}} m_a = c_e - c_{e'}, \qquad (e,e') \in A_{\text{station}}, e \in E_{\text{ar}} \backslash E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, \tag{26}$$

$$\sum_{\substack{a\in A_{\text{turn}},\\ tail(a)=e}} m_a = c_{e'} - c_e + y_{e'}, \qquad (e,e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, \tag{27}$$

$$\sum_{\substack{a\in A_{\text{turn}},\\ head(a)=e'}} m_a = c_e - c_{e'} + y_e, \qquad (e,e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}. \tag{28}$$

where m_a is a binary decision with value 1 indicating that the short-turn activity a is selected. Constraints (25) and (26) are for the cases where a train is affected by one disruption only, while constraints (27) and (28) are for the cases where a train is affected by two or more disruptions. In (27), it may happen that $c_{e'} = 0$ and $c_e = 1$, which makes $c_{e'} - c_e = -1$ while the left term of this equality must be non-negative. Considering this, $y_{e'}$ is added to the right side, of which the value must be 1 in this case due to constraints (15) and (17). A similar reasoning is applied for adding y_e to the right side of (28).

If a short-turn activity is selected, the minimum short-turn duration must be respected, which is formulated by

$$M_1 c_e + 2D(1 - m_a) + x_{e'} - x_e \geq m_a L_a, \qquad a = (e,e') \in A_{\text{turn}}, \tag{29}$$

where A_{turn} is the set of all possible short-turn activities, and L_a represents the minimum duration required for short-turn activity a. If a short-turn activity $a \in A_{\text{turn}}$ is not selected (i.e. $m_a = 0$) while event e is not cancelled (i.e. $c_e = 0$), $2D$ is added to $x_{e'}$ to make constraint (29) still feasible, as $x_{e'}$ could be smaller than x_e. In this case, $2D$ is sufficient enough to make (29) feasible according to the definition of a short-turn activity given in Section 4.1.

4.3.7. Respecting realized train services

Recall that the current emerging disruption is the nth disruption ($n \geq 2$) starting at time t_{start}^n. Then, each departure or arrival event e that has occurred before t_{start}^n must be respected:

$$c_e = 0, \qquad e \in E_{\text{ar}} \cup E_{\text{de}}, r_e < t_{\text{start}}^n, n \geq 2, \tag{30}$$

$$x_e - r_e = 0, \qquad e \in E_{\text{ar}} \cup E_{\text{de}}, r_e < t_{\text{start}}^n, n \geq 2, \tag{31}$$

where r_e is a known value that refers to the previous rescheduled time of event e. Besides, each departure or arrival event e of which the previous rescheduled time r_e was after t_{start}^n cannot be rescheduled to before t_{start}^n in the current rescheduling procedure:

$$x_e \geq t_{\text{start}}^n, \qquad e \in E_{\text{ar}} \cup E_{\text{de}}, r_e \geq t_{\text{start}}^n, n \geq 2. \tag{32}$$

4.3.8. Handling the recovery phase

The train departures that are originally planned to occur up to R s after the maximum disruption ending time cannot be cancelled and should run as scheduled:

$$c_e = 0, \qquad e \in E_{\text{de}}, o_e \geq \max\left\{t_{\text{end}}^1, \ldots, t_{\text{end}}^n\right\} + R, n \geq 2, \tag{33}$$

$$x_e - o_e = 0, \qquad e \in E_{\text{de}}, o_e \geq \max\left\{t_{\text{end}}^1, \ldots, t_{\text{end}}^n\right\} + R, n \geq 2, \tag{34}$$

where R is the required recovery time length.

Also the train arrivals, of which the corresponding departures from the same running activities are originally planned to occur up to R s after the maximum disruption ending time, cannot be cancelled and should run as scheduled:

$$c_{e'} = 0, \qquad (e,e') \in A_{\text{run}}, o_e \geq \max\left\{t_{\text{end}}^1, \ldots, t_{\text{end}}^n\right\} + R, n \geq 2, \tag{35}$$

$$x_{e'} - o_{e'} = 0, \qquad (e,e') \in A_{\text{run}}, o_e \geq \max\left\{t_{\text{end}}^1, \ldots, t_{\text{end}}^n\right\} + R, n \geq 2, \tag{36}$$

Note that an arrival event, which was originally planned to occur after $\max\left\{t_{\text{end}}^1, \ldots, t_{\text{end}}^n\right\} + R$, may also be unable to run as planned, because its corresponding departure event from the same running activity could be originally planned to occur before $\max\left\{t_{\text{end}}^1, \ldots, t_{\text{end}}^n\right\} + R$ and then would possibly be cancelled.

4.3.9. Station capacity

Each arriving train must be assigned with a track to stop at or pass through a station, and the track has to be a platform track if the train stops at the station. These are formulated by

$$\sum_{i=1}^{N^p_{st_e}} u_{e,i} + \sum_{j=1}^{N^{th}_{st_e}} v_{e,j} = 1 - c_e, \qquad e \in E_{\mathrm{ar}}, \tag{37}$$

$$\sum_{i=1}^{N^p_{st_e}} u_{e,i} \geq 1 - s_a - c_e - c_{e'} \qquad e \in E_{\mathrm{ar}}, a = (e, e') \in A_{\mathrm{station}}, \tag{38}$$

$$\sum_{i=1}^{N^p_{st_e}} u_{e,i} \geq \sum_{\substack{a \in A_{\mathrm{turn}},\\ tail(a)=e}} m_a \qquad e \in E^{\mathrm{turn}}_{\mathrm{ar}}, \tag{39}$$

$$\sum_{i=1}^{N^p_{st_e}} u_{e,i} = 1 - c_e, \qquad e \in E^{\mathrm{odturn}}_{\mathrm{ar}}, \tag{40}$$

where $u_{e,i}$ ($v_{e,j}$) is a binary variable with value 1 indicating that train tr_e occupies the ith (jth) platform (pass-through) track at station st_e and 0 otherwise, $N^p_{st_e}$ ($N^{th}_{st_e}$) represents the number of platform (pass-through) tracks at station st_e, and s_a is a binary variable to realize adding stops. A station activity $a = (e, e') \in A_{\mathrm{station}}$ corresponds to a *true* stop (either a kept scheduled stop or an added stop), if and only if $s_a = 0, c_e = 0$ and $c_{e'} = 0$. For the constraints of determining s_a we refer to Zhu and Goverde (2019). Constraint (37) requires one station track to be assigned to an arriving train tr_e if event e is not cancelled. A platform track must be assigned to an arriving train tr_e if (1) it stops at the station ((37) and (38)); (2) it short-turns at the station ((37) and (39)); or (3) it reaches the destination ((37) and (40)). $E^{\mathrm{odturn}}_{\mathrm{ar}}$ is the set of arrival events that occur at the destinations and thus the corresponding rolling stock turns to operate the trains in the opposite directions. For the details of $E^{\mathrm{odturn}}_{\mathrm{ar}}$ we refer to Zhu and Goverde (2019).

If two trains occupy the same track of a station, there must be a minimum time interval to be respected between their occupations. In other words, the arrival of a train has to be a certain time later than the departure of another train that uses the same station track earlier. This is formulated by

$$x_{e'} - x_{e''} \geq h_{e,e'} + M_2(q_{e,e'} - c_e - c_{e'} - c_{e''} + u_{e,i} + u_{e',i} - 3), \qquad e, e' \in E_{\mathrm{ar}}, st_{e'} = st_e, (e, e'') \in A_{\mathrm{station}}, \tag{41}$$

$$x_{e'} - x_{e''} \geq h_{e,e'} + M_2(q_{e,e'} - c_e - c_{e'} - (1 - m_a) + u_{e,i} + u_{e',i} - 3), \quad e, e' \in E_{\mathrm{ar}}, st_{e'} = st_e, a = (e, e'') \in A_{\mathrm{turn}} \cup A_{\mathrm{odturn}}, \tag{42}$$

$$x_{e'} - x_{e''} \geq h_{e,e'} + M_2(q_{e,e'} - c_e - c_{e'} - c_{e''} + v_{e,j} + v_{e',j} - 3), \qquad e, e' \in E_{\mathrm{ar}}, st_{e'} = st_e, (e, e'') \in A_{\mathrm{station}}, \tag{43}$$

$$x_{e'} - x_{e''} \geq h_{e,e'} + M_2(q_{e,e'} - c_e - c_{e'} - (1 - m_a) + v_{e,j} + v_{e',j} - 3), \quad e, e' \in E_{\mathrm{ar}}, st_{e'} = st_e, a = (e, e'') \in A_{\mathrm{turn}} \cup A_{\mathrm{odturn}}, \tag{44}$$

where M_2 is a large positive number set to twice of M_1, $h_{e,e'}$ is a given parameter representing the minimum time interval required between the occurring times of e and e' if corresponding to trains occupying the same station track, and $q_{e,e'}$ is a binary variable with value 1 indicating that event e occurs before event e' and 0 otherwise. For the constraints of determining $q_{e,e'}$ we refer to Zhu and Goverde (2019), as well as the set A_{odturn} that contains all OD turn activities. Constraint (41) means that if arrival event e occurs before arrival event e' (i.e. $q_{e,e'} = 1$), events e, e' and e'' are all not cancelled (i.e. $c_e = 0, c_{e'} = 0$ and $c_{e''} = 0$) and both events e and e' occupy the same platform track (i.e. $u_{e,i} = 1$, and $u_{e',i} = 1$), then event e' must occur at least $h_{e,e'}$ later than the departure event e'' in the station activity corresponding to e. Constraint (42) means that if arrival event e occurs before arrival event e' (i.e. $q_{e,e'} = 1$), events e and e' are both not cancelled (i.e. $c_e = 0$ and $c_{e'} = 0$), the short-turn (OD turn) activity a relevant to e is selected (i.e. $m_a = 1$), and both events e and e' occupy the same platform track (i.e. $u_{e,i} = 1$ and $u_{e',i} = 1$), then event e' must occur at least $h_{e,e'}$ later than the departure event e'' in the short-turn (OD turn) activity corresponding to e. Constraint (43) ((44)) is similar to (41) ((42)), but considers a pass-through track.

5. Rolling horizon solution method

The multiple-disruption rescheduling model can be solved to optimality or near-optimality, if the disruption durations are not long (e.g. 2-hour disruptions). However in some disruption scenarios, a solver may not find high-quality solutions in an acceptable time if the disruption durations become rather long (e.g. 6-hour disruptions). Therefore, we propose a rolling-horizon solution method to the multiple-disruption model, which considers the periodic pattern of the rescheduled train services in the second phase of a disruption to speed up the computation.

A disruption consists of three phases: the 1st phase from the planned timetable transiting to the disruption timetable, the 2nd phase where the disruption timetable is in use, and the 3rd phase from the disruption timetable recovering to the planned timetable (Ghaemi et al., 2017b). A periodic short-turning/cancelling pattern exists among the rescheduled train services in the 2nd phase, due to the periodicity of the planned timetable (Zhu and Goverde, 2019). That means, for example, if a train is short-turned at station A then another train that serves the same train line in a later period will be short-turned at the same station as well. Taking such a periodic pattern into account is helpful to release the computational burden by first obtaining the pattern considering a relatively short time horizon and then applying this pattern to the following train services gradually over time. How often the pattern will repeat varies with train lines. It is observed that for the train lines that are only affected by one disruption the length of

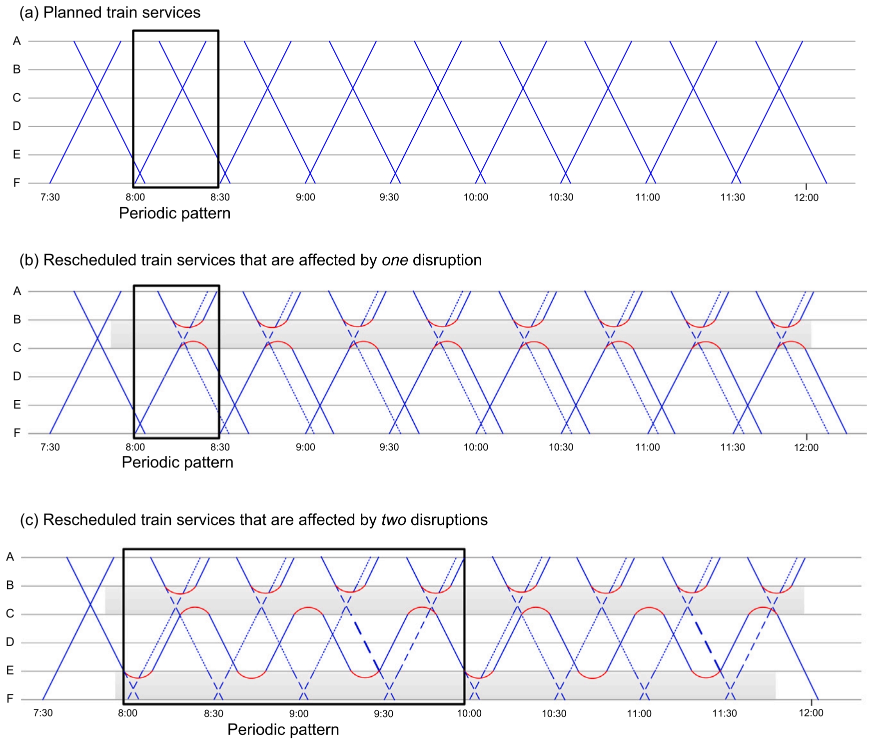

Fig. 8. Illustration of the periodic pattern of train services. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

the period is equal to the cycle time of the planned timetable, while for the train lines that are affected by at least two disruptions the period of the disruption solution may take longer than the planned cycle time and varies with disruption scenarios. An example is given in Fig. 8, where (a) shows that a train line is planned to operate between station A and station F periodically, (b) shows that the rescheduled train services due to one disruption have a periodic pattern and the length of the period is the same as the planned cycle time, and (c) shows that the rescheduled train services due to two disruptions also have a periodic pattern but the length of the period could be much longer than the planned cycle time. Note that the length of the period may be further increased by increasing the short-turning durations to reflect the possible lower passenger demand. The dotted (dashed) lines in Fig. 8(b) and (c) represent the original train services that are delayed (cancelled) in the rescheduled timetable, and the red arcs refer to the short-turning activities. In Fig. 8(b) and (c), train services are delayed to respect the minimum short-turning durations, and all train services that were originally planned to operate in the disrupted sections are cancelled. Also in Fig. 8(c), some train services from station C to station E are cancelled (the thick dashed lines), because they can only be kept if delayed by one planned cycle time to be operated by the rolling stock of the previous short-turned train, but another train service belonging to the same train line will already operate at that time.

The rolling-horizon method divides the time horizon, from the starting time of a new connected disruption until the latest ending time among all connected disruptions, into successive stages. For the train lines that are only affected by one disruption, the periodic pattern is computed at stage 1, which is applied to the corresponding train services in the following stages. For the train lines that are affected by at least two disruptions, no periodic pattern will be computed at stage 1, because as explained before the length of the corresponding period varies with disruption scenarios and thus determining the pattern with an assumed length may affect the solution quality. Therefore, the train services corresponding to these train lines are rescheduled at each stage from scratch.

An illustration of the rolling-horizon method is given in Fig. 9, while the details are given in Algorithm 1. The notation used in the algorithm is listed in Appendix B. Algorithm 1 needs the following inputs: the set of ongoing disruptions $DIS = \{1, \ldots, n\}$, the starting (ending) time t^i_{start} (t^i_{end}) of the ith disruption, the set $TL^i_{\text{dis},1}$ containing the train lines that are only affected by the ith disruption, the set ST_{tl} containing the planned stopping and passage stations of train line $tl \in TL^i_{\text{dis},1}$, the length of a disruption h_r considered at a stage, the maximum allowed delay per event D, and the determined rescheduled time r^1_e of each event $e \in E$ when dealing with the first disruption only. All ongoing disruptions in DIS are sorted in ascending order according to their starting times, and the nth disruption is the emerging disruption. The setting of h_r affects the solution quality as well as the computation time. The value of h_r is set to at least bigger than D. A larger h_r may lead to a better solution but meanwhile could cost longer computation time. The influence of h_r on a solution is investigated in Section 6.3.

Algorithm 1: A rolling-horizon solution method to the multiple-disruption timetable rescheduling model

Input: $DIS = \{1, \cdots, n\}, \left\{\left(t^i_{\text{start}}, t^i_{\text{end}}, TL^i_{\text{dis},1}\right)\right\}_{i \in DIS}, \{ST_{tl}\}_{tl \in TL^i_{\text{dis},1}}, h_r, D, \{r^1_e\}_{e \in E}$

Output: The rescheduled timetable for n connected disruptions

1 $k = 1$; `// Stage 1`

2 $DIS^k = \{1, \cdots, n_k\}, n_k = n$;

3 $\tilde{t}^k_{\text{start}} = t^{n_k}_{\text{start}}, i \in DIS^k$;

4 $\tilde{t}^{i,k}_{\text{end}} = \min\{\tilde{t}^k_{\text{start}} + h_r, t^i_{\text{end}}\}, i \in DIS^k$;

5 Solve the multiple-disruption model considering the ith disruption with duration $[\tilde{t}^k_{\text{start}}, \tilde{t}^{i,k}_{\text{end}}]$, $i \in DIS^k$ to obtain the set of the cancelled events E^k_{cancel}. When solving the model, in constraints (30)-(32) t^n_{start} is set to $\tilde{t}^k_{\text{start}}$ and r_e is set to the previous rescheduled time r^1_e determined for event e when handling the first disruption only, and in constraints (33)-(36) the maximum considered disruption ending time is $\max\{\tilde{t}^{1,k}_{\text{end}}, \cdots, \tilde{t}^{n_k,k}_{\text{end}}\}$;

6 $E^{\text{ar}}_{\text{cancel}} = \emptyset, E^{\text{ar}}_{\text{keep}} = \emptyset$; `// Extract the periodic pattern (lines 6--20)`

7 **for** $i = 1 : n_k$ **do**

8   **foreach** $tl \in TL^i_{\text{dis},1}$ **do**

9   Define $E^{tl,i}_{\text{ar}} = \left\{e \,\middle|\, e \in E_{\text{ar}}, tl_e = tl, \tilde{t}^k_{\text{start}} \leq o_e \leq \tilde{t}^{i,k}_{\text{end}}, \tilde{t}^{i,k}_{\text{end}} < t^i_{\text{end}}\right\}$;

10   **foreach** $st \in ST_{tl}$ **do**

11   Define $E^{st,tl,i}_{\text{ar}} = \{e \mid e \in E^{tl,i}_{\text{ar}}, st_e = st\}$;

12   Find $e' = \arg\min\{o_{e'} : e' \in E^{st,tl,i}_{\text{ar}}\}$;

13   **if** $e' \in E^k_{\text{cancel}}$ **then**

14         $E^{\text{ar}}_{\text{cancel}} = E^{\text{ar}}_{\text{cancel}} \bigcup e'$;

15   **else**

16   Find $e'' = \arg\min\{o_{e''} : e'' \in E^{st,tl,i}_{\text{ar}} \backslash e'\}$;

17   **if** $e'' \in E^k_{\text{cancel}}$ **then**

18           $E^{\text{ar}}_{\text{cancel}} = E^{\text{ar}}_{\text{cancel}} \bigcup e''$;

19   **else**

20           $E^{\text{ar}}_{\text{keep}} = E^{\text{ar}}_{\text{keep}} \bigcup e''$;

21 Remove the ith disruption from DIS^k if $\tilde{t}^{i,k}_{\text{end}} = t^i_{\text{end}}, i \in DIS^k$, and then update the number of the remainder disruptions as n_{k+1} and define $DIS^{k+1} = \{1, \cdots, n_{k+1}\}$;

22 **while** $n_{k+1} \geq 1$ **do**

23   $k = k + 1$; `// Stage 2 and onwards`

24   $\tilde{t}^k_{\text{start}} = \tilde{t}^{j,k-1}_{\text{end}} - D, i \in DIS^k$, j corresponds to the sequence of the current ith disruption at the previous stage;

25   $\tilde{t}^{i,k}_{\text{end}} = \min\{\tilde{t}^k_{\text{start}} + h_r, t^i_{\text{end}}\}, i \in DIS^k$;

26   **for** $i = 1 : n_k$ **do** `// Determine the events that will follow the pattern (lines 26--30)`

27   **foreach** $tl \in TL^i_{\text{dis},1}$ **do**

28   Define $E^{tl,i,k}_{\text{fix}} = \left\{e \,\middle|\, e \in E_{\text{ar}}, tl_e = tl, \tilde{t}^k_{\text{start}} \leq o_e \leq \tilde{t}^{i,k}_{\text{end}} - D\right\}$;

29   Define $E^{tl,i,k}_{\text{cancel}} = \left\{e \,\middle|\, e \in E^{tl,i,k}_{\text{fix}}, e' \in E^{\text{ar}}_{\text{cancel}}, tl_e = tl_{e'}, st_e = st_{e'}, dr_e = dr_{e'},\right\}$;

30   Define $E^{tl,i,k}_{\text{keep}} = \left\{e \,\middle|\, e \in E^{tl,i,k}_{\text{fix}}, e' \in E^{\text{ar}}_{\text{keep}}, tl_e = tl_{e'}, st_e = st_{e'}, dr_e = dr_{e'},\right\}$;

31   Add constraints $\{c_e = 1, e \in \bigcup_{i \in DIS^k} E^{tl,i,k}_{\text{cancel}}\}$ to the multiple-disruption model; `// Apply the pattern`

32   Add constraints $\left\{c_e = 0, e \in \bigcup_{i \in DIS^k} E^{tl,i,k}_{\text{keep}}\right\}$ to the multiple-disruption model; `// Apply the pattern`

33   Solve the multiple-disruption model considering the ith disruption with duration $[\tilde{t}^k_{\text{start}}, \tilde{t}^{i,k}_{\text{end}}], i \in DIS^k$. When solving the model, in constraints (30)-(32) t^n_{start} is set to $\tilde{t}^k_{\text{start}}$ and r_e is set to the rescheduled time determined at the previous stage for event e, and in constraints (33)-(36) the maximum considered disruption ending time is $\max\{\tilde{t}^{1,k}_{\text{end}}, \cdots, \tilde{t}^{n_k,k}_{\text{end}}\}$;

34   Remove the ith disruption from DIS^k if $\tilde{t}^{i,k}_{\text{end}} = t^i_{\text{end}}, i \in DIS^k$, and then update the number of the remainder disruptions as n_{k+1} and define $DIS^{k+1} = \{1, \cdots, n_{k+1}\}$;

35 **Return** the rescheduled timetable obtained at final stage k; `// Terminate`

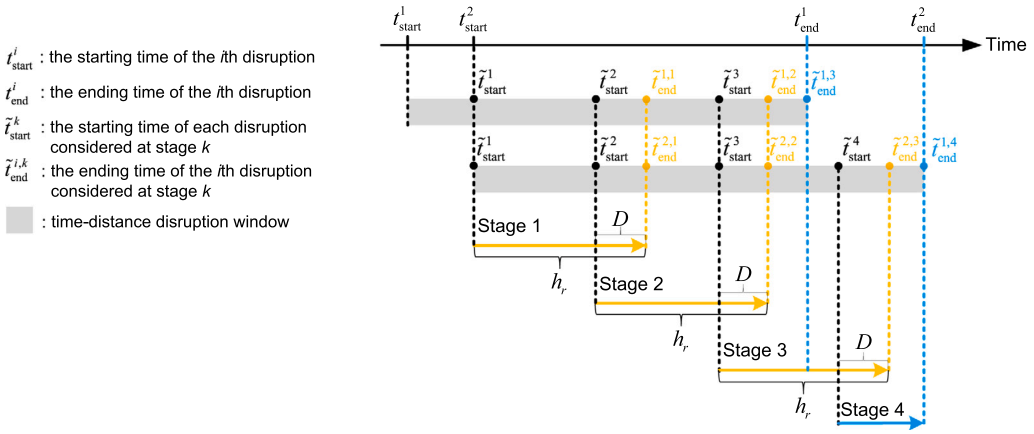

Fig. 9. Illustration of the rolling-horizon solution method with two connected disruptions: the method is called when the 2nd disruption occurs.

Algorithm 1 is called every time a new connected disruption occurs. The algorithm starts in stage 1 by defining the set of ongoing disruptions at the current stage as DIS^1 where the number of ongoing disruptions n_1 is set to the value n (lines 1–2). For each disruption $i \in DIS^1$, the starting time $\tilde{t}^1_{\text{start}}$ considered at stage 1 is set to equal to the starting time of the emerging disruption (line 3), while the ending time $\tilde{t}^{i,1}_{\text{end}}$ considered at stage 1 is set to the minimal value among $\tilde{t}^1_{\text{start}} + h_r$ and t^i_{end}, where in the latter case $\tilde{t}^1_{\text{start}} + h_r$ is larger than the ending time of the disruption t^i_{end} (line 4). The multiple-disruption model is solved considering that each disruption i lasts from $\tilde{t}^1_{\text{start}}$ to $\tilde{t}^{i,1}_{\text{end}}$ to obtain the set E^1_{cancel} containing all cancelled events at stage 1 (line 5). Note that at stage 1 (and at each following stage), the rescheduling solution is computed until the normal schedule has been recovered. Based on E^1_{cancel}, the periodic pattern of the rescheduled train services is obtained into two sets, $E^{\text{ar}}_{\text{cancel}}$ and $E^{\text{ar}}_{\text{keep}}$, which include the representative arrival event e at each stopping/passage station $st \in ST_{tl}$ of train line $tl \in TL^i_{\text{dis},1}$, of which the determined cancellation decision c_e should be followed by the same kind of event in the following periods (lines 6–20). Recall that any arrival and departure events that constitute the same running activity are cancelled or kept simultaneously due to constraint (7), which is why only $E^{\text{ar}}_{\text{cancel}}$ and $E^{\text{ar}}_{\text{keep}}$ are defined.

Before Algorithm 1 proceeds to the next stage $k+1$, the disruption of which the total duration has been considered completely in the current stage will be excluded from the ongoing disruptions of which the number is then updated as n_{k+1} (line 21). If there is at least one disruption remaining (line 22), then the algorithm will proceed to the next stage (line 23). For each disruption $i \in DIS^k$ at the current stage, the considered starting time $\tilde{t}^k_{\text{start}}$ is set to its previous considered ending time minus the maximum allowed delay per event D (line 24). Recall that the previous considered ending time $\tilde{t}^{j,k-1}_{\text{end}}$ is the previous considered starting time $\tilde{t}^{k-1}_{\text{start}}$ plus h_r (see line 4) while h_r is set larger than D. In that sense, $\tilde{t}^k_{\text{start}} = \tilde{t}^{j,k-1}_{\text{end}} - D$ is equivalent to $\tilde{t}^k_{\text{start}} = \tilde{t}^{k-1}_{\text{start}} + h_r - D$, in which $h_r - D$ is always positive. Note that a disruption of which the previous considered duration is smaller than h_r has already been removed from the ongoing disruptions before proceeding to the current stage (see line 21), and is not considered at the current stage, nor any following stages. Setting the considered starting time at the current stage as in line 24 avoids unnecessary train delays/cancellations due to the recovery phase at the previous stage. This is explained in Fig. 10 where case (a) is the example of setting the starting time of a disruption considered at stage k to its ending time considered at stage $k-1$, and case (b) is the example of setting the starting time of a disruption considered at stage k to its ending time considered at stage $k-1$ minus D. As the train departures/arrivals to be rescheduled at the current stage cannot occur before the start time of this stage (the ones outside the blue shadow), two train services (the thick lines) are delayed longer in case (a) than in case (b).

The ending time of a disruption considered at the current stage is set in the same way as introduced before (line 25). Recall that only the periodic pattern of the train lines that are affected by one disruption is computed. Thus for each disruption i, we iterate over the train line $tl \in TL^i_{\text{dis},1}$ that is only affected by the ith disruption to define the set $E^{tl,i,k}_{\text{fix}}$, which includes the events that should follow the determined periodic pattern of train line tl at the current stage k. The set $E^{tl,i,k}_{\text{cancel}} \subseteq E^{tl,i,k}_{\text{fix}}$ ($E^{tl,i,k}_{\text{keep}} \subseteq E^{tl,i,k}_{\text{fix}}$) that includes the events that should be cancelled (kept) at the current stage k is defined according to $E^{\text{ar}}_{\text{cancel}}$ ($E^{\text{ar}}_{\text{keep}}$) (lines 26–30). $E^{tl,i,k}_{\text{fix}}$ does not contain the events that were originally planned to occur during the recovery phase of a disruption, in which the periodic pattern may not be applicable. A recovery phase may start at D minutes before the disruption ending time due to constraints (5) and (6), in which a train can be delayed to the end of a disruption rather than short-turned at a station before the blocked tracks like the similar trains in the previous periods (as Fig. 10 shows). The constraints that demand the events in $\bigcup_{i \in DIS^k} E^{tl,i,k}_{\text{cancel}}$ ($\bigcup_{i \in DIS^k} E^{tl,i,k}_{\text{keep}}$) to be cancelled (kept) are added to the multiple-disruption model, which is then solved considering that each disruption i lasts from $\tilde{t}^k_{\text{start}}$ to $\tilde{t}^{i,k}_{\text{end}}$ (lines 31–33). Next, the disruption of which the total duration has been completely considered at the current stage will be excluded (line 34). If there is at least one disruption remaining, the algorithm proceeds to the next stage. Otherwise, the algorithm terminates by returning the rescheduled timetable obtained at the final stage (line 35).

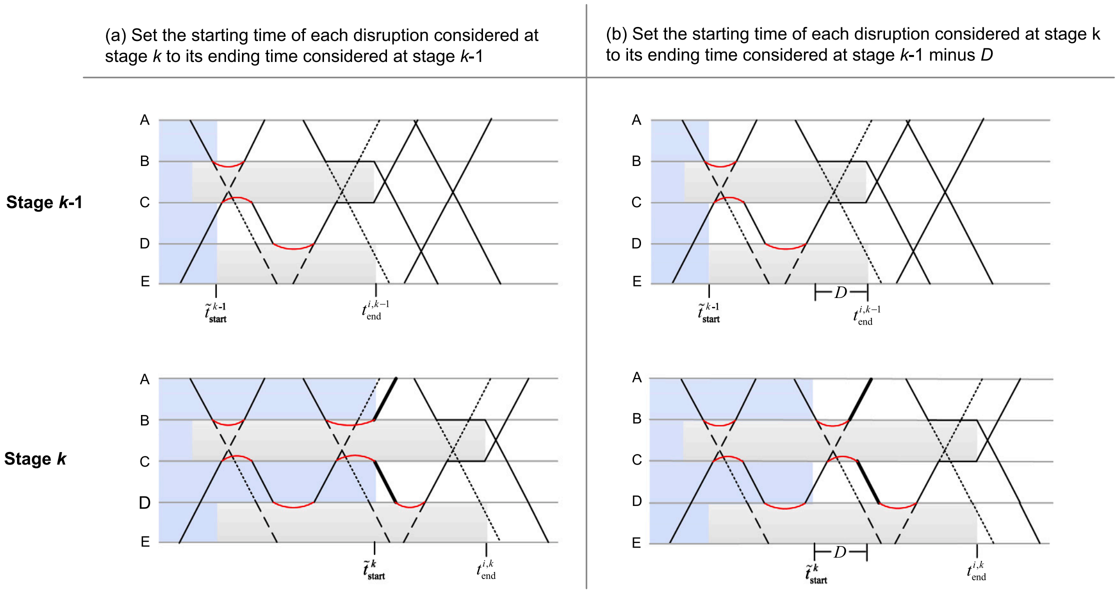

Fig. 10. Two examples of setting the starting time of a disruption considered at a stage (case (b) is used by the rolling-horizon approach). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 1
Train lines in the considered network.

Train line	Terminals in the considered network
IC800	Maastricht (Mt)
IC1900	Venlo (Vl)
IC3500	Heerlen (Hrl)
SPR6400	Eindhoven (Ehv) and Wt
SPR6800	Roermond (Rm)
SPR6900	Sittard (Std) and Hrl
SPR9600	Ehv and Dn
SPR32000	–
IC32100	Mt and Hrl
SPR32200	Rm

6. Case study

We tested the model on a subnetwork of the Dutch railways. There are 38 stations located in this network with 10 train lines operating half-hourly in each direction. The train lines operating in the network are shown in Fig. 11. We distinguish between intercity (IC) and local (called sprinter (SPR) in Dutch) train lines. In the model, trains turning at the terminals to operate the opposite operations (i.e. OD turnings) are taken into account. Table 1 lists the terminals of the train lines that are located in the considered network, while the terminals outside the considered network are neglected. The model was developed in MATLAB on a desktop with Intel Xeon CPU E5-1620 v3 at 3.50 GHz and 16 GB RAM. The solver GUROBI release 7.0.1 was used either to solve the model directly or called by the rolling-horizon method to solve the model gradually over time.

The schematic track layout of the considered network is shown in Fig. 12 where stations Tg, Rv and Sm are located on single-track railway lines while the others are located on double-track railway lines. Due to the infrastructure layouts, some stations do not allow short-turning trains that operate in a specific direction or even both directions. In Fig. 12, the stations that prohibit short-turning trains to both sides are coloured in full grey, the stations that allow short-turning trains to both sides are coloured in full green, and the stations that allow (prohibit) short-turning trains to one side are coloured in half green (grey).

We set the minimum duration required for short-turning or OD turning to 300 s, the minimum duration required for each headway to 180 s, and the penalty of cancelling a service to 100 min. Recall that a service refers to a train run between two adjacent stations. The maximum delay allowed for a train departure or arrival event $e \in E_{\mathrm{ar}} \cup E_{\mathrm{de}} \backslash E^{\mathrm{NMdelay}}$ is set to 25 min. This is because we use a periodic planned timetable that has a cycle time of 30 min. Under this circumstance, delaying a train arrival/departure by 30 min might be unnecessary since at that time there will be a same kind of train departure/arrival originally scheduled. We allow extra stops to be added, considering that a train may dwell at a station where it originally passes through to wait for the platform capacity to be released in a downstream station where it will be short-turned. The minimum dwell time of an extra stop is set to 30 s. The required recovery duration R is set to 2 h in this paper. Note that R is the maximum recovery duration allowed, which means that disruptions could take a shorter time than R to be completely recovered.

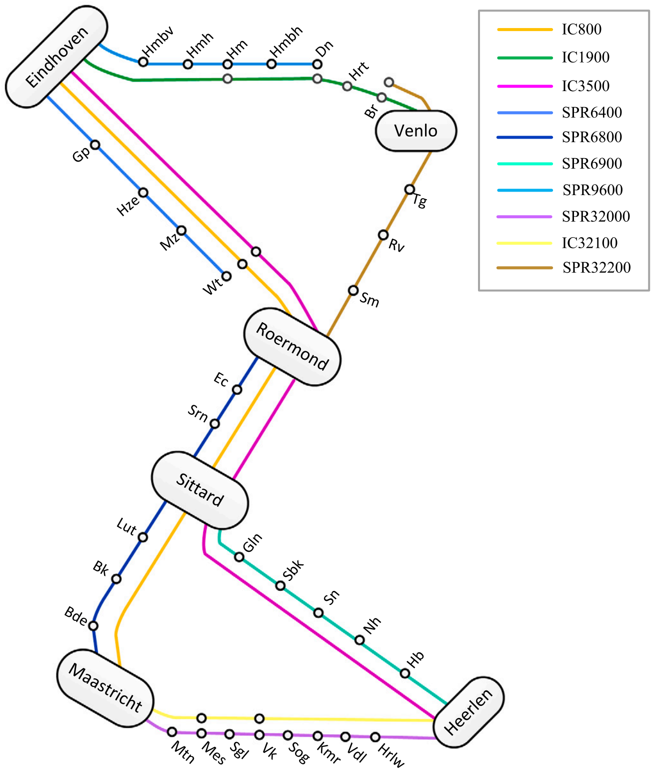

Fig. 11. The train lines operating in the considered network.

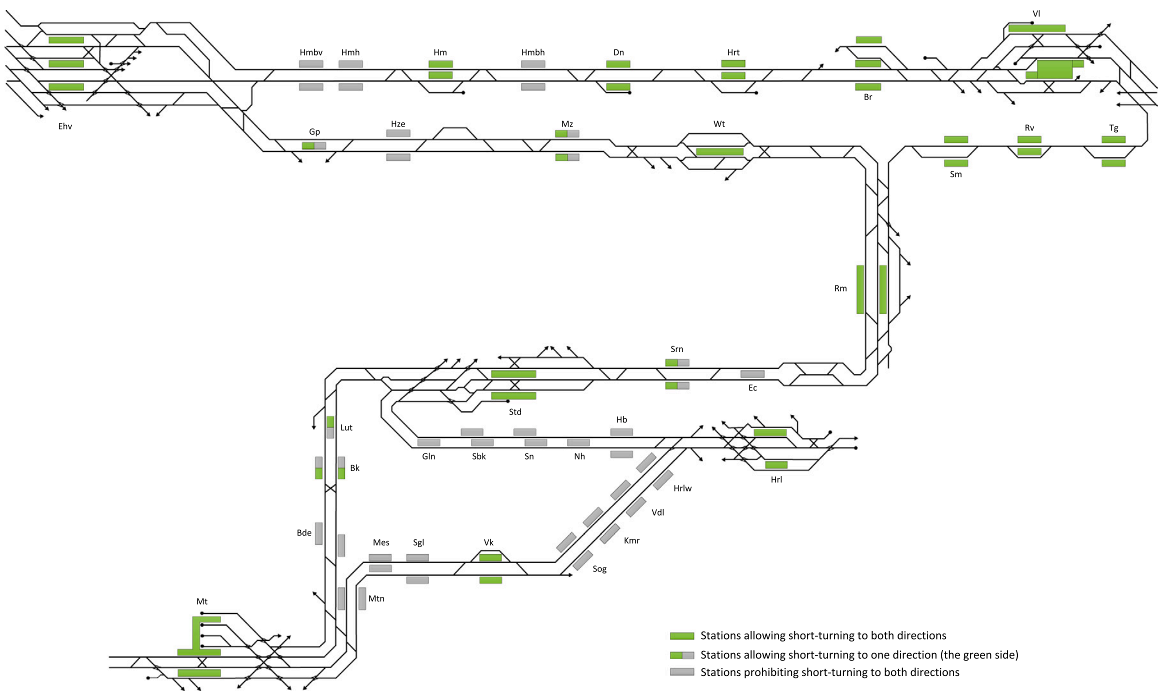

Fig. 12. The schematic track layout in the considered network. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 2
Characteristics of scenarios 1–32.

Scenario	First disruption	Second disruption	Scenario	First disruption	Second disruption
1	Bk - Lut	Rm - Wt	17	Hze - Gp	Bk - Lut
2	Bk - Lut	Wt - Mz	18	Hze - Gp	Lut - Std
3	Bk - Lut	Gp - Ehv	19	Hze - Gp	Mt - Bde
4	Bk - Lut	Hze - Gp	20	Hze - Gp	Srn -Ec
5	Lut - Std	Rm - Wt	21	Rm - Wt	Bk - Lut
6	Lut - Std	Wt - Mz	22	Rm - Wt	Lut - Std
7	Lut - Std	Gp - Ehv	23	Rm - Wt	Mt - Bde
8	Lut - Std	Hze - Gp	24	Rm - Wt	Srn -Ec
9	Mt - Bde	Rm - Wt	25	Wt - Mz	Bk - Lut
10	Mt - Bde	Wt - Mz	26	Wt - Mz	Lut - Std
11	Mt - Bde	Gp - Ehv	27	Wt - Mz	Mt - Bde
12	Mt - Bde	Hze - Gp	28	Wt - Mz	Srn -Ec
13	Srn -Ec	Rm - Wt	29	Gp - Ehv	Bk - Lut
14	Srn -Ec	Wt - Mz	30	Gp - Ehv	Lut - Std
15	Srn -Ec	Gp - Ehv	31	Gp - Ehv	Mt - Bde
16	Srn -Ec	Hze - Gp	32	Gp - Ehv	Srn -Ec

In the following, Section 6.1 explores the performance of the sequential and combined approaches on two connected disruptions occurring in different locations. Each of the two disruptions is considered to last for 2 h approximately, and their durations are almost fully overlapped. Section 6.2 investigates whether and how the length of the overlapping duration would affect the performance of the sequential and combined approaches. Section 6.3 analyzes the performance of the proposed rolling-horizon solution method when dealing with two connected disruptions with longer durations.

6.1. Multiple connected disruptions occurring in different sections

We establish 32 scenarios where each has two complete blockages occurring in different sections as shown in Table 2. For each scenario, we consider that the first disruption starts at 8:06 and ends at 10:06, while the second disruption starts at 8:12 and ends at 10:16. Both disruptions are connected by at least one train line. Considering the real-time requirement for computation, we set 300 s as the upper time limit to get a solution from either the sequential approach or the combined approach by a solver. Table 3 shows the results of handling scenarios 1–32 by both sequential and combined approaches.

In Table 3, the objective value, the number of cancelled services, the total train delay, the computation time, and the optimality gap are indicated for each solution obtained by either approach for each scenario. Recall that a service refers to a train run between two adjacent stations. For each solution, the optimality gap is the difference between the current best integer objective (i.e. the upper bound) and the current lower objective bound of the solution divided by the upper bound. We use "↓" to highlight the cases where smaller values were obtained in the objectives, the numbers of cancelled services, and the total train delays by the combined approach (compared to the sequential approach), while using "↑" to highlight the cases where larger values were obtained. In terms of objective values, the combined approach generated the solutions that were at least as good as the sequential approach. In 20 of 32 scenarios, the combined approach generated better solutions that resulted in less cancelled services and/or less train delays. For example in scenarios 1 and 5, the combined approach reduced both cancelled services and train delays. In some scenarios, it cancelled less services at the expense of introducing more train delays (e.g. scenarios 3); while in one scenario (i.e. scenario 4), it resulted in less train delays at the expense of cancelling more services.

Under the computation time limit of 300 s, the combined approach found optimal solutions for 30 of 32 scenarios, and high-quality solutions with an optimality gap of less than 0.60% for the other two scenarios. Scenarios 1 and 5 are the hardest to solve, which are the two cases resulting in less cancelled services and less train delays at the same time. This is due to the wider search spaces in both scenarios, helping to find better solutions but costing more computation times. The size of the search space is relevant to the location of each disruption. Compared to the combined approach, the sequential approach took less times to compute optimal solutions, which however cannot find feasible solutions for scenarios 13 and 24 where the disrupted sections are the same though the sequence of the occurrence is the other way around. In both scenarios, some services that were required to be cancelled when handling the first disruption cannot be cancelled when handling the second disruption, due to the starting times and locations of both disruptions. This is in conflict with that the sequential approach relies on the previous cancellation decisions, and thus leads to infeasible solutions.

Using scenario 5 as an example, we show the time–distance diagrams of rescheduling solutions obtained by the sequential and combined approaches. The 1st rescheduled timetable corresponding to the 1st disruption obtained by the sequential or combined approach is the same, which is shown in Fig. 13. The solid lines represent the rescheduled services, the dotted (dashed) lines represent the original scheduled services that are delayed (cancelled) in the rescheduled timetable, and the red triangles indicate extra stops. Due to the infrastructure layout, station Lut prohibits short-turning the trains coming from station Bk, which is why these trains short-turn earlier at station Bk. As station Bk has two tracks only, a minimum headway has to be respected between the arrival of a train and the departure of another train that previously arrives at station Bk from the same direction. Thus, three trains from SPR6800 (in dark blue) have to be delayed at station Bde to respect the minimum headway between their arrivals and

Table 3
Results of scenarios 1–32 with 1st disruption in [8:06,10:06] and 2nd disruption in [8:12,10:16].

Scenario	Sequential (solver)					Combined (solver)				
	Obj [min]	# Cancelled services	Total train delay [min]	Time [s]	Gap [%]	Obj [min]	# Cancelled services	Total train delay [min]	Time [s]	Gap [%]
1	6,286	50	1286	39	0.00	5,621 ↓	44↓	1221↓	300	0.56
2	6,802	52	1602	37	0.00	6,260 ↓	52	1060↓	166	0.00
3	13,003	120	1003	24	0.00	12,850 ↓	114↓	1450↑	223	0.00
4	11,930	104	1530	17	0.00	11,843 ↓	108↑	1043↓	139	0.00
5	6,568	54	1168	30	0.00	6,121 ↓	50↓	1121↓	300	0.25
6	7,035	56	1435	32	0.00	6,916 ↓	56	1316↓	125	0.00
7	13,357	124	957	24	0.00	13,290 ↓	116↓	1690↑	58	0.00
8	12,274	108	1474	22	0.00	12,274	108	1474	55	0.00
9	6,618	58	818	18	0.00	6,618	58	818	25	0.00
10	7,670	68	870	28	0.00	7,571 ↓	66↓	971↑	50	0.00
11	14,485	138	685	23	0.00	14,403 ↓	130↓	1403↑	50	0.00
12	13,410	116	1810	27	0.00	13,410	116	1810	85	0.00
13	Infeasible	–	–	–	–	13,705	78	5905	183	0.00
14	11,507	80	3507	52	0.00	11,093 ↓	80	3093↓	166	0.00
15	18,044	150	3044	31	0.00	17,676 ↓	142↓	3476↑	95	0.00
16	16,955	134	3555	26	0.00	16,679 ↓	134	3279↓	170	0.00
17	11,572	90	2572	30	0.00	11,572	90	2572	55	0.00
18	12,372	98	2572	28	0.00	12,372	98	2572	80	0.00
19	13,138	104	2738	20	0.00	13,138	104	2738	50	0.00
20	15,345	114	3845	19	0.00	15,334 ↓	114	3934↓	70	0.00
21	5,220	42	1020	13	0.00	5,214 ↓	42	1014↓	125	0.00
22	6,014	50	1014	13	0.00	6,008 ↓	50	1008↓	91	0.00
23	6,788	56	1188	11	0.00	6,774 ↓	56	1174↓	40	0.00
24	Infeasible	–	–	–	–	12,769	68	5969	160	0.00
25	6,075	52	875	17	0.00	6,075	52	875	69	0.00
26	6,875	60	875	20	0.00	6,875	60	875	71	0.00
27	7,655	66	1055	11	0.00	7,641 ↓	66	1041↓	83	0.00
28	10,010	78	2210	7	0.00	9,999 ↓	78	2199↓	76	0.00
29	12,968	116	1368	19	0.00	12,968	116	1368	53	0.00
30	13,768	124	1368	16	0.00	13,768	124	1368	38	0.00
31	14,548	130	1548	11	0.00	14,534 ↓	130	1534↓	45	0.00
32	16,748	140	2748	19	0.00	16,737 ↓	140	2737↓	37	0.00

the departures of previous arriving trains from IC800 (in orange) at station Bk. The similar reasoning is applied for the extra stops and delays happening to three trains from IC800 (in orange) at station Bde.

The 2nd rescheduled timetable obtained by the sequential approach is shown in Fig. 14. Compared to Fig. 13, there are more train services from IC800 (in orange) cancelled between stations Std and Rm in Fig. 14. This is because trains from IC800 (in orange) have to be short-turned at station Rm due to the emerging disruption (disrupted section Rm-Wt), which however may be inoperable due to their short-turnings at station Std. An earlier short-turning is observed at station Sm between trains from IC800 (in orange). This is because if this short-turning occurs at station Rm instead, although there would be four services cancelled less, the resulting train delays are more than the penalty on cancelling four services. At the top of the disrupted section Rm-Wt, four trains from IC3500 (in pink) additionally dwell at station Mz. This is because station Wt has four tracks while only two of them are alongside platforms. Thus, each of these four trains from IC3500 (in pink) has to wait at station Mz to ensure the headway between its arrival and the departure of a short-turned train from SPR6400 (in light blue) at station Wt where a train from IC800 (in orange) is still occupying another platform at that time. At station Wt, the departures of four upstream trains from IC800 (in orange) are delayed more than necessary. This is because in the sequential approach, the delaying decisions made for the previous disruption are kept. Hence, the adjusted arrival and departure times from the previous step are now the reference timetable, while early arrivals/departures are not allowed, which now is with respect to this timetable.

The 2nd rescheduled timetable obtained by the combined approach is shown in Fig. 15 where different short-turning patterns of trains from IC800 (in orange) are observed. For example in Fig. 15 trains from IC800 (thick solid lines in orange) were short-turned at station Rm around 10:10 instead of at station Srn around 10:15 as in the sequential approach (Fig. 14) in which four more services were cancelled (thick dashed lines in orange). With the combined approach (Fig. 15), four upstream trains from IC800 between Wt and Ehv (thick solid lines in orange) were less delayed than when using the sequential approach (Fig. 14).

From these results it is concluded that the combined approach is able to handle more kinds of multiple-disruption scenarios and find better solutions than the sequential approach in some cases. This is because the combined approach does not rely on previously taken decisions, thus having a wider search space that helps to find a better solution but also costs longer computation time.

6.2. Multiple connected disruptions with different overlapping durations

Section 6.1 considers two disruptions that last for around 2 h, respectively, and the overlapping duration is 1 h and 54 min (almost fully overlapping). To explore whether the length of the overlapping duration affects the performance of the combined approach

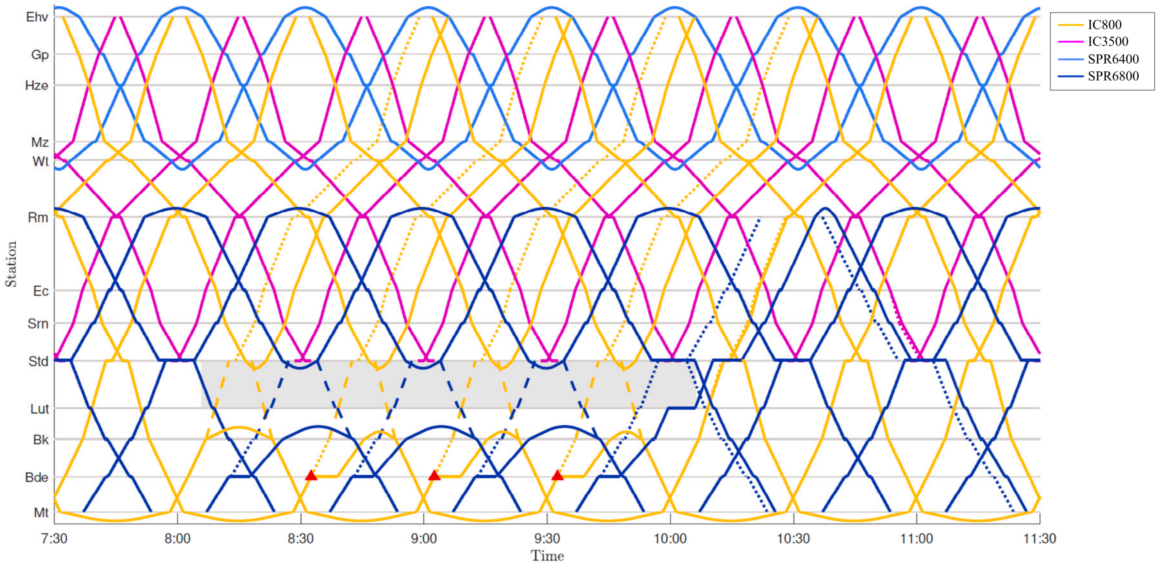

Fig. 13. The 1st rescheduled timetable obtained by the sequential/combined approach for scenario 5: from Eindhoven (Ehv) to Maastricht (Mt). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

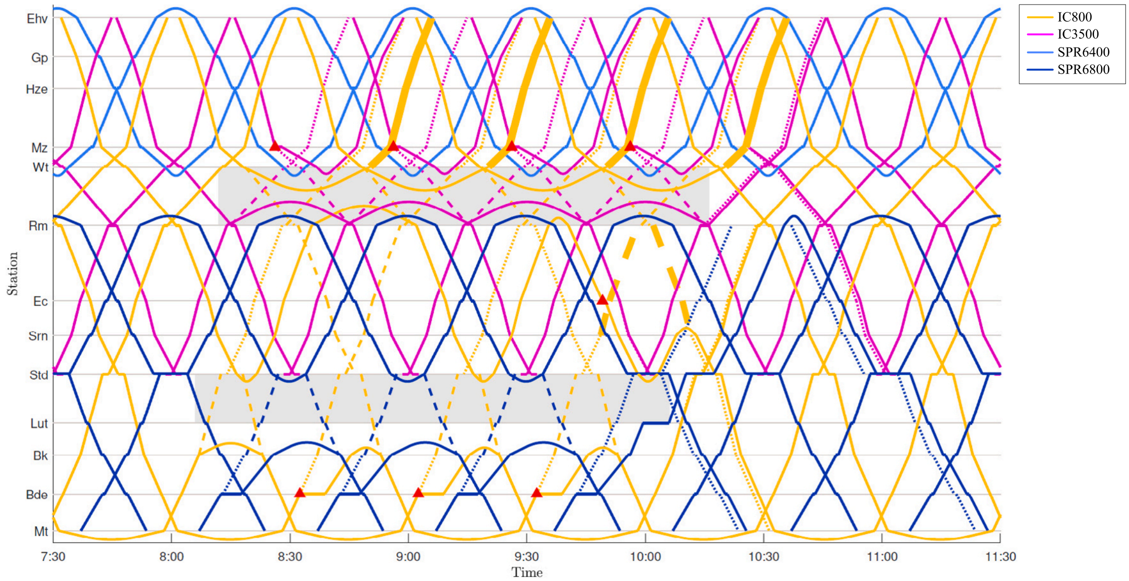

Fig. 14. The 2nd rescheduled timetable obtained by the sequential approach for scenario 5: from Eindhoven (Ehv) to Maastricht (Mt). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

and the corresponding computation time, this sections considers two disruptions that have the same durations as in Section 6.1 but are overlapping to different extents. Several instances differing in the overlapping durations are established as shown in Table 4, in which instance * represents the duration setting used in Section 6.1. In Table 4, the fourth column indicates the total durations of the first and the second disruptions. The first disruption lasts for 2 h and the second disruption last for 2 h and 4 min, which in total is 4 h and 4 min (02:00 + 02:04).

From Table 3 we know that compared to the sequential approach, the combined approach performed much better in scenario 1 , slightly better in scenario 6, and the same in scenario 9 when considering overlapping duration instance *. Hence, we take scenarios 1, 6 and 9 as examples to test whether the performance of the combined approach would be different when considering different overlapping duration instances in the same scenario. We implemented instances a–f in these scenarios, and displayed the results in

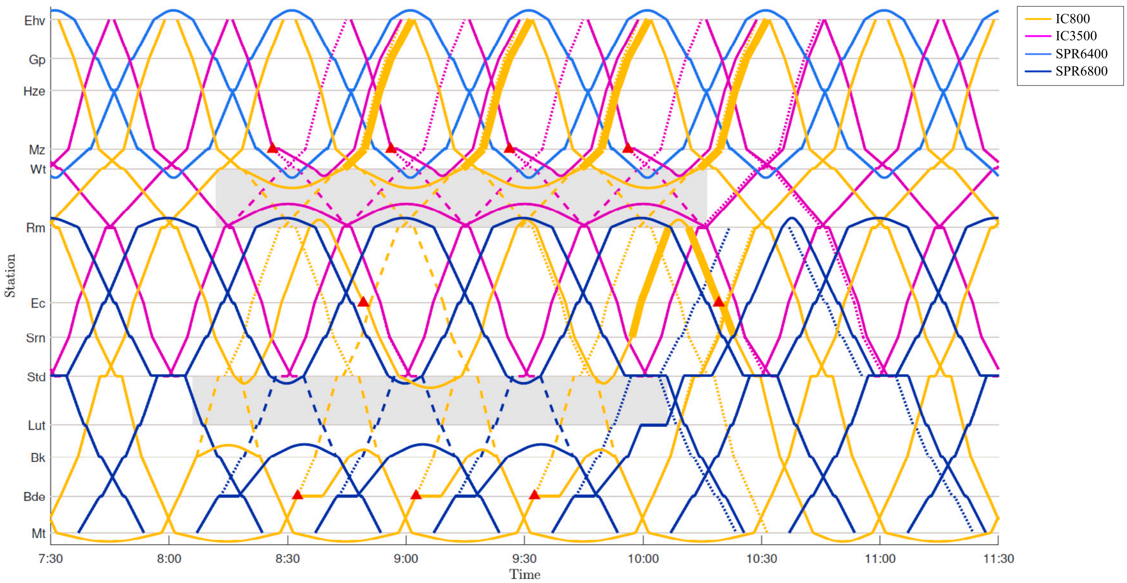

Fig. 15. The 2nd rescheduled timetable obtained by the combined approach for scenario 5: from Eindhoven (Ehv) to Maastricht (Mt). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 4
Two disruptions with different lengths of overlapping durations.

Instance	First disruption period	Second disruption period	Total disruption duration (in HH:MM format)	Overlapping duration (in HH:MM format)
*	[8:06,10:06]	[8:12,10:16]	02:00 + 02:04	01:54
a	[8:06,10:06]	[8:32,10:36]	02:00 + 02:04	01:34
b	[8:06,10:06]	[8:52,10:56]	02:00 + 02:04	01:14
c	[8:06,10:06]	[9:12,11:16]	02:00 + 02:04	00:54
d	[8:06,10:06]	[9:32,11:36]	02:00 + 02:04	00:34
e	[8:06,10:06]	[9:52,11:56]	02:00 + 02:04	00:14
f	[8:06,10:06]	[10:12,12:16]	02:00 + 02:04	00:00

Table 5. The result of implementing instance * on each of these scenarios has already been shown in Table 3, which is also displayed in Table 5.

Table 5 indicates that the performance of the combined approach can change with the overlapping duration between disruptions, and that the change is scenario dependent. Recall that a scenario is different from another scenario in terms of the disrupted sections (see Table 2). In scenario 1 (scenario 6), the combined approach performed the best in terms of the objective under instance * (instance a) in which disruptions were time overlapping to a large extent. In these scenarios longer overlapping duration means more interactions between disruptions, and therefore more interdependent decisions relevant to multiple disruptions need to be decided. These interdependent decisions do not exist in the sequential approach that is unable to consider the combined effects of multiple disruptions. Thus with the increase of interdependent decisions, the solution space of the combined approach becomes larger so that it is more likely to generate a better solution than the sequential approach but meanwhile requires longer computation time. For example either in scenario 1 or scenario 6, the longest computation time happened in the instance where the largest objective decrease was obtained by the combined approach. When considering a much shorter or even zero overlapping duration (instance e or f), the computation times of the combined approach were shorter, and there were few or no differences between the performances of the combined and the sequential approaches in scenarios 1 and 6. Compared to scenario 1 or 6, scenario 9 showed less objective decrease from the combined approach in most instances. The performance of the combined approach in scenario 9 was not relevant to the length of the overlapping disruption duration as in scenario 1 or 6. This is because in scenario 1 or 6 the number of trains that were less delayed or of which less services were cancelled due to the combined approach increased with the overlapping duration, whereas in scenario 9 only one train was delayed less due to the combined approach, which occurred in specific instances depending on the starting/ending times of the disruptions but not on the length of the overlapping duration.

6.3. Multiple connected disruptions with longer (overlapping) durations

In Sections 6.1 and 6.2, the duration considered for each disruption is 2 h approximately. For two connected disruptions with such durations, the combined approach outperforms the sequential approach in terms of solution quality by up to 300 s computation. For

Table 5
Results of considering the duration instances of Table 4.

	Instance	Sequential (solver)			Combined (solver)			
		Obj [min]	Gap [%]	Time [s]	Obj [min]	Gap [%]	Time [s]	Obj decrease
Scenario 1	*	6286	0.00	39	5621↓	0.56	*300*	*665*
	a	5172	0.00	10	5038↓	0.00	30	134
	b	5102	0.00	9	5015↓	0.00	14	87
	c	5778	0.00	40	5518↓	0.00	45	260
	d	5312	0.00	12	5265↓	0.00	18	47
	e	5242	0.00	12	5242	0.00	11	0
	f	5350	0.00	15	5350	0.00	16	0
Scenario 6	*	7035	0.00	32	6916↓	0.00	125	119
	a	7987	0.00	67	7675↓	0.00	*165*	*312*
	b	6974	0.00	40	6902↓	0.00	50	72
	c	7021	0.00	45	6934↓	0.00	60	87
	d	7658	0.00	40	7452↓	0.00	50	206
	e	6671	0.00	42	6670↓	0.00	38	1
	f	6680	0.00	28	6680	0.00	28	0
Scenario 9	*	6618	0.00	18	6618	0.00	25	0
	a	6931	0.00	22	6872↓	0.00	*141*	*59*
	b	6530	0.00	10	6530	0.00	11	0
	c	6770	0.00	35	6721↓	0.00	40	49
	d	6947	0.00	71	6888↓	0.00	50	59
	e	6530	0.00	11	6529↓	0.00	11	1
	f	6778	0.00	25	6730↓	0.00	30	48

Table 6
Two connected disruptions with longer (overlapping) durations.

Case	First disruption period	Second disruption period	Total disruption duration (in HH:MM format)	Overlapping duration (in HH:MM format)
*	[8:06,10:06]	[8:12,10:16]	02:00 + 02:04	01:54
I	[8:06,10:26]	[8:12,10:36]	02:20 + 02:24	02:14
II	[8:06,10:46]	[8:12,10:56]	02:40 + 02:44	02:34
III	[8:06,11:06]	[8:12,11:16]	03:00 + 03:04	02:54
IV	[8:06,11:26]	[8:12,11:36]	03:20 + 03:24	03:14
V	[8:06,11:46]	[8:12,11:56]	03:40 + 03:44	03:34
VI	[8:06,12:06]	[8:12,12:16]	04:00 + 04:04	03:54
VII	[8:06,12:26]	[8:12,12:36]	04:20 + 04:24	04:14
VIII	[8:06,12:46]	[8:12,12:56]	04:40 + 04:44	04:34
IX	[8:06,13:06]	[8:12,13:16]	05:00 + 05:04	04:54
X	[8:06,13:26]	[8:12,13:36]	05:20 + 05:24	05:14
XI	[8:06,13:56]	[8:12,13:56]	05:40 + 05:44	05:34
XII	[8:06,14:06]	[8:12,14:16]	06:00 + 06:04	05:54

longer disruptions, whether this still holds should be investigated. This is for the consideration that the combined approach needs longer computation time than the sequential approach and thus may generate sub-optimal solutions under the required time limit, which then could be worse than the solutions obtained by the sequential approach. This section tests both approaches on longer disruptions using 300 s as the computation time limit still. Particularly in the combined approach, the multiple-disruption model is solved by the rolling-horizon approach proposed in Section 5, as well as an optimization solver for comparison.

According to Table 3, scenario 1 is chosen as an example, because it is the most difficult scenario to be solved by the combined approach. Twelve cases of disruption durations are considered for this scenario, which are shown in Table 6.

The results of applying the sequential/combined approach to deal with duration cases I–XII in scenario 1 are indicated in Table 7. These results are obtained by the optimization solver GUROBI. We use O-gap to indicate the percentage difference between the obtained solution and the optimal solution. If no optimal solution was obtained by the solver up to 24 h computation, we calculated U-gap and L-gap to represent the percentage difference between the obtained solution and the best found upper bound, and the percentage difference between the obtained solution and the best found lower bound, respectively. The sequential approach found optimal solutions within 300 s for most cases, except case XII (the longest disruption case) for which it took 572 s to find the optimal solution. Although the combined approach computed sub-optimal solutions within 300 s, these solutions were still better than the optimal solutions by the sequential approach. By up to 24 h computation, the combined approach obtained optimal solutions for cases I–III, and near-optimal solutions for cases IV–XII.

The proposed rolling-horizon solution method was also applied for the combined approach to solve cases I–XII in scenario 1. The computation time at each stage of the rolling-horizon method is restricted to 300 s. The results under different settings of h_r are shown in Table 8. Recall that h_r represents the length of a disruption considered at each stage (except the final stage). By comparing Table 8 with Table 7, we found that the solutions obtained by the rolling-horizon method under whichever setting of h_r were better

Table 7
Results of scenario 1 by using a solver.

Case	Sequential (solver)			Combined (solver)					Combined (solver up to 24 h)			
	Obj [min]	O-gap [%]	Time [s]	Obj [min]	O-gap [%]	U-gap [%]	L-gap [%]	Time [s]	Optimal	Upper bound	Lower bound	L-gap [%]
I	7,787	0.00	21	7,084 ↓	2.67			300	6895			0.00
II	8,456	0.00	48	7,653 ↓	1.74			300	7520			0.00
III	10,096	0.00	300	8,765 ↓	4.48			300	8372			0.00
IV	11,663	0.00	300	10,027 ↓		3.42	3.55	300		9,684	9,671	0.13
V	12,229	0.00	300	11,111 ↓		7.61	8.16	300		10,266	10,204	0.61
VI	13,196	0.00	300	11,982 ↓		7.26	7.65	300		11,112	11,065	0.43
VII	14,578	0.00	300	12,635 ↓		2.12	2.94	300		12,368	12,263	0.85
VIII	15,115	0.00	300	13,671 ↓		5.60	6.48	300		12,905	12,785	0.93
IX	16,036	0.00	300	15,654 ↓		12.04	12.79	300		13,769	13,652	0.85
X	17,417	0.00	300	16,505 ↓		8.86	10.10	300		15,043	14,838	1.36
XI	17,954	0.00	300	16,362 ↓		4.58	5.65	300		15,612	15,438	1.11
XII	21,338	11.50	300	17,537 ↓		6.44	7.79	300		16,407	16,170	1.44
	(18,875	0.00	572)									

Table 8
Results of scenario 1 by using the rolling-horizon solution method for the combined approach (up to 300 sec computation at each stage).

Case	h_r = 1 h				h_r = 1.5 h				h_r = 2 h			
	Obj [min]	O-gap [%]	U-gap [%]	L-gap [%]	Obj [min]	O-gap [%]	U-gap [%]	L-gap [%]	Obj [min]	O-gap [%]	U-gap [%]	L-gap [%]
I	6,991	1.37			6,895	0.00			6,895	0.00		
II	7,595	0.99			7,520	0.00			7,520	0.00		
III	8,498	1.48			8,372	0.00			8,372	0.00		
IV	9,810		1.28	1.42	9,684		0.00	0.13	9,684		0.00	0.13
V	10,415		1.43	2.03	10,289		0.22	0.83	10,289		0.22	0.83
VI	11,318		1.82	2.24	11,191		0.71	1.13	11,191		0.71	1.13
VII	12,630		2.08	2.91	12,452		0.68	1.52	12,504		1.10	1.93
VIII	13,235		2.49	3.40	13,041		1.04	1.96	13,109		1.56	2.47
IX	14,318		2.61	3.44	13,961		1.38	2.21	14,011		1.73	2.56
X	15,450		2.63	3.96	15,235		1.26	2.61	15,272		1.50	2.84
XI	16,055		2.76	3.84	15,861		1.57	2.67	15,861		1.57	2.67
XII	16,957		3.24	4.64	16,712		1.83	3.24	16,781		2.23	3.64

than the ones obtained by the solver up to 300 s computation. When increasing h_r from 1 h to 1.5 h, optimal solutions were found for cases I–III, and solutions with improved U-gaps and L-gaps were obtained for cases IV–XII. When increasing h_r from 1.5 h to 2 h further, the solutions obtained for cases I–VI were the same, but the solutions found for cases VII–XII mostly became worse (U-gaps and L-gaps both increased). This is due to the computation limit of 300 s required at a stage of the rolling-horizon method. When h_r was set to 1 h or 1.5 h, an optimal solution was always obtained at each stage within the required time limit. When h_r was set to 2 h, sub-optimal solutions were obtained at specific stages due to the time limit, which affected the overall solution optimality.

Fig. 16 shows the stage computation times of cases I–XII under different settings of h_r. Each circle indicates the computation time at a specific stage that is distinguished by colour. The circles on the same vertical line correspond to the same case. Because the disruption durations are different among cases, the number of stages needed at a case can be different from one to another case although both cases were under the same setting of h_r. When $h_r = 1$ h, the stage computation times were mostly below 25 s with 7 exceptions that ranged from 35 s to 210 s and all corresponded to the final stages of the relevant cases. When $h_r = 1.5$ h, stage computation times increased due to longer disruption durations considered, and the most time-consuming stages took 225 s, which were the first stages in all cases. A stage computation time is very sensitive to the starting and ending times of disruptions considered at the stage, which is why it varied with stages although under the same setting of h_r. When h_r increased to 2 h, most stage computation times reached the limit of 300 s, and in some cases only the circles that indicated the final stage computation times are visible, because the ones that represented the previous stages were overlapping due to the same computation times. The total computation time of the rolling-horizon method is the sum of the computation times required at all stages. Table 9 shows the minimum, average and maximum total computation times across cases under the same setting of h_r in scenario 1. These values all increase with the growth of h_r. For example when $h_r = 1$ h the maximum total computation time was below 300 s, while when $h_r = 2$ h the minimum total computation time was over 300 s Although the total computation times were mostly (all) longer than 300 s when setting h_r to 1.5 h (2 h), the corresponding stage computation times were all below 300 s as shown in Fig. 16. Therefore in practice, a rescheduling solution can be rapidly obtained at a stage and immediately delivered to traffic controllers, and then updated gradually over time for the following stages. Although we assume that the disruption durations will not change over time, with minor changes the proposed rolling-horizon method can be used to deal with the dynamic variations regarding the disruption durations.

In scenario 1, the combined approach performs much better than the sequential approach, and thus the sub-optimal solutions by the combined approach can still be better than the optimal solutions by the sequential approach. For the scenarios where

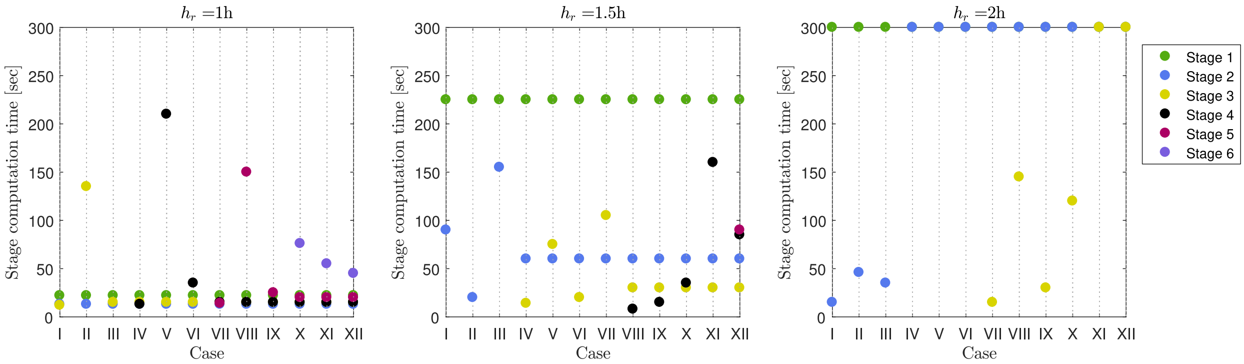

Fig. 16. Stage computation times [sec] under different settings of h_r in the rolling-horizon method. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 9
The minimum, average and maximum total computation times [sec] of the rolling-horizon method for the combined approach across cases in scenario 1.

h_r	Min	Avg	Max
1 h	47	124	260
1.5 h	245	355	490
2 h	315	609	900

Table 10
Results of scenario 6.

Case	Sequential (solver)			Combined (solver)			Combined (rolling-horizon)		
	Obj [min]	O-gap [%]	Time [s]	Obj [min]	O-gap [%]	Time [s]	Obj [min]	O-gap [%]	Max stage time [s]
I	8,569	0.00	162	8,462 ↓	0.00	93	8,462 ↓	0.00	170
II	9,588	0.00	188	9,366 ↓	0.00	100	9,366 ↓	0.00	170
III	10,568	0.00	145	10,425 ↓	0.00	69	10,425 ↓	0.00	170
IV	12,165	0.00	282	12,064 ↓	0.64	300	11,987 ↓	0.00	170
V	13,185	0.00	241	12,967 ↓	0.59	300	12,904 ↓	0.10	170
VI	14,204	0.00	300	14,305 ↑	2.49	300	13,965 ↓	0.11	170
VII	15,785	0.00	300	18,314 ↑	15.25	300	15,580 ↓	0.38	170
VIII	16,804	0.00	300	16,764 ↓	2.09	300	16,451 ↓	0.22	170
IX	17,888	0.58	300	17,564 ↓	0.52	300	17,539 ↓	0.38	170
X	19,825	2.24	300	19,755 ↓	3.59	300	19,063 ↓	0.09	170
XI	20,490	0.43	300	23,552 ↑	15.34	300	19,980 ↓	0.21	170
XII	21,451	0.15	300	21,205 ↓	0.99	300	21,039 ↓	0.21	170

the combined approach performs at least as good as the sequential approach, it is able to generate solutions below the required computation time limit. According to Table 3, scenarios 6 and 9 are chosen as two more example instances, and the corresponding results are shown in Tables 10 and 11, respectively. In these two scenarios, we set h_r to 2 h, under which optimal solutions were always obtained at stage level under the required time limit in all cases.

Table 10 shows that for scenario 6, the combined approach found better solutions than the sequential approach in all cases when using the rolling-horizon solution method, which however was not achieved when using a solver. The optimality gaps of the solutions by the rolling-horizon method were all below 0.40%. Table 11 shows that for scenario 9, the combined approach found the solutions that were at least as good as the ones obtained by the sequential approach in all cases when using either a solver or the rolling-horizon method. In both scenarios, the maximum stage computation time of the rolling-horizon method was below the required time limit of 300 s These results indicate that the computational complexity of the combined approach is scenario dependent, and that the proposed rolling-horizon method is able to generate high-quality solutions in an acceptable time.

From these results we conclude that the proposed rolling-horizon method is helpful to solve longer multiple connected disruptions by high-quality rescheduling solutions in an acceptable time. The value of h_r used in the rolling-horizon method affects the overall solution optimality due to the time limit of 300 s required for a stage computation. In different scenarios the appropriate setting of h_r can be different, but under whichever setting of h_r (1 h, 1.5 h, or 2 h), the rolling-horizon solution method performs well regarding the solution quality.

Table 11
Results of scenario 9.

Case	Sequential (solver)			Combined (solver)			Combined (rolling-horizon)		
	Obj [min]	O-gap [%]	Time [s]	Obj [min]	O-gap [%]	Time [s]	Obj [min]	O-gap [%]	Max stage time [s]
I	8,090	0.00	15	8,090	0.00	93	8,090	0.00	45
II	9,170	0.00	21	9,170	0.00	100	9,170	0.00	45
III	10,089	0.00	45	10,089	0.00	69	10,089	0.00	45
IV	11,561	0.00	73	11,561	0.00	300	11,561	0.00	45
V	12,641	0.00	105	12,641	0.00	300	12,641	0.00	45
VI	13,560	0.00	231	13,560	0.00	260	13,560	0.00	45
VII	15,032	0.00	300	15,032	0.00	300	15,032	0.00	45
VIII	16,112	0.00	300	16,112	0.00	300	16,112	0.00	45
IX	17,031	0.00	300	17,031	0.00	300	17,031	0.00	45
X	18,503	0.00	300	18,503	0.00	300	18,503	0.00	45
XI	19,583	0.00	300	19,583	0.00	300	19,583	0.00	90
XII	20,518	0.08	300	20,506 ↓	0.02	300	20,502 ↓	0.00	45

Table 12
Notation.

Symbol	Description
o_e	The original scheduled time of event e
tl_e	The corresponding train line of event e
tr_e	The corresponding train of event e
st_e	The corresponding station of event e
dr_e	The operation direction of event e
w	Cancellation penalty
n	The nth disruption that currently emerges
r_e	The previous rescheduled time of event e
r_e^1	The determined rescheduled time of event e when handling the first disruption only
t^i_{start}	The start time of the ith disruption, $1 \leq i \leq n$
t^i_{end}	The end time of the ith disruption, $1 \leq i \leq n$
$st^{i,dr}_{\text{en}}$	The entry station of the ith disrupted section in direction $dr_e \in \{up, down\}$
$st^{i,dr}_{\text{ex}}$	The exit station of the ith disrupted section in direction $dr_e \in \{up, down\}$
$tail(a)$	The tail of activity a: which is the event a that starts from
$head(a)$	The head of activity a: which is the event a that points to
R	The maximal recovery duration after the disruption end time
D	The maximum allowed delay per event
L_a	The minimum duration of an activity a
M_1	A positive large number that is set to 1440
M_2	A positive large number that is set to twice of M_1: $M_2 = 2M_1$
$h_{e,e'}$	A minimum interval between the occurring times of events e and e' if corresponding to trains occupying the same station track
A_{run}	Set of running activities
A_{dwell}	Set of dwell activities
A_{pass}	Set of pass-through activities
A_{station}	Set of station activities: $A_{\text{station}} = A_{\text{dwell}} \cup A_{\text{pass}}$
A_{turn}	Set of short-turn activities
A^i_{turn}	Set of short-turn activities for the ith disruption: $A^i_{\text{turn}} \subset A_{\text{turn}}$
A_{odturn}	Set of OD turn activities
E_{ar}	Set of arrival events
E_{de}	Set of departure events
E^{NMdelay}	Set of events that do not have upper limit on their delays
$E^{\text{turn}}_{\text{ar}}$	The subset of E_{ar}, which includes all tails of activities in A_{turn}: $E^{\text{turn}}_{\text{ar}} = \bigcup_{a\in A_{\text{turn}}} tail\,(a)$
$E^{i,\text{turn}}_{\text{ar}}$	The subset of $E^{\text{turn}}_{\text{ar}}$, which includes all tails of activities in A^i_{turn}: $E^{i,\text{turn}}_{\text{ar}} \subset E^{\text{turn}}_{\text{ar}}$
$E^{\text{odturn}}_{\text{ar}}$	The subset of E_{ar}, which includes all tails of activities in A_{odturn}: $E^{\text{odturn}}_{\text{ar}} = \bigcup_{a\in A_{\text{odturn}}} tail\,(a)$
$E^{\text{turn}}_{\text{de}}$	The subset of E_{de}, which includes all heads of activities in A_{turn}: $E^{\text{turn}}_{\text{de}} = \bigcup_{a\in A_{\text{turn}}} head\,(a)$
$E^{i,\text{turn}}_{\text{de}}$	The subset of $E^{\text{turn}}_{\text{de}}$, which includes all heads of activities in A^i_{turn}: $E^{i,\text{turn}}_{\text{de}} \subset E^{\text{turn}}_{\text{de}}$
$ST^{dr_e}_{\text{en}}$	Set of entry stations of all disrupted sections in direction $dr_e \in \{up, down\}$
$ST^{dr_e}_{\text{ex}}$	Set of exit stations of all disrupted sections in direction $dr_e \in \{up, down\}$
TR_{turn}	Set of trains that correspond to the events contained in $E^{\text{turn}}_{\text{ar}} \cup E^{\text{turn}}_{\text{de}}$
TR^i_{turn}	Set of trains that correspond to the events contained in $E^{i,\text{turn}}_{\text{ar}} \cup E^{i,\text{turn}}_{\text{de}}$

Table 13
Notation used in the rolling-horizon method (Algorithm 1).

Symbol	Description
$\tilde{t}^{k}_{\text{start}}$	The considered starting time of a disruption at stage k
$\tilde{t}^{i,k}_{\text{end}}$	The considered ending time of the ith disruption at stage k
h_r	The duration of a disruption considered at a stage (except the final stage)
n_k	The number of ongoing disruptions at stage k
DIS^k	The list of ongoing disruptions at stage k
$TL^{i}_{\text{dis},1}$	The set of train lines that is only affected by the ith disruption
ST_{tl}	The set of planned stopping and passage stations of train line tl
E^{k}_{cancel}	The set of events that are cancelled at stage k
$E^{\text{ar}}_{\text{cancel}}$	The set of cancelled arrival events
$E^{\text{ar}}_{\text{keep}}$	The set of kept arrival events
$E^{tl,i}_{\text{ar}}$	The set of arrival events from train line tl that is affected by the ith disruption
$E^{st,tl,i}_{\text{ar}}$	The set of arrival events occurring at station st and belonging to train line tl that is affected by the ith disruption
$E^{tl,i,k}_{\text{fix}}$	The set of events that should follow the determined periodic pattern of train line tl at stage $k \geq 2$
$E^{tl,i,k}_{\text{cancel}}$	The set of events from train line tl, which should be cancelled at stage $k \geq 2$
$E^{tl,i,k}_{\text{keep}}$	The set of events from train line tl, which should be kept at stage $k \geq 2$

7. Conclusions and future research

To deal with multiple connected disruptions that occur unexpectedly, this paper proposed two approaches, the sequential approach and the combined approach. The sequential approach is based on the single-disruption rescheduling model proposed by Zhu and Goverde (2019), which solves disruptions one by one with the previous rescheduling decisions as reference. The combined approach is based on the multiple-disruption rescheduling model developed in this paper, which reschedules all train services together each time an extra disruption occurs. Both approaches were applied to a subnetwork of the Dutch railways with 38 stations and 10 train lines operating half-hourly in each direction. Numerous experiments revealed that the combined approach resulted in less cancelled train services and/or train delays than the sequential approach. The outperformance of the combined approach may change with the overlapping duration between disruptions, and the change is relevant to the disruption locations. To deal with long multiple connected disruptions in a more efficient way, we proposed a new rolling-horizon method that is able to generate high-quality rescheduling solutions in an acceptable time. The case study applied both approaches to deal with two connected disruptions. In future work, we will test larger railway networks where three or more connected disruptions are more likely to happen. This may need the technique of decomposing the large-scale network into several coordinated local rescheduling zones to release the potential computational burden considering that the network scale and the number of disruptions both increase. In addition, it is important to take into account the uncertainty of disruption durations, for which both the technique of stochastic programming and a rolling-horizon method need to be employed. This is better to be explored from single-disruption cases first and then extended to multiple-disruption cases due to its complexity.

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Appendix A

See Table 12.

Appendix B

See Table 13.

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